Agile multiscale decompositions for automatic image registration

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Background on Image Registration

- Image registration is the process of aligning two or more images of approximately the same scene, possibly captured with different sensors or at different times.
- The registration of multimodal images is a particular challenge; a variety of approaches to the multimodal registration problem have been proposed.
- Some are based on SIFT and related features, while others attempt to efficiently represent the images to be registered in a common feature space.
- For images with very different information content, there is often very little local similarity between the two images. This renders local feature descriptors ineffective for image registration, though robust outlier detection can compensate to some extent.

Wavelets and their Limitations

- Methods that construct global features have been proposed for image registration.
- In particular, using wavelets to isolate important features in images has been successful for automatic image registration.
- However, wavelets are *isotropic*, meaning that they do not emphasize directional features. Indeed, it has been mathematically known for over ten years that wavelets are theoretically suboptimal for a large class of images with edges, i.e. *cartoon-like images*.
- This suggests looking to alternative representation systems for extracting features.

Anisotropic Systems: Shearlets

- Several directionally sensitive systems have been proposed, beginning in the late 1990's, to address the theoretical suboptimality of wavelets.
- Among these are ridgelets (Donoho and Candés), curvelets (Donoho et al), contourlets (Do and Vetterli), and shearlets (Labate, Kutyniok, Weiss, et al).
- Curvelets and shearlets are provably near-optimal for representation certain images with edges, and both are numerically implemented in stable packages.
- However, shearlets have the advantage of not needing to interpolate rotations, because shearlets implement directionality via *shearing*, not rotations.

Use of Shearlets

- In recent work, we proposed to incorporate shearlets in an automatic wavelet registration algorithm, with the hope of utilizing the theoretical properties of shearlets for edges.
- We did so by computing shearlet features for each image pair, then aligning these features via least squares optimization. This registration output was then used as an initial guess for another call to the registration algorithm, this time using wavelet features.

Figure: A 256×256 grayscale optical image of a mixed land-cover area in Washington state containing both textural and edge-like features.

Wavelets versus Shearlets

To illustrate the directional character of discrete shearlet algorithms, and its utility for image registration, consider the features produced by a MATLAB discrete wavelet algorithm using the ‘db2’ wavelet, and the shearlet feature algorithm we have developed.

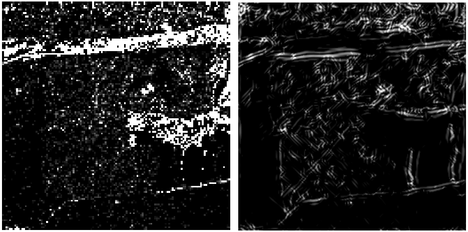

Figure: Wavelet (left) and shearlet (right) features extracted from optical image, emphasizing textural and edge features, respectively.

Towards a More Agile Algorithm

- The order of the shearlets and wavelets was simply because shearlets seemed to have a larger radius of convergence with respect to the initial registration guess, but suffered from lower accuracy in some cases.
- Registering shearlets first provides a good first approximation, which is refined by registering with wavelets second.
- However, we were interested in a more flexible ordering and integration of the shearlets and wavelets.
- By utilizing isotropic wavelets to further decompose the shearlet features, we hoped to further capture the most significant features in the image.
- We presently consider only decompositions of the low-pass shearlet features.

Full Algorithm Integration

- Our prototype shearlet+wavelet registration algorithm enjoyed improved robustness over wavelets alone, but was partially coded in C, and partially in MATLAB.
- The shearlet features component was based on the MATLAB FFST library, while the wavelet features and optimization components were written in C.
- Moreover, the optimization scheme was designed for a decimated (non-redundant) wavelet transform, not a redundant frame like shearlets.
- We present results from the fully integrated in C wavelets+shearlets algorithm.

Summary of Proposed Algorithm (1/2)

1. Input a reference image, I^r, an input image I^i and an initial registration guess $(\theta_0, T_{x_0}, T_{y_0})$.
2. Apply shearlet features algorithm and wavelet features algorithms to I^r and I^i. This produces a set of shearlet features for both, denoted $S_1^r, ..., S_n^r$ and $S_1^i, ..., S_n^i$, respectively, as well as a set of wavelet features for both, denoted $W_1^r, ..., W_n^r$ and $W_1^i, ..., W_n^i$.
3. Apply the wavelet features algorithm to S_1^r, S_1^i to acquire decompositions of these coarse shearlet features. These are denoted $S_{1,1}^r, ..., S_{1,k}^r$ and $S_{1,1}^i, ..., S_{1,k}^i$, respectively. Here, k denotes the number of scales used in this wavelet decomposition; for the present experiments, $k = 2$.
4. Match $S_{1,1}^r$ with $S_{1,1}^i$ with a least squares optimization algorithm and initial guess $(\theta_0, T_{x_0}, T_{y_0})$ to get a transformation $T_{1,1}^S$.
5. Using $T_{1,1}^S$ as an initial guess, match $S_{1,2}^r$ with $S_{1,2}^i$ with lest squares to acquire a transformation $T_{1,2}^S$. Iterate this process by matching $S_{1,j}^r$ with $S_{1,j}^i$ using $T_{1,j-1}^S$ as an initial guess, for $j = 2, ..., k$. At the end of this iterative matching, we acquire a *decomposed shearlet-based registration*, call it $T_1^S = (\theta^{S,1}, T_x^{S,1}, T_y^{S,1})$.

Summary of Proposed Algorithm (2/2)

6. Using T_1^S as an initial guess, match S_2^r with S_2^i with least squares to acquire a transformation T_2^S. Using T_2^S as an initial guess, match S_3^r with S_3^i with least squares to acquire a transformation T_3^S. Iterate this process by matching S_j^r with S_j^i using T_{j-1}^S as an initial guess, for $j = 2, ..., n$. At the end of this iterative matching, we acquire a *full shearlet-based registration*, call it $T^S = (\theta^S, T_x^S, T_y^S)$.
7. Using T^S as an initial guess, match W_1^r with W_1^i with least squares to acquire a transformation T_1^W. Using T_1^W as an initial guess, match W_2^r with W_2^i with least squares to acquire a transformation T_2^W. Iterate this process by matching W_j^r with W_j^i using T_{j-1}^W as an initial guess, for $j = 2, ..., n$. At the end of this iterative matching, we acquire the *final hybrid registration*, call it $T^H = (\theta^H, T_x^H, T_y^H)$.
8. Output T^H.

Outline of Experiments

- We consider experiments with the algorithm just described, denoted *shearlet+wavelet with decomposition*. This is compared to wavelets-only and the previously studied shearlets+wavelets algorithm, with improved optimization for shearlets.
- To evaluate the algorithm, different choices of initial guess are compared with respect to output RMSE. We have seen in previous work that using shearlets+wavelets allows for a poorer initial guess, while retaining acceptable RMSE, thus improving algorithm robustness.
- While our optimization procedure works for general affine transformations, we consider the simpler case of searching for transformations that consist only of translations and rotations.
- Moreover, we couple rotation and translations together for the initial guess, to make the parameter space one-dimensional, and thus easier to visualize.

Synthetic Experiments (1/2)

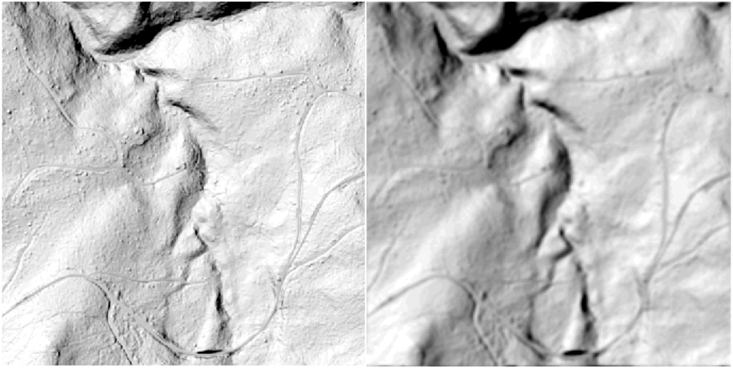

Figure: 512 $\times$ 512 lidar shaded relief images of Mossy Rock without (left) and with (right) synthetic radiometric distortion. The images have been converted to grayscale.

Synthetic Experiments (2/2)

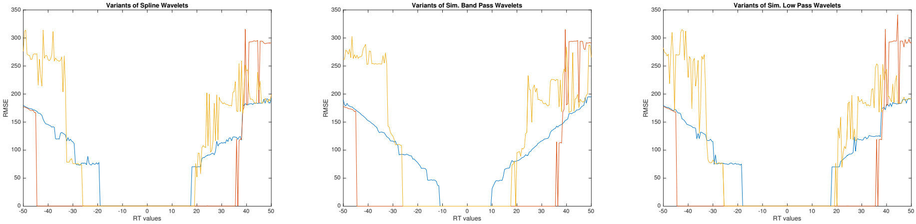

Figure: Comparison of algorithms for Mossy Rock synthetically warped experiments (from left to right: splines, Simoncelli band-pass, Simoncelli low-pass); blue is wavelets, yellow is hybrid shearlets+wavelets with decomposition, and red is shearlets+wavelets without decomposition.

Lidar-to-Optical Experiments (1/2)

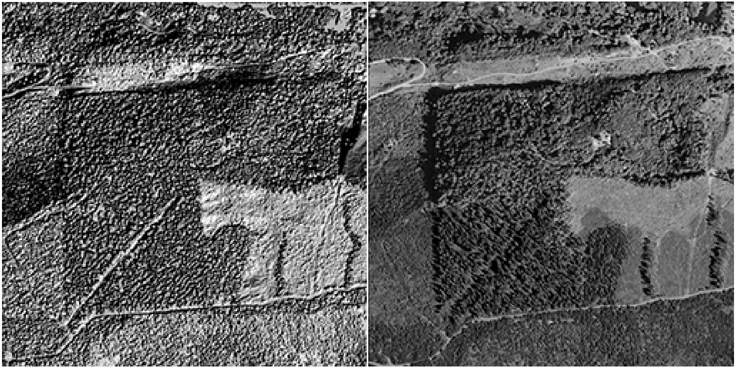

Figure: Lidar DEM (left), and aerial photograph (right) for a scene in WA state. The shaded relief image, illuminated in the same direction as in the optical image, depicts similar patterns of textures and edges. All images are 256×256. The images have been converted to grayscale.

Lidar-to-Optical Experiments (2/2)

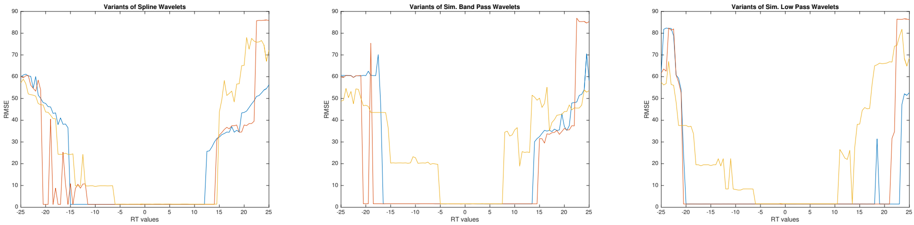

Figure: Comparison of algorithms for WA lidar-to-optical experiments (from left to right: splines, Simoncelli band-pass, Simoncelli low-pass); blue is wavelets, yellow is hybrid shearlets+wavelets with decomposition, and red is shearlets+wavelets without decomposition.

Conclusions

- The experiments affirm the effectiveness of using shearlets for image registration.
- In concert with wavelets, improved robustness can generally be achieved, with little cost inaccuracy.
- The impact of decomposing the low-pass shearlet features with wavelets appears, however, unfavorable.
- This is perhaps due to the fact that the resultant features will be very low-pass indeed, thus having insufficient information content for registration.

Future Work

- It remains of interest to consider the impact of decomposing high-pass shearlet features, instead of the low-pass features.
- Recent theoretical developments with anisotropic Gabor theory suggests that frames of directional Gabor systems exist.
- Early numerical experiments indicate these frames can perform well for textures, which is a weakness of shearlets.
- The use of such systems could improve image registration of highly textural images.

Citations

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