Improving Accuracy of Structural Dynamic Modifications Using Augmented Mode Residual Vectors

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Spacecraft and Launch Vehicles Dynamic Environments Workshop
June 26–28, 2018

Overview

- Review of structural dynamic modification (SDM) method for reduced order representation of parameter changes on dynamic results
- Review of Augmented Mode Residual Vector (AMRV) extension to SDM
- Application to launch vehicle component modes
- Application to SLS integrated modal test (IMT) modes

- References

  - *Coppolino, R. "Sensitivity method for uncertainty and reconciliation analysis," Proceedings of the 29th IMAC, Jacksonville, FL, 2011*
  - *Coppolino, R. "FEM sensitivity technique for dynamic response uncertainty analysis," Proceedings of the 30th IMAC, Garden Grove, CA, 2012*
  - *Coppolino, R., "Methodologies for Verification and Validation of Space Launch System (SLS) Structural Dynamic Models, NASA/CR-2018-219800, Jan. 2018*

Structural Dynamic Modification (SDM) Used to Approximate Parameter Changes in Dynamic Results

- Consider a "parameterized" FEM where mass and stiffness matrix can be expressed as follows:
  - $[M] = [M_0] + \sum p_i[\Delta M_i]$, $[K] = [K_0] + \sum p_i[\Delta K_i]$
  - *Parameters can be anything that result in linear change in mass or stiffness*
    - e.g. material density, Young's modulus, Spring stiffness, etc.
- SDM reduces problem to Modal Space
  - $\left[\widehat{M}\right] = [I] + \sum p_i[\Phi]^T[\Delta M_i][\Phi]$, $\left[\widehat{K}\right] = [\Lambda] + \sum p_i[\Phi]^T[\Delta K_i][\Phi]$
    $\left[\widehat{K}\right][\Psi] = \left[\widehat{M}\right][\Psi]\left[\widetilde{\Lambda}\right]$, $\left[\widetilde{\Phi}\right] = [\Phi][\Psi]$ - *Matrices size of # modes*
  - $[\Phi]^T[\Delta M_i][\Phi]$ *and* $[\Phi]^T[\Delta K_i][\Phi]$ *are not diagonal so SDM captures effect of parameters on coupling modes (changes mode shapes as well as frequencies)*
  - *Fundamental assumption is that modified mode shapes are a linear combination* $[\Psi]$ *of nominal mode shapes –* <u>*carry along extra modes*</u>
  - *Excellent approximation for moderate (± 10% - ± 20% parameter changes)*

Lots of Applications for SDM

- In vocabulary of uncertainty quantification we call this a “surrogate model” – i.e. a reduced model that approximates full model
- Model Updating
  - *Model updating programs use SDM to find a set of parameters that minimize difference between measured and predicted modes (shape and frequency)*
  - *Can run 10,000’s of different parameter values in a genetic algorithm*
- Uncertainty Quantification
  - *Allows for rapid Monte Carlo evaluations of parameter uncertainty*
  - *10,000’s of uncertain parameters can be run in seconds or minutes*
- Design Studies
  - *Rapidly evaluate effect of many parameters on dynamic response*
  - *Structural optimization*
- Best Model Estimate (BME)
  - *Prepare a “library” of parameterized models prior to a modal test in order to be in position to rapidly select best model when data is available*

Augmented Mode Residual Vectors (AMRVs) Address Biggest Draw Back of SDM (limitation to moderate parameter variations)

- Limitation comes from assumption that perturbed modes are linear combination of nominal modes
  - *Poor assumption for large parameter variations*
- What if we included modes for large parameter variations in our nominal mode set?
  - *We'll call these shapes "augmented" modes*
- We can't exactly do that, because augmented modes aren't orthogonal w.r.t. nominal modes
  - *They aren't even necessarily independent*
- Nastran already has machinery to turn static shapes into "residual vectors"
  - *Residual vectors the part of the static shapes that are orthogonal to the nominal modes and to each other*
- Approach is to hijack Nastran residual vector calculation to turn augmented modes into residual vectors
  - *Hence augmented residual vectors (AMRVs)*

Calculating Augmented Mode Residual Vectors

- Start with a set of augmented mode shapes Ψ
  - *Can come from any number of modal solutions with large parameter changes (these are not static solutions)*
- Orthogonalize augmented modes w.r.t. normal modes
  - $\Psi' = (I - \Phi(\Phi^T M))\Psi$ *(i.e.* $\Phi^T M \Psi' = 0$*)*
- Calculate "coherence" since columns of Ψ' are not independent
  - $\hat{C} = \Psi'^T M \Psi'$
- Calculate singular values (neglect those below threshold)
  - $\hat{C} = [U_r \quad U_n] \begin{bmatrix} \Sigma_r & 0 \\ 0 & \Sigma_n \end{bmatrix} [U_r \quad U_n]^T$
- Transform augmented modes to retained space
  - $\widehat{\Psi} = \Psi' U_r$
- Pass $\widehat{\Psi}$ through Nastran residual vector calculation to turn into residual vectors ($\widehat{\Psi}^T M \widehat{\Psi} = \mathrm{I}$, $\widehat{\Psi}^T K \widehat{\Psi} = \Lambda_r$)
  - *Just a linear transformation*
- Treated just like modes for rest of solution
- Number of AMRV's typically << than number of augmented modes

Implementing AMRVs in Nastran

1. Run any number of augmented modal solutions
   - *Save mode shapes in OP2 or OP4 format*
   - *Augmented modes should represent "extreme" cases*
   - *Don't need to run augmented modes for every parameter – just those that will vary a lot*
   - *Only requirement is that the DoF (G-set) be the same*
2. Execute nominal modes with "trial_resvec" DMAP alter
   - *OP2/OP4 files from all augmented mode solutions ASSIGNed to INPUTT2/INPUTT4 units*
   - *PARAM,TFILT sets threshold for retaining residual vectors*
   - *Output is nominal modes plus augmented mode residual vectors*
3. Nastran process works on its own, but is most useful when combined with SDM calculation (e.g. ATTUNE)
   - *Another DMAP alter does the SDM step for model updating*
   - *Running SDM for model updating remains unchanged*

1st Example is Launch Vehicle Component Modal Test

- FEM reflects modal test configuration
- Small model runs quickly
  - *Modes to 50 Hz takes < 20 seconds on desktop*
- Parameterized by 24 groups of "E" and 4 groups of "K" values (DESVARs)
  - *Total of 28 parameters to "tune"*
  - *Assume that K's can decrease by 90% but not increase*
  - *Assume that E's can increase or decrease by 50%*
- 52 sets of augmented modes (2*24 for E + 4 for K)
  - *Augmented modes run to 50 Hz*
  - *Total of 3,248 augmented modes*
- Consider one case
  1. *62 nominal modes to 50 Hz + 678 AMRVs based on singular value cutoff of 1E-6 (total of 740 'modes' used for sensitivity calculation*

Nastran Run to Calculate AMRVs

```
ASSIGN INPUTT2='mode_v12_d01_p50.op2' UNIT=101
                                :
ASSIGN INPUTT2='mode_v12_d28_m90.op2' UNIT=152
SOL 200
INCLUDE 'trial_resvec_op2.dmap'
INCLUDE 'attune22.dmap'
CEND
DSAPRT(END=SENS) = ALL
ANALYSIS = MODES
METHOD=909
RESVEC = YES
BEGIN BULK
PARAM,VUNIT1,101
PARAM,VUNIT2,152
PARAM,TFILT,1.E-6
```

- ASSIGN lines: Assign 52 OP2 files from augmented modes runs
- INCLUDE lines: Include ‘trial_resvec_op2’ and ATTUNE (SDM) DMAP alters
- DSAPRT / ANALYSIS / METHOD: Standard modal sensitivity request
- RESVEC = YES: Turn on residual vectors
- PARAM,VUNIT1 / PARAM,VUNIT2: 1st and last unit number for OP2 files
- PARAM,TFILT: Singular value threshold for retaining residual vectors

Note: OP2 files written using PARAM,POST,-1 only works for models without SPOINTs. For models with SPOINTs you need to use DMAP to write mode shapes in OUTPUT4 format and use ‘trial_resvec’ DMAP alter

Selected Output from AMRV Run

```
                                   R E A L   E I G E N V A L U E S
                           (BEFORE AUGMENTATION OF RESIDUAL VECTORS)
   MODE    EXTRACTION      EIGENVALUE            RADIANS             CYCLES            GENERALIZED         GENERALIZED
    NO.       ORDER                                                                        MASS              STIFFNESS
        1         1        9.028311E+03        9.501742E+01        1.512249E+01        1.000000E+00        9.028311E+03
                                                                  :
       62        62        9.793706E+04        3.129490E+02        4.980738E+01        1.000000E+00        9.793706E+04
```

Nominal Modes to 50 Hz

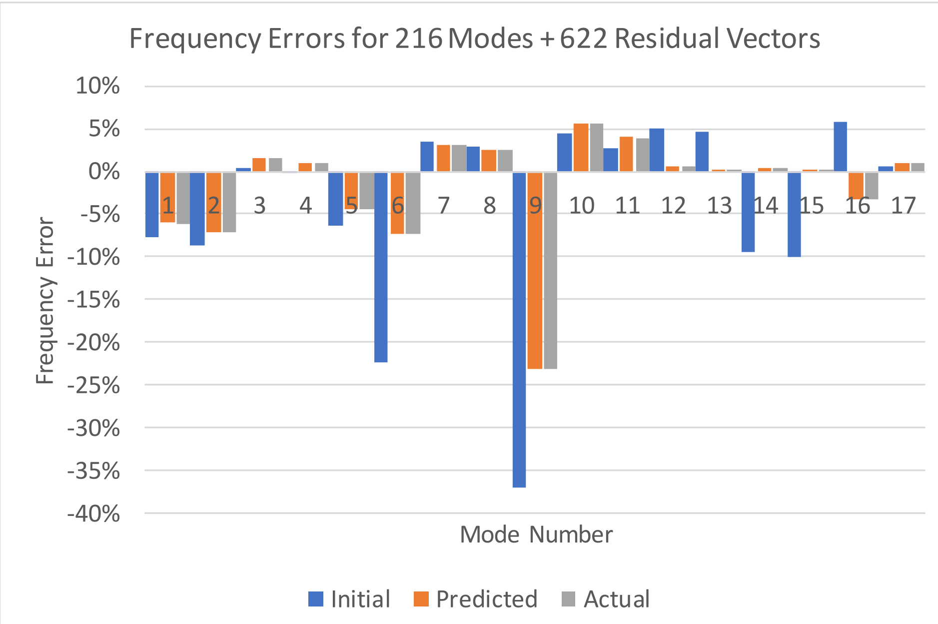
```
                                   R E A L   E I G E N V A L U E S
   MODE    EXTRACTION      EIGENVALUE            RADIANS             CYCLES            GENERALIZED         GENERALIZED
    NO.       ORDER                                                                        MASS              STIFFNESS
        1         1       -2.412605E-17        4.911828E-09        7.817417E-10        1.000000E+00       -2.412605E-17
                                                                  :
     3248      3248        6.016507E+00        2.452857E+00        3.903843E-01        1.000000E+00        6.016507E+00
```

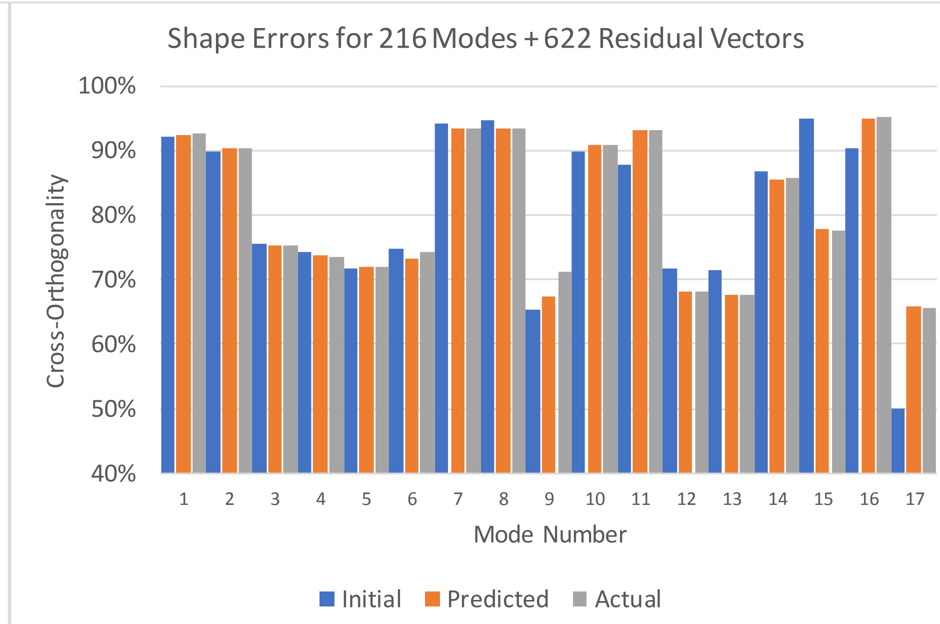
3248 singular values from -2E-17 to 6.02

```
^^^RETAINING          678 OF       3248 RESIDUAL TRIAL VECTORS
^^^BASED ON A FILTER OF  1.000000E-06
                                   R E A L   E I G E N V A L U E S
                           (AFTER AUGMENTATION OF RESIDUAL VECTORS)
   MODE    EXTRACTION      EIGENVALUE            RADIANS             CYCLES            GENERALIZED         GENERALIZED
    NO.       ORDER                                                                        MASS              STIFFNESS
        1         1        9.028311E+03        9.501742E+01        1.512249E+01        1.000000E+00        9.028311E+03
                                                                  :
      740       740        1.811352E+09        4.255998E+04        6.773631E+03        1.000000E+00        1.811352E+09
```

62 Normal modes + 678 Residual Vectors based on 1E-6 cutoff

Look to see how well Model Updating estimates changes

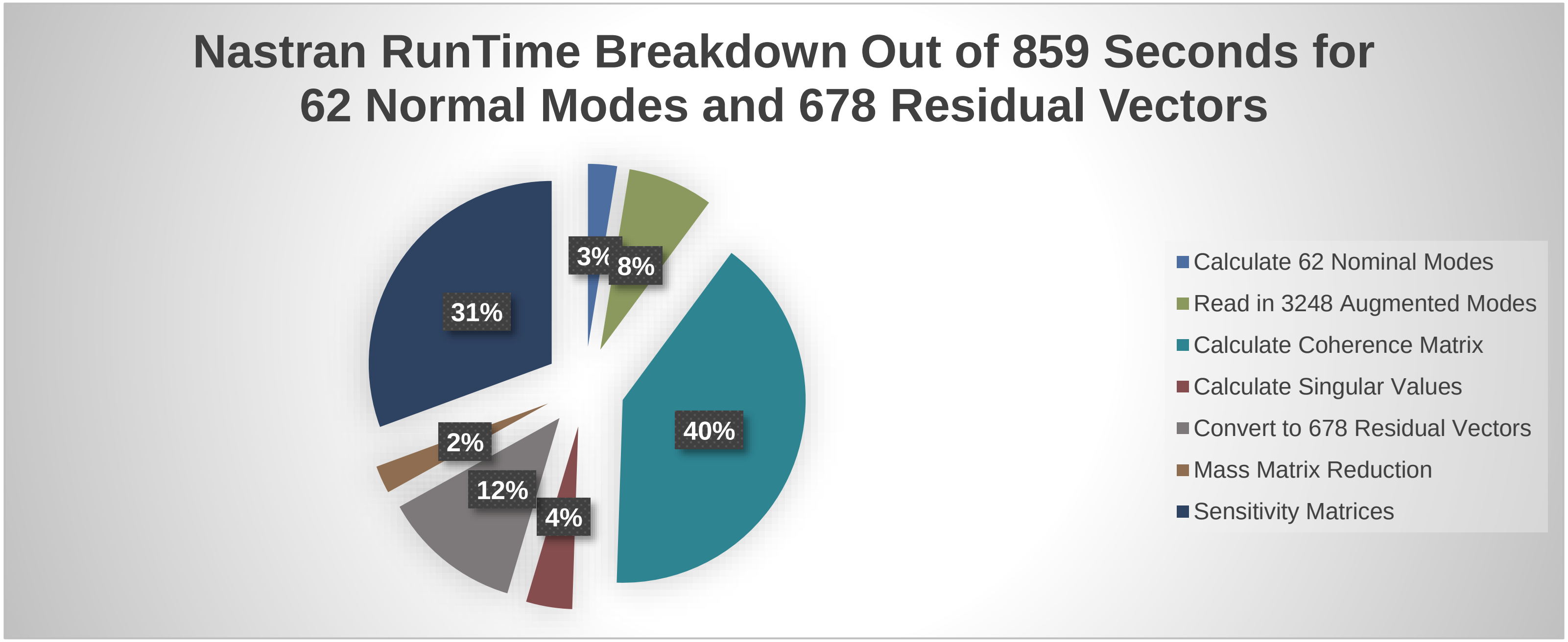




- Model updating software makes "random" changes in parameters based on matching test data
  - *Parameters allowed to move within range of augmented mode solutions*
- Evaluate predicted vs. actual comparison to test data after a single iteration of model updating software
- Results indicate that predicted frequencies and shapes in model updating software matched the actual shapes calculated in Nastran very accurately

Computational Cost of AMRVs Can Be Significant

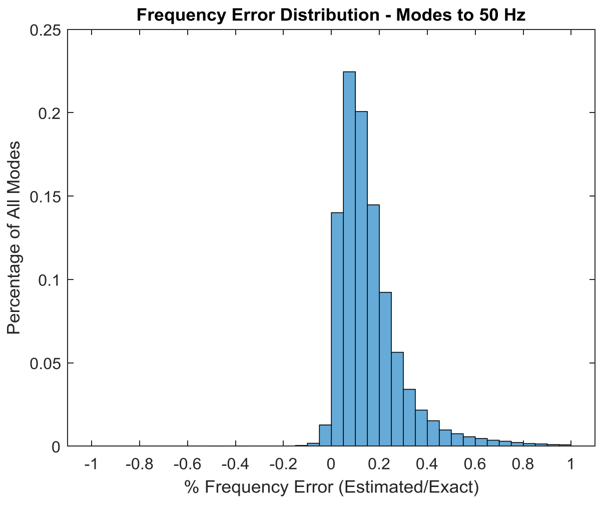
- Modal solution for each set of augmented modes (up to 2x number of parameters)
- Sensitivity solution with AMRVs increases cost from 1 minute to 15 minutes
- Largest computational cost is $\hat{C} = \Psi'^{T} M \Psi'$

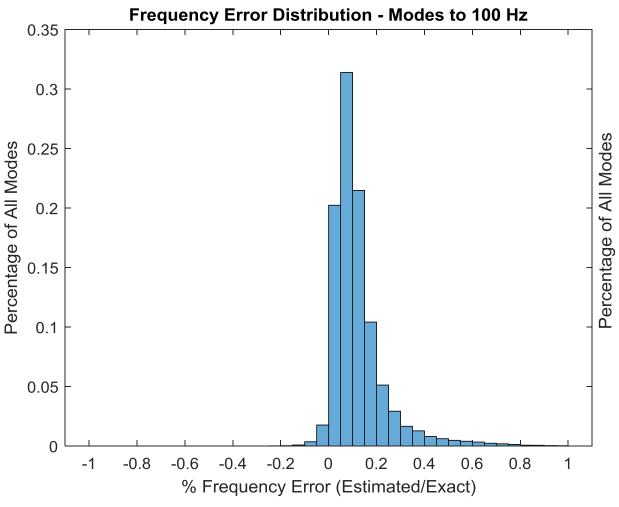


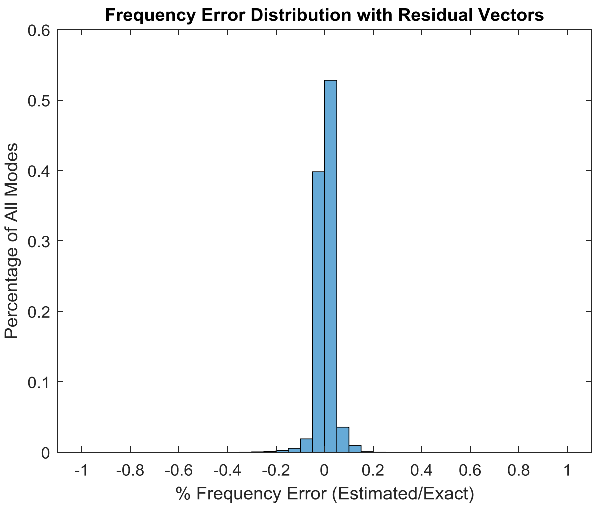
Evaluation of Random Variations

- First case varied all 28 variables uniformly by ± 20%
- Second case varied 24 variables uniformly by ± 50%
  - *Spring variables varied -90% + 0%*
- Run all 1000 solutions to 50 Hz and compare with sensitivity based solutions
  - *Solution with 62 normal modes to 50 Hz + 678 AMRVs (740 modes total)*
  - *Straight modes (62 to 50 Hz, 216 to 100 Hz, 740 to 200 Hz)*
- Gather statistics on frequency error and mode shape fit for all modes for all models
  - *Total of 61,485 modes across 1,000 +/- 20% random models*
  - *Total of 65,540 modes across 1,000 +/- 50% random models*
  - *Modes matched using cross-orthogonality*
  - *Mode shape error based on RSS cross-orthogonality for +/- 2% frequency band*

AMRVs Not Really Necessary for 1000 ± 20% Variations



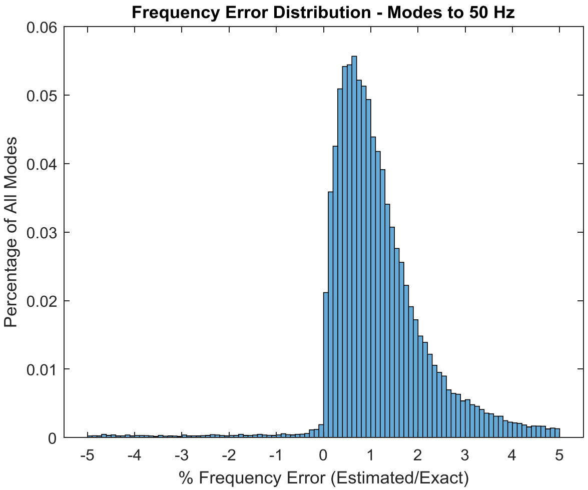




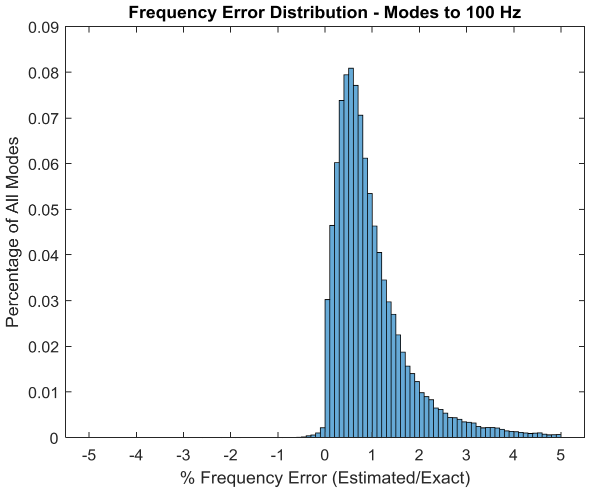
- With 216 modes to 100 Hz and no AMRVs 95% of modes are within 0.34% and 99% are within 0.61%
- With AMRVs 95% of modes are within 0.05% and 99% are within 0.10%
- AMRVs improve situation, but having 99% of the modes well within 1% is probably close enough

AMRVs Really Helpful for ± 50% Variations

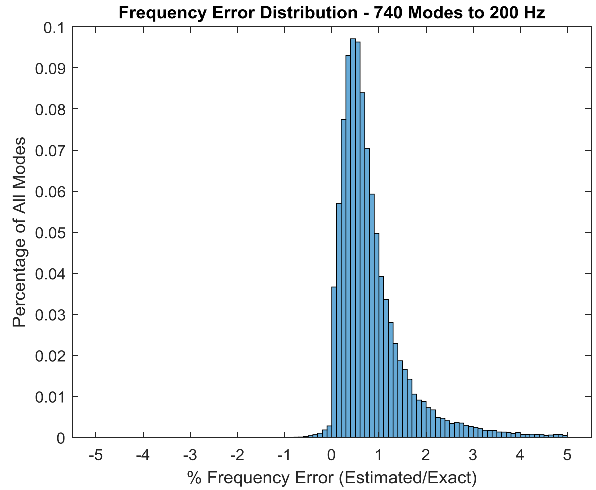
62 Modes to 50 Hz



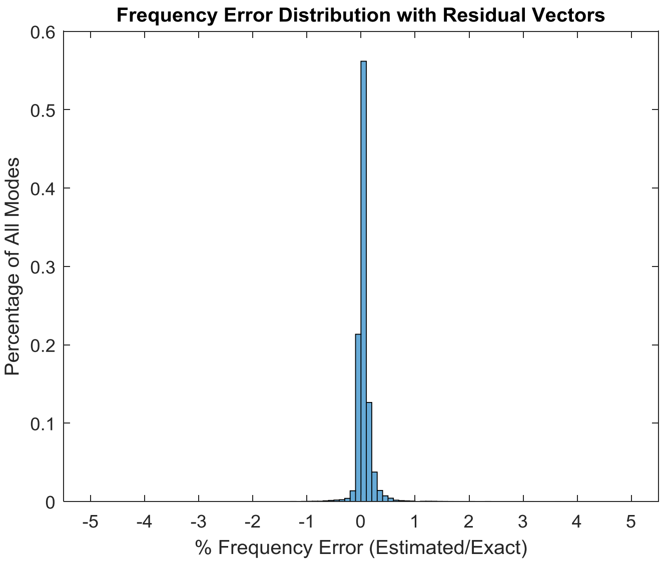
216 Modes to 100 Hz



740 Modes to 200 Hz

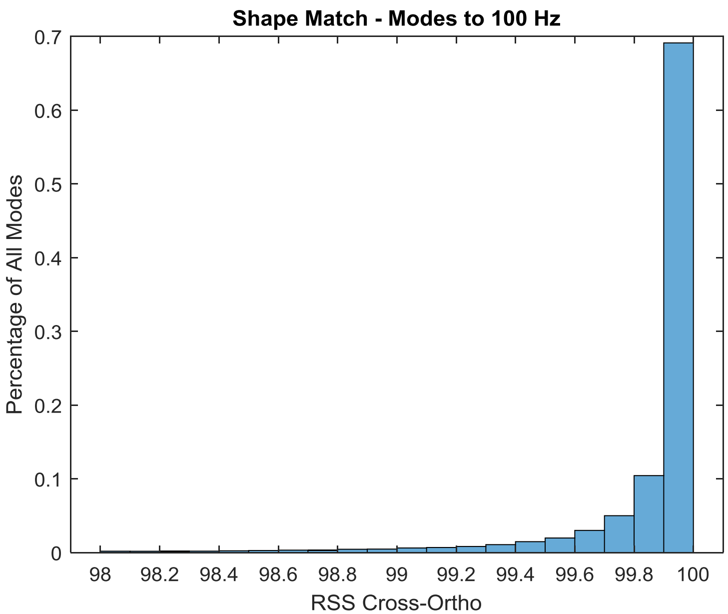


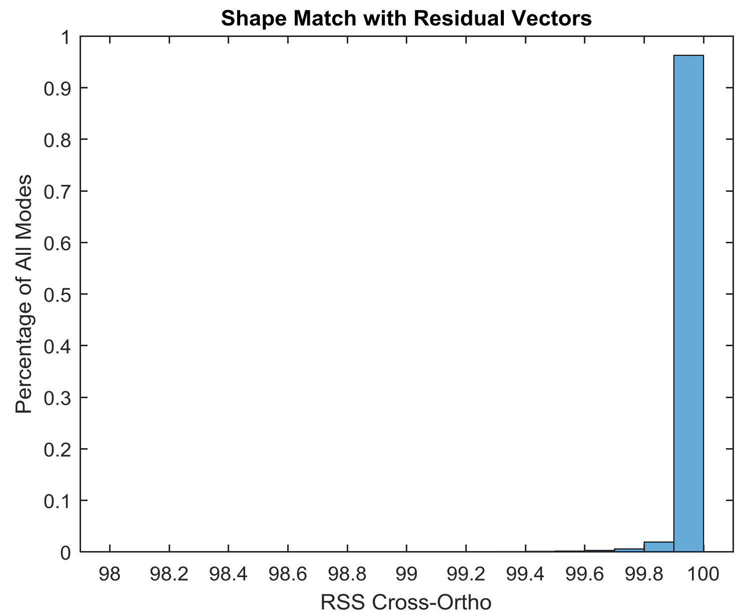
62 Modes + 678 AMRVs



- Adding modes increases frequency accuracy very slowly
  - *Even with 740 modes to 200 Hz only 95% of modes within 2.32% and 99% within 4.08%*
- Adding AMRVs dramatically improves frequency match
  - *With 740 modes almost all modes within 0.5% (95% within 0.24%, 99% within 0.55%)*
- For this case errors acceptable with AMRVs but unacceptable without

Mode Shapes Also Match Better (RSS in +/-2% Frequency Band)





- RSS match looks at all modes without ± 2% of frequency of best fit mode (deals with 'rotated' modes)
- Adding AMRVs improves RSS shape match
  - *Almost all modes better than 99.5% RSS cross-ortho*
- Without AMRVs most results are still not too bad
  - *This is mostly because parameterized variation doesn't move mode shapes around that much anyway*

Observations from Component Modal Data

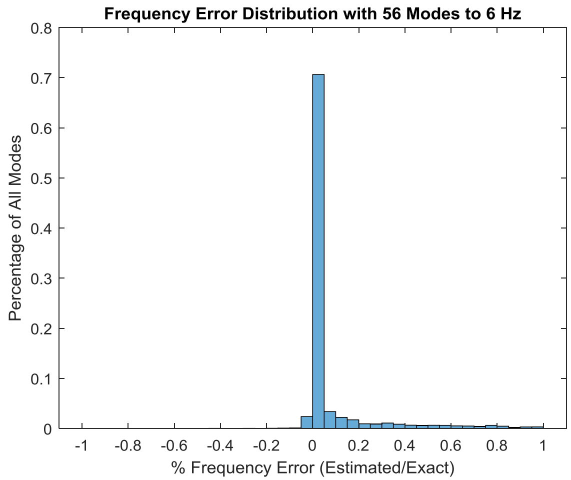
- For ± 20% changes, AMRVs aren't really necessary
  - *Do improve shape match a little bit but probably not worth it*
- AMRVs do a good job of predicting changes within the range of changes accounted for
  - *± 50% changes in materials and -90% changes in springs for this test case*
- AMRVs don't necessary improve solution beyond range of parameter accounted for
- Recommended approach
  - *Don't bother calculating AMRVs for parameters that vary within ± 20%*
  - *For parameters that do vary more than ± 20% calculate AMRVs for max and min range*

Second Case is SLS on Mobile Launcher (ML)

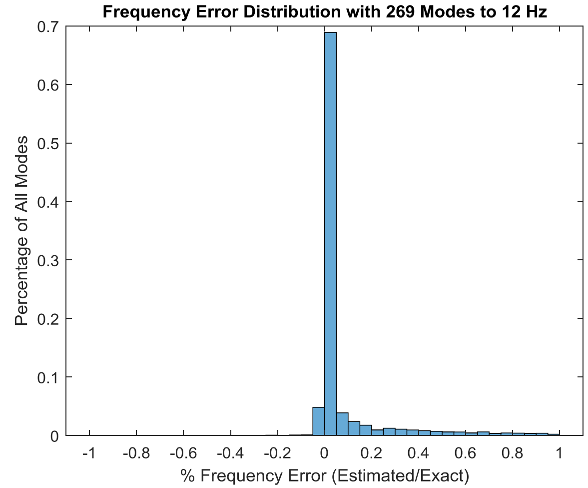
- Much larger model
- 53 parameters
    - *44 E's – allowed to vary ± 50%*
    - *9 interface K's – allowed to vary ± 90%*
- 106 augmented mode sets run in steps due to limitation in old version of Nastran
    - *Run augmented modes in groups through DMAP alter until all 106 sets accounted for*
- Final AMRV run with sensitivities took about 2 hours
- 56 nominal modes to 6 Hz + 213 AMRVs for total of 269 "modes"
- Ran 133 cases using DOE
    - *106 cases for extremes*
    - *27 cases for intermediate values within extremes*
- Consider all modes to 5 Hz
- Modes aligned using MAC with 132 DOF

Results for 133 ± 50%/90% Parameter Variations

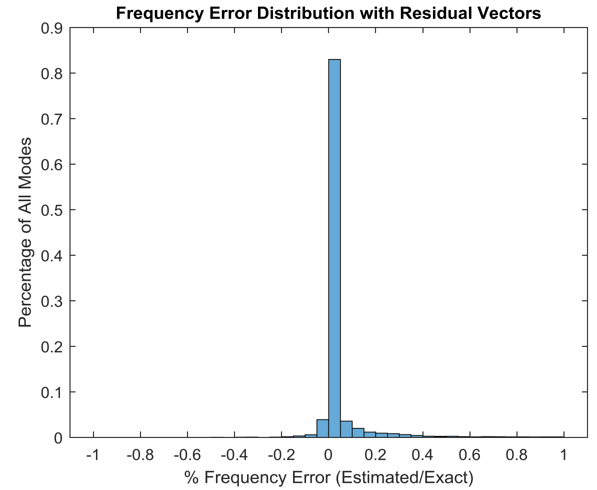
56 Modes to 6 Hz



269 Modes to 12 Hz



56 Modes to 6 Hz + 213 AMRVs



- With just 56 modes to 6 Hz 95% of frequencies are within 1.73% and 99% are within 3.76%
- With 269 modes to 12 Hz 95% of frequencies are within 1.38% and 99% are within 2.98%
- With AMRVs 95% of frequencies are within 0.21% and 99% are within 0.92%
- For this case solutions without AMRVs are not acceptable, but solutions with AMRVs are
  - *Adding modes from 6 Hz to 12 Hz didn't help much*

Summary

- SDM is a very powerful way of generating a reduced order model to track effect of parameter variations on modal properties
- Method typically works well for ±10% to ±20% parameter variations using modes slightly beyond frequency range of interest
  - *Outside this range typically requires multiple steps or accepting significant errors relative to "exact" solution*
- Adding more modes doesn't help SDM very much
- Augmented Modal Residual Vectors (AMRVs) Extend Applicability of SDM to much larger parameter variations
  - *Order of magnitude variations are possible*
  - *Cost in setting up runs and generating AMRVs is significant*
- Recommendation is to use AMRVs judiciously for those variables that will undergo large variations
  - *Typical examples are springs at joints*

Thank you