# Computational Investigation of the Near-Field Plasma Plume in Ion-Ion Propulsion

K. Sankaran[*], T. Widmer[†], and T. Dale[‡]
*Whitworth University, Spokane, WA 99251*

K. A. Polzin[§]
*NASA Marshall Space Flight Center, Huntsville, AL 35763*

**A two-fluid computational model of plasma flows was developed to investigate the plume of an ion-ion propulsion system. The densities of positive and negative ions, along with the associated values of net charge, electric field, and electric potential were calculated throughout the domain. The computational domain was chosen to be large enough (25 thruster diameters downstream of the accelerating grids) to examine the neutralization of the plume. The resulting plasma electric potential and charge neutrality at the downstream end of the domain are shown. The results from this simulation are compared to existing literature on ion-ion plasma thrusters.**

## Nomenclature

$(\ldots)_{p,n}$ = positive and negative ions
$e$ = elementary charge (C)
$\mathbf{f}_{\text{ext}}$ = external force per unit volume (N/m$^3$)
$\bar{\bar{I}}$ = identity matrix
$m_{p,n}$ = mass of a particle (kg)
$n_{p,n}$ = number density of particles (#/m$^3$)
$p$ = pressure (Pa)
$\mathbf{u}$ = velocity (m/s)
$V$ = electric potential (V)
$\epsilon_\circ$ = electrical permittivity (C$^2$/N m$^2$)
$\nu_{\text{coll}}$ = collision frequency (#/s)
$\rho_{p,n}$ = mass density of particles (kg/m$^3$)

## I. Introduction

Electrostatically accelerated propulsion systems, such as conventional ion thrusters, expel a beam of positive ions and thus require neutralization of the exhaust. This is typically accomplished by the emission of electrons from a cathode that is placed slightly downstream of the accelerating grids. To reduce the complexity of the thruster system and eliminate a potential point of failure, an electrostatic acceleration mechanism that does not require a neutralizer would be beneficial. To that end, a thruster in which packets of positive ions and negative ions are successively expelled by varying the electric potential of the accelerating grid was proposed by Aanesland[1]. Further refinements to this ion-ion propulsion concept have led to devices such as the PEGASES[2] that have been experimentally and theoretically investigated[3]. The possibility of neutralizing the plasma plume[4] without the need for an external neutralizer leads us to investigate the viability of that process. Oudini *et al.*[5] investigated this mechanism using a two-dimensional particle-in-cell (2D-PIC) simulation and show that neutralization of the plume occurs approximately five grid diameters downstream (10 cm downstream of a 2 cm grid). We seek to examine the results of Oudini *et al.*[5] using a two-fluid simulation to clarify three questions:

---

[*] Professor; Senior Member of AIAA.
[†] Student.
[‡] Student.
[§] Research Engineer; Associate Fellow of AIAA.

1) How do the results change when the model is changed from a 2D-PIC with weighted particles to a two-fluid simulation?
2) What are the effects of collisions (which are neglected in Ref.[5]) on the behavior of the plume?
3) What is the effect of the downstream computational boundary condition on the results?

This paper describes our methodology to examine those three questions and presents the results of our investigation.

Section II describes the characteristics of the specific thruster that was simulated. Section III describes the mathematical and numerical model of the plasma flow in the plume. Section IV presents the results of our simulations.

## II. Thruster Characteristics

To allow for a direct comparison with the results of Ref. [5], the same thruster specifications were chosen in this work. Thus, the propellant in this simulation was iodine, with alternating fluxes of $I^+$ and $I^-$ ions. The thruster's accelerating grid had a radius of 1 cm. The bias voltage of the grid was set to $V_{\text{accel}} = \pm 500$ V. Though Oudini *et al.*[5] considered a range of bias frequencies from 0.5-2.0 MHz, we set the bias frequency to be $f_{\text{bias}} = 1$ MHz.

The plasma upstream of the grid was assumed to be accelerated electrostatically and then injected into the computational domain. The plasma characteristics immediately downstream of the grid (upstream boundary of the simulation) were also chosen to match the values used in Ref. [5]. The temperature of both the positive and negative ions at the inlet were set to $T_{p,n} = 0.1$ eV. The inlet velocity was purely axial at $\mathbf{u}_{p,n} = \sqrt{2eV_{\text{accel}}/m_{p,n}}\,\hat{\mathbf{z}}$. Based on the values of density and ion thermal velocity upstream of the grid, the downstream value of density was calculated from the exhaust velocity and the mass conservation relationship to be $n_{p,n} = 10^{15}/\text{m}^3$.

## III. Computational Model

Because the ion-ion propulsion system described in Aanesland *et al.* [1] uses magnetic filtering to ensure that the free electron density is negligible in the exhaust, we follow the approach of Oudini *et al.*[5] and treat the plasma as comprising only of positive and negative iodine ions. Furthermore, following Ref. [5], we treat the plasma as isothermal and thereby ignore the need for an energy equation. Under these assumptions, the governing equations for conservation of mass and momentum in an infinitesimal volume are,

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho \mathbf{u} \end{bmatrix}_{p,n} + \nabla \cdot \begin{bmatrix} \rho \mathbf{u} \\ \rho \mathbf{u}\mathbf{u} + p\bar{\bar{I}} \end{bmatrix}_{p,n} = \begin{bmatrix} 0 \\ \mathbf{f}_{\text{ext}} \end{bmatrix}_{p,n} . \tag{1}$$

Here, the mass density of the positive and negative fluids are $\rho_{p,n} = n_{p,n} m_{p,n}$, and the external force per volume due to electric fields and collisions is $\mathbf{f}_{\text{ext}} = \pm n_{p,n}\left(e\mathbf{E} + m_{p,n}\nu_{\text{coll}}\mathbf{u}\right)$.

The electric potential in the domain is calculated using the Poisson equation,

$$\nabla^2 V = -\frac{e}{\epsilon_\circ}\left(n_p - n_n\right), \tag{2}$$

and the field is calculated using the electrostatic definition of $\mathbf{E} = -\nabla V$.

This model assumes cylindrical symmetry and, hence, all azimuthal ($\hat{\theta}$) components and all azimuthal derivatives ($\partial/\partial\theta$) of all pertinent quantities ($\mathbf{u}$, $\mathbf{E}$, etc.) are set to zero.

### A. Initial and Boundary Conditions

The initial values of all number densities in the background are set to $n_p = n_n \leq 10^{13}/\text{m}^3$, a value much smaller than the density of the inlet plasma. The initial electric field is set to zero everywhere in the domain.

### B. Numerical method

The time varying aspect of Eq. (1) was numerically solved using an explicit time-marching scheme. We used a standard Euler scheme with a Courant–Friedrichs–Lewy (CFL) value of 0.2. The plasma flow was explicitly calculated in time for hundreds of cycles of the accelerating grid's bias frequency.

The spatial derivatives for the hyperbolic convection problem were calculated using a standard forward difference with numerical dissipation, as described in Ref. [6].

The Poisson equation (Eq. (2)) was solved using a standard Red-Black Successive Over Relaxation (SOR) scheme[7] with a relaxation parameter of $\omega_{\text{opt}} \simeq 1.99$.

### C. GPU Computation

To speed up the computation, GPU-accelerated computing was used in this work. Therefore, a series of verifications were conducted to validate computational method to solve the equations listed above. First, basic linear algebra operations (specifically, multiplication and inversion) were conducted and benchmarked for speed to verify the accuracy and the efficiency of CPU-GPU communications. Then, the above-mentioned numerical solution techniques to solve elliptic, parabolic, and hyperbolic partial differential equations were tested (in cylindrical coordinates) by solving the Laplace equation, the diffusion equation, and the nonlinear advection equation, respectively. Based on the successes for these test cases, the GPU-accelerated computing platform was then utilized to examine the main questions of this research.

## IV. Results

Resulting contours of number densities of positive and negative ions, $n_{p,n}(r,z)$, their velocities, $\mathbf{u}_{p,n}(r,z)$, and the electric potential, $V(r,z)$ will be calculated and shown here. Preliminary results, shown in Fig. 1, indicate the alternating injections of positives and negatives is captures correctly by the code.

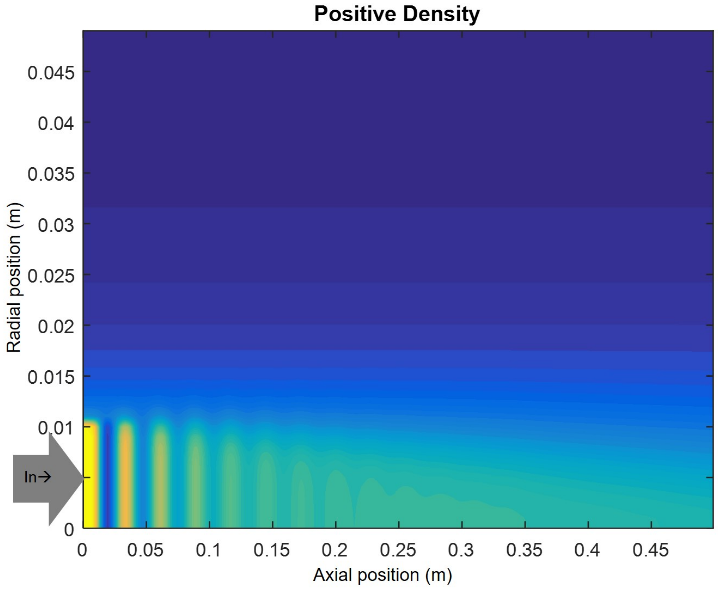

**Fig. 1 Contours of density of positive charges.**

## V. Concluding Remarks

This will be included in the final version of the paper.

## Acknowledgments

This work was supported by the NASA Washington Space Grant. Computational hardware to support this research was donated by the NVIDIA Corporation.

## References

[1] Aanesland, A., Meige, A., and Chabert, P., "Electric propulsion using ion-ion plasmas," *Journal of Physics: Conference Series*, Vol. 162, 2009, p. 012009. doi:10.1088/1742-6596/162/1/012009.

[2] Aanesland, A., Rafalskyi, D., Bredin, J., Grondein, P., Oudini, N., Chabert, P., Garrigues, L., and Hagelaar, G., "The PEGASES gridded ion-ion thruster performance and predictions," *33rd International Electric Propulsion Conference*, ERPS, Washington, DC, 2013.

[3] Gerst, D., Renaud, D., Mazouffre, S., Chabert, P., and Aanesland, A., "E× B probe investigation of the PEGASES thruster ion beam in Xe and $SF_6$," *33rd International Electric Propulsion Conference*, ERPS, Washington, DC, 2013.

[4] Rafalskyi, D., Popelier, L., and Aanesland, A., "Experimental validation of the dual positive and negative ion beam acceleration in the plasma propulsion with electronegative gases thruster," *Journal of Applied Physics*, Vol. 115, 2014, p. 053301. doi:10.1063/1.4863876.

[5] Oudini, N., Aanesland, A., Chabert, P., Lounes-Mahloul, S., and Bendib, A., "Near-field plume properties of an ion beam formed by alternating extraction and acceleration of oppositely charged ions," *Plasma Sources Sci. Technol.*, Vol. 25, 2016, p. 055013. doi:10.1088/0963-0252/25/5/055013.

[6] Sankaran, K., Martinelli, L., Jardin, S., and Choueiri, E., "A Flux-Limited Numerical Method for the MHD Equations to Simulate Propulsive Plasma Flows," *International Journal of Numerical Methods in Engineering*, Vol. 53, No. 5, 2002, p. 1415. doi:10.1002/nme.343.

[7] Press, W. H., *Numerical Recipes in C*, $3^{rd}$ ed., Cambridge University Press, 2007, pp. 863–882.