

Development Status of a 3-D Electron Fluid Model for Hall Thruster Plume Simulations

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Background

- Simulation results are most often validated through comparisons with ground facility measurement data, while models need to predict on-orbit EP thruster operations
 - Predicting behavior of flight system requires deep understanding of components and system-level interaction (e.g., thruster, cathode, facility effects, backspattering)
- While the background pressure effects have been extensively studied in the past,¹⁻⁶ we still have limited understanding electrical facility effects in vacuum chambers
 - The presence of conducting chamber walls provides alternate pathways for electrons to travel from the cathode and serve as a recombination site⁷⁻⁸
 - Need to better understand how electrons in Hall thruster plumes travel through and interact with the metallic walls of vacuum chambers

¹ Walker, *et.al.*, AIAA-4253, 2002.

² Walker, *et.al.*, *Journal of Propulsion and Power*, 2005.

³ Walker, Ph.D. Dissertation, University of Michigan, 2005.

⁴ Reid, Ph.D. Dissertation, University of Michigan, 2009.

⁵ Kamhawi, *et.al.* AIAA-3707, 2014.

⁶ Huang, W. *et.al.*, AIAA-3708, 2014.

⁷ Frieman, *et.al.*, *Journal of Propulsion and Power*, 2015.

⁸ Frieman, *et.al.*, *Journal of Propulsion and Power*, 2014.

Development of an Electron Plume Model

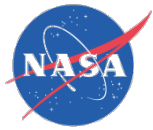


- In general, the Boltzmann relation is used to simulate electrons in EP thruster plumes
 - Simple and useful for isothermal, collisionless, & unmagnetized regions
 - Assumes zero-electron current everywhere
 - Unable to capture strong gradients in the near-field plume
- A detailed electron fluid model was developed by Boyd & Yim using a current conservation and Generalized Ohm's law⁹⁻¹⁰
 - Can predict plume characteristics better than the Boltzmann relation
 - Still could not capture steep gradients in the near-field plume
- This study was further improved by including a full electron mobility tensor to account for anisotropies in electron transport due to magnetic field effects (Choi 2016)
 - Similar approach in 3-D is being pursued at NASA GRC

⁹ Boyd, I.D. and Yim, J.T., AIAA-3952, 2004.

¹⁰ Cai, C and Boyd, I.D., AIAA-4462, 2005.

¹¹Choi, M., Space Propulsion, 2016



3-D Electron Plume Model for TURF

- The Thermophysics Universal Research Framework (TURF) is developed by Air Force Research Laboratory (AFRL)
 - Many capabilities including but not limited to:
 - » Vlasov, particle, fluid, and hybrid simulation;
 - » Flexible geometry import;
 - » Surface interaction and charging modules¹²
- This electron model is currently being implemented in TURF to perform:
 - Full vacuum facility simulations with magnetic field effects
 - Full spacecraft integration modeling with surface charging capability

¹²Araki, S.J., and Barrie, A.C., AIAA-4906, 2018.

Physical Model

- Generalized Ohm's law:

$$\mathbf{j}_e = \mu_e (\mathbf{j}_e \times \mathbf{B}) + \sigma_e \left(\mathbf{E} + \frac{1}{en_e} \nabla P_e \right)$$

- Steady-state current conservation:

$$\nabla \cdot (\mathbf{j}_e + \mathbf{j}_i) = 0$$

- Rearranging, we get:

$$\nabla \cdot (\bar{\mu}^{-1} \sigma_e \nabla \phi) = \nabla \cdot \left(\bar{\mu}^{-1} \frac{\sigma_e}{en_e} \nabla P_e \right) + \nabla \cdot \mathbf{j}_i$$

where the electron mobility is $\mu_e = \frac{q}{m_e \nu_{ce}}$ and the full tensor is

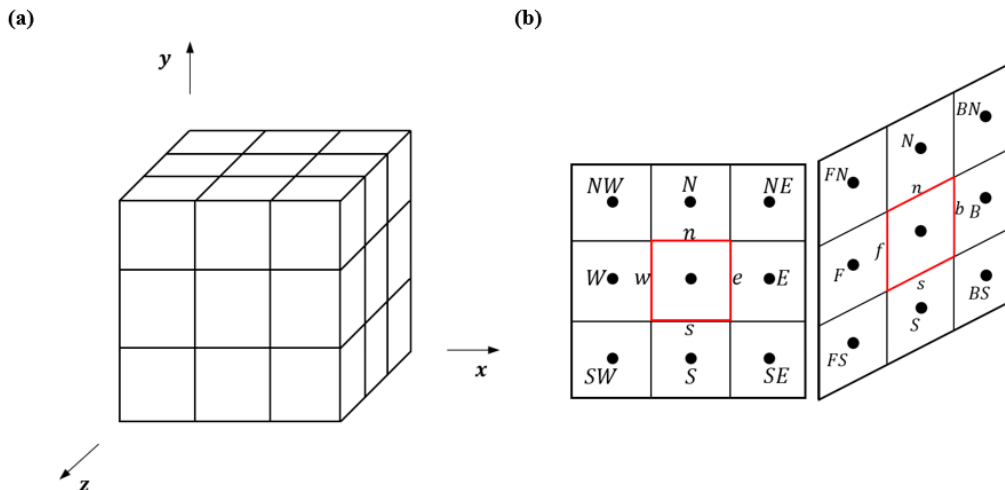
defined as:
$$\bar{\mu} = \begin{bmatrix} 1 & -\mu_e B_x & \mu_e B_z \\ \mu_e B_x & 1 & -\mu_e B_y \\ -\mu_e B_z & \mu_e B_y & 1 \end{bmatrix}$$

Numerical Model

- 2nd order cell-centered finite volume approach is used

$$\int_V [\nabla \cdot (\bar{\mu}^{-1} \sigma_e \nabla \phi)] dV = \int_V \left[\nabla \cdot \left(\bar{\mu}^{-1} \frac{\sigma_e}{en_e} \nabla P_e + \mathbf{j}_i \right) \right] dV$$

- Using the Green's theorem, the volume integral is transformed into surface integrals along all faces
- The final discretized equation becomes a linear system of equations that is solved explicitly ($A\mathbf{x} = \mathbf{b}$)
- FVM = FDM for this equation



Boundary Treatment: Dirichlet Boundary

- (a) Cell-center: the simplest way -> use ghost cells
- (b) Cell-face:

- FVM:
$$\left. \frac{\partial \phi}{\partial x} \right|_{\frac{1}{2}} = \frac{\phi_1 - \phi_{\frac{1}{2}}}{\left(\frac{\Delta x}{2}\right)}$$

- FDM:
$$\phi_0 = 2\phi_{\frac{1}{2}} - \phi_1 ; O(h)$$

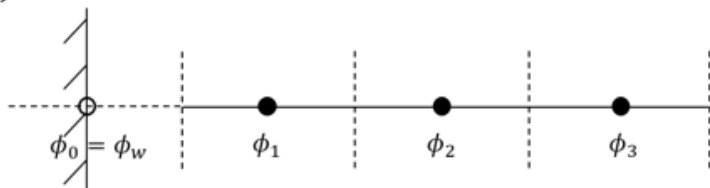
$$\phi_0 = \frac{8}{3}\phi_{\frac{1}{2}} - 2\phi_1 + \frac{1}{3}\phi_2 ; O(h^2)$$

- Flux at the boundary for (b):

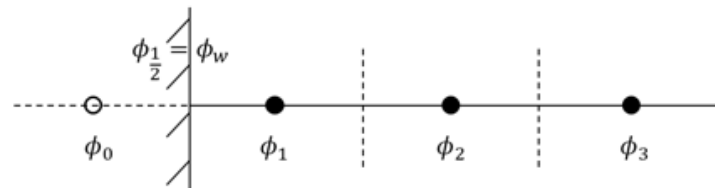
- 0th order:
$$(\sigma \bar{\mu}^{-1})_{\frac{1}{2}} = (\sigma \bar{\mu}^{-1})_1$$

- 2nd order:
$$(\sigma \bar{\mu}^{-1})_{\frac{1}{2}} = \frac{3(\sigma \bar{\mu}^{-1})_1 - (\sigma \bar{\mu}^{-1})_2}{2}$$

(a) Cell-center



(b) Cell-face



Corner Boundaries (in 2-D Example)

- Need to approximate the value of the ghost cell ϕ_{00}

- For $O(h)$:

- 2-point distance-weighted averaging

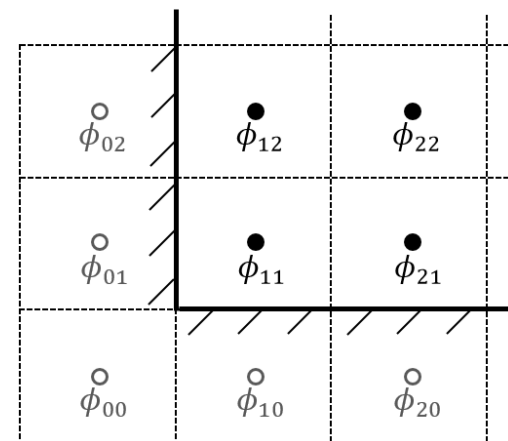
$$\phi_{00} = \frac{\phi_{10}\Delta y + \phi_{01}\Delta x}{\Delta x + \Delta y} \text{ which reduces to } \frac{\phi_{10} + \phi_{01}}{2} \text{ for } \Delta x = \Delta y$$

- 3-point distance-weighted averaging

$$\phi_{00} = \frac{\phi_{10}r\Delta y + \phi_{01}r\Delta x + \phi_{11}\Delta x\Delta y}{\Delta x + \Delta y + r} \text{ where } r = \sqrt{\Delta x^2 + \Delta y^2}$$

- For $O(h^2)$:

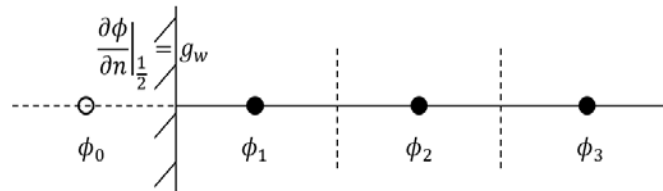
$$\begin{aligned} - \phi_{00} &= \frac{\phi_{10} + \phi_{01}}{2} \\ &= \frac{1}{2} \left[\frac{8}{3} \left(\phi_{\frac{1}{2}1} + \phi_{1\frac{1}{2}} \right) - 4\phi_{11} + \frac{1}{3} (\phi_{12} + \phi_{21}) \right] \end{aligned}$$



Boundary Treatment: Neumann Boundary



- Need to calculate cell-face values
 - $\phi_0 = \phi_1$ for zero gradient
 - $\phi_0 = \phi_1 + g_w \Delta x$ (1st order) for non-zero gradient



Verification Tests

- Method of Manufactured Solutions

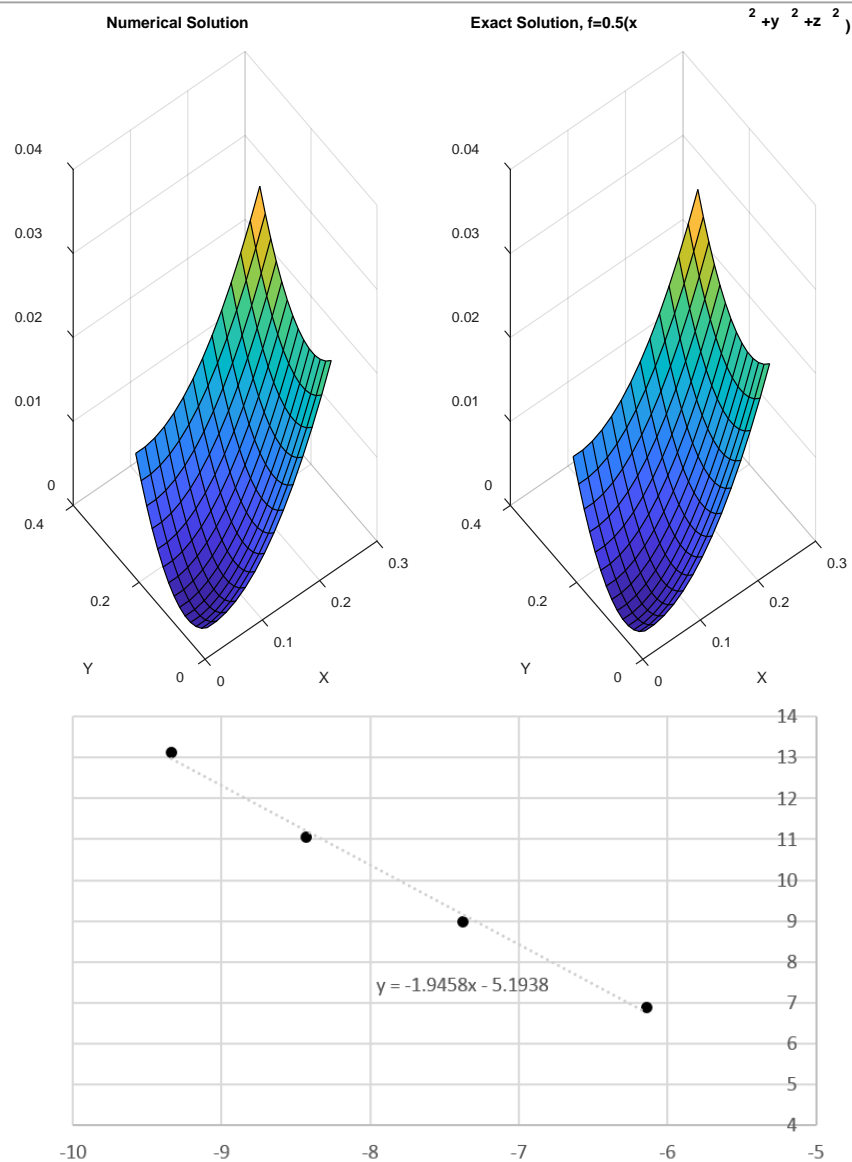
- $\phi_{exact} = \frac{K}{6}(x^2 + y^2 + z^2)$ where K is a constant

- $F = \frac{K}{3}\sigma_e(\mu_{11} + \mu_{22} + \mu_{33})$

- Grid convergence study

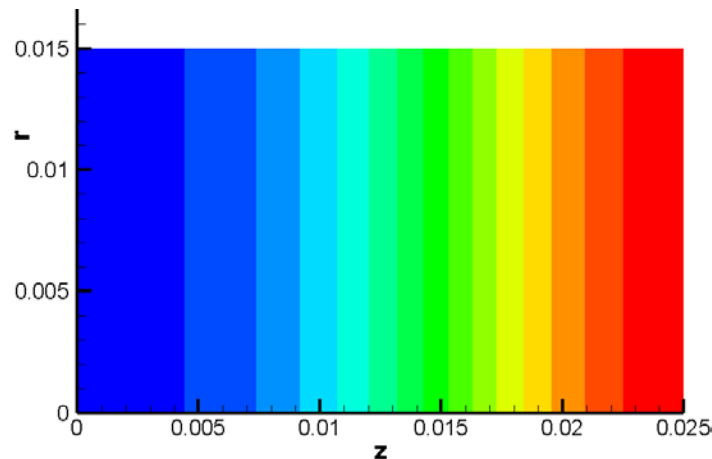
- $\|e\|_{L_2} = \sqrt{\sum_{k=1}^{N_k} \int_{\Omega_k} (\phi_h - \phi_{exact})^2 d\Omega_k}$

Manufactured Solution Result



A Hall Thruster-Like Testcase

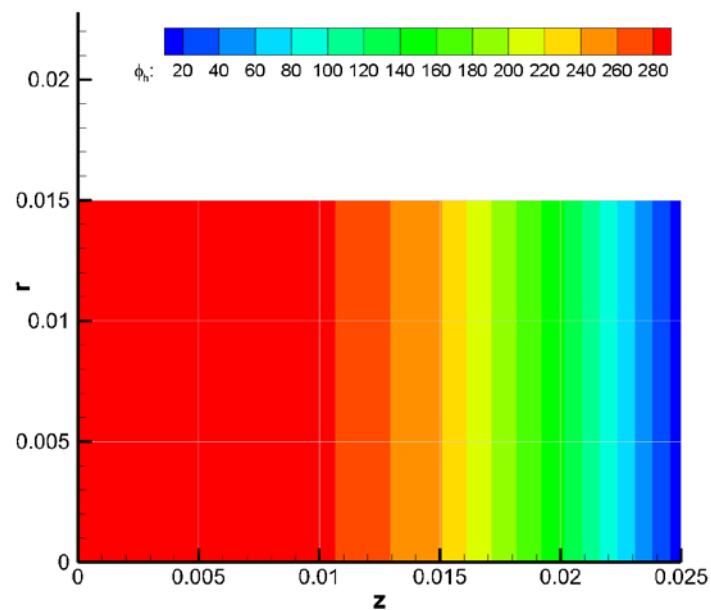
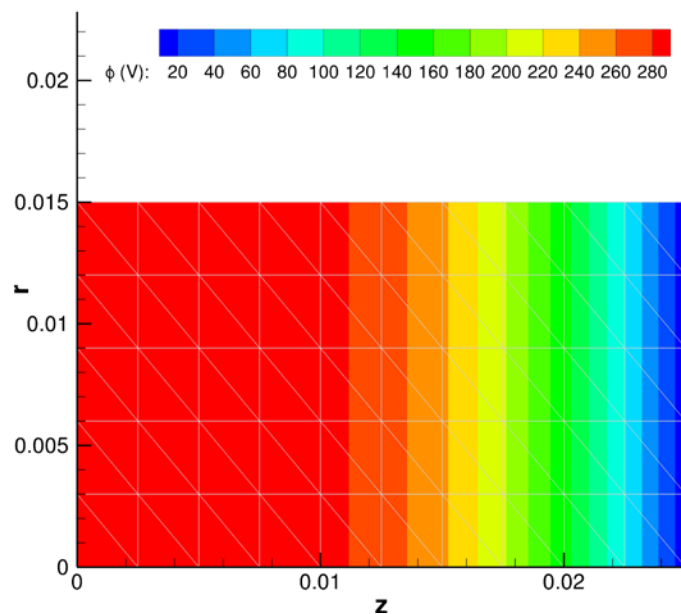
- Simple testcase made by Dragnea¹³
- Assumptions:
 - $T_e = 25 \text{ eV}$, $n_e = 1e17 \text{ m}^{-3}$; ideal gas law
 - $\mathbf{j}_i = 0$
 - 300 V on the left (anode) and 10 V on the right (cathode) surfaces
 - Zero-flux on the top and the bottom walls
 - Infinitely long plate in z-direction (periodic BC)
 - Magnetic field strength as shown below:



¹³Dragnea, *et al.*, AIAA-4632, 2017.

Plasma Potential Calculation

- Qualitative comparison between the 2-D axisymmetric finite element model (left) and the 3-D finite volume model (right)

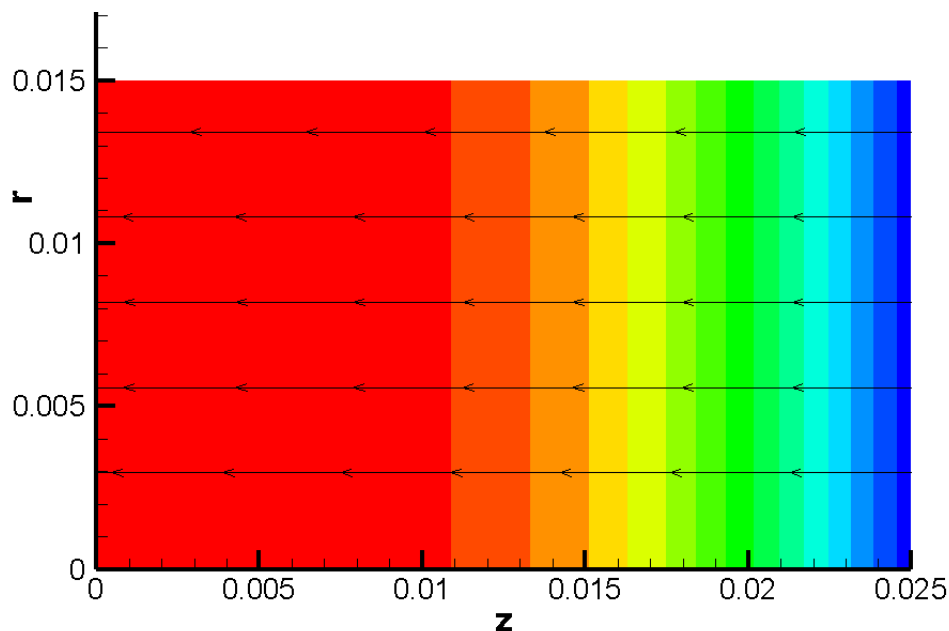


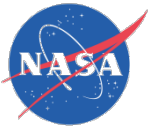
Electron Current Flow Calculation

- Use the plasma potential gradient in Generalized Ohm's law:

$$\mathbf{j}_e = \bar{\mu}^{-1} \sigma_e \left(-\nabla \phi + \frac{1}{en_e} \nabla P_e \right)$$

- Electron current density streamline (j_{e_z}, j_{e_r}) :





Summary & Future Work

- A 3-D electron fluid model has been developed as a stepping stone to simulate
 - the electrical facility effects in a conducting vacuum chamber
 - predict electron flux values to solar arrays and other components of a spacecraft
- The model is also being implemented in TURF in collaboration with AFRL