IMPORTANCE OF SUBGLACIAL WATER TO ICE SHEET FLOW AND CONFIGURATION; A TEST USING DATA FROM MARIE BYRD LAND, ANTARCTICA

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B.Sc. Thesis
June 1979

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II. Introduction

The mechanisms involved in the flow of continental ice sheets are of interest for many reasons. The movement of these huge ice sheets is a complex problem, because many varied factors work together to produce the flow characteristics observed in glaciers.

The causes of variation in ice sheets are an area of current research. A question exists as to the true stability of some glaciers, and the assumptions regarding their equilibrium. A study of the dynamic situation of Antarctic glaciers may not only help in the understanding of contemporary ice sheets but also of ancient glaciers, such as the Laurentide Ice Sheet in North America.

There are certain calculable properties of ice sheets that help explain the dynamic situation of a glacier. Such characteristics as the forces causing subglacial water flow, the basal shear stress, and ice velocity may be calculated and a model constructed describing glacial movement and shape.

The importance, specifically, of subglacial water is apparent when correlated to other calculable properties and overall ice sheet behavior. Basal water provides lubrication to the glacier and affects the mode of ice movement, and the presence of water may explain the variations observed in glacial characteristics. The tendency of water to flow at the base of a glacier is described by the hydrologic potential, which may be assessed in order to test the argument that the dynamics of the ice sheet may be largely controlled by the movement or accumulation of basal water.
III. Previous Work

The calculation of hydrologic potential and the study of its relationships to other observed or calculable characteristics depends in part upon the use of the results of previous research.

The work done by K.E. Rose of the Scott Polar Research Institute was used in the calculation of hydrologic potential values. During the 1974-1975 season he compiled contour maps of both the basal and surface elevations of Marie Byrd Land through the use of radio echo profiling.

The existence of subglacial lakes was suggested by Drewry (1977) based on interpretation of radio echo reflections. Subglacial lakes were defined by the planar water-ice interface, but Reynolds (1979) has suggested the possibility of non-planar interfaces, which would require different interpretive methods.

A number of authors have discussed the importance of subglacial water to ice sheet flow. Weertman (1964) and Paterson (1969) emphasize the possibility that if the water thickness should exceed a certain critical value and submerge obstacles below a discrete threshold, the glacier may surge. Budd (1977) contended that the water production rate rather than water thickness was important and used that concept to explain the fast sliding velocities of some glaciers.
IV. Description of the Ice Sheet

The portion of the sheet studied is located in West Antarctica, and is bordered by the Ross Ice Shelf to the west, Byrd Station (longitude 120° w) to the east, the Transantarctic mountains to the south, the parallel 80° s to the north. This is a rough square of 0.5 x 10^6 \text{km}^2 (see Map 1).

The ice sheet varies by large amounts in thickness, shape, and elevation within Marie Byrd Land. The thickness ranges from approximately 400 meters near the grounding line at the Ross Ice Shelf to nearly 3000 meters nearer the ice divide. The glacial substrate is generally very rough inland, with bedrock elevations as low as 1500 meters below sea level. The bedrock rises and becomes more smooth seaward and longitudinal trenches begin about 600 km from the ice divide and extend to the Ross Sea (Map 2). Between these trenches, high areas of bedrock reach elevations of approximately 400 meters below sea level.

The ice surface is much smoother than the bedrock. Near Byrd Station, the surface elevation is approximately 1500 meters above sea level, and assumes a convex upward elevation profile (Map 3). Approaching the Ross Ice Shelf, the surface elevation drops to 100 meters above sea level and the ice assumes a convex downward longitudinal profile. There are domes and ridges on the surface corresponding to the elevated areas of the bedrock.

Inland, the glacier may be described as having sheet flow, but near the ice shelf distinct ice streams appear. They are recognized as having higher than average velocities and by increased numbers of
crevasses. These may be described as glaciers within an ice sheet. The ice adopts the shape and profile it does as a response to the dynamic forces acting upon it, so a model incorporating these forces must explain the observed configuration and movement of the glacier.
V. Objective and General Idea

Any model of ice mass behavior must include a discussion of the relationships between various properties of the glacier with regard to the overall dynamic situation. One of these resolvable characteristics which may, to some extent, control or affect glacial dynamics and shape, is the presence of subglacial water. Any water under the ice mass will have the effect of lubricating the base, and of smoothing the apparent basal interface. This in turn affects other measurable or calculable elements of the glacier, such as basal shear stress, various velocity vectors, surface configuration, and ice thickness.

To describe the possible existence of accumulated or flowing subbasal fluid, the concept of subglacial hydrologic potential is defined and utilized. The potential of water beneath a glacier has two main components. One is the effect due to the elevation of the bedrock at any point, and the other term includes the contribution of the downward pressure of the ice column above that point (Fig. 1). Because of this, subglacial water need not flow down the elevation gradient, as subglacial water pressure may force the water up a basal slope.

The potential gradient is also useful in describing basal water flow. Potential gradient is the difference in hydrologic potential over a unit of distance. The higher the gradient, the more and/or faster any water present will flow. Areas of low potential gradient may conceivably be accumulating water in the form of subglacial lakes.
If the area to be studied is isopotentially contoured (Map 4), with respect to subglacial hydrologic potential, an idea as to the direction and velocity of basal water flow is obtained. The water will flow normal to the contours, from higher to lower values of hydrologic potential, and the water velocity will be directly proportional to the potential gradient, assuming constant permeability.

Within a specific area, known values of basal shear stress may be correlated with respect to hydrologic potential. In addition, the shape and thickness of the ice sheet can be related to basal water movement, and the relationship of basal sliding velocity to deformation velocity may be examined for possible correlations.
VI. Calculations of Hydrologic Potential and Correlative Methods

The subglacial hydrologic potential, used to describe the flow of water beneath an ice sheet, is calculated by using conventional physical relationships. The hydrologic potential of water at any point is given by:

\[ P_{\text{total}} = P_1 + P_2 \]  

where \( P_1 \) is the potential energy of water due to its elevation relative to a convenient datum, such as mean sea level, and \( P_2 \) is the water pressure due to the weight of the column of ice above it.

The potential energy of subglacial water is dependent upon its elevation above sea level, and this component is expressed by:

\[ P_1 = \rho w g (E_b) \]  

where \( \rho w \) is the density of water (1000 kg/m\(^3\)); \( g \) is the acceleration due to gravity, (9.8 m/s\(^2\)); and \( E_b \) is the bedrock elevation in meters. Using the MKS system of units, \( P_1 \) will be expressed in Joules per cubic meter, or Pascals.

If there is subbasal water present, its pressure, \( P_2 \), must be roughly equal to the weight of the overlying ice column. If the column weight were larger the water would be forced out. If the water pressure exactly balanced the ice weight, the glacier would be floating on the water layer.

The basal water pressure, \( P_2 \), is expected to be a little less than the weight of the ice, and the glacier is almost floating. In
places the glacier makes direct contact with the substrate, and it is there that the friction is manifest. Nye (1972) has discussed the water pressure at the base of a sliding glacier and it depends, in part, upon the basal shear stress; however, we assume here that the water pressure is less by a constant amount, $C$, from the ice overburden pressure.

$$P_2 = \rho_i g (E_s - E_b) - C$$  \hspace{1cm} (3)

where $\rho_i$ is the density of ice, 900 kg/m$^3$; and $E_s$ the ice surface elevation in meters; $P_2$ will be expressed in J/m$^3$, or Pascals.

Our interest here is in potential gradients, and in calculating gradients the quantity $C$ cancels out, and the value of $C$ is not important to the calculations, so for simplicity it is set equal to zero.

Combining the potential energy term with the pressure term:

$$P_T = \rho w g (E_b) + \rho_i g (E_s - E_b) - C.$$  \hspace{1cm} (4)

The values of $E_b$ and $E_s$ used in the calculation of $P_T$ were taken from contour maps. The region is contoured at intervals of 250 meters for $E_b$ and 100 meters for $E_s$, and the data are from radio echo profiling during the 1974-75 season by Rose. The elevations were obtained by subtracting the terrain clearance and basal reflections from the plane altitude.

In this work, the basal elevation values for points not on contour lines were taken to be the average of the adjacent contours. For example, any point between the -500 m. contour and the -750 m. contour was assigned a value of -625 meters. The values of $E_s$ were derived by the same method over the entire area, but within ice stream
A more complex method was used to minimize errors. The errors involved are discussed in a separate section.

Values of $P_T$ were calculated for a grid spacing of 25 km. The grid utilized in part the flight lines used in the radio echo profiling of Eb and Es. The results were contoured by hand and are illustrated in Map 4.

The water beneath a glacier will, for the most part, flow normal to the isopotential contours. The potential gradient of water flowing in this manner is given by:

$$\lim_{\Delta x \to 0} \left( \frac{P_{T1} - P_{T2}}{\Delta x} \right)$$

where $\Delta x$ is the measured linear separation of two points of hydrologic potential, $P_{T1}$ and $P_{T2}$. This gradient value is a measure of the tendency of subglacial water to flow.

There are at least three methods of quantitatively approximating the differential expression $dP/dX$. The first uses a constant value of $\Delta x$, which may be assigned according to the context wherein it will be used (Fig. 2). The values of $P_T$ at each end of a $dX$ line are determined, and the gradient is given by the expression:

$$\frac{dP_T}{dX} = \frac{|P_{T1} - P_{T2}|}{\Delta x}$$

The gradient is in bars per kilometer.

An alternate method is to measure the distance between isopotential contours each crossing a flow band and calculating $dP/dX$ by the expression

$$\frac{dP_T}{dX} = \frac{|P_{T4} - P_{T3}| + |P_{T2} - P_{T1}|}{2 \left[ X_4 - X_3 + X_2 - X_1 \right]}.$$
Shown in Figure 3, this method assumes a constant $\Delta P_T$, while varying the linear distance.

A third technique was developed for the area because of the scale and errors involved. Specific bands of variable widths and $\Delta P_T$ values are defined, and a semi-quantitative value of $dP/dX$ was calculated. The expression

\[
\frac{dP_T}{dX} = \frac{|P_{T2} - P_{T1}|}{X_2 - X_1}
\]  

shown graphically in Figure 4, approximates the gradient across the band. An advantage of this method is that only small errors are generated by spanning two gradient zones (Fig. 5).

Glacial properties which were examined for possible correlations to hydrologic potential were surface slope, basal shear stress, and balance velocity. These were all calculated along Ice Stream C in Marie Byrd Land, and graphed respective to linear distance from the ice divide.

The surface slope was determined from the surface elevation along the flow band. Shown graphically in Figure 7, it is an indication of the ice surface configuration as a result of forces which disrupt the glacial equilibria.

The basal shear stress was calculated by the use of the equation (Paterson, 1969)

\[
S_s = \rho ig (E_s - E_b) \sin \alpha
\]

where $\alpha$ is the surface slope, $S_s$ will be in units of $N \cdot m^{-2}$. 
The balance velocities \( q \) were found through the equation (Whillans, 1977)

\[
\frac{\sum w b \Delta x}{w z} = q \text{ (m} \cdot \text{a}^{-1})
\]  

(10)

where \( \bar{w} \) is the mean width of the flow band for a distance \( \Delta x \), \( b \) is the surface accretion in kg/m\(^2\)/a; \( w \) is the width of the flow band where \( q \) is calculated, and \( z \) is the thickness of the ice (m) (see Figure 10).

In addition, the relative contributions to total velocity by basal slippage and internal deformation were calculated (Paterson, 1969),

\[
q = \frac{2A (\rho i q) n (\sin \alpha)^n}{h (n + 1)} \int_0^h (h - y)^{n+1} dy + u_s
\]  

(11)

where \( A \) is a constant in the flow laws of the system; \( h \) is the ice thickness (m); \( y \) is the subsurface distance (here equal to \( h \)); and \( u_s \) is the basal sliding velocity (m \cdot a^{-1}).

The correlations were made along a well-defined flow band. The graphs were then analyzed for correlations, and interpretations of the results as they apply to a glacial flow model were made.
VII. Errors

The calculation of errors was accomplished by considering the reliability of the data and its importance to calculated quantities. The uncertainty in the elevation of the bedrock is not critical to the calculation of $P_T$. By using a mean value method of determining elevation an error of $\pm 125$ meters is realized. However, this maximum error within the calculation, acting alone, alters the calculated values of $P_T$ by only 1.0 bar.

Far more critical is the determination of the ice surface elevation. This was taken directly from the surface map which is contoured in 100 meter intervals (Rose, 1977). The absolute error using a mean value method is 4.0 bars. This method was used for the greater part of Marie Byrd Land to develop the contour map.

Within a flow band, the ice surface contour is very smooth (Whillans, 1979) and a linear interpolation method is possible. The surface elevations were determined to the nearest 10.0 meters, and a computer program was designed to calculate the $P_T$ values for 10 meter surface elevation intervals and 125 meter basal intervals.

The error in $\frac{dP}{dX}$ is given by the expression (Whillans, 1979)

$$e \frac{dP_T}{dX} = \pm \left( 2 \frac{eP_T}{\Delta x} + \frac{\Delta P_T}{\Delta x^2} \cdot e\Delta X \right)$$

where $eP_T$ is the absolute error in $P_T$, $\Delta x$ is the change in linear distance, $\Delta P_T$ is the change in potential, $e\Delta X$ is the linear measurement error.

The absolute error in $\frac{dP_T}{dX}$ was found to be about $\pm 0.068$ bar/Km, using the mean value method; using a 100 Km $\Delta X$ unit an error of $\pm$
0.021 bar/Km was calculated within the ice stream where linear interpolation of $E_s$ was used.

Accuracy might be further improved by using a more sensitive bed measurement, but this is complicated by the extreme roughness of the basal surface.

A possible source of error which has been neglected for the purposes of this study is the vectoral product of ice movement and subglacial flow. Comparison of the direction of ice flow (Fig. 11) and the direction of subglacial fluid movement reveals that two vectors are nearly parallel, and any errors introduced are small. Since the calculations of hydrologic potential, balance velocity, shear stress and surface slope were all based on the same elevation measurements, the accuracy ranges are compatible and acceptable within the scope of the investigation.

In addition to errors associated with data quality there are certain errors introduced by assumptions in the model used to calculate subglacial water flow. The model assumes the bedrock permeability to be constant throughout the area, when in fact the area bordering the Ross Ice Shelf is suggested to be an extensive sedimentary basin, while the domes and ridges are granitic intrusives (Rose). Possible basal substructure is also ignored. Faults, joints and other structural controls could cause alteration of the drainage characteristics as could influx of ground water from another source.
VIII. Results

Comparison of data along the flow band leading to Ice Stream C shows correlations between basal water quantity and flow to glacial movement and shape. These correlations show the importance of basal fluid to the mechanism of sheet ice flow and indicate the controlling influence of subglacial water on the glacier.

Comparison of the graphs of shear stress and hydrologic potential (figures 6, 8), indicate that basal fluid controls the shear stress by altering the ice-water-rock interface characteristics. A high $dP/dX$ value is reflected by a large shear stress value; indicative of increased bed resistance as a result of increased water drainage. The explanation of the lowered magnitude of the peak at Km. 610 may be explained by looking at Figure 9, the plot of basal slip percentage. This shows the ice to be almost entirely afloat at this point, and the effect due to increased drainage would be minimized.

The graph (Figure 7) of surface slope values shows an increase in slope where high potential gradient values are encountered, indicating the glacier is maintaining equilibrium at that point. The equation of surface slope angle is very similar to the basal shear stress formula, so it may be seen from equation 9 that an increase in shear stress due to water drainage must be accompanied by a thickening of the ice, or an increase in the surface slope.

The glacier exhibits multiple flow modes, changing from wet to semi-wet based and back again. On a smaller scale, there is variation due to bed configuration. The shear stress increases with larger obstacle size, but only to a maximum threshold value. When the obstacle
encountered is larger than this value, the ice adopts a different flow mode, and the shear stress drops (Figure 13).

There is a strong possibility of subglacial water accumulation in the area seaward of Km. 610, due to a combination of factors. Here, the hydrologic potential gradient is low, indicating slow drainage from the area. The shear stress values are also low, suggesting low bed friction, and the velocity values increase rapidly.

Radar echo profiles have not indicated the presence of these lakes, which may be on the order of tens of meters across. The radio echo method of recognizing these lakes as suggested by Drewry (1977), and others may be invalid in that the ice-water interface is probably not planar. The ice, moving over a lake after encountering an obstacle on the shore of the lake, will retain the shape of the lake surface (Figure 12). These nonplanar interfaces may well be indistinguishable from ice-rock interfaces by present methods.
IX. Conclusion

Subglacial water seems to have an appreciable effect upon the flow mechanism of ice sheets. The correlations of hydrologic potential gradient to basal shear stress and to surface slope are clear, but the comparison of potential gradient to balance velocity is not what was expected.

One possible reason the balance velocity does not relate well to the basal water movement is that the water provides lubrication, but may only increase the velocity up to a certain threshold value. Any additional water accumulation will not result in increased ice velocity (Figure 13). In this case, the entire flow band may be above the limiting value.

Another explanation may be that there is very little, if any, water accumulation subglacially, and any water produced is immediately drained away so as not to affect ice velocity. A problem with this hypothesis is that examination of Figure 9 shows that much of ice stream C is moving by basal slippage, indicating the presence of at least some basal fluid.

The concept of hydrologic control of ice sheet dynamics is relevant to various facets of glaciology. The determination of water velocity and direction may be important to the investigation of thermal gradients within the ice. The water could conceivably alter the geothermal heat flux into the ice from the bedrock. Water under the ice would also alter its erosive power enormously. Thus, there is a distinct geomorphic effect related to subglacial water flow.
X. References


Rose, K.E. (nd). 'Radio Echo Studies of Bedrock in Marie Byrd Land, Antarctica.' (unpublished)


Figure 1.

SCHEMATIC REPRESENTATION OF SUBGLACIAL HYDROLOGIC POTENTIAL
Figure 2.

CONSTANT ΔX METHOD OF CALCULATING POTENTIAL GRADIENTS.
Figure 3.

CONSTANT ΔP, METHOD OF CALCULATING POTENTIAL GRADIENTS.
FIGURE 4.

REGIONAL METHOD OF CALCULATING SUBGLACIAL POTENTIAL GRADIENTS.
EXAMPLE OF ERRORS GENERATED BY CROSSING GRADIENT ZONES.
Figure 6.

SUBGLACIAL HYDROLOGIC POTENTIAL GRADIENT VS. DISTANCE FROM ICE DIVIDE  FLOW BAND C
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SURFACE SLOPE VS. DISTANCE FROM ICE DIVIDE

FLOW BAND C
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BASAL SHEAR STRESS VS. DISTANCE FROM ICE DIVIDE

FLOW BAND C
Figure 9.

BASAL SLIP PERCENTAGE VS. DISTANCE FROM ICE DIVIDE

FLOW BAND C
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BALANCE VELOCITY VS. DISTANCE FROM ICE DIVIDE

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SUMMATION OF MOTION VECTORS OF SUBGLACIAL WATER AND ICE FLOW.
Figure 12.

NON PLANAR ICE TO WATER INTERFACE MODEL.
Figure 13.

BASAL OBSTACLE SIZE VS. BASAL SHEAR STRESS