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## Constructing tensegrity frameworks and related applications in multi-agent formation control

Yang, Qingkai

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**Constructing Tensegrity Frameworks  
and Related Applications in  
Multi-Agent Formation Control**

Qingkai Yang

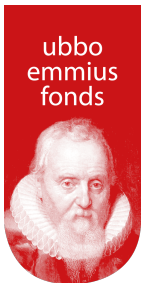


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The research described in this dissertation has been carried out at the Faculty of Science and Engineering, University of Groningen, The Netherlands, within collaboration between the Engineering and Technology Institute Groningen (ENTEG) and the School of Automation, Beijing Institute of Technology (BIT), China.

# disc

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# Constructing Tensegrity Frameworks and Related Applications in Multi-Agent Formation Control

PhD thesis

to obtain the degree of PhD at the  
University of Groningen  
on the authority of the  
Rector Magnificus Prof. E. Sterken  
and in accordance with  
the decision by the College of Deans.

This thesis will be defended in public on

Friday 25 May 2018 at 11.00 hours

by

**Qingkai Yang**

born on 3 August 1988  
in Hebei, China

**Supervisors**

Prof. M. Cao

Prof. J.M.A. Scherpen

**Assessment Committee**

Prof. H.L. Trentelman

Prof. Z. Lin

Prof. H.S. Ahn

*To my wife, Yuhan Wang, and our family without whose supports this thesis would never have been written.*

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---

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Groningen

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<sup>1</sup>The majority of this section is taken from one of Prof. Brian D. O. Anderson's unpublished technical reports. For completeness of the strategy for growing locally rigid tensegrity frameworks, we put it in this thesis.

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## List of symbols

|   |  |    |
|---|--|----|
| $\mathbb{R}^n$                          | $n$ -dimensional Euclidean space .....   | 9  |
| $\mathbb{R}^{m \times n}$               | the set of $m \times n$ real matrices .....  | 9  |
| $I_n$                                   | identity matrix with dimension $n$ .....   | 9  |
| $\mathbf{1}_n$                          | $n$ -dimensional column vector with all ones .....                                 | 9  |
| $\mathbf{0}_n$                          | $n$ -dimensional column vector with all zeros .....                                | 9  |
| $X^\top$                                | transpose of matrix $X$ .....  | 9  |
| $\text{rank}(X)$                        | rank of matrix $X$ .....   | 9  |
| $\text{col}(X)$                         | column space (i.e., image) of matrix $X$ .....                                     | 9  |
| $\text{null}(X)$                        | null space of matrix $X$ .....   | 9  |
| $\det(X)$                               | determinant of matrix $X$ .....  | 9  |
| $\text{sign}(\cdot)$                    | signum function .....  | 9  |
| $\ x\ $                                 | Euclidean norm of vector $x$ .....   | 9  |
| $\text{diag}(x)$                        | diagonal matrix with the vector $x$ on its diagonal .....                          | 9  |
| $\text{span}(v)$                        | linear span of $v$ .....   | 9  |
| $ \mathcal{S} $                         | cardinality of set $\mathcal{S}$ .....   | 9  |
| $A \otimes B$                           | Kronecker product of two matrices $A$ and $B$ .....                                | 9  |
| $\mathcal{G}(\mathcal{V}, \mathcal{E})$ | graph $\mathcal{G}$ with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$ ..... | 10 |
| $A$                                     | adjacency matrix .....   | 10 |
| $L$                                     | Laplacian matrix .....   | 10 |
| $H$                                     | incidence matrix .....   | 10 |
| $\mathcal{A}(q)$                        | affine transformation of $q$ .....   | 11 |
| $R(q)$                                  | rigidity matrix .....  | 12 |
| $\omega$                                | stress .....   | 16 |
| $\Omega$                                | stress matrix .....  | 16 |

