2009 International Nuclear Atlantic Conference – INAC 2009
Rio de Janeiro, Rj, Brazil, September 27 to October 2, 2009
Associação Brasileira de Energia Nuclear – ABEN
ISBN: 978-85-99141-03-8

PARTICLE TRAJECTORY TRACING FOR ELECTROSTATIC AND MAGNETOSTATIC FIELDS.

Mairo Cunha de Carvalho[1] and Reinaldo J. Jospin[1]
[1]Instituto de Engenharia Nuclear (IEN / CNEN - RJ)
Rua Hélio de Almeida n. 75 – Cid. Universitária – Ilha do Fundão – Cx.P: 68.550
21945-970 Rio de Janeiro, RJ
mairocc@gmail.com
rj.jospin@ien.gov.br

ABSTRACT

This work reports a numerical method for single charged particle trajectories computation in 2D electrostatic and magnetostatic stationary fields, in other words, fields that do not change in time. This is approached by the finite element method domain discretisation, and numerical computation of particle trajectory, calculated by the two step centred in time method, which calculates the particle position on the next step using a dummy step in order to increase the accuracy for the same step size. Given particle's coordinates, the finite element that contains that particle is found based on Lohner's algorithm. The examples used to test the method are a electric deflector for the electric case and cyclotron for the magnetic case. Both are very important devices to science and technology, being used in a variety of domestic and industrial appliances and in several scientific and technologic researches. Other particle optics devices can benefit of the method proposed in this paper, as beam bending devices and spectrometers, among others. This method can be easily extended for particle trajectories computation in 3D domains, can be extended also for dynamic fields and for the relativistic case, which is ideal for the typical speed involved when working with particles near the atomic level.

1. INTRODUCTION

This work presents the two step centred time method to calculate the trajectory described by charged particles under the influence of an electric or magnetic field. These fields are calculated using the finite element method [2] with first order triangular elements that leads to the discrete formulation of the corresponding Maxwell differential equation [3]. The field values in the 2D cartesian referential systems are obtained using the Lohner's algorithm [1]. The method is a numeric one that was chosen inspired on the Yasmara De Poli work [4].

The method presented here can calculate particle trajectories, and other variables, like momentum and speed with a good precision. To ascertain this paper, it will be compared with the already tested analytical method [5].

In section 2, the method will be presented with it's algorithm and equations for each case studied. The results obtained in the numerical examples will be compared with those obtained with the analytical method [5]. In section 3, these results will be commented and some future work will be enhanced.

2. ALGORITHM AND RESULTS

In this chapter the two step centred in time method [6] is explained and the numerical results are presented in some test cases.

2.1. Algorithm

The two steps centred in time method follows a simple principle, very similar to the Euler method [6]. Knowing the starting point and the equation of the trajectory tangent in any point, the trajectory can be approached by a succession of straight lines, each one leading to a new point, where another tangent value will be used etc.

The starting point is the starting position of the particle and the equation for the tangent is an equation for the speed of the particle in any point. The position is given and the speed is achieved using the given force field value and the particle properties.

For the calculations to become faster and to require a lesser computational effort, the momentum equation is used instead of speed.

With the starting position and the starting momentum, a dummy step will be calculated at a step of $t/2$, where a new momentum will be assigned and used to calculate the next position. This new position will be plotted in the graph, while the half step will be ignored, being used just to increase the precision of the method.

2.2. Equations for the Method

The formulation of this method is similar for both electric and magnetic cases. The essential differences like the speed dependence and the tendency of not changing the absolute momentum value that occurs in the magnetic case is not present in the electric case.

2.2.1. Electric case

For the electric case, each direction's movement is independent to the others, the following equations are used for each one of them in each step. Dummy steps are used but only the real steps are kept.

The classical value of force is given below [7]:

$$\boldsymbol{F} = q\,\boldsymbol{E} \tag{1}$$

where q the charge value and E the electric field.

From (1), the momentum is calculated as:

$$\boldsymbol{p}=\boldsymbol{p}_0+q\,\boldsymbol{E}\,t \tag{2}$$

where *p is* the momentum and *t* is the time step length for the presented method.
One defines a step sufficiently small so that the momentum can be considered constant, so, the position with respect to the momentum is:

$$\boldsymbol{r}=\boldsymbol{r}_0+\frac{\boldsymbol{p}}{m}t \tag{3}$$

for a particle mass *m.*

2.2.2. Magnetic case

In the magnetic case, the force field will affect the direction of the particle trajectory. That means, each component of the momentum will interact with the force applied to the other particles [7]:

$$\boldsymbol{F}=q\,\boldsymbol{v}\times\boldsymbol{B} \tag{4}$$

for a particle with speed *v* under influence of a magnetic field *B*.

From the force, the momentum can be deduced [4]

$$\boldsymbol{p}_x=\boldsymbol{p}_0+\frac{q}{c}\left[\frac{\boldsymbol{p}_{0y}}{\gamma m}\boldsymbol{B}_z-\frac{\boldsymbol{p}_{0z}}{\gamma m}\boldsymbol{B}_y\right]t \tag{5}$$

$$\boldsymbol{p}_y=\boldsymbol{p}_0+\frac{q}{c}\left[\frac{\boldsymbol{p}_{0z}}{\gamma m}\boldsymbol{B}_x-\frac{\boldsymbol{p}_{0x}}{\gamma m}\boldsymbol{B}_z\right]t \tag{6}$$

$$\boldsymbol{p}_z=\boldsymbol{p}_0+\frac{q}{c}\left[\frac{\boldsymbol{p}_{0y}}{\gamma m}\boldsymbol{B}_x-\frac{\boldsymbol{p}_{0x}}{\gamma m}\boldsymbol{B}_y\right]t \tag{7}$$

Where γ is the Lorentz transformation factor.

The z-axis momentum equation is used in this formulation to show how easy the method can be converted to a 3D environment. In the 2D case, the same equation can be used, since the z-momentum component is considered null.

Dependence between different components may be noticed, as it was said before. Again, the momentum can be considered constant for small steps, and the position for any component will be:

$$\boldsymbol{r}=\boldsymbol{r}_0+\frac{\boldsymbol{p}}{m}t \tag{8}$$

2.3. Results

In the following, we present the results obtained with the two steps centred in time method, for the electric and magnetic cases and the comparison between them and results obtained through the analytical method.

2.3.1. Electric case

An example of electrostatic deflector was used in order to test the method in the electrostatic case. Electrostatic deflectors are charged structures inside which an electrostatic field will interact with the charged particles that pass through there, increasing the momentum component orthogonal to the walls of the deflector.

The studied case is a simplest rectangular two-dimensional deflector. The difference of potential in the walls of this deflector is $5000V$, the lower and upper walls are $0.02m$ distant from each other. The particle within is an electron and the positively charged wall is the lower one.

Fig. 1 was plotted from the analytical method while Fig. 2. and Fig. 3. were plotted from the two steps centred in time method.

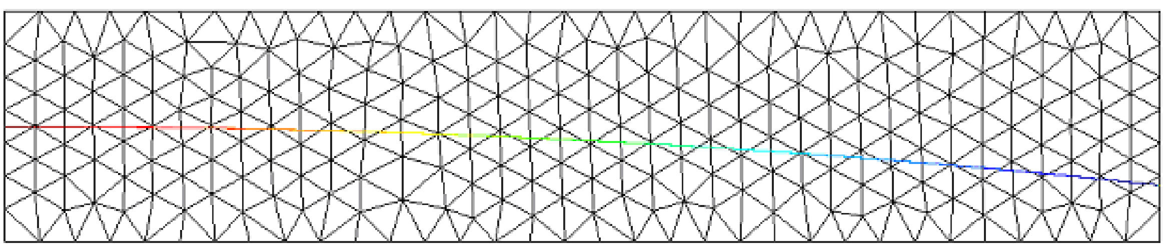

Figure 1. Trajectory of a charged particle in a deflector plotted from the analytical method

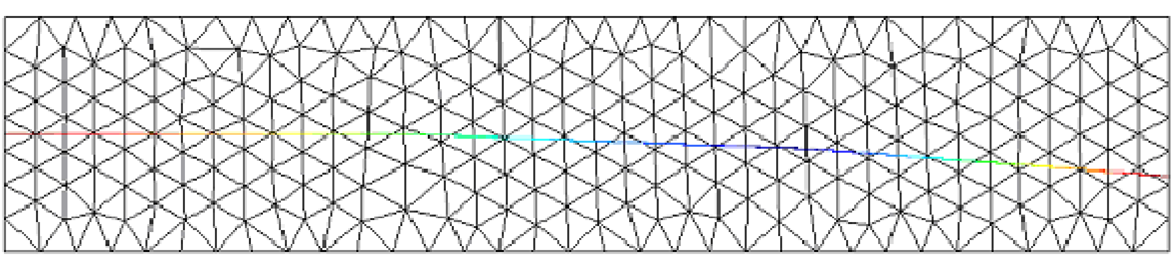

Figure 2. Trajectory of a charged particle in a deflector plotted form the two steps centred in time method with a small step size

Comparing Fig. 1 and Fig. 2 a significant difference in the particle deflection can be seen. A sensitive part of this difference is due to the numeric instability that is bigger when the step size increases. In this example the effects of the instability do not cause major problems, this is because the trajectory is short enough so the deviation is not so intense.

To solve this problem the step size must be decreased. In this case, the precision of the method will increase. There is an ideal step size so that the precision is good and the numerical instability is not significant. This size tends to be as the same order so that to reach the required precision. Fig. 3 shows the trajectory plotted for such step size.

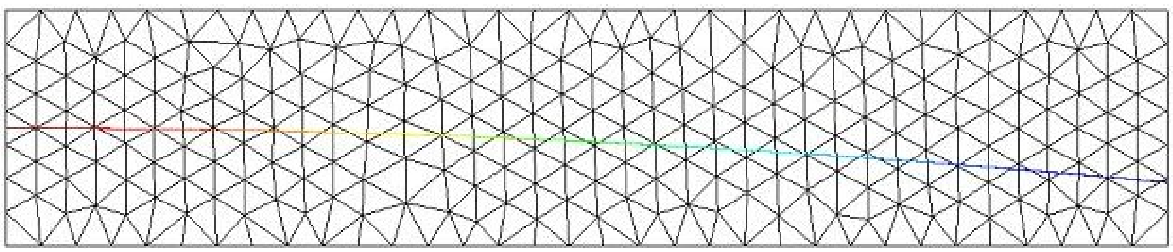

Figure 3. Trajectory of a charged particle in a deflector plotted form the two steps centred in time method with a proper step size

Comparing Fig. 1 and Fig. 3 the difference perceived in Fig. 2. is largely vanished, suggesting the two methods exhibit a good agreement. In order to check the accordance, follows some numerical results.

Table 1. Numerical results to the deflector case

Method	Deflection
Analytical	$4.90271.10^{-3}$
Time Centred Two Steps	$4.68434.10^{-3}$

Table 1 compares the results of each method. In the simulation, a 478 elements and 280 nodes mesh was used. The difference in the results is small, so both methods have a good agreement.

2.3.2. Magnetic case

To test the two steps centred in time method for this case a cyclotron was used as an example. A cyclotron is a structure that causes charged particles to describe a trajectory that resembles a circumference under the influence of a magnetic field, making possible to keep the particle accelerating for a long time. For it is constrained in a limited area in a

predictable trajectory. Being so, the compelling potential can be applied only where the particle is, saving space and increasing the efficiency of the compelling force.
Fig. 4 was plotted from the analytical method while Fig. 5 was plotted from the two steps centred in time method.

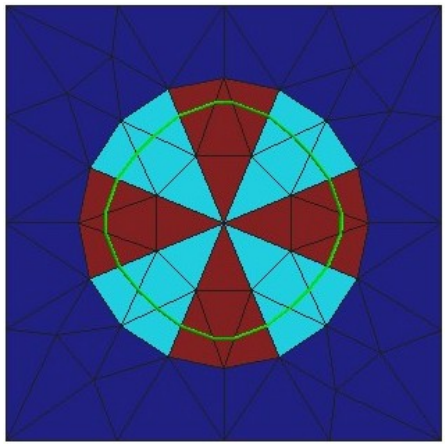

Figure 4. Trajectory of a charged particle in a cyclotron plotted from the analytical method.

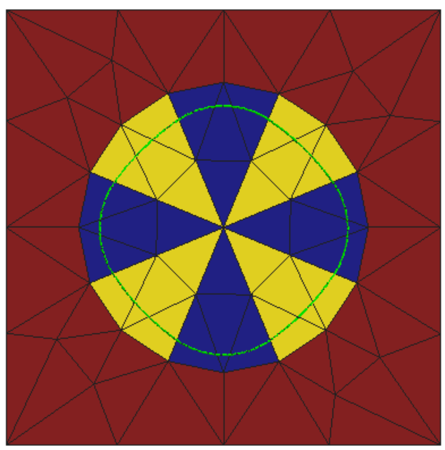

Figure 5. Trajectory of a charged particle in a cyclotron plotted from the two steps centred in time method with the proper step size.

Comparing Fig. 4 and Fig. 5 a good agreement can be noticed. The curve is approximately the same.

3. CONCLUSIONS

The two steps centred in time method presents a good approach to particle trajectories like the analytical method. The good precision of the results allows this numerical method to be applied so that the analytical method was used with approximately the same results.

As shown in section 2.2., the method is easily converted to a 3D environment as a possible and interesting application for future work. Another possible application is to convert both cases to the relativistic particles case.

ACKNOWLEDGEMENTS

We acknowledge CNPQ for the financial support provided, Domingos Eugênio de Sá Nery for the help in this paper and Altivo Monteiro and Rafael Araujo for the support at the development of this work.

REFERENCES

1.J. Peraire, M. Vahdati, K. Morgan, O. C. Zienkiewicz, "Adaptative remeshing for compressible flow computation", *J.Comput. Phys.*, **Vol. 72**, pp. 449-466 (1987).

2.G. Dhatt, G. Touzot, *Une Présentation de la Méthode des Éléments Finis*, Maloine S. A. Éditeur, Paris, France (1981).

3.S. J. Salon, *Finite Element Analysis of Electrical Machines*, Kluwer Academic Publishers, Boston, MA (1995).

4.Y. C. Polli; *Cálculos de Trajetórias de Elétrons em Estruturas Magnéticas*, Tese de Mestrado, Universidade de São Paulo, 1999.

5.C. A. F. Jorge, R. J. Jospin, J. A. D. Furlanetto, "Particle Trajectory Tracing and Computational Electromagnetics for Accelerator and Related Technologies", *Journal of Microwaves and Optoelectronics*, **vol. 6**, number 1, pp. 278-294, http://www.sel.eesc.usp.br/jmo/issues/vol_6/v6_n1/v6_n1_paper_pdf/momag2006/V6n1a23.pdf (2006).

6.W. H. Press, B. P. Flannery, S. A. Teulousky, W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing*, Cambridge University Press, United States of America (1987).

7.J. P. A. Bastos, Eletromagnetismo para Engenharia: Estática e Quase-Estática, Florianópolis: Editora da UFSC, 2004.