Numerical Solutions on Mixed Convection Boundary Layer and Heat Transfer of Jeffrey Fluid over a Horizontal Circular Cylinder by using Keller-box Method

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Abstract— This article discusses the numerical solutions on the problem of mixed convection boundary layer and heat transfer of Jeffrey fluid which passing over a horizontal circular cylinder. The governing boundary layer equations are first transformed into a system of non-dimensional equations using the non-dimensional variables, and then into non-similar equations before solving numerically using an implicit finite-difference scheme known as Keller-box method. Numerical solutions are obtained for the velocity and temperature profiles as well as the reduced Nusselt number and skin friction coefficient for various values of mixed convection parameter and Prandtl number, respectively.

Keywords— mixed convection boundary layer; Jeffrey fluid; horizontal circular cylinder

1. Introduction

Recently, the convection boundary layer flow and heat transfer has engrossed the attention of researchers worldwide. This attentiveness stems from the broad industrial and technological applications of non-Newtonian fluids especially in various branches of science, engineering and technology, particularly in material processing, chemical and petroleum industries, geophysics as well as biological sciences [1]. The governing equations of non-Newtonian fluid are more complex as compared to the Navier–Stokes equations which represent the Newtonian fluid. As with numerous versatile in nature of non-Newtonian fluids models developed [2-4], the Jeffrey fluid model has founded quite relevant. This sophisticated rheological model was presented primarily by Jeffrey [5] to simulate the problems of earth crustal flow. Jeffrey fluid has been considered lately due to its ability of describing the features of relaxation and retardation times.

For instance, Prasad et al. [6] studied the boundary-layer flow and heat transfer of an incompressible Jeffrey's viscoelastic fluid from a permeable horizontal circular cylinder. The graphical results show that the velocity and temperature profiles are decreased with an increase in Prandtl number. Next, the effect of heat generation/absorption on magnetohydrodynamic (MHD) of Jeffrey fluid is discussed by Kasim et al. [7] via numerical approach called Keller-box method. Also, the variation of skin friction coefficient is discussed for various values of Prandtl number. Further, Prasad et al. [8] extended his study on heat and mass transfer from a permeable horizontal isothermal cylinder in a non-Darcy porous medium for simulating the effects of linear porous media drag and second-order Forchheimer drag. In recent times, Gaffar et al. [9] considered the Buoyancy-driven convective heat and mass transfer in boundary layer flow of a viscoelastic Jeffrey fluid from a permeable isothermal sphere embedded in a porous medium. The effects of thermal radiation and heat generation/absorption effect are employed in this model.

According to the literature surveys, the viscous dissipation effects in free convection flow is primarily studied by Gebhart [10]. This effect is then applied in unsteady case for free convection flow by Soundalgekar [11]. Later on, Vajravelu and Hadjinicolaou [12] carried out a study on the flow and heat transfer over stretching sheet by implementing viscous dissipations effect. Meanwhile, the MHD free convection from a vertical surface with Ohmic heating and viscous dissipation is studied by Chen [13] while Partha et al. [14] considered the effect of viscous dissipation from an exponentially stretching surface. Further, Yirga and Shankar [15] present the effects of thermal radiation and viscous dissipation in MHD stagnation point flow. Very recently, Mohamed et al. [16, 17] implement the effect of viscous dissipation both in free and mixed convection boundary layer flow over a horizontal circular cylinder. Therefore, it is worth mentioning that the viscous dissipation is also known as an internal friction. Physically, viscous dissipation happens due to the friction between fluid particles, thus can be regarded as a source of heat [18]. Its presence can be noticed when the induced kinetic energy is more significant in comparison to the amount of heat transferred [19].

Motivated by the above mentioned literature, the aim of this paper is to carry out a numerical study on the mixed convection boundary layer flow and heat transfer over a horizontal circular cylinder towards one type of non-Newtonian fluid, i.e. Jeffrey fluid with viscous dissipation effect by using Keller-box method. To the best of authors' knowledge, no one has conducted a study of Jeffrey fluid flow over a horizontal circular cylinder with viscous dissipation effect, so the reported results are considered new.

2. MATHEMATICAL FORMULATION

The horizontal circular cylinder of radius a, which is heated to a constant temperature T_w embedded in a Jeffrey fluid with ambient temperature T_∞ as shown in Fig. 1. The coordinates of $\bar{x}$ and $\bar{y}$ are measured along the cylinder surface, starting with the lower stagnation point $\bar{x}=0$, and normal to it, respectively. Under the assumptions that the boundary layer approximations is valid, the dimensional governing equations of steady two-dimensional mixed convection boundary layer flow and heat transfer in the presence of viscous dissipation are ([20],[21], [22]):

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\bar{u}_e\frac{d\bar{u}_e}{d\bar{x}}+\frac{\nu}{1+\lambda}\left[\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\lambda_1\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2}+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right)\right]+g\beta(T-T_\infty)\sin\frac{\bar{x}}{a}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2, \tag{3}$$

subject to the boundary conditions [23]

$$\begin{aligned}&\bar{u}(\bar{x},0)=\bar{v}(\bar{x},0)=0,\ T(\bar{x},0)=T_w \ \text{ at } \ \bar{y}=0\\&\bar{u}(\bar{x},\infty)\to\bar{u}_e,\ \bar{v}(\bar{x},\infty)\to 0,\ T(\bar{x},\infty)\to T_\infty \ \text{ as } \ \bar{y}\to\infty\end{aligned} \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively. μ is the dynamic viscosity, ν is the kinematic viscosity, λ is the ratio of relaxation and retardation times, λ_2 is the relaxation time, g is the gravity acceleration, α is the thermal diffusivity, β is the thermal expansion, T is the local temperature, ρ is the fluid density and C_p is the specific heat capacity at a constant pressure. Further, $\bar{u}_e(x)$ is given by

$$\bar{u}_e(x)=U_\infty\sin\left(\frac{\bar{x}}{a}\right) \tag{5}$$

The governing non-dimensional variables are introduced as follows:

$$x=\frac{\bar{x}}{a},\quad y=\mathrm{Re}^{1/2}\frac{\bar{y}}{a},\quad u=\frac{\bar{u}}{U_\infty},\ v=\mathrm{Re}^{1/2}\frac{\bar{v}}{U_\infty},\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad u_e(x)=\frac{\bar{u}_e(x)}{U_\infty}. \tag{6}$$

Using non-dimensional variables of (6), (1) - (3) become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{1}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right]+\gamma\theta\sin x, \tag{8}$$

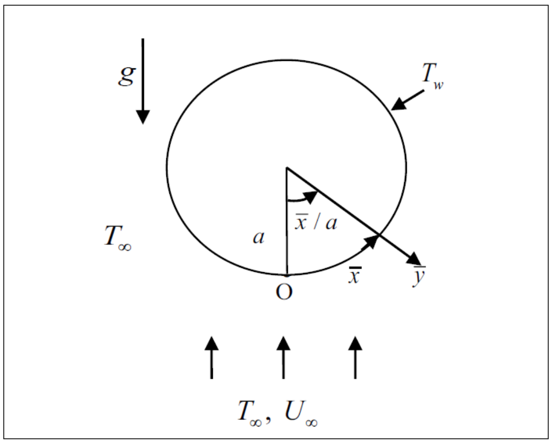

Figure 1: Physical model of the coordinate system

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2, \tag{9}$$

subject to the boundary conditions

$$\begin{aligned}&u(x,0)=0,\ v(x,0)=0,\ \theta(x,0)=1 \text{ at } y=0\\&u(x,\infty)\to u_e,\ v(x,\infty)\to 0,\ \theta(x,\infty)\to 0 \text{ as } y\to\infty\end{aligned} \tag{10}$$

where $\Pr=\dfrac{\mu}{\alpha\rho}$ is the Prandtl number, $\lambda_2=\dfrac{\lambda_1 U_\infty}{a}$ is Deborah number, $Ec=\dfrac{U_\infty{}^2}{C_p\left(T_w-T_\infty\right)}$ is an Eckert number. Note that γ is mixed convection parameter which is given by $\gamma=\dfrac{Gr}{\mathrm{Re}^2}$ where $\mathrm{Re}=\dfrac{U_\infty a}{\nu}$ is Reynolds number and $Gr=\dfrac{g\beta(T_w-T_\infty)a^3}{\nu^2}$ is the Grashof number. In order to solve (7) - (9), we seek a solution by following Merkin [24], which represent

$$\psi=xf(x,y),\qquad \theta=\theta(x,y), \tag{11}$$

Where ψ is the stream function defined as $u=\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{\partial\psi}{\partial x}$ which identically satisfies (7) and θ is the rescaled dimensionless temperature of the fluid. Substitute (11) into (7)-(9), then the subsequent partial differential equations are attained

$$\begin{aligned}&\frac{1}{1+\lambda}\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x}{x}\left(\gamma\theta+\cos x\right)+\frac{\lambda_2}{1+\lambda}\left[\left(\frac{\partial^2 f}{\partial y^2}\right)^2-f\frac{\partial^4 f}{\partial y^4}\right]=\\&x\left[\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{\lambda_2}{1+\lambda}\left(\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}-\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3}+\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}-\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}\right)\right],\end{aligned} \tag{12}$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+f\frac{\partial\theta}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}-xEc\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right), \tag{13}$$

with the boundary conditions

$$\begin{aligned}&f(x,0)=0,\ \ \frac{\partial f}{\partial y}(x,0)=0,\ \ \theta(x,0)=1 \text{ at } y=0\\&\frac{\partial f}{\partial y}(x,\infty)\to\frac{\sin x}{x},\ \ \frac{\partial^2 f}{\partial y^2}(x,\infty)\to 0,\ \theta(x,\infty)\to 0 \text{ as } y=\infty\end{aligned} \tag{14}$$

It can be easily presented that the position, $x\approx 0$, corresponds to the lower stagnation point of the cylinder. Therefore, $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=1$. For this situation, the model defined by (12) and (13) are reduced to an ordinary differential boundary value problem:

$$\frac{1}{1+\lambda}\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\gamma\theta+1+\frac{\lambda_2}{1+\lambda}\left[\left(\frac{\partial^2 f}{\partial y^2}\right)^2-f\frac{\partial^4 f}{\partial y^4}\right]=0 \tag{15}$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+f\frac{\partial\theta}{\partial y}=0 \tag{16}$$

with the boundary conditions:

$$\begin{aligned}&f(x,0)=0,\quad \frac{\partial f}{\partial y}(x,0)=0,\quad \theta(x,0)=1 \ \text{ at } \ y=0\\&\frac{\partial f}{\partial y}(x,\infty)\to 1,\quad \frac{\partial^2 f}{\partial y^2}(x,\infty)\to 0,\quad \theta(x,\infty)\to 0 \ \text{ as } \ y\to\infty\end{aligned} \tag{17}$$

In practical applications, the physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x [25] are:

$$C_f=\frac{\tau_w}{\rho U_\infty^2},\qquad Nu_x=\frac{aq_w}{k(T_w-T_\infty)}. \tag{18}$$

The surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w=\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},\quad q_w=-k\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0} \tag{19}$$

with $\mu=\rho\nu$ and k being the dynamic viscosity and the thermal conductivity, respectively. Using (11) and (19), the reduced skin friction, $C_f\,\mathrm{Re}_x^{1/2}$ and reduced Nusselt number, $Nu_x\,\mathrm{Re}_x^{-1/2}$ are:

$$C_f\,\mathrm{Re}_x^{1/2}=x\frac{\partial^2 f}{\partial y^2}(x,0)\quad\text{and}\quad Nu_x\,\mathrm{Re}^{-1/2}=-\frac{\partial\theta}{\partial y}(x,0) \tag{20}$$

Furthermore, the velocity profiles and temperature distributions can be obtained from the following relations:

$$u=f'(x,y),\qquad \theta=\theta(x,y). \tag{21}$$

3. METHODOLOGY

The nonlinear boundary value problem defined by (12) and (13) with boundary conditions (14) are solved using the Keller-box method. This method has been applied successfully in many rheological flow problems in recent years and thus, it is regarded as one of the powerful numerical methods for fluid mechanics due to its unconditionally stable and outstanding accuracy. The Keller box scheme consists of four (4) steps:

I. First Order System
II. Finite Difference Scheme,
III. Newton Method,
IV. Block Tridiagonal Elimination Technique.

I. First Order System

Firstly, the new dependent variables is introduced in order to transform (12) and (13) to a system of first order differential equations.

$$\begin{aligned}&u(x,y)=f',\quad v(x,y)=f'',\quad p(x,y)=f''',\\&s(x,0)=\theta \text{ and } \theta'=t.\end{aligned} \tag{22}$$

Next, (12) and (13) are solved as a set of seven simultaneous differential equations:

$$f'=u \tag{23a}$$

$$u'=v \tag{23b}$$

$$v'=p \tag{23c}$$

$$s'=t \tag{23d}$$

$$\begin{aligned}&\frac{1}{1+\lambda}v'+fv-u^2+\frac{\sin x}{x}(\gamma s+\cos x)+\frac{\lambda_2}{1+\lambda}\left[v^2-fp'\right]=\\&x\left[u\frac{\partial u}{\partial x}-v\frac{\partial f}{\partial x}+\frac{\lambda_2}{1+\lambda}\left(p'\frac{\partial f}{\partial x}-u\frac{\partial p}{\partial x}+p\frac{\partial u}{\partial x}-v\frac{\partial v}{\partial x}\right)\right],\end{aligned} \tag{23e}$$

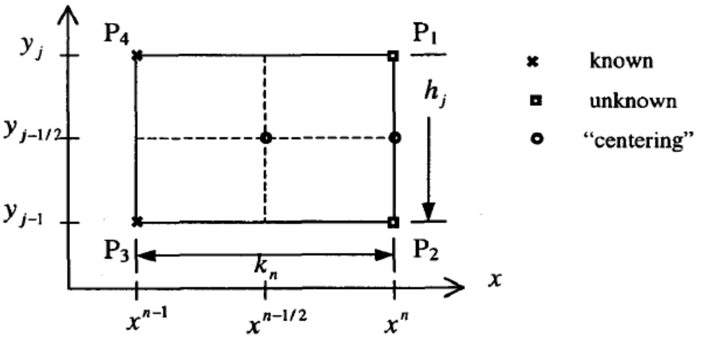

Figure 2: The net rectangle in the $x-y$ plane

$$\frac{1}{\Pr}t' + ft = x\left(u\frac{\partial s}{\partial x} - t\frac{\partial f}{\partial x} - xEcv^2\right), \tag{23f}$$

where primes denote differentiation with respect to y. In terms of new dependent variables, the boundary conditions become

$$\begin{aligned} &f(x,0)=0, \quad u(x,0)=0, \quad s(x,0)=1, \\ &u(x,\infty)\to\frac{\sin x}{x}, \; v(x,\infty)\to 0, \; s(x,\infty)\to 0. \end{aligned} \tag{24}$$

II. Finite Difference Method

Fig. 2 displays the net rectangle employed in the $x-y$ plane and the net points are denoted by:

$$x^0 = 0, \; x^n = x^{n-1} + k_n, \; n = 1,2,...N, \tag{25}$$

$$y^0 = 0, \; y_j = y_{j-1} + h_j, \; j = 1,2,...J, \; y_j \equiv y_\infty \tag{26}$$

where k_n is the Δx - spacing and h_j is the Δy -spacing. Note that n and j are just sequence numbers that indicate the coordinate location, not tensor indices or exponents.

The quantities (f,u,v,p,s,t) are approximated at points (x^n, y_j) of the net by $(f_j^{\,n}, u_j^{\,n}, v_j^{\,n}, p_j^{\,n}, s_j^{\,n}, t_j^{\,n})$. The notation $(\;)_j^n$ for points and quantities midway between net points and for any net function are:

$$x^{n-1/2} = \frac{1}{2}\left(x^n + x^{n-1}\right), \tag{27a}$$

$$y_{j-1/2} = \frac{1}{2}\left(y_j + y_{j-1}\right), \tag{27b}$$

$$(\;)_j^{n-1/2} = \frac{1}{2}\left[(\;)_j^n + (\;)_j^{n-1}\right], \tag{27c}$$

$$(\;)_{j-1/2}^{n} = \frac{1}{2}\left[(\;)_j^n + (\;)_j^{n-1}\right], \tag{27d}$$

$$\left(\frac{\partial(\;)}{\partial x}\right)_{j-1/2}^{n-1/2} = \frac{(\;)_{j-1/2}^n - (\;)_{j-1/2}^{n-1}}{k_n}, \tag{27e}$$

$$\left(\frac{\partial(\;)}{\partial y}\right)_{j-1/2}^{n-1/2} = \frac{(\;)_{j}^{n-1/2} - (\;)_{j-1}^{n-1/2}}{h_j}. \tag{27f}$$

For the midpoint $\left(x^n, y_{j-1/2}\right)$ of the segment $P_1 P_2$, we start by writing the finite difference form of equations (23a, b, c, d) using centred-difference derivatives, which is called as "centering about $\left(x^n, y_{j-1/2}\right)$". The resulting finite-difference approximation of (23a)-(23f) are as follows:

$$f_j - f_{j-1} - \frac{h_j}{2}\left(u_j + u_{j-1}\right) = 0 \tag{28a}$$

$$u_j - u_{j-1} - \frac{h_j}{2}\left(v_j + v_{j-1}\right) = 0 \tag{28b}$$

$$v_j - v_{j-1} - \frac{h_j}{2}\left(p_j + p_{j-1}\right) = 0 \tag{28c}$$

$$s_j - s_{j-1} - \frac{h_j}{2}\left(t_j + t_{j-1}\right) = 0 \tag{28d}$$

$$\begin{aligned}
&\frac{1}{1+\lambda}\left(v_j - v_{j-1}\right) + \frac{(1+\alpha)}{4} h_j\left(f_j + f_{j-1}\right)\left(v_j + v_{j-1}\right) - \frac{(1+\alpha)}{4} h_j\left(u_j + u_{j-1}\right)^2 + \frac{B\gamma}{2} h_j\left(s_j + s_{j-1}\right) \\
&+ h_j M + \frac{\lambda_2}{1+\lambda}\frac{(1+\alpha)}{4} h_j\left(v_j + v_{j-1}\right)^2 - \frac{\lambda_2}{1+\lambda}\frac{(1+\alpha)}{2}\left(f_j + f_{j-1}\right)\left(p_j - p_{j-1}\right) \\
&-\frac{\alpha}{2} h_j\left[f_{j-1/2}^{n-1}\left(v_j + v_{j-1}\right) - v_{j-1/2}^{n-1}\left(f_j + f_{j-1}\right)\right] + \frac{\lambda_2}{1+\lambda}\alpha f_{j-1/2}^{n-1}\left(p_j - p_{j-1}\right) \\
&-\frac{\lambda_2}{1+\lambda}\frac{\alpha}{2} h_j\left(p'\right)_{j-1/2}^{n-1}\left(f_j + f_{j-1}\right) - \frac{\lambda_2}{1+\lambda}\alpha h_j\left[p_{j-1/2}^{n-1}\left(u_j + u_{j-1}\right) - u_{j-1/2}^{n-1}\left(p_j + p_{j-1}\right)\right] = \left(R_1\right)_{j-1/2}^{n-1}
\end{aligned} \tag{28e}$$

$$\begin{aligned}
&\frac{1}{\Pr}\left(t_j - t_{j-1}\right) + \frac{(1+\alpha)}{4} h_j\left(f_j + f_{j-1}\right)\left(t_j + t_{j-1}\right) - \frac{\alpha}{4} h_j\left(u_j + u_{j-1}\right)\left(s_j + s_{j-1}\right) \\
&+\frac{\alpha}{2} h_j s_{j-1/2}^{n-1}\left(u_j + u_{j-1}\right) - \frac{\alpha}{2} h_j u_{j-1/2}^{n-1}\left(s_j + s_{j-1}\right) - \frac{\alpha}{2} h_j f_{j-1/2}^{n-1}\left(t_j + t_{j-1}\right) + \frac{\alpha}{2} h_j t_{j-1/2}^{n-1}\left(f_j + f_{j-1}\right) \\
&+\frac{G}{8}\alpha Ec h_j\left(v_j + v_{j-1}\right)^2 + \frac{G}{2}\alpha Ec h_j v_{j-1/2}^{n-1}\left(v_j + v_{j-1}\right) = \left(R_2\right)_{j-1/2}^{n-1}
\end{aligned} \tag{28f}$$

where

$$\alpha = \frac{x^{n-1/2}}{k_n},\; B = \frac{\sin x^{n-1/2}}{x^{n-1/2}},\; M = B\cos x^{n-1/2},\; G = x^{n-1/2}k_n \tag{28g,h,i}$$

$$\left(R_1\right)_{j-1/2}^{n-1} = -h_j\left[\begin{aligned}&\frac{1}{1+\lambda}\left(\frac{v_j - v_{j-1}}{h_j}\right) + (1-\alpha) f_{j-1/2} v_{j-1/2} + (\alpha-1)\left(u_{j-1/2}\right)^2 \\ &+(1-\alpha)\frac{\lambda_2}{1+\lambda}\left(v_{j-1/2}\right)^2 + (\alpha-1)\frac{\lambda_2}{1+\lambda} f_{j-1/2}\left(\frac{p_j - p_{j-1}}{h_j}\right) + B\gamma s_{j-1/2} + M\end{aligned}\right]^{n-1} \tag{28j}$$

$$\left(R_2\right)_{j-1/2}^{n-1} = -h_j\left[\frac{1}{\Pr}\left(\frac{t_j - t_{j-1}}{h_j}\right) + (1-\alpha) f_{j-1/2} t_{j-1/2} + \alpha u_{j-1/2} s_{j-1/2} + \frac{G}{2}\alpha Ec\left(v_{j-1/2}\right)^2\right]^{n-1} \tag{28k}$$

Suppose that the solution is known on $x^n = x^{n-1}$, then it is noted that $\left(R_1\right)_{j-1/2}$ and $\left(R_2\right)_{j-1/2}$ encompass the only known quantities. In terms of new dependent variables, the boundary conditions yield at $x = x^n$ are:

$$\delta f_0^n = 0,\; \delta u_0^n = 0,\; \delta s_0^n = 1,\; \delta u_J^n = 1,\; \delta v_J^n = 0,\; \delta s_J^n = 0. \tag{29}$$

III : Newton Method

If we assume $f_j^{\,n-1}, u_j^{\,n-1}, v_j^{\,n-1}, p_j^{\,n-1}, s_j^{\,n-1}, t_j^{\,n-1}$ to be known for $0 \le j \le J$, this leads to a system of $6J+6$ unknowns $f_j^{\,n}, u_j^{\,n}, v_j^{\,n}, p_j^{\,n}, s_j^{\,n}, t_j^{\,n}, j = 0,1,2,...,J$. By means of Newton Method, this non-linear system of equations is linearized [26]. Hence, the following iterates are initiated:

$$\begin{aligned}
&f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)}, u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)}, v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)}, \\
&p_j^{(i+1)} = p_j^{(i)} + \delta p_j^{(i)}, s_j^{(i+1)} = s_j^{(i)} + \delta s_j^{(i)}, t_j^{(i+1)} = t_j^{(i)} + \delta t_j^{(i)}.
\end{aligned} \tag{30}$$

The above expressions are substituted into (28a)-(28f). For simplification, the terms that are quadratic in $\left(\delta f_j^{\,i}, \delta u_j^{\,i}, \delta v_j^{\,i}, \delta p_j^{\,i}, \delta s_j^{\,i}, \delta t_j^{\,i},\right)$ and superscript i are dropped. After some algebraic manipulations, the following linear tridiagonal system of equations are attained:

$$\delta f_j - \delta f_{j-1} - \frac{1}{2} h_j\left(\delta u_j + \delta u_{j-1}\right) = \left(r_1\right)_{j-1/2},$$

$$\delta u_j-\delta u_{j-1}-\frac{1}{2}h_j\left(\delta v_j+\delta v_{j-1}\right)=(r_2)_{j-1/2},$$
$$\delta v_j-\delta v_{j-1}-\frac{1}{2}h_j\left(\delta p_j+\delta p_{j-1}\right)=(r_3)_{j-1/2},$$
$$\delta s_j-\delta s_{j-1}-\frac{1}{2}h_j\left(\delta t_j+\delta t_{j-1}\right)=(r_4)_{j-1/2},$$
$$\left(a_1\right)_j\delta v_j+\left(a_2\right)_j\delta v_{j-1}+\left(a_3\right)_j\delta f_j+\left(a_4\right)_j\delta f_{j-1}+\left(a_5\right)_j\delta u_j+\left(a_6\right)_j\delta u_{j-1}+\left(a_6\right)_j\delta u_{j-1}+\left(a_7\right)_j\delta s_j+\left(a_8\right)_j\delta s_{j-1}$$
$$+\left(a_9\right)_j\delta p_j+\left(a_{10}\right)_j\delta p_{j-1}=\left(r_5\right)_{j-1/2},$$
$$\left(b_1\right)_j\delta t_j+\left(b_2\right)_j\delta t_{j-1}+\left(b_3\right)_j\delta f_j+\left(b_4\right)_j\delta f_{j-1}+\left(b_5\right)_j\delta u_j+\left(b_6\right)_j\delta u_{j-1}+\left(b_7\right)_j\delta s_j+\left(b_8\right)_j\delta s_{j-1}$$
$$+\left(b_9\right)_j\delta v_j+\left(b_{10}\right)_j\delta v_{j-1}=\left(r_6\right)_{j-1/2},\tag{31}$$

where

$$\left(a_1\right)_j=\frac{1}{1+\lambda}+\frac{\left(1+\alpha\right)}{2}h_jf_{j-1/2}+\frac{\lambda_2}{1+\lambda}\left(1+\alpha\right)h_jv_{j-1/2}-\frac{\alpha}{2}h_jf_{j-1/2}^{n-1},$$
$$\left(a_2\right)_j=-\frac{1}{1+\lambda}+\frac{\left(1+\alpha\right)}{2}h_jf_{j-1/2}+\frac{\lambda_2}{1+\lambda}\left(1+\alpha\right)h_jv_{j-1/2}-\frac{\alpha}{2}h_jf_{j-1/2}^{n-1}=\left(a_1\right)_j-2\left(\frac{1}{1+\lambda}\right)$$
$$\left(a_3\right)_j=\frac{\left(1+\alpha\right)}{2}h_jv_{j-1/2}-\frac{\lambda_2}{1+\lambda}\frac{\left(1+\alpha\right)}{2}\left(p_j-p_{j-1}\right)+\frac{\alpha}{2}h_jv_{j-1/2}^{n-1}-\frac{\lambda_2}{1+\lambda}\frac{\alpha}{2}h_j\left(p^{'}\right)_{j-1/2}^{n-1},\quad\left(a_4\right)_j=\left(a_3\right)_j,$$
$$\left(a_5\right)_j=-\left(1+\alpha\right)h_ju_{j-1/2}-\frac{\lambda_2}{1+\lambda}\alpha h_jp_{j-1/2}^{n-1},$$
$$\left(a_6\right)_j=\left(a_5\right)_j,\ \left(a_7\right)_j=\frac{B\gamma}{2}h_j,\quad\left(a_8\right)_j=\left(a_7\right)_j,$$
$$\left(a_9\right)_j=-\frac{\lambda_2}{1+\lambda}\left(1+\alpha\right)f_{j-1/2}+\frac{\lambda_2}{1+\lambda}\alpha f_{j-1/2}^{n-1}+\frac{\lambda_2}{1+\lambda}\alpha h_ju_{j-1/2}^{n-1},$$
$$\left(a_{10}\right)_j=\frac{\lambda_2}{1+\lambda}\left(1+\alpha\right)f_{j-1/2}-\frac{\lambda_2}{1+\lambda}\alpha f_{j-1/2}^{n-1}+\frac{\lambda_2}{1+\lambda}\alpha h_ju_{j-1/2}^{n-1},\tag{32a}$$

$$\left(b_1\right)_j=\frac{1}{\Pr}+\frac{\left(1+\alpha\right)}{2}h_jf_{j-1/2}-\frac{\alpha}{2}h_jf_{j-1/2}^{n-1},$$
$$\left(b_2\right)_j=-\frac{1}{\Pr}+\frac{\left(1+\alpha\right)}{2}h_jf_{j-1/2}-\frac{\alpha}{2}h_jf_{j-1/2}^{n-1},=\left(b_1\right)_j-\frac{2}{\Pr},$$
$$\left(b_3\right)_j=\frac{\left(1+\alpha\right)}{2}h_jt_{j-1/2}+\frac{\alpha}{2}h_jt_{j-1/2}^{n-1},\qquad\left(b_4\right)_j=\left(b_3\right)_j,$$
$$\left(b_5\right)_j=-\frac{\alpha}{2}h_js_{j-1/2}+\frac{\alpha}{2}h_js_{j-1/2}^{n-1},\quad\left(b_6\right)_j=\left(b_5\right)_j$$
$$\left(b_7\right)_j=-\frac{\alpha}{2}h_ju_{j-1/2}-\frac{\alpha}{2}h_ju_{j-1/2}^{n-1},\quad\left(b_8\right)_j=\left(b_7\right)_j$$
$$\left(b_9\right)_j=\frac{G}{2}\alpha Ech_jv_{j-1/2}+\frac{G}{2}\alpha Ech_jv_{j-1/2}^{n-1},\quad\left(b_{10}\right)_j=\left(b_9\right)_j\tag{32b}$$

$$\left(r_1\right)_{j-1/2}=f_{j-1}-f_j+h_ju_{j-1/2},$$
$$\left(r_2\right)_{j-1/2}=u_{j-1}-u_j+h_jv_{j-1/2},$$
$$\left(r_3\right)_{j-1/2}=v_{j-1}-v_j+h_jp_{j-1/2},$$
$$\left(r_4\right)_{j-1/2}=s_{j-1}-s_j+h_jt_{j-1/2},$$

$$\begin{aligned}
\left(r_5\right)_{j-1/2} &= \frac{1}{1+\lambda}\left(v_{j-1}-v_j\right)-(1+\alpha)h_j f_{j-1/2}v_{j-1/2}+(1+\alpha)h_j u_{j-1/2}^2-B\gamma h_j s_{j-1/2}-h_j M \\
&-\frac{\lambda_2}{1+\lambda}(1+\alpha)h_j v_{j-1/2}^2+\frac{\lambda_2}{1+\lambda}(1+\alpha)f_{j-1/2}\left(p_j-p_{j-1}\right)+\alpha h_j f_{j-1/2}^{n-1}v_{j-1/2}-\alpha h_j v_{j-1/2}^{n-1}f_{j-1/2} \\
&-\frac{\lambda_2}{1+\lambda}\alpha f_{j-1/2}^{n-1}\left(p_j-p_{j-1}\right)+\frac{\lambda_2}{1+\lambda}\alpha h_j\left(p'\right)_{j-1/2}^{n-1}f_{j-1/2}+\frac{\lambda_2}{1+\lambda}\alpha h_j\left[2p_{j-1/2}^{n-1}u_{j-1/2}-2u_{j-1/2}^{n-1}p_{j-1/2}\right] \\
&+\left(R_1\right)_{j-1/2}^{n-1}. \\
\left(r_6\right)_{j-1/2} &= \frac{1}{\Pr}\left(t_{j-1}-t_j\right)-(1+\alpha)h_j f_{j-1/2}t_{j-1/2}+\alpha h_j u_{j-1/2}s_{j-1/2}-\alpha h_j s_{j-1/2}^{n-1}u_{j-1/2}+\alpha h_j u_{j-1/2}^{n-1}s_{j-1/2} \\
&+\alpha h_j f_{j-1/2}^{n-1}t_{j-1/2}-\alpha h_j t_{j-1/2}^{n-1}f_{j-1/2}-\frac{G}{2}\alpha Ech_j\left(v_{j-1/2}\right)^2-G\alpha Ech_j v_{j-1/2}^{n-1}v_{j-1/2}+\left(R_2\right)_{j-1/2}^{n-1}.
\end{aligned} \tag{33}$$

Next, we take the following boundary conditions in order to keep the correct values in all iterates.

$$\delta f_0^n=0,\ \delta u_0^n=0,\ \delta s_0^n=0,\ \delta u_J^n=0,\ \delta v_J^n=0,\ \delta s_J^n=0. \tag{34}$$

IV : Block Tridiagonal Elimination Technique

The linearized difference equations of the system (31) have a block-tridiagonal structure which consist of block matrices. As for constant wall temperature, the element of the matrces are outlined as follows:

$$\boldsymbol{A\delta}=\boldsymbol{r}, \tag{35}$$

where

$$\boldsymbol{A}=\begin{bmatrix} [A_1] & [C_1] & & & \\ [B_2] & [A_2] & [C_2] & & \\ & & \ddots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{bmatrix},\ \boldsymbol{\delta}=\begin{bmatrix}[\delta_1]\\ [\delta_2]\\ \vdots \\ [\delta_{J-1}]\\ [\delta_J]\end{bmatrix},\boldsymbol{r}=\begin{bmatrix}[r_1]\\ [r_2]\\ \vdots \\ [r_{J-1}]\\ [r_J]\end{bmatrix}.$$

The element of matrices are

$$\left[A_1\right]=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{2}h_1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -\frac{1}{2}h_1 & 0 & 0 & -\frac{1}{2}h_1 & 0 \\ 0 & 0 & -\frac{1}{2}h_1 & 0 & 0 & -\frac{1}{2}h_1 \\ \left(a_2\right)_1 & \left(a_{10}\right)_1 & 0 & \left(a_3\right)_1 & \left(a_9\right)_1 & 0 \\ \left(b_{10}\right)_1 & 0 & \left(b_2\right)_1 & \left(b_3\right)_1 & 0 & \left(b_1\right)_1 \end{bmatrix}, \tag{36a}$$

$$\left[A_j\right]=\begin{bmatrix} -\frac{1}{2}h_j & 0 & 0 & 1 & 0 & 0 \\ -1 & -\frac{1}{2}h_j & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -\frac{1}{2}h_j & 0 \\ 0 & 0 & -1 & 0 & 0 & -\frac{1}{2}h_j \\ \left(a_6\right)_j & \left(a_2\right)_j & \left(a_8\right)_j & \left(a_3\right)_j & \left(a_9\right)_j & 0 \\ \left(b_6\right)_j & \left(b_{10}\right)_j & \left(b_8\right)_j & \left(b_3\right)_j & 0 & \left(b_1\right)_j \end{bmatrix}, \quad 2\le j\le J, \tag{36b}$$

$$\left[B_j\right]=\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2}h_j & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}h_j \\ 0 & 0 & 0 & \left(a_4\right)_j & \left(a_{10}\right)_j & 0 \\ 0 & 0 & 0 & \left(b_4\right)_j & 0 & \left(b_2\right)_j \end{bmatrix}, \quad 2\le j\le J, \tag{37}$$

$$\left[C_j\right]=\begin{bmatrix} -\frac{1}{2}h_j & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{1}{2}h_j & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \left(a_5\right)_j & \left(a_1\right)_j & \left(a_7\right)_j & 0 & 0 & 0 \\ \left(b_5\right)_j & \left(b_9\right)_j & \left(b_7\right)_j & 0 & 0 & 0 \end{bmatrix}, 1\le j\le J-1 \tag{38}$$

$$[\delta_1]=\begin{bmatrix} \delta v_0 \\ \delta p_0 \\ \delta t_0 \\ \delta f_1 \\ \delta p_1 \\ \delta t_1 \end{bmatrix}, \quad [\delta_j]=\begin{bmatrix} \delta u_{j-1} \\ \delta v_{j-1} \\ \delta s_{j-1} \\ \delta f_J \\ \delta p_J \\ \delta t_J \end{bmatrix}, \quad 2\le j\le J \tag{39}$$

and

$$[r_j]=\begin{bmatrix} (r_1)_{j-1/2} \\ (r_2)_{j-1/2} \\ (r_3)_{j-1/2} \\ (r_4)_{j-1/2} \\ (r_5)_{j-1/2} \\ (r_6)_{j-1/2} \end{bmatrix}, \quad 1\le j\le J. \tag{40}$$

Equation (35) can be solved by assuming that A is nonsingular. Thus, it can be forced into

$$\boldsymbol{A=LU}, \tag{41}$$

where

$$\boldsymbol{L}=\begin{bmatrix} [\alpha_1] & & & \\ [B_2] & [\alpha_2] & & \\ & & & \\ & & [\alpha_{J-1}] & \\ & & [B_J] & [\alpha_J] \end{bmatrix} \text{ and } \boldsymbol{U}=\begin{bmatrix} [I] & [\Gamma_1] & & \\ & [I] & [\Gamma_2] & \\ & & & \\ & & & [I][\Gamma_{J-1}] \\ & & & [I] \end{bmatrix},$$

where $\left[I\right]$ is the identity matrix of order 6 and $\left[\alpha_j\right]$, and $\left[\Gamma_i\right]$ are $6x6$ matrices. The element of matrices can be determined by the subsequent equations:

$$\left[\alpha_1\right]=\left[A_1\right], \tag{42}$$

$$\left[A_1\right]\left[\Gamma_1\right]=\left[C_1\right], \tag{43}$$

$$\left[\alpha_j\right]=\left[A_j\right]-\left[B_j\right]\left[\Gamma_{j-1}\right],\ j=2,3,...,J, \tag{44}$$

$$\left[\alpha_j\right]\left[\Gamma_j\right] = \left[C_j\right], \quad j = 2,3,...,J-1. \tag{45}$$

Thus, (41) can now be substituted into (35). This will yield

$$LU\delta = r, \tag{46}$$

If we define

$$U\delta = W, \tag{47}$$

then (47) becomes

$$LW = r, \tag{48}$$

where

$$W = \begin{bmatrix} [W_1] \\ [W_2] \\ \\ [W_{J-1}] \\ [W_J] \end{bmatrix},$$

and $\left[W_j\right]$ are column matrices with 6x1. Now, the elements W can be solved from (48) as

$$[\alpha_1][W_1] = [r_1], \tag{49}$$

$$\left[\alpha_j\right]\left[W_j\right] = \left[r_j\right] - \left[B_j\right]\left[W_{j-1}\right], \quad 2 \le j \le J \tag{50}$$

The solution of (41) by the block- elimination method involves of two sweeps. The forward sweep referred to the step where Γ_j, α_j and W_j are calculated. They are computed from the recursion formulas given by (42), (43), (44), (45), (49) and (50). Once the elements of $\boldsymbol{W}$ are found, (47) will give the solution of δ in the so-called backward sweep. Therefore, the elements are gained by the following relations:

$$[\delta_J] = [W_J], \tag{51}$$

$$[\delta_J] = [W_J] - \left[\Gamma_j\right]\left[\delta_{j+1}\right], \quad 1 \le j \le J-1. \tag{52}$$

These calculations are repeated until some convergence criterion is satisfied. Calculations are stopped when

$$\left|\delta v_0^{(i)}\right| < \varepsilon_1, \tag{53}$$

where $\varepsilon_1 = 10^{-7}$ is a too small fixed value.

4. Result and Discussions

Both the systems of partial differential equations (12) and (13) subject to the boundary conditions (14) and the ordinary differential equations (15) and (16) subject to boundary conditions (17) have been solved numerically by using Keller-box method. In this study, two parameters will be considered for discussions, i.e. mixed convection parameter, $\gamma = -1.0, 0.0, 1.0, 5.0$ and Prandtl number, $\Pr = 0.5, 0.71, 1.0, 1.5$. To obtain the results in graphical form, the step size $\Delta x = \Delta y = 0.02$ and boundary layer thickness $y_\infty = 7$ are considered. Table 1 presents the comparison values of the present solutions with the existing publication reported by Eckert [27] and Salleh et al. [28]. Since the results obtained demonstrate a very excellent agreement, therefore we are confident that the results presented here are accurate.

Figs. 3 and 4 show the profiles for velocity $f'(y)$ and temperature $\theta(y)$ for different values of γ. It is noticed that the velocity is significantly enhanced due to the increase in γ. On the contrary, an increase in γ has strongly decelerate the temperature. A similar trend of graph has been observed by Gaffar et al. [29] on the problem of flow and heat transfer of Jeffrey fluid from a non-isothermal wedge. Physically, a large value of γ produces large buoyancy forces which produce large kinetic energy. As a result, the momentum boundary layer thickness is increase while the thermal boundary layer thickness is decrease.

Figs. 5 and 6 present the influence of Prandtl number, $\Pr$ on reduced skin friction coefficient $C_f \mathrm{Re}^{1/2}$ and Nusselt number $Nu_x \mathrm{Re}^{-1/2}$. In Fig. 5, it is observed that an increase in Pr results in a slight decrease of $C_f \mathrm{Re}^{1/2}$. Since the graph obtained does not noticeably vary and also Pr is not directly appear in equation (20), it is concluded that Pr does not give much effect on $C_f \mathrm{Re}^{1/2}$. As can be seen in Fig. 6, a small $\Pr$ will contribute a pronounced effect on $Nu_x \mathrm{Re}^{-1/2}$ where $Nu_x \mathrm{Re}^{-1/2}$ increases as $\Pr$ increases. This is because, the fluid is highly thermal conductive for a small values of $\Pr$. As Pr increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing of energy ability that reduces the thermal boundary layer thickness. Hence, the convective heat transfer process is enhanced.

Table 1 : Comparison between the present solutions of (12) and (13) with previous published results for various values of Pr when $\lambda = 0,\ \lambda_2 \to 0$ (very small), γ=0 and $Ec = 0$ (Newtonian Fluid)

Pr	$-\theta'(0)$		
	Eckert [27]	**Salleh et al. [28]**	**Present**
0.6		0.4663	0.466338
0.7	0.496	0.4959	0.495877
0.8	0.523	0.5228	0.522756
1	0.57	0.5705	0.570486
5	1.043	1.0436	1.043597
7		1.1786	1.178610
10	1.344	1.3391	1.339139
100		2.9897	2.989934
1000		6.5662	6.566698

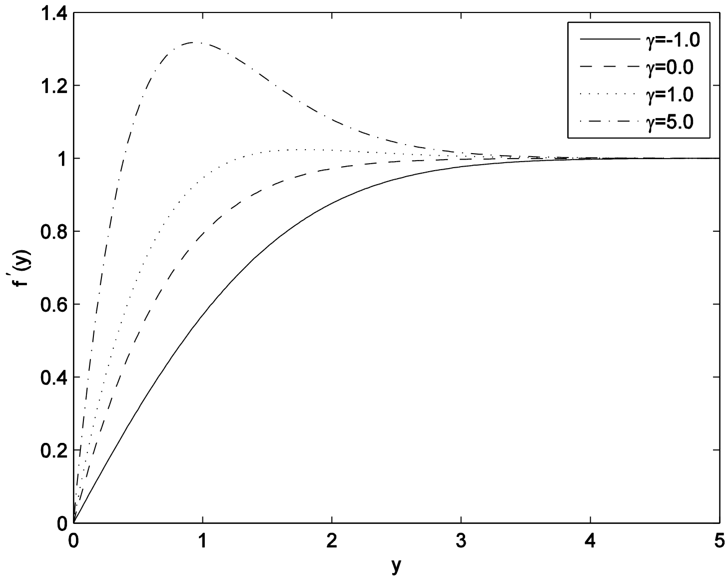

Figure 3 : Velocity profile $f'(y)$ against y for various values of γ when $\Pr = 0.71,\ \lambda = 0.1,\ \lambda_2 = 0.1,\ Ec = 0.1$

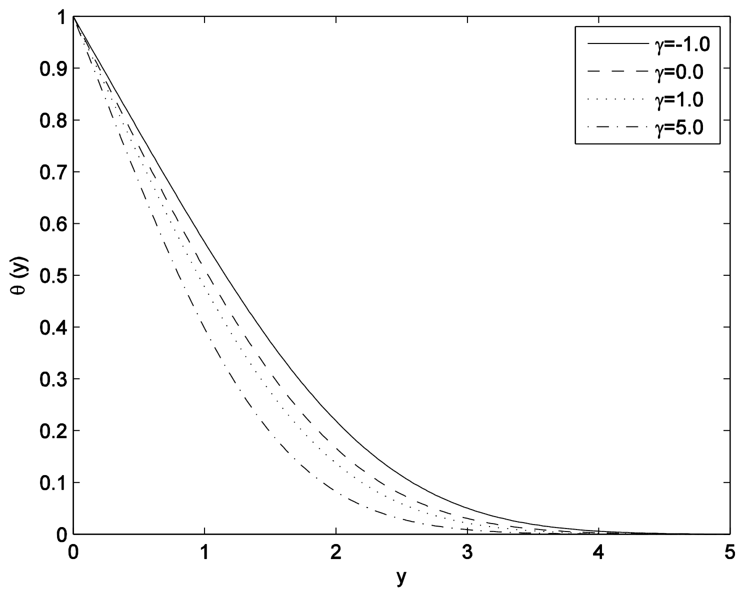

Figure 4 : Temperature profile $\theta(y)$ against y for various values of γ when $\Pr = 0.71,\ \lambda = 0.1,\ \lambda_2 = 0.1,\ Ec = 0.1$

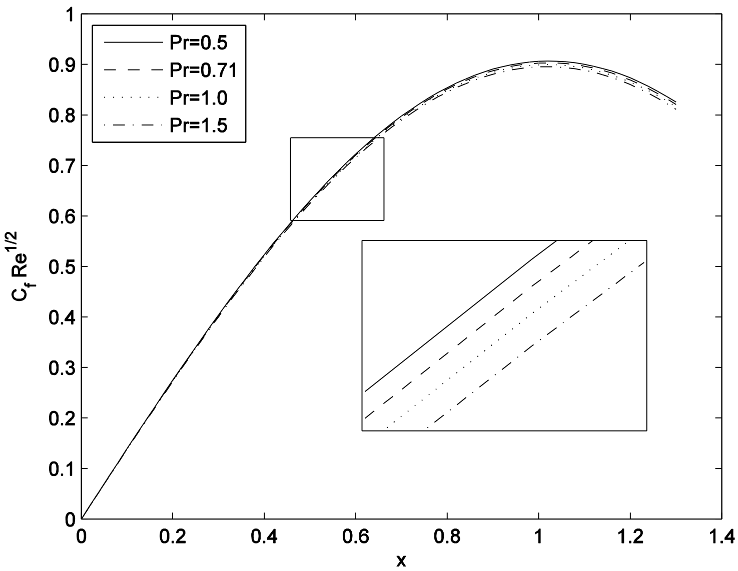

Figure 5 : Reduced skin friction $C_f \,\mathrm{Re}^{1/2}$ against x for various values of $\Pr$ when $\lambda = 0.1,\ \lambda_2 = 0.1,\ \gamma = 0.1,\ Ec = 0.1$

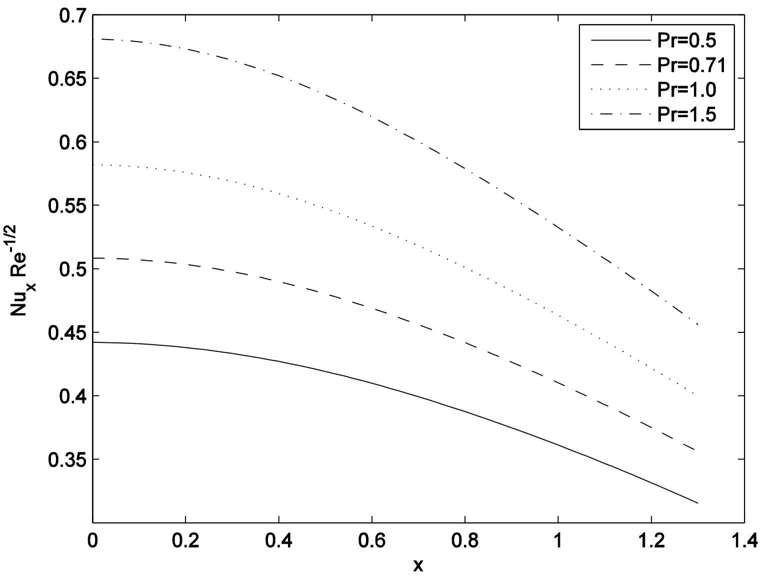

Figure 6 : Reduced Nusselt number $Nu_x \,\mathrm{Re}^{-1/2}$ against x for various values of $\Pr$ when $\lambda = 0.1,\ \lambda_2 = 0.1,\ \gamma = 0.1,\ Ec = 0.1$

5. CONCLUSION

The solutions of mixed boundary layer and heat transfer over horizontal circular cylinder embedded in Jeffrey fluid with viscous dissipation has been computed using numerical approach. The conclusions of this study are given as follows:

I. Increasing mixed convection parameter will increase the velocity profile and decrease the temperature profile.
II. An increase in $\Pr$ results in a decrease of the skin friction coefficient and will increase the values of the Nusselt number.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial supports received from the research grant (RDU140111, RDU150101 and RDU160330) from the Universiti Malaysia Pahang and Ministry of Higher Education (MoHE).

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