Folded Yukawa interaction potential model
description of heavy ion elastic scattering

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Abstract: The surface interaction potential obtained from the knowledge of the nuclear
densities and effective interactions is reviewed. The corresponding ion-ion potential and its
theoretical estimates with special reference to folding potentials, proximity potentials are
discussed. The comparison of folded Yukawa interaction potential with other phenomenological
potentials in the description of heavy ion elastic scattering is presented with reference to the
standard Woods-Saxon form.

Keywords: Heavy ion elastic scattering, Yukawa interaction potential
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1. Introduction

The interaction between two nuclei is determined by using singly Yukawa-folded sharp,
uniform distributions for the nucleon densities, and effective nucleon-nucleon forces in
Yukawa forms. When these are all folded in, one obtains a result in closed form, which,
when the centers of the nuclei are not allowed to approach for more than the sum of their
radii, is defined to be the surface interaction. Sometime ago Christensen and Winther [1]
have proposed an empirical potential to be used for elastic ion-ion collisions. A similar
potential could also be derived in the so-called proximity model [2], which is related to the
liquid drop model. Both are exponential functions of surface separations of the colliding
nuclei, and differ only in the mass dependences of their respective parameters [3]. Broglia
and Winther [4] have tried to establish a basis for such potentials through a folding model,
but due to the asymmetric representation of the nuclei in the folding, the results could vary
up to 50% upon the exchange of the types of the representations.

Several models have been suggested for the calculation of the nuclear interaction
potential. For example, Krappe and Nix [5] have proposed a model in which the interaction
energy is calculated as the Yukawa interaction between two nuclear distributions with
sharply defined surfaces and uniform interior. The Yukawa interaction is supposed to contain the combined effect of two diffuse matter distributions interacting via some short range interaction. This procedure leads to a simple analytic potential.

A different approach is represented by the proximity formula [2] which expresses the force between two gently curved leptodermous surfaces as a product of a geometrical factor proportional to the mean radius of curvature of the gap between the surfaces and a universal function equal to the interaction energy per unit area between two parallel surfaces. This latter approach is very general and has the advantage of being simple to use, once the problem involving the parallel surfaces has been solved.

In the present talk an analytical model is discussed which enables one to gain insight into the accuracy of some of the various approaches, including the two mentioned above. In the model studied, each of the two interacting objects has a diffuse surface which is generated by folding a Yukawa function into a generating sharp-surface distribution. The interaction energy is subsequently obtained on the basis of a two-body Yukawa interaction.

![Figure 1. Comparison of the Woods-Saxon, Hill-Ford and Folded-Yukawa potential forms.](image)

This model can be considered a generalization of the Krappe-Nix [5] model. Hence it permits a test of the idea that the interaction can be represented as a single effective Yukawa interaction acting between sharp-surface distributions. Moreover, the model is sufficiently realistic to present a good test case for the proximity formula [2]. So far, such tests have only been carried out for the extreme cases of zero-diffuseness distributions (the Krappe-Nix model) or a zero-range interaction between diffuse surface [2].
2. The folded-Yukawa model

The interaction energy $V$ between two matter distributions $\rho_1$ and $\rho_2$ is given by

$$V = -C \int \int \rho_1(r_1) \rho_2 (r_2) \, d^3r_1 \, d^3r_2$$

(1)

where the notation

$$\rho_1(r_1) = \frac{1}{4\pi a^3} \exp \left( -\frac{r_1}{a} \right)$$

(2)

has been introduced. The strength of the interaction is governed by the constant $C$ which is positive for an attractive interaction. The matter density distribution $\rho_i (i = 1, 2)$ is obtained by folding a Yukawa function of some range $a_i$ into a generating sharp distribution

$$\rho_i (r_i) = \int \chi_i (r_1) \rho_i (r_2) \, d^3r_2$$

(3)

The starting point for the analytical treatment of this model is the observation that the interaction energy may be calculated as the interaction between the two sharp
generating densities arising from a composite tow-body interaction-cy. This composite interaction is given as the folding product of the three entering Yukawa interactions.

\[ y = y_{u0} \ast y_{u1} \ast y_{u2}, \]  

(4)

(the symbol \( \ast \) denotes the folding).

The above observation implies that the formulated generalized folded-Yukawa model due to Krappe and Nix [5] in that it calculates the energy by folding some Kernel into generating sharp densities. The generalized model thus applies to all cases covered by the Krappe-Nix model. In particular, the modified surface-energy prescription suggested by Krappe and Nix [5] can be generalized by employing the composite kernal \( y \) rather than a single Yukawa function.

**Figure 3.** Comparison of the fits to elastic scattering angular distributions for \( ^{10}\text{B} + ^{25}\text{Mg} \) at 87.4 MeV(Lab).

Adopting the above mentioned procedure it is possible to obtain an ion-ion interaction potential which is outlined as follows [7]. The folded-Yukawa parametrized form factor \( F(K, r) \) for the charge density \( \rho(r) \) as well as the interaction potential \( \nu(r) \) can be given as

\[ \rho(r) = \rho_0 F(K, R), \]
\[ \nu(r) = \nu_0 F(K, R), \]

(5)

where \( \rho_0 \) is the charge density at the centre and \( \nu_0 \) is the corresponding strength of the potential.
The function \( F(K, r) \) is obtained by folding a Yukawa function together with a step function \( \theta(R - r') \).

\[
F(K, R) = K^2 / 4\pi \int d^3r \, \theta(R - r') \frac{\exp(-|r - r'|)}{|r - r'|}
\]

The above expression is evaluated by following the procedure outlined above by folding the Yukawa function to a step function defined as

\[
(H)(r) = \theta(r - r') = \begin{cases} 
1 & \text{for } r < R \\
0 & \text{for } r > R
\end{cases}
\]

The folding of two Yukawa functions centered at different points is denoted by the symbol (*), e.g.,

\[
Y(K_1) \ast Y(K_2) = \int \frac{\exp(-K_1|r_1 - r'|)}{|r_1 - r'|} \frac{\exp(-K_2|r_2 - r'|)}{|r_2 - r'|} \, (d^3r') \
\]

\[
= \frac{4\pi}{K_2^2 - K_1^2} \frac{\exp(-K_1|r_1 - r'|) - \exp(-K_2|r_2 - r'|)}{|r_1 - r_2|}
\]

The result of integration is a function of distance \( |r_1 - r_2| \) only. Equation above can therefore be written as

\[
Y(K_1) \ast Y(K_2) = 4\pi/\left(K_1^2 - K_2^2\right) [Y(K_1) - Y(K_2)]
\]
The folding of $Y$ with can also be evaluated explicitly. One finds

$$ Y(k) \ast \Theta(R) = \int \Theta(R-r') \frac{\exp(-K|r-r'|)}{|r-r'|} d^3r' $$

$$ = 4\pi/k^2 \cdot F(K,R) $$

with

$$ F(K,R) = \begin{cases} 
1 - \frac{1}{2} \exp\left(-\frac{1}{2}KR\right) & r < R \\
R\cos hKR - (\frac{1}{2}k) \sin h(KR) \left(\exp(-kr)\right)/r & r > R 
\end{cases} \quad (9) $$

for $KR >> 1$ we find

$$ F(K,R) = 1 - \frac{1}{2} \exp\left(-\frac{1}{2}KR\right) \quad (10) $$

The function $F(K,R)$ is similar to fermi distribution $[1 + \exp((r-R)/a)]^{-1}$ in the tail region. The advantage of this parametrisation is that one may evaluate explicitly multiple folded integrals.

The fermi distribution

$$ f(r) = f_0 \left[1 + \exp(r-c)/a\right]^{-1} \quad (11) $$
Folded Yukawa interaction potential model description etc

may be approximately written as

\[ f(r) = f_0 F(K, R) = f_0 K^2 / 4\pi \left[ Y(K) \cdot Y(R) \right] \]  

(12)

\[ f(r) = f_0 F(K, R) = f_0 K^2 / 4\pi Y(K) \cdot Y(R) \]  

(13)

where a fairly good parametrization in the tail region is obtained by Akyuz and Winther [81]

for

\[ K = 2/3 \left( 1/a + 1.2/c \right) \text{ and } R = 0.99c + 0.5a - 0.1 + 0.2/c \]

using these approximations we have analysed the Heavy ion elastic scattering data and

found that such approximation was not giving a better fit to the experimental data. We have

Figure 6. Comparison of the fits to elastic scattering angular distributions for

\[ 11B + 25Mg \text{ at } 79.6 \text{ MeV(Lab).} \]

incorporated the procedure of obtaining the better fit to the experimental data by matching

the folded Yukawa interaction potential with the woods-saxon potential at the strong

absorption radius to obtain the relevant depth and other parameters involved in the analysis.

The best fit data so obtained is listed in Table 1.

3. Remarks and conclusions

As expected in the case of single folded potentials the estimation of the real and imaginary

strength in folded Yukawa interaction potential is almost double that of Woods-Saxon

potential. The analysis is expected to improve by taking density distributions obtained for

folded Yukawa in the double folded potential. Such analysis was undertaken in the case of
Table 1. $^{10}$B + $^{24}$Mg Scattering at 87.4 MeV(Lab).

<table>
<thead>
<tr>
<th>Potential form</th>
<th>$V$</th>
<th>$r_n$</th>
<th>$a$</th>
<th>$W$</th>
<th>$r_n$</th>
<th>$a_n$</th>
<th>$x^2$</th>
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<tbody>
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<td>F - Y</td>
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<td>1.059</td>
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$^{10}$B + $^{25}$Mg Scattering at 87.4 MeV(Lab)

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<th>$W$</th>
<th>$r_n$</th>
<th>$a_n$</th>
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$^{10}$B + $^{60}$Ni Scattering at 87.4 MeV(Lab)

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<td>F - Y</td>
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<td>0.960</td>
<td>141.62</td>
<td>1.014</td>
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$^{11}$B + $^{24}$Mg Scattering at 79.6 MeV(Lab)

<table>
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<th>$W$</th>
<th>$R_n$</th>
<th>$a_n$</th>
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$^{11}$B + $^{25}$Mg Scattering at 79.6 MeV(Lab)

<table>
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<tbody>
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$^{11}$B + $^{27}$Al Scattering at 79.6 MeV(Lab)

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<th>$R_n$</th>
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$^{11}$B + $^{59}$Co Scattering at 79.6 MeV(Lab)

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<th>$R_o$</th>
<th>$a$</th>
<th>$W$</th>
<th>$R_n$</th>
<th>$a_n$</th>
<th>$x^2$</th>
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<tbody>
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$^{11}$B + $^{60}$Ni Scattering at 79.6 MeV(Lab)

<table>
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<th>Potential form</th>
<th>$V$</th>
<th>$R_o$</th>
<th>$a$</th>
<th>$W$</th>
<th>$R_n$</th>
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<td>F - Y</td>
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<td>114.55</td>
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Nucleon-Nucleus Scattering with the heavy ion probes. The effects of coupling between channels and the inclusion of quadrupole moment contributions are expected to improve the analysis in inelastic scattering when compared to the conventional Woods-Saxon analysis. The F-Y interaction model predictions will be more useful while dealing with the data where the large angle oscillations are prominent.

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References

8. Akyuz and Winther (Private Communication) (1979)