Analyses of tunnel stability under dynamic loads

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Abstract
Tunnels as any other structures are affected by different loads and stresses. Generally, the loads on tunnels consist of both static and dynamic loads. These loads must both be considered in the tunnel design process. In this paper, the stability state of Jiroft water-transform tunnel is evaluated. Firstly, the in-situ stresses and then using Kirsch’s equations the induced stresses due to static loads in walls and crown of tunnel are calculated. Consequently, the strain caused by probable earthquake without considering the interaction between the concrete lining of tunnel and rock mass using seismic analysis is calculated based on free-field deformation. When the strain is determined, the simplified method of closed-form solution, Wang equations and Penzien equations are used to calculate (estimate) the applied forces on the tunnel due to earthquake such as axial force and bending moment on a tunnel section taking into account the interaction of the tunnel concrete lining and rock mass. Results of this study show that the loads applied to the tunnel as a result of earthquake waves can be affect the stability of underground structures, especially long structures such as tunnels.

Introduction
In process design, the stability of tunnels and other underground structures under the influence of seismic waves and dynamic load is one of the important issues that should be studied carefully. Although seismic waves are not the only cause of earthquakes, however, earthquakes are the most known source of seismic waves. In addition, the movement of trains in underground tunnels, the operation of machinery on ground surface and many other activities as such produce seismic waves that if to be neglected, may cause different damages such as subsidence. Such damages not only result in an increase in costs, but also remain to be a source of danger to the human lives.

Earthquake waves
The released energy from earthquakes are scattered in the earth as waves and therefore affect the stability of structures. Usually, the studies of general form of earthquake waves are very difficult therefore for simplicity, these waves are divided into simple waves. The earthquake waves are elastic types and based on strain, are divided to Body waves (P and S-) and Surface waves (Love and Rayleigh) [1].

The generation mode of elastic waves

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As mentioned above, earthquake waves are of elastic type. When a force is applied at a certain point of a piece of rock, as this force does not exceed the elastic limit of rock, very little deformation occurs on that specific point. This deformation is transformed to the surrounding points and therefore propagates. If the amplitude of vibration (i.e. instantaneous value of the particle movement of equilibrium state) is showed by $u$, based on Newton’s second law:

$$\sum dF = m \frac{d^2u}{dt^2}$$

where $m$ is the mass of particle in vibration, $t$ is the time, and $F$ is the force.

The velocity of propagation and absorption of waves in rocks

The properties of elastic waves propagation in rock by characteristics such as wave velocity, absorption coefficient, wave amplitude, coefficient of reflection and diffraction at interface of rocks are determined. The velocity of P-wave and S-wave are calculated as follow:

$$V_p = \sqrt{\frac{\lambda + 2G}{\lambda}} = \sqrt{\frac{E\nu}{\lambda g(1 + \nu)(1 - 2\nu)}}$$

$$V_s = \sqrt{\frac{G}{\lambda}} g = \sqrt{\frac{E}{\lambda g}} \frac{1}{2(1 + \nu)}$$

where $V_p$ and $V_s$ are velocity of P-wave and S-wave respectively, $E$ is modulus of elasticity, $\nu$ is Poisson ratio, $g$ is the gravity acceleration, $\lambda$ is the Lame constant that is defined as follow:

$$\lambda = \frac{E\nu}{(1 + \nu)(1 - 2\nu)}$$

The velocity of P-wave is always greater than the S-wave. It should be noted that the amount of this difference depends on the Poisson ration of rock, namely:

$$\frac{V_p}{V_s} = \sqrt{\frac{2(1 - \nu)}{1 - 2\nu}}$$

The ratio for metamorphic and crystalized igneous rocks ranges from 1.7 to 1.9, for sedimentary rocks from 1.4 to 1.5 and for poor and loose rocks is 0.5. Usually, the velocity propagation of surface waves is less than body waves, as the velocity propagation of Love waves is:

$$V_L = 0.919V_s$$

where $V_L$ and $V_s$ are the velocity propagation of Love waves and S-waves respectively.

It should be mentioned that the velocity of elastic waves is actually independent of vibration. The intensity of elastic waves in rocks increases as the distance from the source decreases due to:

1. Partial absorption of the elastic energy due to friction between two particles vibrator and convert to thermal energy.
2. Propagation of energy in different direction due to heterogeneous and anisotropic (porosity and fractures) in rocks.
The amplitude of earthquake waves increases the less the distance from the source is. Therefore, the furthest the tunnel is from the center of the earthquake, the less damage is done to the tunnel. Generally, the energy reduction of surface waves is inversely proportional to distance, whereas the energy reduction of body waves is inversely proportional to the squared distance.

Earthquake and damage to the tunnels

The damages to tunnels due to earthquakes include: a) slip of fault, b) ground failure, and c) ground motion.

a) Slip of fault

In this case, failure occurs when the fault zone passes through the tunnel. In such situations, the failure is confined to the fault zone and the damage on the tunnel can be changed from minor cracking to complete collapse. Because the tunnels are long and linear structures, they may cross the fault zones and increase the damageability. Hence in order to choose the direction and site of the tunnel construction, the state of fault zones must be in considered carefully.

b) Ground failure

Ground failure may cause rock mass or soil sliding, liquefaction, subsidence and many other such phenomena. The ground fails by the creation of discontinuities which reduces the strength and cohesion of rock mass and consequently causes fractures, sliding and popping in rock mass. Furthermore, the liquefaction occurs when the tunnel is constructed in loose sediment or alluvial. For example, metro tunnels are usually constructed in these sort of locations.

c) Ground motion

Ground motion is usually the results of an earthquake and it causes serious damage to the portal of tunnels. The response of tunnels to the ground motion is dependent on several factors such as shape, size, depth and geomechanical properties of surrounding rock mass.

Generally, many factors affect the dynamic damage in tunnels. Based on statistical investigation and historical studies, usually in depths of over 50m a lower level of damage is expected and in depths of over 300m the damage is ignorable. However, in shallow depths (less than 50 m) a high risk of damage due to earthquake waves exists. Furthermore, the other factors such as distance from the earthquake’s center, acceleration, intensity and magnitude of the earthquake are also affected. Therefore, if the design of the tunnel is based primarily on the static parameters such as overburden, in-situ stress and stress distribution, without the consideration of dynamic parameters, the tunnel therefore would become very unstable and susceptible earthquake waves.

Deformation of tunnels in earthquake

The types of deformations of tunnels due to an earthquake include axial, curvature and convolute. The axial deformation together with compression and tensile strain and with the passing of the elastic wave through the tunnel, the displacement in axial of tunnel takes place. The curvature deformation shows a positive and negative curve along the tunnel. In the positive curvature, the roof of tunnel will be under compression whereas the floor will be under tension. In contrast, in the case of a negative
curvature, the situation is vice versa. The convolute deformation results of impact of the wave as a vertical or near vertical to axial of the tunnel. This deformation occurs only when the length of the seismic wave is less than the tunnel radius. The propagation of elastic wave through the rock mass causes various stresses. The most notable of which is tensile stress. It is a known fact that the shear strength of rock mass decreases when a tensile stress is applied. Therefore the bearing capacity of support system (especially active support system such as rockbolt) is reduced and consequently the potential for the tunnel to fail increases.

**Seismic analysis of Jiroft water-transform tunnel**

In order to analyze the stability of Jiroft water-transform tunnel, firstly the in-situ stresses are calculated. Then, based on Kirsch’s equations, the induced stresses due to static loads in walls and crown of the tunnel are calculated. In the next step, the state of strain distribution surrounding the tunnel is calculated using the free-field deformation seismic analysis method, closed form solutions, Wang’s equations and Penzien’s equations.

**The technical characteristic of Jiroft water-transform tunnel**

The depth of Jiroft water-transform tunnel is 80 m, the radius of tunnel excavation 2.3 m and the radius of lining is 2 m. The specific gravity of rock mass is 0.027 $MN/m^3$, the Poisson ratio of rock mass and lining are 0.3 and 0.2 respectively. Furthermore, the elasticity modulus of rock mass and lining are $0.01 \times 10^5 MPa$ and $0.2 \times 10^5 MPa$ respectively.

**Calculation of the in-situ and induced stresses**

The in-situ stresses (before the excavation of the tunnel) are calculated using equations (7) to (9).

\[
\sigma_v = \gamma \cdot h \quad (7)
\]

\[
k = 0.25 + 7E(0.001 + \frac{1}{h}) \quad (8)
\]

\[
\sigma_h = k \cdot \sigma_v \quad (9)
\]

where $\sigma_v$ is the vertical in-situ stress, $\gamma$ the average density of overburden, $z$ is the depth below ground surface, $k$ the ratio of horizontal to vertical in-situ stress, $E_h$ the average horizontal deformability modulus and $\sigma_h$ is the horizontal in-situ stress.

The induced stresses are calculated (after excavation) using equations (10) to (12)

\[
\sigma_r = \frac{\sigma_H + \sigma_V}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_H - \sigma_V}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta \quad (10)
\]

\[
\sigma_\theta = \frac{\sigma_H + \sigma_V}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_H - \sigma_V}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \quad (11)
\]

\[
\tau_{r\theta} = \frac{\sigma_V - \sigma_H}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right) \sin 2\theta \quad (12)
\]
Where $\sigma_r$ is the induced radial stress, $\sigma_\theta$ is the induced tangential stress, $\tau_{r\theta}$ is the induced shear stress, $a$ is the radial of tunnel, $r$ is the distance from tunnel center and $\theta$ is the angle from horizontal (clockwise).

Tables (1) and (2) show the results of the in-situ and induced stresses calculations respectively.

**Table (1), the in-situ stresses in Jiroft water-transform tunnel field**

<table>
<thead>
<tr>
<th>$\sigma_v$ (MPa)</th>
<th>$k$ (Unitless)</th>
<th>$\sigma_h$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.16</td>
<td>0.34</td>
<td>0.74</td>
</tr>
</tbody>
</table>

**Table (2), the induced stress in Jiroft water-transform tunnel**

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_r$ (MPa)</th>
<th>$\sigma_\theta$ (MPa)</th>
<th>$\tau_{r\theta}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall of tunnel</td>
<td>0</td>
<td>5.74</td>
<td>0</td>
</tr>
<tr>
<td>Crown of tunnel</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
</tr>
</tbody>
</table>

**Seismic analysis of Jiroft water-transform tunnel**

For seismic analysis, firstly the surface strains due to an earthquake are calculated using free-field deformation method without considering the interaction between the concrete lining of the tunnel and rock mass. Secondly, the strain are calculated using closed-form solution method, Wang’s equation and Penzien’s equation. At this stage, the interaction between the concrete lining of tunnel and rock mass are considered too. Thus, the axial force and bending moment in the full-slip and no-slip assumptions are calculated. The interaction between the concrete lining of the tunnel and rock mass using the ratios of compressibility and flexibility are applied. When the rigidity of the tunnel is much more than surrounding rock mass or the intensity of earthquake is very high, the full-slip occurs.

In order to calculate the axial strain of the tunnel, the Newmark’s formulation in simplified form as equation (13) is used:

$$\varepsilon^{ab} = \pm \frac{V_S}{C_S} \sin \phi \cos \phi \pm \frac{a_S r}{C_S} \cos^3 \phi$$

(13)

where, $\varepsilon^{ab}$ is the axial strain of tunnel, $V_S$ is the peak particle velocity associated with S-wave, $C_S$ is the apparent velocity of S-wave propagation, $\phi$ is the angle of incidence of wave with respect to tunnel axis, $a_S$ is the peak particle acceleration associated with S-wave and $r$ is the radius of tunnel.

The Penzien’s equations in full-slip assumptions are:

$$a^n = 12E_I(5 - 6v_m) \frac{d^3G_m(1 - v^2)}{d^3}$$

(14)

$$R^n = \frac{4(1 - v_m)}{a^n + 1}$$

(15)

$$\gamma_{max} = \frac{V_S}{C_S}$$

(16)
The Penzien’s equations in no-slip assumption are:

\[
\Delta d_{\text{lining}}^n = R \gamma_{\text{max}}^n \frac{d}{2}
\]  

\[
T(\theta) = \frac{12E_l\Delta d_{\text{lining}}^n}{d^3(1 - \nu_l^2)} \cos^2(\theta + \frac{\pi}{4})
\]  

\[
M(\theta) = \frac{6E_l\Delta d_{\text{lining}}^n}{d^2(1 - \nu_l^2)} \cos^2(\theta + \frac{\pi}{4})
\]

The Wang’s equations in full-slip assumption are:

\[
\alpha = \frac{24E_l(3 - 4\nu_m)}{d^3G_m(1 - \nu_l^2)}
\]  

\[
R = \frac{4(1 - \nu_m)}{\alpha + 1}
\]  

\[
\Delta d_{\text{lining}} = R \gamma_{\text{max}} \frac{d}{2}
\]  

\[
T(\theta) = \frac{24E_l\Delta d_{\text{lining}}}{d^3(1 - \nu_l^2)} \cos^2(\theta + \frac{\pi}{4})
\]  

\[
M(\theta) = \frac{6E_l\Delta d_{\text{lining}}}{d^2(1 - \nu_l^2)} \cos^2(\theta + \frac{\pi}{4})
\]

where, \(\alpha^n\) is the coefficient used in calculation of lining-rock mass racking ratio of circular tunnels under normal loading only, \(E_l\) is the modulus of elasticity of tunnel lining, \(I\) is the moment of inertia of the tunnel lining (per unit width) for circular lining, \(\nu_m\) is the Poisson’s ratio of soil or rock mass, \(\nu_l\) is the Poisson’s ratio of tunnel lining, \(d\) is the diameter or equivalent diameter of tunnel lining, \(G_m\) is the shear modulus of soil or rock mass, \(R^n\) is the lining-rock mass racking ratio under normal loading only, \(\gamma_{\text{max}}\) is the maximum free-field shear strain of soil or rock mass, \(\Delta d_{\text{lining}}^n\) is the lining diametric deflection under normal loading only, \(T(\theta)\) is the circumferential thrust force in tunnel lining at angle \(\theta\), \(M(\theta)\) is the circumferential bending moment in tunnel lining at angle \(\theta\), \(\alpha\) is the coefficient used in calculation of lining-rock mass racking ratio of circular tunnels, \(R\) is the lining-rock mass racking ratio and \(\Delta d_{\text{lining}}\) is the lining diametric deflection.

The Wang’s equations in full-slip assumptions are:

\[
C = \frac{E_m(1 - \nu_l^2)r}{E_l(1 + \nu_m)(1 - 2\nu_m)}
\]  

\[
F = \frac{E_m(1 - \nu_l^2)r^3}{6E_l(1 + \nu_m)}
\]  

\[
k_1 = \frac{12(1 - \nu_m)}{2F + 5 - 6\nu_m}
\]  

\[
T_{\text{max}} = \frac{k_1}{6} \frac{E_m}{(1 + \nu_m)} r\gamma_{\text{max}}
\]

\[
T(\theta) = \frac{12E_l\Delta d_{\text{lining}}}{d^3(1 - \nu_l^2)} \cos^2(\theta + \frac{\pi}{4})
\]  

\[
M(\theta) = \frac{6E_l\Delta d_{\text{lining}}}{d^2(1 - \nu_l^2)} \cos^2(\theta + \frac{\pi}{4})
\]
The Wang’s equations in no-slip assumption are:

\[ M_{\text{max}} = \frac{1}{6} k_1 \frac{E_m}{(1 + \nu_m)} r^2 \gamma_{\text{max}} \]  

(29)

\[ K_2 = 1 + \frac{F[(1 - 2\nu_m) - (1 - 2\nu_m)C] - \frac{1}{2}(1 - 2\nu_m)^2 + 2}{F[(3 - 2\nu_m) + (1 - 2\nu_m)C] + C\left[\frac{5}{2} - 8\nu_m + 6\nu_m^2\right] + 6 - 8\nu_m} \]  

(30)

\[ T_{\text{max}} = K_2 \frac{E_m}{2(1 + \nu_m)} r\gamma_{\text{max}} \]  

(31)

where, \( C \) is the compressibility ratio of tunnel lining, \( r \) is the radius of circular tunnel, \( t \) is the thickness of tunnel lining, \( F \) is the flexibility ratio of tunnel lining, \( k_1 \) is the full-slip lining response coefficient, \( T_{\text{max}} \) is the maximum thrust in tunnel lining, \( M_{\text{max}} \) is the maximum bending moment in tunnel cross-section due to shear waves and \( K_2 \) is the no-slip lining response coefficient.

In seismic analysis of Jiroft water-transform tunnel, the intensity of earthquake is assumed to be 7.5 \( M_w \), the tunnel distance from center of earthquake (epicenter) is 15 \( km \), the ratio of peak ground particle acceleration at surface than maximum ground acceleration is 97, the maximum acceleration of S-wave is 3.44 \( m/s^2 \), the angle of wave propagation is 25 degrees, and the axial strain via Newmark’s formulation is calculated to be 0.0003. Therefore:

\[ V_s = \left(\frac{97}{9.81}\right) \times 3.44 \approx 34 \frac{cm}{s} = 0.34 \frac{m}{s} \]  

(31)

The results of seismic analysis of Jiroft water-transform tunnel in full-slip and no-slip assumptions based on Penzien’s equations and Wang’s equations are shown in table (3).

<table>
<thead>
<tr>
<th></th>
<th>Penzien</th>
<th>Wang</th>
<th>Penzien</th>
<th>Wang</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending moment ((KN.m))</td>
<td>85.1</td>
<td>42.6</td>
<td>85.2</td>
<td>42.6</td>
</tr>
<tr>
<td>Axial force ((KN))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending moment ((KN.m))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial force ((KN))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table (3), the results of seismic analysis

**Conclusion**
In full-slip assumption, the axial (thrust) force and the bending moment due to vibrations of earthquake that are calculated by Penzien’s and Wang’s equations are the same. Furthermore, in the no-slip assumption, the bending moments of either equations are the same, however, the axial force that is calculated by Wang’s equations is much greater than Penzien’s equation’s result. The reason behind this high difference is taking into consideration the compressibility and flexibility coefficients in Wang’s equations. The flexibility coefficient represents the ability of tunnel lining under the vibration of an earthquake. On the other hand, the rigidity of the tunnel and surrounding soil and rock mass by compressibility ratio and flexibility ratio of the tunnel lining are also considered. Whenever the flexibility ratio decreases, the potential of deformation and failure of tunnel is increased. Therefore, the tunnel lining properties, especially rigidity or flexibility of lining, play an important role in the stability of the tunnel under such dynamic loads as an earthquake.

Reference

[1]
[2]
[3]
[4]