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## MULTIPLANE COHESIVE-ZONE MODELS ACCOUNTING FOR FRICTION, FINITE DILATION AND ASPERITY DEGRADATION UNDER MIXED-MODE CYCLIC LOADING

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**Abstract.** *Multi-plane Cohesive-Zone Models (M-CZMs), based on the concept of Representative Multiplane Element (RME), provide an intermediate option between macroscale CZMs and full multiscale approaches for the analysis, within the mechanics of generalized continua, of mixed-mode fracture over micro-structured interfaces where initiation and propagation is expected. In M-CZM-based numerical methods, the number of planes in the RME can be selected as a trade-off between the computational effort and the sought level of detail in representing damage distribution over different 3D space orientations within the material across the process zone. The nonlinear features of the response over the individual elementary planes can be chosen, as well, according to the cohesive/viscous or brittle/ductile nature of the inelastic degradation phenomena occurring in the fracture process zone.*

*Some recent advances are presented concerning: 1) the extension of the basic M-CZM formulation from 2D kinematics to 3D kinematics to address problems of mixed-mode crack propagation in presence of finite dilation, and 2) the response of M-CZMs under mixed-mode repeated cyclic loading, when the finite depth of the micro-structured asperities is accounted for as a reduction in contact area between interfacing elementary planes and when degradation of asperities is modelled as a progressive reduction of the inclination of the planes belonging to the RME.*

## 1 INTRODUCTION

Multi-plane Cohesive-Zone Models (M-CZMs) [9, 11], based on the concept of Representative Multiplane Element (RME), provide an intermediate option between macroscale CZMs [4, 8] and full multiscale approaches for the analysis of mixed-mode fracture over micro-structured interfaces. In M-CZM-based numerical methods, the number of planes in the RME can be selected as a trade-off between the computational effort and the sought level of detail in representing damage distribution over different 3D space orientations within the material across the process zone. Similarly, the nonlinear features of the response over the individual elementary planes can be chosen according to the cohesive/viscous or brittle/ductile nature of the inelastic degradation phenomena occurring in the fracture process zone.

When a cohesive-frictional response over the individual elementary planes is employed [1], M-CZM formulations capture the increase of measured fracture energy, under increasing mode II/mode I ratio, as a natural effect of multiscale coupling between cohesion, friction and interlocking [14]. This coupling ultimately results into an increasing amount of energy dissipated by friction, which is obtained employing a reduced set of micromechanical parameters each characterized by a well-defined micromechanical interpretation, thus permitting to devise neat calibration and identification procedures for fracture problems in quasi-brittle materials, such as concrete [15], masonry [18] and composites [16].

The authors recently proposed enhancements of the basic M-CZM formulation aiming at: 1) extending the model from 2D kinematics to 3D kinematics to address problems of mixed-mode crack propagation [10, 11], and 2) account for the finite depth of the surface asperities while capturing their progressive rupture and wear, in terms of a progressive reduction of the inclination of the planes belonging to the RME [12]. In the present contribution the results related to the response under cyclic loading are briefly reviewed illustrating the potentiality of the model in describing fatigue-like phenomena in quasi-brittle materials.

## 2 MULTIPLANE INTERFACE FORMULATION

A synoptic account of the M-CZM framework is hereby provided. The basic M-CZM formulation, which predicts an indefinitely dilating response in mode II, employs the formulation by Alfano and Sacco [1] as component model to incorporate damage and friction, and which represents the response of the individual *ideally flat* (i.e., without local dilatancy) elementary planes composing the RME. The RME is a repeating unit deputed to describe the microstructured geometry of the interface (see Figures 1 and 2). When some of the elementary planes, in number  $N_p$ , have orientation different from the orientation of the average plane, the RME, as a repeating unit, permits to represent asperities.

Hypotheses on the mechanical behavior are added aiming at preserving a formulation within the framework of the mechanics of generalized continua and at limiting the addition of phenomenological ansatzs to a minimum number and to maximum simplicity. Accordingly, the kinematics of the the RME is described by a unique relative-displacement vector  $\mathbf{s}$  which is the same for every elementary  $k$ -th plane  $\mathbf{s}^{(k)}$  ( $\mathbf{s}^{(k)} = \mathbf{s}$ ). In absence of inelastic processes the elementary planes are assumed infinitely stiff and with fixed orientation.

Relying on the extensive character of the free energy and denoting by  $\gamma_k = A_k/A_P$  the ratio of the effective area of the  $k$ -th microplane over the overall projected onto the average plane,  $A_P$ , the macroscale free energy density,  $\Psi$ , of the interface is defined as the sum of the free energies associated with each elementary plane,  $\Psi^{(k)}$ , weighted by the respective area fractions:  $\Psi = \sum_{k=1}^{N_p} \gamma_k \Psi_k$ . A damage parameter  $\alpha^{(k)}$  and an internal variable of local tangential relative

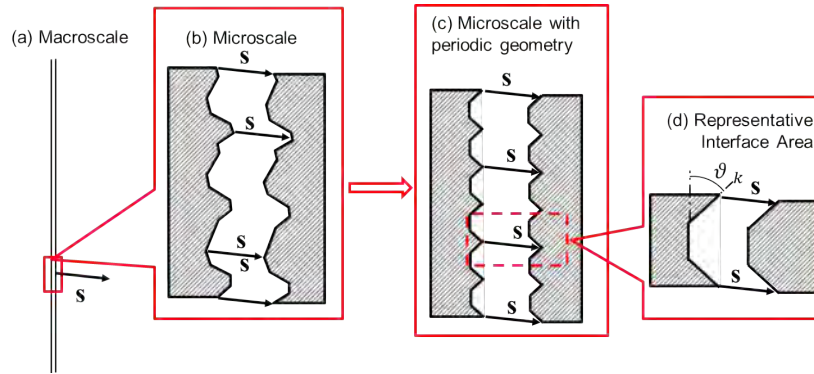


Figure 1: Multiscale scheme: (a) flat macro-scale geometry; (b) geometry of the asperities accounted for at the micro-scale; (c) micro-scale geometry with simplified periodic pattern; (d) representative interface area (repeating unit).

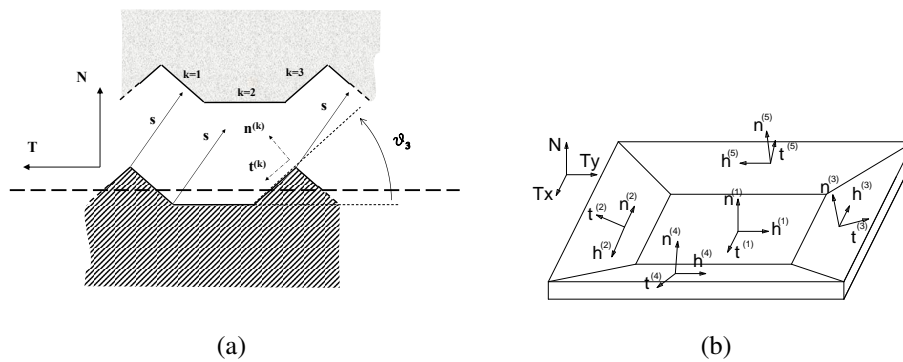


Figure 2: (a) example of 2D trapezoidal RME; (b) example of 3D 5-plane RME with truncated pyramid shape.

displacement  $s^{di(k)}$ , related to friction, are introduced, associated with each  $k$ -th elementary plane.

Under the above summarized assumptions, the macro-scale cohesive law relating  $s$  to the macro-scale stress  $\sigma$  is obtained upon solving the nonlinear micro-scale equilibrium problem for the RME. These hypotheses, although very simple, determine a rich multi-scale mechanical behavior which is capable of recovering an increase of mode II fracture energy due to friction [14]. This basic version of the M-CZM is suitable for modelling decohesion processes in which relative displacements are small compared to the characteristic dimension of the interface irregularities, and determines in mode II an indefinitely dilating response (which, in short, is referred to as *infinite dilation*).

### 3 FINITELY DILATING MODEL

To encompass decohesion processes in which relative displacements are not small compared to the dimension of asperities, a *finitely dilating* enhancement of basic M-CZM has been proposed in [13], essentially based on the consideration of a finite height of asperities  $H_N$  and on the statement of the equilibrium of the RME in the current displaced configuration of the interacting elementary planes. Denoting by  $\gamma_{0k}$  the initial microplane area fraction, the microplane

area fraction is updated by the following relation:

$$\gamma_k = \gamma_{0k} \mathcal{A}_g \left( \frac{s_N}{H_N} \right), \quad (1)$$

in which  $\mathcal{A}_g$  is a function accounting for the opening-induced reduction of the area of potentially contacting elementary planes belonging to the opposite crack lips:

$$\mathcal{A}_g(x) = \langle 1 - \langle x \rangle \rangle. \quad (2)$$

An extension of the finitely dilating M-CZM to 3D kinematics has been developed and implemented for the FEM prediction of the mixed mode I-II and mixed mode I-III fracture resistance by simulating tests with Double-Cantilever Beams (DCBs) made of E-glass laminates [11] and with arms having a rectangular cross section with aspect ratio  $H/B$ . Results from a related campaign of numerical simulations provide, at a computational cost considerably lower than full multi-scale approaches [2,3,6,7,19], insights on the mechanics of those mixed-mode debonding and delamination processes in quasi-brittle materials such that friction and small scale irregularities are significant in the fracture process. Examples of typical deformed meshes obtained in pure mode III debonding and in mixed mode I-II debonding are reported in Figure 3.

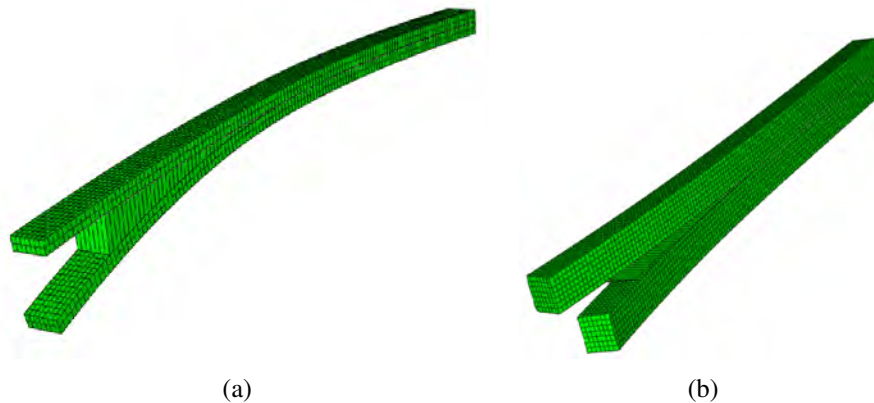


Figure 3: Deformed meshes of simulated DCB tests: (a) simulation under mixed mode I-II loading with  $H/B=0.33$ , and (b) simulation under pure mode III loading with  $H/B=1.00$ .

Numerical results obtained with the 3D finitely dilating M-CZM show that the height of the asperities plays a fundamental role in determining the energy release rate both in mixed mode I-II and mixed mode I-III interaction. The results of simulations also permit to identify an important factor of dependence of the crack growth resistances upon the test-setup geometry when  $H_N$  is different from zero. Specifically, different DCB sections result in significantly different energy release rate in mixed mode I-mode III interaction. Consequently, when friction and irregularities generated within the process zone are non-negligible the analyses show that it is not possible to define mode-II and mode-III fracture toughness as objective material properties of a structural interface independent from the employed test setup.

The 3D finitely dilating M-CZM has been also employed to describe the response of masonry joints. A modelling strategy is proposed in [10], in which a zero-thickness 3D multiplane-CZM is used to embed all nonlinear features of the response of the masonry joint, and is coupled with an elastic layer of finite thickness and stiffness incorporating the elastic properties which the mortar layer shows before the onset of any inelastic process.

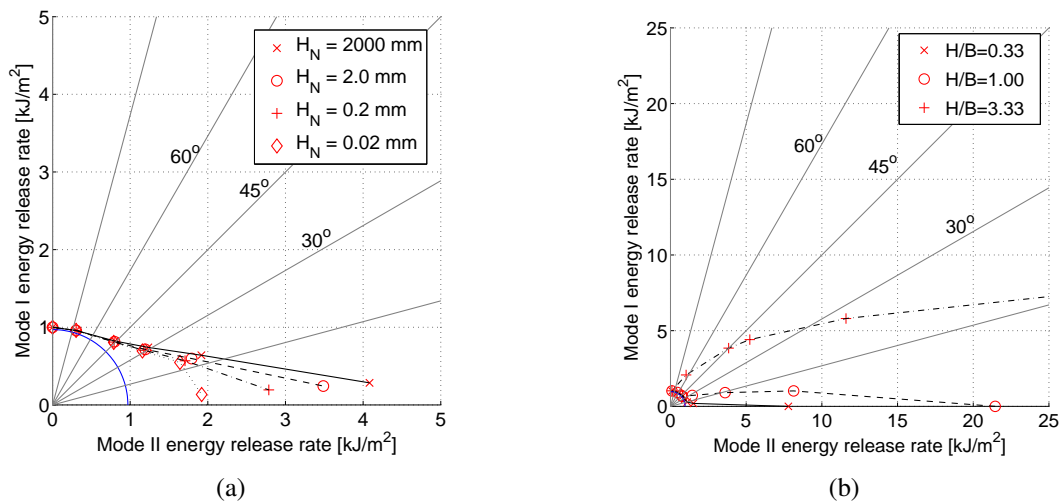


Figure 4: (a): polar plot of the plateau value of the mode-II energy release rate as function of mixed mode loading angle, for the mixed mode I-II delaminations with  $H_N$ , spanned in the range 0.2 mm-2000 mm; (b): Polar plot of the plateau value of the mixed-mode I-III energy release rate as function of the mode mixity angle in mode III with  $H_N = 2.00$  mm.

A numerical-experimental assessment of the capability of this strategy in predicting the flexural response of masonry wallettes has been also presented by the authors, employing the data available from the experimental campaign of Van der Pluijm [17]. This assessment was conducted by applying a consistent and reproducible calibration procedure whereby only the data from the meso-scale response of units and mortar joints are employed as a source for material parameters. Conversely, the data from the structural bending tests on masonry wallets of the same campaign are used, subsequently, to assess the simulated flexural behavior of masonry wallettes, taking also into account the dispersion of experimental data. This protocol for the numerical-experimental comparison aims to mimic a blind-like procedure which avoids fine tuning of the mesoscale model parameters on the basis of experimental structural data.

The analyses have shown a reasonably satisfactory numerical-experimental correlation. The simulations with the individual set of material parameters stemming from the meso-scale calibration procedure were found to capture with reasonable agreement the initial stiffness and the collapse point of two different wallette typologies, coded JO.VER and JO.HOR, tested in [17]. In the specimen JO.VER brick alignments are directed transversally to the flexure plane, as shown in Figure 5a, while, in the JO.HOR specimen, bricks are aligned as the longitudinal axis of the wallette (see Figure 5c). Figure 6 compares the numerical and experimental output data available for the JO.VER and JO.HOR setups. The numerically simulated moment-curvature curves are contained within confidence regions defined on a statistical basis from experimental data, see Figure 6. This comparison appears to be even more satisfactory by considering that the different alignment of bricks in the two wallette typologies determines significantly different mode-mixity in crack formation. Numerical simulations show that damage in JO.VER specimens is mode-I dominated while damaging in JO.HOR is developed with a prevailing mode-III cracking which determines a more brittle and dilating response with the lateral expulsion of the most external bricks.

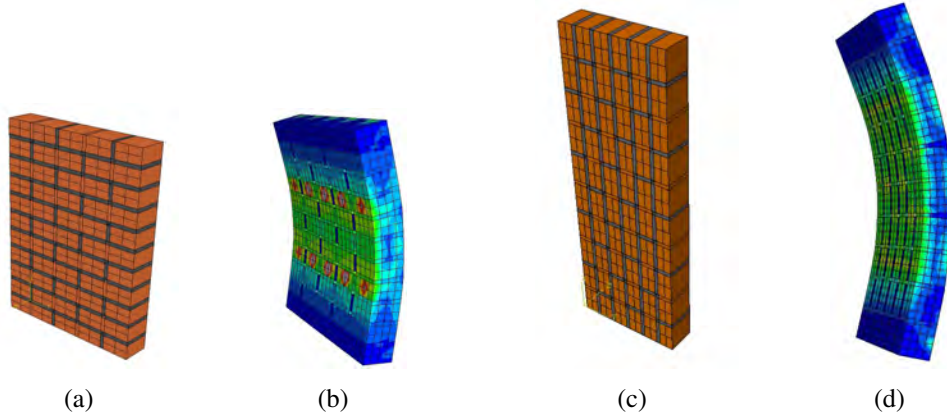


Figure 5: Simulated wallette four point bending tests: (a) JO.VER undeformed; (b) JO.VER deformed; (c) JO.HOR undeformed; (d) JO.HOR deformed.

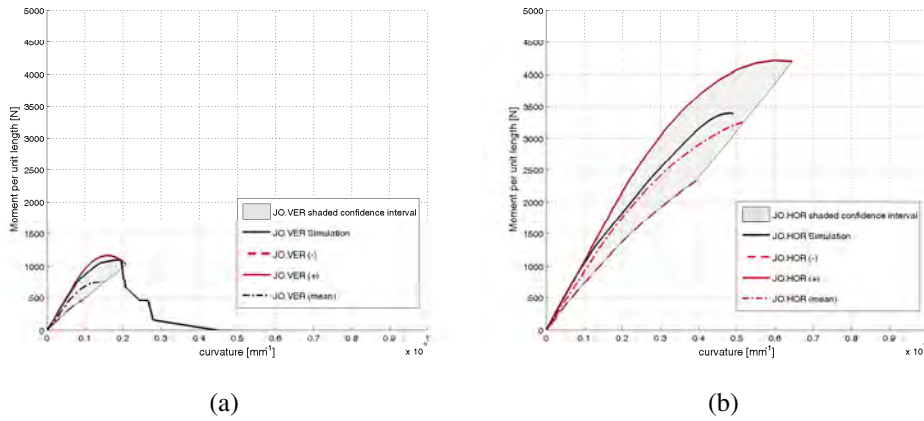


Figure 6: Numerical-experimental comparison for Moment-Curvature curves in four point bending tests over masonry wallettes: (a) JO.VER wallette typology; (b) JO.HOR wallette typology.

#### 4 MODELLING OF PROGRESSIVE INTERLOCKING DEGRADATION

In [13], account of degradation of asperities has been also included in a M-CZM to capture the decrease of the interlocking effect induced by damage. This effect is addressed by considering the following exponential law for the decrease of the current value  $\theta_k$  of the inclination angle of a given elementary plane  $k$  :

$$\theta_k = (\theta_{k0} - \theta_{kf}) e^{-\frac{\zeta_k}{\zeta_{k0}}} + \theta_{kf}. \quad (3)$$

In (3)  $\zeta_k$  is the frictional work spent in sliding along the local tangential direction of the  $k$ -th plane:

$$\zeta_k = \int_{history} \sigma_{kt} ds_{kf}. \quad (4)$$

Quantity  $\zeta_{k0}$  is a characteristic energy value controlling the rate of asperities degradation whereas  $\theta_{k0}$  is the microplane inclination angle at the beginning of the analysis and  $\theta_{kf}$  is the angle asymptotically reached in the  $k$ -th microplane when  $\zeta_k \rightarrow \infty$ .

A numerical-experimental comparison of the tangential stress-slip and dilation-slip curves describing the response obtained in the first two cycles of the tests over rough granite joints, reported in [5], is shown in Figures 7 and 8. The numerical response shown in these figures is obtained employing the above described strategy for capturing damage-induced interlocking decrease into the finitely dilating 2D M-CZM. For all details concerning the calibration procedure adopted to set the material parameters of the RME in these analyses the reader is referred to reference [13].

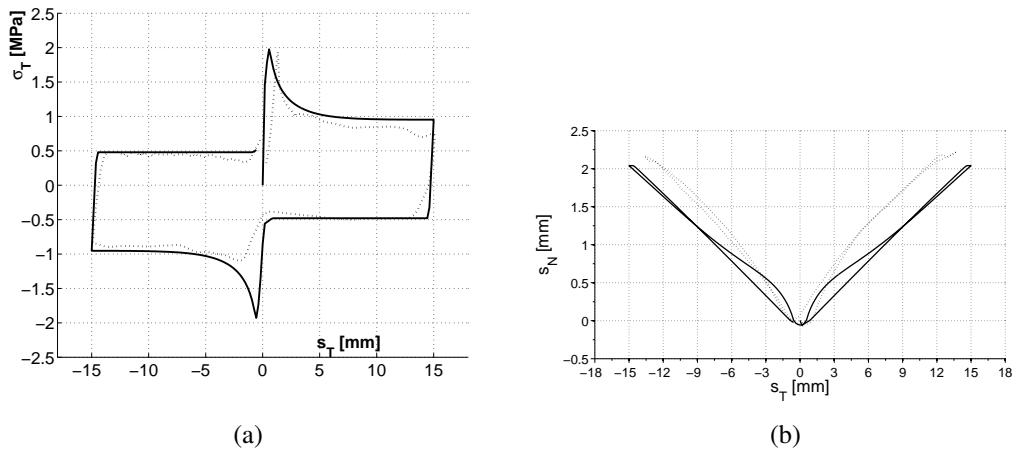


Figure 7: Experimental (dotted lines) and numerical (solid lines) shear stress-slip curves (a) and dilation-slip curves (b) for the rough granite joints tested by Lee et al. [5] in the first loading cycle.

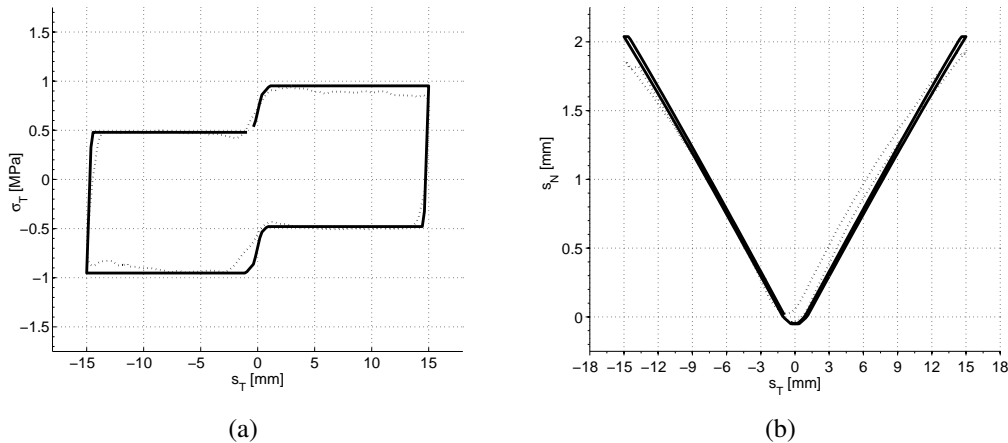


Figure 8: Experimental (dotted lines) and numerical (solid lines) shear stress-slip curves (a) and dilation-slip curves (b) for the rough granite joints tested by Lee et al. [5] in the second loading cycle.

An interesting feature of the finitely dilating M-CZM with asperity degradation is finally highlighted by Figure 9. Repeated cycles of applied tangential stress of constant amplitude determine a damaging response with accumulation of damage and residual displacement. The stability properties of this behavior, which is detected only above a certain threshold of the maximum applied tangential stress, are currently under investigation.

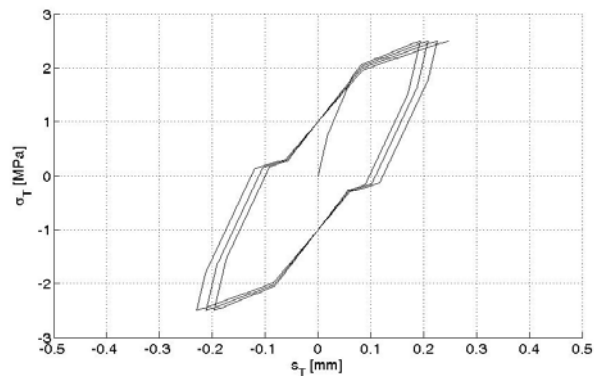


Figure 9: Tau-slip curves obtained with three tangential stress cycles of constant amplitude.

## 5 CONCLUSIONS

Some recent advances have been surveyed concerning the extension of the basic M-CZM formulation from 2D kinematics to 3D kinematics to address problems of mixed-mode crack propagation in presence of finite dilation, and the response of M-CZMs under mixed-mode repeated cyclic loading. The results recollected in this short memory from published records show that M-CZMs:

- have provable predictive capabilities in determining the mixed-mode decohesion response in FRP and masonry structures;
- permit to describe complex 3D mechanics of mixed-mode fracture propagation;
- permit to elucidate and separate, in the design of experimental test standard for structural interfaces, the role of friction in contributing to the overall fracture toughness, and to clarify the capability of a given test setup to measure mode-II and mode-III fracture toughness as objective material properties independent from the employed test setup;
- under repeated cyclic loading, have potentiality to analyze fatigue-like behaviors in quasi-brittle materials originated by the interaction between friction and damage.

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