

Arc Phenomena in low-voltage current limiting circuit breakers

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1 Introduction

This problem was brought to the Study Group by Dr. Suleiman Sharkh, Electromechanical Engineering, University of Southampton.

Circuit breakers are an important safety feature in most electrical circuits, and they act to prevent excessive currents caused by short circuits, for example. Low-voltage current-limiting circuit breakers are activated by a trip solenoid when a critical current is exceeded. The solenoid moves two contacts apart to break the circuit. However, as soon as the contacts are separated an electric arc forms between them, ionising the air in the gap, increasing the electrical conductivity of air to that of the hot plasma that forms, and current continues to flow. The currents involved may be as large as 80,000 amperes.

Critical to the success of the circuit breaker is that it is designed to cause the arc to move away from the contacts, into a widening wedge-shaped region. This lengthens the arc, and then moves it onto a series of separator plates called an arc divider or splitter.

The arc divider raises the voltage required to sustain the arcs across it, above the voltage that is provided across the breaker, so that the circuit is broken and the arcing dies away. This entire process occurs in milliseconds, and is usually associated with a sound like an explosion and a bright flash from the arc. Parts of the contacts and the arc divider may melt and/or vapourise.

See Fig. (1) for a view of the inside of a typical low-voltage circuit breaker.

The question to be addressed by the Study Group was to mathematically model the arc motion and extinction, with the overall aim of an improved understanding that would help the design of a better circuit breaker.

Further discussion indicated that two key mechanisms are believed to contribute to the

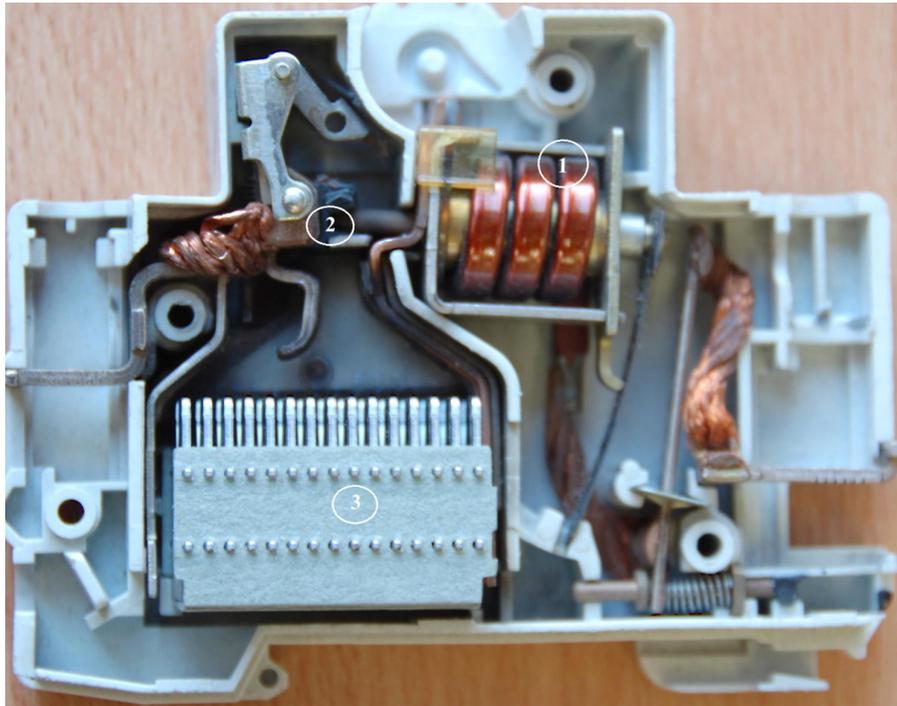


Figure 1: A photograph of the inside of a low-voltage circuit breaker, showing trip solenoid (1), contacts (2), and arc divider (3). Source: Mark McGuinness

movement of the arc away from the contacts, one being self-magnetism (where the magnetic field associated with the arc and surrounding circuitry acts to push it towards the arc divider), and the other being air flow (where expansion of air combined with the design of the chamber enclosing the arc causes gas flow towards the arc divider).

Further discussion also indicated that a key aspect of circuit breaker design was that it is desirable to have as fast a quenching of the arc as possible, that is, the faster the circuit breaker can act to stop current flow, the better. The relative importance of magnetic and air pressure effects on quenching speed is of central interest to circuit design.

2 Circuit breaker design

With reference to Fig. (1), we can describe the action of a circuit breaker as follows:

- Initially the contacts (2) are closed and the circuit filled with air at room temperature.
- A large current flowing in the circuit will trigger the solenoid (1), which forces the

contacts apart.

- The current is able to leap across the gap between the contacts provided that the voltage drop between the contacts is not too large.
- The passage of the current heats and ionises the air (Ohmic heating), greatly increasing the conductivity of the air.
- Due to this increase in conductivity, the current prefers to travel along paths which are already heated, and thus heats them further. Temperatures inside the circuit breaker are of the order of 10,000 K. Movement of the hot plasma is critical to moving the arc away from the contacts and onto the splitter.
- The circuit breaker is designed to increase the voltage drop between the contacts with time, until it eventually becomes greater than the voltage source and the current stops. However, the voltage drop decreases with time in a single arc, since it gets hotter and hence more conductive.
- The breaker is designed to move the arc away from the contact region towards the stack of parallel V-shaped plates (3), so that the distance between the anode and cathode increases. There is a risk of further arcs forming in the narrower region between the contacts, especially if the air between the contacts is still ionised. The formation of further arcs hinders the function of the circuit breaker and should be avoided if possible.
- The primary contribution to the voltage drop comes from a boundary layer near the electrodes, which provides a drop of 10 V.
- In order to pass through the stack of plates at (3), the arc would have to overcome the voltage drop from the boundary layer at each of these plates. Eventually, this causes the current to stop.

We would like to determine the mechanism by which the arc is moved towards the stack. Possible candidates include self-magnetism by the arc itself, attraction to the V-shaped plates, air flow from the vents in the circuit breaker, and shock waves from the heated air.

3 3D equations of resistive compressible MHD

From the description of the action of the circuit breaker, it seems that the temperature, electrical conductivity, electric field and fluid velocity are all important in understanding the formation and motion of the arc. We use continuum equations to link these quantities,

in the form of Maxwell's equations, the Navier-Stokes equations, the equation of state for an ideal gas and an equation for conservation of entropy.

3.1 Maxwell's equations

We begin our analysis by considering Maxwell's equations, which are:

$$\nabla \cdot \mathbf{B} = 0, \quad (1)$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_{charge}}{\epsilon_0}, \quad (2)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad (3)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \quad (4)$$

Here \mathbf{B} is the magnetic field, \mathbf{E} the electric field, \mathbf{j} the current density and ρ_{charge} the electric charge density. The constants ϵ_0 and μ_0 are the permittivity and permeability of free space.

We neglect the displacement current term in (3) because our fluid flow is much slower than the speed of light (note that $\epsilon_0 \equiv \mu_0^{-1} c_0^{-2}$). We may therefore replace (3) with

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}. \quad (5)$$

Away from a thin layer near the electrodes we assume charge neutrality, so that $\rho_{charge} \equiv \rho_+ - \rho_- \approx 0$. At leading order we take $\rho_{charge} = 0$ and neglect equation (2). In general, however, we may not assume $\nabla \cdot \mathbf{E} = 0$.

The electric field \mathbf{E} and the current density \mathbf{j} are linked by Ohm's law, which for a moving conductor gives

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \mathbf{j}/\sigma. \quad (6)$$

Here $\sigma \equiv \sigma(T)$ is the conductivity of air, which depends strongly on temperature (see Belbel and Lauraire [1]), and \mathbf{u} is the fluid velocity.

3.2 Fluid equations

We now turn our attention to the fluid flow. We can write mass conservation as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0. \quad (7)$$

In a similar way, we write conservation of momentum as

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla \cdot (\rho\mathbf{u}\mathbf{u}) = -\nabla p + \mathbf{j} \times \mathbf{B} + \nabla \cdot \hat{\sigma}. \quad (8)$$

Here $\mathbf{j} \times \mathbf{B}$ is the Lorentz force. (There would also be an electric force if we were not assuming charge neutrality.) We can use the equation for mass conservation (7) to simplify (8) to

$$\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \frac{1}{\rho}\mathbf{j} \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \hat{\sigma}. \quad (9)$$

The viscous stress tensor $\hat{\sigma}$ is defined by

$$\hat{\sigma}_{ik} = \eta \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) + \left(\zeta - \frac{2}{3}\eta \right) \delta_{ik} \nabla \cdot \mathbf{u}. \quad (10)$$

This stress tensor involves both the dynamic viscosity η and the bulk viscosity ζ . The bulk viscosity is difficult to measure, though there is some speculation that $\zeta = 2\eta/3$. We will assume that ζ is of the same order of magnitude as η .

3.3 Gas model

We need two further scalar equations to close the system. We need to relate the pressure of the gas to its temperature, and also find how the temperature evolves.

The simplest model we can take is to assume that the heated air acts like an ideal gas. The equation of state is then

$$p = \rho RT, \quad (11)$$

where R is the specific gas constant for air.

Following Landau et al. [3, (Vol. 8, 2nd Ed., p229)], we can write the equation for conservation of total energy as

$$\rho T \left(\frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s \right) = \nabla \cdot (K(T)\nabla T) + \frac{\mathbf{j} \cdot \mathbf{j}}{\sigma(T)} - k_S B T^4 + \hat{\sigma}_{ik} \frac{\partial u_i}{\partial x_k}, \quad (12)$$

where s is the entropy per unit mass.

For a gas of fixed particle number N , pressure P , temperature T and volume V , we can derive an expression for total entropy using the Maxwell relation:

$$\left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V \quad (13)$$

and the definition of the total heat capacities of the system:

$$\hat{C}_V = T \left(\frac{\partial S}{\partial T} \right)_V, \quad \hat{C}_p = T \left(\frac{\partial S}{\partial T} \right)_P. \quad (14)$$

Using the equation of state (11), we obtain

$$\frac{S}{\hat{C}_V} = \frac{s}{C_V} = \log \left(\frac{p}{\rho^\gamma} \right). \quad (15)$$

Here $\gamma = C_p/C_v$. For an ideal monatomic gas, $\gamma = 5/3$, while for a diatomic gas $\gamma = 1.4$. γ varies from these theoretical values depending on temperature and the gas involved, but is generally $O(1)$.

We can use this definition of entropy, together with (11) and (7), to write the left hand side of (12) as

$$\rho T \left(\frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s \right) = \rho C_V \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) + p \nabla \cdot \mathbf{u}. \quad (16)$$

The terms on the right hand side of (12) represent various sources of heat. The term

$$\nabla \cdot (K(T) \nabla T)$$

is due to heat conduction. The thermal conductivity K is dependent on T , but varies more slowly with temperature than $\sigma(T)$ does. Ohmic heating is given by the term

$$\frac{\mathbf{j} \cdot \mathbf{j}}{\sigma(T)}.$$

Viscous dissipation for a compressible fluid is given by

$$\hat{\sigma}_{ik} \frac{\partial u_i}{\partial x_k},$$

where the stress tensor $\hat{\sigma}_{ik}$ is given by (10). We have also included a radiative heat loss term

$$k S_B T^4. \quad (17)$$

Here S_B is the Stefan-Boltzmann constant, and k is a constant with units of m^{-1} . This radiation model was adopted from Karetta and Lindmayer [2].

The temperature equation that results from (12), for an ideal gas, is

$$\rho C_V \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (K(T) \nabla T) + \frac{\mathbf{j} \cdot \mathbf{j}}{\sigma(T)} - k S_B T^4 + \hat{\sigma}_{ik} \frac{\partial u_i}{\partial x_k} - p \nabla \cdot \mathbf{u}. \quad (18)$$

3.4 Summary of dimensional equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (19)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho} + \frac{1}{\rho} \mathbf{j} \times \mathbf{B} + \frac{1}{\rho} \nabla \cdot \hat{\sigma}. \quad (20)$$

$$\rho C_V \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (K(T) \nabla T) + \frac{\mathbf{j} \cdot \mathbf{j}}{\sigma(T)} - k_S B T^4 + \hat{\sigma}_{ik} \frac{\partial u_i}{\partial x_k} - p \nabla \cdot \mathbf{u}. \quad (21)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (22)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (23)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \quad (24)$$

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \mathbf{j} / \sigma(T). \quad (25)$$

$$p = \frac{\rho R T}{m}, \quad (26)$$

3.5 Nondimensionalisation

We now nondimensionalise the equations given in Section 3.4. This gives

$$\frac{\partial \rho}{\partial t} + M \nabla \cdot (\rho \mathbf{u}) = 0, \quad (27)$$

$$N \left(\frac{\partial \mathbf{u}}{\partial t} + M \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\frac{\nabla p}{\rho} + \frac{2}{\beta} \frac{1}{\rho} \mathbf{j} \times \mathbf{B} + \frac{MN}{R_V} \frac{1}{\rho} \nabla \cdot \hat{\sigma}, \quad (28)$$

$$H_1 \rho \left(\frac{\partial T}{\partial t} + M \mathbf{u} \cdot \nabla T \right) = H_2 \nabla \cdot (K(T) \nabla T) + \frac{\mathbf{j} \cdot \mathbf{j}}{\sigma(T)} - \left(\frac{T}{T_R} \right)^4 \quad (29)$$

$$+ H_3 \hat{\sigma}_{ik} \frac{\partial u_i}{\partial x_k} - H_4 p \nabla \cdot \mathbf{u}, \quad (30)$$

$$-\Gamma \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (31)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (32)$$

$$\nabla \times \mathbf{B} = \mathbf{j}, \quad (33)$$

$$\mathbf{E} + R_m \mathbf{u} \times \mathbf{B} = \mathbf{j}/\sigma, \quad (34)$$

$$p = \rho T. \quad (35)$$

This nondimensionalisation has used the following rescalings:

$$T = T_0 \tilde{T}, \quad p = p_0 \tilde{p}, \quad \rho = \rho_0 \tilde{\rho}, \quad \mathbf{u} = U_0 \tilde{\mathbf{u}}, \quad \mu \approx \mu_0, \quad (36)$$

$$\mathbf{j} = j_0 \tilde{\mathbf{j}}, \quad \mathbf{E} = E_0 \tilde{\mathbf{E}}, \quad \mathbf{B} = B_0 \tilde{\mathbf{B}}, \quad \mathbf{x} = L_0 \tilde{\mathbf{x}}, \quad (37)$$

and all tildes have been dropped after the rescaling. Parameter values are listed in tables (1) and (2). Further relationships between scaling parameters are:

$$B_0 = L_0 \mu_0 j_0, \quad j_0 = \sigma_0 E_0, \quad p_0 = \rho_0 R T_0. \quad (38)$$

We have used the speed of sound to rescale plasma velocities, $U_0 = 340 \text{ m.s}^{-1}$, since there is an audible bang when the circuit breaker triggers.

The nondimensional parameters R_M and R_V are the magnetic and viscous Reynolds numbers, and β is the ratio between fluid and magnetic pressure in the plasma. These are defined as

$$R_m \equiv \mu_0 \sigma_0 U_0 L_0 \approx 10^{-3}, \quad (39)$$

$$R_V \equiv \frac{\rho_0 U_0 L_0}{\eta_0} \approx 10^3, \quad (40)$$

$$\beta \equiv \frac{p_0}{B_0^2 / 2\mu_0} \approx 10^4, \quad (41)$$

Equations (32) to (35) involve three further dimensionless parameters, Γ , M and N . Definitions and values of these parameters are below:

$$M \equiv \frac{U_0 t_0}{L_0} \approx 70, \quad (42)$$

$$\Gamma \equiv R_m/M \approx 10^{-5}, \quad (43)$$

and

$$N \equiv \frac{U_0 L_0 \rho_0}{t_0 p_0} \approx 10^{-3}. \quad (44)$$

The four parameters H_1 , H_2 , H_3 and H_4 describe the sizes of terms in the heat equation relative to Ohmic heating. Values of these parameters, using values tabulated in Tables (1) and (2), are as follows:

$$H_1 \equiv \frac{\rho_0 C_V T_0 \sigma_0}{t_0 j_0^2} = O(1), \quad (45)$$

$$H_2 \equiv \frac{K T_0 \sigma_0}{L_0^2 j_0^2} = O(1), \quad (46)$$

$$H_3 \equiv \frac{\eta U_0^2 \sigma_0}{L_0^2 j_0^2} \approx 10^{-3}, \quad (47)$$

$$H_4 \equiv \frac{p_0 U_0 \sigma_0}{L_0 j_0^2} \approx 20. \quad (48)$$

Finally, T_R is the nondimensional temperature at which radiation balances Ohmic heating.

$$T_R \equiv T_0^{-1} \left(\frac{k S_B \sigma_0}{j_0^2} \right)^{-1/4} \approx 1. \quad (49)$$

4 Discussion

4.1 Leading order equations

At leading order with these scalings, the equations for temperature and the electromagnetic fields uncouple from the fluid velocity. We are left with:

$$\nabla \cdot \mathbf{B} = 0, \quad (50)$$

$$\nabla \times \mathbf{B} = \mathbf{j}, \quad (51)$$

$$\nabla \times \mathbf{E} = 0, \quad (52)$$

$$\mathbf{E} = \mathbf{j}/\sigma \quad (53)$$

and

$$0 = \nabla \cdot (K(T)\nabla T) + \frac{\mathbf{j} \cdot \mathbf{j}}{\sigma(T)} - T^4. \quad (54)$$

From (52) we can write $\mathbf{E} = \nabla\phi$, and from (51) we have $\nabla \cdot \mathbf{j} = 0$. We combine these with (53) and (54) to obtain

$$\nabla \cdot (\sigma(T)\nabla\phi) = 0 \quad (55)$$

and

$$0 = \nabla \cdot (K(T)\nabla T) + \sigma(T)|\nabla\phi|^2 - T^4. \quad (56)$$

4.2 Geometric considerations of heat loss

We began by looking for a solution to (55) in 1D. In particular we are seeking a solution for $T(x)$ that decays as $x \rightarrow \infty$. In one dimension, we can integrate (55) immediately to give $\sigma(T)\phi_x = A_0$, so that (56) becomes

$$0 = \frac{d}{dx} \left(K(T) \frac{dT}{dx} \right) + \frac{A_0^2}{\sigma(T)} - T^4. \quad (57)$$

The solution of this equation depends on how the thermal diffusivity K and electrical conductivity σ vary with temperature. We expect both will increase with temperature, with $\sigma(T)$ being more sensitive to changes in temperature than $K(T)$. We therefore take K to be constant.

For large x , we want $T \rightarrow 0$, so that the arc has finite width. However, the solution to $\nabla^2 T = 0$ in 1D is $T = Ax + B$; the far field sees a source at the origin. In 2D, the solution to $\nabla^2 T = 0$ grows as $\log(x)$. We need 3D effects to obtain a solution to $\nabla^2 T = 0$ that is bounded at infinity.

The conclusion from this analysis of the leading order equations is that we must include either heat loss in 3D or through the casing in the form of finite boundary conditions in order to obtain a satisfactory solution.

4.3 Another Scaling

There remains some uncertainty about the appropriate scalings to use, given that the gap between contacts changes from zero to about 5 mm over the timescale of 1 ms involved, and that the voltage and currents change during this time.

The voltage across the contacts typically increases linearly with time, and so does the distance between the contacts. In an experimental study with various speeds of contact separation and a variety of contact materials and currents, Belbel and Lauraire [1] find that the electric field does not vary much, and is typically measured to be $E_0 \approx 10^4 \text{ V.m}^{-1}$.

Altering E_0 from the value 10^3 V.m^{-1} used previously, to Belbel and Lauraire's value 10^4 V.m^{-1} , significantly changes these parameters:

$$H_1 \approx 10^{-2}, \quad H_2 \approx 10^{-2}, \quad H_4 \approx 0.2.$$

This modifies the leading order equations obtained earlier, so that fluid velocity is no longer decoupled from temperature and electromagnetic field equations. The energy equation (56) is modified to leading order to read

$$\rho \mathbf{u} \cdot \nabla T = \sigma(T) |\nabla \phi|^2 - T^4 - 0.2p \nabla \cdot \mathbf{u}. \quad (58)$$

This is coupled with equations (50) to (53), equation (55), and with the equations

$$\nabla \cdot (\rho \mathbf{u}) = 0, \quad (59)$$

$$\mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho}, \quad (60)$$

$$p = \rho T. \quad (61)$$

In 1D, eqn. (59) simplifies to $\rho u = c_0$, and eqn. (60) gives $c_0 u + p = c_1$, where c_0 and c_1 are $O(1)$ constants. Multiplying the latter result by ρ leads to a quadratic equation for ρ in terms of T and constants,

$$\rho^2 T - \rho c_1 + c_0^2 = 0. \quad (62)$$

Now the energy equation can be written in the form

$$c_0 T_x = \frac{A_0^2}{\sigma(T)} - T^4 - 0.2T c_0 \frac{\rho_x}{\rho}. \quad (63)$$

Since density is given in terms of T by the quadratic equation, this energy equation can in principle be integrated to obtain $T(x)$. Then density and fluid velocity follow immediately.

Differentiating the quadratic (62) gives

$$\rho_x = \frac{\rho^2 T_x}{c_1 - 2\rho T},$$

so that the differential equation (63) for T becomes

$$c_0(1.8\rho T - c_1)T_x = (2\rho T - c_1) \left(\frac{A_0^2}{\sigma(T)} - T^4 \right). \quad (64)$$

Temperature increases from small values to a steady state. The steady state occurs either when $T \rightarrow c_1/(2\rho)$, or when ohmic heating balances radiative heat loss, and $\sigma T^4 \approx A_0^2$, whichever happens first.

If $T \approx c_1/(2\rho)$, then $\rho \approx 2c_0^2/c_1$, and $u \approx c_1/(2c_0)$.

5 Ionisation model for the arc under contact opening

In this section we will look at a simple, ionisation driven model of the arc, and will use this to predict the arc-current I and the arc-voltage V as the contacts are opened. The model also gives an estimate for the arc-radius.

Firstly we consider a simple model for an arc-formation. Arcs are formed when the air is sufficiently ionised to conduct electricity. Ionisation is (in this context at least) largely driven by the action of a high electric field causing the separation of the air molecules into ions and electrons. The ionisation charge S^+ is generally given by an Arrhenius law with

$$S^+ = S e^{-\alpha/(E-E_I)}$$

where E_I is the ionisation field. This field depends upon the nature and composition of the gas. For dry air at room temperature it is about $3 \times 10^6 \text{V.m}^{-1}$, but for the arc in our circuit breaker the work of Belbel and Lauraire [1] suggests a value of about 10^4V.m^{-1} . Hence, from Maxwell's equations, in steady state, a one-dimensional electric field satisfies

$$-\mu E_x = S^+. \quad (65)$$

Note that μ is a small parameter. Typically what is then seen is a very large electric field close to one of the electrodes, where there is a large amount of ionisation and an accumulation of space charge in a thin boundary layer. This acts to suppress the electric field which then drops to a value close to the constant value of E_I away from the boundary later. Hence away from the electrodes we have

$$V_x \approx E_I. \quad (66)$$

Integrating E_x over the boundary layer, we see a very rapid voltage drop (typically about 10V) close to the electrode. Thus if the overall potential difference across the contacts between which the arc is formed is given by V and the length of the arc is L_A then

$$\frac{V - 10}{L_A} \approx E_I. \quad (67)$$

In the initial state of the device, the contacts are slowly opened so that

$$L_A = \beta t. \quad (68)$$

We deduce that there is a constant γ so that

$$V = 10 + \gamma t. \quad (69)$$

To determine the arc current I and also the arc dimensions, we must consider the external circuitry linking the arc to the remainder of the device. In particular the arc is formed by driving potential E , with the arc current I passing through an inductor L and a resistance R so that

$$L \frac{dI_A}{dt} + RI_A = E - V = (E - 10) - \gamma t. \quad (70)$$

Initially $I_A = 0$. Thus it is easy to deduce that

$$I_A(t) = \left(\frac{E - 10}{R} + \frac{\gamma L}{R^2} \right) \left(1 - e^{-Rt/L} \right) - \frac{\gamma t}{R}. \quad (71)$$

Observe that I rises rapidly and reaches a maximum value at a time

$$t = \mathcal{O} \left(\frac{L}{R} \right). \quad (72)$$

A representative plot of I , V and L_A is given in Fig. (2). These results compare well with experimental values for voltage across the contacts, and current through the fuse, obtained by Belbel and Lauraire [1]. A representative plot from their paper is the inspiration for Fig. (3). Note that during the time that voltage increases linearly (up to about 5 ms in the plot), the arc dwells on the contacts. The rapid increase in voltage over a timespan of about 1 μ s that follows this corresponds to a rapid movement of the arc off the contacts, along the rail and onto the splitter plates. A rapid decrease in current also occurs at this time, as the external circuit does not provide such a large driving voltage, and the circuit breaker is finally effective at breaking the current.

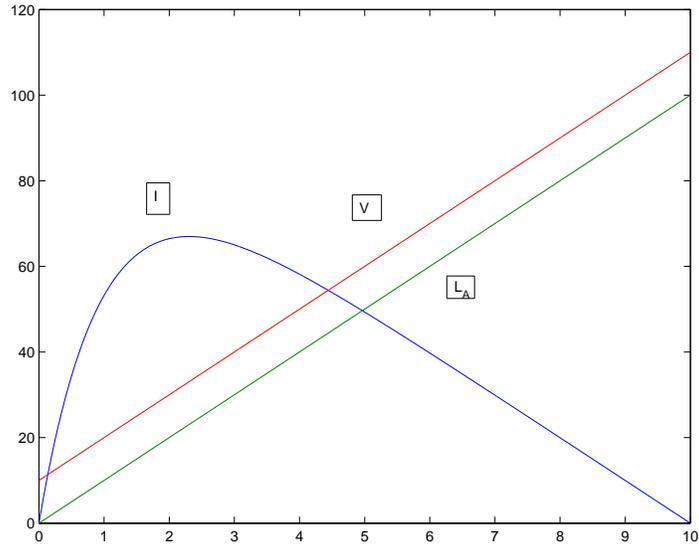


Figure 2: A representative plot of I , V and L_A for ionisation model. Source: Chris Budd

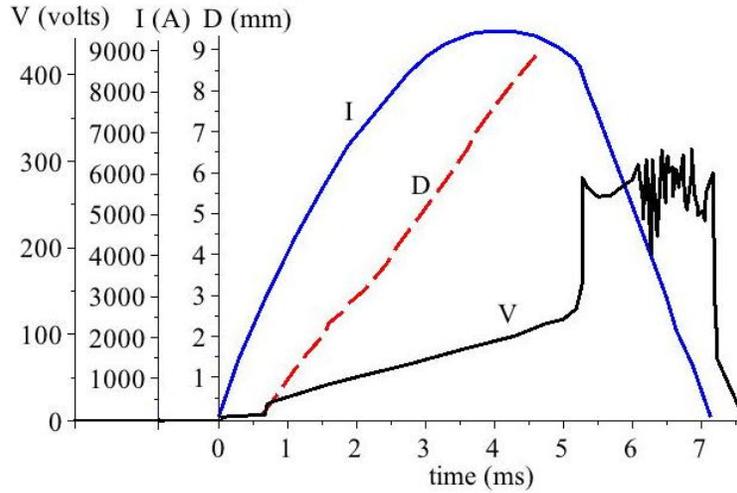


Figure 3: A representative plot of experimentally obtained values of I , V and L_A for a low-voltage circuit breaker. The upper curve shows current I through the device, the dashed line shows the contact separation distance D , and the lowest line shows the voltage V across the contacts, versus time in milliseconds. Source: Mark McGuinness, after Belbel and Lauraire [1]

From these simple estimates it is possible to derive an approximate formula for the dimensions of the arc which we will assume is roughly cylindrical of length L_A and of cross-sectional area A . For most of its length, the arc acts as a simple conductor of unit conductance σ which usually depends upon the temperature, with σ increasing with temperature. Thus the overall conductance of the arc is given by

$$\sigma_A = \sigma \frac{A}{L}.$$

Thus the arc current can be estimated from arc voltage drop away from the boundary layer and is given by

$$I_A = (V_A - 10)\sigma \frac{A}{L}.$$

Note that $(V_A - 10)/L = E_I$ is a constant. Thus

$$A = I_A/\sigma.$$

We deduce that the arc area initially grows as I_A grows and σ stays roughly constant. However, later on, as the temperature increases and as I_A starts to drop, we expect to see a reduction in the arc cross-section.

6 Conclusion

Further work is needed, to reduce the system of equations (27–35) to an understandable and easily soluble system. They also need to be properly coupled to the external circuitry, which is clearly important in explaining the behaviour of current through the device.

Much of previous work on modelling low-voltage circuit breakers has resorted to numerical simulation to explore dependency on circuit breaker design. Some conclusions can be made as the equations stand, however.

An explicit formula for the cross-sectional area of the arc has been obtained, by using a simple ionisation model coupled with the external circuit driving the breaker. This area increases with current flow, and decreases with arc temperature.

The relatively large size of M indicates that air velocity is significant in the total time derivative terms on the time scales considered, that is, temporal changes are dominated by advective effects. The large size of β , the ratio between fluid and magnetic pressures, clearly indicates that pressure differences generated by ohmic heating are more important than magnetic field effects due to current flows. This points the way for design considerations in circuit breakers — controlling air flow is indicated to be the most important design issue for improving breaker speed, given that contact points are being mechanically opened as rapidly as possible.

7 Acknowledgements

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A Parameter values

Variable	Symbol	Value	Reference
Temperature in an arc	T_0	10^4 K	
Current density	j_0	10^6 A · m ⁻²	
Electric field	E_0	10^3 kg · m · s ⁻³ · A ⁻¹	
Magnetic field	B_0	6×10^{-3} T	
Electrical conductivity of hot air	σ_0	10^3 S.m ⁻³	[1] [4]
Lengthscale	L_0	5×10^{-3} m	
Timescale	t_0	10^{-3} s	
Fluid velocity	U_0	340 m · s ⁻¹	
Density of hot air	ρ_0	0.1 kg · m ⁻³	
Hot air pressure	p_0	2×10^5 kg · m ⁻¹ · s ⁻²	

Table 1: Parameter values used to rescale/nondimensionalise the model.

Constant	Symbol	Value	Reference
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ m · kg · s ⁻² · A ⁻²	
Stefan-Boltzmann constant	S_B	6×10^{-8} kg · s ⁻³ · K ⁻⁴	
Radiation lengthscale	k^{-1}	13^{-1} m	[2]
Dynamic viscosity of air	η_0	1.5×10^{-4} kg · m ⁻¹ · s ⁻¹	[4]
Bulk viscosity of air	ζ_0	$\approx 10^{-4}$ kg · m ⁻¹ · s ⁻¹	
Gas constant for air	R	287 m ² · s ⁻² · K ⁻¹	
Heat capacity of (hot) air	C_p	3×10^3 J.kg.K ⁻¹	[4]
Thermal conductivity of hot air	$K(T_0)$	2 W.m ⁻¹ .K ⁻¹	[4]

Table 2: Other parameters and values.