University Bremen
Center for Cognitive Sciences

Theta rhythmicity enhances learning in adaptive STDP

Christian Albers, Joscha Schmiedt and Klaus Pawelzik
University of Bremen -- Institute for Theoretical Physics -- Department of Neurophysics
{calbers, schmiedt, pawelzik}@neuro.uni-bremen.de

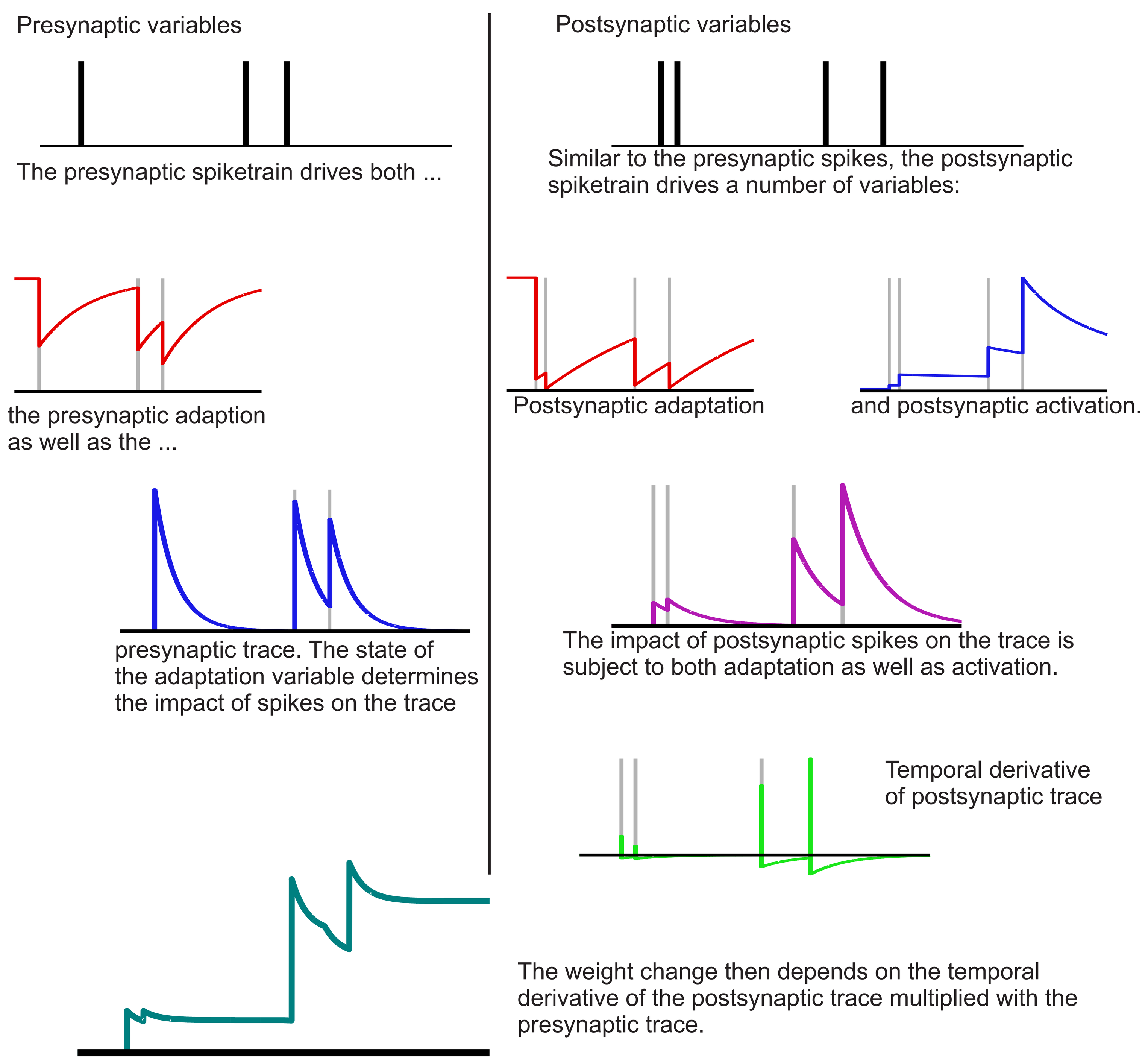
Bernstein Group for Computational Neuroscience Bremen

I. Introduction

Many theoretical studies on STDP model the synaptic change with the "double exponential" STDP curve, which is used as a kernel (but see [1], [2]) This approach does not take into account the nonlinear effects of spike interaction that have been observed in a number of experiments (e.g. [3], [4], [5]). We propose a new phenomenological model of STDP that is formulated as a set of differential equations, that reproduces a number of these effects. The benefit of this approach that we can use the equations to investigate synaptic changes also in response to abitrary continuous valued firing rates.

II. The model

Schematics of the behavior of the variables



The equations

(More details can be found in [7])

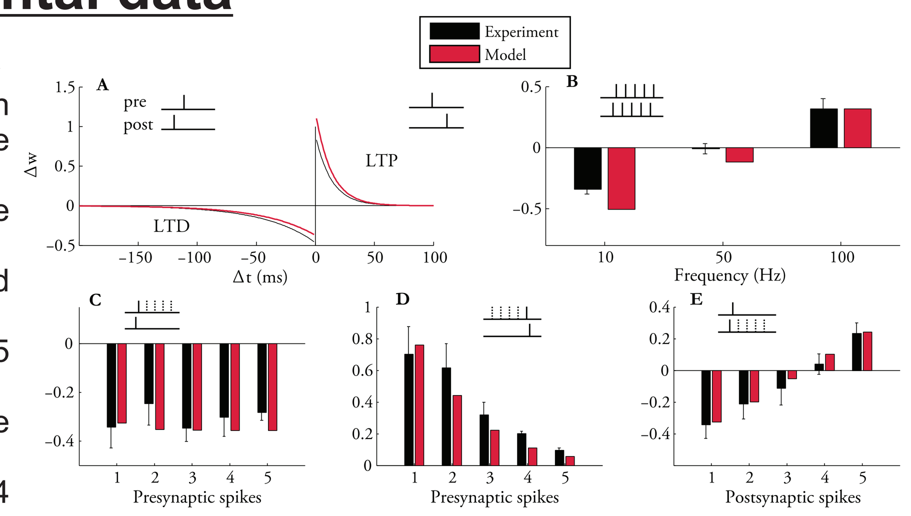
Spike train (delta-pulses)	$x_{pre} = \sum_i \delta(t - t^i_{pre})$	$x_{post} = \sum_j \delta(t - t^j_{post})$
Adaptation	$\dot{u}_{pre} = \frac{1 - u_{pre}}{\tau^{rec}_{pre}} - c_{pre} u_{pre} x_{pre}$	$\dot{u}_{post} = \frac{1 - u_{post}}{\tau^{rec}_{post}} - c_{post}(u_{post} - u_0) x_{post}$
Activation (only postsynapse)		$\dot{z} = -\alpha (z - z_0)^2 + c_{act} z x_{post}$
Trace	$\dot{y}_{pre} = -\frac{y_{pre}}{\tau_{pre}} + u_{pre} x_{pre}$	$\dot{y}_{post} = -\frac{y_{post}}{\tau_{post}} + u_{post} z x_{post}$

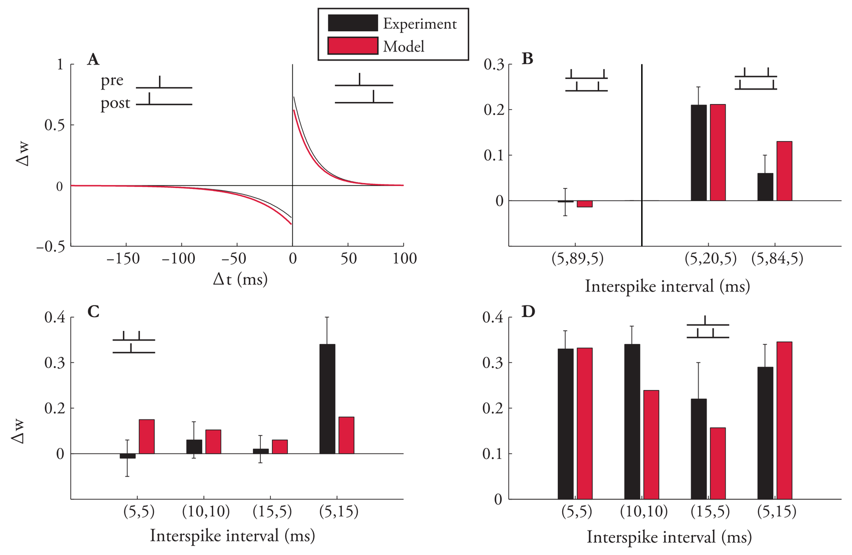
Weight change (differential Hebbian Learning)

$$\dot{w} = c_w y_{pre} \dot{y}_{post}$$

III. Fitting to experimental data

Fig. 1 ▶
Measurements of synaptic change in cortical slices form rats. Data from Froemke el al. [3]
A Synaptic change in response to spike pairs of different timing.
B 5-5 bursts, 5 pre-post spike pairs induced with different interpair intervals.
C A postsynaptic spike followed by 1 to 5 presynaptic spikes.
D 1 to 5 presynaptic spikes followed by one postsynaptic spike.
E First one post-pre pair, then 0 to 4 postsynaptic spikes.



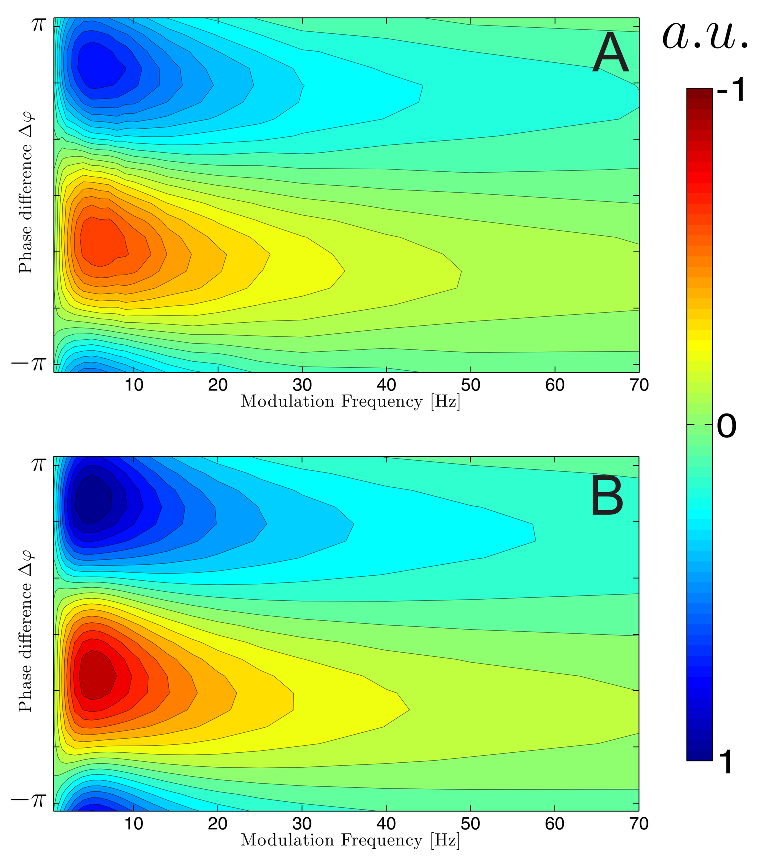


◀ Fig. 2
Measurements of synaptic change in cultured hippocampal cells. Data from Wang et al. [2]
A Synaptic change in response to spike pairs of different timing.
B Spike quadruplets. A pre-post pair followed by a post-pre pair or post-pre pair followed by a pre-post pair
C Spike triplets of the order pre-post-pre.
D Spike triplets of the order post-pre-post

	τ_{pre}	τ_{post}	τ^{rec}_{pre}	τ^{rec}_{post}	c_{pre}	c_{post}	u_0	z_0	c_{act}
$Hippocampus$	$16.8ms$	$33.7ms$	$0.5s$	$0.5s$	0.6	0.4	0.7	0.2	3.5
$Cortex$	$13.5ms$	$42.8ms$	$2s$	$0.2s$	0.9	1	0.01	1	1.5

◀ Table
Parameters of the fits to the data of Froemke et al. (Cortex) and Wang et al. (Hippocampus)

IV. Susceptibility of the synapse to sinusoidal rate modulations



▲ Fig. 3: Comparision of synaptic change in hippocampus per time as a function of modulation frequency and phase difference for a sinusoidal time course of the firing rate with $\varepsilon = 1$. **A** Average synaptic change for poissonian spike trains. **B** Synaptic change for the continuous equations. Same time course and parameters as above. The colorbar is the same for both plots.

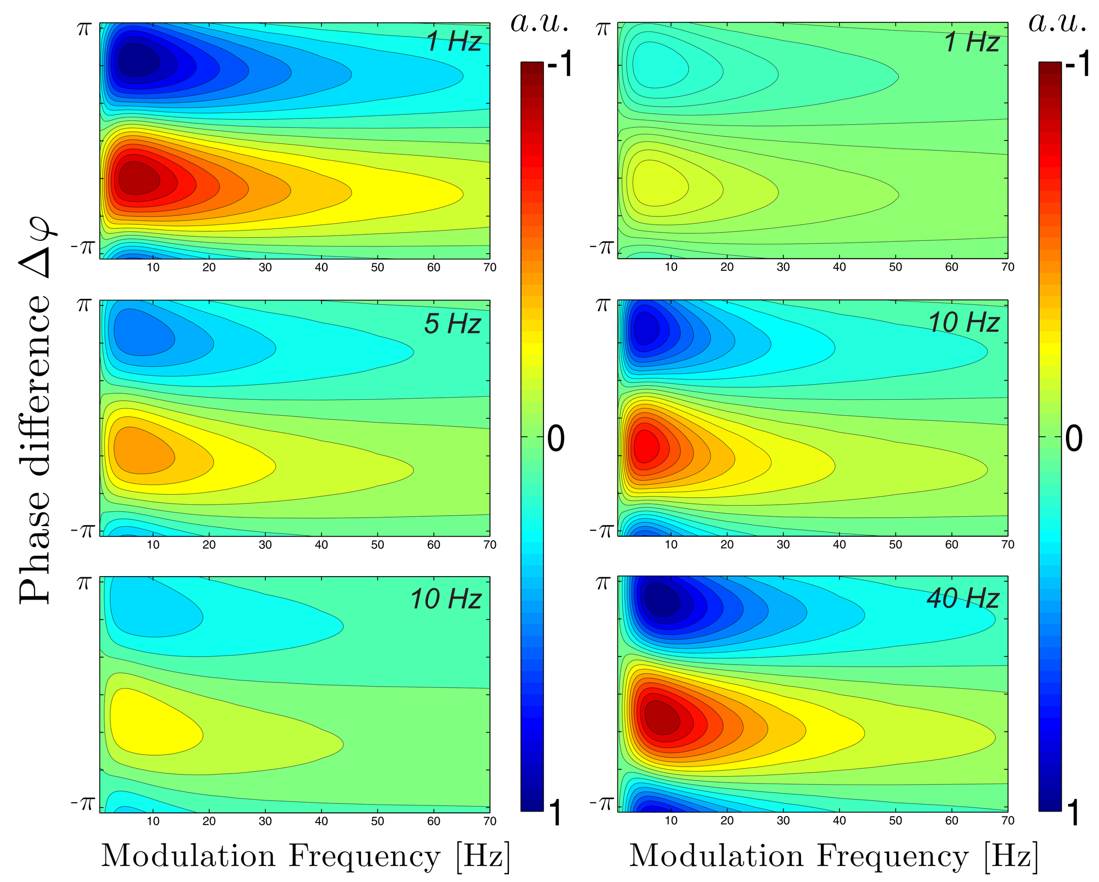
Under the assumption of poissonian firing of the neurons and neglecting correlations, the formulation of the model allows to use mean versions of the variables to investigate the synaptic change in response to continuous valued firing rates.
With that, we investigated the synaptic change in response to sinusoidal time courses of the firing rates of both the presynaptic and postsynaptic neuron, which are given by

$$r_{pre}(t) = r_0 \big(1 + \varepsilon \cos(2\pi f_{mod} t)\big)$$

$$r_{post}(t) = r_0 \big(1 + \varepsilon \cos(2\pi f_{mod} t + \Delta\varphi)\big)$$

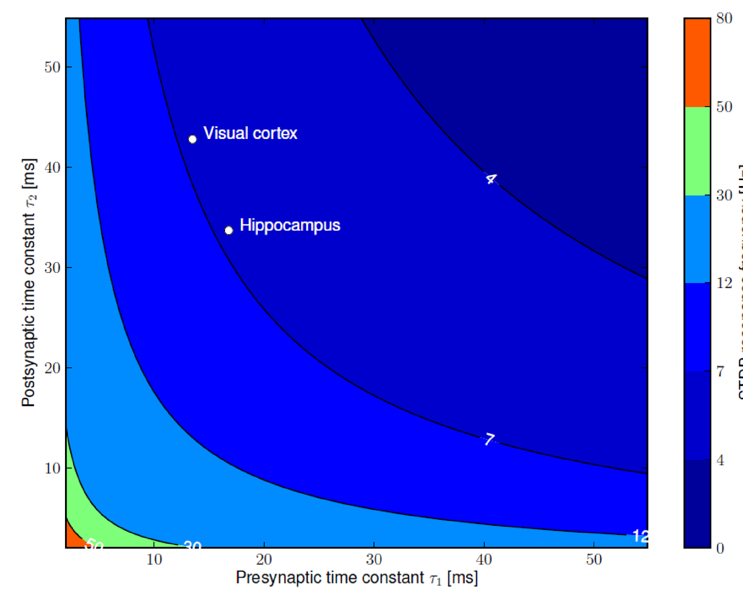
Where r_0 is the mean firing rate of both neurons, f_{mod} the modulation frequency and ε the deviation from a constant firing rate. The comparison in **Fig. 3** shows that the mean description deviates little from averaged poissonian spiking.
Our analysis shows that the synapse is most susceptible for rate modulations in the theta range (around 7 Hz), which is known to play a role in learning ([6]).

V. Comparison of Hippocampus and Cortex



◀ Fig 4: Synaptic change per time as a function of modulation frequency and phase difference in the cortex (left) and hippocampus (right) for small ε. The respective baseline firing rate is given in the plot.
The position of the maximal synaptic change (Potentiation and Depression) is the same for all baseline frequencies and lies at 7 Hz. The same is true in the case of $\varepsilon \approx 1$ (not shown).

Fig 5 ▶:
Analysis shows that the modulation frequency the synapse reacts most to depends almost only on the inverse product of the pre- and postsynaptic filter constants τ_{pre} and τ_{post}. This plot shows the modulation frequency the synapse is most susceptible to as a function of these two parameters. The white dots show the position of hippocampal and cortical parameters. The similarity in the resonance frequency is remarkable.



VI. Summary

- We introduced a model that captures a number of nonlinear effects in experiments on STDP
- The formulation allows the investigation the effects of delta spikes as well as continuous firing rates on the synapse with the same set of equations
- For regular sinusoidal modulations of a background firing rate, our analysis shows that both hippocampal and cortical synapses show maximal change for rate modulations at the same frequency in the theta-range
- Beyond the specific set of differential equations presented here, the formulation as differential equations allows adjusting the model to other experiments

VII. References

[1] Pfister and Gerstner, J. Neurosci. 26, 9673, 2006
[2] Shouval et al., Biol. Cybern., 87, 383, 2002
[3] Froemke et al., J. Neurophysiol. 95, 1620, 2006
[4] Wang et al., Nature Neuroscience, 8, 187, 2005
[5] Nevian and Sakmann, J. Neurosci., 26, 11001, 2006
[6] Fell and Axmacher, Nature Reviews Neuroscience, 12, 105, 2011
[7] Schmiedt et al., Advances in Neu. Inf. Proc. Systems (NIPS 2010), 23, 2010