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Paying for Loyalty: Product Bundling in Oligopoly

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In recent times, pairs of retailers such as supermarket and retail gasoline chains have offered bundled discounts to customers who buy their respective product brands. These discounts are a fixed amount off the headline prices that allied brands continue to set independently. In this paper, we model this bundling using Hotelling competition between two brands of each product. We show that a pair of firms can profit from offering a bundled discount to the detriment of firms who do not bundle and consumers whose preferences are farther removed from the bundled brands. Indeed, when both pairs of firms negotiate bundling arrangements, there are no beneficiaries (the effect on equilibrium profits is zero) and consumers simply find themselves consuming a sub-optimal brand mix. If the two separate products are owned by the same firm, additional complications arise although if both product sets are integrated, no bundled discounts are offered in equilibrium. *Journal of Economic Literature* Classification Numbers: L13, L41.

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1.   Introduction

Bundling has long been used as a business strategy and the benefits of bundling, particularly for a price discriminating monopoly selling complementary products, are well understood in economics.[1] An increasingly popular business strategy, however, involves firms in oligopolistic environments encouraging customer loyalty by offering interlocking discounts between particular brands of seemingly unrelated products. If customers buy one product then they can receive a discount if they buy a particular brand of some other product. The earliest examples of this strategy are reward points offered by credit card companies that can be redeemed as discounts or free offers from particular airlines, hotel chains, car rental companies or in some cases car manufacturers.[2] More recently, supermarket chains in the U.K., France and Australia have offered their grocery customers discount vouchers that can be redeemed when purchasing gasoline from particular retail petroleum chains.

At first blush, these discounts might appear to involve the bundling of complementary goods. Intensive credit card users may tend to be frequent travellers and those with large supermarket expenses also may tend to consume relatively more petrol. However, recent bundling by supermarkets and credit card companies has involved exclusive brand-specific relationships. For example, gasoline discount coupons offered

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[1] For example see Stigler 1968, Adams and Yellen 1976, Schalensee 1982 and McAfee, McMillan and Whinston 1989.

[2] Both GM and Ford have adopted such 'co-branding.' See Clark (1997) for an early review of these strategies.

by supermarkets are only redeemable at specific branded petroleum outlets. They cannot be redeemed on just any gasoline purchase. While a consumer's demand for groceries might be related to their purchase of gasoline, there is no particular reason to expect that a customer of a specific supermarket chain will gain any intrinsic value by also buying gasoline at a particular petroleum chain. For this reason, traditional explanations of bundling based on relationships between demands for alternative products are inadequate to explain this trend in exclusive co-branding.

Two other features also characterise recent bundling in credit cards and supermarkets. First, the bundling has occurred for both horizontally integrated and non-integrated (and indeed otherwise-unrelated) firms. For example, in the U.S., Walmart has experimented with bundled discounts by owning its own petrol pumps. In contrast, the Albertson supermarket chain teamed up with Arco (who are owned by BP Amoco) to offer loyalty discounts (Barrionuevo and Zimmerman, 2001). Similar mixtures of bundled discounts by integrated and by non-integrated and otherwise unrelated firms have arisen in Europe and Australia.

Second, the bundling involves a set discount (usually for one of the products) that is offered regardless of the prices offered for the particular products. That is, mileage redemption rates from credit card use are fixed in advance even as interest rates and airline ticket prices change. Similarly, supermarket basket and petrol pump prices change on a daily basis whereas the bundled discount may be unchanged for months or years. This inflexibility of discounting stands in contrast to the usual assumption in the economics literature where firms – if they opt to have both bundled and separate prices –

choose both sets of prices simultaneously.[3] Here, however, the bundled discount is chosen prior to the actual store or pump prices that emerge in competition for consumers. For this reason, the bundled discount represents an ex ante commitment to the price for customer loyalty.

In this paper we model the interaction between four producers of two products to investigate the consequences of bundled discounts in an oligopoly setting. Our model is an extension of the standard differentiated goods framework used, for example, by Matutes and Regibeau (1992), Denicolo (2000) and Nalebuff (2003), and is designed to capture the key features described above. The products are unrelated in that both consumer demands and production costs for the two products are independent. Each product is produced by two firms and we explore situations where firms are either unrelated except for the bundling or are horizontally integrated. Pairs of firms may negotiate to set a bundled discount across the two products and to share the costs of that discount, with any discount being a publicly observable commitment that is set prior to any competition for customers.

We show how offering a bundled discount for two otherwise unrelated products creates a strategic interdependence between those products. For example, if only one pair of firms offers a bundled discount then in the eyes of the customers those two products are like complements. A lower price for one of the products raises demand for that product and, through the discount, also raises demand for its bundled pair. Importantly, bundling by one pair of firms also creates a strategic interdependence between the prices

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[3] For example see Chen (1997).

of the other two products – even if those firms offer no equivalent bundled discount. A rise in the price of one unbundled product increases consumption of the bundled pair and reduces demand for the other unbundled product.

By creating an externality in pricing between otherwise unrelated firms, bundling allows firms to alter the intensity of price competition.[4] If only one pair of independent firms sets a bundled discount then it gains a strategic advantage through price discrimination, similar to that shown by McAfee, McMillan, and Whinston (1989). The discount leads to an aggressive pricing response by the pair of firms that do not offer the bundled discount, but this response is tempered by the inability to coordinate prices. Unilateral bundling is profitable in this situation as the increase in the intensity of competition is muted by the coordination failure. While the co-branded firms increase profits, both the profits of the other firms and social welfare fall.

If both pairs of firms can establish a bundled discount but are otherwise unrelated, then both pairs will co-brand, even though, in equilibrium, there is no increase in profits. Retaliatory co-branding is an effective competitive response to bundled discounts offered by other firms, albeit only returning profits to their pre-bundling levels. At the same time, mutual co-branding greatly diminishes social welfare. The market is divided into two mutually exclusive sets of customers who buy both products of one pair of firms. The bundled discounts are sufficiently high so that even customers who otherwise would have a strong preference for unpaired products find it in their interest to buy a bundled pair.

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[4] Issues of pricing between complementary products has been analyzed, for example, by Economides and Salop (1992). A key feature of bundled discounts, however, is that this complementarity is endogenously created by the discount between otherwise unrelated products.

The key role of price competition and pricing coordination is shown when we allow for horizontal integration between firms. As Matutes and Regibeau (1992) show, bundled discounting is mutually unprofitable in duopoly with full market coverage. We show this result but also show how it critically depends on integration. One integrated firm facing a pair of non-integrated firms finds bundling profitable while co-branding is never profitable for the non-integrated pair. The non-integrated pair is unable to coordinate specific product pricing making them 'soft' from the perspective of the integrated firm. Retaliatory bundling is not profitable for the non-integrated firms due to the aggressive pricing response by their integrated rival. However, retaliatory horizontal integration can be a useful strategic response; eliminating bundling in equilibrium.

The analysis presented below significantly extends the existing bundling literature. Other related papers tends to focus on duopoly (for example Matutes and Regibeau, 1992), albeit sometimes with a competitive fringe (Chen, 1997). While Denicolo (2000) considers three firms, similar to our situation with one integrated pair, his focus is on compatibility rather than bundling. In contrast, our model shows the key role played by the endogenous pricing interdependence created by co-branding and bundled discounts. In particular, we highlight the potential adverse welfare outcomes that can arise through the type of bundling between otherwise unrelated firms and products that has been growing in popularity in a variety of countries.

2.   Model Set-Up

We model the interaction between four firms that produce and sell two products, X and Y. Firms A_X and B_X produce X and firms A_Y and B_Y produce Y. There are no costs

associated with the production of either product and firms are otherwise symmetric. Let P_n^i and Q_n^i be the (headline) price charged and quantity sold by firm n_i for product i.

There is a population of customers who may choose to buy the products. Depending on the prices and their preferences, a consumer may choose to buy one unit of one product, one unit of both products or neither product. Consumers also choose which firms to buy from. We use a standard 'linear city' model to capture consumers' preferences of each product. Thus, with regards to product X, consumers can be viewed as arrayed along the unit interval. A particular consumer's location on this line is denoted by x, with firm A_X is located at $x = 0$ and firm B_X is located at $x = 1$. If a consumer located at x purchases from firm A then that consumer gains net utility, $v_X - P_A^X - xd$, where v_X is the consumer's gross value of product and d is the disutility associated with the difference between the purchased product and the consumer's most preferred product. If that same consumer purchases from firm B then that consumer gains net utility, $v_X - P_B^X - (1-x)d$. We assume that v_X is the same for all customers and is at least equal to $2d$ so that in equilibrium all customers will choose to buy one (but only one) unit of product X.

We use an analogous structure for good Y, where customers' preferences are denoted by their location y along a unit interval with firm A_Y located at $y = 0$ and firm B_Y located at $y = 1$. Thus, customers can be viewed as arrayed over a unit square according to their preferences. For simplicity, we normalise the population of customers to unity. Again, we assume that all customers value Y sufficiently high so that all customers will buy one (but only one) unit of this product.

The products are independent in the sense that customers' preferences for the two products are independent. Thus, if $G(x, y)$ is the joint distribution function of customers over preferences for the two products, we can represent G by $G(x, y) = f(x)h(y)$ where f and h are the distributions of customers over preferences for product X and Y respectively. Thus, there is no reason why a customer who tends to prefer firm A_X for product X will tend to prefer either firm A_Y or firm B_Y for product Y. Similarly, there is no reason why a customer who tends to prefer firm A_Y for product Y will tend to prefer either firm A_X or firm B_X for product X. This is a reasonable assumption, for example, with regards to consumers' preferences for particular supermarket and retail petrol chains. For ease of analysis we assume that both f and h are uniform distributions.

Firms simultaneously set the prices for their products, P_n^i. However, firms might also agree to a 'bundled' discount γ_n for consumers who purchase X from n_X and Y from n_Y. Thus, if a consumer buys both products from A_X and A_Y, that customer pays $P_A^X + P_A^Y - \gamma_A$. The bundled discount is like a voucher that the consumer receives when purchasing product X from A_X that enables that consumer to a discount of γ_A when that customer also purchases Y from A_Y. Each consumer can only receive one discount. We are interested in the profitability of relatively small discounts; thus, we assume that the discount is non-negative but no greater than the price of a single product. Thus, we assume that $\gamma_n \in [0, d]$.[5]

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[5] As we show below, the equilibrium price for each good in the absence of any bundled discount is given by d.

In setting the discount, we assume that firms n_X and n_Y are natural partners and that only a single exclusive relationship between producers of either product are possible. The partnered firms choose their discount to maximise their expected joint profits. This would arise naturally from any efficient bargaining game (such as Nash bargaining) where ex ante (lump sum) side payments are possible. Nonetheless, ex post, the costs of the discount might be shared between the two firms. We assume that firm n_X bears a proportion $\alpha_X \in [0,1]$ of the discount in any relationship. We will explore how α_X might be set so as to maximise the joint expected profits of the partners.

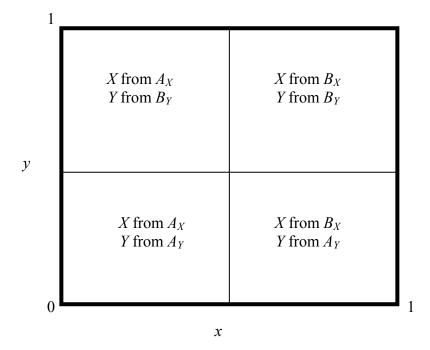The timing of the game played between the firms is as follows:

1. Firms simultaneously agree to their bundled discount if any.

2. Given the bundled discount(s), all firms simultaneously announce their prices.

3. Given prices and any bundled discounts, customers decide where to make their purchases. Firms receive payments and profits.

A key assumption here is that firms find it easier to change their retail prices than their agreed bundled discount. This assumption contrasts with the prior literature on bundling and compatibility. In that literature, while some choices may be made initially by firms, such as to whether to make their products compatible or not, the prices of individual and bundled products are determined simultaneously (Matutes and Regibeau, 1992; Chen, 1997). Here, the discount to the bundled product is set first. Hence, that discount is a commitment that impacts upon later price competition.

Equilibrium without Bundling

As a benchmark, suppose that no bundled agreements have been made (i.e., $\gamma_A = \gamma_B = 0$). Consumers will make their choice over the two products independently.

The marginal consumer for product X will be located at $\hat{x}$ such that

$v_X - P_A^X - d\hat{x} = v_X - P_B^X - d(1-\hat{x})$ or $\hat{x} = \frac{1}{2} + \frac{1}{2d}\left(P_B^X - P_A^X\right)$. Thus, all consumers located at

$x \le \hat{x}$ purchase X from firm A_X while all other consumers purchase from firm B_X.

Similarly, for product Y, all consumers located at $y \le \hat{y} = \frac{1}{2} + \frac{1}{2d}\left(P_B^Y - P_A^Y\right)$ purchase from

firm A_Y while all other consumers purchase from firm B_Y. Each firm simultaneously and

independently sets prices to maximise profits. For example,

$\hat{P}_A^X = \arg\max_{P_A^X} P_A^X\left(\frac{1}{2} + \frac{1}{2d}(\hat{P}_B^X - P_A^X)\right)$. Notice that this does not depend upon the prices

charged for product Y.

In the unique Nash equilibrium, prices are given by $\hat{P}_A^X = \hat{P}_B^X = \hat{P}_A^Y = \hat{P}_D^Y = d$, with

one half of consumers buying product X from firm A_X with the rest buying this product

from firm B_X. Similarly, for product Y. Thus, $\hat{Q}_A^X = \hat{Q}_B^X = \hat{Q}_A^Y = \hat{Q}_D^Y = \frac{1}{2}$. Given this, it is

easy to see that each firm makes profits of $\frac{1}{2}d$ but, more significantly, this outcome

maximises social welfare in that each consumer purchases both products from their

nearest respective retailer (see Figure 1).

Figure 1: No Bundling

3. Unilateral Bundling by Independent Firms

We begin by considering the effects of bundling by one coalition of producers. Suppose that firms A_X and A_Y unilaterally decide to offer a bundled discount on their products, but that firms B_X and B_Y do not set a discount. Thus, we fix $\gamma_B = 0$.

If $\gamma_A > 0$, the resulting division of consumers is as in Figure 2. Given retail prices and the bundled discount, a consumer who would have purchased product Y from A_Y and X from B_X might now purchase product X from A_X as well. Given that they are going to buy Y from A_Y anyway, the effective price of product X from A is reduced by γ_A. Thus, a consumer who is located at $y \leq \hat{y}$ and at $x \leq \frac{1}{2} + \frac{1}{2d}\left(P_B^X - P_A^X + \gamma_A\right) = \hat{x} + \frac{1}{2d}\gamma_A$ will

purchase both X and Y from A_X and A_Y. A similar increase in sales of Y from firm A_Y also holds. Finally, some consumers who, in the absence of the discount would have bought neither product from firms A_X and A_Y will now find it in their interest to do so. Any consumer with preferences for X and Y such that $x > \hat{x}$, $y > \hat{y}$ but $x + y \leq \hat{x} + \hat{y} + \frac{1}{2d}\gamma_A$ will now prefer to buy both products from A_X and A_Y even though in the absence of the bundled discount they would buy neither product from them.
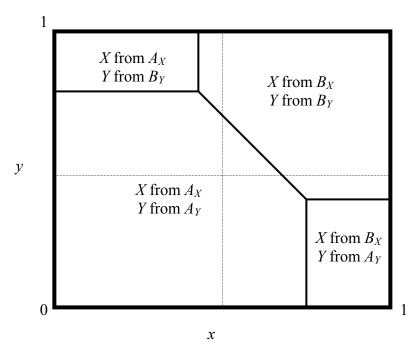
Figure 2: A Bundles

The existence of a bundled discount alters the nature of price competition by endogenously creating interdependence between the otherwise-independent customer demands. To see this, note that sales for each firm are given by:

$$Q_A^X = \tfrac{1}{2} + \tfrac{1}{2d}\left(P_B^X - P_A^X\right) + \tfrac{\gamma_A}{2d}\left(\tfrac{1}{2} + \tfrac{1}{2d}\left(P_B^Y - P_A^Y\right)\right) + \tfrac{\gamma_A^2}{8d^2}$$

$$Q_A^Y = \tfrac{1}{2} + \tfrac{1}{2d}\left(P_B^Y - P_A^Y\right) + \tfrac{\gamma_A}{2d}\left(\tfrac{1}{2} + \tfrac{1}{2d}\left(P_B^X - P_A^X\right)\right) + \tfrac{\gamma_A^2}{8d^2}$$

$$Q_B^X = \tfrac{1}{2} + \tfrac{1}{2d}\left(P_A^X - P_B^X\right) - \tfrac{\gamma_A}{2d}\left(\tfrac{1}{2} + \tfrac{1}{2d}\left(P_B^Y - P_A^Y\right)\right) - \tfrac{\gamma_A^2}{8d^2}$$

$$Q_B^Y = \tfrac{1}{2} + \tfrac{1}{2d}\left(P_A^Y - P_B^Y\right) - \tfrac{\gamma_A}{2d}\left(\tfrac{1}{2} + \tfrac{1}{2d}\left(P_B^X - P_A^X\right)\right) - \tfrac{\gamma_A^2}{8d^2}$$

In the absence of bundling, demand for units of X sold by A_X only depend on P_A^X and P_B^X. With bundling, the demand for X sold by A_X, Q_A^X, depends on both the prices for product X and the prices for product Y. A decrease in P_A^Y, given P_B^Y, leads to more sales of Y by A_Y and this increases the number of customers able to benefit from the bundled discount by also purchasing units of X from A_X. As such, a fall in P_A^Y relative to P_B^Y increases the demand for X sold by A_X. In contrast, a rise in P_A^Y relative to P_B^Y lowers the demand for X sold by A_X. A similar relationship holds between prices P_A^X and P_B^X and the demand for Y sold by A_Y.

While bundling by firm A creates a dependency between the prices of A_X and A_Y it also creates a dependency between the prices of the non-bundled products. Sales of firm B_X, Q_B^X, also depend on the prices of Y-sellers. Thus a fall in P_B^Y relative to P_A^Y makes B_X better off by increasing its sales. A similar relationship holds between P_B^X and the sales of A_Y. The creation of these pricing externalities between otherwise independent products by bundled discounts is a key factor in our analysis.

We denote the total number of consumers who purchase from both A_X and A_Y (and so receive the discount γ_A) by D_A where:

$$D_A = \left(\tfrac{1}{2} + \tfrac{1}{2d} \left(P_B^X - P_A^X \right) \right) \left(\tfrac{1}{2} + \tfrac{1}{2d} \left(P_D^Y - P_A^Y \right) \right) + \tfrac{\gamma_A}{2d} \left(1 + \tfrac{1}{2d} \left(P_D^Y - P_A^Y + P_B^X - P_A^X \right) \right) + \tfrac{\gamma_A^2}{8d^2}.$$

The individual profits of firms A_X and A_Y are $\pi_A^X = P_A^X Q_A^X - \alpha_X D_A \gamma_A$ and $\pi_A^Y = P_A^Y Q_A^Y - \left(1 - \alpha_X \right) D_A \gamma_A$ respectively. The profits of firms B_X and B_Y are $\pi_B^X = P_B^X Q_B^X$ and $\pi_B^Y = P_B^Y Q_B^Y$ respectively. The joint profits of A_X and A_Y is $P_A^X Q_A^X + P_A^Y Q_A^Y - D_A \gamma_A$.

Proposition 1. *Regardless of the exact level of bundled discount, the (joint) profit maximising level of α_X for firms A_X and A_Y is equal to 0.5.*

PROOF: Each firm unilaterally sets its own price to maximise own profit, given γ_A and α_X then the equilibrium prices are given by:

$$P_A^X = \frac{144d^4 + 12d^2\gamma^2\left[\alpha_X^2 + 2\alpha_x - 2\right] + 4d\gamma^3\left(\alpha_x - 2\right)\left(2\alpha_x + 1\right) + \gamma^4\alpha_x\left(\alpha_x - 3\right) + 24d^3\gamma\left(2\alpha_x + 1\right)}{144d^3 + 4d\gamma^2\left(\alpha_x - 3\right)\left(\alpha_x + 2\right)}$$

$$P_A^Y = \frac{144d^4 + 12d^2\gamma^2\left[\alpha_X^2 - 4\alpha_x + 1\right] + 4d\gamma^3\left(\alpha_x + 1\right)\left(2\alpha_x - 3\right) + \gamma^4\left(\alpha_X^2 + \alpha_x - 2\right) - 24d^3\gamma\left(2\alpha_x - 3\right)}{144d^3 + 4d\gamma^2\left(\alpha_x - 3\right)\left(\alpha_x + 2\right)}$$

$$P_B^X = \frac{144d^4 + 4d^2\gamma^2\left[2\alpha_X^2 + \alpha_x - 6\right] + 4d\gamma^3\left(\alpha_x - 2\right)\left(\alpha_x - 1\right) + \gamma^4\alpha_x\left(\alpha_x - 3\right) + 24d^3\gamma\left(\alpha_x - 1\right)}{144d^3 + 4d\gamma^2\left(\alpha_x - 3\right)\left(\alpha_x + 2\right)}$$

$$P_B^Y = \frac{144d^4 + 4d^2\gamma^2\left[2\alpha_X^2 - 5\alpha_x - 3\right] + 4d\gamma^3\alpha_x\left(\alpha_x + 1\right) + \gamma^4\left(\alpha_X^2 + \alpha_x - 2\right) - 24d^3\gamma\alpha_x}{144d^3 + 4d\gamma^2\left(\alpha_x - 3\right)\left(\alpha_x + 2\right)}$$

Substituting these prices into quantities and then into profit, the equilibrium value of joint profit to firms A_X and A_Y is given by:

$$\frac{\left(\begin{array}{c}20736d^8 + 3456d^7\gamma + 576d^6\gamma^2\left(\Lambda - 19\right) - 144d^5\gamma^3\left(4\Lambda + 3\right) - 24d^4\gamma^4\left(5\Lambda - 91\right) \\ -12d^3\gamma^5\left(\Lambda\left(4\Lambda - 43\right) + 20\right) - 4d^2\gamma^6\left(\Lambda\left(11\Lambda - 27\right) + 22\right) - 2d\gamma^7\left(\Lambda\left(5\Lambda + 18\right) - 1\right) - \gamma^8\left(\alpha_X\left(\alpha_X - 2\right) + 2\right)\left(\alpha_X^2 + 1\right)\end{array}\right)}{16d^3\left(36d^2 + \gamma^2\left(\alpha_X + 2\right)\left(\alpha_X - 3\right)\right)^2}$$

where $\Lambda = \alpha_X\left(\alpha_X - 1\right)$.

Maximising the joint profits with respect to α_X gives a relevant solution at $\alpha_X = 0.5$. Further, remembering that $\gamma \le d$, it is easy to confirm that the second order conditions on the joint profit equation are negative at $\alpha_X = 0.5$ for all feasible discounts.

As noted above, we would expect A_X and A_Y to negotiate both a level of bundled discount and a sharing rule for the discount to maximise their joint profits. Proposition 1 shows that, regardless of the actual bundled discount, joint profit maximisation involves an equal sharing of the cost of the bundled discount. This result is intuitive given the symmetry of both the firms' production functions and consumers' preferences.

Given $\alpha_X = \frac{1}{2}$ and the level of bundled discount γ_A, firms individually set prices to maximise their own profits. The equilibrium prices are:

$$\hat{P}_A^X = \hat{P}_A^Y = \frac{1}{3}\gamma_A + \frac{\gamma_A^2}{20d} + d + \frac{\gamma_A^2\left(0.0611111d - 0.025463\gamma_A\right)}{d^2 - 0.173611\gamma_A^2} \quad \text{and}$$

$$\hat{P}_B^X = \hat{P}_B^Y = -\frac{1}{12}\gamma_A + \frac{\gamma_A^2}{20d} + d + \frac{\gamma_A^2\left(0.0152778d - 0.00636574\gamma_A\right)}{d^2 - 0.173611\gamma_A^2}.$$

It is easy to see that each P_B^i is decreasing and each P_A^i is increasing in γ_A. However, $P_A^X + P_A^Y - \gamma_A$ is decreasing in γ_A. As the bundled discount rises, each of firms A_X and A_Y has an incentive to raise their individual prices. However, overall, an increase in the bundled discount reduces the total price associated with the bundled products so that consumers who do in fact buy the bundle are made better off. A rise in the bundled discount raises the pricing pressure on firms B_X and B_Y and they respond by lowering their prices. Again, consumers who buy both products from these firms are made better off by the fall in prices even though they do not receive a bundled discount. This is reflected in the ranking of price combinations that consumers can pay for the two products. In equilibrium, for $\gamma_A > 0$:

$$\hat{P}_A^X + \hat{P}_B^Y = \hat{P}_A^Y + \hat{P}_B^X > 2d > \hat{P}_B^X + \hat{P}_B^Y > \hat{P}_A^X + \hat{P}_A^Y - \gamma_A.$$

Relative to the benchmark with no bundled discount, consumers of the bundled product pay a reduced price as do those who do not consume products from A_X and A_Y. However, consumers who purchase one product from A_X and A_Y are worse off when there is a bundled discount. Moreover, it is easy to see from Figure 2, that overall social welfare is reduced as there are some consumers who no longer consume their nearest product.

What will be A_X and A_Y's choice of γ_A? Maximising $\hat{P}_A^X \hat{Q}_A^X + \hat{P}_A^Y \hat{Q}_A^Y - \hat{D}_A \gamma_A$ with respect to γ_A gives $\hat{\gamma}_A = 0.576578d$. This, in turn, implies that:

$$\hat{P}_A^X = \hat{P}_A^Y = 1.22528d \text{ and } \hat{P}_B^X = \hat{P}_B^Y = 0.964472d$$

$$\hat{Q}_A^X = \hat{Q}_A^Y = 0.517764 \text{ and } \hat{Q}_B^X = \hat{Q}_B^Y = 0.482236$$

$$\pi_{A_X} = \pi_{A_Y} = 0.521545d \text{ and } \pi_{B_X} = \pi_{B_Y} = 0.465103d$$

This outcome is summarised in the following proposition.

Proposition 2. *If all firms are non-integrated and only two firms can offer a bundled discount then, in equilibrium, relative to the situation without bundling:*

  (a) *The (headline) prices for the bundling firms will rise and the prices for the other firms will fall;*

  (b) *Profits of the bundling firms rise while profits for each of the other firms fall and total industry profits fall;*

  (c) *Consumers who either purchase the bundle or make no purchases from the bundling firms pay a lower total price while other consumers pay a higher total price;*

  (d) *Social welfare falls as more than half of the consumers of product i purchase that product from firm A_i for i = X, Y.*

Proposition 2 shows that two firms selling otherwise unrelated products to the same consumer base have an incentive to offer a bundled discount for their products. This discount has the effect of increasing their total sales and profits by allowing them to price discriminate between consumer types; especially those who strongly prefer one of their products but not the other. In this sense, the outcome here is similar to the case of monopoly bundling analysed by McAfee, McMillan and Whinston (1989). However, our result holds for oligopolistic competition and is valid even for relatively intense competition as *d* approaches zero.

4.   Bilateral Bundling by Independent Firms

Unilateral bundling benefits the firms who initiate the bundling but harms other firms. For this reason it is natural at ask whether the other pair of firms wish to follow suit and also offer a bundled discount or not? If there is bilateral bundling, how does this affect prices, sales and welfare in equilibrium? In this section, we answer these questions

by considering the equilibrium choices of (γ_A, γ_B) when two partnering arrangements are possible.

Suppose that both the coalitions of firms A and the coalitions of firms B simultaneously announce their bundled discounts, then each firm simultaneously and independently announces its price.[6] The equilibrium outcome is characterised in the following proposition.

Proposition 3. *The unique subgame perfect equilibrium involves all consumers receiving a bundled discount, $\hat{\gamma}_A = \hat{\gamma}_B = d$ with each firm's profits and output the same as the case where there are no bundled discounts.*

PROOF: Let $\gamma_A = ad$ and $\gamma_B = bd$ where both a and b are elements of $[0,1]$. Given a and b the Nash equilibrium prices are unique and are given by:

$$\hat{P}_A^X = \hat{P}_A^Y = \frac{d\left(a(6+a)^2 + 2ab(5+2a) + 4ab^2 + b^3 + 16(3+b)\right)}{4(12+5a+5b)} \text{ and}$$

$$\hat{P}_B^X = \hat{P}_B^Y = \frac{d\left(b(6+b)^2 + 2ba(5+2b) + 4ba^2 + a^3 + 16(3+a)\right)}{4(12+5a+5b)}.$$

Nash equilibrium quantities are given by:

$$\hat{Q}_A^X = \hat{Q}_A^Y = \frac{48 + 24a - a^3 + 16b - a^2 b + ab^2 + b^3}{96 + 40a + 40b} \text{ and}$$

$$\hat{Q}_B^X = \hat{Q}_B^Y = \frac{48 + 24b - b^3 + 16a - b^2 a + ba^2 + a^3}{96 + 40b + 40a}.$$

Substituting these values into the profit function and differentiating gives the first order condition for a as $\dfrac{d\Gamma(a,b)}{16(12+5a+5b)^3}$ where

$$\Gamma(a,b) = 4608 - 20a^6 + a^5(258 - 105b) + a^4 b(1680 + 15(82 - 13b))$$
$$+ 6a^2(12 + 5b)(7b(10 + 11b) - 96) + a^3(768 + 2b(3062 + 7b(171 - 10b)))$$
$$+ b\left(1920 + b\left(-160 + b\left(720 + b(668 + 15b(12 + b))\right)\right)\right)$$
$$+ a\left(-4608 + b\left(-6432 + b\left(2208 + b(3868 + 3b(356 + 15b))\right)\right)\right)$$

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[6] Throughout the remainder of this paper we assume that the cost of any bundled discount is shared equally by firms in a coalition.
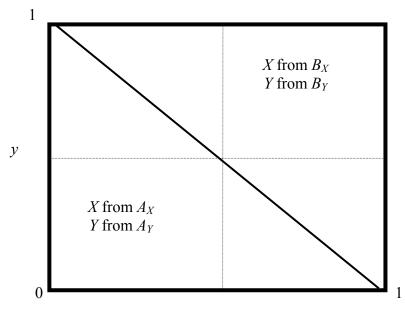
The symmetric first order condition for b is $\dfrac{d\Gamma(b,a)}{16(12+5a+5b)^3}$.

First, consider symmetric equilibria. Substituting $b = a$ into the first order condition for a gives the first order condition as $\frac{(8-a(a(a-12)-1))d}{48+40a}$. However, it is easy to confirm that this is positive for all $a \in [0,1]$. Thus the unique symmetric equilibrium is the corner solution where $a = b = 1$.

Second, consider asymmetric equilibria. From the first order conditions for a and b, $\hat{a}$ and $\hat{b}$ only form a subgame perfect equilibrium if $\Gamma(\hat{a},\hat{b}) = \Gamma(\hat{b},\hat{a}) = 0$. Numerical approximation over $[0,1]^2$ shows that no such values of $\hat{a}$ and $\hat{b}$ exist in the relevant domain. Thus the unique equilibrium involves $a = b = 1$, or in other words, $\hat{\gamma}_A = \hat{\gamma}_B = d$ with symmetric prices and quantities. The remainder of the proposition follows from simple substitution demonstrating that $\hat{P}_n^i = \frac{3}{2}d$, $\hat{Q}_n^i = \frac{1}{2}$ and $\hat{\pi}_{A_i} = \hat{\pi}_{B_i} = \frac{1}{2}d$.

Figure 3 illustrates the outcome under bilateral bundling with independent firms. All consumers either buy both products from firms A_i or both products from firms B_i. There are no consumers who buy one product from each pair of firms. In this sense, the equilibrium bundled discounts are ubiquitous in our model. All consumers receive a discount.

Figure 3: Both Bundle

It is also clear that bilateral bundling has significant adverse welfare consequences in our model. Comparing the outcome with the 'no bundling' situation, firm profits are unchanged but social welfare is significantly lower under bilateral bundling. While each pair of firms sells to exactly one half of the market, consumers are wasting surplus by purchasing from firms in less desirable 'locations'. For example, a consumer located at x close to unity but y closer to zero will buy both products of firms A. This is despite the fact that purchasing product X from A_X imposes a cost of almost d on the consumer relative to purchasing product X from B_X. The consumer still finds it individually desirable to purchase X from A_X given that she purchases Y from A_Y because of the size of the bundled discount. This discount, d, more than offsets the personal loss associated with purchasing X from the personally less desirable firm.

Formally, total social welfare in the absence of bundling is given by $v_X + v_Y - \frac{1}{2}d$. This is divided into total firms' profits of d with the remaining $v_X + v_Y - \frac{3}{2}d$ being consumers' surplus. In contrast, under bilateral bundling, total social welfare is $v_X + v_Y - \frac{2}{3}d$, with producers' profits still equal to d but consumers' surplus falling to $v_X + v_Y - \frac{5}{3}d$. Thus social welfare falls by $\frac{1}{6}d$ under bilateral bundling relative to no bundling. Further, all of this welfare loss falls on the consumers.

Despite leading to a welfare loss, there are strong pressures on firms to introduce bundling. As we have seen from Section 3, unilateral bundling is profitable for firms. Thus, if one pair of firms is not going to offer a bundled discount then it always pays the other pair of firms to offer such a discount. There is no equilibrium where neither pair of firms offers a bundled discount. Further, given that one pair of firms has introduced a bundled discount, it always pays the other pair of firms to copy this strategy and also

introduce a bundled discount. Given that one pair of firms offers a discount, the profits of the other pair of firms rises if they too offer a discount. Thus, offering a bundled discount is a dominant strategy for both pairs of firms.

Our result here contrasts with Chen's (1997) model of price competition without product differentiation, in that both pairs of firms find it optimal to bundle their products. However, so far we have not allowed for any integration between pairs of firms. Given that bundled discounts create pricing externalities within our model, we would expect that coordinated pricing by integrated firms will significantly alter the industry effects of bundled discounts.

5. Integration and Bundling

In contrast to the above analysis, suppose that pairs of firms can not only offer a bundled discount but can also horizontally merge. Such merger does not alter the timing of the interaction between firms – pairs of firms still commit to setting bundled discounts prior to setting their prices. However, unlike a bundled pair involving two separate firms, a single merged firm can explicitly set prices of both product X and Y to maximise total profits of the integrated firm. The merger allows for coordinated pricing as well as a coordinated bundled discount. There are clearly two situations of interest – where both pairs of firms A_i and B_i are merged, and where only one pair of firms is merged.

Two Integrated Firms

We first consider the case where there are two integrated firms. There is a single firm A that sells both A_X and A_Y and a single firm B that sells both B_X and B_Y. If neither

integrated firm offers a bundled discount then the equilibrium is the same as in the non-integrated base case without bundled discounting. In the absence of discounting, the demands for each of the firm's products are independent so that there is no additional benefit from the ability of an integrated firm to coordinate pricing.

If we consider the bundled discounts offered by *A* and *B*, it is easy to show that the unique equilibrium involves no bundled discounting.

Proposition 4. *If both A_X and A_Y are integrated and B_X and B_Y are integrated, then the unique subgame perfect Nash equilibrium involves no bundled discounting.*

PROOF: To show that this is an equilibrium, suppose that *B* does not set any bundled discount and consider firm A's best response. If *A* sets $\gamma_A > 0$ then equilibrium prices are given by $\hat{P}_A^X = \hat{P}_A^Y = d + \frac{\gamma_A^2}{2\gamma_A + 4d}$ and $\hat{P}_B^X = \hat{P}_B^Y = \frac{2d^2}{\gamma_A + 2d}$. Firm *A*'s profits are $\pi_A = d - \frac{1}{4}\gamma_A + \frac{\gamma_A^2(\gamma_A - 2d)}{16d^2(\gamma_A + 2d)}$. But this is falling in γ_A. Thus, given that firm *B* is not offering a bundled discount, firm *A* maximises profits by also offering no discount. By symmetry, the same holds for firm *B* if firm *A* offers no discount. Thus, setting $\gamma_A = \gamma_B = 0$ is a mutual best response for the two integrated firms.

To show that this equilibrium is unique, suppose that both *A* and *B* set positive bundled discounts. *A*'s profits in this situation are given by $\pi_A = \frac{\gamma_A(\gamma_A + \gamma_B)\left((\gamma_A + \gamma_B)^2 - 2d(\gamma_A + \gamma_B) - 4d^2\right) + 8d^3(\gamma_A + 4d)}{16d^2(\gamma_A + \gamma_B + 2d)}$. It is easy to verify that for $\gamma_B \leq d$, π_A is decreasing in γ_A. Thus, the unique equilibrium is where $\hat{\gamma}_A = \hat{\gamma}_B = 0$.

This proposition mirrors the relevant result from Proposition 1 of Matutes and Regibeau (1992).[7] Mutual integration changes the benefits from bundled discounting by changing the nature of price competition. Price competition is 'tougher' under integration when one firm offers a discounted bundle than in the absence of integration. This is reflected in the prices. As in the non-integrated case, when γ_A is positive but $\gamma_B = 0$, each of P_B^i is decreasing in γ_A, each P_A^i is increasing in γ_A and $P_A^X + P_A^Y - \gamma_A$ is decreasing in γ_A.

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[7] As Matutes and Regibeau show, this result depends on the specific timing of the duopoly interaction.

However, in the integrated case, $2d > \hat{P}_A^X + \hat{P}_B^Y = \hat{P}_A^Y + \hat{P}_B^X > \hat{P}_B^X + \hat{P}_B^Y = \hat{P}_A^X + \hat{P}_A^Y - \gamma_A$. Thus, in contrast to the non-integrated case, unilateral bundling under integration lowers prices for all consumers.

The increased intensity of price competition arises under integration because the interdependence of pricing is internalised. As discussed in Section 3, bundled discounting creates interdependence between prices. Sales of B_X are decreasing in P_B^Y and vice-versa. If firms B_X and B_Y are non-integrated and cannot coordinate their prices then this interdependence is ignored. Each firm sets its price too high from the perspective of the other firm. Under integration, however, this interdependence is internalised, resulting in more aggressive pricing by firm B. Further, as can be seen from Section 3 and the prices given above, the interdependence in pricing for firms B_X and B_Y is increasing in γ_A. The higher is the bundled discount offered by firm A the greater is the cross price effect between the products sold by firm B and the more aggressive is the pricing by firm B. Bundled discounting is thus self defeating for each firm. It lowers profits because of the co-ordinated aggressive response by the rival integrated firm.

One Integrated Firm

The analysis above suggests an important asymmetry when only one pair of firms is horizontally integrated. Because it can internalise the price interdependency created by a bundled discount, an integrated firm will respond aggressively to any discount offered by other non-integrated firms, making such discounting unprofitable for those non-integrated firms. But the reverse does not hold. A non-integrated pair of firms cannot co-ordinate their pricing response to bundled discounts offered by an integrated firm. From

our analysis so far we would expect that this pricing externality would make bundled discounting profitable for the integrated firm. Thus, we would expect that if only one pair of firms is integrated, those firms would have a strong incentive to offer bundled discounts to create and exploit a pricing externality between the non-integrated firms. The non-integrated firms would not, however, find it profitable to respond by creating their own discounted bundle because this would lead to a strong response by the integrated rival.

Proposition 5 confirms this intuition.

Proposition 5. *If A_X and A_Y are integrated but B_X and B_Y are not integrated then:*
- *(1) Regardless of the level of γ_A, B_X and B_Y always set $\gamma_B = 0$;*
- *(2) The integrated firm offers a bundled discount. However, compared with the unilateral bundling case, the discount is lower, headline prices are lower but market share of the integrated firm is higher under integration by a single pair of firms.*

PROOF: Let $\gamma_A = ad$ and $\gamma_B = bd$ where both a and b are elements of $[0,1]$. Solving for the first order conditions in prices where A jointly sets P_A^X and P_A^Y to maximize the total profit of firm A given the bundled discounts, and each firm B sets its own price to unilaterally maximise its own profits given the bundled discounts, gives equilibrium prices as:

$$\hat{P}_A^X = \hat{P}_A^Y = \frac{\left(2a(4+a)(6+a)+ab(16+7a)+6ab^2+b^3+16(3+b)\right)d}{4a^2+10a(4+b)+6(2+b)(4+b)}$$

$$\hat{P}_B^X = \hat{P}_B^Y = \frac{\left(48+5a^2b+2a(16+b(9+5b))+b(44+b(18+5b))\right)d}{4a^2+10a(4+b)6(2+b)(4+b)}$$

and equilibrium quantities as:

$$\hat{Q}_A^X = \hat{Q}_A^Y = \frac{a^3b+(2+b)(48+16b+b^3)+a^2(16+b(2+3b))+a(96+b(32+b(4+3b)))}{8\left(2a^2+5a(4+b)+3(2+b)(4+b)\right)}$$

$$\hat{Q}_B^X = \hat{Q}_B^Y = \frac{(2+b)(48+8b-b^3)+a(8-3b)(8+b(4+b))-a^3b-a^2b(2+3b)}{8\left(2a^2+5a(4+b)+3(2+b)(4+b)\right)}$$

Substituting these values into the profit equations and maximising the joint profits of B_X and B_Y with respect to b, gives a first order condition that is negative for all d and for all $a,b \in [0,1]$. Thus, for any value of d and any feasible values of a the optimal value of b is always equal to zero. This proves (1).

(2) is shown by substitution and maximisation with regards to a; yielding,
$\hat{\gamma}_A = 0.25981d$, $\qquad$ $\hat{P}_A^X = \hat{P}_A^Y = 1.05444d$, $\qquad$ $\hat{P}_B^X = \hat{P}_B^Y = 0.959966d$,
$\hat{Q}_A^X = \hat{Q}_A^Y = 0.5441$, $\qquad$ $\hat{Q}_B^X = \hat{Q}_B^Y = 0.479983$, $\qquad$ $\pi_A^* = 1.01064d$ and

$\pi^*_{B_X} = \pi^*_{B_Y} = 0.460768d$ and the proposed comparisons with the unilateral bundling case.

It is useful to note here that the bundled discount offered by the integrated firm here is less than the discount that is unilaterally offered by non-integrated firms. However, as we would expect, due to the price coordination created by integration, the prices P^X_A and P^Y_A are also lower than in the non-integrated case leading to a reduction in P^X_B and P^Y_B. Thus, all (headline) prices are lower in the integrated case, although those customers buying a single product from A face a (slightly) higher price. Nonetheless, overall welfare is lower in the integrated case, as the integrated firm's market share is above that it would achieve if it were not integrated.

Incentives and Effects of Integration

The above results allow us to consider the incentives for horizontal integration by firms selling unrelated products. Suppose that prior to negotiating on bundled discounts, each pair of firms simultaneously chooses whether or not to horizontally integrate. When bundled discounting is preceded by horizontal integration, the unique equilibrium outcome involves both firms merging with no horizontal discounting. If neither pair integrated then both pairs of firms would offer bundled discounts with total pair-wise profits of d. But in this situation, it would pay one pair to pre-empt the other pair and merge. The merged pair would still engage in bundled discounting but the non-integrated pair would not find it profitable to bundle. However, the profits of the non-merged pair of firms falls in this situation, leading to incentives for them to also merge. In equilibrium,

both pairs merge but there are no bundled discounts. The outcome from the consumers'
perspective is the same as in the absence of integration and bundled discounts.

This analysis suggests that merger might be used as a defensive strategy in the
presence of bundled discounting. In the absence of horizontal integration, if one pair of
firms begins to discount then the other pair of firms can either respond by also
discounting or by merging. So long as the bundled discount is reversible, integration will
promote an aggressive pricing response and result in the initial bundling being
unprofitable. Of course, an equivalent response (in terms of profits) would be for one pair
of firms to respond to the other pair's bundled discount by matching that discount. In this
sense, either integration or matching bundled discounts could be used as defensive
strategies to the introduction of a bundled discount by one pair of firms. Of course, the
welfare consequences of these alternative responses differ significantly. Mutual
integration leads to no bundled discounts and an efficient allocation of customers
between firms. No integration with bundled discounts results in an inefficient allocation
of customers. Firms make the same profit is both cases but welfare is significantly lower
with bundled discounts and no horizontal integration.

6.   Conclusions

The strategy of introducing bundled discounts to encourage customer loyalty has
become widespread. This paper demonstrates why. Even for unrelated products, a
bundled discount has the effect of tying customers to particular product brands and
improving the profitability of the firms involved. However, once the full competitive
responses are included, the net effect on profits is zero although the allocation of

customers to brands is dramatically altered. In the case of supermarket-gas deals, in equilibrium, many customers find themselves consuming one type of product potentially far away from their most preferred brand. To the extent that physical location drives those choices, those customers will incur higher transport costs.

For this reason, we believe that bundled discounts of unrelated products should be regarded with suspicion. In contrast to some statements by regulators (e.g., ACCC, 2004), a bundled discount cannot in itself be considered a pro-competitive act as one also has to take into account the effect on headline prices. Our paper has demonstrated that those headline prices adjust (perhaps fully) for the discount; leaving only distorted customer choices. Ironically, when the unrelated products are sold by the same firm, this reduces the incentives for welfare-reducing bundling. Indeed, merger is a potential commitment device against the distorted pricing strategy.

While our model is simple, the widespread existence and introduction of bundled discounts suggests an opportunity for empirical testing. Bundled arrangements will be introduced over time by different firms in an industry. The effect on prices can therefore by discerned by examining their movement in response to the timing of bundling events. This variation will also assist in establishing whether the driving forces of consumer harm as a result of bundling actually exist. However, that empirical exercise is well beyond the scope of this paper.

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