Operations Research for

Decision Support in Wildfire Management

by

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Abstract

The February 2009 'Black Saturday' bushfires resulted in 173 fatalities, caused AUD$4 billion in damage and provided a stark reminder of the destructive potential of wildfire. Globally, wildfire-related destruction appears to be worsening with observed increases in fire occurrence and severity. Wildfire management is a difficult undertaking and involves a complex mix of interrelated components operating across varying temporal and spatial scales. This thesis explores how operations research methods may be employed to provide decision support to wildfire managers so as to reduce the harmful impacts of wildfires on people, communities and natural resources. Some defining challenges of wildfire management are identified, namely complexity, multiple conflicting objectives and uncertainty. A range of operations research methods that can resolve these difficulties are then presented together with illustrative examples from the wildfire and disaster literature. Three mixed integer programming models are then proposed to address specific real-world wildfire management problems. The first model incorporates fuel treatment and supression preparedness decisions within an integrated framework. The second model schedules fuel treatments across multiple time periods to maintain fire resistant landscape patterns while satisfying various ecological requirements. The third model aggregates fuel treatment units to minimise total perimeter requiring management.

1. Introduction

The February 2009 'Black Saturday' bushfires in Victoria, Australia provided a stark reminder of the destructive potential of wildfire. The fires resulted in 173 fatalities and damage to property, infrastructure and the natural environment with an estimated total cost of over A$4 billion (Teague, Mc Leod, & Pascoe, 2010). While fire is a natural component of many forest ecosystems, uncontrolled wildfires can cause loss of human life and destruction of property and natural resources (King, Bradstock, Cary, Chapman, & Marsden-Smedley, 2008).

Globally, wildfire-related destruction is a problem that appears to be worsening. In the Mediterranean basin a sharp increase in wildfire events has been observed over the past several decades despite increased investment in fire prevention and suppression measures (Carmel, Paz, Jahashan, & Shoshany, 2009; Pappis & Rachaniotis, 2010a). In Canada there has been an observed rise in both fire occurrence and area burnt (Podur, Martell, & Knight, 2002). Increased fire activity has also been seen in U.S. forests with more frequent large fires, longer fire durations and longer fire seasons (Westerling, Hidalgo, Cayan, & Swetnam, 2006). Other countries to experience extreme fire seasons in recent years include Australia (Teague, et al., 2010) and Russia (Kharuk, Kasischke, & Yakubailik, 2007). This upward trend appears set to continue due to rising temperatures and changed weather conditions associated with climate change (Westerling, et al., 2006; Wotton, Martell, & Logan, 2003).

Wildfire management involves a complex mix of components and processes including: fire occurrence prediction, fuel management, fire prevention, fire detection and fire suppression (Martell, 2007). Wildfire managers operate in a difficult decision environment and are faced with limited time, constrained resources, extreme uncertainty and multiple objectives that may conflict (Martell, Gunn, & Weintraub, 1998). As fire suppression expenditures continue to rise, governments seek wildfire management approaches that are economically efficient and that take into account both market and non-market benefits (Venn & Calkin, 2011). However, there appears to be a large and growing gap between the decision support needs of wildfire managers and the decision support tools currently available (Martell, 2011).

Operations research (OR) is the use of analytical techniques such as mathematical modelling to analyse complex interactions between people, resources and the environment to aid decision-making and the design and operation of systems (Altay & Green, 2006). As a discipline, OR has its origins in World War II Great Britain where it helped guide the allocation of scarce resources against the enemy (Larson, 2005). OR methods have subsequently been applied to complex problems in a wide range of industries including: transportation, logistics, telecommunications, manufacturing, mining, health care and forestry. This thesis explores how OR methods might be applied in the wildfire management context. In particular, how OR methods can assist fire management agencies in assessing alternatives and making decisions that will reduce the impact of wildfires on people, communities and natural resources.

In *Chapter 2* some of the defining challenges of wildfire management are identified, namely complexity, multiple conflicting objectives and uncertainty. A range of OR methods that can resolve these difficulties are then presented, with illustrative examples drawn from the wildfire and disaster OR literature. The work presented in this chapter is the first detailed review of wildfire OR undertaken since 1998. A paper based on the contents of this chapter has been published in the *International Journal of Wildland Fire* (Minas, Hearne, & Handmer, 2012).

In *Chapter 3* a mixed integer programming model for fuel management and fire suppression preparedness planning is presented. The model makes fuel treatment and fire suppression resource allocation decisions simultaneously, so as to maximise the complementary effect of these two components of fire management. This is the first optimisation model to incorporate both fuel treatment and suppression preparedness planning decisions within an integrated framework. Despite the strong interrelation between these two elements of wildfire management, previous optimisation models have considered these components in isolation from one another. A paper based on the contents of this chapter has been published in *Annals of Operations Research* (Minas, Hearne, & Martell, 2013).

In *Chapter 4* a mixed integer programming model for multi-period spatially explicit fuel treatment scheduling is presented. The model schedules fuel treatments over time to generate spatial patterns that fragment the landscape fuel complex with a view to moderating wildfire behavior. It is the first multi-period landscape-level fuel treatment model to be formulated and solved using exact optimisation methods. The model provides a flexible framework that allows for incorporation of landscape heterogeneity, as well as a range of ecological and operational constraints. A paper based on the contents of this chapter (Minas, Hearne, & Martell, 2012) was submitted to the *European Journal of Operational Research* in December 2012, a revised version of the paper addressing reviewer comments was resubmitted in May 2013.

In *Chapter 5* a mixed integer programming model is presented for aggregation of fuel treatment units. The model aggregates existing ‘fundamental’ fuel treatment units into larger units or ‘clusters’. The aim being to improve efficiency of prescribed burning activities through a reduction in the total perimeter requiring management.

Finally in *Chapter 6* we conclude with a summary of the research findings.

2. A review of operations research methods applicable to wildfire management

2.1 Introduction

Wildfire managers operate in a difficult decision environment. They are faced with limited time, constrained resources, extreme uncertainty and multiple objectives that may conflict (Martell, et al., 1998). In recent years, wildfire management has become increasingly complex with the advent of inter-agency resource sharing arrangements and the recognition of the beneficial effects of fire on ecosystems (Martell, 2011). Operations research (OR) is a discipline that is uniquely placed to assist managers operating in this challenging environment. Wildfire managers have access to a proliferation of data from a variety of sources including geospatial databases and fire behaviour and climatology models. OR methods can provide a framework to help wildfire managers make sense of this information and use it to guide decision-making.

A large body of emergency OR work has been undertaken. Most of this work has focused on the allocation, deployment and dispatch of police, fire and ambulance resources for routine emergencies in an urban context (Simpson & Hancock, 2009). However, wildfire agencies must cover much larger areas than urban fire departments and wildfire occurrence and behaviour displays large spatial and temporal variation (Martell, et al., 1998). Due to these key differences, material from the urban emergency OR literature will not be considered here. There is a large body of disaster management OR work relating to non-routine emergency events such as: earthquakes, floods and hurricanes (Altay & Green, 2006). There is also a substantive literature on the application of OR to wildfire-specific management problems. Martell (1982) conducted a comprehensive review of wildfire OR work from 1961 to 1981 with elements of this review updated in 1998 (Martell, et al., 1998), as such this chapter will focus on post-1998 wildfire OR work. The remainder of the chapter is structured as follows. A range of OR methods will be discussed in terms of their ability to address some of the defining challenges of wildfire management, namely: complexity, multiple conflicting objectives and uncertainty. Illustrative examples and case studies drawn from the wildfire and disaster OR literature will be presented for each of the OR methods discussed.

2.2 *Methods for handling complexity*

2.2.1 Mathematical programming

Wildfire managers are often faced with complex problems consisting of a large number of inter-related decisions together with resourcing and other operational constraints. Mathematical programming (MP) is a field of OR that can assist with such problems. MP methods are concerned with the optimisation, that is maximisation or minimisation, of some explicit and quantifiable objective (Williams, 2009). In an MP model this objective is defined as a mathematical function of the decision variables in the form of an 'objective function' and is optimised subject to a series of related constraints (Hillier & Lieberman, 2005). Several categories of MP: linear programming (LP), integer programming (IP), nonlinear programming (NLP) and dynamic programming (DP) are described in further detail below together with examples from the wildfire and disaster OR literature.

Linear programming (LP) can be used when a problem's objective function and constraints can be formulated as a linear combination of the decision variables (Ragsdale, 2008). Hof et al. (2000) developed a timing-oriented LP model for the spatial allocation of suppression effort for an existing fire. Their model's objective was to delay the ignition of "protection areas" such as population centres. In an extension of this work Hof and Omi (2003) described the application of a similar timing-oriented LP model to a fuel management scheduling problem. In their model, spatial application of fuel-reduction treatments were determined so as to mitigate the effects of a particular "target fire" with a known origin and spread behaviour. When a LP model is solved a "shadow price" is generated for each constraint as a standard model output. Shadow prices can be interpreted as the marginal effect that tightening or relaxing a constraint has on the objective value obtained (Williams, 2009). Armstrong and Cumming (2003) used shadow prices obtained from a timber-harvesting LP model to estimate the potential cost of land based changes due to wildfire. Spatially explicit values-at-risk information like this can be useful for fuel treatment and suppression preparedness planning.

Integer programming (IP) models feature inputs or outputs that are required to take on discrete whole number values. IP can be useful for modelling problems that feature: indivisible resources, "yes or no" decisions or logical connections such as "if" and "then" (Wolsey, 1998). IP methods have been applied to a range of wildfire management problems. The maximal covering location model (MCLM) is a classic IP model that has been used extensively in emergency service deployment (Church & ReVelle, 1974). Dimopoulou and Giannikos (2001, 2004) described the use of an MCLM model for suppression resource deployment as part of a decision support system that also included a simulation module and a GIS interface. Kirsch and Rideout (2005) presented an IP model for initial attack preparedness planning. Their model deployed initial attack resources across a user-defined set of fires with the objective being to maximise the weighted area protected (WAP) for a given level of budget funding, with weights assigned based on protection priorities. Donovan (2006) presented a model for determining the optimal mix of agency and contract fire crews to minimise costs and satisfy demand across a fire season. A multi-period transportation formulation was used with the fire season modelled as a series of discrete time periods with differing levels of demand. This approach leads to reduced computational complexity for this type of problem as compared to a standard IP formulation. Donovan and Rideout (2003) described an IP model for determining the optimal mix of fire-fighting resources to dispatch to a given fire to achieve containment with minimal resultant costs and damages. Wei et al. (2008) formulated an IP model for optimal allocation of fuel treatment across a landscape based on spatially explicit ignition risk, fire spread probability, fire intensity levels and values-at-risk. Higgins et al. (2011) used an IP approach to develop a seasonal resource allocation model for planning fuel reduction burning on public lands in Victoria, Australia.

Nonlinear programming (NLP) methods are used when a problem features a nonlinear objective function or nonlinear constraints. The probability of containing a wildfire and the suppression time required to do so are nonlinear functions of fire size at the start of initial attack. This means small delays in dispatch of initial attack resources can result in dramatic fire loss increases (MacLellan & Martell, 1996). Rachaniotis and Pappis sought to incorporate this element of fire behaviour in an NLP model via the use of the "deteriorating jobs" concept. Their model tackled the problem of scheduling a single fire-fighting resource when there are several existing fires to be controlled (Pappis & Rachaniotis, 2010a; Rachaniotis & Pappis, 2006; Rachaniotis & Pappis, 2011). The model was subsequently extended to allow scheduling of multiple fire-fighting resources (Pappis & Rachaniotis, 2010b). Minciardi et al. (2009) formulated two related NLP models, one for deployment of wildfire suppression resources in the pre-operational phase and the other for dispatch of resources to fires in the operational phase.

Dynamic programming (DP) is an optimisation method that is particularly useful when sequences of interrelated decisions need to be made. In deterministic DP the state of the system at the next stage is completely determined by the current system state and the policy decision made (Hillier & Lieberman, 2005). Wiitala (1999) used a DP approach in his model for determining the most efficient mix of available initial attack resources to dispatch to a fire.

2.2.2 Problem structuring methods

Traditional 'OR' methods such as mathematical programming are suited to well-structured problems that can be clearly formulated in terms of performance measures, constraints and relations between action and consequence. However, many wildfire and disaster management problems lack structure and are typified by multiple perspectives, disagreement amongst experts and the presence of intangibles and uncertainties. Problem structuring methods (PSM) are a suite of techniques that can assist in resolving some of these difficulties. Compared to traditional 'hard' OR methods PSM typically employ rudimentary mathematical or statistical techniques (Mingers & Rosenhead, 2004). Two PSM methods, decision conferencing and expert judgment elicitation, are discussed in further detail below.

Decision conferencing can be an effective method for assisting with longer-term collaborative decision making. A decision conference is typically a two-day event that brings together decision makers from various organisations to discuss issues and work out a way forward. A facilitator is present to keep the discussion focused. An analyst is also present to build a series of analytical decision models with a view to developing a shared understanding of the problem (French, 1996). A series of decision conferences were held in the USSR following the 1986 Chernobyl nuclear accident. The aim was to identify the major factors influencing decision-making on relocation and other long term protective measures. The decision conferences helped develop a common understanding amongst participants including government ministers, policy-makers and scientists and successfully identified a number of key medical, socio-economic and political factors influencing protective measures undertaken (French, Kelly, & Morrey, 1992). Decision conferencing could be similarly used following major wildfires to facilitate dialogue between stakeholders and aid recovery-phase planning.

Expert judgement elicitation (EJE) is the use of structured methods to elicit expert opinions in a planned, formal manner that attempts to minimise bias. EJE typically involves interviewing or surveying “subject experts” and then analysing their answers together with information about their background and experience. EJE methods can provide an understanding of the degree of and reasons for consensus or disagreement amongst experts and can be useful in facilitating learning and dialogue (Gregory, Failing, Ohlson, & Mcdaniels, 2006). Furthermore, EJE studies are often a cost-effective and practical means of obtaining valuable information. In the wildfire context, EJE methods have been used to estimate fire containment probabilities and fire-line construction rates. In these instances, alternate methods such as observation of actual or experimental fires are often deemed to be too expensive, time-consuming and dangerous (Hirsch, Corey, & Martell, 1998). One of the earliest applications of EJE methods to wildfire management involved eliciting information from experienced fire managers in Ontario to derive subjective probability assessments of daily forest fire occurrence (Cunningham & Martell, 1976). Hirsch et al. (1998) used an EJE approach to model the relationship between fire size, fire intensity and probability-of-containment by a 5-7 person initial attack crew. In their study they interviewed crew leaders from four Canadian forest fire agencies and elicited probability-of-containment estimates for various fire scenarios (Hirsch, et al., 1998). Gilless and Fried (2000) surveyed California fire-fighters and used their responses to estimate probability distributions for fire-line construction rates for different fire-fighting resources under a range of conditions. These fire-line construction rate distributions were subsequently incorporated into the CFES2 simulation model used for initial attack planning in California. Similarly, Hirsch et al. (2004) interviewed crew leaders in Ontario and developed probability distributions for production rates of three and four person initial attack crews for a range of fuel types and fire intensities. Rideout et al. (2008) used EJE methods in their Marginal Attribute Rate of Substitution (MARS) approach to assessing values-at-risk for initial attack planning.

2.2.3 System dynamics

In complex systems, components can interact with one another via a web of feedback loops meaning a small change to input parameters can produce a drastic change to the whole system (Anderson, 1999). These feedback effects can be modelled using system dynamics (SD). Unlike many traditional 'hard' OR approaches that are static and linear in character, SD can accept the nonlinearity and feedback loop structures of real world social and physical systems. Whilst SD uses a 'soft' PSM-like approach for information elicitation and problem structuring, it includes two additional 'hard' steps: model definition using rate and level equations and the running of model simulations. An SD model initially serves to demonstrate how the problem under consideration is being generated in the real world, it is subsequently used to test alternative policies and structures (Forrester, 1994). Hoard et al. (2005) discussed the application of SD methods to disaster preparedness planning in rural areas with a focus on hospital surge capacity for a variety of disaster types. A similar SD approach could be used in wildfire preparedness planning to explore surge capacity considerations in suppression resource deployment and rostering of fire-fighting personnel.

2.2.4 Hyper-projects

Wildfire incident controllers are dealing with a problem that is emergent in nature. They are faced with a 'moving target' or a dynamic set of changing circumstances. The incident trajectory is influenced by actions taken such as fire suppression and external forces such as weather (Faraj & Xiao, 2006). Simpson (2006) defined a class of project, the 'hyper-project', that captures these emergent characteristics. Hyper-projects are characterised by the presence of a dynamic, external 'pacing function' and a set of defined tasks and resource requirements that interact with this pacing function. Time pressure is an inherent feature of hyper-projects with tasks measured in minutes or hours. Simpson (2006) used the hyper-project construct to model response to a residential structure fire, a similar approach could be used to model real-time wildfire suppression decision-making. In such a model various suppression resources would be dispatched and tactical fire-fighting decisions made relative to an external pacing function, which in this case would be the growth and lifecycle of the uncontained wildfire. The hyper-project approach can capture threshold effects, a key feature of complex biophysical systems. Thresholds are breakpoints that occur in systems with multiple stable states where crossing a threshold results in a shift from one state to another (Berkes, 2007). An example being when a wildfire crosses the 4000 kW/m threshold it can be said to have changed state from a controllable fire to a spot generating fire (Gill, 2005) thus requiring a different suppression response. The hyper-project provides a framework for responding to state changes via the execution of a flexible set of tasks that vary in a pre-defined manner relative to the pacing function.

2.3 *Methods for handling multiple conflicting objectives*

2.3.1 Multi-objective optimisation

Wildfire management involves various agencies and groups with different priorities and objectives including: reduction of impacts on public safety, private property and ecosystem processes as well as cost minimisation (Martell, 2007). Instances will often arise where multiple objectives conflict with one another, for example frequent planned burning can provide additional protection to built assets but may have a negative impact on biodiversity in some ecosystems (Driscoll et al., 2010). Where multiple objectives can be expressed in terms of market values they can be aggregated into a single cost minimisation objective. However wildfire managers are required to consider potential impacts on non-market values such as: ecosystem health, conservation of flora and fauna, air quality, water quality, recreational opportunities and cultural heritage (Venn & Calkin, 2011). In many cases ascribing a monetary value to these items would be an expensive, time-consuming and uncertain exercise. This lack of a common currency makes it difficult to evaluate and compare the outcomes of decisions or strategies. Multi-objective optimisation (MO) is a technique that is suited to these types of problems. MO models are formulated with more than one objective function to find a set of Pareto optimal solutions. A solution is Pareto optimal if none of the objectives can be improved without making another objective worse. Decision-makers can assess alternatives from this set of Pareto optimal solutions by examining trade-offs amongst the various objective values. This explicit identification and structured exploration of trade-offs provides a level of transparency in the decision process (Gregory, et al., 2006). Lehmkuhl et al. (2007) described FUELSOLVE a prototype decision support system that incorporates MO modelling into fuel management decision-making to consider both ecological and cost objectives. Kennedy et al. (2008) demonstrated the use of the FUELSOLVE MO model with a fuel treatment case study with trade-offs assessed between protection of endangered species habitat, preservation of old growth forest reserves and minimisation of area treated.

Goal Programming (GP) is a branch of multi-objective optimisation in which each of the multiple objectives takes the form of a goal. Goals are formulated as 'soft constraints' each with a target value it is desirable to satisfy. A penalty function is then specified that seeks to minimise deviations from this set of target values. Adjustments to the penalty function parameters allows the exploration of trade-offs between objectives (Ragsdale, 2008). Calkin et al. (2005) used a GP approach to analyse trade-offs between fire threat reduction and habitat preservation in silvicultural treatment scheduling. A goal programming module is currently under development as part of the United States Fire Program Analysis (FPA) project (Kumar, Carty, Parija, & Soni, 2010). The FPA project has been undertaken by the US Forest Service and other federal land management agencies in an attempt to develop a wildfire management planning and budgeting decision-support tool that will incorporate a full range of both market and non-market objectives (Venn & Calkin, 2011).

2.4 *Methods for handling uncertainty*

2.4.1 Simulation

Wildfire managers are required to make difficult decisions in conditions of uncertainty. Simulation is arguably the most robust and easily applied method for consideration of uncertainty in decision support systems (Mowrer, 2000). Simulation is an approach used to model real-life stochastic systems that evolve probabilistically over time. The real-life system's performance is imitated by using probability distributions to generate various events that occur in the system (Hillier & Lieberman, 2005). Prior to implementation, simulation models require validation to ensure they realistically represent the system being analysed and that the results they provide are reliable (Winston, 1994). Simulation models feature in a number of decision support systems used by wildfire agencies for strategic planning purposes. The California Fire Economics Simulator version 2 (CFES2) is a stochastic simulation model that simulates fire occurrence and suppression on a daily basis. Simulation of many years of "data" makes it possible to undertake "what if" analysis for changes to organisational components such as: resource stationing, dispatch rules and staff schedules (Fried, Gilless, & Spero, 2006). The Level of Protection Analysis System (LEOPARDS) is underpinned by a simulation model that is spatially conscious and incorporates temporal queuing conflicts. LEOPARDS has evolved from an initial attack simulation model developed in the early 1980s by Martell et al. (1984). LEOPARDS can model daily fire suppression activities and is used in Ontario to assess initial attack performance under a range of policy and budget conditions (McAlpine & Hirsch, 1999). The USDA Forest Service's National Fire Management Analysis System (NFMAS) Interagency Initial Attack Assessment (IIAA) is a simulation model that has been used in the past to test alternative initial attack organisations and strategies at various budget levels with a view to determining the Most Efficient Level (MEL) of funding (Lundgren, 1999). Manipulation of simulation models can provide valuable insights into a problem, however the primary shortcoming of this approach is that it is only possible to find "the best" management alternative from those investigated. For large problems with many management alternatives it is unlikely that a near-optimal solution can be found in this manner. For this reason, mathematical programming (MP) methods that systemically explore the solution-space can add significant value to complex wildfire management problems (Hof & Haight, 2007).

2.4.2 Stochastic programming

Stochastic programming (SP) is a method that combines mathematical programming methods with probability techniques to provide a constructive approach to tackling optimisation problems that feature uncertain data. SP can be used when there are uncertain model parameters with probability distributions that are known or can be estimated (Kall & Wallace, 1994). These parameter distributions can be either continuous or described by discrete scenarios and in some cases are generated using simulation techniques. The most common SP objective is optimisation of the mean outcome or expected value of the system. An alternate formulation incorporating decision maker risk preferences is the optimisation of a weighted sum of expected value and variance (Snyder, 2006). SP models generate solutions that are less sensitive to data uncertainty than deterministic MP models, however large SP models can prove difficult to solve.

One of the earliest uses of SP methods in forest fire management was Boychuk and Martell's (1996) multi-stage model for forest-level timber management that considered uncertain losses that could result from fires. A common SP formulation is the two-stage model with recourse. In such models a first-stage decision is made after which a random event occurs, a recourse decision can then be made in the second-stage that compensates for any undesirable effects. Hu and Ntaimo (2009) modelled the wildfire initial attack dispatch problem as a two-stage SP model with recourse. In their model the first stage decisions related to dispatch of suppression resources to reported wildfires, with recourse decisions made on fire-fighting tactics in the second stage. Stochastic parameters in the model included: fire growth scenarios, fire-line production rates, arrival times to fires and suppression resource operating costs. Ntaimo (2010) described an alternate application of a two-stage SP approach with deployment of suppression resources to bases in the first-stage and dispatch of resources to wildfires in the second stage. Two-stage SP models have been applied to a range of disaster management problems including: transportation of first-aid commodities on a disaster effected road network (Barbarosoglu & Arda, 2004), pre-positioning of emergency supplies in a hurricane-threatened region (Rawls & Turnquist, 2010) and locating storehouses and developing transportation plans for flood-relief logistics (Chang, Tseng, & Chen, 2007).

Probabilistic SP approaches, such as chance-constrained programming, require the probability of a constraint holding to be above a specified threshold (Snyder, 2006). Bevers (2007) demonstrated the use of chance-constrained programming for a fire organisation budgeting problem. In his model formulation the probability of total fire costs exceeding the budget had to be less than a specified risk level.

Stochastic dynamic programming (SDP) is a method used for problems with sequential decisions that are subject to uncertainty. SDP differs from deterministic DP in that state-to-state system transitions are governed by probability distributions

(Hillier & Lieberman, 2005). Konoshima et al. (2010; 2008) demonstrated the use of an SDP approach for determining optimal spatial patterns of fuel treatment and timber harvesting in a theoretical landscape subject to fire risk. Spring and Kennedy (2005) developed an SDP model with decisions made at the beginning of each stage as to which stands of trees are harvested and what level of fire protection is applied.

2.4.3 Robust optimisation

Like stochastic programming (SP), robust optimisation (RO) provides a constructive approach to solving optimisation problems that feature uncertain data (Vladimirou & Zenios, 1997). However RO differs from SP in that probability distributions of uncertain parameters are not required. All that needs to be known about the uncertain parameters is that they belong to some 'uncertainty set" which may be described as either a continuous interval or as set of discrete scenarios (Ben-Tal & Nemirovski, 2002). RO models are a great deal less sensitive to data perturbations than deterministic MP methods but substantially more difficult to solve. RO models can be formulated in a number of ways. The Minimax formulation seeks to minimise the maximum cost or damage across all possible scenarios. This is a highly conservative approach that provides costly solutions that cater for worst-case outcomes (Snyder, 2006). Unless a model has significant built-in redundancies a solution is unlikely to remain both feasible and optimal across all scenarios (Vladimirou & Zenios, 1997). Model and solution robustness approaches seek to balance optimality and feasibility based on the decision maker's degree of risk aversion. Restricted scenario approaches minimise the maximum cost or damage across a restricted 'reliability set' of scenarios. This reliability set is specified by the decision maker based on risk preferences (Snyder, 2006). Haight and Fried (2007) presented a scenario-optimisation IP model for suppression resource deployment based on the classical maximal covering model (MCLM). Their formulation included a binary "standard response" variable that serves as a proxy for fire-line construction. The model's objective was to minimise the number of fires not receiving a "standard response" across a defined set of scenarios. Mercer et al.(2008) modified Haight and Fried's standard-response model to incorporate the effects of fuel treatment. Other problems with relevance to wildfire and disaster management that RO methods have been applied to include evacuation transportation planning (Yao, Mandala, & Chung, 2009) and facility location under uncertainty (Snyder, 2006).

2.4.4 Fuzzy models

Stochastic programming and robust optimisation methods are appropriate for problems where uncertainty is mostly due to randomness, however uncertainty is sometimes due to other factors such as imprecision and ambiguity (Verderame, Elia, Li, & Floudas, 2010). Fuzzy set theory is an approach that can tackle problems that feature fuzzy predicates such as 'small' or 'safe', fuzzy quantifiers such as 'most' or 'often', and fuzzy probabilities such as 'likely' or 'unlikely' (Smithson, 1991). In classical set theory membership of a set is assessed in binary terms, that is an element either belongs to a set or it doesn't. In fuzzy set theory 'degrees of membership' ranging from 0 to 1 are permitted based on a fuzzy membership function (Dubois & Prade, 1988). Models based on fuzzy set theory have been used to classify areas into risk-zones for both fire prevention planning (Iliadis, Papastavrou, & Lefakis, 2002; Iliadis, Papastavrou, & Lefakis, 2002b; Iliadis, 2005; Iliadis & Spartalis, 2005; Kaloudis, Tocatlidou, Lorentzos, Sideridis, & Karteris, 2005; Kaloudis, Costopoulou, Lorentzos, Sideridis, & Karteris, 2008; Tsataltzinos, Iliadis, & Stefanos, 2009; Iliadis, Vangeloudh, & Spartalis, 2010) and disaster relief purposes (Sheu, 2007; Tan, Huang, Wu, Cai, & Yan, 2009; Sheu, 2010).

2.5 *Summary and discussion*

In this chapter we have presented a range of OR methods and discussed their ability to address some of the major challenges of wildfire management including: complexity, multiple conflicting objectives and uncertainty. Many of these OR methods are complementary and can be used in conjunction with one another. Problem structuring methods (PSM) can be used to elicit objectives and opinions and to help develop a common understanding. Simulation and system dynamics (SD) methods can be used to model the dynamics of complex systems to gain insights into the problem structure and possible management prescriptions through the use of "what-if" analysis. Whilst optimisation methods such as mathematical programming (MP) can be used to explore the decision space and seek good solutions from the many alternatives.

The many wildfire OR examples discussed in this chapter range from those that are largely theoretical in nature to those that have been successfully implemented, such as the LEOPARDS model (McAlpine & Hirsch, 1999). The Victorian Bushfires Royal Commission investigated the catastrophic 2009 bushfires and made a series of recommendations aimed at reducing the risk and impacts of fire and minimising fire-related loss of life (Teague et al. 2010). Of the 67 recommendations made, fifteen could be addressed with the use OR methods, including: consideration of multiple objectives in fuel treatment planning, pre-emptive risk-based deployment of aerial resources and the location of refuges and shelters.

With wildfire related destruction a worsening global problem and wildfire management becoming increasing complicated. There nonetheless exists a concerning and sizeable gap between the decision support needs of wildfire managers and the decision support tools currently available (Martell, 2011. We have demonstrated with the use of examples from the literature the role OR techniques can play in bridging this gap. However it is apt to recall Martell's (1982) reminder that OR specialists can develop decision-making aids that will enhance but not replace the experience and intuition of wildfire managers, and that the successful application of OR methods will require the OR analyst to work closely with wildfire agency personnel.

3. An integrated optimisation model for fuel management and fire suppression preparedness planning

3.1 Introduction

Wildfire management involves a complex mix of components and processes including: fire occurrence prediction, fuel management, fire prevention, fire detection and fire suppression (Martell, 2007). Despite many of these components being interrelated previous wildfire management decision support models have tended to consider these components in isolation from one another, often in the interest of model tractability. In this chapter we present a modelling approach that considers elements of fuel management and fire suppression planning in an integrated manner.

Fire and land management agencies establish fire suppression systems to control and extinguish destructive forest fires. Fire suppression activities can be divided into two distinct subsystems: initial attack and extended attack (also referred to as large fire management). Initial attack refers to the early phase of suppression action during which fire agencies try to contain fires while they are still small (Martell, 1982). When a fire's size and intensity grows such that it is beyond the capabilities of initial attack resources it is called an escaped fire.

The goal of the initial attack subsystem is to prevent fires from escaping. Escaped fires can grow to hundreds of thousands of hectares in size and cause significant damage, the goal of the extended attack subsystem is to mitigate the impact of these large fires. Management of escaped fires is resource intensive and can tie up large numbers of fire agency personnel and equipment for weeks on end (Martell, 2007). The model presented in this chapter is concerned with improving initial attack subsystem effectiveness with a view to reducing the number of escaped fires. Consideration of the spread and suppression of large escaped fires is beyond the scope of the model.

To make initial attack success possible, fire authorities must look ahead and make preparedness planning decisions. These decisions include determining what type and amount of suppression resources to acquire and where to base these resources in order to best satisfy demand (Martell 1982). Optimisation methods have been applied to a range of initial attack preparedness planning problems. MacLellan & Martell (1996) developed an integer programming model for evaluating airtanker home-basing strategies in Ontario. Their model minimised the average annual cost of meeting daily airtanker demands, based on subjective daily deployment rules and historic fire weather data. Dimopoulou & Giannikos (2001 & 2004) determined the optimal location of fire-fighting resources for a region near Athens using a variant of the maximal covering location model (Church & ReVelle, 1974). In their approach a GIS application was used to classify sub-regions based on vegetation and slope, with different classes needing different levels of coverage. Kirsch & Rideout (2005) formulated an integer programming model for determining the optimal set of initial attack resources for an upcoming fire season. Their model optimised the weighted area protected for a user-defined set of fires, with weights assigned based on protection priorities. Haight & Fried (2007) developed a scenario-based integer programming model for exploring optimal initial attack resource deployment levels and locations. Their model minimised the weighted sum of suppression resources deployed and the expected number of fires not receiving a "standard response", with a standard response defined as the desired number of resources that can reach a fire within a specified response time.

Fire behaviour is influenced by three factors: fuel, weather and topography. Of these factors only fuel can be actively managed. In many locations the continued successful containment of fires by initial attack resources has led to fuel build-ups resulting in highly flammable forest landscapes (Schmidt, Taylor, & Skinner, 2008). Fire managers are tasked with reducing the flammability of these landscapes by applying fuel treatments to modify fuel patches (Martell, 2007). A number of optimisation models have been developed to aid in spatial allocation of fuel treatment across a landscape. Hof, et al. (2000) formulated a linear programming model to schedule fuel treatments to mitigate the effects of a defined "target fire" with a known origin and spread behaviour. Wei, et al. (2008) developed an integer programming model for efficiently locating fuel treatments across a landscape based on spatially explicit ignition risk, fire spread probability, fire intensity levels and values-at-risk. Konoshima, et al. (2008 & 2010) used a stochastic dynamic programming model to explore optimal fuel treatment and timber harvesting spatial patterns across a hypothetical landscape subject to fire risk.

The optimisation models discussed above consider fire suppression and fuel treatment planning in isolation from one another. However, these two elements of forest fire management are strongly interrelated. They are implicitly interrelated in a budgetary sense in that funding allocated to one element often reduces funding available to the other. But perhaps more importantly, they are interrelated in a productivity sense in that fuel treatment positively affects suppression efforts by reducing fire spread rates and fire intensity (Rideout, Wei, Kirsch, & Botti, 2008). In this way fuel treatment can enhance the effectiveness of both the initial and extended attack subsystems, by increasing the likelihood of initial attack success and making large fires easier to control. Our model is concerned with the effect fuel treatment has on the efficacy of the initial attack subsystem. That is if a forest patch has been modified by fuel treatment, containment of a fire will generally require less suppression resources and these resources will have more time to get there before the fire escapes. In this way the spatial allocation of fuel treatment has implications for initial attack preparedness planning, and vice versa.

Some recent models have considered elements of both fire suppression and fuel treatment. Mercer, et al. (2008) presented a framework for assessing trade-offs between investments in fuel treatment and fire suppression resources using an integer programming model. However their approach was not fully integrated, in that one-at-a-time adjustment of model parameters was used to incorporate the effect of alternate fuel treatment locations and levels into an initial attack deployment and dispatch model. Wei (2012) developed an integer programming model for selecting fuel treatment locations with a view to providing control opportunities for future fires. However while the fuel treatment patterns generated by the model are intended to be complementary to suppression efforts, the model does not contain explicit consideration of suppression decisions.

Here we present an integrated integer programming model for fire suppression preparedness and fuel management planning. Our model is fully integrated, so that fuel treatment and suppression resource allocation decisions are made simultaneously so as to maximise the complementary effect these two fire management components have on initial attack effectiveness. The motivation for the development of this model came from the Australian bushfire context, where large fuel build-ups in the vicinity of heavily populated urban areas are characteristic. In this environment, some fire and land management agencies are seeking to prioritise short-term fuel management activities in the wildland-urban interface with the aim of increasing initial attack effectiveness so as to protect human life and assets. So with this motivation in mind, a single-period model for planning year-ahead fuel treatment and initial attack resource deployment was deemed most appropriate. We appreciate that in the broader context fuel management is an activity that is typically planned over many years, consideration of the multi-period case is discussed in *Section 3.4 – Summary and discussion* and is modelled in *Chapter 4*.

The remainder of the chapter is structured as follows. The mathematical formulation of the model is presented and explained. The model's functionality is then demonstrated using a series of hypothetical test landscapes. We then conclude by discussing possible extensions where the model could be used as the basis for analysing more complex problem instances.

3.2 Model formulation

Our formulation of an integer programming model for year-ahead suppression preparedness and fuel management planning appears below. We consider a landscape divided into a number of cells representing potential fire locations and candidate locations for fuel treatment. These cells need not be uniform in shape or size. Rather this partitioning would be done based on logical fuel treatment units for the specific landscape in question. We also define a set of potential bases for suppression resource deployment. This set could include existing permanent and temporary bases, as well as locations deemed suitable for “forward deployment” of suppression resources. The main decisions to be considered are: where to base suppression resources and where to undertake fuel treatment. The model is formulated with the following notation.

3.2.1 Indices and sets

i, I = index, set of cells (demand points and candidate locations for fuel treatment);

j, J = index, set of potential base locations where suppression resources can be deployed;

$\Psi \subset I$ = set of cells where fuel treatment is not permitted;

$\Phi_i \subset J$ = set of potential base locations capable of covering cell i if untreated;

$\Omega_i \subset J$ = set of potential base locations capable of covering cell i if treated;

3.2.2 Parameters

w_i = cells weights;

c_j^X = seasonal cost of deploying a suppression resource to base j;

c_i^Y = cost of treating cell i;

b^X = seasonal budget for suppression deployment;

b^Y = seasonal budget for fuel treatment;

r_i^u = suppression resources needed to contain a fire originating in cell i if untreated;

r_i^Y = suppression resources needed to contain a fire originating in cell i if treated;

m_j = maximum number of suppression resources that can be deployed to base j;

3.2.3 Variables

X_j = number of suppression resources deployed to base *j*.

Y_i = 1 if cell *i* is treated,

0 otherwise;

Z_i = 1 if cell *i* is suitably covered by deployed resources,

0 otherwise;

Z_i^u = 1 if an untreated cell *i* is suitably covered by deployed resources,

0 otherwise;

Z_i^Y = 1 if a treated cell *i* is suitably covered by deployed resources,

0 otherwise;

3.2.4 Model

$$\text{Maximise } z = \sum_{i \in I} w_i Z_i \tag{3.1}$$

Subject to:

$$r_i^u Z_i^u \leq \sum_{j \in \Phi_i} X_j \qquad \forall i \in I \tag{3.2}$$

$$r_i^Y Z_i^Y \leq \sum_{j \in \Omega_i} X_j \qquad \forall i \in I \tag{3.3}$$

$$Z_i^Y \leq Y_i \qquad \forall i \in I \tag{3.4}$$

$$Z_i \leq Z_i^u + Z_i^Y \qquad \forall i \in I \tag{3.5}$$

$$Y_i = 0 \qquad \forall i \in \Psi \tag{3.6}$$

$$X_j \leq m_j \qquad \forall j \in J \tag{3.7}$$

$$\sum_{j \in J} c_j^X X_j \leq b^X \tag{3.8}$$

$$\sum_{i \in I} c_i^Y Y_i \leq b^Y \tag{3.9}$$

$$Z_i^u \in \{0,1\} \qquad \forall i \in I \tag{3.10}$$

$$Z_i^Y \in \{0,1\} \qquad \forall i \in I \tag{3.11}$$

$$Z_i \in \{0,1\} \qquad \forall i \in I \qquad (3.12)$$

$$X_j \in INTEGER \qquad \forall j \in J \qquad (3.13)$$

$$Y_i \in \{0,1\} \qquad \forall i \in I \qquad (3.14)$$

The objective function (3.1) maximises the weighted number of cells covered. Assignation of cell weights could be based upon factors such as: "ignition probability" and "values threatened" if a fire originating in that cell is not contained by initial attack resources.

Constraints (3.2) – (3.5) define whether or not a cell *i* is covered. Constraint (3.2) defines the coverage criteria for untreated cells based upon sufficiency and proximity of suppression resources. That is, an untreated cell *i* is considered covered if the sum of suppression resources deployed to bases $j \in \Phi_i$ meets or exceeds the resource requirement r_i^u needed to contain a fire originating in cell *i*. A base *j* is a member of set Φ_i if the response time from base *j* is less the than escape time for a fire originating in cell *i*. With response time defined as the time taken for resources from base *j* to mobilise and travel from to cell *i* and undertake line construction activities. While escape time is defined as the time taken for a fire to reach a pre-defined escaped fire threshold size (e.g. five hectares), the implication being that fires larger than this are considered beyond the capabilities of initial attack resources.

Constraint (3.3) defines the coverage criteria for treated cells in an analogous manner. However the expression includes a different suppression resources required parameter r_i^Y and a different set of proximate bases $j \in \Omega_i$. Since fuel treatment tends to reduce fire intensity, for any given cell the resources required post treatment (r_i^t) are typically lower than those required pre-treatment (r_i^u). Similarly as fuel treatment tends to increase fire escape time, the post-treatment set of proximate bases $j \in \Omega_i$ is typically larger than the pre-treatment set $j \in \Phi_i$. Constraint (3.4) ensures that only treated cells are assessed against the treated cell coverage criteria. Finally, constraint (3.5) defines a cell *i* as covered if it meets either the untreated or treated coverage criteria.

Constraint (3.6) identifies a set of cells where fuel treatment is not permitted. This type of restriction could apply for a range of reasons, for example fuel reduction burning may not be permitted in localities close to airports due to smoke hazard. Constraint (3.7) specifies maximum resource deployment levels for each base. In practical terms this type of restriction would relate to a base's size or capacity, for example a large base may have the capacity to accommodate four fire crews while a small base may only be able to house two crews.

Constraint (3.8) imposes a budget on suppression resource deployment expenditures that cannot be exceeded. The model allows deployment costs to vary on a cell-by-cell basis. In practice this cost variability could be due to factors such as whether it is a permanent or temporary base and how remote its location is. Constraint (3.9) imposes a budget on fuel treatment expenditures that cannot be exceeded. The model allows fuel treatment costs to vary on a cell-by-cell basis. In practice this cost variability could be due to a range of factors such as: site accessibility, fuel type, fuel load and proximity to human settlements.

Constraints (3.10) – (3.12) restrict coverage variables to binary values. Constraint (3.13) restricts the number of resources deployed to a base to integer values, this reflects the fact that suppression resources usually take the form of indivisible quantities. Constraint (3.14) restricts treatment to binary values, such that cell i is either treated or it is not.

In defining Constraints (3.2 and 3.3) with differing "suppression resources required" and "sets of potential base locations capable of covering a cell", we are assuming that the application of fuel treatment has a measurable effect on both fire intensity and rate of spread. The model allows for this effect to vary on a cell by cell basis to take into account local factors such as fuel type, fuel load and topography. In practice both suppression resources required and potential base locations capable of covering a cell will be fire-weather dependent. Thus, the model would need to be parameterised based on a target fire weather scenario. Consideration of multiple fire weather scenarios is discussed further in *Section 3.4 – Summary and discussion*.

This formulation of the model also contains an implicit "no-congestion" assumption as resources deployed at a base j are permitted to help cover more than one cell. That is, we assume there will be no concurrent fires in cells that are covered by resources from a common base. For implementation purposes, the validity of this "no-congestion" assumption would need to be verified for the landscape being modelled, this is discussed further in *Section 3.4 – Summary and discussion*.

The model has been formulated with separate deployment and fuel treatment budgets as this reflects operating conditions for most fire agencies. However the model could be reformulated with a "pooled budget" to allow decision makers to explore optimal expenditure levels for fuel treatment and suppression deployment programs. This could be done by replacing Constraints (3.8) & (3.9) with Constraint (3.15), in which b = total pooled budget for both suppression deployment and fuel treatment.

$$\sum_{j \in J} c_j^X X_j + \sum_{i \in I} c_i^Y Y_i \leq b \tag{3.15}$$

Similarly we could consider a more constrained but more realistic case, where there is a suppression deployment budget, a fuel treatment budget as well as a discretionary budget (b^d) that can be spent on either. In this case, Constraints (3.8) & (3.9) would be replaced by Constraints (3.15), (3.16) & (3.17) with total budget (*b*) equal to $b^X + b^Y + b^d$.

$$\sum_{j \in J} c_j^X X_j \leq b^X + b^d \qquad (3.16)$$

$$\sum_{i \in I} c_i^Y Y_i \leq b^Y + b^d \qquad (3.17)$$

The use of these different levels of budget flexibility to allow decision makers to explore optimal expenditure levels is demonstrated in in *Section 3.3 – Model demonstration*.

In order to determine the minimum resources required to cover the entire landscape the model could be reformulated as a set covering model. In such a formulation the "maximise coverage" objective function (1) would be replaced with a "cost minimisation" objective function (3.18).

$$\text{Minimise } \sum_{j \in J} c_j^X X_j + \sum_{i \in I} c_i^Y Y_i \qquad (3.18)$$

While budget constraints (3.8) and (3.9) would be replaced by a constraint that requires all cells are covered (3.19), where n is the total number of cells in the landscape.

$$\sum_{i \in I} Z_i = n \tag{3.19}$$

In practice, covering the entire landscape may not be cost effective. For example, the cost of covering a geographically remote or difficult to access cell may exceed the expected damage or loss that would result if this cell was left uncovered. If this expected loss (l_i) was known, the model could be reformulated as a "minimisation of cost plus loss" problem with the following objective function (3.20).

$$\text{Minimise } \sum_{j \in J} c_j^X X_j + \sum_{i \in I} c_i^Y Y_i + \sum_{i \in I} l_i \, (1 - Z_i) \tag{3.20}$$

In such a formulation, no budget or level-of-coverage constraints need to be specified. Rather, solving the model to minimise "cost plus loss" will determine the optimal (i.e. most cost effective) budget and resultant level-of-coverage for the landscape in question.

3.3 Model demonstration

In order to demonstrate the functionality of the model 20 hypothetical 100-cell test landscapes were created. The parameter values for the test landscapes are summarised in Table 3.1 below.

Parameters	**Values**
Set of cells: I	100 cells (10 x 10 grid)
Set of potential base locations: J	Corresponds to the set of cells I
Set of cells where fuel treatment is not permitted: Ψ	Empty set
Set of potential base locations capable of covering cell i if untreated: Φ_i	Bases within one-cell distance
Set of potential base locations capable of covering cell i if treated: Ω_i	Bases within two-cell distance
Cell weights: w_i	Between 1 and 9 (random integer)
Seasonal cost of deploying a suppression resource (crew) to base j: c_j^X	\$20,000
Cost of treating cell i: c_i^Y	\$10,000
Budget for suppression deployment: b^X	\$500,000
Budget for fuel treatment: b^Y	\$100,000
Suppression resources needed to contain a fire originating in cell i if untreated: r_i^u	Between 2 and 6 crews (random integer)
Suppression resources needed to contain a fire originating in cell i if treated: r_i^Y	2 crews
Maximum number of suppression resources that can be deployed to base j: m_j	25 crews

Table 3.1: Test landscape parameter values

For all test cases the set of bases *J* corresponded with the set of cells *I*, meaning suppression resource deployment was permitted at all locations in the landscape. Similarly, there were no restrictions applied as to permissible fuel treatment locations. For each cell *i* two parameters: cell weight and suppression resources required if untreated were independent random variables. For simplicity it was assumed that post-treatment suppression resources required were common across all cells irrespective of the cell's pre-treatment condition. In another simplifying assumption, relative positions of cells in the landscape were used to determine the set of base locations capable of covering a cell such that an untreated cell could be covered by a base located one cell away, while a treated cell could be covered by a base located two cells away. With the fuel treatment budget set at $100,000 and with a common fuel treatment cost of $10,000 applied to all cells, the fuel treatment component of the problem amounts to deciding which ten of the 100 cells to treat. Likewise, with the suppression deployment budget set at $500,000 and with a common deployment cost of $25,000 per crew applied to all potential base locations, the suppression preparedness component of the problem amounts to deciding where to locate 25 crews amongst the 100 potential base locations.

In our initial testing the performance of the integrated model was compared to two alternate non-integrated approaches. In the first "independent" approach, fuel treatment and suppression resource deployment decisions were made in a rational manner but independently from one another. Cells were selected for fuel treatment based on largest cell weight values, while suppression resource deployment was optimised with no consideration given to fuel treatment. In the second "coordinated" approach, cells were selected for fuel treatment based on largest cell weight values. Suppression resource deployment was then optimised with the cells selected for fuel treatment treated as an input parameter. The "independent", "coordinated" and "integrated" approaches were applied to each of the 20 test landscapes. In addition to these three approaches, suppression resource deployment was also optimised with no fuel treatment permitted. This provided a baseline measure of the level of coverage the 25 crews were able to deliver in the absence of fuel treatment. Test results appear below in Table 3.2, results are presented in terms of percentage of total cell weights covered.

Landscape No.	No Treatment	Independent	Coordinated	Integrated
1	51.2%	56.9%	59.5%	67.0%
2	56.4%	59.7%	63.7%	71.5%
3	57.6%	61.0%	65.0%	72.5%
4	54.9%	61.6%	64.1%	70.2%
5	52.5%	57.9%	61.8%	68.2%
6	59.8%	61.6%	66.9%	73.5%
7	56.5%	61.9%	65.7%	71.7%
8	55.5%	59.2%	65.2%	71.4%
9	54.8%	60.1%	62.6%	70.7%
10	55.1%	60.6%	66.0%	70.6%
11	54.4%	56.2%	63.5%	70.6%
12	51.9%	53.5%	60.7%	67.0%
13	53.2%	56.9%	62.2%	69.5%
14	57.1%	60.8%	64.0%	72.0%
15	59.3%	64.8%	69.1%	76.4%
16	54.8%	61.8%	64.1%	71.2%
17	53.6%	53.6%	65.7%	69.5%
18	60.7%	62.5%	68.4%	75.5%
19	61.1%	63.0%	67.9%	75.7%
20	57.1%	58.9%	64.0%	72.8%
Average	**55.9%**	**59.6%**	**64.5%**	**71.4%**

Table 3.2: Test results – performance of integrated model vs. non-integrated approaches in terms of percentage of total cell weights covered

As mentioned previously, the “no treatment” results provide a baseline measure of the level of coverage available in the absence of fuel treatment. In the “independent”, “coordinated” and “integrated” approaches fuel treatment was applied to ten of the 100 cells or 10% of the landscape. Since fuel treatment has a complementary effect on initial attack effectiveness, it is not surprising that the “independent”, “coordinated” and “integrated” approaches provided a higher level of coverage than the “no treatment” baseline.

The same amount of fuel treatment and deployment resources were available in the “independent” and “coordinated” approaches and the same method was used to select fuel treatment locations. However in the “coordinated” approach, suppression resource deployment optimisation incorporated previously selected fuel treatment locations, this resulted in the “coordinated” approach outperforming the “independent” approach by 8.2% on average. The “integrated” approach also had the same amount of fuel treatment and deployment resources available as both the “independent” and “coordinated” approaches. However the “integrated” model’s ability to make fuel treatment and suppression resource deployment decisions simultaneously to maximise initial attack effectiveness led to it on average outperforming the “coordinated” approach by 10.7% and the “independent” approach by 19.7%. While numerical results will dependent on landscape configurations and on the costs and effects of fuel treatment and suppression deployment actions as specified by model parameters, in general the “integrated” model will always provide a level of coverage greater than or equal to the “independent” and “coordinated” approaches.

This is because in the "integrated" approach we are simultaneously optimising two related sets of decisions as compared to the "coordinated" approach where these decisions are made sequentially, and the "independent" approach where these are made independently. An illustrative example was selected from amongst the twenty test cases to demonstrate how the "integrated" model outperforms the "coordinated" approach.

$w = 8$ / $r = 2$	7 / 4	3 / 3	**9 / 6**	2 / 6	2 / 2	1 / 5	**9 / 3**	6 / 3	7 / 3
1 / 4	4 / 2	9 / 6	**9 / 3**	6 / 3	3 / 6	7 / 2	8 / 2	3 / 5	3 / 4
6 / 4	4 / 6	**9 / 3**	6 / 4	4 / 5	5 / 2	3 / 4	7 / 2	2 / 3	8 / 5
6 / 5	3 / 3	5 / 5	7 / 2	**9 / 4**	1 / 3	5 / 4	5 / 3	2 / 3	**9 / 2**
6 / 3	6 / 2	5 / 3	7 / 5	7 / 2	7 / 4	**9 / 4**	7 / 3	1 / 5	7 / 2
7 / 4	6 / 2	4 / 6	2 / 4	8 / 4	7 / 6	4 / 4	4 / 2	3 / 4	**9 / 3**
6 / 6	1 / 3	8 / 5	4 / 4	1 / 2	4 / 3	2 / 4	1 / 3	7 / 6	5 / 2
5 / 6	3 / 3	8 / 6	2 / 2	3 / 2	6 / 3	2 / 2	8 / 2	4 / 6	6 / 4
5 / 5	**9 / 4**	**9 / 4**	4 / 6	2 / 3	7 / 2	8 / 4	8 / 4	6 / 4	6 / 6
7 / 3	2 / 2	5 / 3	3 / 5	8 / 5	6 / 5	2 / 2	2 / 3	5 / 2	4 / 3

"Coordinated" approach

$w = 8$ / $r = 2$	**7 / 4**	3 / 3	**9 / 6**	2 / 6	2 / 2	1 / 5	9 / 3	6 / 3	7 / 3
1 / 4	4 / 2	**9 / 6**	9 / 3	6 / 3	3 / 6	**7 / 2**	8 / 2	3 / 5	3 / 4
6 / 4	4 / 6	9 / 3	6 / 4	4 / 5	5 / 2	3 / 4	7 / 2	2 / 3	8 / 5
6 / 5	3 / 3	5 / 5	7 / 2	9 / 4	1 / 3	5 / 4	5 / 3	2 / 3	9 / 2
6 / 3	6 / 2	5 / 3	**7 / 5**	7 / 2	7 / 4	9 / 4	7 / 3	1 / 5	7 / 2
7 / 4	6 / 2	4 / 6	2 / 4	**8 / 4**	**7 / 6**	4 / 4	4 / 2	3 / 4	**9 / 3**
6 / 6	1 / 3	8 / 5	4 / 4	1 / 2	4 / 3	2 / 4	1 / 3	**7 / 6**	5 / 2
5 / 6	3 / 3	8 / 6	2 / 2	3 / 2	6 / 3	2 / 2	8 / 2	4 / 6	6 / 4
5 / 5	9 / 4	9 / 4	4 / 6	2 / 3	7 / 2	8 / 4	8 / 4	6 / 4	6 / 6
7 / 3	2 / 2	5 / 3	3 / 5	**8 / 5**	6 / 5	2 / 2	2 / 3	5 / 2	4 / 3

"Integrated" approach

Cell selected for fuel treatment

w = relative cell weight

r = suppression resources required

Figure 3.1: Cells selected for fuel treatment (test landscape 3)

In the test instance in Figure 3.1 it is can be seen that two of the cells selected for treatment are common for both modelling approaches while the other eight cells selected differ. The cells selected for fuel treatment using the "coordinated" approach have higher cell weight values than those selected using the "integrated" approach. Despite this, as seen in Figure 3.2 below, the combination of treatment and deployment decisions employed by the "integrated" model provided a higher level of coverage than the sequential approach. In this test case the use of the "integrated" model resulted in an additional two cells receiving coverage and an 11.5% higher objective value than the "coordinated" approach.

w = 8 *r* = 2	7 *X*= 2 4	3 3	9 2	2 6	2 2	1 5	9 2	6 3	7 3
1 4	4 2	9 6	9 2	6 3	3 6	7 2	8 2	3 5	3 4
6 4	4 6	9 2	6 4	4 5	5 2	3 *X*= 2 4	7 2	2 3	8 5
6 5	3 3	5 5	7 2	9 2	1 3	5 4	5 3	2 3	9 2
6 3	6 2	5 3	7 5	7 *X*= 2 2	7 4	9 *X*= 2 2	7 3	1 5	7 2
7 4	6 *X*= 3 2	4 6	2 4	8 4	7 6	4 4	4 2	3 *X*= 2 4	9 2
6 6	1 *X*= 1 3	8 5	4 4	1 *X*= 2 2	4 3	2 4	1 3	7 6	5 2
5 6	3 *X*= 2 3	8 6	2 2	3 2	6 3	2 *X*= 1 2	8 2	4 6	6 4
5 5	9 *X*= 3 2	9 2	4 6	2 3	7 2	8 4	8 *X*= 2 4	6 4	6 6
7 3	2 2	5 3	3 5	8 5	6 5	2 *X*= 1 2	2 3	5 2	4 3

"Coordinated" approach

w = 8 *r* = 2	7 2	3 3	9 2	2 6	2 2	1 5	9 3	6 3	7 3
1 4	4 2	9 2	9 3	6 3	3 6	7 2	8 2	3 *X*= 3 5	3 4
6 4	4 6	9 3	6 *X*= 3 4	4 5	5 2	3 4	7 2	2 3	8 5
6 5	3 3	5 5	7 2	9 4	1 *X*= 2 3	5 4	5 3	2 *X*= 2 3	9 2
6 3	6 2	5 3	7 2	7 2	7 4	9 *X*= 2 4	7 3	1 5	7 2
7 4	6 *X*= 3 2	4 6	2 4	8 2	7 2	4 4	4 2	3 4	9 2
6 6	1 *X*= 1 3	8 5	4 4	1 2	4 3	2 4	1 3	7 2	5 2
5 6	3 *X*= 2 3	8 6	2 2	3 2	6 3	2 *X*= 3 2	8 *X*= 1 2	4 6	6 4
5 5	9 *X*= 3 2	9 4	4 6	2 3	7 2	8 4	8 4	6 4	6 6
7 3	2 2	5 3	3 5	8 2	6 5	2 2	2 3	5 2	4 3

"Integrated" approach

Cell covered by suppression resources

X = suppression resources deployed (crews)

Figure 3.2 : Deployment locations and cells covered (test landscape 3)

Our next set of testing demonstrates how the integrated model can be used to allow decision makers to explore optimal expenditure levels for fuel treatment and suppression deployment programs. With this in mind, the integrated model was run with three different levels of budget flexibility. In the "fixed" budget case there was a fuel treatment budget of $100,000 and a suppression deployment budget of $500,000 as per the previous round of testing reported in Table 3.2. In the "discretionary" budget case there was a fixed fuel treatment budget of $50,000, a suppression deployment budget of $450,000 and a discretionary budget of $100,000 that could spent on either suppression deployment or fuel treatment. In the "pooled" case there was a total budget of $600,000 that could be spent on suppression deployment or fuel treatment with no restrictions. We tested these varying degrees of budget flexibility using the same 20 test landscapes employed in the previous round of testing. Other than the budget differences described above all other parameter values were as per Table 3.1. Test results appear below in Table 3.3, results are presented in terms of percentage of total cell weights covered and proportion of the budget spent on fuel treatment.

Landscape No.	Fixed		Discretionary		Pooled	
	Coverage	Proportion spent on fuel treatment	Coverage	Proportion spent on fuel treatment	Coverage	Proportion spent on fuel treatment
1	67.0%	16.7%	68.0%	25.0%	70.15%	60.0%
2	71.5%	16.7%	74.2%	23.3%	78.77%	46.7%
3	72.5%	16.7%	73.6%	23.3%	75.53%	40.0%
4	70.2%	16.7%	71.9%	23.3%	74.67%	46.7%
5	68.2%	16.7%	69.6%	23.3%	73.36%	60.0%
6	73.5%	16.7%	75.1%	23.3%	75.90%	40.0%
7	71.7%	16.7%	72.1%	23.3%	74.15%	53.3%
8	71.4%	16.7%	73.0%	23.3%	74.25%	53.3%
9	70.7%	16.7%	71.5%	25.0%	75.00%	56.7%
10	70.6%	16.7%	72.4%	23.3%	73.24%	46.7%
11	70.6%	16.7%	71.8%	23.3%	73.77%	40.0%
12	67.0%	16.7%	69.6%	23.3%	73.52%	56.7%
13	69.5%	16.7%	72.0%	23.3%	74.23%	36.7%
14	72.0%	16.7%	73.2%	23.3%	74.59%	50.0%
15	76.4%	16.7%	78.0%	23.3%	80.89%	56.7%
16	71.2%	16.7%	72.7%	23.3%	75.05%	40.0%
17	69.5%	16.7%	71.5%	23.3%	75.97%	60.0%
18	75.5%	16.7%	76.2%	23.3%	77.78%	40.0%
19	75.7%	16.7%	77.0%	23.3%	78.72%	43.3%
20	72.8%	16.7%	74.4%	23.3%	78.35%	53.3%
Average	**71.4%**	16.7%	**72.9%**	**23.5%**	**75.4%**	**49.0%**

Table 3.3: Test results – integrated model performance with varying amounts of budget flexibility in terms of percentage of total cell weights covered and proportion of the budget spent on fuel treatment

Unsurprisingly as the level of budget flexibility increased so did the level of coverage achieved, with the least constrained "pooled" budget case on average outperforming the "fixed" budget case by 5.7% and the "discretionary" budget case by 3.5%. Similarly the "discretionary" budget case outperformed the "fixed" budget case by 2.1% on average. In the "pooled" budget case the lowest proportion spent on fuel treatment was 36.7% or $220,000, this means there would be 22 cells treated and 19 crews deployed. While the highest proportion spent on fuel treatment was 60% or $360,000 equating be 36 cells treated and only 12 crews deployed. The level of fuel treatment expenditure observed in the "pooled" budget case suggests that in the "discretionary" case the maximum allowable 25% of expenditure would be allocated to fuel treatment. However, interestingly this only occurs in two of the 20 test instances. In the other18 instances $90,000 of the discretionary budget is spent on fuel treatment and the remaining $10,000 is added to the suppression deployment budget. This somewhat counter-intuitive result is due to the interactive effect of fuel treatment and suppression deployment decisions. Whereby, application of fuel treatment to a single additional cell is of no benefit in the absence of sufficient and proximal suppression resources.

In general the proportion of the budget spent on fuel treatment versus suppression deployment will depend on the relative costs and effects of these fire management components as specified by model parameters, as well as the attributes and spatial arrangement of the landscape the model is applied to. However the less constrained "pooled" budget model will always provide a level of coverage greater than or equal to the "discretionary" budget model and likewise the "discretionary" budget model will always perform as well or better than the more constrained "fixed" budget model.

The integrated model was solved to optimality for test landscapes of various sizes with differing levels of fuel treatment on a regular PC (Intel 2Duo 3.6 GHz processor and 3.49 GB RAM) using CPLEX 12.2 OPL-IDE with standard settings. Computation times are reported below in Table 3.4.

Landscape		Percentage of landscape treated		
		2%	5%	10%
100 cells (10 by 10) 25 crews deployed	computing time (s)	31	56	71
144 cells (12 by 12) 36 crews deployed	computing time (s)	126	242	430
196 cells (14 by 14) 49 crews deployed	computing time (s)	519	1099	>3000

Table 3.4: Computational test results

3.4 *Summary and discussion*

In this chapter we have presented the first optimisation model that incorporates both fuel treatment and suppression preparedness planning decisions. In the preceding section we demonstrated the use of this model on a set of hypothetical landscapes. While further testing is required on more realistic landscapes, the initial test results suggest that an integrated approach to fuel management and suppression preparedness planning can lead to improved initial attack coverage outcomes. Given the link between fuel treatment, fire behaviour and resultant suppression effort required, it makes sense intuitively that a modelling approach that captures this interrelation would outperform approaches that treat these elements in isolation from one another. We have also demonstrated how an integrated model can be used by decision makers to explore optimal expenditure levels in fuel treatment and suppression deployment programs.

Implementation of the model on real landscapes will require model parameterisation. The model has been designed to incorporate inputs that are currently available to Australian fire and land management agencies from a range of sources including geospatial databases, fire behavior models and meteorological data. Good estimates are generally available for fuel treatment and suppression deployment costs. Location-specific fire escape times can be readily estimated for target fire weather conditions using fire spread models and geospatial fuel data. Similarly, response times can be generated by combining geospatial travel time data with mobilisation and line-construction time estimates based on historic data. Escape and response time estimates can then be used to calculate potential base locations capable of covering a locality. Suppression resources required will be more difficult to estimate and will likely require the development of rules-of-thumb based on expert judgment elicitation. While designation of cell weights could be aided by fire simulation modelling and analysis of spatially explicit historic ignition and values-at-risk data.

In this chapter, the integrated model has been presented in a very simple and general form. However the model could be readily adapted without significantly altering its structure to consider several different suppression resource types with varying costs, travel speeds and levels of suppression effectiveness. Similarly, a number of different fuel treatment types with varying costs and levels of effectiveness could be included. There are a number of ways the model could be extended to cater for special features arising in specific implementation instances, a few of these possible extensions are discussed briefly here. With the exception of the multi-year formulation, all the model extensions discussed below have the same fundamental mathematical structure as the general integrated model.

In some implementation instances congestion may be identified as an issue. That is, the occurrence of concurrent fires in cells that are covered by a single base. To allow for congestion a probabilistic reliability level formulation could be employed (Marianov & ReVelle, 1992). In such an approach, a "busy fraction" is estimated for each suppression resource and then used to determine the number of resources required at a base to cover demand points with a given reliability level (α). These calculations are done exogenously with the resultant parameter values then incorporated into an adapted model that maximises the level of " α -reliable" coverage.

In addition to fire occurrence, other model elements that could be treated as stochastic variables include location specific fire escape times and suppression resources needed to contain a fire. Both of these elements will be dependent on fire-weather conditions. That is, as conditions become increasingly hot, dry and windy, fire escape time will tend to decrease and suppression resources required will tend to increase. Where decision makers are interested in system performance across a range of defined fire-weather scenarios, the problem could be formulated as a two-stage stochastic programming model with recourse. In such an approach, integrated fuel treatment and suppression resource deployment decisions would be made in the first stage taking into account the full range of defined fire-weather scenarios, with the opportunity for adjustments to deployment of suppression resources in the second stage based on observed fire-weather outcomes.

A multi-year model formulation could be developed for instances where decision makers wish to consider longer time horizons. Such a model would need to track “time since fire” on a cell-by-cell basis so as to incorporate “diminishing returns” on fuel treatment effect over time due to vegetation regrowth. A multi-year model would lend itself to consideration of ecological considerations such as restrictions on burn frequency and desired spatio-temporal post fire seral stage landscape composition.

4. A spatial optimisation model for multi-period landscape level fuel management to mitigate wildfire impacts

4.1 Introduction

Fire is a natural component of many terrestrial ecosystems. However, uncontrolled wildfires can cause loss of human life and destruction of property and natural resources (King, et al., 2008). This is of special concern in localities such as southern Australia, California and Mediterranean Europe where major cities are situated in close proximity to highly flammable vegetation (Bradstock et al., 2012). Wildfire incidence requires the co-occurrence in time and space of three factors: fire-conducive weather, an ignition source and fuel (i.e. flammable vegetation) (Parisien, Junor, & Kafka, 2007). In recent decades an increase in wildfire extent and severity has been observed in many countries including the USA, Canada, Australia and southern Europe (Boer, Sadler, Wittkuhn, McCaw, & Grierson, 2009; McCaw, 2013). This is due in part to uncharacteristically high fuel loads arising from suppression focused 20th century fire management practices (Loehle, 2004; Reinhardt, Keane, Calkin, & Cohen, 2008; Schmidt, et al., 2008; Hessburg, Reynolds, Keane, James, & Salter, 2007).

In an attempt to reduce the risk posed by wildfires, land management agencies in Australia and the USA have implemented extensive fuel management programs (Ager, Vaillant, & Finney, 2010; Boer, et al., 2009; Collins, Stephens, Moghaddas, & Battles, 2010; McCaw, 2012). Fuel management is defined as the process of altering the amount and structure of forest fuels through the application of treatments such as prescribed fire and mechanical thinning (Finney, 2001; King, et al., 2008). Fuel management programs typically aim to reduce risk in two ways: (1) by forming fuel-breaks adjacent to communities to facilitate the establishment of fire-lines by suppression forces such as fire crews or air tankers, or (2) by altering fuels in the surrounding landscape to modify fire behaviour and lessen the potential for severe fires (Bevers, Omi, & Hof, 2004; Kim, Bettinger, & Finney, 2009). Large destructive wildfires typically occur in hot, dry and windy weather conditions and tend to be resistant to suppression efforts due to their rapid growth, sheer size, and crown fire and spotting behaviours. Under such conditions a program of the second type that manages fuel in the wider landscape is thought to offer the best possible means for resisting fire growth (Finney, 2007; Reinhardt, et al., 2008).

Large wildfires cover an area greater than a treated forest stand, meaning a single large fire could encounter several fuel treatments before extinguishment. Hence, a landscape-level fuel management strategy that considers the layout of all fuel treatments in relation to one another is likely to be more effective than a 'greedy' selection of treatment locations (Rytwinski & Crowe, 2010). The potential benefits of landscape-level fuel treatment has been recognised in wilderness areas of the western United States where free-burning fires have generated mosaics of differing fuel ages and this pattern of historic burns has been seen to delay and detour large fires in subsequent years (Finney, 2007). This type of landscape-level effect was observed in a study of two large Arizona wildfires, where the fires circumvented treated areas resulting in fire-shadows on the lee-side of fuel treatments and an overall reduction in fire severity (Finney, McHugh, & Grenfell, 2005). In the eucalypt forests of south-western Australia prescribed burning has been practised at large spatial scales over the past five decades. Analysis of historic data in this region has revealed that the connectedness of 'old' untreated fuel patches is the strongest contributing variable to wildfire extent, highlighting the need to consider spatial arrangement of fuels when planning fuel treatment regimes (Boer, et al., 2009). This finding is consistent with observations made in the USA's Sierra Nevada forests that indicate spatial fragmentation of fuels can modify wildfire size and behavior (van Wagtendonk 1995, Parsons and van Wagtendonk 1996).

Despite this small but growing body of field evidence, current understanding of fire behaviour responses to landscape-level fuel treatment is largely based upon simulation studies. Probabilistic models based on percolation theory and cellular automata have demonstrated the importance of fuel connectivity for landscape-level fire spread dynamics (Miller & Urban, 2000), with fragmentation of the fuel complex by treatment resulting in a reduction in average fire size (Loehle, 2004). Simulation studies indicate that fire spread is affected by the amount of fuel treatments as a proportion of the landscape and their spatial configuration (Gonzalez, Palahi, & Pukkala, 2005; Parisien, et al., 2007; Schmidt, et al., 2008; King, et al., 2008). Nonlinear relationships have been identified between area treated and fire behaviour outputs (Ager, et al., 2010), including 'threshold' effects where if fuel treatment exceeds some critical level a marked reduction in fire propagation is realised (King, et al., 2008). Geometrically derived fuel treatment patterns have been shown to reduce fire spread rate and fire-line intensity (Finney, 2001). However, real-life application of such patterns is complicated by the heterogeneity of landscapes with respect to fuels, weather and topography (Finney, 2007).

While findings from empirical and simulation studies can inform strategies for spatial fuel treatment configurations, in practice the performance of such strategies can be significantly degraded by operational constraints that restrict treatment extent and location (Ager, et al., 2010). These restrictions arise due to factors such as: land ownership, funding limitations, inadequate road access, habitat preservation regulations and prescribed burning weather requirements (M. A. Finney, 2001; Fernandes & Botelho, 2003; Collins, et al., 2010; McCaw, 2013). Such constraints make fuel management a problem amenable to optimisation (Finney, 2007) and accordingly a number of models for spatial allocation of treatment effort have been proposed. Hof, et al. (2000) formulated a linear programming model to delay a defined target fire's spread to nominated "protection areas". Finney (2007) developed an iterative procedure incorporating the minimum travel time algorithm for locating fuel treatments in major fire flow paths. Palma, Cui, Martell, Robak, & Weintraub (2007) proposed a heuristic approach using shortest path methods to select individual harvest blocks to disrupt critical paths between potential ignition points and values at risk. Wei, et al. (2008) formulated a mixed integer programming model for locating fuel treatments that reduce fire intensity so as to minimise the expected loss incurred on a flammable landscape. Rytwinski & Crowe (2010) used an iterative procedure that paired a fire spread simulator with a metaheuristic scatter-search algorithm to select fuel break location. Wei (2012) developed a mixed integer programming model to locate fuel treatments to set up potential control locations for future fires.

A limitation of the models described above is that they handle spatial allocation of fuel treatments as a single period problem. However, in practice treatment effects are transient because most vegetation eventually recovers and begins to re-grow after it has been treated. This means that the generation and maintenance of desirable landscape-level fuel configurations requires a multi-period schedule that takes longevity of individual treatments into account (Finney, 2001; Reinhardt, et al., 2008). Spatially explicit multi-period fuel treatment scheduling is a complicated problem and most of the modelling efforts to date have either employed heuristic approaches or considered very small landscapes. Gonzalez, et al. (2005) used a heuristic procedure to schedule harvesting activities to optimise a number of landscape metrics and combinations thereof. Konoshima, et al. (2008 & 2010) formulated a stochastic dynamic programming model to explore optimal fuel treatment and timber harvesting spatial patterns across a small hypothetical landscape. Kim et al. (2009) explored the use of a heuristic for multi-period scheduling of fuel management activities across a large landscape in north-eastern Oregon. Their model was used to generate both dispersed and clustered fuel treatment patterns in an attempt to mitigate the effects of wildfires whilst maintaining evenly distributed annual harvest volumes. A follow-up paper examined the effects of these spatial fuel treatment patterns on simulated, human-caused fires in the same study area in north-eastern Oregon (Kim and Bettinger 2008). González-Olabarria & Pukkala (2011) developed an iterative procedure that used a simulated annealing algorithm and a fire spread simulator to generate fuel treatment schedules with a view to stabilising fire risk over time. Longer term fuel management planning often involves the consideration of a number of ecological considerations (Ager, et al., 2010), these can include burn frequency constraints based on vital attributes of ecosystems and species and requirements to maintain post-fire seral stage heterogeneity to support biodiversity (Burrows, 2008). Calkin, et al. (2005) used a simulated annealing algorithm to solve a goal programming model for reducing wildfire threat while maintaining late seral forest for faunal habitat. Lehmkuhl, et al. (2007) used fire spread models and an evolutionary algorithm to simultaneously minimise potential fire behaviour and loss of faunal habitat.

In this chapter we present a spatially explicit mixed integer programming model for fuel treatment scheduling. The model accounts for the transient nature of fuel by keeping track of the age of the age of both treated and untreated patches of fuel or vegetation. It is, we believe, the first multi-period landscape-level fuel treatment model to be formulated and solved using exact optimisation methods. The model provides a flexible framework that allows for incorporation of landscape heterogeneity, as well as a range of ecological and operational constraints.

The integrated model presented in *Chapter 3* considered the complementary effect of fuel management on the effectiveness of initial attack activities undertaken by suppression resources. As such, the model's focus was short-term fuel management in the wildland-urban interface. In contrast, the focus of the model presented in this chapter is longer term fuel treatment scheduling so as to modify fuel structure in the wider landscape with a view to mitigating large fire behavior. The remainder of the chapter is structured as follows. The mathematical formulation of the model is presented and explained. The model's functionality is then demonstrated on a series of hypothetical test landscapes. This is followed by some computational testing and discussion of implementation issues.

4.2 Model formulation

Our formulation of a mixed integer programming model for multi-year landscape level fuel treatment planning appears below. We consider a landscape divided into a number of cells representing candidate locations for fuel treatment. . In *Section 4.5* the model is implemented on a series of landscapes composed of regular grid cells. However, cells need not be uniform in shape or size and in practice this partitioning would be done based on what constitutes suitable management units for the specific landscape in question. In practical implementations these cells are likely to be irregular polygons of various sizes. The key decision to be made is - which cells should be treated in each time period (i.e. each year). In order to account for the transience of fuel treatment effect, fuel age (years) or time since treatment is tracked. Each cell's fuel age is a discrete-time step function, where at annual intervals a cell's fuel age increases by one year if untreated and resets to zero if treated. It is assumed that fuel treatment has an inhibitory effect that lasts for a defined period of time. A cell is classified as an 'old fuel cell' if its fuel age exceeds this inhibition period (Boer, et al., 2009). Since the spatial nature of fire origin is difficult to predict and as fire behaviour is complex we have not tried to explicitly capture fire dynamics within our model. Instead we have chosen to focus our efforts on generating desirable spatial fuel patterns (Hof & Omi, 2003). Our model therefore schedules fuel treatments so as to reduce the connectivity of 'old fuel cells' in the belief that fragmentation of the landscape fuel complex will inhibit fire spread. The model is formulated with the following notation.

4.2.1 Sets

I is the set of all cells in the landscape;

$\Psi \subset I$ is the set of cells where fuel treatment is not permitted;

$\Lambda \subset I$ is the set of cells where fuel treatment is permitted (where $\Lambda = I - \Psi$);

$\Phi_i \subset I$ is the set of cells connected to cell *i*;

T is the number of time periods in the planning horizon;

4.2.2 Parameters

a_i = initial fuel age of cell *i*;

b_t = fuel treatment budget for time period *t*;

c_{ti} = cost of treating cell *i* in time period *t*;

u_{ti} = fuel age upper bound of cell *i* at time period *t* (where $u_{ti} = a_i + t$);

o_i = fuel age threshold for 'old fuel cell' classification of cell *i*;

4.2.3 Variables

X_{ti} = 1 if cell i is treated in time period t,

0 otherwise;

A_{ti} = fuel age of cell i in time period t;

O_{ti} = 1 if cell i is classified as an 'old fuel cell' in time period t, 0 otherwise;

Q_{tij} = 1 if cell i and connected cell j are both classified as 'old fuel cells' in time period t,

0 otherwise;

4.2.4 Model

$$\text{Minimise } z^* = \sum_{t=1}^{T}\sum_{i\in I}\sum_{j\in\Phi_i} Q_{tij} \tag{4.1}$$

Subject to:

$$\sum_{i\in\Lambda} c_{ti} X_{ti} \le b_t \qquad t = 1 \ldots T \tag{4.2}$$

$$A_{ti} = a_i \qquad t = 0 \quad \forall i \in I \tag{4.3}$$

$$A_{ti} = A_{(t-1)i} + 1 \qquad t = 1 \ldots T \quad \forall i \in \Psi \tag{4.4}$$

$$A_{ti} >= A_{(t-1)i} + 1 - u_{ti} * X_{ti} \qquad t = 1....T \quad \forall i \in \Lambda \qquad (4.5)$$

$$A_{ti} - u_{ti} * O_{ti} \le o_i \qquad t = 1....T \quad \forall i \in I \qquad (4.6)$$

$$O_{ti} + O_{tj} - Q_{tij} \le 1 \qquad t = 1....T \quad \forall i \in I \quad \forall j \in \Phi_i \qquad (4.7)$$

$$X_{\mathrm{ti}} \in \{0,1\} \qquad t = 1....T \quad \forall i \in I \qquad (4.8)$$

$$O_{\mathrm{ti}} \in \{0,1\} \qquad t = 1....T \quad \forall i \in I \qquad (4.9)$$

$$Q_{tij} \in \{0,1\} \qquad t = 1....T \quad \forall i \in I \quad \forall j \in \Phi_i \qquad (4.10)$$

$$A_{ti} \ge 0 \qquad t = 1....T \quad \forall i \in I \qquad (4.11)$$

The objective function (4.1) minimises the number of 'connected old fuel cells' across all time periods. A set of connected cells is defined for each cell in the landscape. In the simplest case each set would be composed of all immediately adjacent cells. An alternative case is where these sets are constructed to take into account heterogeneous landscape features such as the prevailing wind direction associated with severe burning conditions. In addition to prevailing wind direction, other considerations in defining connectivity sets in practical implementations may include topographic features and anticipation of spotting behaviour. If a cell contains a fuel type conducive to spotting, it may be considered functionally connected to other cells with which it does not share a common boundary. At any rate, the specification of connectivity sets on a cell-by-cell basis provides a flexible means for these various locality specific connectivity requirements to be included in the model as required.

Constraint (4.2) imposes a fuel treatment budget for each time period. The model allows fuel treatment costs to vary on a cell-by-cell basis. In practice this cost variability could be due to a range of factors such as site accessibility, fuel type and proximity to the wildland urban interface.

Constraints (4.3) – (4.5) track the fuel age of each cell. Constraint (4.3) initialises each cell's fuel age. Constraint (4.4) applies to the set of cells where fuel treatment is not permitted and ensures that the fuel age of these cells increments by one each year. This treatment restriction could apply for a range of reasons for example: the land may be privately owned by individuals that do not wish their land to be treated or proximity to an airport or busy highway may preclude prescribed burning due to smoke hazard. It should be noted that these cells can still be classified as 'old fuel cells' and as such can contribute to the connectivity of the fuel complex. Constraint (4.5) applies to the set of cells where fuel treatment is permitted. Here the choice between treating and not treating a cell in a given time period and the resultant fuel age is modelled as a disjunctive constraint (Hooker 2009). In the absence of fuel treatment this constraint ensures that a cell's fuel age increments by one. The fuel age upper bound (u_{ti}) acts as a Big-M and has a sufficiently large value so that when a cell is treated the disjunct is not constraining and consequently the cell's fuel age resets to zero. A fuel age upper bound (u_{ti}), calculated for each cell *i* at each time period *t,* is used rather than an arbitrarily large Big-M in the interests of formulation strength and pursuant solvability (Williams 2013).

Constraint (4.6) uses a binary indicator variable (O_{ti}) to classify a cell as an 'old fuel cell' if its fuel age exceeds a threshold value based on the fuel treatment inhibition period (Williams 2013). The model allows this threshold to vary on a cell-by-cell basis to take into account, different fuel types. Fuel types with a shorter inhibition period will regain their fuel load more quickly.

When two connected cells *i* and *j* are both are classified as 'old fuel cells' we can express this as the product of two binary variables (i.e. $O_{ti} * O_{tj}$). In constraint (4.7) we replace this product with a new binary variable (Q_{tij}) that takes the value one when a connected pair of cells *i* and *j* are both classified as 'old fuel cells' in time period *t* (Williams 2009). As mentioned earlier, the specification of a connectivity set on a cell-by-cell basis allows for directional connectivity to be included in the model as required.

Constraints (4.8), (4.9) and (4.10) restrict 'fuel treatment', 'old fuel cell' and 'connected old fuel cell' variables to binary values. Constraint (4.11) restricts 'fuel age' to positive values.

4.3 Ecological model extensions

In the preceding section the model was presented in its basic form. This formulation can however be extended to include a number of ecological considerations. One such consideration is treatment frequency or tolerable fire interval (TFI) (Burrows, 2008; Cheal, 2010). Minimum and maximum TFIs are assigned to treatment units according to ecological vegetation classes. The minimum TFI refers to the minimum time required between successive fire events at a site and is often based upon the juvenile period(s) of sensitive species in the vegetation class. While the maximum TFI is the maximum time required between fire events and takes into account the requisite fire interval for rejuvenation of fire adapted species. Tolerable fire interval restrictions can be incorporated into the model with the following constraints.

$$A_{ti} \geq r_i * X_{ti} \qquad t = 1....T \quad \forall i \in I \tag{4.12}$$

$$A_{ti} \leq s_i - u_{ti} * X_{ti} \qquad t = 1....T \quad \forall i \in I \tag{4.13}$$

Constraint (4.12) precludes a cell from being treated if its fuel age is less than the minimum TFI (r_i). Constraint (4.13) ensures that all cells are treated before their fuel age exceeds the maximum TFI (s_i). Both minimum TFI and maximum TFI are specified on a cell specific basis, thus allowing the modelling of landscapes with multiple vegetation classes or even site specific TFI requirements.

Another ecological consideration is the desire to maintain the correct proportion of vegetation in different stages of maturity so as to support biodiversity (Burrows 2008, Cheal 2010). Distinct 'habitat growth' or 'seral' stages are identified for each ecological vegetation class based on fuel age. Cells can be classified into ecological vegetation classes in the following manner. A set (Π) of all ecological vegetation classes (k) is specified, with any cell i permitted to be a member of strictly one vegetation class (i.e. $i \in \Pi_k$ for some k). Constraints (4.14) – (4.21) classify individual cells into one of three seral stages based on fuel age using binary indicator variables and thresholds (Williams 2013). In this case three indicator variables are required, one for each seral stage: 'early' (E_{ti}), 'mid' (F_{ti}) and 'late' (G_{ti}). With seral stage classification based on two thresholds, one used to indicate the upper limit of the 'early' (e_i) stage and one used to indicate the upper limit of the 'mid' (f_i) stage.

$$A_{ti} + e_k * E_{ti} >= e_k \qquad t = 1....T \quad \forall i \in \Pi_k \quad \forall k \in \Pi \qquad (4.14)$$

$$A_{ti} - u_{ti} * (F_{ti} + G_{ti}) \le e_k + 1 \qquad t = 1....T \quad \forall i \in \Pi_k \quad \forall k \in \Pi \qquad (4.15)$$

$$A_{ti} + f_k * (E_{ti} + F_{ti}) >= f_k \qquad t = 1....T \quad \forall i \in \Pi_k \quad \forall k \in \Pi \qquad (4.16)$$

$$A_{ti} - u_{ti} * (G_{ti}) \le f_k + 1 \qquad t = 1....T \quad \forall i \in \Pi_k \quad \forall k \in \Pi \qquad (4.17)$$

$$E_{ti} + F_{ti} + G_{ti} = 1 \qquad t = 1....T \quad \forall i \in I \qquad (4.18)$$

$$E_{ti} \in \{0,1\} \qquad t = 1....T \quad \forall i \in I \tag{4.19}$$

$$F_{ti} \in \{0,1\} \qquad t = 1....T \quad \forall i \in I \tag{4.20}$$

$$G_{ti} \in \{0,1\} \qquad t = 1....T \quad \forall i \in I \tag{4.21}$$

The following numerical example illustrates how these constraints function. Consider a set of cells belonging to an ecological vegetation class that has an 'early' seral stage threshold of five years and a 'mid' seral stage threshold of ten years. In this case, cells with a fuel age between zero and five years will be classified as 'early' seral stage, cells with a fuel age between six and ten years will be classified as 'mid' seral stage, and cells with a fuel age of eleven years or greater will be classified as 'late' seral stage. While three 'seral stages' have been defined here, the same approach can be used to formulate constraints to define any number of 'seral stage' categories.

With a mechanism for classifying individual cells into seral stages we can now formulate constraints to maintain a desired proportion of the landscape in any of the various seral stages.

$$\sum_{i \in \Pi_k} G_{ti} * z_i \geq p_k * \sum_{i \in \Pi_k} z_i \qquad t = 1....T \quad \forall k \in \Pi \tag{4.22}$$

Constraint (4.22) ensures that for a given ecological vegetation class (*k*) the summation of the area of each cell (z_i) of 'late seral stage' classification (G_{ti}) is greater than some target proportion (p_k).

In some instances it may be preferable to formulate desired 'seral stage' proportions as goal constraints. If, for example initial landscape conditions are such that 'late seral stage' vegetation is well below the desired proportion. In this case, a hard 'late seral stage' constraint could not be satisfied and would result in infeasibility. A goal constraint, on the other hand, would guide subsequent treatment decisions and over time redress this shortage of 'late seral stage' vegetation.

$$\sum_{i \in \Pi_k} G_{ti} * z_i + D_t \geq p_k * \sum_{i \in \Pi_k} z_i \qquad t = 1....T \quad \forall k \in \Pi \qquad (4.23)$$

In constraint (4.23) the 'late seral stage' proportion requirement has been reformulated as a goal constraint with the inclusion of a deficit variable (D_t). A penalty function composed of the weighted sum of this deficit variable over all time periods would then be added to the objective function (Tamiz et al. 1998).

4.4 Other model extensions

In addition to the ecological considerations discussed in the preceding section, there are a number of other straightforward extensions to the basic formulation that can augment the model's flexibility and usefulness. One such extension relates to the concept of leverage, which is the idea that a single hectare of fuel treatment can protect additional hectares of land. In heterogeneous landscapes fire may spread farther than usual due to spotting in locations with topographic features such as ridge lines or canyons, or those with fuel types with loose, combustible bark. These locations can be described as high leverage points and there is likely to be a benefit in focusing fuel treatment here (Loehle, 2004). Leverage can be incorporated into the model formulation through the application of a weight w_i to each cell based on relative leverage values.

$$\text{Minimise } z = \sum_{t=1}^{T} \sum_{i \in I} \sum_{j \in \Phi_i} w_i * Q_{tij} \tag{4.24}$$

The objective function (4.24) has been reformulated so that it now minimises the weighted number of 'connected old fuel cells' across all time periods. This will result in high leverage cells being prioritised for treatment.

It is possible to partition a landscape into a number of zones with differing treatment emphases. In wildland urban interface areas the priority may be reduction and fragmentation of the fuel complex for asset protection purposes and as such ecological constraints may be relaxed. While in wilderness areas the primary concern may be satisfaction of ecological constraints and it may be appropriate for these cells in these zones to be given a lower weighting or excluded from the objective function. This partitioning into zones is done by defining a number of disjoint sets such that each cell is an element of one such set.

$$r_i^{\Theta} * X_{ti} - A_{ti} - \leq 0 \qquad t = 1....T \quad \forall i \in \Theta \qquad (4.25)$$

$$r_i^{\Omega} * X_{ti} - A_{ti} - \leq 0 \qquad t = 1....T \quad \forall i \in \Omega \qquad (4.26)$$

In constraints (4.25) & (4.26) the minimum TFI constraint has been split so that there are different TFI requirements for the urban interface zone, denoted by Θ, and the wilderness zone, denoted by Ω.

There can be significant benefits in incorporating non-flammable features such as lakes into landscape-level patterns (Parisien et al. 2007). This can be done by considering these features as cells and ascribing them an 'old fuel cell' threshold k_i greater than the maximum possible fuel age upper bound.

4.5 *Model demonstration*

4.5.1 Homogenous landscape

In order to demonstrate its functionality we applied the model to a number of hypothetical landscapes with different attributes. We first consider a homogenous 100 cell landscape. The landscape is composed of a single fuel type with an 'old fuel cell' classification threshold of four years. The initial fuel age of all cells is greater than this threshold, meaning all 100 cells are classified as 'old'. For every cell the 'set of connected cells' is defined as the neighbourhood of immediately adjacent cells. For simplicity the treatment cost is set at a constant value of one unit per cell across the entire landscape and the annual treatment budget is set at fifteen units. No treatment restrictions or ecological constraints are imposed. As can be seen in Figure 4.3 below, after five years 75 cells have been treated and the landscape has been completely fragmented with all old fuel cells disconnected. In the sixth year, the cells treated in the first year have exceeded the treatment inhibition period and they are reselected for treatment. Similarly the cells treated in the second year are retreated in the seventh year and so forth. It is apparent that in this homogenous landscape with no ecological constraints or treatment restrictions, the optimal solution amounts to the creation of an initial pattern and then the maintenance of this pattern through a recurring treatment cycle. Indeed, this generation of a stationary pattern that is maintained by a recurring cycle is a general result that would apply to homogenous landscapes of any size.

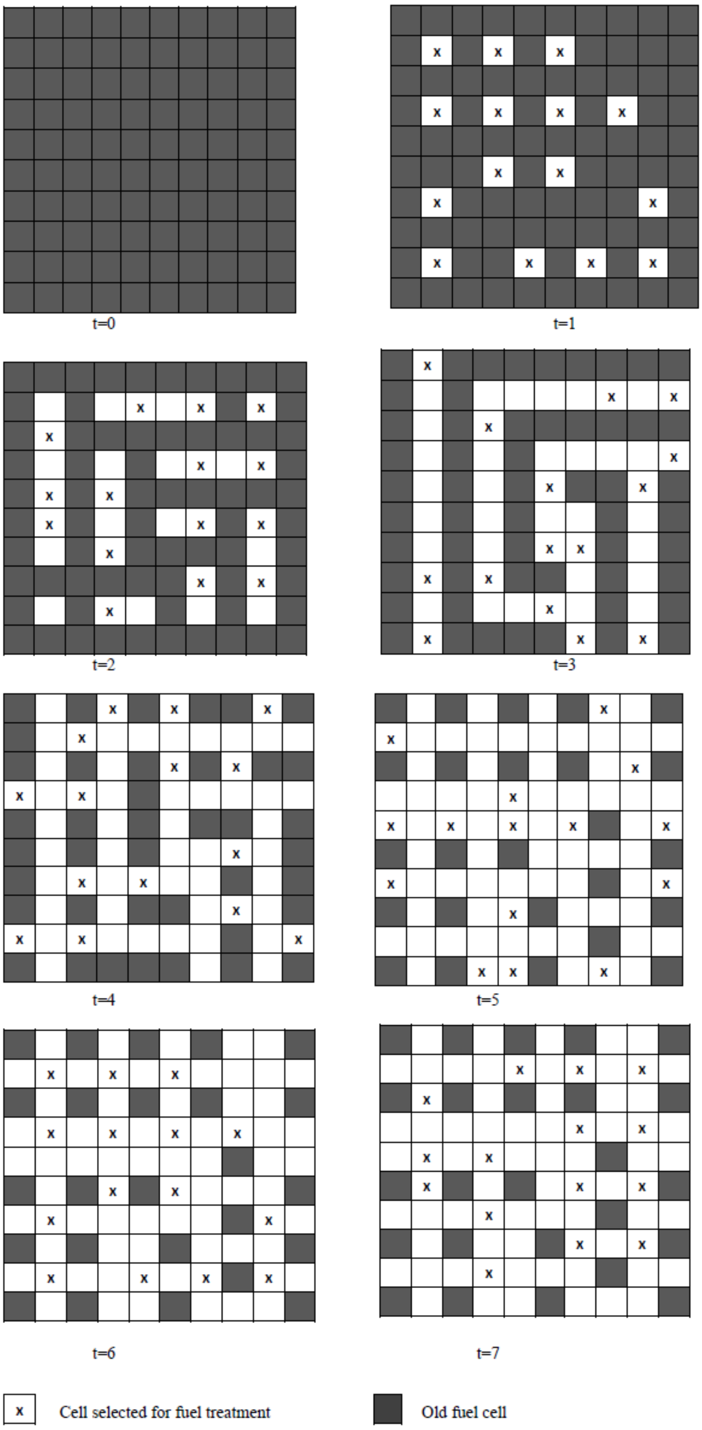

Figure 4.3: Fuel treatment schedule for a homogenous landscape with no ecological constraints

4.5.2 Heterogeneous landscape

In practice most landscapes will have some degree of heterogeneity. To demonstrate how the model handles this, we introduce a second fuel type with an 'old fuel cell' classification threshold of five years. One of the two fuel types is randomly assigned to each cell in our next landscape. We also introduce a maximum tolerable fire interval (TFI) constraint, the maximum TFI for the first fuel type is seven years and for second fuel type is nine years. The initial fuel age of each cell is a randomly assigned value between two and six, meaning at time period zero not all cells are classified as 'old'. The treatment costs, annual budget and 'set of connected cells' definition remain the same as in the previous example. With the homogenous landscape we were able to generate and then maintain a static landscape pattern by treating cells in a five year cycle. However when we consider more complex heterogeneous landscapes instead of a static pattern we tend to see a dynamic mosaic, this is illustrated in Figure 4.4 below. Indeed it appears that as the landscapes under consideration become increasingly complicated, determining optimal treatment schedules becomes less intuitive and a model like ours starts to prove its worth.

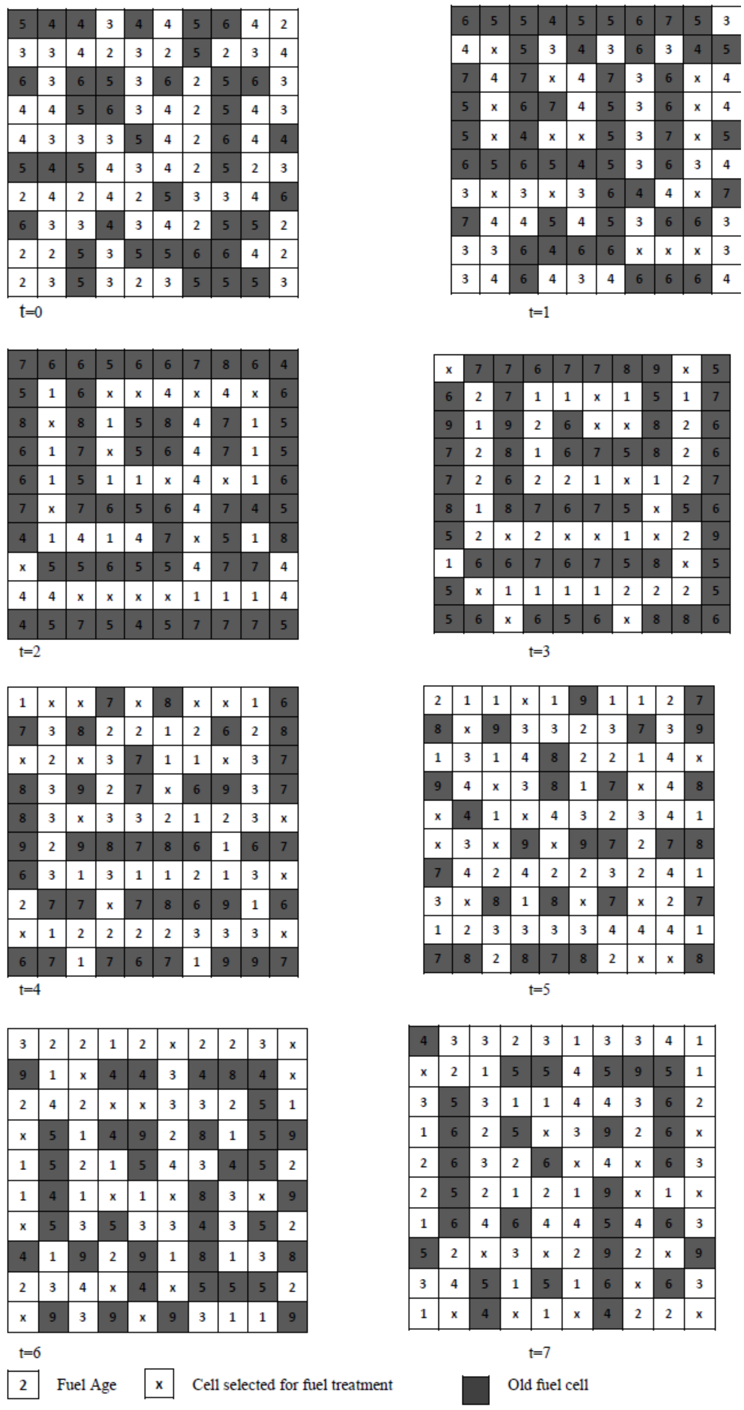

Figure 4.4 : Fuel treatment schedule for a heterogeneous landscape with ecological constraints

4.5.3 Heterogeneous landscape with different land use zones

In the next example we take the initial heterogeneous landscape from the previous example and partition it into two zones. The bottom half of the landscape is designated an urban interface zone and the maximum TFI constraint is specified such that it does not apply here. The top half of the landscape is designated a wilderness zone and the objective function is formulated to exclude cells from this zone. All other parameters remain the same as in the previous example. As can be seen in Figure 4.5, from the third year onward the lower half of the landscape is completely fragmented with all old fuel cells disconnected. While in the upper half of the landscape treatment is only undertaken when required to satisfy the maximum TFI constraint. This example provides an illustration of how the model might be used to manage a single budget to simultaneously achieve various management aims that vary spatially across a landscape.

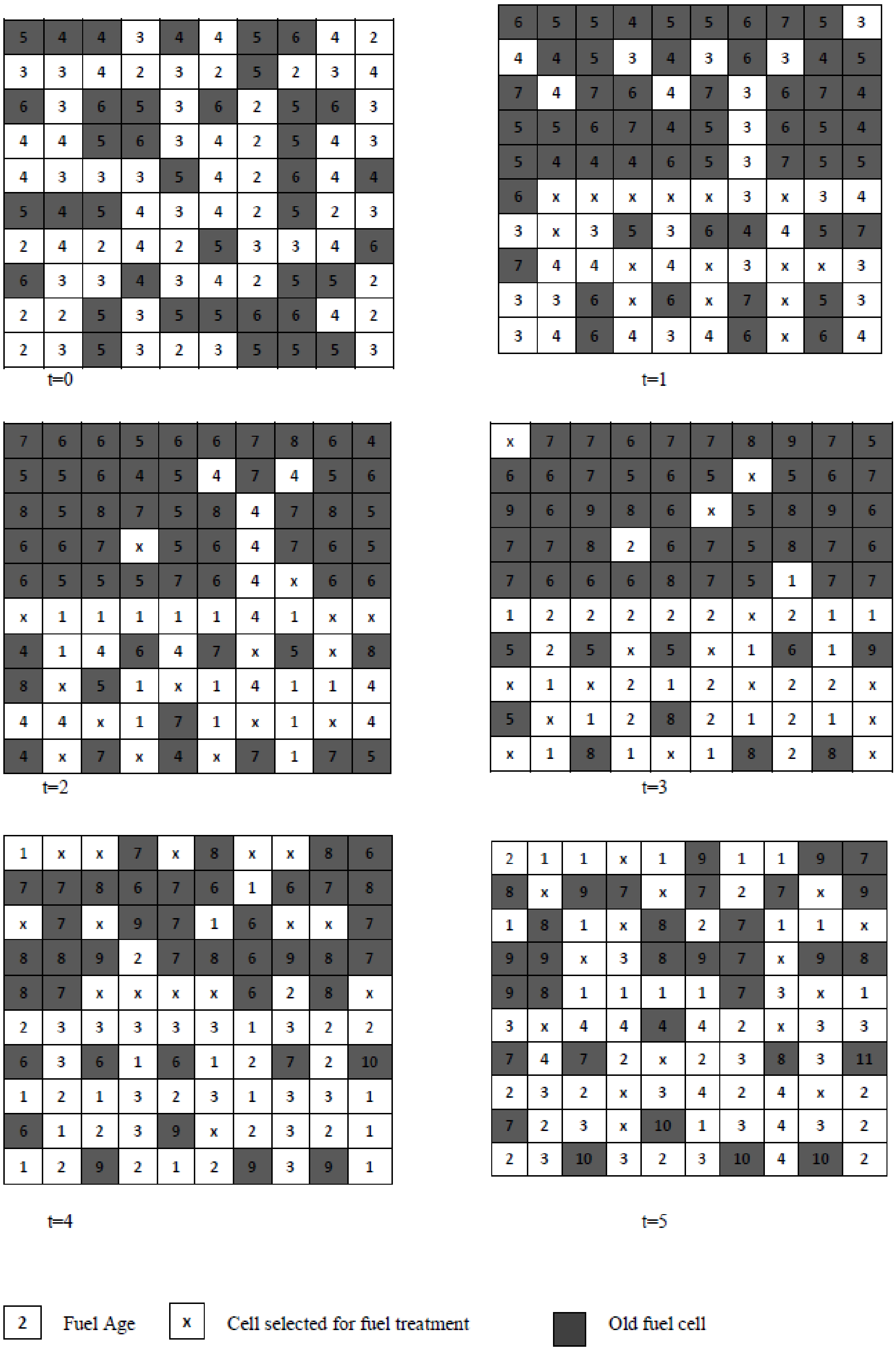

Figure 4.5: Fuel treatment schedule for a heterogeneous landscape with ecological constraints and different land use zones

4.5.4 Computational Testing

Some computational testing was undertaken on a series of randomly generated test landscapes with a couple of aims in mind. The first of these aims being, to provide some indication of the size of problems that the model can solve. The second aim being, to gain some insight into the ease (or difficulty) of implementing the ecological and other model extensions detailed in *Sections 4.3* and *4.4*.

In our initial testing, the model was implemented in its 'basic' form including expressions (4.1) – (4.11) from *Section 4.2-Model Formulation*. The model was run on a series of test landscapes of six sizes (25, 100, 225, 400, 900 and 1225 cells). In each landscape, cells were randomly assigned to one of three fuel types with differing 'old fuel cell' classification thresholds of four, eight and twelve years, with the initial fuel age of each cell an independent random integer value between one and twelve years. Connectivity was defined based on a north-westerly prevailing wind direction, with each cell connected to three neighbouring cells. The 'set of cells where fuel treatment is not permitted' was defined as the 'the empty set', meaning no restrictions were placed on permissible fuel treatment locations. The budget was adjusted to allow for three different annual treatment levels (5%, 10% and 15%) across a ten year time horizon. The model was solved for ten instances for each of the six landscape sizes and three treatment levels, meaning there were 180 model runs in this phase of testing. The model was implemented in the OPL modelling language and solved with CPLEX 12.5. All tests were performed on a Lenovo E530 notebook with a single quad-core Intel i7-3612QM processor at 2.10GHz and with 16 GB RAM memory. Computation results appear below in Table 4.1 these are reported as either solution time to optimality in wall-clock time (based on a relative MIP gap tolerance of 0.01%) or optimality gap at 1800 seconds.

Landscape size			Percentage of landscape treated		
			5%	10%	15%
25 cells (5 by 5)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	2.3 s	1.9 s	0.5 s
		- minimum	1.0 s	1.1 s	0.3 s
		- maximum	6.1 s	4.2 s	1.7 s
100 cells (10 by 10)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	69.7 s	12.7 s	2.7 s
		- minimum	41.1 s	8.2 s	1.1 s
		- maximum	323.6 s	138.4 s	120.5 s
225 cells (15 by 15)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	1424.1 s	274.8 s	4.1 s
		- minimum	226.7 s	36.7 s	2.3 s
		- maximum	(1.41 %)	(1.25 %)	(1.19 %)
400 cells (20 by 20)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	(0.69 %)	(0.57%)	13.1 s
		- minimum	(0.16 %)	220.2 s	8.6 s
		- maximum	(2.27 %)	(1.69 %)	(0.29 %)
900 cells (30 by 30)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	(17.63 %)	(0.60 %)	41.4 s
		- minimum	(16.01 %)	(0.43 %)	30.8 s
		- maximum	(23.61 %)	(1.07 %)	74.9 s
1225 cells (35 by 35)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	(18.27 %)	(0.89 %)	57.9 s
		- minimum	(16.64 %)	(0.44 %)	45.9 s
		- maximum	(21.32 %)	(31.09 %)	150.9 s

Table 4.5: Computational test results – basic model formulation

The results in Table 4.1 indicate that as landscape size increases and the percentage of landscape treated decreases the model becomes more difficult to solve. For all landscapes up to 400 cells in size, solutions within 2% of optimal were obtained within 1800 seconds. For landscapes larger than 900 cells there were typically sizable optimality gaps at 1800 seconds at the 5% treatment level. A small number of these larger landscape instances were allotted a longer run time of 6 hours and in all cases solutions within 1% of optimal were obtained. In discussions held during model development, fire agency personnel indicated that landscapes divided into several hundred to a thousand management units were of practical interest. The indicative computational testing undertaken here suggests that with modest computing power it is possible to model landscapes in this size range.

In our next phase of testing, we wished to consider the ecological and other model extensions detailed in *Sections 4.3* and *4.4* and their effect on model tractability and computation times. Pursuant to this aim, the model was implemented in three different configurations. In the first configuration, the 'basic' formulation was implemented including expressions (4.1) – (4.11) from *Section 4.2-Model Formulation*. In the second 'TFI' configuration, minimum and maximum tolerable fire interval constraints were added using expressions (4.12) and (4.13).With minimum (3, 7 and 11 years) and maximum (18, 22 and 26 years) tolerable fire intervals defined according to fuel type. In the third 'PFSS' configuration, cells were classified into three post fire seral stages (early, mid and late) using expressions (4.14) – (4.21). With 'early' (4, 8 and 12 years) and 'mid' (9, 13 and 17 years) seral stage thresholds defined according to fuel type. Expressions (4.23), (4.25) and (4.26) were then used to partition the landscape into two zones with differing objectives. In the upper half of each landscape the objective was to maintain target 'mid' (20%) and 'late' (20%) seral stage proportions. While in the lower half of each landscape the objective was to minimise the number of 'connected old fuel' with no heed paid to ecological considerations.

Landscape size			Model configuration		
			Basic	TFI	PFSS
900 cells (30 by 30)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	(17.63 %)	506.2 s	(0.56 %)
		- minimum	(16.01 %)	402.1 s	(0.24 %)
		- maximum	(23.61 %)	1697.4 s	(1.63 %)
1225 cells (35 by 35)	**solution time (s)** or **optimality gap (%) at 1800s**	- median	(18.27 %)	805.0 s	(0.87 %)
		- minimum	(16.64 %)	652.1 s	(0.56 %)
		- maximum	(21.32 %)	(0.02 %)	(1.23 %)

Table 4.2: Computational comparison by model configuration – 5% treatment level

The three model configurations were implemented at a 5% annual treatment levels on the 900 and 1225 cell test landscapes used in the first phase of testing. Computation results appear above in Table 4.2, with either solution time to optimality or optimality gap at 1800 seconds reported. The results in Table 4.2 suggest that the additional expressions that appear in the 'TFI' and 'PFSS' configurations have served to further constrain these problems and accordingly have led to reduced solution times when compared to the 'basic' formulation.

4.6 Summary and discussion

Scheduling fuel treatment activities to maintain the landscape in a fire resistant state is a challenging problem with important societal implications. In this chapter we have presented a spatially explicit optimisation model for multi-period scheduling of fuel treatments. The model tracks treatment decisions and fuel age over time and thus is able to capture the transience of treatment effect due to vegetation regrowth. The mixed integer programming formulation allows for heterogeneity of landscape features such as: fuel type, topography and prevalent wind direction. The model also allows for the incorporation of ecological considerations such as: tolerable fire intervals and seral stage landscape composition required to support biodiversity. A number of mechanisms for adapting the model to specific features of a given implementation environment have been presented. These include the use of zones to accommodate spatially variable land uses and management aims, as well as the use of weights to prioritise treatment of high leverage locations. Some of features of the model were demonstrated in the previous section and computational testing suggests that the model is able to handle problem sizes of practical interest.

It is important to note that though we have formulated this problem deterministically there are in fact a number of stochastic elements, the most important of these being the effects of 'unplanned' wildfires. Probability of fire ignition and escalation modelling based on historic fire data and knowledge of physical fire processes could be used to ascertain localities with a higher likelihood of fire occurrence. While simulation models that incorporate both fire spread and fire suppression components could shed some light on the potential impacts on human life, property and other values arising from fires in various localities in a range of weather conditions. Insights gained from probability and simulation modelling could then be used to assist in parameterising the optimisation model. For example, spatially-explicit probability of fire occurrence and consequence of fire escape could form the basis for partitioning a landscape into zones with differing treatment emphases.

While wildfires can have undesirable destructive effects in the immediate term, they also result in additional 'unscheduled' fuel reduction that can be beneficial in future periods. In the advent of a significant wildfire event it would be desirable to take the resultant fuel reduction effect into account when scheduling treatments for subsequent years. Indeed, even scheduled fuel treatment is stochastic since more or less than the planned amount may be achieved in a given year due to weather and various operational issues. Thus in a practical setting, fuel treatment scheduling would be treated as a rolling horizon problem with the model re-run annually with updated information on the current state of the landscape. More problematic is the effect that 'unplanned' wildfire can have on the proportion of late seral stage vegetation in the landscape. Stochastic optimisation methods could be employed here. Though for landscapes with a low annual burn fraction, a simpler and more computationally tractable mean-value approach may be sufficient. A straightforward way to implement such an approach would be to strengthen the late seral stage constraint by adjusting the right hand side based on predicted or historic burn fractions (Savage, Martell, & Wotton, 2011).

It has been noted that scientists and managers often overlook the need to translate complex science into practical fire management prescriptions and that this can result in a gap between the state of knowledge and current management practices (Burrows, 2008). In this research we have applied integer programming methods to the combinatorially complex problem of fuel treatment scheduling. Our hope is that the modelling approach developed here can assist in closing this knowledge-practice gap.

5. An integer programming model for aggregation of fuel treatment units

5.1 Introduction

In *Chapter 3* a mixed integer programming model was proposed for integrated fire suppression preparedness and fuel management decision making. In *Chapter 4* a mixed integer programming model for multi-period fuel treatment scheduling was presented. Both of these models are designed for implementation on a landscape that has been divided into a number of fuel treatment units that need not be uniform in shape or size. This partitioning is typically a division of the landscape into 'fundamental' units based on features including topography, fuel type and presence of barriers such as roads and creeks.

In Victoria, Australia in recent years there has been a three-fold increase in the annual statewide fuel treatment target as a result of recommendations arising from the Victorian Bushfires Royal Commission (Teague, et al., 2010). Meeting this revised target is made difficult by operational constraints including limited numbers of: personnel, equipment and suitable burn days (Higgins, et al., 2011). This has motivated fire authorities in Victoria to start exploring the idea of aggregating existing 'fundamental' fuel treatment units into larger units or 'clusters'. The rationale being that managing or patrolling the burn unit perimeter is one of the most costly and labour intensive elements of prescribed burning. As such, aggregation of burn units into larger clusters will reduce the total perimeter requiring management and therefore enable more burning to be done with the same resources. However, this efficiency improvement needs to be balanced against the heightened risk of an escaped fire that can result from conducting larger prescribed burns (Fogarty 2012, pers. comm 19 December).

The aggregation of basic spatial units (areas) into larger units (regions) has been identified as a general problem class referred to as 'supervised regionalisation' (Duque, Ramos, & Suriñach, 2007) or the '*p*-Regions problem' (Duque, Church, & Middleton, 2011). In such problems one is typically trying to aggregate geographical areas into a smaller number of contiguous spatial regions while optimising some aggregation criterion. Problems of this type have been solved using both heuristic and exact optimisation methods. The main challenge apparent in using an exact approach is finding an efficient means for ensuring contiguity of regions (Duque, Ramos, & Suriñach, 2007). In the burn unit problem presented here, in order to minimise total perimeter we seek to aggregate burn units that share a common boundary. As such there is no incentive for aggregation of disjoint units and thus our optimality criterion ensures contiguity of clusters. This feature allows for quite a compact mixed integer programming assignment problem formulation.

The remainder of the chapter is structured as follows. The mathematical formulation of the model is presented and explained. The model's functionality is then demonstrated on a 35 cell test landscape. This is followed by some computational testing and discussion of implementation issues.

5.2 Model formulation

The formulation of a mixed integer programming model for aggregation of prescribed burning fuel treatment units appears below. We consider a landscape divided into a number of cells representing fuel treatment units. These cells need not be uniform in shape or size. Rather this initial partitioning would be done based on what constitutes practical management units for the specific landscape in question. The key decision to be made is how to aggregate these fuel treatment units into larger units or clusters. Our primary motivation for aggregating fuel treatment units into clusters is to reduce the amount of perimeter to be managed when conducting prescribed burning. Accordingly the model assigns units to clusters in a way that leads to the greatest reduction in perimeter for a given cluster size constraint. The model is formulated with the following notation.

5.2.1 Indices and sets

i, I = index, set of all cells in the landscape;

$j, \Phi_i \subset I$ = index, set of cells adjacent to cell i;

k, K = index, number of clusters;

5.2.2 Parameters

a_i = area of cell i;

b_{ij} = length of shared boundary between cell i and adjacent cell j;

m = maximum permissible cluster area;

5.2.3 Variables

X_{ik} = 1 if cell i is assigned to cluster k,

0 otherwise;

Y_{ijk} = 1 if cell i and adjacent cell j are both assigned to cluster k,

0 otherwise;

5.2.4 Model

$$\text{Maximise } z = \sum_{k=1}^{K} \sum_{i \in I} \sum_{j \in \Phi_i} b_{ij} Y_{ijk} \tag{5.1}$$

Subject to:

$$\sum_{i \in I} a_i X_{ik} \leq m \qquad k = 1....K \tag{5.2}$$

$$\sum_{k=1}^{K} x_{ik} = 1 \qquad \forall i \in I \tag{5.3}$$

$$X_{ik} - Y_{ijk} \geq 0 \qquad k = 1....K \quad \forall i \in I \quad \forall j \in \Phi_i \tag{5.4}$$

$$X_{jk} - Y_{ijk} \geq 0 \qquad k = 1....K \quad \forall i \in I \quad \forall j \in \Phi_i \tag{5.5}$$

$$X_{ik} \in \{0,1\} \qquad k = 1....K \quad \forall i \in I \tag{5.6}$$

$$Y_{ijk} \in \{0,1\} \qquad k = 1....K \quad \forall i \in I \quad \forall j \in \Phi_i \tag{5.7}$$

The objective function (5.1) maximises the length of shared boundary for cells assigned to the same cluster. When cells are aggregated together their shared boundary is subtracted from the perimeter of the newly formed, larger treatment unit. Therefore the objective function is effectively minimising the total length of treatment unit perimeter that needs to be managed across the entire landscape.

Constraint (5.2) imposes a size limit for each cluster that cannot be exceeded.

Constraint (5.3) ensures each cell is assigned to a single cluster.

Constraints (5.4) and (5.5) classify a pair of adjacent cells i and j as a 'clustered pair' if they are both assigned to the same cluster k.

Constraints (5.6) and (5.7) restrict cluster assignment and 'clustered pair membership' variables to binary values.

In the formulation above we are required to specify the number of clusters (*K*). However since there is no constraint forcing each cluster *k* to be used, this means we can have empty clusters with no cells allocated to them. This suits our purposes for the application at hand, as our interest is in minimising the total amount of perimeter to be managed across the landscape irrespective of the number of clusters employed. So long as *K* is set to a sufficiently large value, an optimal solution will be obtained with only the necessary number of clusters used. A *K* value that is too small will result in infeasibility or a sub optimal solution. The simplest way to deal with this is to set *K* equal to the number of cells in the landscape (*n*). However model size is a function of the number of cells (*n*), the cardinality of the set of adjacent cells $\left(\left|\Phi_i\right|\right)$ and the number of clusters (*K*). With the number of variables equal to $n * K + \sum_{i \subset I} \left|\Phi_i\right| * K$ and the number of constraints equal to $2 * \sum_{i \subset I} \left|\Phi_i\right| + n + K$. Thus for implementation of large problem instances setting *K* equal to *n* is likely to be too computationally costly and more care will be required in specifying an appropriate *K* value.

If aggregation of cells into a predetermined fixed number of clusters *K* is desired, this can be handled with the following constraint.

$$\sum_{i \subset I} x_{ik} \geq 1 \qquad\qquad k = 1....K \qquad\qquad (5.8)$$

Constraint (5.8) ensures that all clusters *k* have at least one cell allocated to them.

The model has been presented here in a very simple, general form. This formulation can however be extended in a number of ways. For example, as discussed in S*ection 4.3* in the preceding chapter, tolerable fire intervals will differ according to ecological vegetation class (EVC). Thus it may be desirable to preclude cells from EVCs that require different fire intervals from being assigned to the same cluster. This is done by assigning each cell to an EVC (*e*) by defining a number of disjoint sets (Λ_e) such that each cell is an element of one such set.

$$X_{ik} - X_{jk} \leq 1 \qquad k = 1....K \quad \forall i \in \Lambda_e \quad \forall j \in \Gamma_e \qquad (5.9)$$

Constraint (5.9) precludes a cell *i* belonging to ecological vegetation class *e* from being assigned to the same cluster as a cell *j* from the set of cells belonging to a non-complementary EVC denoted by Γ_e .

5.3 Model demonstration

In order to demonstrate the model's functionality we implemented it on a 32,579 hectare test landscape composed of 35 irregular shaped burn units ranging in size from 29 to 2211 hectares. The test landscape is based on a real landscape in southwestern Victoria, Australia. A visual representation of the test landscape appears below in Figure 5.7 with each burn unit labeled with a unique identifying number between 0 and 34.

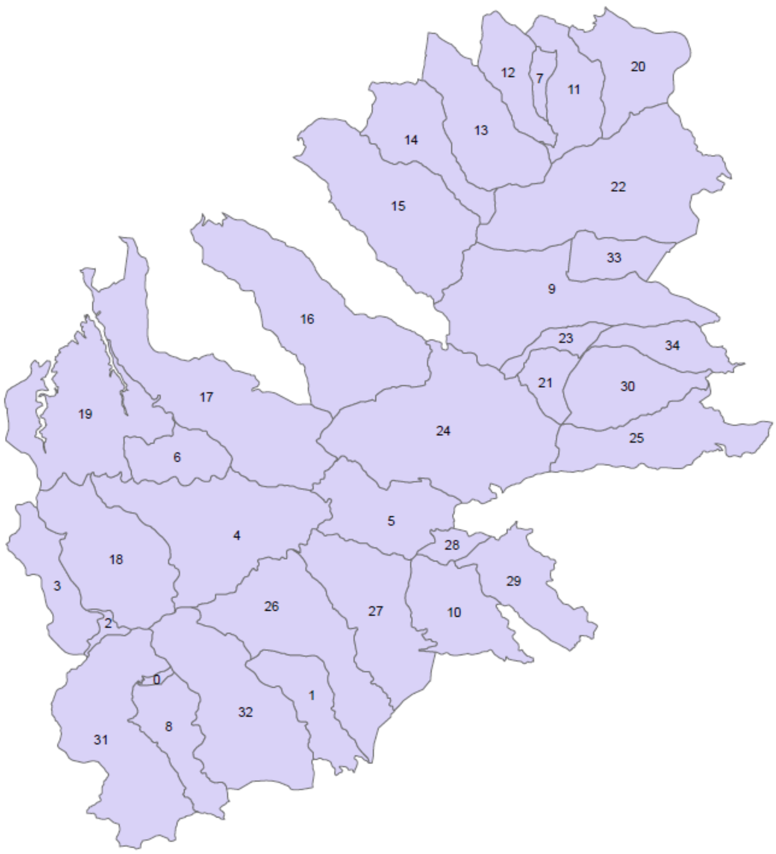

Figure 5.6: Test landscape

The area and perimeter of each burn unit in the landscape is listed below in Table 5.5.

Burn Unit No.	Area (Ha)	Perimeter (m)
0	29	3100
1	559	13912
2	66	6064
3	547	13722
4	1656	23450
5	851	15319
6	443	10438
7	116	5762
8	595	14726
9	1697	23991
10	764	15088
11	601	13901
12	476	11200
13	961	14615
14	738	15291
15	1568	19270
16	1868	23933
17	1875	30768
18	1239	16629
19	1489	37317
20	763	14082
21	272	7777
22	2026	23127
23	191	9089
24	2211	24088
25	935	19718
26	1374	22702
27	1165	18951
28	143	6073
29	697	15060
30	764	12857
31	1592	23379
32	1530	19965
33	366	9182
34	413	11854
	32579	**566403**

Table 5.6: Initial burn unit areas and perimeters

A table detailing the length of shared boundary between all adjacent burn units appears in *Appendix A*. These lengths of shared boundary range in value from 222 to 6958 meters.

The model was run for various maximum permissible cluster area values with results reported below in Table 5.6. Other than permissible cluster area there were no further restrictions placed on assignment of burn units to clusters. The model was implemented in the OPL modelling language and solved with CPLEX 12.2 on a Lenovo E530 notebook with a single quad-core Intel i7-3612QM processor at 2.10GHz and with 16 GB RAM memory. With some tuning of settings solution times ranging from a fraction of a second to 30 seconds were obtained.

Permissible Cluster Area (Ha)	No. of Clusters	Perimeter Eliminated (m)	Perimeter Remaining (m)
33000	1	387164	179240
17000	2	371070	195333
12000	3	336506	229898
9000	4	321128	245275
8000	5	308309	258094
6000	6	275182	291221
5000	8	260686	305718
4000	10	233084	333320
3500	12	200087	366316
3000	14	168058	398345
2500	17	147803	418600

Table 5.7: Test results - for various permissible cluster area values

The objective value reported in the third column of Table 5.6 represents the total amount of perimeter eliminated as a result of aggregation of burn units into clusters. The total remaining perimeter post-aggregation is reported in the fourth column of the table. For example, a permissible cluster area of 33,000 Ha allows the burn units to be aggregated into one single 32,579 Ha cluster that encompasses the entire landscape. Thus the objective value obtained in this case, 387,164 m, is equal to the total shared boundary between all neighbouring burn units in the landscape. The 179,240m of remaining perimeter is just the difference between the 566,403 m of pre-aggregation perimeter and the eliminated perimeter expressed in the objective value. It is apparent that as the permissible cluster area is reduced the number of clusters that can be formed decreases and the amount of perimeter to be managed increases. Figures 5.8 – 5.14 below illustrate the clusters formed for various maximum permissible cluster area values.

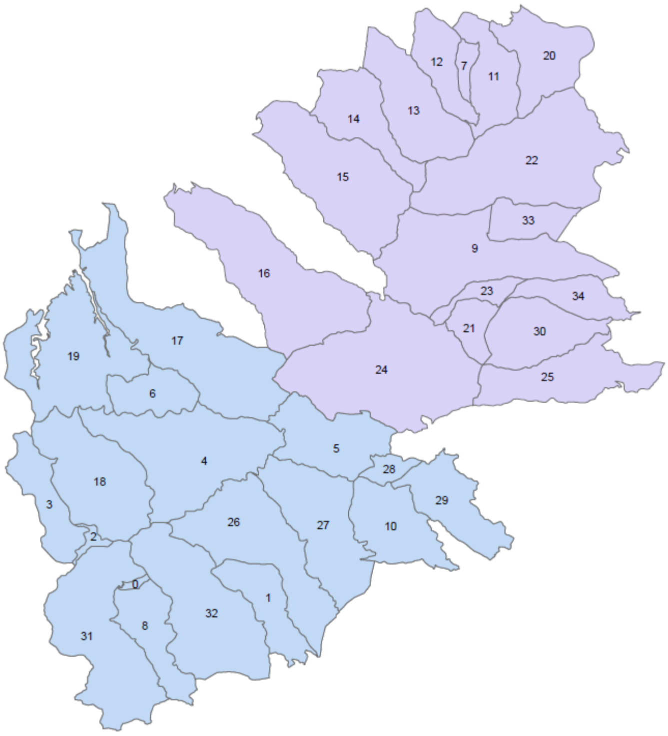

Figure 5.7: Test landscape – 17,000 Ha maximum permissible cluster area

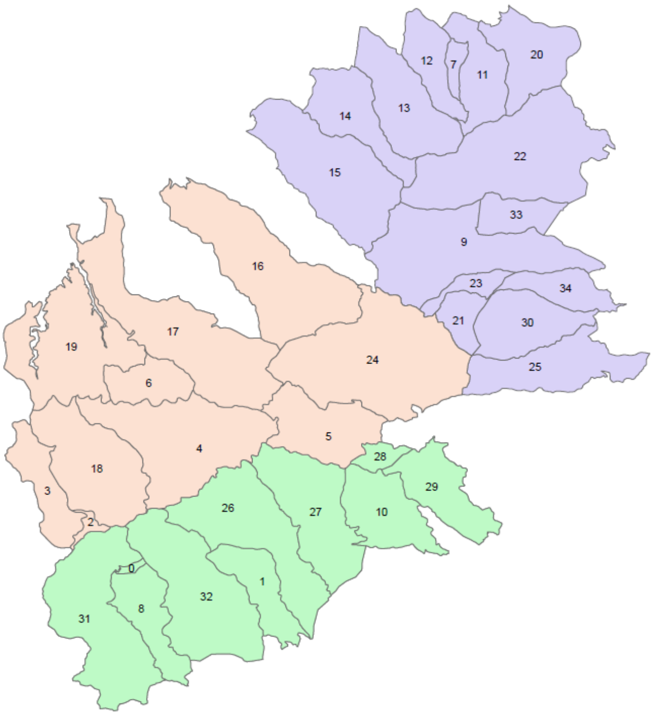

Figure 5.8: Test landscape – 12,000 Ha maximum permissible cluster area

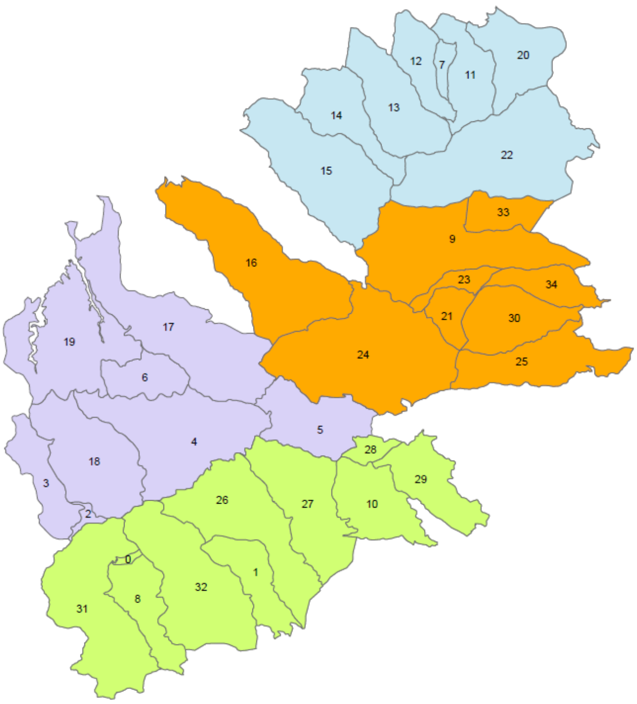

Figure 5.9: Test landscape – 9,000 Ha maximum permissible cluster area

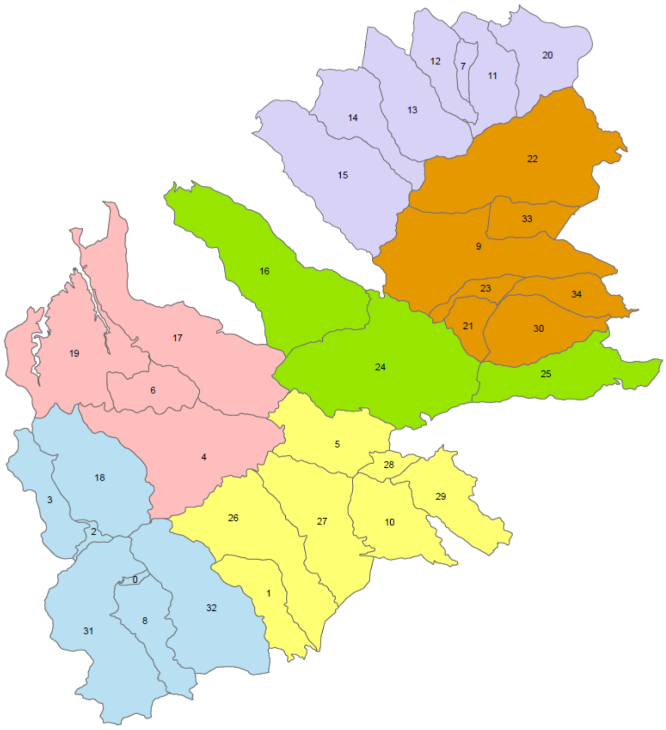

Figure 5.10: Test landscape – 6,000 Ha maximum permissible cluster area

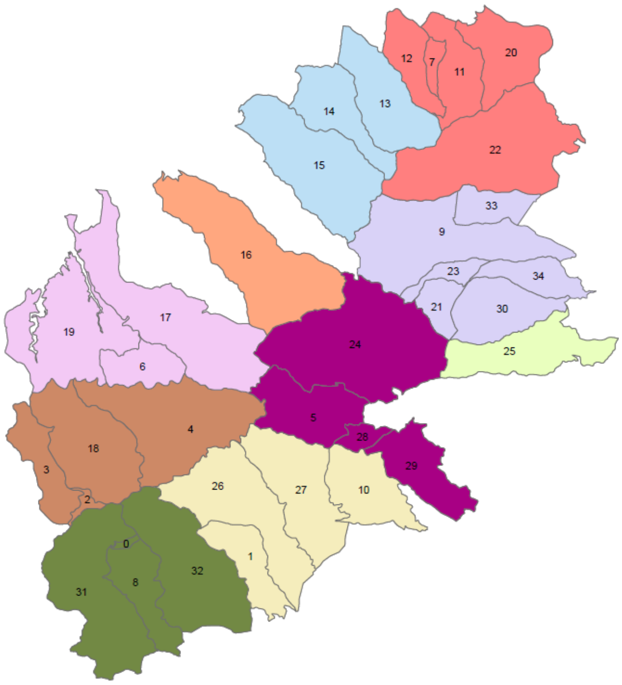

Figure 5.11: Test landscape – 4,000 Ha maximum permissible cluster area

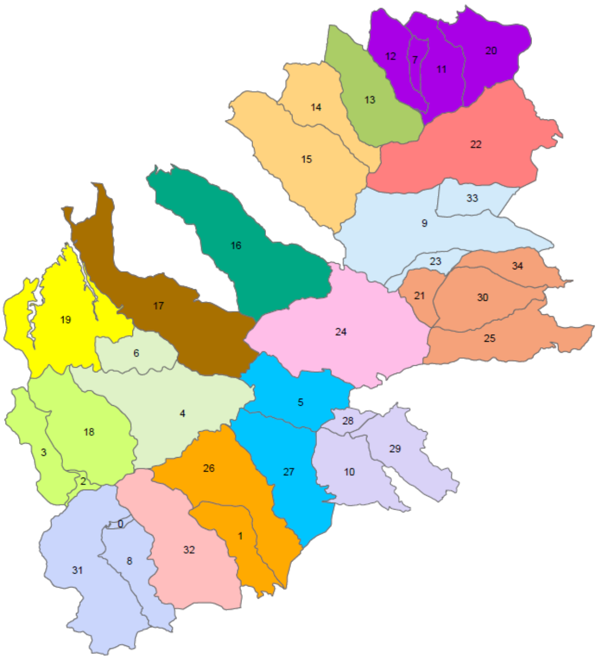

Figure 5.12: Test landscape – 2,500 Ha maximum permissible cluster area

In Figure 5.14 below, the perimeter-to-area ratio of clusters is plotted against the maximum permissible cluster area. It can be seen that perimeter-to-area ratio decreases as maximum permissible cluster area increases. The gradient of the curve is quite steep at first and then proceeds to flatten out. As discussed in *Section 5.1* the potential problem with larger burn unit sizes is an increased risk of an escaped fire during prescribed burning. Fire authorities must balance this risk against efficiency gains when deciding on what maximum permissible cluster area to implement. This decision may be influenced by fuel type and proximity to values at risk. For example, highly flammable fuel types close to the urban interface may require smaller burn units.

Whereas for less flammable fuels in more remote localities larger units may be appropriate.

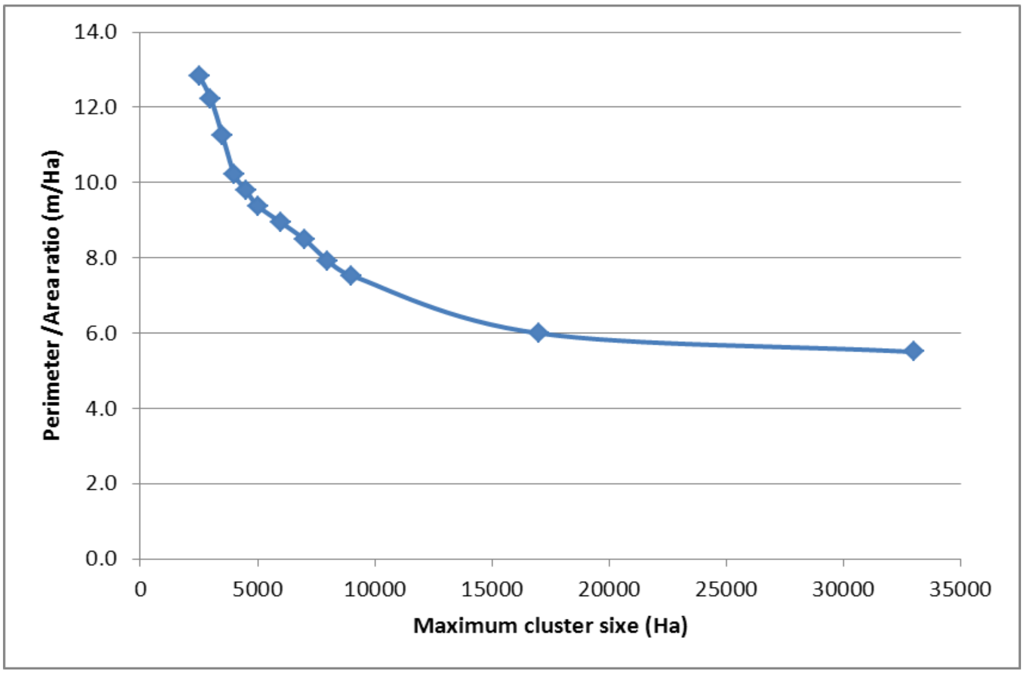

Figure 5.13: Cluster perimeter to area ratio as a function of maximum permissible cluster area

5.4 Summary and discussion

In this chapter we have presented a mixed integer programming model for aggregating 'fundamental' burn units into larger burn units or clusters. Features of the problem structure have been exploited to produce a compact model formulation. Firstly, aggregation of burn units to maximise shared boundary means that only adjacent burn units need be considered in the objective function (5.1) and in the clustered pair constraints (5.4) & (5.5). Secondly, the optimality criterion obviates the need for additional constraints to ensure contiguity of clusters. This results in a tractable model that can be solved to optimality, as seen with the model demonstration undertaken in *Section 5.3.*

In this chapter, the burn unit aggregation model has been presented in a very simple, general form. This formulation can however be extended to include various ecological considerations. One such ecological consideration explained in *Section 4.3* is tolerable fire intervals (TFIs). As discussed in S*ection 5.3* it is quite straightforward to add a constraint such as (5.9) to preclude burn units with non-complementary TFIs from being included in the same cluster. Another ecological consideration discussed in *Section 4.3* is the desire to maintain the correct proportion of each ecological vegetation classes (EVC) in the various habitat growth or post fire seral stages (PFSS). To achieve this PFSS balance is may be necessary to include constraints to limit the allowable proportion of land of any given EVC that can be included in a single cluster. Finally, as discussed in *Section 4.3* constraints to partition a landscape into zones with spatially variant treatment emphases. This would allow permissible burn unit sizes and application of ecological constraints to vary according to zone. For example in urban interface areas where community and asset protection is a priority permissible burn units may be smaller and some ecological constraints may be relaxed.

6. Conclusion

While fire is a natural component of many ecosystems, uncontrolled wildfires can cause large scale devastation in the form of loss of life and destruction of private property, infrastructure and natural resources. Over the past several decades, an increase in wildfire occurrence and severity has been observed across the globe. Changed weather conditions associated with climate change suggest this upward trend is set to continue. Wildfire management is an expensive and difficult undertaking and involves a complex mix of interrelated components operating across varying temporal and spatial scales. There currently exists a concerning and sizeable gap between the decision support needs of wildfire managers and the decision support tools currently available (Martell, 2011). The detailed review undertaken in *Chapter 2* suggests there is considerable scope for the use of OR methods in bridging this gap.

In this thesis, three models were developed to address a series of wildfire management challenges. The three proposed models all used mixed integer programming methods to tackle combinatorially complex spatial optimisation problems. The first initial attack coverage model incorporated two types of decision variables, fuel treatment and suppression resource deployment within a single integrated framework. The second model scheduled fuel treatments across multiple time periods to maintain fire resistant landscape patterns while satisfying various ecological requirements. The third model aggregated fuel treatment units to minimise total perimeter requiring management.

The proposed models have all been presented in a very simple and general form. There is however plenty of opportunity for future research to extend these models in various ways. All three models could be adapted using a multi-objective optimisation approach to consider a range of land use requirements such as: ecosystem function, water catchment integrity and tourism values. While the models have been formulated deterministically, parameter uncertainty could be considered in various ways. The integrated fuel treatment and suppression preparedness model presented in *Chapter 3* could be reformulated as a two-stage stochastic programming model with recourse to account for various fire-weather scenarios. Similarly, congestion resulting from concurrent fires could be accounted for in a probabilistic reliability formulation. For the multi-period fuel treatment model presented in *Chapter 4*, the effects of unplanned fire on habitat could also be explored through a comparison of rolling horizon and stochastic programming approaches.

In this thesis we've demonstrated the use of OR methods to generate insights into the management of complicated systems that require the consideration of a host of diverse factors. While we have applied these methods in the realm of wildfire management, the insights gained in this research could be applied to a broader range of disciplines. For example, we have demonstrated the performance benefits that result from integrating interrelated management decisions within a single model, in our case fuel treatment and suppression resource deployment decisions. Further, we have developed an approach for the extremely complicated task of determining management strategies that link across space and time, in our case to generate fire resistant landscape mosaics. As more frequent and destructive wildfire events threaten lives and homes in an expanding wildland-urban interface, now more than ever we need to apply best practice analytical methods to assist wildfire managers in assessing alternatives and making decisions. Here we have demonstrated how OR methods can be used to formulate challenging real-world problems into coherent and solvable models. As OR formulation methods and algorithms continue to improve and greater computing power become available, it will be possible to tackle increasingly complex wildfire problems using OR methods.

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Appendix A – Initial burn unit shared boundaries

Burn Unit No.	Adjacent Burn Unit	Shared Boundary (m)
0	8	1513.7
0	31	1363.2
0	32	223.2
1	26	6279.6
1	32	4492.5
2	3	2231.6
2	18	1837.1
2	31	1848.0
3	18	4270.3
4	5	2104.5
4	6	3925.6
4	17	2797.5
4	18	6179.0
4	19	1894.3
4	26	4510.6
4	27	1421.9
4	32	614.8
5	10	221.7
5	17	1399.7
5	24	5312.9
5	27	3352.1
5	28	1975.3
6	17	2731.0
6	19	3780.6
7	11	3105.9
7	12	2654.9
8	31	5789.6
8	32	5341.0

Burn Unit No.	Adjacent Burn Unit	Shared Boundary (m)
9	15	2765.9
9	22	3130.5
9	23	4206.4
9	24	2045.7
9	33	4027.3
9	34	2817.3
10	27	4830.7
10	28	1347.4
10	29	2506.5
11	12	2045.4
11	13	855.6
11	20	4192.5
11	22	1878.9
12	13	3266.7
13	14	4852.4
13	22	2125.0
14	15	5384.2
14	22	1446.3
15	22	584.4
16	17	862.1
16	24	5069.2
17	19	4073.4
17	24	1871.7
18	19	2248.1
18	31	541.0
18	32	979.1
20	22	2621.2
21	23	2855.8
21	24	2279.0
21	30	2640.8

Burn Unit No.	Adjacent Burn Unit	Shared Boundary (m)
22	33	3609.7
23	24	788.7
23	34	1236.1
24	25	1696.4
25	30	5458.2
26	27	6957.9
26	32	2979.3
28	29	1418.4
30	34	4350.8
31	32	1563.5
		193581.8