Quasi-synchronization of Delayed Coupled Networks with Non-Identical Discontinuous Nodes*

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Abstract. *This paper is concerned with the quasi-synchronization issue of linearly coupled networks with discontinuous nonlinear functions in each isolated node. Under the framework of Filippov systems, the existence and boundedness of solutions for such complex networks can be guaranteed by the matrix measure approach. A design method is presented for the synchronization controllers of coupled networks with non-identical discontinuous systems. Numerical simulations on the coupled chaotic systems are given to demonstrate the effectiveness of the theoretical results.*

Key words: Quasi-synchronization; Filippov systems; Discontinuous functions; Non-identical nodes

1 Introduction

Over the past decades, complex networks have been studied intensively in various fields, such as physics, mathematics, engineering, biology, and sociology [1,2]. A complex network is a large set of interconnected nodes, which represent individuals in the system and among them, the edges, represent the connections. Each node is a fundamental unit having specific contents and exhibiting dynamical behavior. A complex network can exhibit complicated dynamics which may be absolutely different from those of a single node. Hence, the investigation

* This work was supported by the National Natural Science Foundation of China under Grant No. 61104145, the Natural Science Foundation of Jiangsu Province of China under Grant No. BK2011581, the Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20110092120024, the Fundamental Research Funds for the Central Universities of China, and the scientific research support project for teachers with doctor's degree, Jiangsu Normal University under Grant No. 11XLR19.

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of complex dynamical networks is of great importance, and many large-scale systems in nature and human societies, such as biological neural networks, the Internet, the WWW, electrical power grids, etc., can be described by complex networks.

On the other hand, synchronization, a typical collective behavior in nature, means two or more systems share a common dynamical behavior, which can be induced by coupling or by external forces. Synchronization certainly is a basis to understand an unknown dynamical system from one or more well-known dynamical systems [3–7]. However, it is known that dynamical systems with discontinuous and/or unbounded nonlinear functions do frequently arise in the real applications. For the well known neural networks, there have been extensive results on the global stability of neural networks with discontinuous activations in the existing literature [8–14]. In these references, the first problem to be resolved is giving the definition of solution for the discontinuous systems under the framework of Filippov solution. By constructing the Filippov set-valued map, the differential equation could be transformed into a differential inclusion, which is also called as the Filippov regularization (the details can be founded in Definition 2). Such a notion has been utilized as a feasible approach in the field of mathematics and control for discontinuous dynamical systems.

The behavior of a network is determined by two main features: the dynamics of the isolated nodes, and the connections between the nodes. In order to analyze the network synchronization, most works in the literature assume that all the node dynamics are identical which mainly origins from physical connections in biology, physics and social science [15, 16]. Nowadays, the interest of synchronization issue is shifting to networks of coupled non-identical dynamical systems mainly due to the above assumption that the identical nodes is a highly unlikely circumstance for technological networks in the real world. Indeed, almost all complex dynamical networks in engineering have different nodes [17]. In addition, the behavior of networks with non-identical nodes is much more complicated than the identical-node case. For instance, there does not exist a common equilibrium for all nodes even if each isolated node has an equilibrium. Therefore, a network with non-identical nodes still show some kind of synchronization behaviors which are far from being fully understood. Certain reasonable and satisfactory boundedness [15, 18] of state motion errors between different nodes can be taken as useful synchronization properties, which is usually called as quasi-synchronization [19].

Motivated by the above discussions, we aim (i) to formulate a mathematical model considering discontinuous dynamics of each isolated node for the coupled complex networks; (ii) to use the concept of Filippov solution to describe the solutions' existence and boundedness of coupled networks; (iii) to utilize matrix measure method to cope with the quasi-synchronization issue of network with non-identical nodes.

2 Model Formulation and Preliminaries

In this paper, we consider a complex dynamical network consisting of N linearly coupled identical nodes. Each node is an n-dimensional system composed of linear and nonlinear terms. The i-th node can be described by following differential equation:

$$\dot{x}_i(t) = Dx_i(t) + Bf(x_i(t)), \ i = 1, 2, \cdots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in \mathbb{R}^n (i = 1, 2, \cdots, N)$ is the state vector representing the state variables of node i at time t; $D \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$; and $f(x_i(t)) = [f_1(x_i), f_2(x_i), \cdots, f_n(x_i)]^T : \mathbb{R}^n \to \mathbb{R}^n$.

Consider the dynamical behavior of the complex dynamical network described by the following linearly coupled differential equations:

$$\dot{x}_i(t) = Dx_i(t) + Bf(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \ i = 1, 2, \cdots, N, \tag{2}$$

where Γ is the inner coupling positive definite matrix between two connected nodes i and j; c is the coupling strength; a_{ij} is defined as follows: if there is a connection from node j to node i ($j \to i$), then $a_{ij} = a_{ji} > 0$; otherwise, $a_{ij} = a_{ji} = 0 (j \neq i)$; and the diagonal elements of matrix A are defined by

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}. \tag{3}$$

Unlike the previous studies on synchronization of complex networks, the nonlinear function f of each isolated node in this paper does not hold the Lipschitz condition [4, 19, 20] or QUAD condition [7] any more. Moreover, the basic continuous conditions are also removed. The marked difference between this paper and the existing work is that the node dynamics in our model are admitted to be discontinuous.

From the theoretical point of view, the basic and natural question is about the solution of the discontinuous dynamical systems. The existence of solutions for discontinuous dynamical systems is a delicate problem, as can be seen from our previous work [11, 12]. Firstly, we need some preliminaries to introduce the new definition for the solutions.

Definition 1. Class $\mathcal{F}$ of functions: we call $f \in \mathcal{F}$, if for all $i = 1, 2, \cdots, n$, $f_i(\cdot)$ satisfies: $f_i(\cdot)$ is continuously differentiable, except on a countable set of isolated points $\{\rho_k^i\}$, where the right and left limits $f_i^+(\rho_k^i)$ and $f_i^-(\rho_k^i)$ exist, $k = 1, 2, \cdots$.

In the following, we apply the framework of Filippov [21] in discussing the solution of each node (1) with the discontinuous function f.

Definition 2. A set-valued map is defined as

$$F(x_i) = \bigcap_{\delta>0} \bigcap_{\mu(N)=0} K[f(B(x_i, \delta) \setminus N)], \tag{4}$$

where $K(E)$ is the closure of the convex hull of set E, $B(x_i,\delta)=\{y:\|y-x_i\|\leq\delta\}$, and $\mu(N)$ is Lebesgue measure of set N. A solution in the sense of Filippov of equation (1) with initial condition $x_i(s)=\theta_s,\ \forall s\in[-\tau,0]$, is an absolutely continuous function $x_i(t), t\in[0,T]$, which satisfies differential inclusion:

$$\frac{dx_i}{dt}\in Dx_i(t)+B\mathbb{F}(x_i),\quad a.e.\ t\in[0,T], i=1,\cdots,N. \tag{5}$$

where $\mathbb{F}(x_i)\triangleq K[f(x_i)]=(K[f_1(x_i)],\cdots,K[f_n(x_i)])$, and $K[f_j(x_i)]=[\min\{f_j(x_i^-),f_j(x_i^+)\},\max\{f_j(x_i^-),f_j(x_i^+)\}],\ i=1,\cdots,N, j=1,\cdots,n.$

It is obvious that, for all $f\in\mathcal{F}$, the set-valued map $x_i(t)\hookrightarrow Dx_i(t)+B\mathbb{F}(x_i(t))$ has nonempty compact convex values. Furthermore, it is upper-semi-continuous [22] and hence it is measurable. By the measurable selection theorem [23], if $x_i(t)$ is a solution of (1), then there exists a measurable function $\alpha_i(t)\in K[f(x_i(t))]$ such that for $a.e.\ t\in[0,+\infty)$, the following equations hold:

$$\dot{x}_i(t)=Dx_i(t)+B\alpha_i(t),\quad for\ a.e.\ t\in[0,T), i=1,\cdots,N. \tag{6}$$

In [11, 12], we have considered the existence and stability (and then the uniqueness) of such solutions for each node. In this paper, we will not repeat the existence results, which can also be found in [9, 10, 13, 14]. We will discuss the uniform boundedness of the complex dynamical networks (2) in the next section.

Next, we introduce the concept of matrix measure which is the main tool in the deduction of this paper.

Definition 3. The matrix measure of a real square matrix $A=(a_{ij})_{n\times n}$ is as follows:

$$\mu_p(A)=\lim_{\varepsilon\to0^+}\frac{\|\mathbf{I}+\varepsilon A\|_p-1}{\varepsilon},$$

where $\|\cdot\|_p$ is an induced matrix norm on $\mathbb{R}^{n\times n}$, $\mathbf{I}$ is the identity matrix, and $p=1,2,\infty$.

3 Uniform Boundedness of Complex Networks

In this section, we establish some basic results on uniform boundedness of solutions in the sense of Filippov for the complex networks (2) under the next hypothesis called as the growth condition [23].

The growth condition (**g.c.**): for $f\in\mathcal{F}$, there exist constants M_1 and M_2, with $M_1\geq0$ such that

$$\|\mathbb{F}(x_i)\|_p=\sup_{\xi\in\mathbb{F}(x_i)}\|\xi\|_p\leq M_1\|x_i\|_p+M_2,\ p=1,2,\infty,\ i=1,2,\cdots,N. \tag{7}$$

Let $A\otimes B$ denote the Kronecker product of matrices A and B, $\mathbf{D}=I_N\otimes D$, $\mathbf{B}=I_N\otimes B$, $\mathbf{\Gamma}_1=A\otimes\Gamma_1$, $x(t)=(x_1^T(t),x_2^T(t),\cdots,x_N^T(t))^T$ and $\mathbf{f}(x(t))=(f^T(x_1(t)),f^T(x_2(t)),\cdots,f^T(x_N(t)))^T$. The linearly coupled dynamical system (2) can be rewritten as

$$\dot{x}(t)=\mathbf{D}x(t)+\mathbf{B}\mathbf{f}(x(t))+c_1\mathbf{\Gamma}_1x(t). \tag{8}$$

Definition 4. The complex system (8) is uniformly bounded with a bound $\omega > 0$ if there exist $\delta_0 > 0$ and $T \geq 0$ such that if $\|x(0)\|_p^\tau \leq \delta_0$ then $\|x(t)\|_p \leq \omega$ for all $t \geq T$, where $\|x(0)\|_p^\tau = \max_{-\tau \leq z \leq 0} \|x(z)\|_p$.

Theorem 1. Under the growth condition (**g.c.**), the complex network (8) will be uniformly bounded, if there exist $\sigma > 0$ and one matrix measure $\mu_p(\cdot)$, $p = 1, 2, \infty$ such that

$$\mu_p(\mathbf{D} + c_1\mathbf{\Gamma}_1) + M_1\|\mathbf{B}\|_p \leq -\sigma < 0. \tag{9}$$

Proof. The proof is omitted for simplicity due to page limit.

4 Quasi-synchronization of Coupled Networks

In the above section, model (2) is a complex dynamical network without delayed coupling. In this section, we consider the synchronization issue of linearly delayed coupled networks with non-identical nodes:

$$\dot{x}_i(t) = D_i x_i(t) + B_i f(x_i(t)) + c_1 \sum_{j=1}^{N} a_{ij} \varGamma_1 x_j(t) + c_2 \sum_{j=1}^{N} a_{ij} \varGamma_2 x_j(t-\tau), \quad i = 1, 2, \cdots, N, \tag{10}$$

which is a general complex network model. It means that each node communicates with other non-identical nodes at time t as well as at time $t - \tau$.

Our goal is to synchronize the states of networks (10) on the manifold

$$\dot{s}(t) = Ds(t) + Bf(s(t)), \tag{11}$$

by introducing a controller $u_i(t) \in \mathbb{R}^n$, $i = 1, 2, \cdots, N$, into each individual node, where $s(t)$ can be any desired state, for example, an equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit. That is, by adding a suitable designed feedback controller to complex networks (10), there exists a constant $t_1 > 0$ such that $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$, for any $t \geq t_1$. The controlled complex networks (10) can be written as :

$$\dot{x}_i(t) = D_i x_i(t) + B_i f(x_i(t)) + c_1 \sum_{j=1}^{N} a_{ij} \varGamma_1 x_j(t) + c_2 \sum_{j=1}^{N} a_{ij} \varGamma_2 x_j(t-\tau) + u_i(t). \tag{12}$$

Subtracting (11) from (12), we obtain the following error dynamical systems:

$$\begin{aligned} \dot{e}_i(t) &= \dot{x}_i(t) - \dot{s}(t) \\ &= D_i e_i(t) + B_i g_i(t) + c_1 \sum_{j=1}^{N} a_{ij} \varGamma_1 x_j(t) + c_2 \sum_{j=1}^{N} a_{ij} \varGamma_2 x_j(t-\tau) + u_i(t) \\ &\quad + \varDelta D_i s(t) + \varDelta B_i f(s(t)), \end{aligned} \tag{13}$$

where $\Delta D_i = D_i - D$, $\Delta B_i = B_i - B$, $g_i(t) = f(x_i(t)) - f(s(t))$.

Based on (3), we have

$$\sum_{j=1}^{N} a_{ij}\Gamma_1 s(t) = \sum_{j=1}^{N} a_{ij}\Gamma_2 s(t-\tau) = 0. \tag{14}$$

Consider the state-feedback control law

$$u_i(t) = k_i e_i(t). \tag{15}$$

By Kronecker product, (14) and (15), the error system (13) can be rewritten as

$$\dot{e}(t) = \mathbb{D}e(t) + \mathbb{B}\mathbf{g}(t) + c_1\mathbf{\Gamma}_1 e(t) + c_2\mathbf{\Gamma}_2 e(t-\tau) + \mathbf{K}e(t) + \Delta\mathbb{D}\mathbf{S}(t) + \Delta\mathbb{B}\mathbf{f}(\mathbf{S}(t)), \tag{16}$$

where $\mathbb{D} = \mathrm{diag}(D_1, D_2, \cdots, D_N)$, $\mathbb{B} = \mathrm{diag}(B_1, B_2, \cdots, B_N)$, $\mathbf{K} = \mathrm{diag}(k_1, k_2, \cdots, k_N)$, $\Delta\mathbb{D} = \mathrm{diag}(\Delta D_1, \Delta D_2, \cdots, \Delta D_N)$, $\Delta\mathbb{B} = \mathrm{diag}(\Delta B_1, \Delta B_2, \cdots, \Delta B_N)$, $\mathbf{\Gamma}_2 = A \otimes \Gamma_2$, $\mathbf{S}(t) = (s^T(t), s^T(t), \cdots, s^T(t))^T$, and $\mathbf{g}(t) = (g_1^T(t), g_2^T(t), \cdots, g_N^T(t))^T$.

Definition 5 [15, 18]. Complex networks (10) and (11) is quasi-synchronized, if there exists a compact set $\Omega \subset \mathbb{R}^{Nn}$ so that $e(t_0) \in \Omega$ and there exists a bound B and a time $T(B, e(t_0))$, which are both independent of $t_0 > 0$, such that $\|e(t)\|_p \le B,\ p = 1, 2, +\infty,\ \forall\ t \ge t_0 + T$.

Before proceeding to the main results, we further assume that the set-valued map $\mathbb{F}$ satisfies:

(**L.**) Suppose $0 \in K[f(0)]$ and there exist constants $\bar{M}_1$ and $\bar{M}_2 \ge 0$ such that for all $\iota(t) \in K[f(x(t))]$, $\kappa(t) \in K[f(y(t))]$, the following holds:

$$\|\iota(t) - \kappa(t)\|_p \le \bar{M}_1 \|x(t) - y(t)\|_p + \bar{M}_2, \quad p \in \{1, 2, \infty\}.$$

Remark 1. Under the assumption (**L.**), the growth condition (**g.c.**) holds. Hence, based on the Theorem 1, the synchronization manifold (11) will be uniformly bounded. In other words, for each orbit in system (11), $\forall \mathbf{S}_0 \in \mathbb{R}^{Nn}$, there exist a time T and a constant $\omega > 0$ such that $\|\mathbf{S}(t)\|_p \le \omega$, $\forall t \ge T$.

Theorem 2. Under the condition (**L.**), if there exists one matrix measure $\mu_p(\cdot),\ p = 1, 2, \infty$ such that (9) and (17) hold

$$\mu_p(\mathbb{D} + c_1\mathbf{\Gamma}_1 + \mathbf{K}) + M_1\|\mathbb{B}\|_p + c_2\|\mathbf{\Gamma}_2\|_p \le -\bar{\sigma} < 0,\ \forall\ t \ge T. \tag{17}$$

Then, complex network (10) quasi-synchronizes (11). Moreover, the bounds on synchronization error can be smaller by increasing the control gain $\mathbf{K}$.

Proof. Consider another positive radially unbounded auxiliary functional for the error system (16) as

$$V_2(t) = \|e(t)\|_p. \tag{18}$$

By the Chain Rule in [26], calculating the upper right-hand derivative of $V_2(t)$ along the positive half trajectory of Eq. (16), we have

$$\begin{aligned} D^+V_2(t) &= \overline{\lim}_{h\to 0^+}\frac{\|e(t+h)\|_p - \|e(t)\|_p}{h} \\ &\leq \overline{\lim}_{h\to 0^+}\frac{\|\mathbf{I}+h(\mathbb{D}+c_1\mathbf{\Gamma}_1+\mathbf{K})\|_p - 1}{h}\|e(t)\|_p + \|\mathbb{B}\beta(t)\|_p \\ &+ c_2\|\mathbf{\Gamma}_2 e(t-\tau)\|_p + \|\varDelta\mathbb{D}\mathbf{S}(t)\|_p + \|\varDelta\mathbb{B}F(\mathbf{S}(t))\|_p. \end{aligned} \tag{19}$$

where $\beta(t) = (\beta_1^T(t), \beta_2^T(t), \cdots, \beta_N^T(t))^T$, and $\beta_i(t) \in K[g_i(t)]$, $i = 1, 2, \cdots, N$. Based on the condition (**L.**) and Theorem 1, when $t \geq T$, we have

$$\begin{aligned} &D^+V_2(x(t)) \\ &\leq (\mu_p(\mathbb{D}+c_1\mathbf{\Gamma}_1+\mathbf{K}) + \bar{M}_1\|\mathbb{B}\|_p)\|e(t)\|_p + c_2\|\mathbf{\Gamma}_2\|_p \cdot \max_{t-\tau\leq z\leq t}\|e(z)\|_p \\ &+ \bar{M}_2\|\mathbb{B}\|_p + \omega\|\varDelta\mathbb{D}\|_p + \omega\bar{M}_1\|\varDelta\mathbb{B}\|_p + \bar{M}_2\|\varDelta\mathbb{B}\|_p. \end{aligned} \tag{20}$$

Then, by (17) and the generalized Halanay inequalities, one obtains

$$\|e(t)\|_p \leq \frac{\bar{\gamma}}{\bar{\sigma}} + (\sup_{-\infty\leq z\leq 0}\|e(z)\|_p - \frac{\bar{\gamma}}{\bar{\sigma}}) \cdot e^{-\bar{\mu}^*(t-t_0)}, \tag{21}$$

where $\bar{\gamma} = \bar{M}_2(\|\mathbb{B}\|_p + \|\varDelta\mathbb{B}\|_p) + \omega(\|\varDelta\mathbb{D}\|_p + \|\varDelta\mathbb{B}\|_p)$ and $\bar{\mu}^* > 0$.

Therefore, for the given sufficient small $\varepsilon > 0$, there exists $\bar{T} \geq 0$ such that

$$\|e(t)\|_p \leq \frac{\bar{\gamma}}{\bar{\sigma}} + \varepsilon, \ \forall\ t \geq \bar{T}. \tag{22}$$

This completes the proof of Theorem 2.

Remark 2. From the matrix measure and Definition 5, we can see that it can have positive as well as negative values, whereas a norm can assume only non-negative ones. Due to these special properties, the results obtained via matrix measure usually are less restrictive than the one using the norm. Furthermore, the matrix measure approach appears simple and clear, which can be verified and applied easily.

5 Numerical Examples

Example 1. Consider the following linearly coupled network model:

$$\dot{x}_i(t) = Dx_i(t) + Bf(x_i(t)) + c_1\sum_{j=1}^{N} a_{ij}\varGamma_1 x_j(t) + c_2\sum_{j=1}^{N} a_{ij}\varGamma_2 x_j(t-\tau), \quad i = 1, 2, 3, \tag{23}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, $c_1 = c_2 = 1$, $D = \mathrm{diag}(-1, -1, -1)$, $\tau = 1$, the discontinuousfunction

$$f(x_{i1}(t), x_{i2}(t), x_{i3}(t)) = \begin{cases} -\frac{11}{21}x_{i1}(t) + 3x_{i2}(t) + \frac{9}{7}\text{sign}(x_{i1}(t)), \\ \frac{2}{3}x_{i1}(t) + 5x_{i2}(t) + \frac{1}{3}x_{3i}(t), \\ -\frac{1}{3}x_{i1}(t) - 10x_{i2}(t) + \frac{1}{3}x_{i3}(t), \end{cases}$$

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, A = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.1 & 0 \\ 0.1 & 0 & -0.1 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 1 & 0.5 & 0.4 \\ 0.8 & 1 & 0.3 \\ 0.2 & 0.7 & 0.9 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 0.6 & 0.3 & 0.4 \\ 0.5 & 1 & 0.3 \\ 0.2 & 0.5 & 0.9 \end{bmatrix}.$$

Based on the detailed discussion in [24], the isolated node dynamics behavior is chaotic (the generalized Chua circuit). From Theorem 1, the linearly coupled network (23) is uniformly bounded, as shown by Fig. 1.

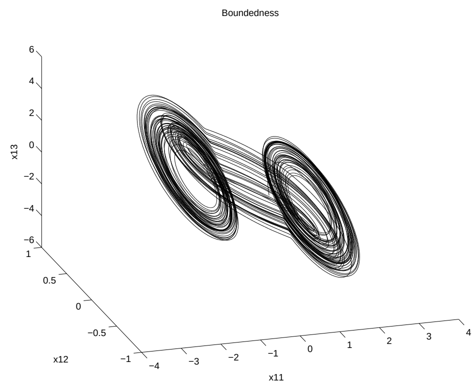

Fig. 1. Trajectories of one node in the coupled networks.

Example 2. Consider the following linearly coupled network model with non-identical nodes:

$$\dot{x}_i(t) = D_i x_i(t) + B_i f(x_i(t)) + c_1 \sum_{j=1}^{N} a_{ij}\Gamma_1 x_j(t) + c_2 \sum_{j=1}^{N} a_{ij}\Gamma_2 x_j(t-\tau), \quad i = 1, 2, 3, \quad (24)$$

where the values of $c_1,\ c_2,\ A,\ \Gamma_1,\ \Gamma_2,\ \tau$ and f are same as those in Example 1,

$$B_1 = \begin{bmatrix} 3.01 & 0 & 0 \\ 0 & 2 & 1.02 \\ 0 & 1 & 1.99 \end{bmatrix}, B_2 = \begin{bmatrix} 2.99 & 0 & 0 \\ 0 & 1.99 & 1.01 \\ 0 & 1 & 2 \end{bmatrix}, B_3 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0.99 \\ 0 & 1.01 & 2.01 \end{bmatrix},$$

$D_1 = diag(-0.99, -0.97, -0.99)$, $D_2 = diag(-1.01, -1, -0.99)$, and $D_3 = diag(-1, -0.99, -1.01)$. The manifold that we want to synchronize to is:

$$\dot{s}(t) = Ds(t) + Bf(s(t)), \quad (25)$$

where the parameters D and B are the same as those in Example 1. Designing the feedback controller $u(t) = \mathbf{K}e(t) = k * \mathbf{I}e(t)$, where $\mathbf{I}$ is the identity matrix with proper dimensions. Based on Theorem 2, the coupled network (24) quasi-synchronizes (25), just as shown as Figs. 2-3.

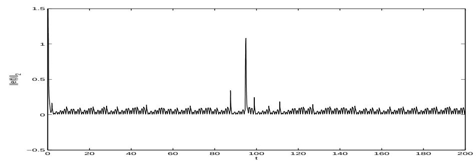

Fig. 2. The quasi-synchronization error $\|e(t)\|_2$ with $k = -5$ in Example 2.

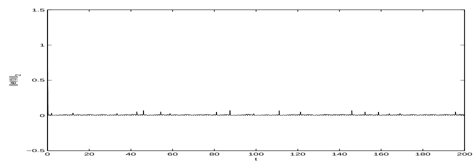

Fig. 3. The quasi-synchronization error $\|e(t)\|_2$ with $k = -20$ in Example 2.

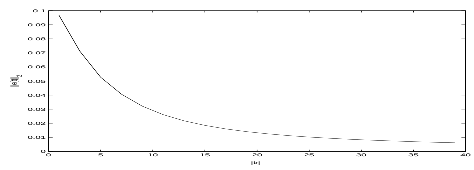

Fig. 4. The trend of quasi-synchronization error $\|e(t)\|_2$ with the increasing $|k|$.

Fig. 4 shows the trend of $\|e(t)\|_2$ with the decreasing gain k. However, for the coupled complex networks with non-identical discontinuous node dynamics, the complete synchronization ($\|e(t)\|_2 \to 0$) can't be realized even for a given sufficiently large $|k|$, unless $|k| \to \infty$.

6 Conclusions

This paper has introduced a general delayed coupled complex networks model with nonlinear functions of possessing jumping discontinuities. Based on the concept of Filippov solution, boundedness and quasi-synchronization problems of such networks have been studied by the matrix measure approach and the generalized Halanay inequalities. Easily testable conditions have been established to ensure synchronization for linearly coupled networks with non-identical nodes. These results are novel since there are few works on the synchronization control of complex networks with discontinuous non-identical systems.

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