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THE LAMINAR /TURBULENT TRANSITION IN A SLUDGE PIPELINE

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ABSTRACT:
Globally, wastewater treatment plants are under pressure to handle high concentration sludge in a sludge treatment line. Unawareness of the non-Newtonian behaviour of the thickened sludge has the potential to cause unexpected problems when the fluid behaviour changes from turbulent to laminar flow.

In this study, sludge apparent viscosity was plotted as a function of Total Suspended Solids concentration (TSS) and shear rate. Then, the transition velocity based on several predictive models in the literature was determined. This analysis provides a practical basis for the prediction of the pipe flow behaviour of thickened sludge in troubleshooting and engineering design.

Keyword: High concentration sludge, Laminar flow, Non-Newtonian fluids, Turbulence, Transport processes, Viscoplastic

INTRODUCTION
Large volumes of primary and secondary sludge are produced on a daily basis at sewage treatment plants. In many cases, this sludge receives tertiary treatment in anaerobic digesters. The main purpose of this tertiary treatment is to reduce the organic content and pathogen levels of the sludge as well as the odour potential prior to any subsequent processing or disposal activities. Anaerobic sludge digesters are typically well mixed reactors operating at 37ºC. The heat load is normally provided by continuously circulating a sludge stream from the digester through an external heat exchanger.
Due to increasing urban populations and associated issues, wastewater treatment plants are under pressure to treat increasing volumes of wastewater with existing treatment plant. For tertiary treatment processes, this inevitably means that a more concentrated sludge will circulate in the anaerobic digesters and the associated pipes, pumps and heat exchangers. A serious exacerbating issue is the fact that viscous stresses increase exponentially with concentration, and become increasingly viscoplastic in character. Underlying these matters is the fact that a fluid's flow behaviour changes fundamentally, depending upon whether the flow is in the laminar or turbulent flow regime. From a fluid mechanics design and operational perspective, a major concern is the particular possibility that laminar flow will be encountered where previously it was customarily assumed to be turbulent. Consequently, accurate prediction of the transition velocity of this thickened sludge from turbulent to laminar flow is a critically important, but analytically elusive, process parameter.

The objective of this study is to provide a practical basis for the prediction of the pipe flow behaviour of highly concentrated sludge for troubleshooting and engineering design, in this context.

THEORY AND LITERATURE REVIEW

The behaviour of a fluid changes fundamentally at the point of the laminar/turbulent transition and it is vitally important either in hydraulic design or in flow control to identify this point accurately. Over the last 50 years, a number of predictive approaches have been proposed. But those such as the Metzner and Reed (1955) model that use simple criteria for the determination of the flow regime in non-Newtonian fluid - similar to that for a Newtonian fluid - have been very popular.

Non-Newtonian sludges are often best modelled as yield pseudoplastic material (Govier and Aziz, 1972 and Hanks, 1979). The constitutive rheological equation for the yield pseudoplastic rheological model is

$$\tau = \tau_y + K\left[-\frac{du}{dr}\right]^n \qquad \text{Eq.1}$$

where τ_y is the yield stress, K is the fluid consistency index and n is the flow behaviour index. This expression can be used to represent both Herschel-Bulkley and Bingham plastic behaviour (Slatter and Wasp, 2000).

For Newtonian fluids, tThe generally accepted value of the Reynolds number at the lower bound of the laminar/turbulent transition is 2100 (Govier and Aziz, 1972) and the critical velocity can easily be formulated, $V_c=2100\mu/\rho D$. In order to make use of standard Newtonian theory, a value for the viscosity of the fluid is required. Usually the term viscosity is meaningless once a non-Newtonian approach has been adopted. However, an apparent viscosity can be defined at the pipe wall (Holland, 1973), and the standard Newtonian theory can be adapted for a non-Newtonian fluid by using the apparent viscosity at the wall in the standard Newtonian Reynolds number equation (Eq.2). Then, the transition velocity from laminar to turbulent flow is obtained when the Reynolds number is equal to 2100.

$$Re_{Newt} = \frac{\rho VD}{\mu'} \quad \text{where } \mu' = \frac{\tau_0}{\left[-\frac{du}{dr}\right]_0} \qquad \text{Eq.2}$$

Metzner and Reed (1955) used the laminar Fanning friction factor, f, as their stability parameter. They proposed that for all time independent non-Newtonian fluids flowing in pipes, transition would take place at the critical value of 0.0076 for the Fanning friction factor or 2100 for Re_{MR}. This generality has made the Metzner and Reed approach popular and is arguably the most widely encountered. They defined a non-Newtonian Reynolds number, $Re_{MR} = 16/f$, as follows:

$$\mathrm{Re}_{MR} = \frac{8\rho V^2}{K'\left(\frac{8V}{D}\right)^{n'}} \qquad \text{Eq.3}$$

The problem with this model is that for a viscoplastic fluid, K' and n' are not constant and must be evaluated for each value of τ_0. This leads to a significant complication in the use of this model (Slatter, 1995).

The Bingham plastic model has been found useful by many researchers to approximate the viscous flow behaviour of non-Newtonian materials (Xu et al., 1993; Slatter, 2001), and a Reynolds number can be formulated as (Grovier and Aziz, 1972);

$$\mathrm{Re}_{BP} = \frac{\rho VD}{K\left(1+\frac{\tau_y D}{6KV}\right)} \qquad \text{Eq.4}$$

It is assumed that the transition from laminar to turbulent flow will occur when Re_{BP}=2100 (Eq.4), from which the critical velocity can be defined as

$$V_c = 19\sqrt{\frac{\tau_y}{\rho}} \; . \qquad \text{Eq.5}$$

The fundamental problem here is that, at larger diameter, the yield stress causes the transition velocity to become independent of the pipe diameter (Slatter, 2007). This is in sharp contrast to the Newtonian hyperbolic condition where $V_C D$ is constant for a given fluid. Skelland (1967) has shown that the laminar/turbulent transition should occur when Re_{BP}=2100.

Ryan and Johnson (1959) suggested using the ratio of input energy to energy dissipation for a fluid element as the stability parameter. Hanks (1981) identified the key mechanism leading to transitional instability is a rotational momentum transfer. They have derived stability functions for laminar flow velocity vector fields (Slatter, 2007), and for axially symmetrical pipe flow the two functions differ by a factor of two. The Ryan and Johnson stability function is:

$$Z = \frac{Ru\rho}{\tau_0}\left[-\frac{du}{dr}\right] , \qquad \text{Eq.6}$$

where R is the internal radius of the pipe.

The maximum value of this function Z_{max} across a given laminar velocity vector field is taken as the stability criterion. For Newtonian pipe flow, Z_{max} = 808 corresponds to Re = 2100 and it is assumed that all fluids will obtain this value of Z_{max} = 808 at the transition limit. The transition criterion is Z_{max} = 808. Experimental data of Slatter (1995) showed that for viscoplastic fluids, the transition from laminar to turbulent flow does not occur at a constant value of Z_{max} = 808.

Hedstrom (1952) proposed a practical approach which uses the intersection of laminar and turbulent friction factor curves (Wilson, 1997). This approach is known as the intersection method. The critical velocity calculated by this approach relies on the accuracy of the turbulent model used. This model is also incompatible with Newtonian behaviour, where the transition point is not the intersection of the laminar and turbulent theoretical lines (Slatter, 1995; Slatter, 1999).

Torrance (1963) modelled yield pseudoplastic fluid flow. He used the following formulation for a Reynolds number, also known as the Clapp Reynolds number (Govier and Aziz, 1972):

$$Re_{nn} = \frac{8\rho V^2}{K\left(\frac{8V}{D}\right)^n} \qquad \text{Eq.7}$$

This Reynolds number gives the same value for a pseudoplastic and yield pseudoplastic fluids, as the yield stress is totally ignored. Therefore the Torrance Reynolds number (Torrance, 1963) does not accurately encompass the full viscous stress, due to the fact that the full rheology is not included (yield stress is excluded). It also should be noted that there is no direct claim in the literature that this Reynolds number should obtain the value 2100 at the transition point.

A recent approach which is popular in the mining industry is the Reynolds number Re_3 (Slatter, 1995; Slatter, 1999). This approach predicts a laminar to turbulent transition in the Reynolds number region of 2100. This approach was specifically developed to place emphasis on the viscoplastic nature of the material (Slatter, 1995). Using the fundamental definition that Re $\propto$ inertial / viscous forces, the final formulation is:

$$Re_3 = \frac{8\rho V_{ann}^{\ 2}}{\tau_y + K\left(\frac{8V_{ann}}{D_{shear}}\right)^n} \qquad \text{Eq.8}$$

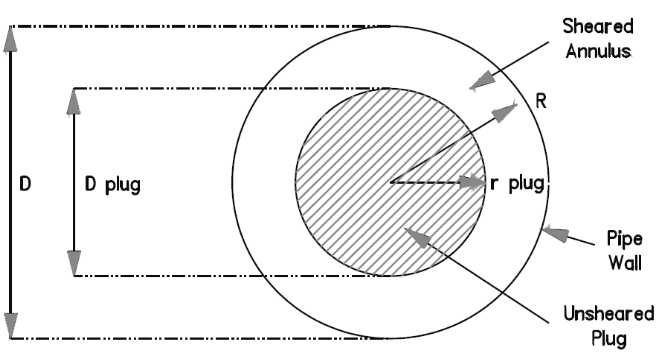

Figure 1: Unsheared plug geometry

As shown in Fig 1, in the presence of a yield stress the central core of the fluid moves as a solid plug which fundamentally affects the stability of flow (Slatter, 1995; Slatter, 1999). The unsheared plug is treated as a solid body in the centre of the pipe. The flow that the plug represents must be subtracted as it is no longer being treated as part of the fluid flow. The corrected mean velocity in the annulus V_{ann} is then obtained as follows:

$$V_{ann} = \frac{Q_{ann}}{A_{ann}} = \frac{Q - Q_{Plug}}{\pi\left(R^2 - r_{plug}^2\right)}$$ Eq.9

and

$Q_{plug} = u_{plug}A_{plug}$. Eq.10

The constitutive rheological equation can be integrated to obtain the plug velocity u_{plug},

$$u_{plug} = \frac{D}{2K^{\frac{1}{n}}\tau_0}\frac{n}{n+1}\left[\left(\tau_0 - \tau_y\right)^{\frac{n+1}{n}}\right]$$ Eq.11

The radius of the plug is

$$r_{plug} = \frac{\tau_y}{\tau_0}R$$ Eq.12

The sheared diameter, D_{shear}, is taken as the characteristic dimension because this represents the zone in which shearing of the fluid actually takes place, and it is defined as:

$D_{shear} = D - D_{plug}$, Eq.13

where

$D_{plug} = 2r_{plug}$. Eq.14

The laminar-turbulent prediction method Re_3 was developed specifically for visco-plastic material, and has been shown to be the most accurate predictive tool for this purpose as yet, (Slatter, 1995; Slatter and Wasp, 2000; Slatter and Wasp, 2004). In particular, Re_3 has been shown to be significantly superior to Z_{max}. Furthermore, all the other Reynolds number approaches ignore the fact that an un-sheared solid plug exists under laminar flow conditions due to the presence of the yield stress.

Recently, Guzel et al. (2009) defined a local Reynolds number, $Re_{G,l}(r)$ as follows:

$$Re_{G,l}(r) = \frac{\rho u(r) D}{\mu(r)}$$ Eq.15

where u(r) is the axial velocity and μ(r) is the effective velocity, which depends on r via the rate of strain, $\dot{\gamma}(r)$. This interpretation of the local Reynolds number is close to the stability parameters postulated by Ryan and Johnson (1959) and (Hanks, 1963).

EXPERIMENTAL DATA AND ANALYSIS

The data for three concentrations of sludge in this study were obtained from previous work (Slatter 1997). Slatter (1997) used a rotary viscometer (HAAKE ROTOVISCO MV1P) for rheological measurements. Figure 2 shows the sludge rheograms, which present with both yield stress and rheogram curvature. Figures 3 and 4 show the apparent viscosity of sludge as a function of shear rate and total suspended solid concentration (TSS), respectively.

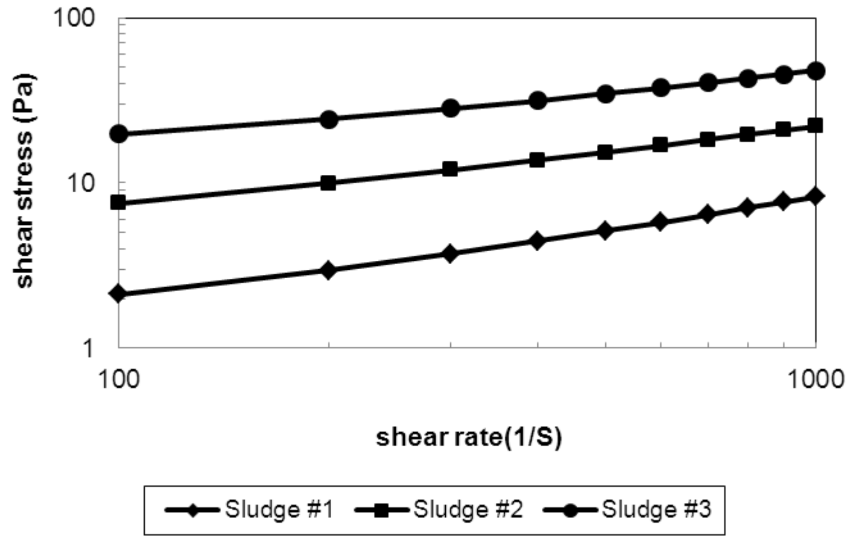

Figure2: Sludge rheograms for three concentrations of sludge

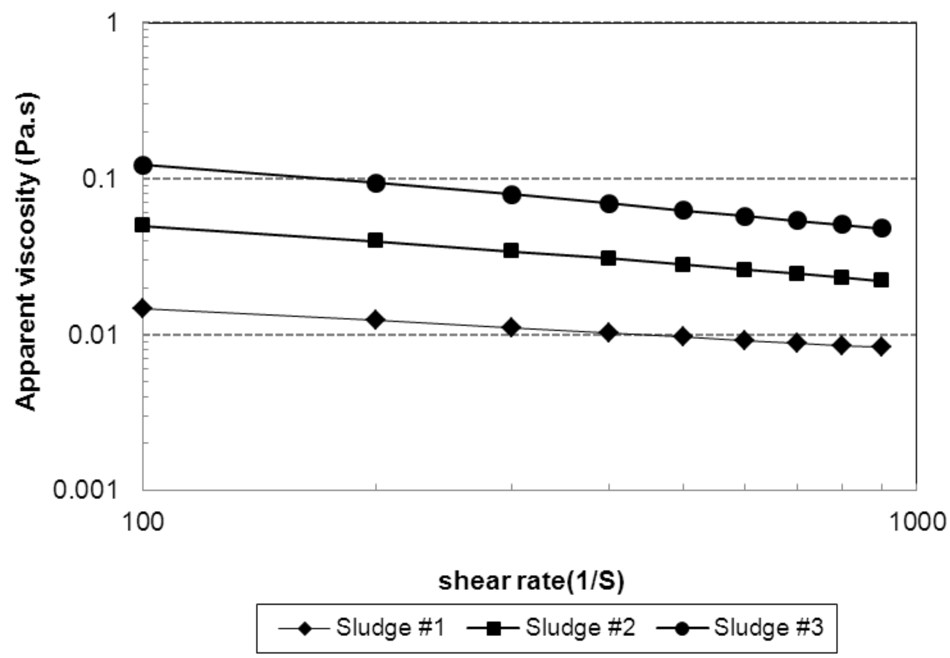

Figure3: Apparent viscosity as a function of shear rate

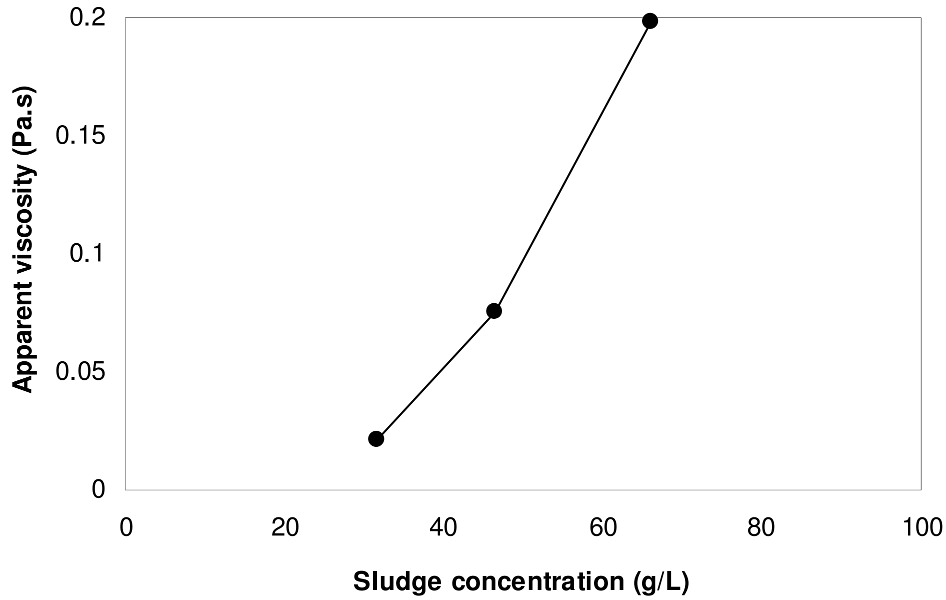

Figure4: Apparent viscosity of sludge at a shear rate of 100 s^{-1}, as a function of total suspended solid concentration (TSS)

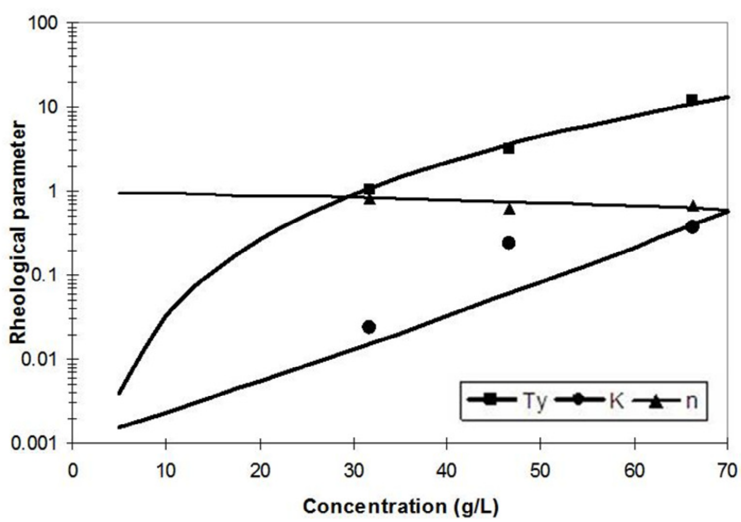

Figure 5: Correlation between rheological parameters and sludge concentration

Figure 5 shows the correlations between the fluid consistency index (K), the yield stress (τ_y) and the flow behaviour index (n) with sludge concentration. The Landel et al. (1965) correlation (Eq.16) and Dabak and Yucel (1987) correlation (Eq.17) were used for correlating the fluid consistency index (K) and the yield stress with sludge concentration, respectively. As there is no proposed correlation for the flow behaviour index with sludge concentration, polynomial regression ($n= a_1 C^2+ a_2 C+1$ with a_1= - 1.56E1, and a_2= -4.59) was used. The rheological constants (τ_y, K, n) of the sludge are presented in Table 1.

Fig

$$K = \mu_W (1 - \frac{C}{C_{Max}})^{-35..3}, C_{Max} = 0.425 \quad \text{Eq.16}$$

$$\tau_y = 13400 \frac{C^3}{C_{Max} - C}, C_{Max} = 0.425 \quad \text{Eq.17}$$

Table 1: The rheological constants for three concentrations of sludge

	TSS (g/l)	Apparent viscosity at 100 s^{-1} shear rate (Pa.s)	τ_y (Pa)	K ($Pa.s^n$)	n
Sludge # 1	31.7	0.021	1.04	0.0239	0.827
Sludge # 2	46.6	0.075	3.13	0.240	0.632
Sludge # 3	66.2	0.198	12.0	0.366	0.664

A 150 mm diameter heat exchanger pipe was considered for calculating transition velocities from turbulent to laminar flow and the calculated values are presented in Table 2. The density of all sludge concentrations was assumed to be similar to the density of water as the highest sludge concentration is 6% which does not produce significant changes in density.

Table2: The calculated transition velocity from different models for three different concentrations of sludge in a 150 mm diameter heat exchanger pipe

	V_c (ms^{-1}) from $Re_{Newt.}$	V_c (ms^{-1}) from $Re_{MR.}$	V_c (ms^{-1}) from Re_{BP}	V_c (ms^{-1}) from $Z_{max.}$	V_c (ms^{-1}) from $Re_{nn.}$	V_c (ms^{-1}) from $Re_{3.}$
Sludge # 1	0.47	0.68	0.61	0.65	0.22	0.85
Sludge # 2	1.11	1.45	1.06	1.49	0.83	1.59
Sludge # 3	1.88	2.58	2.08	2.58	1.25	2.94

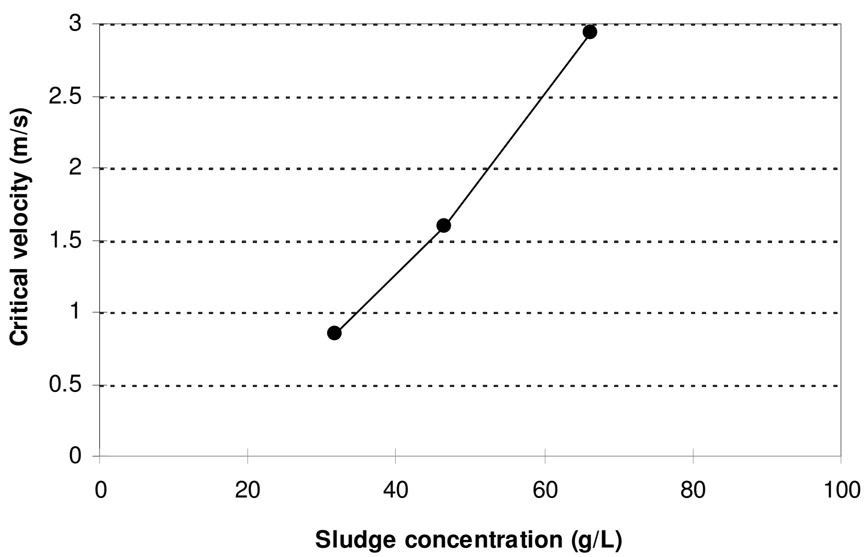

Figure6: Critical velocity as a function of total solid suspension concentration

Table 2 summarizes the calculated values for transition velocities from the different models introduced earlier. As Re_3 is the most successful predictive tool for transition/critical velocity calculation (Slatter, 1995; Slatter and Wasp, 2000; Slatter and Wasp, 2004), all calculated values of the transition velocity from the other models underestimated the transition point from turbulent to laminar flow. Figure 6 shows the calculated critical velocity using the Re_3 model against sludge concentration. This figure indicates that above a concentration of 31.7g/L in this type of sludge, the requirement for turbulent velocity will exceed the standard design velocity, which is 0.8 m/s (Ludwig, 1999), in the 150mm (6 in) heat exchanger pipe with a 15 L/s sludge volumetric flow rate, Also, a small increase in sludge concentration causes a steep increase in the critical velocity. For example, 5g/L increase in sludge concentration changed the critical velocity from 0.8 m/s to 1.1 m/s, which is a 38% increase. The critically important point of note is that ignoring the non-Newtonian character of the fluid can result in blockage and inefficient performance in the heat exchanger pipes when the fluid behaviour changes from turbulent to laminar flow.

CONCLUSION

In this paper, the transition/critical velocity from turbulent to laminar flow for three different concentrations of sludge was calculated for different models in literature. All predicted values of the transition velocity from these models underestimated the transition point in comparison to the most successful predictive model. Also, the data analysis revealed that a small increment in sludge concentration significantly increases the critical velocity. This analysis affords a practical basis for troubleshooting and engineering design because it can provide realistic, useful and accurate prediction of the pipe flow behaviour of highly concentrated sludge.

NOMENCLATURE

D - Pipe inside diameter (m)
f - Fanning friction factor
K - Fluid consistency index ($Pa.s^n$)
K' Metzner and Reed parameter ($Pa.s^{n'}$)
n - Flow behavior index
n' Metzner and Reed parameter
r – Radial position (m)
R – Pipe inside radius (m)
Re_3 - Slatter laminar to turbulent Reynolds Number
$Re_{Newt.}$ Newtonian theory Reynolds Number
$Re_{MR.}$ Metzner and Reed Reynolds Number
Re_{BP} Bingham plastic model Reynolds Number
TSS- Total suspended solid concentration (g/L)
u –Point velocity ($m\ s^{-1}$)
V – Average velocity ($m\ s^{-1}$)
V_C – Critical/ transition velocity ($m\ s^{-1}$)
Z_{max} - Ryan and Johnson stability function
$\dot{\gamma}$ - Shear rate (s^{-1})
ρ – Fluid density ($kg\ m^{-3}$)
τ_0 - Wall shear stress (Pa)
τ_y - Yield stress (Pa)
μ' - Apparent viscosity

$[-\frac{du}{dr}]_0$ Velocity gradient at wall (s^{-1})

$[-\frac{du}{dr}]$ Velocity gradient (s^{-1})

Subscripts:

0	at the pipe wall
ann	of the annulus
c	critical
plug	of the plug
Shear	over the sheared zone

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