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The Sheet Flow Viscometer

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Abstract:

The flow of non-Newtonian materials as a planar sheet is an industrially important topic. In order to characterise the rheological properties of these flows, rotational/tube viscometry can be used successfully. However in most cases, it is generally not possible to achieve low shear rate data with coarse particle slurries, especially in small tube diameters, or normal gap-size couette viscometers, whilst adhering to the empirical criterion that the measuring gap size be greater than ten times the largest particle diameter. A possible solution is to use laminar sheet flows to obtain flow curve data at lower shear rates thereby supplementing the tube rotational viscometer data. One of the primary objectives of viscometry is the establishment of the relationship between shear stress and shear rate. This is often referred to as a rheogram or a flow curve and can be cast in the general form relating the flow curve to the bulk shear rate. The objective of this paper is to develop the basic relationships connecting shear stress to bulk shear rate for sheet flow, and to show how these can be extrapolated for laminar flow open channel design. This process is validated by comparison with established experimental data in the literature. It is concluded that laminar sheet flows can be used as a viscometrically reliable geometry for the rheological characterisation of complex slurry flows. Furthermore, these flows can be conceptually extrapolated to a sheet flow paradigm for laminar flow open channel design. A basis for the scale-up of such laminar flows for engineering design purposes is established. Further research is required to establish the laminar-turbulent transition and the effect of Froude number surface deformation effects.

1. INTRODUCTION

Concentrated suspensions are encountered literally everywhere in nature and in technology. Common examples include mineral and waste slurries, mine waste tailings, and flow of mud and debris. Furthermore, it is readily acknowledged that the free surface sheet flow of rheologically complex fluids is a subject of broad scientific and technological interest. Topics as diverse as mine waste tailings placement and avalanche and landslide flows are centrally related to this subject.

Whilst rotational and tube rheometry can be used successfully in most cases to measure the rheology of concentrated suspensions [1,2], it is generally not possible to achieve low shear rate data, especially in small tube diameters. A possible solution is to use laminar sheet flows to obtain flow curve data at lower shear rates, thereby supplementing the rotational/tube viscometry data.

The earliest work dealing with this topic is that of Astarita *et al*. [3]. However, they did not attempt to generate a flow curve over a range of shear rates or develop an expression for a bulk shear rate for rheological characterisation and scale up purposes. Instead, they found that the power law index values for polymer solutions obtained from inclined plane flow were similar to those found from rotational viscometry.

De Kee et al. [4] took this work further by extracting the values of the viscosity model parameters for power law, Bingham Plastic and Herschel-Bulkley models using the volumetric flow rate data from sheet flow experiments. However, the resulting values of the yield stress were only the fitted values that may or may not correspond to the true yield stress values if any.

Whilst all these workers built on earlier work [5], Uhlherr *et al*. [6], on the other hand, attempted to evaluate true (apparent) yield stress by locating the inclination angle demarcating the flow/no flow condition. The resulting values for a few silica suspensions and carbopol solutions were in line with those found from vane rheometry.

The objective of this paper is to directly use this sheet flow configuration to infer shear rate-shear stress behaviour of time independent fluids, akin to the approach of Rabinowitsch and Mooney [1,2,7] for tube flow.

2. LAMINAR TUBE FLOW OF A GENERAL TIME-INDEPENDENT FLUID

One of the primary objectives of viscometry is the establishment of the relationship between shear stress and shear rate. This is often referred to as a rheogram or a flow curve. For any time-independent fluid, this can be cast in the general form

$$\dot{\gamma} = f(\tau) \tag{1}$$

without reference to any particular rheological model.

For a cylindrical tube in the absence of wall-slip, it can be shown that the volumetric flowrate for steady laminar tube flow of a general time-independent fluid is given by [7]:

$$\frac{8V}{D} = \frac{32Q}{\pi D^3} = \frac{4}{\tau_0^3}\int_0^{\tau_0} \tau^2 . f(\tau)\, d\tau \tag{2}$$

where 8V/D is the bulk shear rate, Q is the volumetric flow rate, τ_0 is the wall shear stress and $f(\tau)$ is the shear rate as defined by Eqn (1).

For Newtonian fluids, the bulk shear rate is equal to the shear rate at the wall. However, this is not the case for non-Newtonian fluids, and the two values are related to each other via the well-known Rabinowitsch and Mooney equation [7].

$$\dot{\gamma}_0 = \frac{8V}{D}\left(\frac{3n'+1}{4n'}\right) \tag{3}$$

where n′ is defined as the local slope of the double logarithmic plot of wall shear stress, τ_0 vs. bulk shear rate, 8V/D:

$$n' = \frac{d \ln \tau_0}{d \ln \frac{8V}{D}}. \qquad (4)$$

The aim of this present work is to develop relationships similar to the above for sheet flow.

3. SHEET FLOW ANALYSIS

Consider the steady incompressible and laminar flow of a time-independent fluid on an inclined plane, as shown in Figure 1.

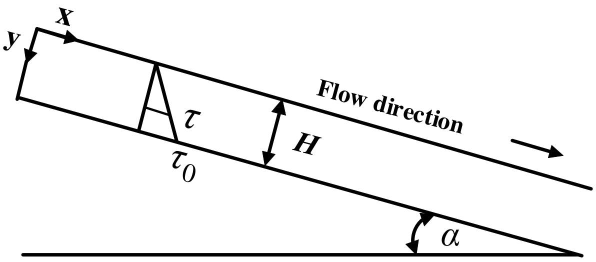

Figure 1: Sheet flow shear stress distribution

The free surface of the sheet is assumed to be smooth and free from ripples. Furthermore, the sheet thickness, H is assumed to be uniform and << W, the width of the plate in the z-direction. Under these conditions, there is only one non-zero shear stress component τ_{xy} given by

$$\tau_{xy} = \rho g y \sin\alpha \qquad (5)$$

The shear stress varies linearly in the y-direction from being zero at y = D at the free surface to its maximum value τ_0 at the wall given by:

$$\tau_0 = \rho g H \sin\alpha \qquad (6)$$

From Eqns (5) and (6), it can be deduced that:

$$\frac{\tau_{xy}}{\tau_0} = \frac{y}{H} \qquad (7)$$

In this case, the only non-zero component of the velocity is u_x which is a function of y only, i.e. $u_x(y)$.

The volumetric flow rate per unit width Q, of the liquid in the sheet is given by

$$Q = \int_0^H u_x \, dy \qquad (8)$$

Integrating by parts yields

$$Q = \left[u_x y\right]_0^H - \int_0^H \left(y \frac{du_x}{dy} \right) dy \qquad (9)$$

Assuming the no-slip condition at the wall, i.e. $u_x = 0$ at y = H, Eqn (9) reduces to

$$Q = \int_0^H y \left(-\frac{du_x}{dy} \right) dy \qquad (10)$$

For a time-independent fluid, we can describe the relationship between shear rate, $\dot{\gamma}$ and shear stress, τ as:

$$-\frac{du_x}{dy} = \dot{\gamma} = f(\tau) \qquad (11)$$

Substituting Equation (11) and changing the variable of integration from y to τ, Eqn (10) can be rewritten as

$$Q = \frac{H^2}{\tau_0^2} \int_0^{\tau_0} \tau f(\tau) d\tau \qquad (12)$$

For a Newtonian fluid, the constitutive relationship is

$$f(\tau) = \frac{\tau}{\mu} \qquad (13)$$

Substitution in Eqn (12) leads directly to the well-known result for Newtonian laminar sheet flow [5]

$$Q = \frac{H^2 \tau_0}{3\mu} . \qquad (14)$$

In introducing the average velocity of flow V as Q/H, Eqn (14), yields

$$V = \frac{Q}{H} = \frac{H\tau_0}{3\mu} \tag{15}$$

Combining Eqns (13) and (15) and expressing this in terms of the wall shear stress and wall shear rate leads to

$$\tau_0 = \mu \frac{3V}{H} = \mu \dot{\gamma}_0 . \tag{16}$$

It follows from Eqn (16) that the term 3V/H is the shear rate at the wall for a Newtonian fluid, i.e.

$$\dot{\gamma}_{0\,\mathrm{Newt}} = \frac{3V}{H} \tag{17}$$

It is further proposed that in general, 3V/H is the bulk shear rate for sheet flow. Hence, Eqn (12) can be rewritten as

$$\frac{3V}{H} = \frac{3}{\tau_0^2} \int_0^{\tau_0} \tau . f(\tau) d\tau \tag{18}$$

As with tube flow, the dilemma now arises as to how the true shear rate at the wall can be expressed in terms of the bulk shear rate. We now develop a relationship similar to the Rabinowitsch-Mooney equation [7] for sheet flow.

From Eqn (18)

$$\tau_0^2 \frac{V}{H} = \int_0^{\tau_0} \tau . f(\tau) d\tau \tag{19}$$

Differentiating both sides of Eqn (19) with respect to τ_0 leads to

$$2\tau_0 \left(\frac{V}{H} \right) + \tau_0^2 \frac{d\left(\frac{V}{H} \right)}{d\tau_0} = \tau_0 \; f(\tau_0) \tag{20}$$

Dividing by τ_0, introducing a factor of 3 in V/H terms and identifying $f(\tau_0)$ as the shear rate at the wall, $\dot{\gamma}_0$, Eqn (20) can be re-written as:

$$\dot{\gamma}_0 = \frac{3V}{H} \left(\frac{2}{3} + \frac{1}{3} \frac{d\ln\left(\frac{V}{H} \right)}{d\ln(\tau_0)} \right) \tag{21}$$

By analogy with tube flow, one can introduce the apparent sheet flow behaviour index n'_* defined as the local slope of the double logarithmic plot of wall shear stress, τ_0 vs bulk shear rate, 3V/H

$$n'_* = \frac{d\ln\tau_0}{d\ln\left(\frac{3V}{H}\right)} \cdot \qquad (22)$$

Finally, the shear rate at the wall is expressed in terms of the bulk shear rate, 3V/H as

$$\dot{\gamma}_0 = \frac{3V}{H}\left(\frac{2n'_* + 1}{3n'_*}\right) \cdot \qquad (23)$$

In summary, Table 1 gives the analogy of the proposed sheet flow analysis with that of Rabinowitsch and Mooney for tube flow.

Table 1: Comparison of key elements of the viscometric analysis of tube to sheet flow

	Bulk Shear Rate	Wall Shear Stress	R-M Factor
Tube Flow	$\frac{8V}{D}$	$\frac{D\Delta p}{4L}$	$\frac{3n'+1}{4n'}$
Sheet Flow	$\frac{3V}{H}$	$\rho g H \sin\alpha$	$\frac{2n'_*+1}{3n'_*}$

4. EXPERIMENTAL

Since detailed descriptions of the flow loop and experimental procedures are available elsewhere [8], only the salient features are recapitulated here.

The experimental work was conducted in a 10 m long, 300 mm wide tilting flume which can be tilted at various slope angles up to 5^{o}. The recirculating flow is driven by a progressive cavity positive displacement pump as well as centrifugal pump delivering a maximum flow rate of 45 l/s. Included in the flow loop is also an in-line tube viscometer with 3 tubes of diameters 13, 28 and 80 mm. Pressure drop is measured with differential pressure transducers. The volumetric flow rate is measured using a magnetic flow meter as well as a mass flow meter on the 13 mm line; the latter can be used to measure density and temperature. The depth of the liquid in the sheet is measured with two digital vernier type depth gauges. These are placed at 5 and 6 m from the entrance of the flume to ensure the measurements are free from any possible entrance and end effects. All data is collected via a data-logger to a PC.

Three fluids were used in the experimental work. An oil was used as a viscous Newtonian fluid, an aqueous carboxymethyl cellulose solution and a kaolin-in-water suspension were used as power law and viscoplastic fluids respectively, to demonstrate the validity and utility of the approach outlined here. The maximum value of the aspect ratio of the flow H/W was in all cases less than 0.1 thereby suggesting the 1-D flow conditions in the sheet [9,10] would be valid and applicable. This is very much in line with the work of previous workers investigating planar sheet flow.

5. RESULTS AND DISCUSSION

Figure 2 shows the shear stress-shear rate data for the Newtonian oil flowing in a 300 mm flume. The viscosity obtained from the flume data was 0.41 Pa s compared to 0.40 Pa s obtained from rotational viscometry. Because of the close correspondence between these two values, this suggests the experimental uncertainty to be around 2 to 3% thereby inspiring confidence in the use of this sheet flow approach as a means of obtaining flow curve data

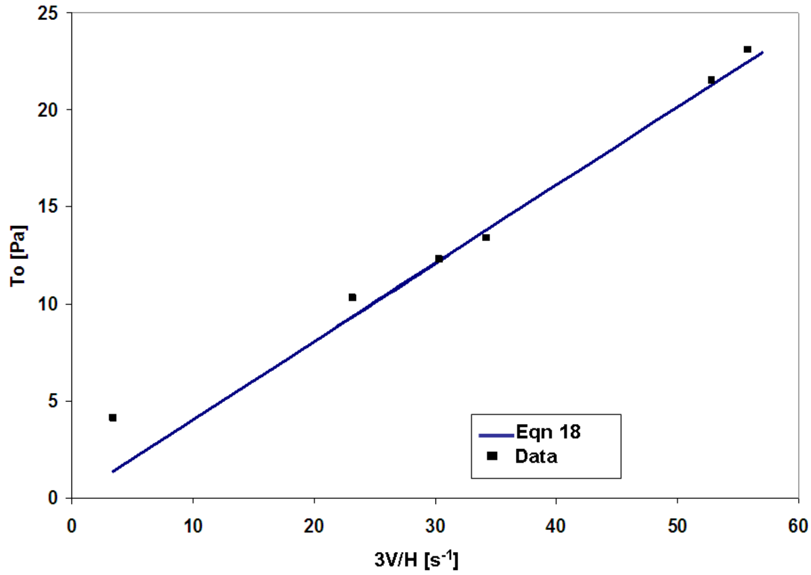

Figure 2: Oil in a 300 mm flume

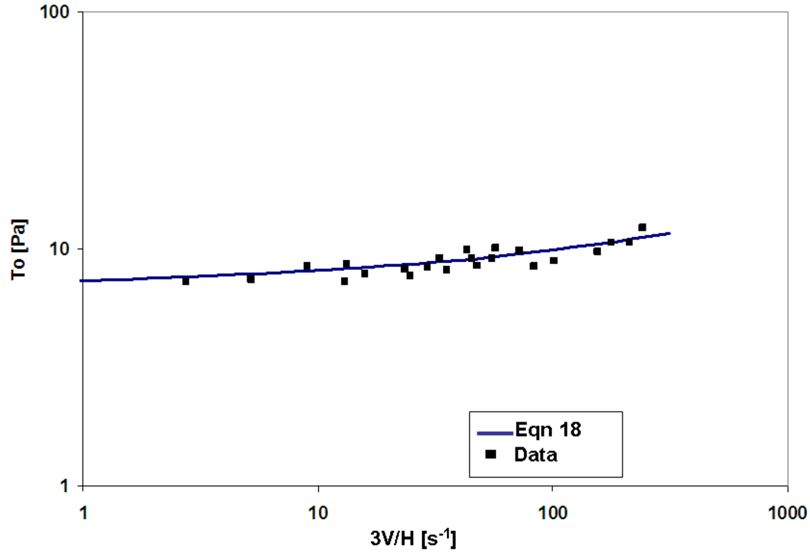

Figure 3: Kaolin 8% in a 300 mm flume

Figure 3 shows the analogous results for the flow of a kaolin clay suspension, whilst the solid line has been generated using conventional rheometer data and Eq. (18). The approach was to integrate Eq. (18) numerically using the rheological constants derived from the conventional rheometer data. As can be seen from Figure 3, there is good agreement between the new theory and the data.

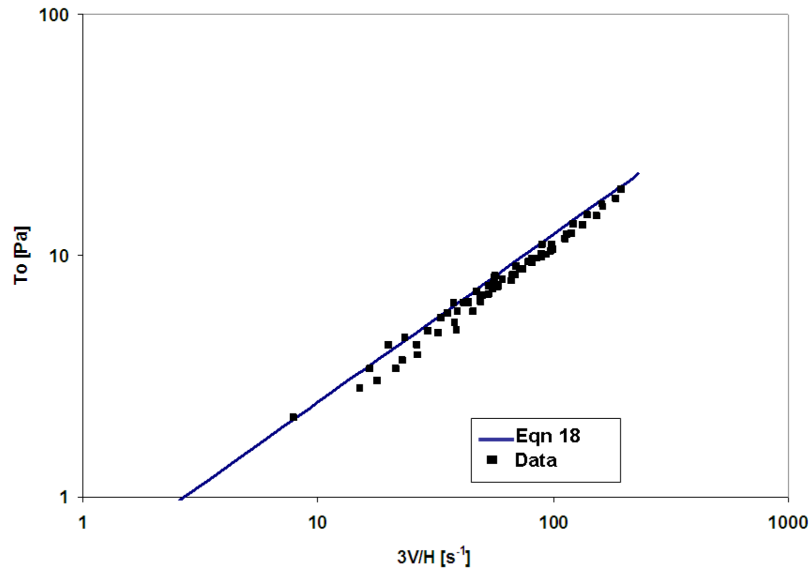

Figure 4: CMC 3.5% in a 300 mm flume

The data depicted in Figure 4 is for the sheet flow of a 3.5% Carboxy Methylcellulose (CMC) solution. As above, the solid line has been generated using conventional rheometer data for the CMC solution and Eq. (18).

This relationship of Eqn (18) is of fundamental importance for several reasons:

- A plot of 3V/H versus τ_0 will give a unique line for a given material for all values of H. This means that in general the bulk shear rate (3V/H) is a unique function of the rheogram and the wall shear stress (τ_0), provided that the fluid is time-independent, there is no slip at the wall and the flow is laminar.

- Being a definite integral, Eqn (18) shows that the relationship between 3V/H and τ_0 can be obtained by numerical integration using data directly from a viscometer without having to resort to using a conventional rheological model.

- Since 3V/H is a unique function of the rheogram and the wall shear stress, it can be used for scale-up and design of open channels at any required slope and depth.

- It provides the link or pathway between the rheogram and the pseudo shear diagram, i.e. given a rheogram, we can use Eqn (18) to construct a pseudo shear diagram (3V/H vs. τ_0 , which can be used for open channel design in laminar flow. Conversely, this can be done in reverse, i.e. differentiating Eqn (18) to provide the link or pathway between the pseudo shear diagram and the rheogram.

A similar overall result (Eqn (23)) was obtained for the viscometric analysis of slit flow [11], indicating that sheet flow can also be considered and analysed as the lower half of a slit flow.

6. CONCLUSIONS

An approach for the viscometric analysis of sheet flow data has been presented and validated with experimental data. An expression for the bulk shear rate, 3V/H, has been derived from first principles. It has been shown that, for Newtonian fluids, the bulk shear rate is equivalent to wall shear rate for sheet flow. A fundamental relationship (Eqn (18)) which directly relates shear rheology to the bulk shear rate has been developed for sheet flow. It has been argued that this relationship has profound significance for both viscometric analysis and practical design. Differentiation of this fundamental relationship (Eqn (18)) leads to a method of obtaining the wall shear rate from the measured bulk shear rate in sheet flow.

7. REFERENCES

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