ANALYSIS OF CURRENT METHODS OF FLEXURAL DESIGN

FOR

HIGH STRENGTH CONCRETE BEAMS

A thesis submitted in fulfillment of the requirements for the degree of Master of Engineering

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DECLARATION

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged.

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Date:

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ABSTRACT

Considerable amount of research was carried out into the properties and structural performance of high strength concrete for more than few decades. Whilst this research has produced relevant and useful results, there are several properties of high strength concrete like compressive and tensile strengths, stiffness, durability etc. that need to be evaluated and investigated to determine an accurate representation for the determination of different structural properties of beams made of high strength concrete.

For this purpose, an investigation into the behaviour of beams made of higher concrete strengths has been carried out and conclusions drawn for the design of high strength concrete beams in flexure. High strength concrete used in this study is defined as concrete with compressive strength exceeding 50MPa. Although there are many publications proposing stress block models for HSC beams, a universally accepted stress block model is yet to be developed. This study emphasizes the analysis of the models proposed in various design standards as well as those published in recent research literature.

Experimental data from previous research was considered for the study to establish some understanding of flexural behavior of HSC beams. A number of spreadsheets in Excel were developed using available data and various graphs were plotted to determine the accuracy of the code provisions for calculating the ultimate moment capacity of beams. Based on this, conclusions are drawn for the design of high strength concrete beams in flexure utilizing different code provisions.

A study on flexural ductility of beams has been carried out using a computer program FRMPHI which generates moment-curvature curves for the beams. Ductility has been studied using ductility factors. The influence of ductility on the value of the depth of neutral axis has been analysed and discussed.

A chapter on the short-term deflection of simply supported high strength concrete beams under instantaneous deflections is presented. This chapter includes analysis of the available formula to calculate deflection to determine if these can be adopted for high strength concrete. 24 methods developed for calculating each of the deem-to-comply and rigorous method forms of deflection have been analysed and some conclusions are drawn.

Extensive ongoing research on the shear strength of beams by several researchers since many years has lead to the generation of a large body of knowledge. Although each author has analyzed the data comparing them with existing relationships, the whole body of information has not been analysed to establish a statistical significance. In this study, regression analysis on experimental data collected from published research is carried out to establish a relationship between the different parameters affecting the shear strength of beams. The level of significance of the association between parameters influencing shear strength is also discussed.

NOTATION

A_{st} Cross-sectional area of steel in tension

A_{sc} Cross-sectional area of steel in compression

A_{sv} Area of stirrups

A Constant = $\beta_1 d_n$

a Stress block depth

b Width of section

b_{ef} Effective width of a compression face

C Compressive force

D Total Depth of section

d Effective depth of the concrete section

d_n Depth to the Neutral Axis

d_0 Depth to the outermost layer of tensile reinforcement

E_c Mean value of the modulus of elasticity of concrete at 28 days

$F_{d.ef}$ effective design service load per unit length or area

f_c' Compressive strength of concrete

f_{cf}' Characteristic flexural tensile strength

f_{cu} Cube compressive strength of concrete

$f'_{c,150}$ Characteristic compressive cylinder strength of concrete at 150 days

f_y Yield Strength of Steel

f_{sty} Yield strength of stirrups

I Second moment of area, of the gross concrete cross-section about the centroidal axis.

Icr Second moment of area of a cracked section with the reinforcement transformed to an equivalent area of concrete

I_{eff} Effective second moment of area

I_g Moment of inertia in uncracked state

k_u Neutral axis depth parameter

L_{ef} Effective depth of a cross-section, taken as L

M_{cr} The bending moment causing cracking of the section

M_e Experimental moment capacity

M_p Predicted moment capacity

M^*_s The bending moment causing cracking of the section

N The ratio between E_s and E_c

s Stirrup spacing

T Tensile steel force ($T=A_{st}$ x f_y)

V_{uc} Shear resisted by concrete

V_{us} Shear resisted by stirrups

V_e Experimental Shear strength

V_p Calculated shear strength

x_y Depth to the neutral axis from the extreme compression fibre

z Lever arm

a/d Shear span to effective depth ratio

α_1, β_1 Rectangular Stress Block Parameters ($\alpha_1 = k_1 k_3$, $\beta_1 = k_2$)

ρ Density of concrete, in kilograms per cubic meter, taken as 2400 kg/m^3

ρ_l Percentage of longitudinal steel

ρ_t Percentage of transverse steel

ρ_l / ρ_b Tension steel to balanced steel ratio

$\rho_v f_y$ Stirrup nominal strength

μ Ductility factor

ε_{sy} Yield strain for steel

ε_y Yield Strain

ε_{cu} Ultimate concrete strain

Φ_u Ultimate curvature

Φ_y Yield curvature ($\Phi_y = \varepsilon_y / d - x_y$)

Δ Deflection at mid-span

ρ Density of concrete, in kg/m^3, taken as 2400 kg/m^3

η Correction factor to determine the modulus of elasticity for high strength concrete.

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

A worldwide use of high strength concrete during the last decade and an expansion in the material technology has made it possible to design concrete having superior mechanical properties, material properties and structural behaviour. Many researchers have conducted investigations into behaviour of structural elements constructed of high strength concrete. However, analysis of these revealed that findings are inconsistent. Consequently, in this research project, attempt has been made to combine findings of all other researchers to develop some rational design methods. Whilst use of high strength concrete has accelerated, the progress in development of revised design standards have not progressed at the same rate. Acceleration is due to its enhanced mechanical properties and better structural performance as compared to Normal Strength Concrete. Also High Strength Concrete offers economy and superior performance in terms of strength and long-term behavior.

1.1.1 A Brief History of High Strength Concrete

High strength concrete (HSC) is a relative term and has changed its meaning with time and the increase in the productivity of concretes with higher strengths. In the present work, concrete with a 28-day characteristic compressive strength of 50MPa or higher is considered as high strength concrete (Ting & Patnaikuni, 1992). In recent years, the applications of high strength concrete have increased and high-strength concrete is now used in many parts of the world. This growth has been made possible as a result of developments in material technology and a demand for higher strength concretes (Russell, 1998).

High strength concrete has found its application in a number of concrete structures, the most common applications being the columns of high rise buildings, long span bridges, longer spans for beams or fewer beams for a given span length, offshore structures, etc. In North America, HSC has been used in a number of buildings and bridges including the Lake Point Tower, Water Tower Place, The Chase Tower, 311 South Wacker Drive, Washington State Bridges, Barker Lane Bridge and Confederation Bridge. Use of high strength concrete in Australia has been found on buildings such as The Rialto Tower, Melbourne Central, Bourke Place, 530 Collins Street, the Eureka Tower (Figure 1) and 120 Collins Street in Melbourne, The R & I Tower in Perth, and the Four Seasons Hotel in Sydney (Kovacic, 1995).

Figure 1.The Eureka Tower in Melbourne (Image courtesy of Angelo Marcina)

The Burj Dubai Tower in Dubai (Figure 2), under construction is another example of a building that is pushing the vertical limits of construction and architecture through its use of high-strength concrete. High strength concrete that was used in the construction was effectively delivered as high as 1900 feet. High strength concrete was used in different elements including caissons (62MPa), core walls and columns <126th floor and floors 154 & 155 (80MPa), and core walls and columns >126th floor (62MPa). Together with its superior engineering and structural properties, high strength concrete has been found to be economically attractive compared to normal strength concrete.

Figure 2. Dubai Tower under construction (Image courtesy of Bernadette Redfern)

1.1.2 Standards

The development of High Strength Concrete (HSC) has occurred over many years and is commercially available today. It is used mainly in the construction of tall buildings, large span of bridges, wider beam spacing, etc. But the design procedures used currently in construction by the practicing engineers for high strength concrete are same as that given for normal strength concrete and are beyond the scope of the Australian Standards. Important design equations found currently in the AS3600 are derived from tests of materials and members with compressive strengths less than 65MPa. The Australian Codes of Practice is still lacking in the design provisions for HSC structural members. The purpose of this research is to analyse the available design equations and present the applicability of these equations for their use in the design of beams made of higher concrete strengths. In the present program concrete having compressive strength of 50MPa or more is considered as high strength concrete. Analytical investigation on HSC beams is made under four different aspects namely flexure, ductility, deflection and shear. Test data is collected from past literature and the design provisions have been analysed for their use in high strength concrete and recommendations are made regarding the practicality of these equations for high strength concrete members.

1.1.3 Scope of Thesis

This thesis aims to provide authenticity of the existing code provisions in view of its applicability to high strength concrete. In order to achieve this, the important concepts need to be considered are flexure, ductility, deflection and shear.

There has been a rapid growth in the use of high strength concrete because of its enhanced material and structural properties and the ability to gain high early strength. But lack of a proper design procedure discourages structural designers to make full use of the material. There are currently no design guidelines in the Australian Standards for the design of concrete members with compressive strength in excess of 65MPa. Much experimental work on High strength concrete (HSC) has been carried out to date; however, findings are diverse and require careful analysis prior to proposing any change to the current provisions. Many suggestions have been made concerning the design rules, and there is now a need for finding the most appropriate and feasible design method for the flexural and shear strength capacities of beams.

This thesis includes theoretical analysis of reinforced High Strength Concrete beams for design of normal weight HSC members with compressive strength greater that 50MPa. It will allow the design of High strength concrete beams for flexure, deflection and shear and propose recommendations to be considered in the revised Australian Standards.

The aims of this thesis are to (a) review the existing literature and identify the gaps in knowledge. Conduct analytical study on flexure, ductility, deflection and shear of High Strength Concrete beams; examine the code equations and suggest recommendations for the higher strength concrete for use by engineers in practice to design concrete members with compressive strength beyond the scope of AS3600 where no guidelines are currently available,

(b) compare NSC with HSC to assess the differences and investigate the applicability of AS3600 code provisions to High Strength Concrete beams, and

(c) suggest the most feasible design method for use by the structural engineers. The general aim is to provide the accuracy of current design provisions to enable industry the use of High Strength concrete with confidence and therefore able to utilize its benefits and acknowledge its limitations.

1.1.4 Organization of Thesis

The work presented in this thesis deals with four main areas of research: flexure, ductility, deflection and shear of reinforced concrete beams. The behavior of high strength concrete beams with different concrete compressive strengths has been investigated. The thesis includes a discussion about the suitability of current design rules for such HSC beams. The validity and applicability of the existing code provisions for HSC with strength more than 50MPa has been examined. The influence of various parameters on the flexural strength and the shear strength of HSC beams has been investigated.

An introduction to the thesis has been given in Chapter 1. A detailed review of previous investigations on the four main aspects of research - flexure, ductility, deflection and shear of HSC beams has been given in Chapter 2.

Chapter 3 deals with the flexural capacity of HSC beams in comparison with the behavior of normal strength concrete (NSC) beams. Various code provisions currently available and those from different publications have also been discussed in this chapter.

Spreadsheets have been developed for each of the different stress block parameters to compare the theoretical moment capacity with the experimental moment capacity of the beams as obtained from past experimental research. The work summarized in the chapter depicts the accuracy and authenticity of the code provisions currently available when used with HSC beams in flexure. It also suggests recommendations to the code provisions for the use with high strength concrete beams.

An investigation on the ductility of HSC beams in flexure is described in the following Chapter 4. This includes development of moment-curvature curves using a computer program developed by Setunge et al (1994) for different beams and the comparison of ultimate moment with the theoretical moment obtained from the flexural analysis using rectangular stress block parameters. The moment curvature curves have been developed in order to investigate the full range moment curvature behaviour of the reinforced concrete section.

Chapter 5 gives the analysis of simply supported high strength concrete beam deflections under short-term loading. Theoretical analysis of the available formula from the Australian Standards AS3600 to calculate deflection has been carried out to verify if these can be accepted for high strength concrete beams.

Investigation of different parameters on shear strength of HSC beams with and without stirrups has been presented in Chapter 6. A review of previous investigations on shear of HSC beams both with and without transverse reinforcement has been reported. An association existing between the shear strength and the parameters influencing the same has been developed using regression analysis. Regression equations for shear strength of HSC beams with stirrups and without stirrups have been developed and conclusions are drawn.

Chapter 7 contains a summary of the thesis and recommendations for future work. A list of references cited in the thesis is also presented. Appendix A consists of the spreadsheets developed using Excel for the investigation of beams in flexure. Appendix B consists of the moment-curvature curves developed for different beams using the computer program FRMPHI. Appendix C consists of the spreadsheets developed in Excel for the Analysis of beams in deflection. Appendix D consists of the spreadsheets developed for HSC beams in shear. Appendix E gives the graphical representation obtained using Minitab for beams with and without web reinforcement.

CHAPTER 2

LITERATURE REVIEW

2.1 FLEXURE

2.1.1 Stress block parameters

Rectangular stress block is generally used to calculate the ultimate moment capacity of reinforced concrete beams. The stress-strain curves for high strength concrete are more linear than parabolic and hence it was reasonable to infer that the rectangular stress block parameters could be different. The rectangular stress block model was first introduced by Hognestad et al (1955) from experimental work involving normal strength concrete. The rectangular stress block parameters covered in the Australian Standards AS3600 are limited to concrete strengths of not more than 65MPa. Whilst many researchers produced modifications to this, none have been included in AS3600.

Different stress block parameters obtained for use with high strength concrete by researchers in the past have been described in Chapter 3. Experimental and theoretical studies on the development of stress block parameters for the design of high strength concrete have been given in different publications.

In the research by Ibrahim and MacGregor (1997), the design procedures in the American concrete code ACI 318-89 for high strength concrete and ultra high strength concrete sections were reviewed and new equations for the stress block parameters were proposed. It was established that the ACI rectangular stress block was not conservative for high strength concrete.

Attard and Stewart (1998) examined the applicability of ACI 318-95 rectangular stress block parameters to high-strength concretes. They have shown that for a ductile singly-reinforced rectangular section, the ultimate moment capacity is relatively insensitive to the stress block model. The researchers also investigated the applicability of the building code ACI318-95 provisions to high strength concrete.

In the research conducted by Marinucci and Patnaikuni (2000), design stress block models were proposed for concrete strengths ranging from 50-100MPa.

From the experimental study by Oztekin et al (2003), it was observed that the rectangular stress block parameters used in ordinary concrete members cannot be used safely for high strength concrete members. Stress block parameters were obtained from modified Hognestad model for ordinary concrete.

Other research studies on the stress block parameters include those by Ozbakkaloglu and Saatcioglu (2004), Bae and Bayrak (2003), Mendis and Pendyala (1997), Wee et al (1996), Azizinamini et al (1994), Swartz et al (1985) and Kaar et al. (1978).

2.1.2 Ultimate moment Capacity of HSC Beams

Based on the review of previous work, it is clear that a systematic investigation on the design recommendations of various codes for determination of strength capacity of HSC beams in flexure is essential. For this purpose, previous experimental studies have been reviewed. It was determined that a large number of investigations have been carried out in the past on the flexural behavior of high-strength concrete beams, some of which include those by Rashid & Mansur (2005), Bernardo & Lopes (2004), by S. A. Ashour (2000), Mansur et al (1997), Sarkar et al (1997), Pendyala et al (1996), Shehata & Shehata (1996) Lin et al (1992), Lambotte & Taerwe (1990), Shin et al (1989), Uzumeri & Basset (1987), Swamy (1985), Pastor et al (1984), Okada & Azimi (1981), Tognon et al (1980), Leslie et al (1976).

Leslie et al. (1976) reported flexural tests on 12 under-reinforced rectangular beams with concrete compressive strengths ranging between 64 to 81MPa. They concluded that the rectangular stress block from the ACI 318-71 did not accurately predict the beam behavior when concrete strengths exceed 55 MPa.

A study of flexural behavior of a number of singly reinforced concrete beams made of normal and high strength concrete beams was carried out by Pam et al. (2001). The test results for flexural strength agreed closely with the theoretical predictions using the stress block, verifying the applicability of British Standard BS 8110 (1997), after modification as per Concrete Society recommendation to high strength concrete beams.

Ashour (2000) has shown that the flexural rigidity increases as concrete compressive strength increases. In his study, experimental tests were carried out on nine reinforced HSC beams to investigate effect of concrete compressive strength and flexural tensile reinforcement ratio on load-deflection behavior and displacement ductility of reinforced concrete beams.

Investigation of the flexural behavior of high strength reinforced concrete beams and an assessment of the BS 8110 (1985) code provisions was undertaken by Sarkar et al. (1997) to verify the BS design recommendation for HSC beams under flexure.

Rashid and Mansur (2005) have shown that the equivalent rectangular stress block specified in the ACI 318-02 Code and the failure criterion ε_{cu} =0.003 have been found to give reasonable conservative predictions for the ultimate moment capacity of HSC beams with concrete strengths as high as 126MPa.

An experimental study on the evolution of depth of neutral axis at failure with the ductility at bending on HSC beams was carried out by Bernardo & Lopes (2004). It was found that the theoretical formulations based on the use of the rectangular block diagram for the concrete to compute the depth of neutral axis at failure gave substantially smaller values as compared to the experimental values. As such, it was concluded that the rectangular stress block diagram proposed by ACI 318-1989 was not adequate for HSC beams.

2.2 Ductility

While designing a member in flexure for adequate strength, it is also necessary to ensure that the member exhibits adequate ductility under overload conditions. For this reason, a number of experimental and theoretical studies on flexural ductility of high strength concrete beams have been carried out previously by various researchers.

The first studies were carried out in 1976 by Leslie et al. on HSC beams subjected to pure bending in the failure zone. In their study, the influence of various parameters including compressive strength of concrete, the longitudinal tensile reinforcement ratio was investigated. Subsequent studies confirming the influence of different parameters including the compressive strength of concrete, the longitudinal tensile and compression reinforcement ratio, and the transverse reinforcement ratio were carried out. Some of the tests include those by Tognon et al. (1980), Pastor et al (1984), Shin (1986), Naaman et al (1986), Shin et al (1989), Lambotte and Taerwe (1990), Shin et al (1990), Ahmad and Barker (1991), Shehata and Shehata (1996) and Bernardo and Lopes (2003).

Theoretical studies on flexural ductility of high strength concrete members include those by Mansur et al.(1997), Mendis et al (2000), Konstantinidis & Kappos (2003), Pam et al (2001), Ho et al (2003) and Kwan et al (2002).

The results from the previous experimental research studying the influence of different parameters generally agree with one another with respect to the influence of these parameters on ductility, it is however still not absolutely certain how concrete strength affects ductility (Bernardo, Lopes; 2003).

2.3 Deflection of HSC beams under short-term loading

The serviceability or deflection of a flexural member is a significant aspect considered in the design of concrete structures. A number of investigations have produced different approaches for calculating deflections of reinforced concrete beams.

In their study on deflections, Alcock and Pauw (1962), presented a procedure for the design of simply supported beams and slabs subjected to uniformly distributed loads. In their study, the researchers describe a design method for reinforced concrete beams and slabs in which the allowable ratio of span to deflection was the criterion.

Valeriu (1964) gave a method for determining deflections in beams of variable stiffness. Branson (1968) presented practical design procedures for deflections of ordinary reinforced and prestressed concrete flexural members.

An experimental research by Lambotte and Taerwe (1990), on both NSC (f'_c= 36MPa) and HSC (f'_c = 83 MPa) beams, showed that the deflections decreased by using high strength concrete due to the increased modulus of elasticity and cracking moment. That is, the deflections of high strength concrete members are smaller than those of normal strength concrete specimens with the same reinforcement ratio.

A review of the design provisions for deflection of reinforced concrete members contained in the ACI Building Code 318-89 and the Canadian Standards CAN3-A23.3-M84 was done by Ghali (1993). His study showed that the approach of the codes yielded accurate prediction of the immediate deflection in some cases; however, it was found that the predicted deflections can be largely in error in beams when the reinforcement ratio is low.

In the research by Ashour et al, 1997, it was observed that the methods of predicting and controlling deflection in the ACI Building Code 318-95 were applicable to normal-strength concrete members with concrete compressive strength in the range of 21 to 42MPa. However, the accuracy of these methods for use with high-strength concrete flexural members has not yet been fully established (ACI Committee 363-IR-87, Ashour et al, 1997).

Gilbert (1999) found that the simplified procedures contained in ACI 318-95 and AS3600 for calculating the deflection of reinforced concrete beams and slabs are inadequate.

2.4 SHEAR CAPACITY OF HSC BEAMS

A reinforced concrete beam under loading is generally subjected to both bending moment and shear force. This aspect of shear has been considered for research since few decades, but there is still no universally acceptable design method for finding the shear strength of beams made of high strength concrete. As the shear design procedures in the various codes and recommendations are based mainly on experimental studies conducted on structural elements made with relatively low compressive strength concrete, there is concern that these procedures may not be adequately safe for reinforced high strength concrete structures. Therefore there is an urgent need to provide additional information on the shear capacity of high strength concrete structural elements, both without and with transverse reinforcement (Galeota and Giammatteo, 1997).

In order to improve the shear capacity of reinforced concrete beams in shear, it is necessary to evaluate the influence of different design parameters on the structural behavior of concrete. The primary variables are the concrete compressive strength, the amount of longitudinal reinforcement, shear reinforcement and shear span-to-effective depth ratio (a/d). The secondary variables include the beam dimensions, concrete cover-to-shear reinforcement cage and overall beam depth, type of load (point load or uniform load), and support conditions (pinned or fixed).

2.4.1 Concrete compressive strength

The definition of concrete strength has been changing with time as higher strengths have been produced and increasingly used. In this thesis, normal strength concrete is defined as concrete having compressive strength less than 50MPa, and high strength concrete having compressive strength equal to or more than 50MPa.

Research has shown that reinforced high strength concrete beams, subjected to shear, tend to fail in more brittle manner, with cracks forming through the aggregates rather than around them (Attard and Mendis, 1993). As a result the surfaces of diagonal tension fracture are relatively smooth and less effective in aggregate interlock in contrast with the rough crack surfaces in lower strength concretes. (Galeota and Giammatteo, 1997).

Johnson and Ramirez (1989) have shown that, as the compressive strength of the concrete increases, the diagonal tension cracking load also increases. The smooth crack surfaces in higher strength concretes result in a decrease in shear carried by the aggregate interlock and hence the shear carried by the concrete.

Roller and Russell (1990) have shown that the ACI318-83 code provisions overestimate the nominal shear strength provided by the concrete when the concrete compressive strength is more than 117MPa. The researchers also proposed a revision to ACI 318-83 requiring an increase in the minimum amount of web reinforcement for concretes with compressive strengths of more than 69MPa.

The effect of parameters –compressive strength of concrete, the shear span-to-depth ratio, and the vertical shear reinforcement ratio on the shear strength of reinforced HSC beams without stirrups was studied by Shin et al. (1999). It was found that ACI 318-95 code provisions underestimated the shear strength more markedly for higher strength concrete beams than for moderately high strength concrete beams.

For beams without web reinforcement, the failure shear strength was found to increase with the increase in the concrete compressive strength (Cladera & Mari, 2005).

2.4.2 Shear reinforcement

It is generally noted that reinforced concrete beams should contain adequate shear reinforcement to prevent sudden and brittle failure and to provide adequate crack control at service conditions. The minimum shear reinforcement is also required to provide somewhat ductile behavior prior to failure (Ozcebe et al, 1999).

HSC cracks at higher shear stresses when compared to the conventional concrete and consequently requires larger amounts of minimum transverse reinforcement (Rahal & Al-Shaleh, 2004).

Cladera & Mari (2005) concluded that HSC beams with stirrups presented a less fragile response than similar beams without web reinforcement.

Researchers in the past showed that as the amount of stirrups increased, the beams became more ductile and failures were less sudden. Beams without any stirrups failed suddenly (Mphonde and Frantz, 1985; Elzanaty et al 1986).

Johnson and Ramirez (1989) proposed that more shear reinforcement be provided with increasing concrete strengths to avoid sudden brittle failure.

In the research for evaluation of minimum shear reinforcement requirements for higher strength concrete, Ozcebe et al (1999) found that the ACI 318-83 code specifications for minimum shear reinforcement are inadequate when high strength concrete is used.

Results from the investigation by Roller and Russell (1990) indicate that for non-prestressed high-strength concrete beams subject to shear and flexure only, the minimum amount of shear reinforcement specified in ACI 318-83 should increase with the increase in concrete compressive strength.

Kong and Rangan (1998) reported in their research, that the shear strength of beams increased with an increase in the shear reinforcement ratio. The test shear strengths were compared with the predicted shear strength by different code provisions and a large coefficient of variation was found due to a considerable scatter in measured shear strength of high performance concrete test beams.

Elzanaty et al (1986) conducted experiments on eighteen beams with compressive strengths ranging from approximately 21 to 83MPa. From their observation, it was noted that the shear strength of the beams increased with increase in compressive strength of concrete.

2.4.3 Shear span-to-effective depth ratio

Elzanaty et al (1986) found ACI Equation (ACI 318R-83) unconservative for several high strength concrete beams without stirrups with high f'_c and a/d values, but low longitudinal reinforcement ratio.

The shear strength of test beam was found to decrease with the increase of *a/d*. It was noted that the ACI code provisions did not account for the effect of shear span-to-depth into consideration.

The shear span-to-depth ratio did not have a significant effect on the shear strength when $a/d \geq 2.50$ (Kong and Rangan, 1998). However, when $a/d < 2.5$, the shear strength was found to increase because of arch action.

It was observed that for estimating the shear contribution of concrete, the ACI 318-83 code provisions gave conservative values for beams with low shear span ratios (a/d less than 2.5). For high *a/d* ratios and relatively low longitudinal steel ratios, the ACI equation may be unconservative for HSC beams (Ahmad et al, 1986).

2.4.4 Longitudinal tensile reinforcement ratio (ρ_l)

Similar to transverse reinforcement, larger amounts of longitudinal steel increase the post cracking reserve strength and provide better control of the propagation of cracks. They also act as the tension member in the truss analogy, used to quantify shear capacity of concrete beams. Hence the available longitudinal steel can influence upon the minimum transverse reinforcement taken into consideration (Rahal & Al-Shaleh, 2004).

Research has shown that the ACI 318-99 shear design provisions provide unconservative results for large beams and beams with low levels of longitudinal reinforcement (Tompos & Frosch, 2002).

As the longitudinal reinforcement ratio ρ_l is decreased, a reduction in the concrete shear strength was observed. From experimental data, it was found that the design provisions in ACI 318-99 are unconservative for beams with reinforcement ratios less than 1%. Moreover, this reduction in shear strength was found to occur in beams of all sizes (Tompos and Frosch, 2002).

Kong and Rangan (1998) showed that the shear strength increased with an increase in the longitudinal tensile reinforcement ratio. It was noted that bundling of bars in the analysis may have contributed to the increased shear resistance.

Effect of longitudinal steel was one of the parameters included in the study of shear capacity of reinforced concrete beams using HSC by Elzanaty et al (1986). It was observed that for all concrete strengths in the range of 21 to 82 MPa, increasing the longitudinal steel ratio increased the shear strength of beams without stirrups.

Ahmad et al (1986) conducted experimental work on high strength concrete beams without stirrups and reported that the ACI 318-83 code provisions for shear strength may be found unconservative for high strength concrete beams with a low percentage of longitudinal steel.

The performance of different levels of transverse steel was found to depend considerably on the amount of longitudinal reinforcement in the beam. The behavior of beams with a higher amount of longitudinal reinforcement was more favourable (Rahal & Al-Shaleh, 2004).

2.4.5 Depth of the beam

For large beams and beams with low longitudinal reinforcement ratios, the shear strength attributed to the concrete has been found to decrease as the depth of the beam increases (Bazant and Kazemi, 1991; Frosch, 2000; Collins and Kuchma, 1999).

In an experimental study by Tompos and Frosch (2002), as the depth of the beams tested was almost doubled, there was a corresponding 18% decrease in the shear strength of concrete. Size of the beam was found to affect the shear strength of the beam.

2.5 Conclusions from review of literature

a) Flexure: Whilst there has been numerous work reported on flexural strength of HSC beams, findings are quite diverse. There have been many publications where experiments are reported. A systematic analysis of the published work is needed to decipher the implication of this body of research.

b) Ductility: With the same ratio of reinforcement, increase in compressive strength of concrete increases ductility of beam. However, to obtain the benefit of increased strength of concrete, some increase in reinforcement ratio is needed. No published work explored this effect.

c) Deflection: There are many different ways of calculating deflection of reinforced concrete beams. Methods range from approximate methods to detailed ones covering cracked second moment of area. No published work has conducted an investigation into the effect of the method of calculation of deflection on predicted deflection on HSC beams.

d) Shear: There have been many published work on shear capacity of HSC beams studying influence of specific parameters on shear strength. However, no published work covers analysis of all the influencing parameters to predict shear capacity.

CHAPTER 3

HIGH STRENGTH CONCRETE BEAMS IN FLEXURE

3.1 INTRODUCTION

The availability and advancement of material technology and the acceptance has led to the production of higher grades of concrete. High strength concrete offers superior engineering properties i.e. compressive strength, tensile strength, durability, modulus of elasticity and overall better performance when compared to the conventional concrete. Due to its enhanced strength and improved structural properties, high strength concrete has been increasingly used for the past two decades. Research on the behavior of HSC beams with concrete strength higher than 50MPa has been carried out in the past and is still continuing, to understand the behavior of HSC beams in flexure. Whilst there are many publications proposing stress block models for HSC beams, a universally accepted stress block model is yet to be developed. In most design standards, the conventional rectangular stress block developed for NSC is still being used for design of HSC beams. In this chapter, published work has been analysed to establish some understanding of flexural behavior of HSC beams. Models proposed in various design standards as well as those published in recent research literature have been analysed to compare the experimental and theoretical moment capacities.

3.2 SIGNIFICANCE

Although different stress block parameters for high strength concrete have been proposed by various codes and other researchers, a rational rectangular stress block model is still unavailable for the design of high strength concrete beams. It is vital to establish if the current design methods for the ultimate strength of beams are applicable to the high strength concrete. There are several methods to calculate the ultimate strength of concrete beams each providing different levels of accuracy and applicability. A complete investigation has been made to study the accuracy of the stress block parameters proposed by different concrete codes including AS3600-2001, ACI318-05, CAN3-A23.3-M94, NZS3101-1995, etc and those proposed by Attard & Stewart (1998), Mendis & Pendyala (1997), Ibrahim & Macgregor (1997), and Bae & Bayrak (2003). A correlation between the ultimate strength of beams tested by Pam et al (2001), Sarkar et al (1997), Bernardo & Lopes (2004), and S. A. Ashour (2000) versus the predicted capacity shows that the expressions given by the various codes and other authors have an accuracy of $\pm 20\%$ which seemed ambiguous. The methods used to determine the ultimate strength of the beams have been found to be unconservative. On the other hand, the stress block parameters do not seem to have a significant effect on the ultimate strength of the concrete beams.

This chapter gives an understanding of the behaviour of beams in flexure when different stress block parameters are used for beams made of higher concrete strengths. The results from the analysis will add to the body of knowledge currently available and is also significant to building code writers since the current stress block parameters were developed for normal strength concrete.

3.3 CONCRETE COMPRESSIVE STRESS BLOCK

For simplicity, a rectangular stress block is preferred for calculation of the ultimate moment capacity of reinforced concrete members. This ultimate strength is assumed to occur at a particular value of extreme fiber concrete strain, ε_{cu}. The stress block model was introduced by Hognestad et al. (1955) from experimental investigations making use of normal strength concrete. The rectangular stress block is defined by two parameters. $\alpha_1 -$ The intensity of stress in the stress block and $\beta_1 -$ the ratio of the depth of the stress block to the depth of the neutral axis. The rectangular stress block is found to be useful only for under-reinforced beams when the neutral axis lies within the cross-section.

The typical stress-strain curve for high strength concrete is more linear than parabolic and the ultimate strain is lower for high strength concrete. Considering the differences in the stress-strain curves and other characteristics of high strength concrete, a modification of the rectangular stress block parameters is necessary. Table 1 summarizes various recommendations for the stress block parameters and ε_{cu} from various design codes.

3.3.1 AS3600 -2001 code provisions (2001)

In AS3600-2001 the current code provisions for the stress block parameters are derived based on the stress-strain curves for concrete with strengths up to 50 MPa. The value of the ultimate concrete strain is taken as 0.003. Figure 3 shows the rectangular stress block parameters of AS3600-01. The intensity of the equivalent stress block is given by $\alpha_1 f_c'$. The depth of the stress block is $\beta_1 d_n$ where d_n is the depth of the neutral axis. According to AS3600-2001, α_1 is assumed to have a constant value of 0.85. The parameter β_1 is particularly limited to concrete strengths of less than or equal to 50MPa and is given by $\beta_1 = 0.85 - 0.007(f_c' - 28)$ within the limits $0.65 \le \gamma \le 0.85$ .The upper limit applies when f_c' is less than 25MPa and the lower limit is relevant when f_c' exceeds 64MPa.

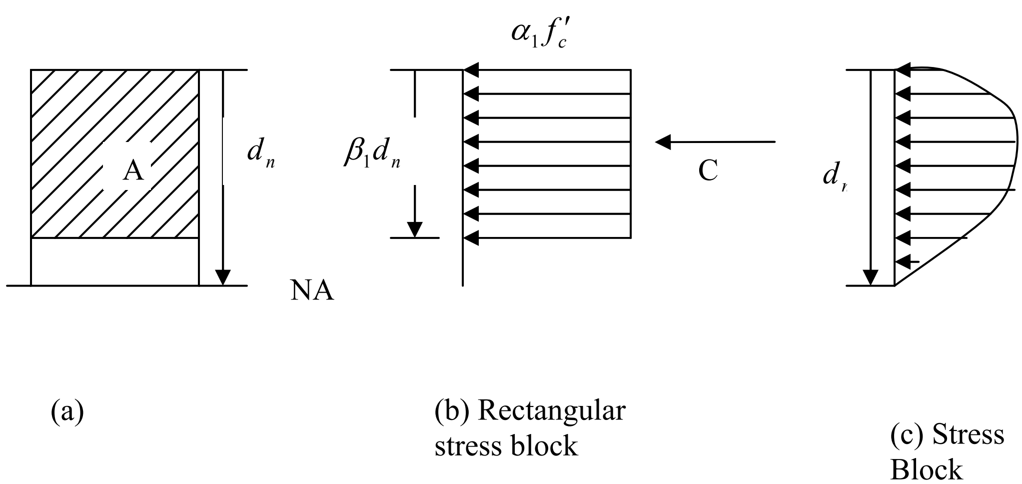


Figure 3. AS 3600 Rectangular Stress Block

3.3.2 ACI318-2005 code provisions (2005)

The ACI Code provisions are based exclusively on the results of experimental studies using concrete in the range of 20-30MPa for reinforced concrete. In the ACI code the stress block width factor α_1 is constant equal to 0.85 with the variable parameter being the stress block depth β_1 which varies with concrete strength. The parameter β_1 is equal to 0.85 for concrete strengths up to 30MPa and reduces linearly at a rate of 0.08 for each 10MPa of concrete strength in excess of 30MPa to the limit that β_1 should not be less than 0.65. A flexural strain limit of 0.003 was assumed for high strength concrete.

3.3.3 CAN 3-A23.3-M94 provisions (1994)

The ultimate concrete compressive strain suggested by the Canadian standards is 0.0035. The stress block parameters are given by the equations as follows:

$$\alpha_1 = 0.85 - 0.0015 f_c' \geq 0.67$$

$$\beta_1 = 0.97 - 0.0025 f_c' \geq 0.67$$

3.3.4 CEB-FIP 1990 (1990)

Unlike the other codes, this model code gives two stress blocks for design. The first stress block consists of a second degree parabola with apex at a stress of $0.85 f_c'$ with a corresponding strain of 0.002. This is followed by a yield plateau with a constant stress of $0.85 f_c'$ for concrete strains between 0.002 and 0.0035. The second is a rectangular stress block with the following parameters

$$\alpha_1 = 0.85\left(1 - \frac{f_c'}{250}\right)$$

$$\beta_1 = 1$$

$$\varepsilon_{cu} = 0.004 - 0.002\frac{f'_c}{100}$$

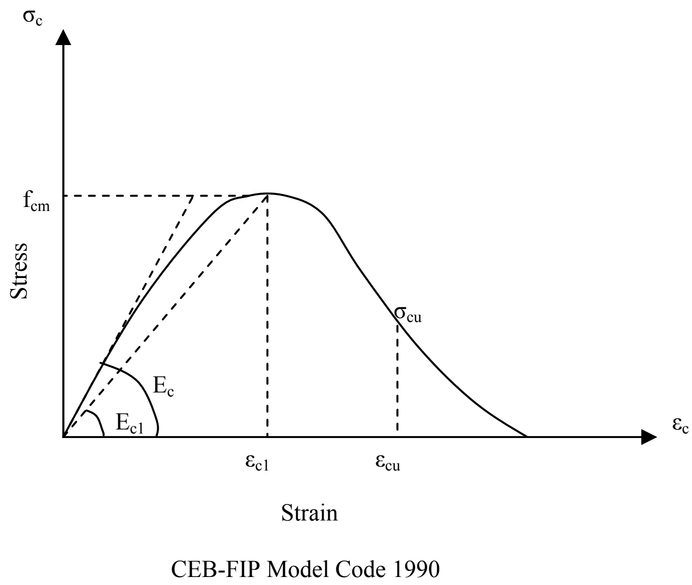


Figure 4. Stress-strain diagram for concrete in compression

3.3.5 NZS3101-1995 provisions (1995)

For practical design, the code allows the use of a rectangular compressive stress block to replace the most exact concrete stress distributions. In the equivalent rectangular stress block, a stress of $\alpha_1 f'_c$ is used with a rectangle of depth $\beta_1 c$, where c is the depth of the neutral axis. The parameter α_1 is equal to 0.85 for concrete strengths up to 55 MPa and reduces linearly at a rate of 0.04 for each 10MPa of concrete strength in excess of 55MPa to the limit that α_1 should not be less than 0.75. The parameter β_1 is equal to 0.85 for concrete strengths up to 30MPa and reduces linearly at a rate of 0.08 for each 10MPa of concrete strength in excess of 30MPa to the limit that β_1 should not be less than 0.65. A flexural strain limit of 0.003 was assumed for high strength concrete. The stress block depth parameter in the ACI318-05 code and the NZS3101 are identical. The New Zealand Code sets the maximum strain limit at 0.003.

3.3.6 Eurocode-2 provisions (1999)

The code suggests a strain limit of 0.0035. In the Eurocode, the stress block width factor is a constant equal to 0.85 for all concrete strengths. The variable parameter is the stress block depth. The parameter β_1 is equal to 0.85 for concrete strengths up to 25MPa and reduces linearly at a rate of 0.02 for each 10MPa of concrete strength in excess of 25MPa.

Rectangular stress block parameters obtained from various codes are given in Table 1 and those from different publications are summarized in Table 2.

Reference	α_1 (=k_1k_3)	β_1 (=k_2)	ε_{cu}
AS3600-2001	0.85	0.85-0.007($f_c' - 28$)	0.003
ACI318-2005	0.85	1.09-0.008 f_c' (0.85 $\geq \beta_1 \geq 0.65$)	0.003
CAN3-A23.3-M94	0.85-0.0015 $f_c' \geq 0.67$	0.97-0.0025 $f_c' \geq 0.67$	0.0035
CEB-FIP 1990	$0.85(1 - f_c'/250)$	1	$0.004 - 0.002 f_c'/100$
NZS3101-1995	$1.07 - 0.004 f_c'$ $0.75 \leq \alpha_1 \leq 0.85$	$1.09 - 0.008 f_c'$ $0.65 \leq \beta_1 \leq 0.85$	0.003
Eurocode-2-1999	0.85	$0.9 - f_c'/500$	0.0035

TABLE 1. Rectangular Stress Block Parameters in Different Design Codes

Reference	α_1 (=k_1k_3)	β_1 (=k_2)	ε_{cu}
Azizinamini et al. (1994)	$0.85-0.00725(f_c'-69)$ $0.60 \le \alpha_1 \le 0.85$	$1.09-0.008f_c'$ $0.65 \le \beta_1 \le 0.85$	0.003
Ibrahim & MacGregor (1997)	$0.85-f_c'/800 \ge 0.725$	$0.95-f_c'/400 \ge 0.70$	0.003
Mendis & Pendyala (1997)	$0.85-0.0025(f_c'-57)$ $57 < f_c' \le 100MPa$	$0.65-0.00125(f_c'-57)$ $57 < f_c' \le 100MPa$	0.003
Attard & Stewart (1998)	$1.29(f_c')^{-0.1} \ge 0.71(DB)$ $0.65(f_c')^{0.032} \ge 0.675(SL)$	$1.095(f_c')^{-0.091} \ge 0.67$	0.003
Bae & Bayrak (2003)	$0.85-0.004(f_c'-70)$ $0.67 \le \alpha_1 \le 0.85$	$0.85-0.004(f'c-30)$ $0.67 \le \beta_1 \le 0.85$	0.0035 for f_c'>55MPa otherwise 0.003
Ozbakkaloglu & Saatcioglu (2003)	$0.85, \; for \; f_c' \le 30MPa$ $0.85-0.0014(f_c'-30) \ge 0.72$	$0.85, \; for \; f_c' \le 30MPa$ $0.85-0.0020(f_c'-30) \ge 0.67$	0.003

TABLE 2. Proposed Rectangular Stress Block Parameters in Different Publications

3.4 EXPERIMENTAL DATA

The experimental data required for the analysis has been sourced from previous research in which HSC beams were tested in flexure. Beams tested for flexure by researchers in the past have been considered with a view to compare the ultimate strength of beams in bending to the capacity predicted by different codes and other available expressions. For a comparison to be made between the actual moment capacities and theoretical moment capacities, the theoretical moment capacities had to be based on the same parameters as the actual beams tested. The details and moment capacities have been given in the following sub-sections.

3.4.1 Pam et al (2001)

A number of singly-reinforced beams made of normal and high strength concrete were tested under monotonically increasing loads. Since the rectangular stress block theory is applicable only for under-reinforced sections, only those beams tested by Pam et al (2001), that were under-reinforced were considered for the purpose of analysis. All of the beams were simply supported and were subjected to monotonically applied point loads near mid-span as shown in Figure 5. The flexural ductility of the beams was also investigated. The cross-section of the reinforced concrete beams is shown in Figure 5. Analysis of test results led to the following conclusions.

- Flexural strength using the rectangular stress block as in British Standard BS 8110 agreed closely with the experimental value.

- Major structural parameters determining the flexural ductility of singly reinforced concrete beams are the tension steel ratio and generally, the tension steel to balanced steel ratio ρ_l/ρ_b and f_c'. A formula was developed for estimating the flexural ductility of normal- and high-strength concrete beams.
- It was proposed to set a maximum limit to the tension steel to balanced steel ratio.
- For HSC with f_{cu} > 80MPa the beam ductility needs to be improved for its adequate performance.

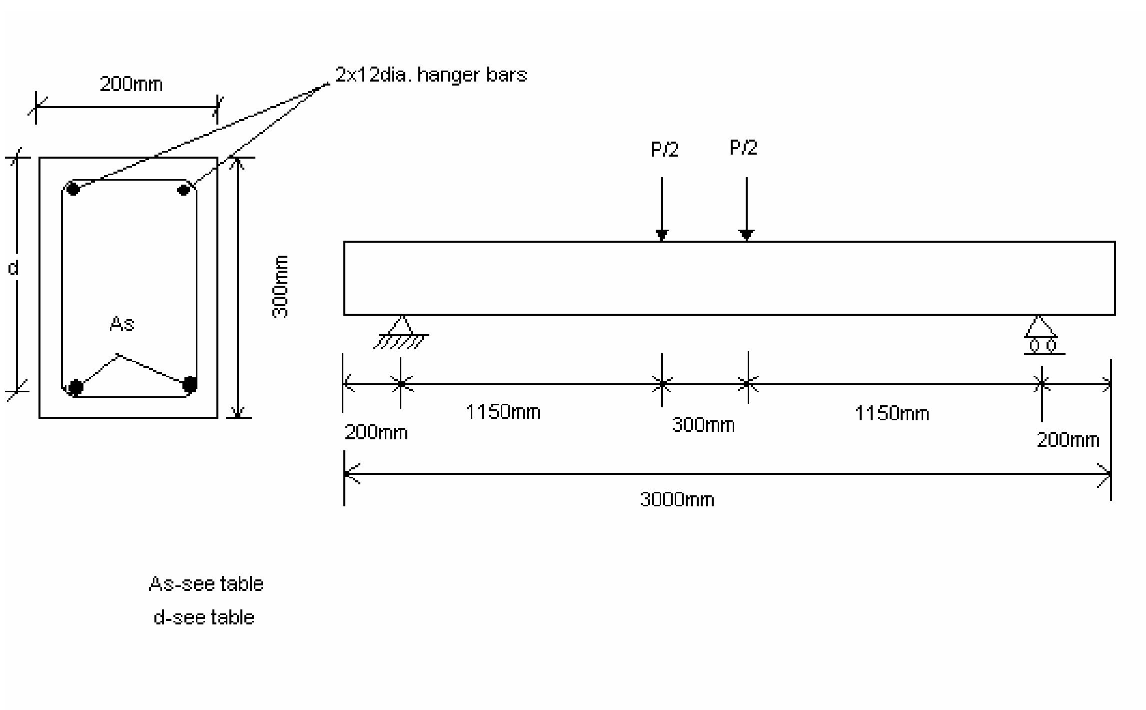


Figure 5 Beam cross-section and loading arrangement (Pam et al, 2001)

Beam No.	*f'$_c$ (MPa)*	*d(mm)*	*Main reinforcement*			ρ_l, %	ρ_b; %	ρ_l/ρ_b
			Layout	*A_{st}(mm^2)*	*f_y (MPa)*			
1	37.4	264	2T16	402	579	0.76	2.04	0.37
2	36.8	264	3T16	603	579	1.14	2.01	0.57
3	36.4	260	2T25	982	578	1.89	1.99	0.95
4	42.3	260	2T25	982	536	1.89	2.55	0.74
5	46.4	260	2T25+1T16	1183	546	2.28	2.70	0.84
7	58.6	260	2T25+1T20	1296	520	2.49	3.57	0.70
8	57.1	260	3T25	1473	520	2.86	3.50	0.82
9	58.6	256	2T32+1T16	1809	520	3.53	3.57	0.99
14	95.5	260	2T25	982	578	1.89	3.91	0.48
15	98.0	260	3T25	1473	578	2.84	3.93	0.72
16	102.5	260	3T25	1473	578	2.84	3.94	0.72
17	87.0	256	2T32	1608	546	3.14	4.15	0.76

TABLE 3. Details of beam specimens (Pam et al, 2001)

(T denotes high-yield steel bar. The number before T is the number of bars and the number after T is the diameter of the bar in mm) (Pam et al; 2001)

3.4.2 Sarkar et al (1997)

Flexural tests were carried out on singly reinforced beams with concrete strength in the range of 80MPa to 120MPa. The variables were concrete strengths, and tensile steel ratios. The choice of the size and the length of test specimens was made in order to make the beam fail in flexure $(a/d = 6)$ and to ensure that the specimen is sufficiently large to simulate a real structural element. Flexural tests were carried out on singly reinforced beams with the variables being the concrete strengths and tensile reinforcement ratios ranging from 1.03% to 4.04%. The code provisions for ultimate strength available in the

British Standards BS 8110 were also examined for their use in high strength concrete beams.

The following conclusions in relation to the flexural behavior of high strength reinforced concrete beams were drawn from this investigation.

- The steel reinforcement ratio ρ and the compressive strength of concrete f_c' , both contribute to the load-deflection characteristics of beams. Higher ρ_l and f_c' cause less deflection.
- The moment-curvature relationship showed a stiff response for HSC beams for the same value of longitudinal reinforcement.
- The value of strain 0.0035 adopted in BS8110 seemed high for its use in compressive strengths over 100MPa. Strain value of 0.003 was recommended.
- The ductility index (μ) of HSRC beams increased with increasing f_c'.
- A stress block is proposed to estimate the ultimate moment capacity of HSRC beams.

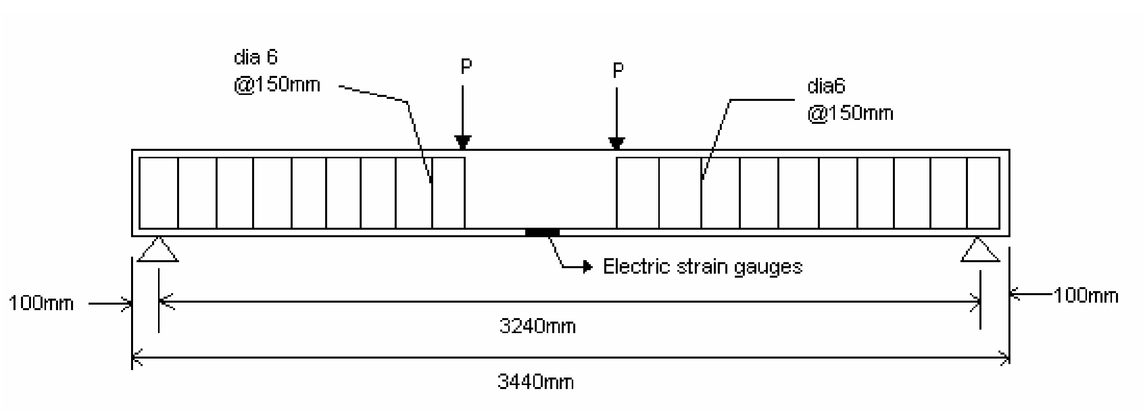


Figure 6 Testing arrangement of beams (Sarkar et al, 1997)

Beam no.	*f'_c (MPa)*	*b (mm)*	*D (mm)*	*d (mm)*	*A_{st} (mm^2)*	*ρ_l (%)*
HSC1-1	107	150	250	220	339	1.03
HSC1-2	97	150	250	220	339	1.03
HSC1-3	85	150	250	220	339	1.03
HSC2-1	105	150	250	212.5	452	1.42
HSC2-2	100	150	250	212.5	452	1.42
HSC2-3	77	150	250	212.5	452	1.42
HSC2-4	90	150	250	212.5	452	1.42
HSC3-1	107	150	250	215	628	1.94
HSC3-2	85	150	250	215	628	1.94
HSC3-3	78	150	250	215	628	1.94
HSC4-1	101	150	250	207.5	1257	4.04
HSC4-2	87	150	250	207.5	1257	4.04
HSC4-3	82	150	250	207.5	1257	4.04

TABLE 4. Details of test beams (Sarkar et al, 1997)

3.4.3 Bernardo & Lopes (2004)

Nineteen simply supported beams were tested to study the evolution of the neutral axis depth at failure with the ductility at bending on high-strength concrete beams. The beams were simply supported and subjected to pure bending. Each beam was loaded by two symmetrical concentrated loads applied approximately at the span's thirds. The concrete strength and the longitudinal tensile reinforcement ratio vary from beam to beam.

The researchers mainly wanted to check if the practical rule of limiting the neutral axis depth in failure, used currently is also applicable for high-strength concrete beams. The influence of ductility index on the value of neutral axis' depth at failure was also analysed.

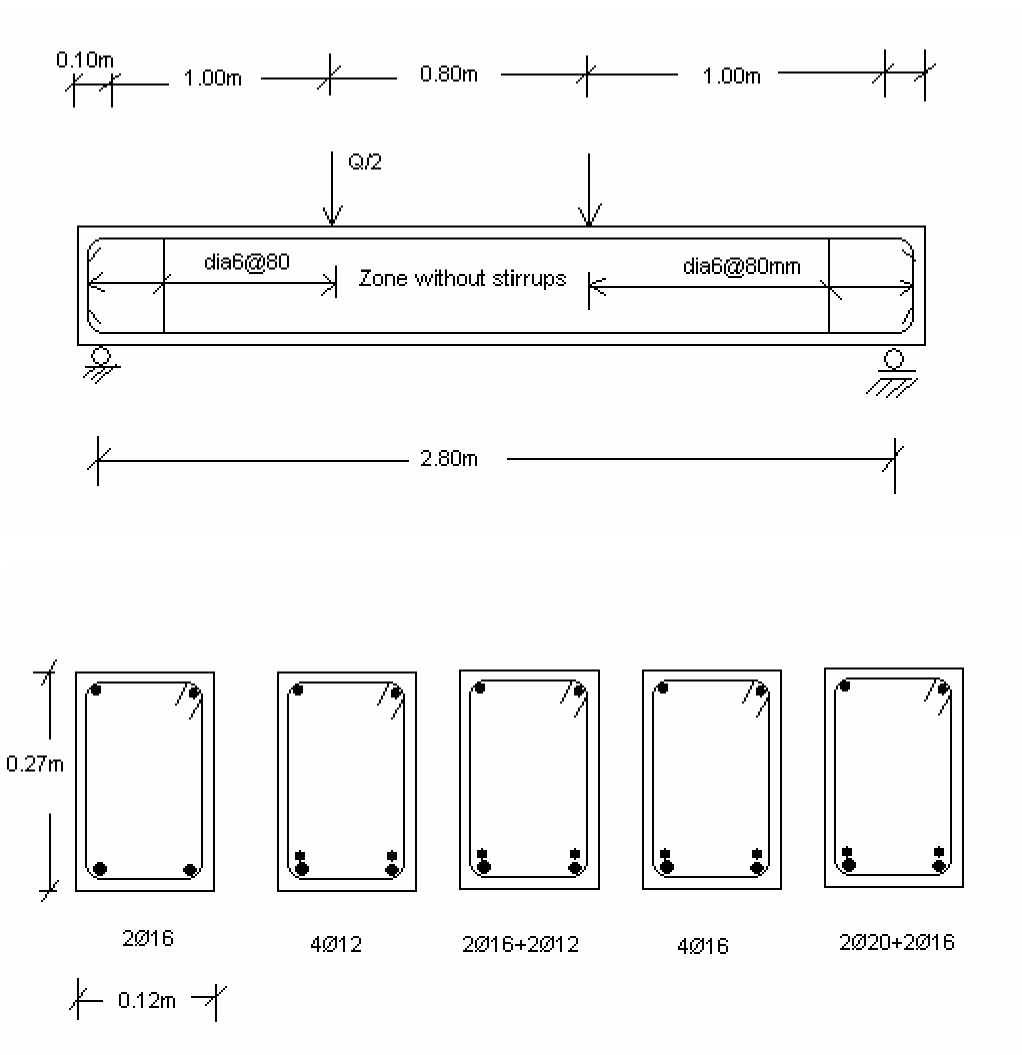


Figure 7. Geometry and detailing of test beams (Bernardo & Lopes, 2004)

From the study by Bernardo & Lopes, the following conclusions were made:

- The neutral axis depth at failure in high strength concrete beams seemed to be a valuable parameter to ascertain the beam ductility.
- The results showed that the theoretical formulation based on the use of rectangular stress block diagram for the concrete to compute the neutral axis' depth at failure does not give a reasonable prediction of those obtained from the tests.

- The theoretical formulation based on the rectangular stress block diagram for the concrete to compute the neutral axis' depth at failure gave substantially smaller values for the depth of the neutral axis at failure when compared to those calculated experimentally. As such, it was concluded that the rectangular stress block diagram proposed by ACI 318-1989 did not seem adequate for high strength concrete beams.

Series	*Beam*	*b(mm)*	*D(mm)*	f'_c*(MPa)*	$A_{st}(mm^2)$	*d(mm)*	ρ_l*(%)*	ρ_b*(%)*	ρ_l / ρ_b
A	A1	125	270	62.9	452	238	1.52	3.44	0.44
	A2	130	270	64.9	628	237	2.04	3.36	0.61
	A3	120	270	64.1	628	237	2.21	3.32	0.67
	A4	120	270	63.2	804	234	2.86	3.10	0.92
	A5	120	270	65.1	804	234	2.86	3.19	0.90
B	B1	120	264	79.2	452	237	1.59	4.33	0.37
	B2	124	270	78.9	628	242	2.09	4.08	0.51
	B3	120	270	78.5	628	242	2.16	4.06	0.53
C	C1	123	270	82.9	628	242	2.11	4.29	0.49
	C2	120	270	83.9	628	242	2.16	4.34	0.50
	C3	125	270	83.6	804	239	2.69	4.10	0.66
	C4	122	275	83.4	804	244	2.70	4.09	0.66
D	D1	120	270	88.0	402	247	1.36	4.32	0.31
	D2	120	270	85.8	1,030	238	3.61	4.22	0.86
	D3	120	270	86.0	1,030	238	3.61	4.23	0.85
E	E1	123	270	94.6	804	239	2.73	4.64	0.59
	E2	120	270	90.2	804	239	2.80	4.42	0.63
F	F1	139	263	100.3	628	230	1.96	5.19	0.38
	F2	129	270	105.2	804	234	2.66	5.16	0.52

TABLE 5. Details of beams tested by Bernardo & Lopes (2004)

3.4.4 Ashour (2000)

Nine reinforced high-strength concrete beams with compressive strengths of 48, 78, and 102MPa and tensile reinforcement ratios of 1.18, 1.77 and 2.37% were tested to investigate the effect of concrete compressive strength and flexural tensile reinforcement ratio on load-deflection behavior and displacement ductility of cracked rectangular reinforced concrete beams. All the beams were singly reinforced. Table 6 gives the details of the beam specimens. Each beam is designated to indicate the concrete compressive strength and tensile reinforcement. Thus B-M4 represents the beam with concrete compressive strength of 78.50MPa and reinforced with 4 bars of 18mm diameter as tension reinforcement.

A formula for the estimation of the effective moment of inertia, previously proposed by the researchers was modified to consider the effect of the parameters involved in the study. Displacement ductility was also investigated. Following the research, Ashour made the following conclusions:

- Flexural rigidity increases as the concrete compressive strength increases.
- The cracking moment increases as f_c' increases for the same longitudinal reinforcement.
- For the same reinforcement ratio, the neutral axis depth was found to be inversely proportional to concrete compressive strength.
- For the same concrete strength, the effective moment of inertia increases as ρ_l increases.

- The displacement ductility increases as f_c' increases for the same ρ_l up to certain limit and thereafter decreases as f_c' increases.

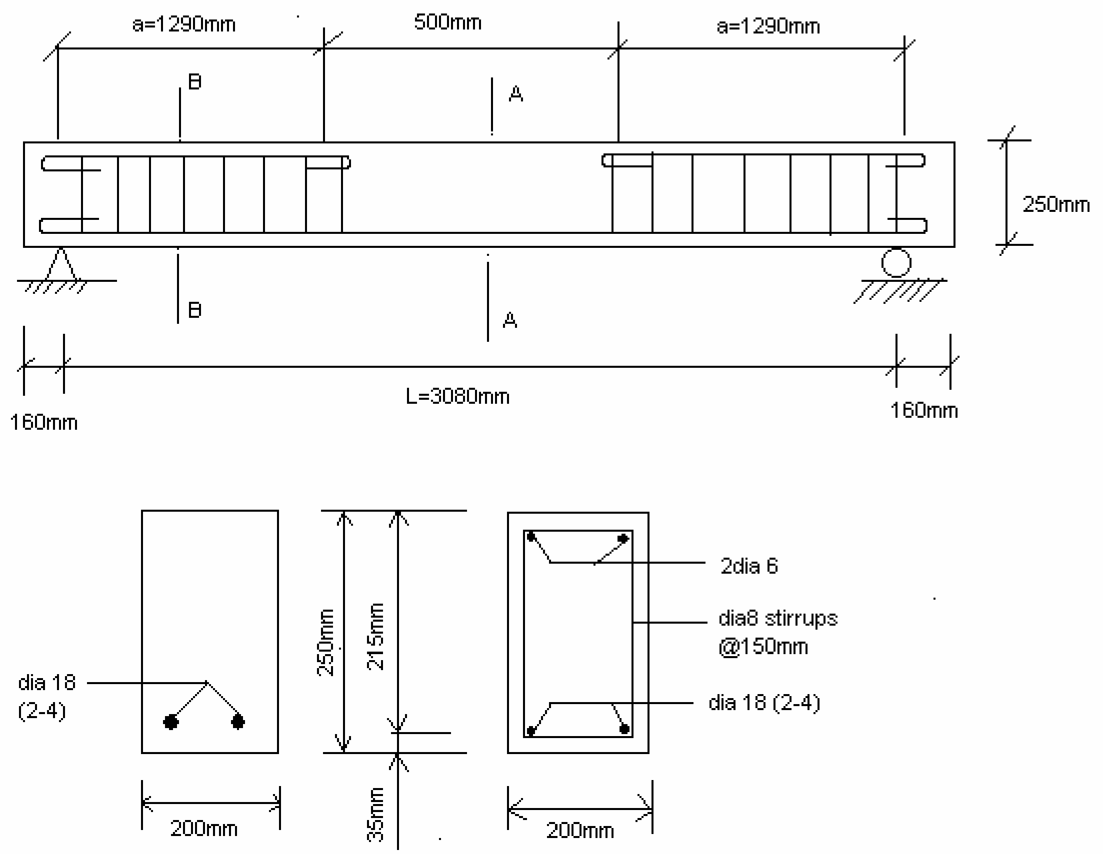


Figure 8. Geometry and detailing of beams (Ashour, 2000)

Beam No.	*f'$_c$ (MPa)*	*b (mm)*	*D (mm)*	*d (mm)*	*Layout (mm)*	*A_{st} (mm^2)*	*ρl (%)*
B-N2	48.61	200	250	215	2Ø18	509	1.18
B-N3	48.61	200	250	215	3 Ø18	763	1.77
B-N4	48.61	200	250	215	4 Ø18	1018	2.37
B-M2	78.50	200	250	215	2 Ø18	509	1.18
B-M3	78.50	200	250	215	3 Ø18	763	1.77
B-M4	78.50	200	250	215	4 Ø18	1018	2.37
B-H2	102.40	200	250	215	2 Ø18	509	1.18
B-H3	102.10	200	250	215	3 Ø18	763	1.77
B-H4	102.40	200	250	215	4 Ø18	1018	2.37

TABLE 6. Details of beams tested by Ashour (2000)

3.5 Correlation of Test Moment Capacity with Predictions by codes and proposed methods

Various code provisions for flexural capacity of concrete beams have been described. The experimental moment capacities of the 46 beams tested by Pam et al (2001), Sarkar et al (1997), Bernardo & Lopes (2004), and Ashour (2000) have been compared to the predictions by the codes and other publications. Although various recommendations made by researchers and the design codes are considerably different in nature, capacities of reinforced concrete members can be predicted with similar level of accuracy by using any of the aforementioned recommendations. The effect of using accurate and conservative stress block parameters for high strength concrete beams will be pronounced. For this purpose, a comparison of test moment capacity to predictions by the various codes and other researchers has been carried out using spreadsheets in Excel. Details of beams and ultimate moment predicted using different codes have been given in

Table 7. A summary of the correlation has been given in Table 8. The summary of correlation indicates significant scatter in the predictions by the above methods. Figures 9 to 19 show the correlation of the test moment capacity versus the predicted moment capacities of the beams. The mean values of M_e/M_p are also given and the coefficient of variation determined. For the eleven methods of prediction, the coefficient of variation ranged from 13.73% (CEB-FIP1990) to 16.98% (ACI318-05 (2005). CEB-FIP1990 gave the best prediction with the smallest scatter. The mean value M_e/M_p value is 0.974 with a coefficient of variation of 14.72%. Most of the results fall either within the ±20% band of the ideal 1:1 test moment capacity versus predicted moment capacity line, or above this band. Comparison of experimental results with predicted strength capacity in bending showed that the expressions given by the various researchers are not quite accurate and display a deviation of ±20%. It is found that for beams tested by Bernardo and Lopes, the predictions from the theoretical point of view were a bit unconservative. All of the methods used to determine the strength of beams in bending tested by Bernardo and Lopes (2004), overestimate the capacity of the beam and produce theoretical moment capacity which is more than the actual capacity of the beam. The reason identified has been the rate of loading and choice of demec gauges used to measure the strains along the height of the central zone of the beams.

It is believed that design guidelines should provide similar levels of conservativeness for NSC as well as HSC (Bae & Bayrak, 2003). In this regard, it is said that the current code provisions are unsuitable for use in designing HSC beams subjected to flexure.

Reference	Beam No.	f'_c	b	D	d	A_{st}	f_y	M_e	M_e/M_p					
		(MPa)	mm	mm	mm	mm^2	N/mm^2	kN.m	AS3600-01	ACI318-05	CAN3-A23.3-M94	NZS3101-95	CEB-FIP1990	EUROCODE-2
Pam et al (2001)	1	37.4	200	300	264	402	579	77.6	1.36	1.36	1.36	1.35	1.37	1.36
	2	36.8	200	300	264	603	579	103.5	1.26	1.26	1.27	1.24	1.28	1.26
	3	36.4	200	300	260	982	578	126.5	1.04	1.04	1.06	1.03	1.08	1.04
	4	42.3	200	300	260	982	536	129	1.10	1.10	1.11	1.12	1.13	1.10
	5	46.4	0.003	0.792	0.834	200	300	142.8	1.00	1.16	1.03	1.01	1.05	1.01
	7	58.6	0.003	0.77675	0.8035	200	300	164.6	1.07	1.09	1.10	1.08	1.13	1.08
	8	57.1	0.003	0.778625	0.80725	200	300	166.2	0.98	1.02	1.00	0.99	1.04	0.98
	9	58.6	0.003	0.77675	0.8035	200	300	171.6	0.871	0.70	0.90	0.90	0.94	0.87
	14	95.5	0.003	0.730625	0.71125	200	300	138	1.00	1.46	1.02	1.01	1.05	1.00
	15	98	0.003	0.7275	0.705	200	300	200.7	1.00	0.91	1.03	1.02	1.08	1.01
	16	102.5	0.003	0.725	0.69375	200	300	181.7	0.90	0.86	0.93	0.92	0.98	0.91
	17	87	0.003	0.74125	0.7325	200	300	172	0.86	0.96	0.89	0.88	0.93	0.87
Sarkar et al (1997)	HSC1-1	107	150	250	220	339	470	38.94	1.14	1.14	1.15	1.15	1.16	1.14
	HSC1-2	97	150	250	220	339	470	35.64	1.05	1.05	1.05	1.05	1.07	1.05
	HSC1-3	85	150	250	220	339	442	37.62	1.18	1.18	1.19	1.18	1.20	1.18
	HSC2-1	105	150	250	213	452	470	46.33	1.07	1.07	1.08	1.07	1.10	1.07
	HSC2-2	100	150	250	213	452	470	46.86	1.08	1.08	1.09	1.09	1.11	1.08
	HSC2-3	77	150	250	213	452	442	43.56	1.08	1.08	1.09	1.08	1.10	1.08
	HSC2-4	90	150	250	213	452	442	48.84	1.20	1.20	1.21	1.21	1.23	1.20
	HSC3-1	107	150	250	215	628	470	67.32	1.12	1.12	1.13	1.12	1.16	1.12
	HSC3-2	85	150	250	215	628	470	66	1.11	1.11	1.12	1.12	1.15	1.11
	HSC3-3	78	150	250	215	628	442	64.68	1.16	1.16	1.17	1.17	1.20	1.16
	HSC4-1	101	150	250	208	1257	470	92.42	0.85	0.85	0.87	0.86	0.93	0.85
	HSC4-2	87	150	250	208	1257	470	89.6	0.84	0.84	0.86	0.86	0.91	0.84
	HSC4-3	82	150	250	208	1257	442	111.63	1.11	1.11	1.14	1.12	1.20	1.11
Bernardo & Lopes	A1	62.9	125	270	238	452	534	100.55	0.95	0.95	0.96	0.95	0.97	0.95

TABLE 7. Details of beams and ultimate moment predictions using different codes

Reference	Beam No.	f'_c	b	D	d	A_{st}	f_y	M_e	M_e/M_p					
		(MPa)	mm	mm	mm	mm^2	N/mm^2	kN.m	AS3600-01	ACI318-05	CAN3-A23.3-M94	NZS3101-95	CEB-FIP1990	EUROCODE-2
Bernardo&Lopes	A2	64.9	130	270	237	628	575	129.88	0.85	0.85	0.86	0.85	0.89	0.85
(2004)	A3	64.1	120	270	237	628	575	129.1	0.85	0.85	0.87	0.86	0.89	0.85
	A4	63.2	120	270	234	804	575	156.96	0.86	0.86	0.88	0.86	0.91	0.86
	A5	65.1	120	270	234	804	575	145.09	0.79	0.79	0.81	0.81	0.84	0.79
	B1	79.2	120	264	237	452	534	95.99	0.90	0.90	0.91	0.90	0.92	0.90
	B2	78.9	124	270	242	628	575	127.33	0.80	0.80	0.81	0.81	0.84	0.80
	B3	78.5	120	270	242	628	575	131.9	0.83	0.83	0.85	0.84	0.87	0.83
	C1	82.9	123	270	242	628	575	125.57	0.79	0.79	0.80	0.80	0.82	0.79
	C2	83.9	120	270	242	628	575	135.92	0.85	0.85	0.87	0.86	0.90	0.85
	C3	83.6	125	270	239	804	575	144.89	0.74	0.74	0.75	0.75	0.78	0.74
	C4	83.4	122	275	244	804	575	155.88	0.78	0.78	0.79	0.79	0.83	0.78
	D1	88	120	270	247	402	575	88.44	0.82	0.82	0.83	0.82	0.84	0.82
	D2	85.8	120	270	238	1030	572	186.68	0.78	0.78	0.80	0.79	0.85	0.78
	D3	86	120	270	238	1030	572	195.22	0.81	0.81	0.84	0.83	0.89	0.81
	E1	94.6	123	270	239	804	575	157.99	0.79	0.79	0.81	0.80	0.85	0.79
	E2	90.2	120	270	239	804	575	157.79	0.80	0.80	0.82	0.81	0.85	0.80
	F1	100.3	139	263	230	628	575	128.46	0.83	0.83	0.84	0.84	0.87	0.83
	F2	105.2	129	270	234	804	575	158.14	0.80	0.80	0.82	0.81	0.86	0.80
S.A.Ashour	B-N2	48.61	200	250	215	509	530	58.17	1.09	1.09	1.09	1.09	1.11	1.09
(2000)	B-N3	48.61	200	250	215	763	530	57.95	1.05	1.05	1.06	1.05	1.08	1.05
	B-N4	48.61	200	250	215	1018	530	56.8	1.01	1.01	1.03	1.01	1.06	1.01
	B-M2	78.5	200	250	215	509	530	80.6	1.05	1.05	1.06	1.05	1.07	1.05
	B-M3	78.5	200	250	215	763	530	79.91	0.99	0.99	1.00	0.99	1.02	0.99
	B-M4	78.5	200	250	215	1018	530	82.76	0.99	0.99	1.00	0.99	1.04	0.99
	B-H2	102.4	200	250	215	509	530	99.55	1.02	1.02	1.02	1.02	1.04	1.02
	B-H3	102.4	200	250	215	763	530	103.77	1.01	1.01	1.02	1.01	1.05	1.01
	B-H4	102.4	200	250	215	1018	530	108.1	1.00	1.00	1.02	1.00	1.06	1.00

TABLE 7. Details of beams and ultimate moment predictions using different codes (Contd).

From Table 8, it is noted that all the code provisions and publications involving different stress block parameters used to predict the ultimate strength show a coefficient of variation (COV). The mean value of M_e/M_p is found to be less than 1 for all the methods except for CEB-FIB1990, where M_e/M_p is found to be 1.011. It is concluded that all the methods involving different stress block parameters are unconservative for use with high strength concrete beams.

	M_e/M_p ratio	
Method	Mean	COV
AS3600-01(2001)	0.966	14.93%
ACI318-05(2005)	0.974	16.98%
CEB-FIP1990(1990)	1.011	13.73%
CAN-A23.3-M94(1994)	0.981	14.47%
NZS3101-1995	0.974	14.45%
Eurocode-2(1999)	0.966	14.93%
Mendis & Pendyala(1998)	0.95	13.86%
Ibrahim& MacGregor(1997)	0.974	14.57%
Attard & Stewart(1998)	0.981	14.74%
Azizinamini et al(1994)	0.976	14.57%
Bae & Bayrak(2003)	0.971	14.74%

TABLE 8 Summary of Correlation

Another analysis of the data excluding the beams tested by Bernardo & Lopes has been done to investigate the effect of the stress block parameters. It is noted that all methods give conservative results as depicted in Table 9. The theoretical moment capacity is found to be less than the actual moment capacity and the M_e/M_p ratio is found to be always greater than 1.

	M_e/M_p ratio	
Method	Mean	COV
AS3600-01 (2001)	1.048	13.03%
ACI318-05 (2005)	1.06	15.62%
CEB-FIP1990 (1990)	1.091	12.65%
CAN-A23.3-M94 (1994)	1.062	12.62%
NZS3101-1995	1.056	12.69%
Eurocode-2 (1999)	1.048	13.03%
Mendis & Pendyala	1.04	11.26%
Ibrahim & MacGregor (1997)	1.053	12.74%
Attard &.Stewart(1998)	1.063	12.86%
Azizinamini et al (1994)	1.057	13.32%
Bae & Bayrak (2003)	1.052	12.82%

TABLE 9. Summary of Correlation (Excluding beams tested by Bernardo & Lopes, 2004)

3.6 Discussion

The use of formulae for calculating the moment capacity of reinforced concrete beams by engineers makes it obvious that the theoretical moment capacity should be less than the actual moment capacity. The results obtained theoretically for the calculation of ultimate strength must be conservative. The design rules should provide similar level of conservativeness for normal and high strength concrete (Bae & Bayrak, 2003). From the Excel spreadsheet in Appendix A, it is seen that the rectangular stress block approach is satisfactory for all the beams tested by Pam et al (2001), Sarkar et al (1997) and Ashour (2000) considered in this study except for the beams tested by Bernardo & Lopes (2004). It has been found that code provisions and the different methods from the literature conservatively predict the ultimate strength of most of the beams tested by Pam et al (2001), Sarkar et al (1997) and Ashour (2000) and unconservatively predict the ultimate strength of beams tested by Bernardo & Lopes (2004). All the methods produce theoretical moment capacities which are less than the actual moment capacity for the beams. In case of beams tested by Bernardo and Lopes (2004), the theoretical moment capacity that is calculated by each method is greater than the actual moment capacity. The reason for this has been attributed to the inaccuracy of the experimental moment capacity that is calculated from the ultimate load obtained from the test results.

It is noted that there is not much difference between the various stress block parameters. The ACI318-05 stress block parameters show the highest coefficient of variation (16.98%). The use of different stress block parameters have been found to yield unconservative estimations for beam capacity as concrete compressive strength increases. Hence, it is concluded that all the stress block parameters used to predict the ultimate strength are unconservative. But the extent of unconservativeness is not large. The degree of accuracy and unconservativeness of all the stress block parameters considered for this study appears similar from the figures. For the predictions by Mendis and Pendyala (1998), only those beams with a concrete strength greater than 57MPa have been considered for the correlation analysis because of the lower limit on the stress block parameters. In the present study, only 54 beams have been considered and if there were more data, different tendencies of the stress block parameters could be clearly examined.

3.7 Conclusion

On the basis of work completed in this chapter, it may be concluded that the rectangular stress block approximation will not give a conservative estimate of the moment capacity of HSC beams. Until future work is completed to address this and develop a conservative model, it is recommended that a reduction factor of 0.8 be used on all HSC beams designed using the rectangular stress block theory.

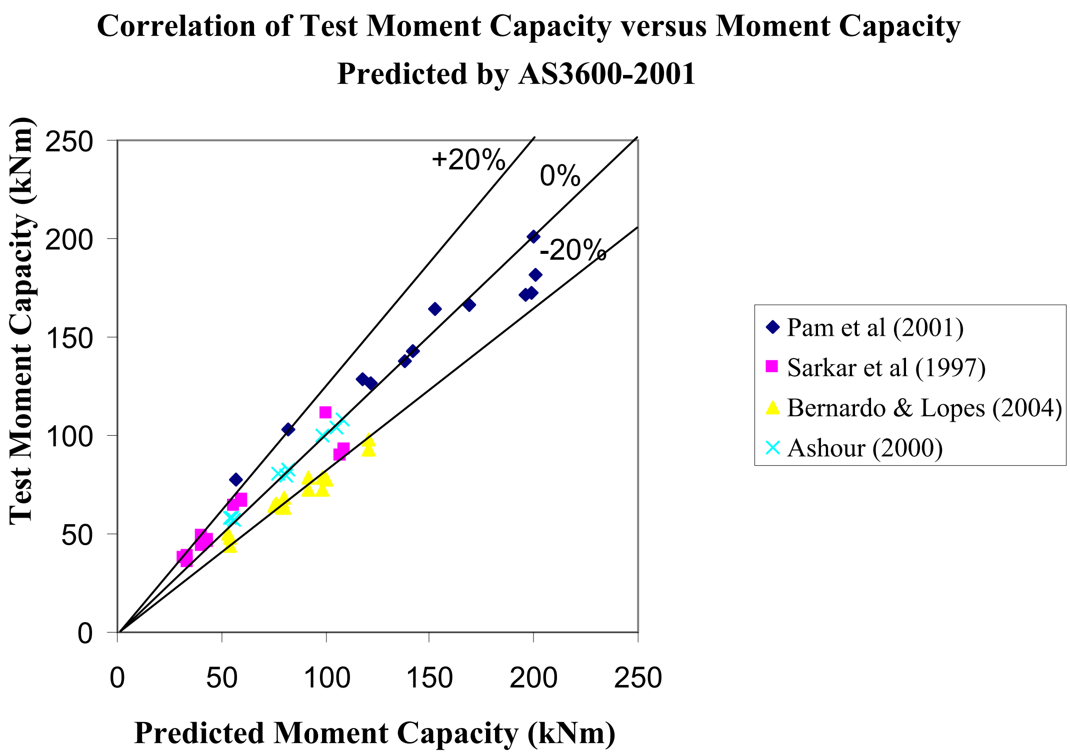


Figure 9. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per AS 3600-2001.

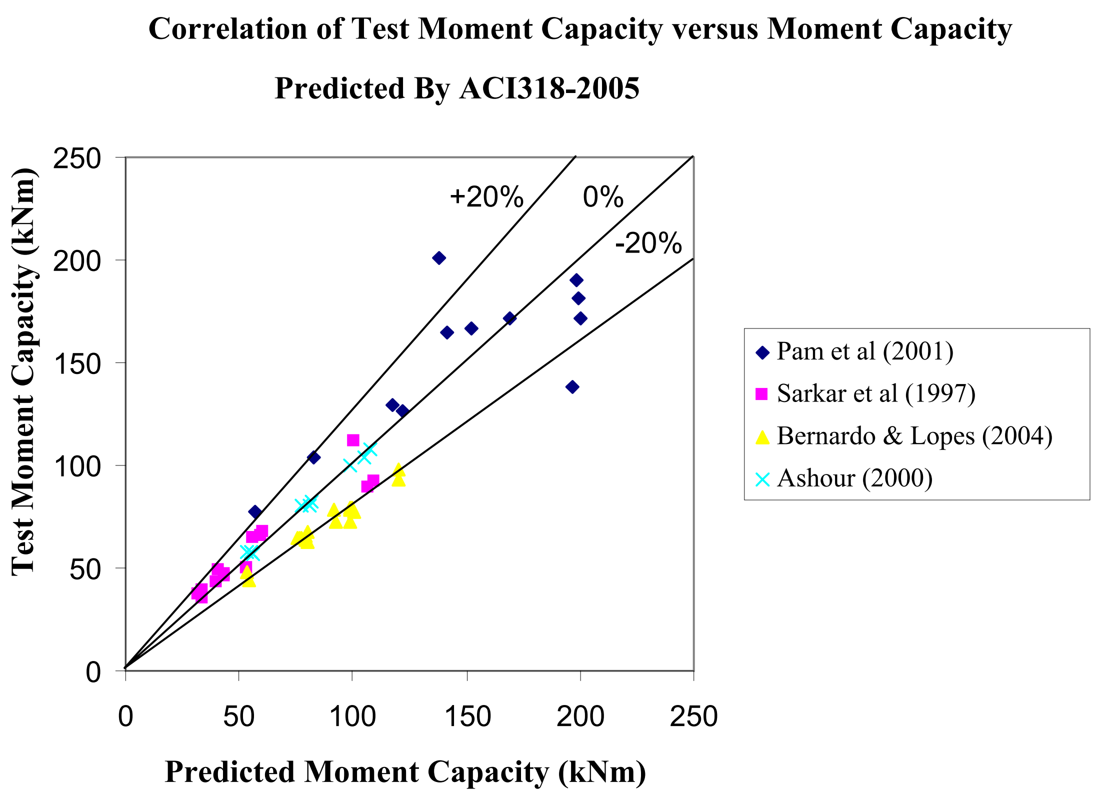


Figure 10. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per ACI318-2005

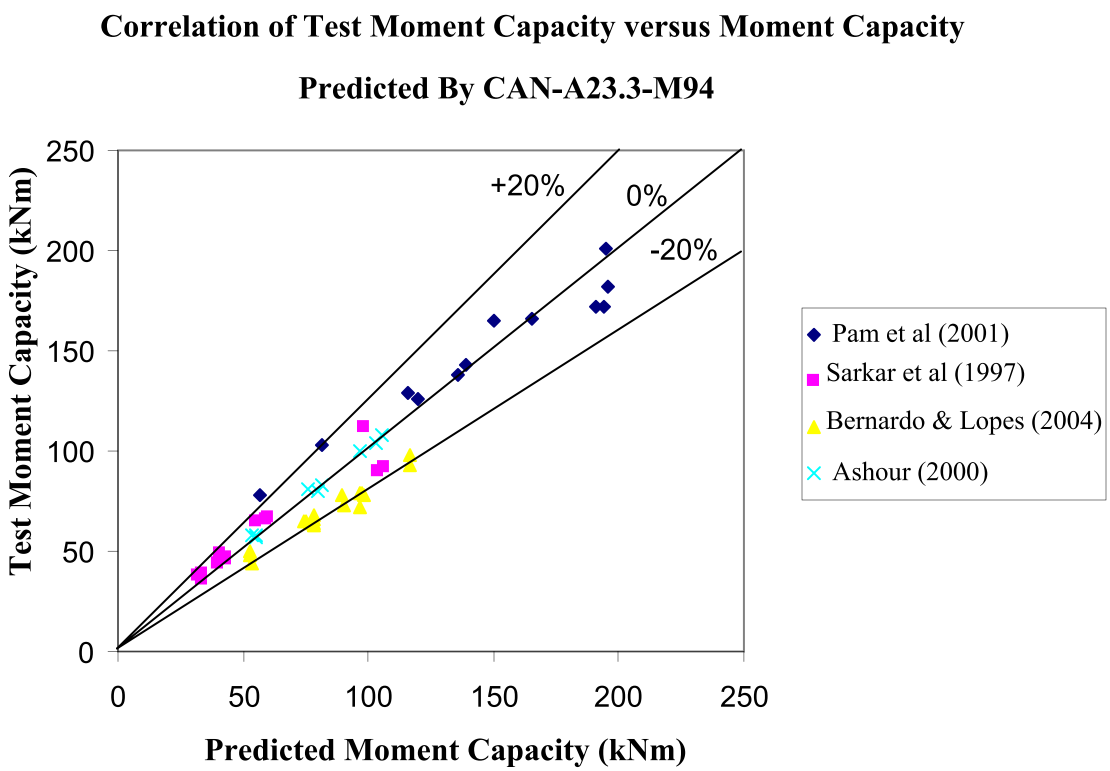


Figure 11. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per CAN-A23.3-M94

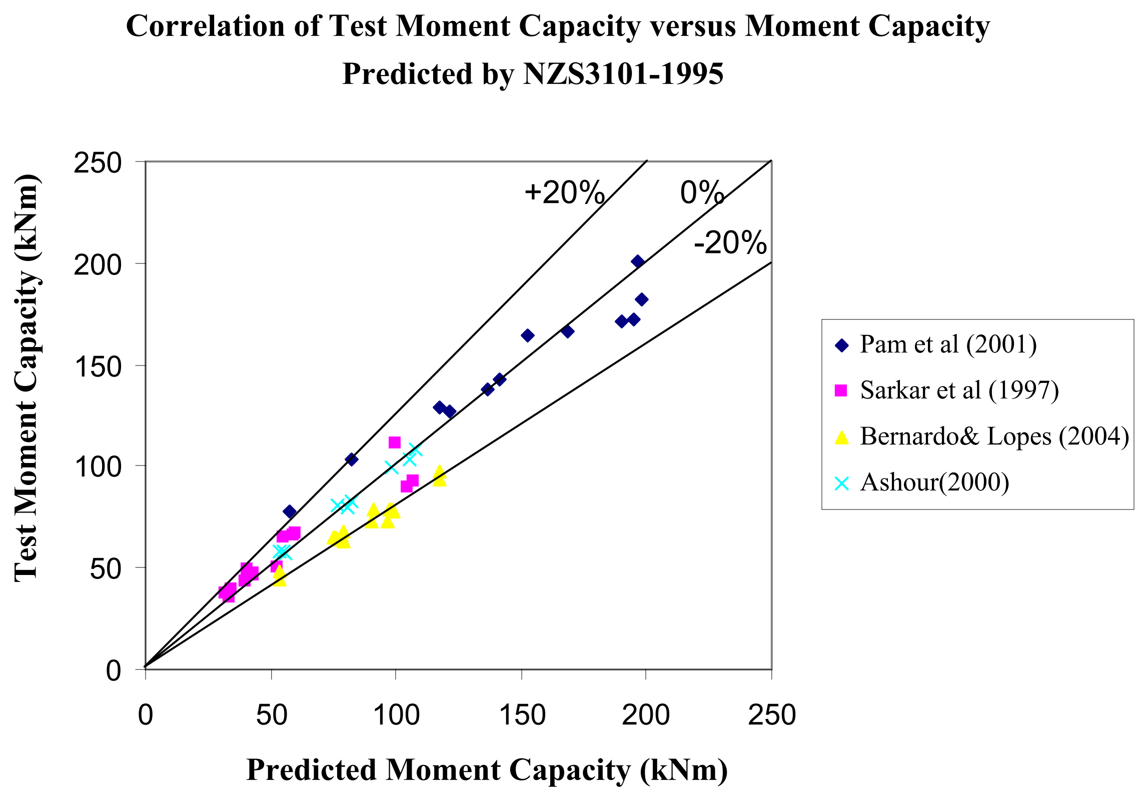


Figure 12. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per NZS 3101-1995

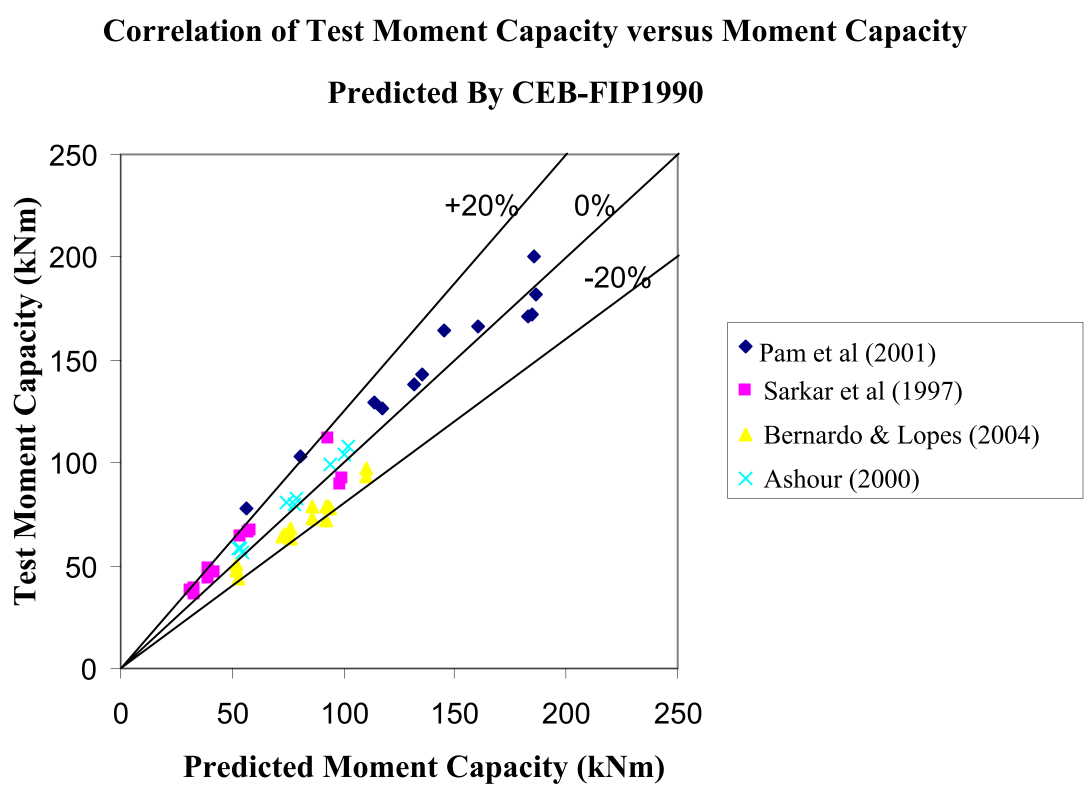


Figure 13. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per CEB-FIP 1990

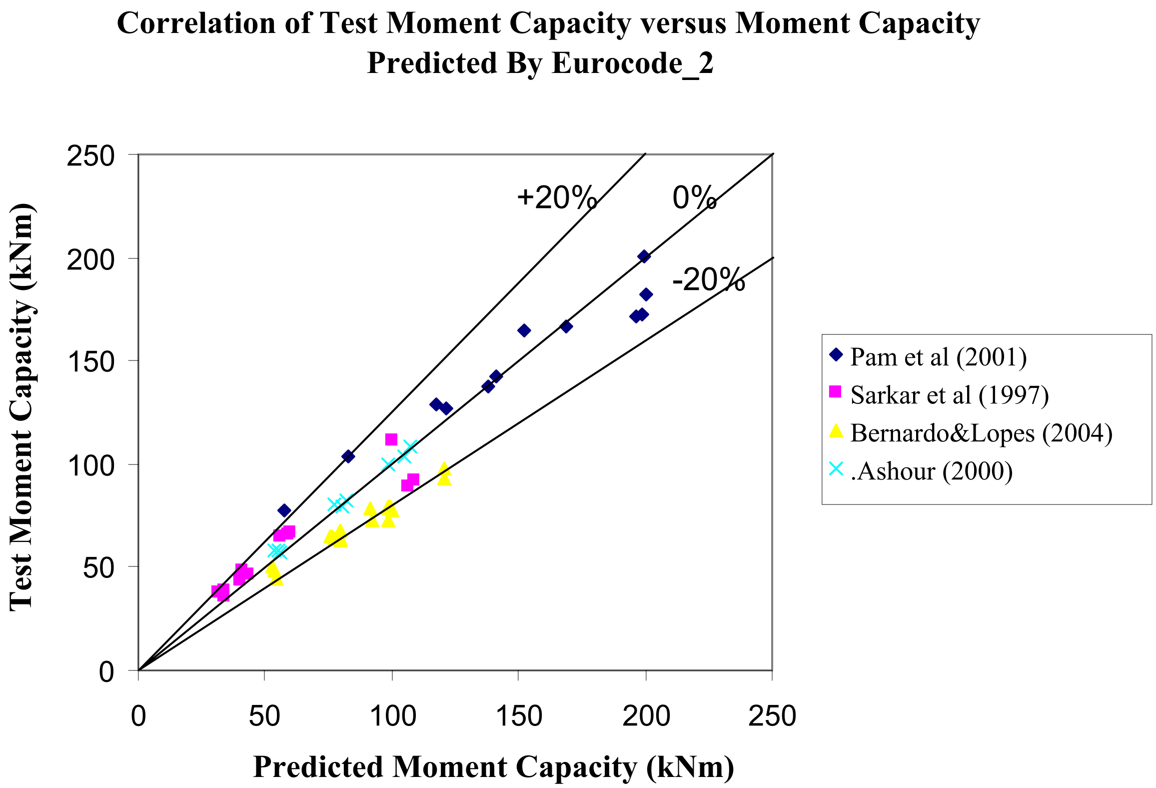


Figure 14. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per Eurocode-2

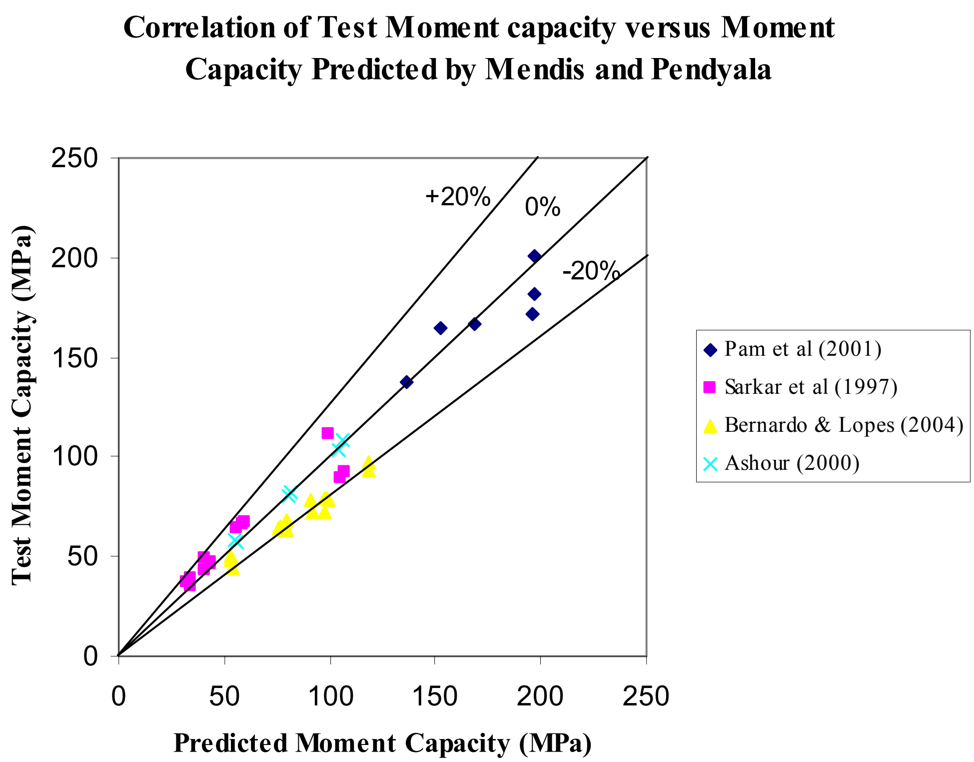


Figure 15. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per Mendis & Pendyals (1997)

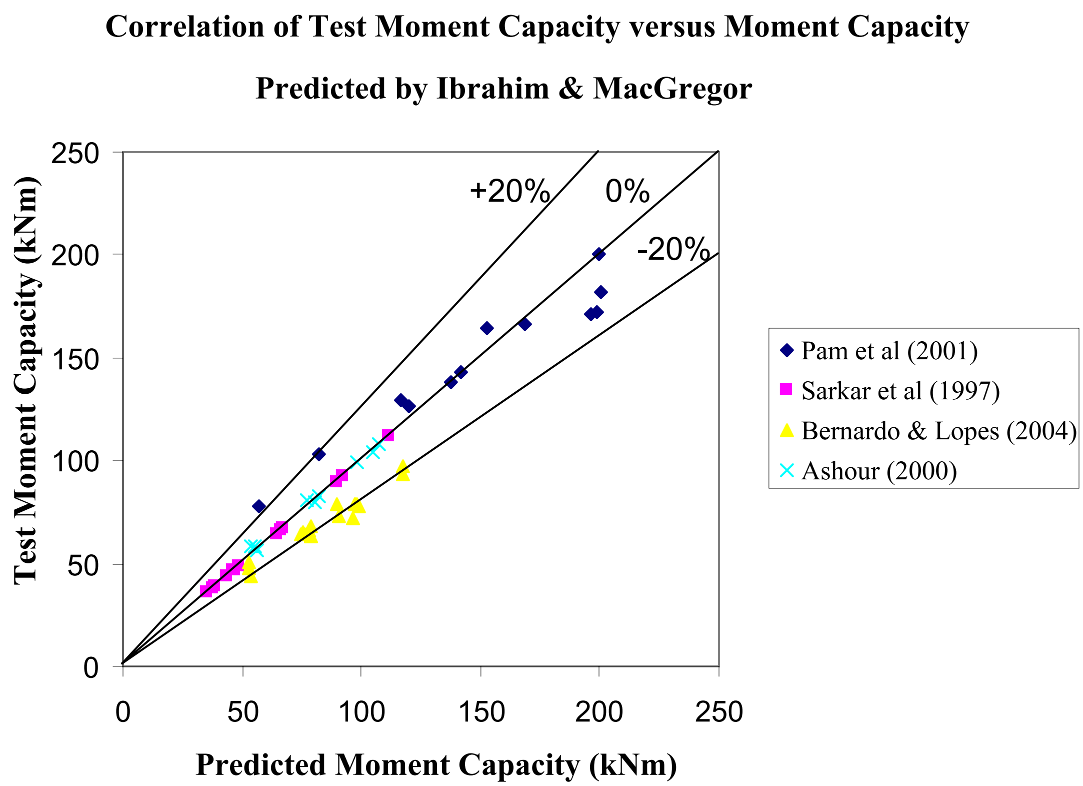


Figure 16. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per Ibrahim & MacGregor (1997)

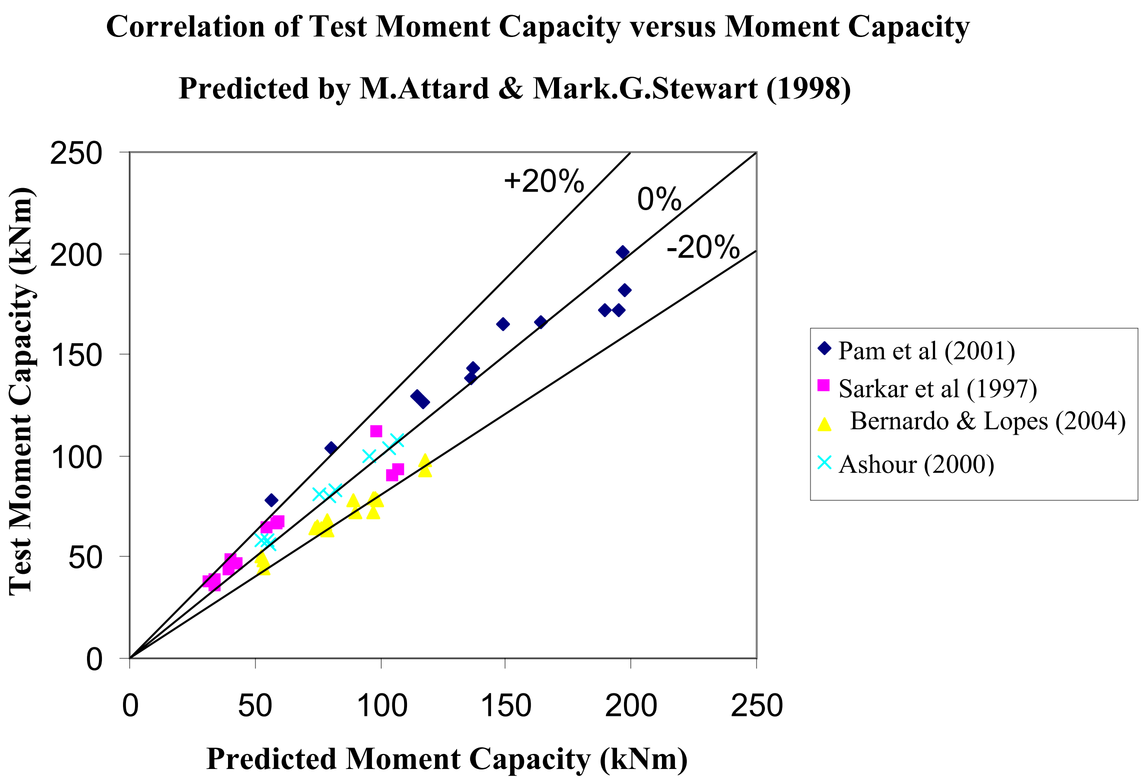


Figure 17. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per Attard & Stewart (1998)

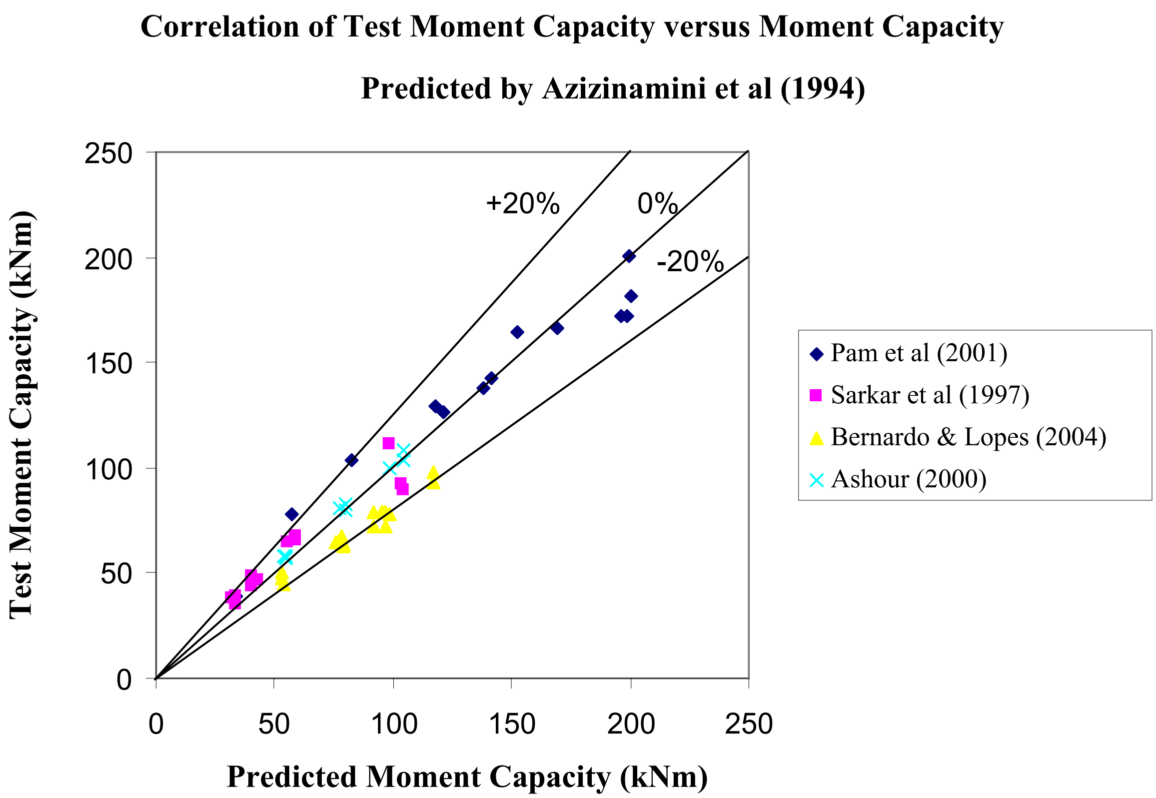


Figure 18. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per Azizinamini et al (1994)

Correlation of Test Moment Capacity versus Moment Capacity predicted by S.Bae & O.Bayrak (2003)

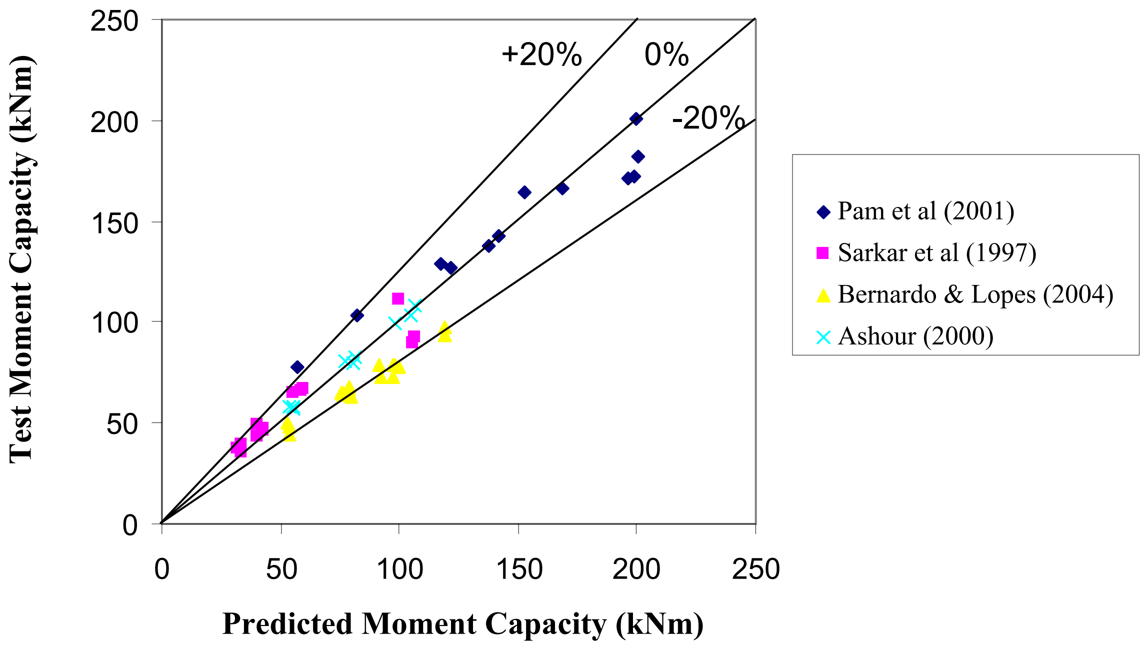


Figure 19. Correlation of Experimental Moment Capacity vs. Moment Capacity predicted as per Bae & Bayrak (2003)

CHAPTER 4

DUCTILITY

4.1 INTRODUCTION

Ductility is an important structural property because it allows stress redistribution and gives warning of the imminence of failure. Ductility is usually measured using the moment curvature relationship for a given beam section. The length of the plastic plateau usually is a direct measure of the ductility of a beam. This chapter presents moment-curvature curves developed theoretically using a computer program. Theoretical curves have been compared with the experimental behavior of beams considered for the analysis.

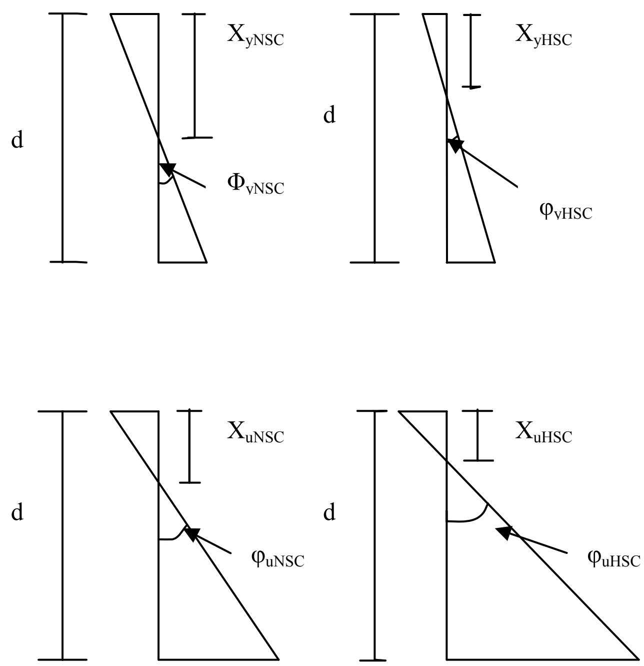
4.2 DEFINITION

Ductility is generally defined as the ability of a reinforced concrete member to undergo deformations without a substantial reduction in the flexural capacity of a member. Different forms of ductility include

- Curvature ductility
- Rotational ductility
- Displacement ductility

In a broad sense, ductility is defined as the ability of a structure to sustain deformations beyond the elastic range without a significant variation of the resistance capacity (Sarkar et al, 1997). For this study, ductility is defined as the ratio of ultimate curvature to curvature at first yield of tension reinforcement; i.e. $\mu=\varphi_u/\varphi_y$. Ultimate curvature is the curvature corresponding to the maximum moment of the beam.

Generally, if the reinforcement ratio is low, the HSC beams are more ductile due to the higher neutral axis. According to Fasching and French (1998), the yield curvature of HSC was found to be smaller than the yield curvature of NSC members, assuming that the two cross-sections have the same amount of tension reinforcement (Fig. 20). If the reinforcement yield stress (and yield strain, ε_y) is the same in both members, the depth to the neutral axis from the extreme compression fiber, should be less in HSC member. It follows that the yield curvature (φ_y), defined as the ratio of ε_y / (d-x_y), and consequently the yield deflection, should be smaller for the HSC member. Assuming that both the members reach the same ultimate concrete strain, ε_{cu}, the ultimate curvature (φ_u) should be greater in the HSC member due to the smaller depth of the neutral axis. As defined above, the larger maximum curvature (φ_u) and smaller yield curvature (φ_y) of the HSC section translates into a larger ductility for the HSC member.



$\Phi_y = \varepsilon_y/(d - x_y)$

$X_{yNSC} > X_{yHSC}$, and $\varepsilon_{yNSC} = \varepsilon_{yHSC}$

Therefore, $\varphi_{yNSC} > \varphi_{yHSC}$

$\Phi_u = \varepsilon_{cu}/\ x_u$

$X_{uNSC} > X_{uyHSC}$, and

$\varepsilon_{cuNSC} = \varepsilon_{cuyHSC}$

Therefore, $\varphi_{uyNSC} > \varphi_{uyHSC}$

Figure 20. Effect of concrete strength on ultimate curvature

A plot of moment versus curvature for a portion of a beam provides a good representation of flexural behavior under short-term loading (Warner et al, 1998). For a given section of a reinforced concrete member, the moment curvature relationship can be theoretically determined, if the section properties and the stress-strain relationship of concrete and reinforcing steel are known. Based on the moment-curvature curve, ductility of a reinforced concrete section is generally defined as the ratio of curvature at the point at which a section starts to soften and the curvature at the point of yield of tensile steel (Setunge et al., 1994). In this study the moment curvature curve diagram of each beam is developed in order to investigate the full range moment curvature behavior of the reinforced concrete section. The ultimate moment capacity obtained from the moment-curvature curve and the moment capacity obtained using different stress block parameters have been compared. A computer program (FRMPHI) developed by Setunge et al. (1994) has been used for this purpose.

4.3 Computer Program- FRMPHI

A computer program has been used to seek a better understanding of the behavior of high strength concrete beams in bending. Assuming plane sections remain plane and using strain compatibility, derivation of a moment curvature behavior of a flexural member with or without axial loads or pre-stress, can be easily achieved using a computer program. This program FRMPHI was developed using the same basic algorithm of the EMPHI program originally written by Darvall in BASIC computer language (Darvall, 1987). The program can be used for sections with variable width and uses the Rectangular Stress Block for the stress-strain relationship for concrete, elastic-plastic-strain hardening stress-strain relationship for reinforcing steel.

Rectangular stress-block is not valid beyond a concrete compressive strain of 0.003 and hence, it cannot be used for prediction of the Full Range Moment Curvature Behavior of a flexural member. In order to determine the full-range moment curvature behavior of reinforced concrete sections, a more realistic stress-strain relationship for concrete has to be used.

If a realistic stress-strain relationship for concrete in compression is used to derive the moment curvature relationship, it shows a linear portion until yield of tensile steel, followed by a non-linear ascending portion until the maximum moment followed by a softening branch. It has been shown that in a redundant structure, failure would not occur until one or more hinges enter the softening region. Furthermore, a "critical softening slope" is defined as the slope of the softening portion of the moment curvature curve, above which, the structure cannot carry further load however redundant the structure may still be (Darvall, 1983; Setunge et al., 1994). Therefore, prediction of full range moment curvature behavior of reinforced concrete sections is essential in determining the available rotation capacity of a critical section to be used in plastic analysis.

The computer program FRMPHI was developed in order to investigate the Full Range Moment Curvature Behavior of a reinforced concrete sections. It deals with both confined and unconfined concrete sections and gives the user a choice of six stress-strain models for the flexural stress-block of concrete. The models incorporated in the program are Scott et al. (1982), Fafitis and Shah (1985), Mander et al (1988), Bjerkeli et al (1990), Setunge et al. (1994) and also the rectangular stress block. The program assumes that plane sections remain plane and can be used for variable width of a section.

According to Mendis (1986), the Scott et al. (1982) model used in the program gave a good prediction of the flexural compressive stress block of reinforced concrete sections. The stress-strain model of Fafitis and Shah (1985) was proposed for concrete with compressive strengths of up to 65MPa and is recommended for both normal and high strength concrete. The stress-stain relationship proposed by Bjerkeli et al. (1990) was shown to be valid for axially loaded high strength concrete columns with compressive strengths of up to 85MPa.

In the present study, theoretical moment-curvature curves were obtained for beams using the stress-strain model of Setunge for beams with concrete strengths greater than 50MPa and using Mander et al. (1988) model for beams made of conventional concrete. Theoretically, the increase in compressive strength of concrete of a flexural member reduces the depth to the neutral axis of the section at ultimate load. Appendix B displays moment curvature curves obtained for beams using the program.

4.3.1 Ultimate Strength

The ultimate strength of the section is determined from the moment-curvature curves using different stress-strain models incorporated into the computer program FRMPHI. The ultimate moment capacity obtained from the curves for each beam is noted. Table B.1 in Appendix B gives a comparison of the ultimate moment capacities predicted using different stress block parameters and that obtained using the program FRMPHI which incorporates different stress-strain models. It has been found that the capacity ratio (experimental/predicted) exceeds 1.0 for few beams and was generally less than 1.0 for the rest of the beams. The comparison revealed that the stress block models are not conservative to predict the ultimate moment capacity for a HSC beam section.

It is also established that the CEB-FIP model has the least COV of 13.7386 with a mean of 1.01. In general the capacity ratio conservatively exceeds 1.0 for less than half of the beams with an average ratio of 1.14 and standard deviation of 1.12. For the beams which exceed the capacity ratio of 1, the moment capacity is 5% to 37% greater than predicted. For the remaining beams, the predicted moment capacity is found to exceed the experimental moment capacity of the beams.

4.3.2 Moment – Curvature Relationship

The moment-curvature curves give a measure of the ductility of a concrete member. The plastic behavior of any structural member is limited by the amount of deformation which can occur at a critical section when subjected to applied loads (Sarkar et al, 1997). In this study, theoretical moment-curvature curves for different beams considered have been derived using Mander et al. (1988) model for normal strength concrete beams and Setunge et al. (1993) model for high strength concrete beams incorporated in the program FRMPHI. Table 10 gives the curvatures of the beams at yield and ultimate stages derived from the moment-curvature curves. Figure 21 presents the moment curvature relationship of the beams tested by Ashour (2000) which had same concrete compressive strength (f'_c=48.61MPa), but different reinforcement ratio, ρ_l= 1.18% (B-N3), ρ_l= 1.77% (B-N3), ρ_l= 2.37% (B-N4). Figure 21 also shows that the beam with higher reinforcement ratio exhibited a slightly stiffer response.

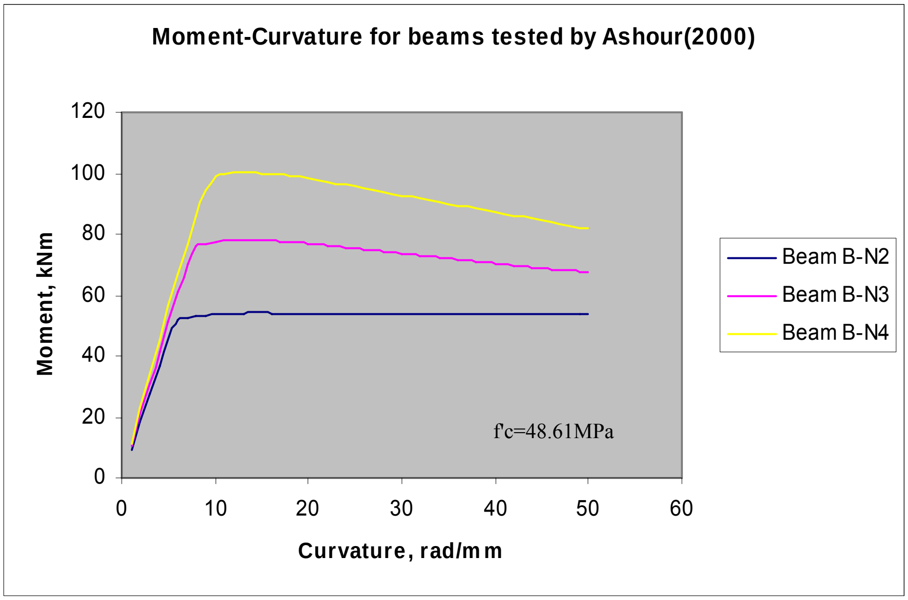


Figure 21. Analytical Moment-Curvature for Beams having same f'_c= 48.61MPa, but different ρ_l ,tested by Ashour (2000)

Figure 22 gives the moment-curvature relationship for beams having same ρ_l= 1.77% but varying compressive strengths, f'_c=48.6MPa (B-N3), f'_c= 78.5 MPa (B-M3), f'_c=102.4 MPa (B-H3). It is found that the beams with higher f'_c demonstrate a rigid response.

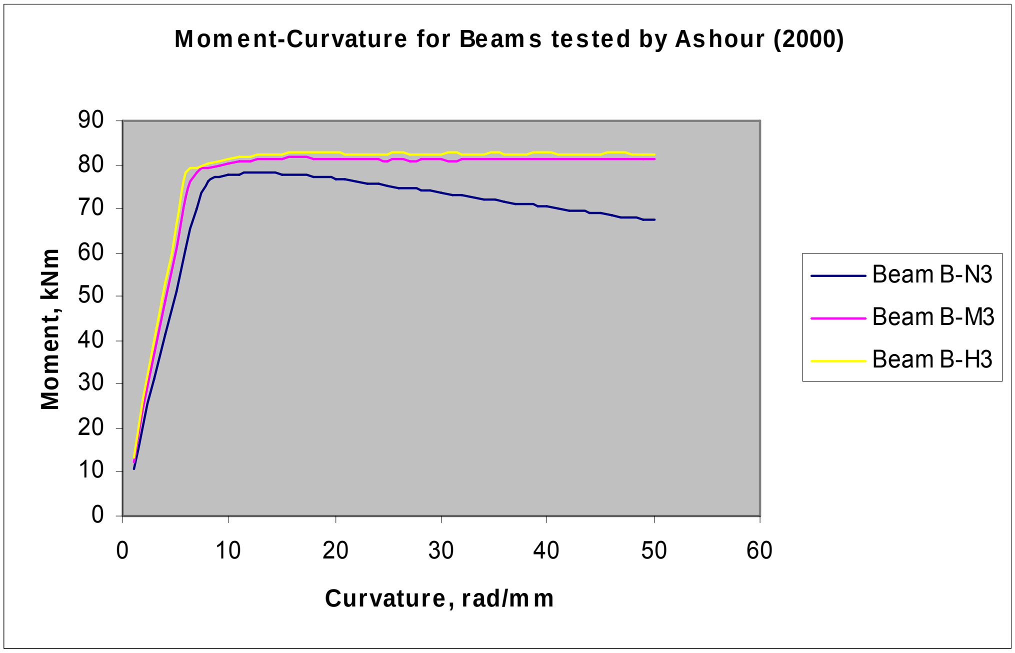


Figure 22. Analytical Moment-Curvature for Beams having same ρ_l but varying f'_c, tested by Ashour (2000)

Figure 23 presents the moment-curvature relationship for beams tested by Pam et al (2001) having approximately same percentage of longitudinal steel, ρ_l=2.84% but different concrete compressive strengths, f'_c =57.1 MPa (PB8), f'_c =98.0 MPa (PB15) and f'_c =102.5 MPa. The behavior is generally elastic-plastic for these beams and the beam with higher f'_c displays a stiffer response.

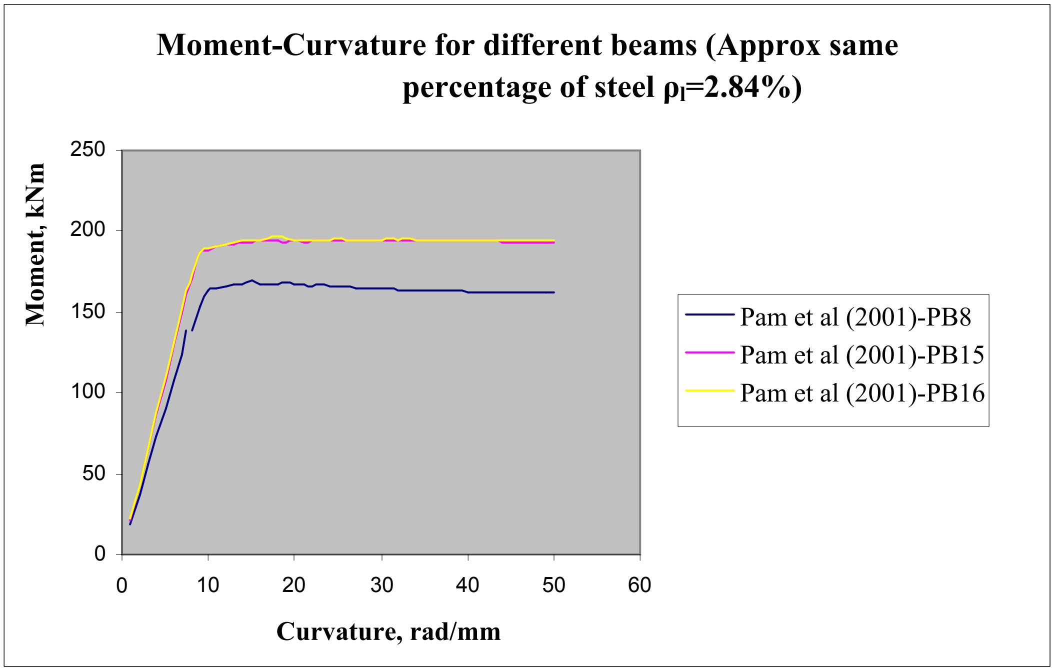


Figure 23. Analytical Moment-Curvature for Beams having similar ρ_l but varying f'_c tested by Pam et al (2001)

A comparison of the four beams tested by Sarkar et al (1997), with different reinforcement ratios, and similar range of concrete compressive strengths, f'_c =90MPa (HSC2-4), f'_c =78 MPa (HSC3-3), f'_c =87 MPa (HSC4-2), f'_c =82 MPa (HSC4-2) shows that the beams with higher tensile reinforcement ratio exhibit a stiffer response (Fig. 24). Each beam is found to exhibit different moment-curvature response depending upon the reinforcement ratio ρ_l, and concrete compressive strength, $f'_{c..}$

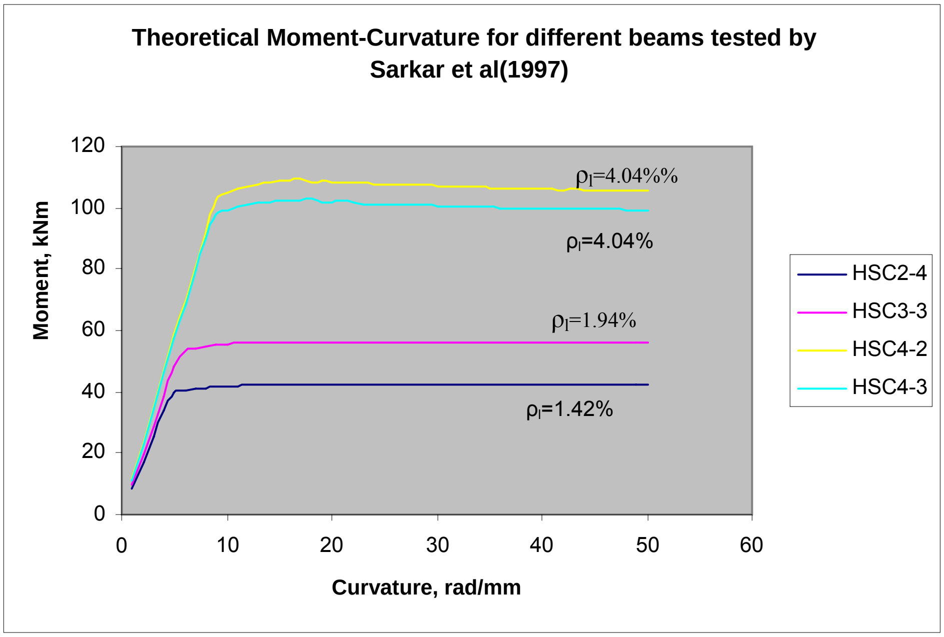


Figure 24. Analytical Moment-Curvature for Beams having similar range of f'_c but varying ρ_l , tested by Pam et al (2001)

4.4 Flexural Ductility Analysis

A comparison of theoretical moment curvature curves of beams obtained with the program FRMPHI using the stress-strain model of Setunge et al (1994), involving different concrete compressive strengths shows a similar trend for all the beams. The general tendency of all the beams is elastic-plastic.

The ductility of a beam can be assessed in terms of its ductility factor, μ, which is measured from the moment–curvature curves. The yield curvature, $\varphi_{y,}$, and the ultimate curvature, φ_u at the ultimate moment M_p, have been determined from the moment-curvature relationship of the beams depicted in Appendix B. The curvature ductility factors obtained for the beam specimens using the equation, $\mu=\varphi_u/\varphi_y$, have been given in Table 10.

Reference	*Beam No.*	*f'c*	*% of steel*	*Yield Curvature (Φ_y)*	*Ult.Curvature (Φ_u)*	*Ductility*
		MPa	ρ_l	*rad/mm*	*rad/mm*	$\mu=\varphi_u/\varphi_y$
Pam et al	1	37.4	0.76	5.57E-05	3.02E-04	5.41
-2000	2	36.8	1.14	5.06E-05	1.75E-04	3.45
	3	36.4	1.89	3.74E-05	9.74E-05	2.60
	4	42.3	1.89	4.19E-05	1.06E-04	2.53
	5	46.4	2.28	4.02E-05	1.05E-04	2.62
	7	58.6	2.49	4.34E-05	1.39E-04	3.20
	8	57.1	2.86	3.80E-05	1.12E-04	2.94
	9	58.6	3.53	3.06E-05	8.62E-05	2.82
	14	95.5	1.89	8.80E-05	3.18E-04	3.61
	15	98	2.84	6.01E-05	1.95E-04	3.24
	16	102.5	2.84	7.00E-05	2.10E-04	3.00
	17	87	3.14	5.78E-05	1.56E-04	2.70
Sarkar et al	HSC1-1	107	1.03	2.36E-04	6.40E-04	2.71
1997	HSC1-2	97	1.03	2.02E-04	5.99E-04	2.97
	HSC1-3	85	1.03	1.98E-04	5.60E-04	2.82
	HSC2-1	105	1.42	2.02E-04	5.45E-04	2.70
	HSC2-2	100	1.42	1.53E-04	5.38E-04	3.51
	HSC2-3	77	1.42	1.30E-04	4.67E-04	3.58
	HSC2-4	90	1.42	1.61E-04	5.04E-04	3.14
	HSC3-1	107	1.94	1.32E-04	4.66E-04	3.53
	HSC3-2	85	1.94	1.03E-04	3.95E-04	3.84
	HSC3-3	78	1.94	1.53E-04	3.96E-04	2.59
	HSC4-1	101	4.04	6.47E-05	1.88E-04	2.90
	HSC4-2	87	4.04	5.78E-05	1.54E-04	2.67
	HSC4-3	82	4.04	6.29E-05	1.55E-04	2.47
Bernardo & Lopes	A1	62.9	1.52	8.05E-05	2.73E-04	3.39
2004	A2	64.9	2.04	5.77E-05	1.61E-04	2.78
	A3	64.1	2.21	4.86E-05	1.57E-04	3.23
	A4	63.2	2.86	3.74E-05	1.13E-04	3.01
	A5	65.1	2.86	4.52E-05	1.17E-04	2.58
	B1	79.2	1.59	9.41E-05	3.65E-04	3.88
	B2	78.9	2.09	6.30E-05	2.03E-04	3.22
	B3	78.5	2.16	6.28E-05	2.02E-04	3.22
	C1	82.9	2.11	7.24E-05	2.18E-04	3.01
	C2	83.9	2.16	7.22E-05	2.19E-04	3.04
	C3	83.6	2.69	6.35E-05	1.56E-04	2.46
	C4	83.4	2.7	6.35E-05	1.56E-04	2.46
	D1	88	1.36	1.19E-04	4.09E-04	3.44
	D2	85.8	3.61	4.06E-05	1.19E-04	2.92
	D3	86	3.61	4.07E-05	1.19E-04	2.92

TABLE 10. Summary of results for curvatures at yield and ultimate moment (continued next page)

Reference	*Beam No.*	*f'c*	*% of steel*	*Yield Curvature* (Φ_y)	*Ult.Curvature* (Φ_u)	*Ductility*
		MPa	ρ	*rad/mm*	*rad/mm*	$\mu=\varphi_u/\varphi_y$
	E1	94.6	2.73	6.82E-05	1.82E-04	2.67
	E2	90.2	2.8	6.29E-05	1.71E-04	2.72
	F1	100.3	1.96	9.52E-05	3.38E-04	3.55
	F2	105.2	2.66	7.82E-05	2.29E-04	2.93
S.A.Ashour	B-N2	48.61	1.18	1.09-04	3.23E-04	2.98
2000	B-N3	48.61	1.77	5.41E-05	1.27E-04	2.34
	B-N4	48.61	2.37	3.38E-05	9.56E-05	2.83
	B-M2	78.5	1.18	1.32E-04	4.56E-04	3.45
	B-M3	78.5	1.77	1.01E-04	3.37E-04	3.33
	B-M4	78.5	2.37	6.84E-05	2.39E-04	3.50
	B-H2	102.4	1.18	1.73E-04	5.69E-04	3.29
	B-H3	102.4	1.77	1.21E-04	4.11E-04	3.40
	B-H4	102.4	2.37	9.94E-05	3.25E-04	3.26

TABLE 10. Summary of results for curvatures at yield and ultimate moment

4.5 Neutral axis depth versus ductility

In order to study the ductility of beams in relation to the depth of neutral axis, some of the beams tested by Pam et al (2001) and Ashour (2000) have been selected as their set of data contained both NSC and HSC beams. The ductility factor of beams with concrete compressive strength less than 50MPa was limited. This is done to determine the neutral axis depth factor for each of the other beams (HSC) with the ductility factor remaining the same as that for NSC beam. The details are listed in Table 11. The neutral axis depth parameter is given by $k_u = d_n / d$

It is known that k_u is also used as an indicator of the ductility of a section, i.e. this term can be used in lieu of d_n to specify the neutral axis position. Generally for beams to ensure good ductile behavior, the value of k_u should not exceed 0.4. A choice of k_u well below 0.4 leads to larger cross-sectional dimensions, but good ductility and a smaller quantity of steel (Warner et al, 1998).

The depth of the neutral axis in failure is a controlling parameter for ductility in the failure section (Bernardo & Lopes, 2003). From this study, in both sets of data, it is observed that at approximately same ductility factor, the value of k_u is scattered and is found to be less than 0.4 in the first set of beams tested by Pam et al (2001). In the second test data (Ashour, 2000), a similar observation is made. The value of k_u for both NSC and HSC beams is found to be less than 0.4. The beam sections seem to possess good ductility, proving to be economic and structurally efficient. It is further observed that the beams with higher concrete strengths show a small value of k_u (Fig. 25), giving good ductility and a smaller quantity of steel but leading to larger cross-sectional dimensions. It may be, therefore, concluded that the value of k_u for high strength concrete beams follows similar limitation already observed and accepted for normal strength concrete beams.

Reference	*Beam No.*	*f'_c*	*% of steel*	*Yield Curvature (Φ_y)*	*Ult.Curvature (Φ_u)*	*Ductility*	*Adjusted Ductility*	*k_u*	*Moment (kNm)*	*N.A Depth*
		MPa	*ρ*	*rad/mm*	*rad/mm*	$\mu=\varphi_u/\varphi_y$	$\mu=\varphi_u/\varphi_y$			
Pam et al (2001)	1	37.4	0.76	5.57E-05	3.02E-04	5.41	5.41	0.152	56.8	39.5
	7	58.6	2.49	4.34E-05	1.39E-04	3.2	5.32	0.337	148.3	84.3
	16	102.5	2.84	7.00E-05	2.10E-04	3	5.3	0.263	193.1	65.6
Ashour (2000)	B-N2	48.61	1.18	1.09-04	3.23E-04	2.98	2.98	0.164	54.2	35.3
	B-M2	78.5	1.18	1.32E-04	4.56E-04	3.45	3.03	0.112	55.5	23.9
	B-M4	78.5	2.37	6.84E-05	2.39E-04	3.5	3.03	0.112	55.5	23.9
	B-H2	102.4	1.18	1.73E-04	5.69E-04	3.29	3.11	0.099	55.8	20.8
	B-H3	102.4	1.77	1.21E-04	4.11E-04	3.4	2.98	0.127	82.6	27.3

TABLE 11. Limited Ductility Factors and the respective k_u values for different beams

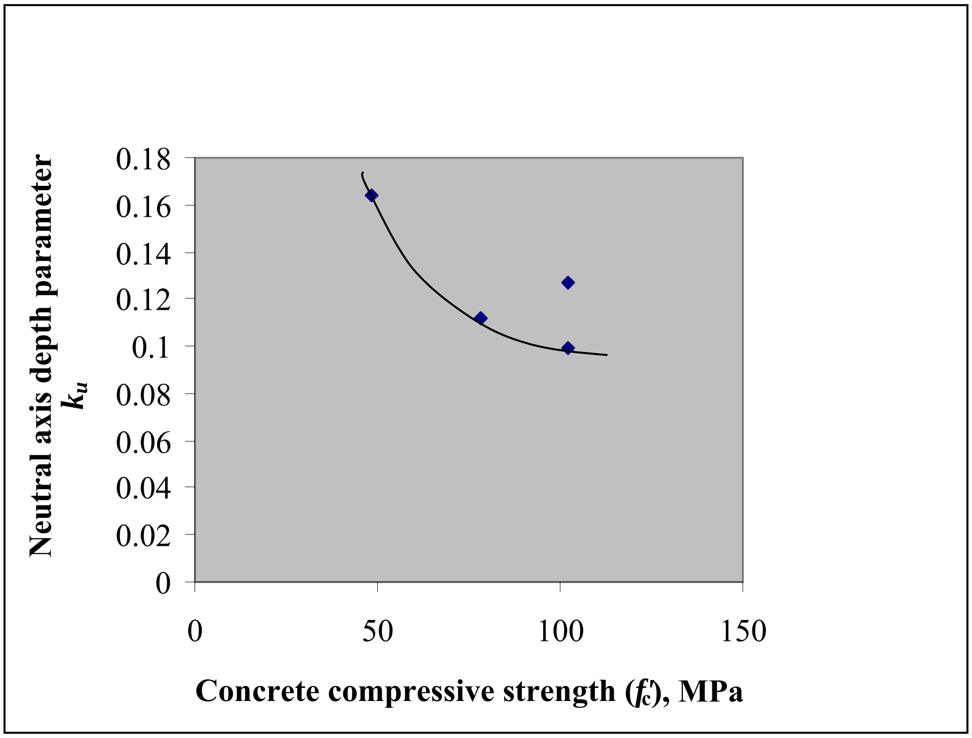


Figure 25. Concrete compressive strength vs. k_u

4.6 Factors affecting ductility:

4.6.1 Influence of f'_c on ductility

The major parameters influencing the ductility of concrete specimens can be analyzed. The ductility factors, μ, have been plotted against the concrete compressive strengths but similar reinforcement ratio (Fig. 26). Except for a few anomalous cases, it is observed that the ductility factors are generally lower at lower concrete strengths and higher at higher concrete strengths. But for the beams tested by Bernardo & Lopes (2004), μ decreased as f'_c increased upto 84MPa and then it is found to increase with the increase in f'_c. Sarkar et al (1997), noted that, although HSC is a less ductile material compared with

NSC, the ductility index for a specified reinforcement ratio, ρ_l of high-strength reinforced concrete section in flexure increases with the increase in compressive strength of concrete.

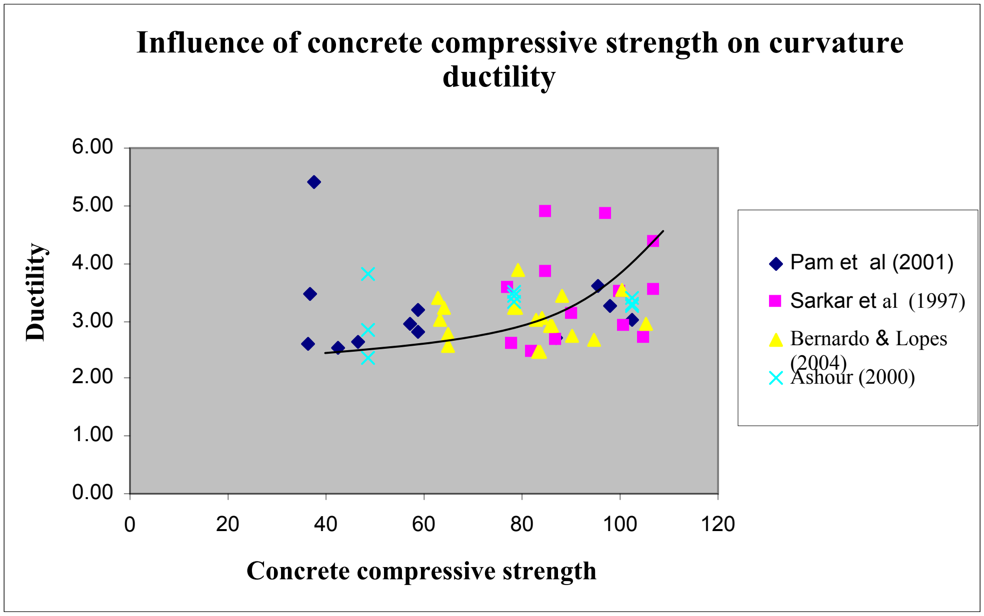


Figure 26. Effect of concrete compressive strength on curvature ductility

4.6.2 Influence of reinforcement ratio on ductility

The curvature ductility factor is plotted against different concrete grades for different steel reinforcement ratio ρ_l. It is observed that the ductility factors are generally lower at higher reinforcement ratios and higher at lower reinforcement ratios (Fig 27). Similar observations were made by Pam et al (2001), in their study on flexural strength and ductility of reinforced normal-and-high strength concrete beams. In an experimental investigation on high strength concrete beams in flexure, Sarkar et al (1997) noted that for almost the same concrete strength, the ductility index, μ decreases as the tensile reinforcement ratio, ρ_l increases. Bernardo & Lopes (2003) confirmed this observation that the reduction in curvature ductility indexes with an increase in longitudinal reinforcement ratio.

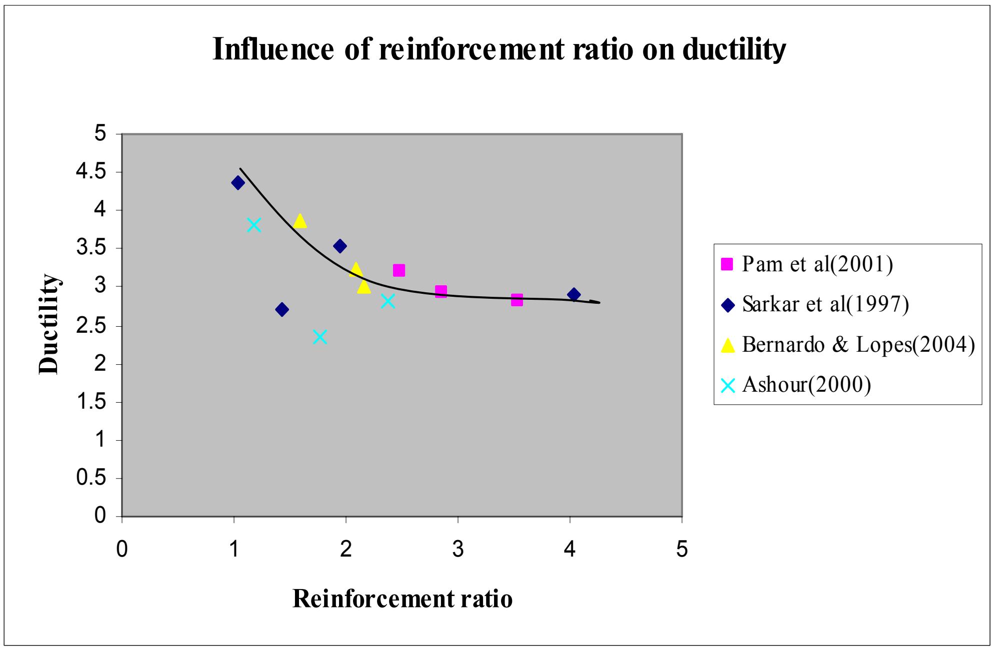


Figure 27. Effect of reinforcement ratio on curvature ductility

The study of the parameters influencing the ductility of high strength concrete beams gives the following conclusions.

- μ of a reinforced concrete section usually increases with increase in concrete compressive strength, and
- μ decreases with an increase in the longitudinal reinforcement (ρ_l).
- The ductility factor obtained was in the range of 2 to 5.

It has been found that the parameter which has a greater influence on the ductility of high strength concrete beams is the longitudinal tensile reinforcement ratio. The analytical research indicates that for approximately constant values of concrete compressive strength, high strength concrete beams experience a moderate reduction in ductility with increasing longitudinal reinforcement ratio.

The results, in confirmation with previously published papers, indicate that the increase in concrete compressive strength causes an increase in the ductility. As mentioned by Bernardo and Lopes (2003), that inspite of high-strength concrete being a more brittle material than normal-strength concrete, the ductility of reinforced concrete section increases with increasing concrete compressive strength. It may be noted that although the ductile behavior of high strength concrete beams in flexure is studied by various authors, the results on the topic are contradictory.

4.7 Conclusion

Work presented in this chapter indicates that at similar values of k_u, both high and normal strength concrete beams have similar levels of ductility. Therefore, it may be concluded that the provision of design codes of $k_u < 0.4$, which ensures ductility of NSC beams will also be applicable to HSC beams.

CHAPTER 5

SHORT TERM DEFLECTIONS OF SIMPLY SUPPORTED REINFORCED HIGH-STRENGTH CONCRETE BEAMS

5.1 INTRODUCTION

In recent years, high-strength concrete (HSC) has found many applications in columns of high-rise buildings and in a broad range of long-span flexural members (ACI Committee 363R-84; Ashour et al, 1997). The use of higher concrete strengths guarantees smaller cross-sections with reduced dead load and longer spans. However, for the members with smaller cross-sections, excessive deflections may become a problem. Unavailability of specific formulae to calculate the deflection of high strength concrete beams makes it difficult for structural engineers and builders to accept higher strength concrete. For an efficient use of high strength concrete, it is necessary for structural engineers to determine the effect of deflections using the current code provisions. This chapter includes analysis of the available formulae to calculate deflection to determine if these can be adopted for high strength concrete.

A number of studies on the performance of high strength concrete have been carried out for more than two decades. Although, these studies have produced significant and constructive results, the application of these to high strength concrete is hampered by gaps in a number of key design criteria. One of these properties is the deflection of simply supported high strength concrete beams.

5.2 Serviceability

5.2.1 Short-term deformations

For flexural members, smaller cross-sections are possible by the use of high strength concrete. This allows for the reduction of dead load and hence permits longer spans. Instantaneous deformations occur when a load is added to a building. Excessive deflections of the smaller cross-sections when subjected to larger loading may become a problem. The reduction in cross-sectional area reflects on the moment of inertia, I, of the member and the corresponding deflection under service load (Ashour, 2000). The flexural rigidity $E_c I_{eff}$ is the main parameter for determining the amount of deformation of a member.

In the evaluation of the deflection of the tested beams, the determination of cracking moment, modulus of elasticity and cracked moment of inertia are the most critical parameters. These parameters control the serviceability and deflection calculation (Ashour, 2000). Theoretically, it would be expected that deflections of HSC members should be lower compared to those of NSC members with the same reinforcement ratio, the reason attributed mainly to the higher modulus of elasticity of the concrete and the increased cracking moment and to a little extent to the increase of the stiffness in the cracked state (Lambotte and Taerwe, 1990).

As the compressive strength of concrete increases, the stiffness also increases, leading to a decrease in overall deflection of flexural members. A difference in the elastic modulus of concrete for different concrete strengths is demonstrated in Fig. 28.

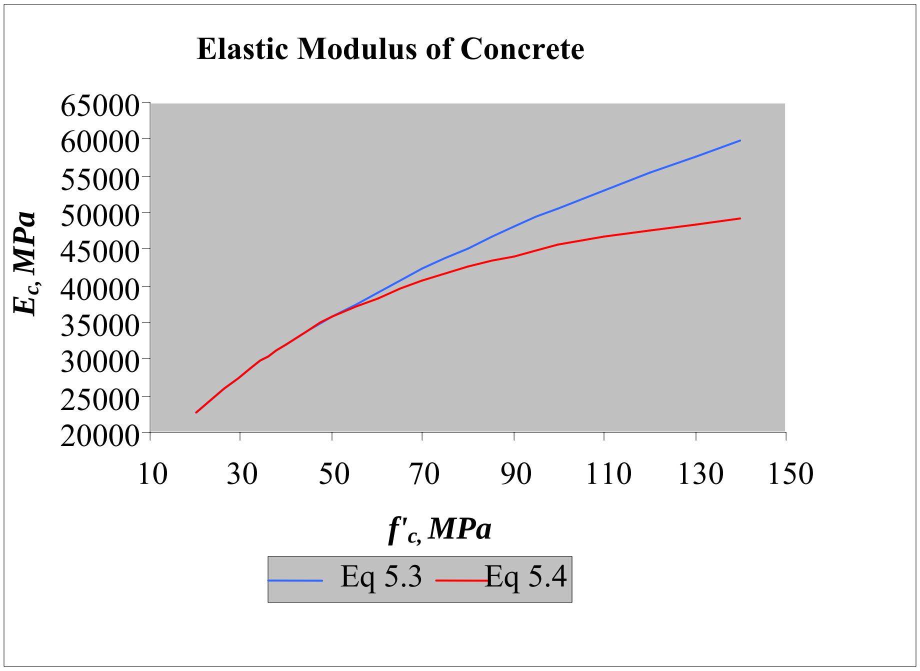


Figure 28. Difference in Elastic Modulus of NSC and HSC

The NSC curve has been calculated using AS3600-2001 code formula, while the HSC curve has been calculated using the formula proposed by Mendis et al (2001) given in Equation 5.4., which is as follows:

$$Ec = 0.043\eta\rho^{1.5}\sqrt{f'_c} \pm 20\%$$

Where $\eta = 1.1 - 0.002 f'_c \leq 1.0$.

For most structures it is often necessary to obtain the deflection at mid-span of a reinforced concrete simply supported beam. In this research, the deflections of high strength concrete simply supported beams have been investigated

5.2.2 Standards

The simplified procedures contained in AS3600-2001, for calculating the deflection of beams are insufficient in some situations. The calculated deflection is often significantly less than the actual deflection, and serviceability problem resulting from excessive deflection are not uncommon for structures designed in accordance with the code (Gilbert, R. I. 1999).

According to the Australian Standard AS3600, there are two methods of calculating the deflection of simply supported beams with uniformly distributed load applied which are shown in Equation 5.1 and Equation 5.2.

$$\frac{L_{ef}}{d} \leq \left[\frac{k_1 \left(\Delta / L_{ef}\right) b_{ef} E_c}{k_2 F_{d.ef}}\right]^{1/3} \ldots\ldots\ldots\ldots\ldots (5.1)$$

DEEM-TO-COMPLY METHOD

$$\Delta = \frac{5}{384} \frac{wL^4}{E_c I_{eff}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.2)$$

RIGOROUS METHOD

The above methods are mostly dependent on the calculation of elastic modulus, E_c and the characteristic flexural tensile strength, f'_{cf}, of concrete to calculate the deflection at mid-span. The effective moment of inertia I_{eff} for a particular section lies between the moment of inertia of that section in the cracked state, I_{cr} and that in the uncracked state, I_g (Warner et al, 1998). The calculation of the effective moment of inertia I_{eff} is also an important variable in these formulae and includes the characteristic flexural tensile strength, f'_{cf} in the calculation.

These above methods of calculating deflection provide different types of results with respect to the deflection. The deem-to-comply method gives a value to which the deflection should be limited to. This suggests that if the members satisfy the span-depth limits, calculation of deflections can be avoided (Warner et al, 1998). The span-to-depth ratio, L_{ef}/d shall not be greater than the value given, which depends on the span, load, elastic modulus, width of the beam and type of span.

The second method or the rigorous method gives a numerical value of the amount of deflection expected depending on the span, load, elastic modulus and effective moment of inertia.

The calculation of deflection using the above formulae depends on the elastic modulus, E_c the characteristic flexural tensile strength, f'_{cf} and the effective moment of inertia, I_{eff}. An evaluation of these variables is needed with respect to the current code and other recent publications. This chapter includes the evaluation of current methods for calculating these variables, for their use in the investigation of deflections of simply supported high strength concrete beams.

The above formulae for calculating the mid-span deflection are applicable for simply supported beams subjected to a uniformly distributed load. Hence, the beams considered in the present study needed to be subjected to uniformly distributed load, or an equivalent uniformly distributed load (UDL) transformed load from two point loads.

5.2.3 ELASTIC MODULUS OF CONCRETE, E_c

Several methods for calculating the elastic modulus of concrete available in the current codes and published literature have been reviewed for this investigation. The modulus of elasticity depends on parameters such as the volume of aggregates, the modulus of the paste and the modulus of the aggregates. Complex equations incorporating these factors and as suggested by various researchers are reviewed by Setunge (1994).

The short-term deflections which occur immediately due to external loads and pre-stressing can be determined from the flexural stiffness given by $E_c I_{ef}$ where E_c is the characteristic modulus of elasticity.

According to the Australian Standard, AS3600 (2001), the elastic modulus can be calculated using the empirical expression

$$E_c = 0.043(\rho)^{1..5}\sqrt{f'_c} \pm 20\% \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(5.3)$$

Where f'_c is the mean value of the compressive strength of concrete at the relevant age.

The above formula has been derived for normal strength concretes. It has been shown that for application to high strength concretes, this formula has been extrapolated. Setunge (1994) has shown that the existing formula for E_c as per AS3600 has the tendency to overestimate the elastic modulus of high strength concrete.

Mendis (2001) proposed the following expression to calculate the elastic modulus for all grades of normal and high strength concrete.

$$Ec = 0.043\eta\rho^{1.5}\sqrt{f'_c} \pm 20\% \text{ .... } (5.4)$$

Where $\eta = 1.1 - 0.002 f'_c \leq 1.0$

The above expression has been developed for use with high strength concrete and has been formulated by calibrating experimental results and comparing them with the Carrasquillo et al (1981) formula given in Eq. 5.5

$$E_c = (3320\sqrt{f'_c} + 6990)\left(\frac{\rho_c}{2320}\right)^{1.5} \text{ .... } (5.5)$$

In the investigation of reinforced high strength concrete beams in flexure, Rashid et al (2002) suggested that the values of modulus of elasticity for normal strength concrete as well as high strength concrete can be estimated using Eqn. (5.6). They concluded that the ACI Code specifications are found to underestimate the maximum deflection of reinforced HSC beams at service load.

$$E_c = 8900(f'_{c,150})^{0.33} \text{ .... } (5.6)$$

Rashid et al (2005) also found that the effect of pre-tension in the beams induced by shrinkage was a substantial factor in the accuracy of results. The fact that pretension induced by shrinkage is sustained in nature, an attempt was made to include the effects of resulting creep to see if the predictions could be improved further. This effect is not taken into account for this investigation as it only deals with short term deflection of beams and not the long term deflection where increased shrinkage would greatly affect the behavior of beams.

A comparison between different formulae for the calculation of elastic modulus of concrete has been shown in Fig. 29. A variation in each formula is observed and the resulting value of E_c is either higher or lower depending on the formula used. From Figure 29, it can be seen for Equation 5.3, that once f'_c increases above 50MPa, the formula given by AS3600 greatly overestimates the value of E_c if extrapolated. Similar is the case for other formulae.

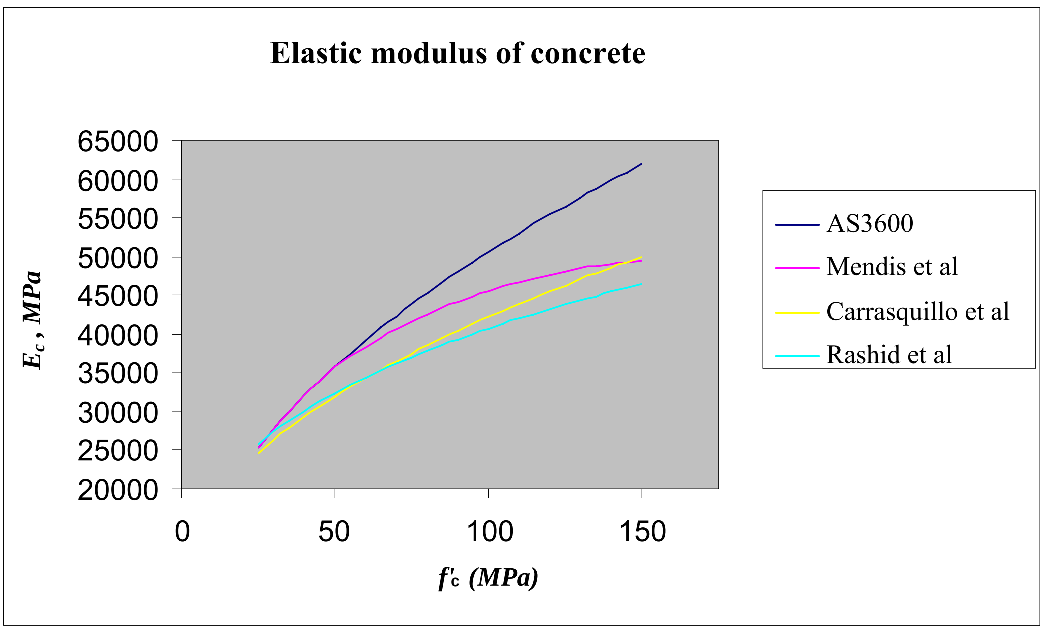


Figure 29. Elastic Modulus of Concrete

5.2.4 EFFECTIVE MOMENT OF INERTIA, I_{eff}

The use of high strength concrete permits the reduction of sizes and weights of structural elements especially for long spans of beams. The reduction in size in turn affects the moment of inertia of the members and hence the deflection under service loads (Ashour, 2000). NSC and HSC have different cracking behavior which may directly affect the effective moment of inertia; therefore, the effect of increased concrete strength must be evaluated because the formula for I_{eff}, was derived statistically from tests on NSC flexural members (Fasching and French, 1998). Concrete is good in compression but weak in tension. As a beam is subjected to flexure, the tensile zone cracks, reducing the second moment of inertia, I_g of the beam. That is, as the concrete cracks in the tensile zone of the beam section, the capacity of the section is reduced to the effect of the compression zone of the concrete beam and the tensile capacity of the tensile steel reinforcement. The approximation of the cracked beam section is known as the effective moment of inertia, I_{eff}, and lies somewhere between the cracked moment of inertia, I_{cr}, and the uncracked moment of inertia, I_g. There are two ways of calculating the effective moment of inertia.

The first is a simplified method of calculating the effective moment of inertia based on an estimation of the cracked moment of inertia and gross-section as in AS3600-2001.

$I_{eff} = 0.045bd^3$ ………………………………………… (5.7)

The above equation does not make allowance for the expected load history and the effects of cracking and the bending stiffness. Hence equation 5.7 may give rise to more conservative and uneconomical design.

The second method stated in AS3600-2001 Cl.8.5.3.1 recommends Branson's formula for calculating the effective moment of inertia. This formula which includes stiffness characteristics has been used to calculate the effective moment of inertia since 1968.

$$I_{ef} = I_{cr} + (I_g - I_{cr})(\frac{M_{cr}}{M_s})^3 \leq I \text{ ....(5.8)}$$

Where

I_g= moment of inertia of the uncracked section.

I_{cr} = moment of inertia of cracked section.

And $I_{cr} = \frac{1}{3} d_n{}^3 + nA_{st}(d - d_n)$ for a rectangular section

M_s= the service bending moment at the section, based on the short-term serviceability or the construction load

M_{cr}= cracking moment at midspan of the beam

$$M_{cr} = f'_{cf} \frac{I_g}{y_t} = Zf'_{cf}$$

y_t = distance form the centroidal axis of the member to the extreme tension fibre.

f'_{cf} = the characteristic flexural tensile strength of concrete and $f'_{cf} = 0.6\sqrt{f^1{}_c}$.

The inclusion of the cracking moment involves the calculation of the characteristic flexural tensile strength, f'_{cf} into the deflection calculations, which can vary depending on the formula used.

Of the above two methods of calculating the effective moment of inertia, the simplified method is easier to calculate but the second method would give more accurate results.

ACI Building Code has adopted, since 1971, the expression developed by Branson for computation of the effective moment of inertia, I_{eff}, over the entire length of a simply supported beam, but recommends a higher value for f'_{cf}.

Branson's formula gives reasonably accurate results for the effective moment of inertia provided the tensile steel ratio $\rho_l = A_{st}/bd$ is not less than 0.005. Low reinforcement ratio can lead to an over-estimation of I_{eff} and under-estimation of short-term deflections (Warner et al, 1998). Lambotte and Taerwe (1990) suggested that while calculating the deflections using Branson's formula, a better fit for the high strength concrete beams might be obtained when taking the exponent of $\frac{M_{cr}}{M_s}$ higher than 3.

5.2.5 CHARACTERISTIC FLEXURAL TENSILE STRENGTH

The tensile strength of high strength concrete is higher than normal strength concrete. The reason being the tensile cracks in high strength concrete which have a smooth interface due to which, the contribution from aggregate interlock is reduced. This may be due to very low water-cement ratios, use of high quality materials and high quality control techniques employed in the production of HSC.

The characteristic flexural tensile strength is critical when calculating the cracking moment, M_{cr}, of a beam.

$$M_{cr} = Zf'_{cf} \quad \text{..} \quad (5.9)$$

According to AS3600, the characteristic flexural tensile strength of concrete, f'_{cf} is given by Eq. 5.10.

$$f'_{cf} = 0.6\sqrt{f'_c} \quad \text{..} \quad (5.10)$$

ACI Committee 364 recommends similar formula for the calculation of the flexural tensile strength. This is given by Eqn. (5.11)

$$f'_{cf} = 0.59\sqrt{f'_c} \quad \text{..} (5.11)$$

An equation for the flexural tensile strength of concrete proposed by ACI State-of-the-art report (1990) can be adopted as an upper bound given by the equation 5.12. This formula is based on high strength concrete and takes into account the effects of the higher tensile strength of the concrete.

$$f'_{cf} = 0.94\sqrt{f'_c} \quad \text{...} \quad (5.12)$$

For high strength concrete, Mendis (2001) recommends the use of AS3600-01 equation 5.10 to reduce any overestimation of the tensile concrete strength.

Experimental tests on high strength concrete cylinders conducted by Sarkar et al (1997) showed that the recommended formulae by Mendis, 2001; Carrasquillo et al,1981; ACI Committee 364, and Shuaib & Shah, 1985, underestimated the tensile strength of high strength concrete and observed that the expression given by Raphael (1981) showed a general agreement with the test results.

The formula proposed by Raphael (1981) follows:

$$f'_{cf} = 0.342(f'_c)^{\frac{2}{3}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5.13)}$$

In the experimental study by Sarkar et al (1997), a linear regression analysis was employed in order to obtain a relationship between f'_{cf} and f'_c of high strength concrete mixes given by Eq 5.14.

$$f'_{cf} = 0.564(f'_c)^{0.55} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5.14)}$$

The above equation is in close agreement with that by Raphael (1981) and provides a more accurate representation of the tensile strength of high strength concrete.

ACI Building Code suggests Equation 5.15 to calculate the flexural tensile strength of high strength concrete.

$$f'_{cf} = 0.42(f'_{c,150})^{0.68} \qquad \ldots\ldots\ldots\ldots\ldots \text{(5.15)}$$

A comparison between the above formulae shows the difference as depicted in Fig. 30. It was seen that the Australian Standard is the most conservative. The two ACI methods are found to be much higher than the rest of the accepted standards.

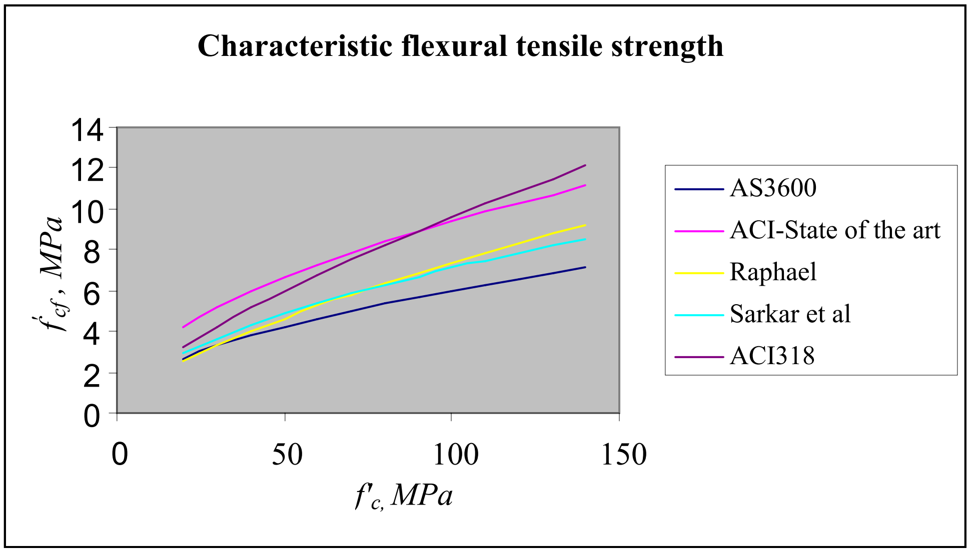


Figure 30. Characteristic Flexural Tensile Strength

5.3 METHODOLOGIES AND DATA COLLECTION

5.3.1 Experimental Data

Two main sets of data have been used for this investigation. The experimental data and the theoretical data calculated using the different formulae and variables involved in the deflection calculations. The experimental data includes beams tested by Rashid and Mansur, 2005 in their investigation on HSC beams in flexure. A comparison between the deflection methods has been made for the purpose of analysis.

The key parameters involved in this investigation were characteristic compressive strength, f'_c, dimensions, tensile and compression steel reinforcement ratios, span of beam tested, type and position of loads and the deflections at yield, load vs. central deflection graphs.

5.3.2 Loading arrangement

It should be noted that the loads applied to the beams in this experiment are in the form of two point loads, rather than a uniformly distributed load as required in the deflection formula. All the investigations carried out in the past, involved concentrated loading of the beams. No experiments could be found which used a uniformly distributed load, therefore the two point loads are transformed to an equivalent uniformly distributed load and applied to the deflection formulae.

5.3.3 DATA COLLECTED

Rashid & Mansur (2005)

In an experimental investigation, sixteen reinforced concrete beams were subjected to flexure to evaluate the implications of high strength concrete. The test parameters involved the concrete compressive strength, ratios of tensile and compressive reinforcements, and spacing of lateral ties. Each of the beam specimens, is designated by a specific number, such as A111 or B312, etc. the letters A, B, C, D and E stand for $f'c$ of 42.8, 72.8 to 77.0, 85.6 to 88.1, 114.5, and 126.2MPa, respectively. The first numeral -1, 2, 3, 4, or 5 indicates the tensile reinforcement ratio in percentage, the second numeral 1, 2, or 3 indicates the minimum, twice the minimum, and thrice the minimum ratio of compression reinforcement, respectively, while the third numeral -1, 2, or 3- stands for the similar quantities of the volumetric ratio of lateral ties. The data obtained from the publication has been used to carry out an analysis between the actual deflections and the calculated deflections. Table 12 gives the details of the beam specimen tested by Rashid and Mansur, 2005.

		BEAM PROPERTIES							LOADS	
			Dimensions			Reinforcement Ratio			At Yield	
SOURCE	BEAM	f'_c, MPa	b, mm	D, mm	d, mm	Tensile Steel, %	Compression Steel, %	L_{eff}, m	Load, kN	Deflection, mm
M.A.RAHSID & M.A.MANSUR MAY/JUNE 2005	A111	42.8	250	400	362	1.25	0.30	3.4	300.78	7.70
	A211	42.8	250	400	362	2.20	0.30	3.4	440.48	15.50
	B211	74.6	250	400	362	2.20	0.30	3.4	456.70	14.60
	B311	72.8	250	400	362	3.46	0.31	3.4	575.68	17.00
	B312	72.8	250	400	362	3.46	0.31	3.4	560.36	16.90
	B313	72.8	250	400	362	3.46	0.31	3.4	580.26	16.10
	B321	77.0	250	400	362	3.46	0.62	3.4	551.16	15.50
	B331	72.8	250	400	362	3.46	0.94	3.4	590.38	16.50
	B411	77.0	250	400	362	4.73	0.32	3.4	621.34	15.50
	C211	85.6	250	400	362	2.71	0.30	3.4	560.94	18.20
	C311	88.1	250	400	362	3.22	0.31	3.4	605.46	18.00
	C411	85.6	250	400	362	4.26	0.32	3.4	722.56	19.30
	C511	88.1	250	400	362	5.31	0.33	3.4	811.82	20.80
	D211	114.5	250	400	362	3.22	0.30	3.4	506.00	16.00
	E211	126.2	250	400	362	2.20	0.30	3.4	506.00	15.90

TABLE 12. Details of beams tested by Rashid & Mansur (2005)

5.3.4 Transforming Two Point Loads to an Equivalent Uniformly Distributed Load (UDL)

In order to transform the two point loads into an equivalent uniformly distributed load (UDL) the following formula was found in the Lysaght - Purlins & Girts User's Manual to convert two symmetrically loaded point loads on a simply supported beam to a uniformly distributed load.

$$w = \frac{8bp}{L^2} \qquad \text{……………………..……… (5.16)}$$

Where: $p = \frac{P}{2}$

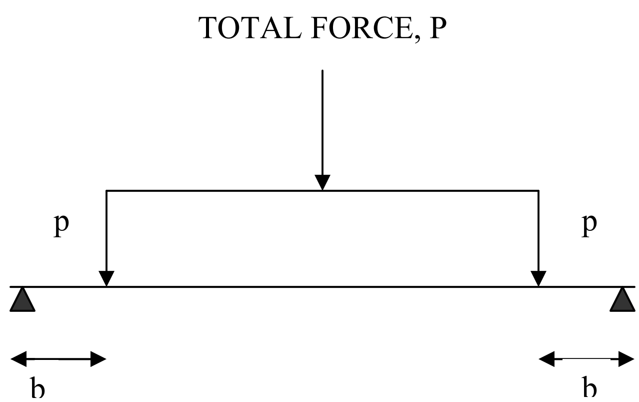


Figure 31. Point Load Equivalent

Using this simple transformation, all the point loads can be represented by an equivalent UDL. This can be seen here in an extract from the deflection calculations (Table 13)

Load At Yield			
Load P (kN)	Deflection, mm	Equivalent Point Loads, p (kN)	Load as UDL, kN/m
P	Δ	=P/2	$\frac{8bp}{L^2}$
300.78	7.70	150.49	125.0
440.48	15.50	220.39	183.0
456.70	14.60	228.51	189.8

TABLE 13. Example of Equivalent UDL

5.3.5 Error in the transformation of two point loads

It is found that when calculating the deflection of a simply supported beam with two symmetrically placed point loads using Eqn. 5.17, there is difference when comparing the results with the deflection calculated using Eqn. 5.2 for an equivalent UDL.

$$\Delta = \frac{pb}{24EI^2}(3L^2 - 4b^2) \qquad \text{………………… (5.17)}$$

When comparing this deflection to the deflection given by an equivalent UDL, load calculated by Eq. 5.2 there is an error of 0.069%. This is evident in the calculation of an arbitrary beam with any given load, as shown in Table 14.

BEAM PROPERTIES:

E_c =35,000MPa I= 988x10^6 mm^4 L=3.4m b=1.2m

1	2	3	4	5	6	7
2 Point Loads, kN	Deflection mm	2 Loads as UDL, kN/m	Deflection mm	Difference mm	Percentage Difference	Adjusted Deflection ,mm
p	$\Delta = \frac{pb}{24EI^2}(3L^2 - 4b^2)$	$w = \frac{8bp}{L^2}$	$\Delta = \frac{5}{384}\frac{wL^4}{E_c I_{eff}}$	2-4	2/4	4x6
10	0.4180	8.30	0.4177	0.00029	1.00069	0.4180
20	0.8361	16.61	0.8355	0.00058	1.00069	0.8361
40	1.6722	33.22	1.6710	0.00116	1.000069	1.6722

TABLE 14. Example of Deflection Correction Factor

By applying the small correction factor of 1.0069 to the deflection calculated using the transformed equivalent UDL, the accuracy of the final deflection is increased.

5.3.6 Theoretical Deflection Calculations

All the data has been collected from an actual experiment by Rashid & Mansur (2005) along with the necessary calculations to adjust or calculate the required information, and the theoretical load vs. deflection curves have been generated. Table 15 compiles all of the variables found during the literature review to calculate E_c and f'_{cf} .

METHOD	ELASTIC MODULUS OF CONCRETE, Ec	CHARACTERISTIC FLEXURAL TENSILE STRENGTH, f'cf
Method 1	$E_c = 0.43\rho^{1.5}\sqrt{f'_c} \pm 20\%$	$f'_{cf} = 0.6\sqrt{f'_c}$
Method 2	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c} \pm 20\%$	$f'_{cf} = 0.94\sqrt{f'_c}$
Method 3	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$f'_{cf} = 0.342(f'_c)^{\frac{2}{3}}$
Method 4	$E_c = 8900(f'_{c,150})^{0.33}$	$f'_{cf} = 0.564(f'_{cf})^{0.55}$
Method 5		$f'_{cf} = 0.42(f'_{c,150})^{0.68}$

TABLE 15. Variables in each formula

As shown in Table 16, there are four methods for calculating each variable, along with I_{eff} which gives a total of 24 methods to calculate each of the deem-to-comply and rigorous method forms of deflection.

Method	E_c	I_{eff}	f'_{cf}
1	$0.043\rho^{1.5}\sqrt{f'_c}$	$0.045bd^3$	-
2	$0.043\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.6\sqrt{f'_c}$
3	$0.043\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.94\sqrt{f'_c}$
4	$0.043\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.342(f'_c)^{\frac{2}{3}}$
5	$0.043\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.564(f'_{cf})^{0.55}$
6	$0.043\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.42(f'_{c,150})^{0.68}$
7	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c}$	$0.045bd^3$	-
8	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.6\sqrt{f'_c}$
9	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.94\sqrt{f'_c}$
10	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.342(f'_c)^{\frac{2}{3}}$
11	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.564(f'_{cf})^{0.55}$
12	$E_c = 0.43\eta\rho^{1.5}\sqrt{f'_c}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.42(f'_{c,150})^{0.68}$
13	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$0.045bd^3$	-

TABLE 16. Summary of different methods to calculate deflections (continued on next page)

Method	E_c	I_{eff}	f'_{cf}
14	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.6\sqrt{f'_c}$
15	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.94\sqrt{f'_c}$
16	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.342(f'_c)^{\frac{2}{3}}$
17	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.564(f'_{cf})^{0.55}$
18	$E_c = (3320\sqrt{f'_c} + 6900)\left(\dfrac{\rho_c}{2320}\right)^{1.5}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.42(f'_{c,150})^{0.68}$
19	$E_c = 8900(f'_{c,150})^{0.33}$	$0.045bd^3$	-
20	$E_c = 8900(f'_{c,150})^{0.33}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.6\sqrt{f'_c}$
21	$E_c = 8900(f'_{c,150})^{0.33}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.94\sqrt{f'_c}$
22	$E_c = 8900(f'_{c,150})^{0.33}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.342(f'_c)^{\frac{2}{3}}$
23	$E_c = 8900(f'_{c,150})^{0.33}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.564(f'_{cf})^{0.55}$
24	$E_c = 8900(f'_{c,150})^{0.33}$	$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$	$f'_{cf} = 0.42(f'_{c,150})^{0.68}$

TABLE 16. Summary of different methods to calculate deflections.

With these methods the individual deflection of each of the 15 beams can be calculated and compared to the actual deflections. This has been completed in an excel spreadsheet which can be found in Appendix C.

In the above mentioned spreadsheet, the deflection of each of the 15 beams has been calculated for several loads, for each of the above methods. The loads, at which the deflections have been calculated, were deduced from the load-deflection curves. The yield load provided in the study is the major focus of the analysis.

5.4 Analysis

From the load-deflection graphs developed for each beam for the 24 different methods of calculating the deflection, it is observed that the deem-to-comply method is satisfied only once in the calculation of each method of deflection. This is for the beam with a concrete compressive strength of 42.8MPa. In all the other cases, the experimental L_{eff}/d ratio is always greater than the allowable ratio. It is, therefore, concluded that the deem-to-comply method is not suitable for the deflection calculations of high strength concrete beams.

The calculated deflection should be greater than the actual deflection for safe design of the beam and for a formula to be acceptable. A comparison of the actual deflection to the deflection found using different methods is shown in Table 17.

Likewise, it is seen that the rigorous method did not give many accurate predictions for the deflection of the beams. It is found that only methods, 1, 7, 13 and 19 conservatively predicted the actual deflection of the beams for 80% of the beams. All the methods involving the calculation of the effective second moment of inertia, overestimate the rigidity of the beam and produce theoretical deflection which are less than the actual deflection for beams with f'_c greater than 50MPa.

The beam which had a concrete compressive strength less than 50MPa shows a theoretical deflection which is more than the actual deflection. On the other hand, in most cases, it is found that the calculated deflection is almost double the actual deflection which is not quite favorable as this would make the design of the beam inefficient.

For methods 13 and 19, the theoretical deflection is conservative on most beams. Similar trend is observed with most beams in methods 1 and 7.

It is observed that methods utilizing the AS3600 method lead to inaccurate deflections, for a small amount of tensile reinforcement. This is because when this method is used for high strength concrete beams, it overestimates the elastic modulus leading to inaccurate values. Similarly, it is noted that methods involving Mendis formula for calculating the elastic modulus for beams with tensile reinforcement ratio less than 2%, produce inaccurate deflections. However, for tensile reinforcement greater than 2%, the deflections are found to be conservative.

Although, methods 13 and 19 produce theoretical deflections, which are greater than the actual deflections for all the beams except for beams A111 and A211, a comparison of these methods reveal that deflection in Method 13 is greater for f'_c ranging between 20 to 65MPa. This is because of a lower elastic modulus in this range. After 65MPa, the elastic modulus of Method 19 was found to be lower, and hence it gives greater deflections for f'_c greater than 65 MPa. It may be therefore, concluded that Method 13 is the most efficient formula to calculate the deflection of concrete beams upto 100MPa, and this uses Carrasquillo et al formula.

$$E_c = (3320\sqrt{f'_c} + 6900)\left(\frac{\rho_c}{2320}\right)^{1.5}$$

From this investigation, it is found that deem-to-comply method as in AS3600-2001 is unconservative for deflection calculation of high strength concrete beams. Only the rigorous method gives substantial results which could be applied in practice. The accuracy of this formula depends on the method used to calculate E_c and I_{eff}.

SOURCE	BEAM	f'c, MPa	Actual Deflection, mm	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7	Method 8	Method 9	Method 10	Method 11	Method 12	Method 13	Method 14	Method 15	Method 16	Method 17	Method 18	Method 19	Method 20	Method 21	Method 22	Method 23	Method 24
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	7.70	12.4	11.3	11.1	11.3	11.2	11.2	12.4	11.2	11.1	11.2	11.2	11.2	13.6	11.5	11.4	11.5	11.5	11.4	13.3	11.4	11.3	11.4	11.4	11.4
	A211	42.8	15.50	18.2	11.0	11.0	11.0	11.0	11.0	18.2	11.0	11.0	11.0	11.0	11.0	19.9	11.4	11.3	11.4	11.4	11.4	19.5	11.3	11.3	11.3	11.3	11.3
	B211	74.6	14.60	14.3	10.5	10.5	10.5	10.5	10.5	15.0	10.6	10.6	10.6	10.6	10.6	16.6	11.0	10.9	11.0	11.0	10.9	16.9	11.0	11.0	11.0	11.0	11.0
	B311	72.8	17.00	18.2	9.8	9.8	9.8	9.8	9.8	19.1	9.9	9.9	9.9	9.9	9.9	21.2	10.3	10.3	10.3	10.3	10.3	21.4	10.3	10.3	10.3	10.3	10.3
	B312	72.8	16.90	17.7	9.5	9.5	9.5	9.5	9.5	18.6	9.6	9.6	9.6	9.6	9.6	20.6	10.0	10.0	10.0	10.0	10.0	20.9	10.1	10.1	10.1	10.1	10.1
	B313	72.8	16.10	18.3	9.8	9.8	9.8	9.8	9.8	19.2	10.0	10.0	10.0	10.0	10.0	21.3	10.4	10.4	10.4	10.4	10.4	21.6	10.4	10.4	10.4	10.4	10.4
	B321	77	15.50	16.9	9.3	9.3	9.3	9.3	9.3	17.9	9.4	9.4	9.4	9.4	9.4	19.8	9.8	9.8	9.8	9.8	9.8	20.1	9.8	9.8	9.8	9.8	9.8
	B331	72.8	16.50	18.7	10.0	10.0	10.0	10.0	10.0	19.5	10.2	10.2	10.2	10.2	10.2	21.7	10.6	10.5	10.6	10.6	10.5	22.0	10.6	10.6	10.6	10.6	10.6
	B411	77	15.50	19.1	8.6	8.6	8.6	8.6	8.6	20.2	8.7	8.7	8.7	8.7	8.7	22.3	9.1	9.1	9.1	9.1	9.1	22.7	9.2	9.2	9.2	9.2	9.2
	C211	85.6	18.20	16.3	10.9	10.9	10.9	10.9	10.9	17.6	11.2	11.1	11.1	11.1	11.1	19.3	11.5	11.5	11.5	11.5	11.5	19.8	11.6	11.6	11.6	11.6	11.6
	C311	88.1	18.00	17.4	10.4	10.4	10.4	10.4	10.4	18.8	10.7	10.7	10.7	10.7	10.7	20.6	11.0	11.0	11.0	11.0	11.0	21.2	11.1	11.1	11.1	11.1	11.1
	C411	85.6	19.30	21.1	10.4	10.4	10.4	10.4	10.4	22.7	10.7	10.7	10.7	10.7	10.7	24.9	11.1	11.1	11.1	11.1	11.1	25.5	11.2	11.2	11.2	11.2	11.2
	C511	88.1	20.80	23.3	10.2	10.2	10.2	10.2	10.2	25.2	10.5	10.5	10.5	10.5	10.5	27.6	11.1	11.1	11.1	11.1	11.1	28.4	11.4	11.4	11.4	11.4	11.4
	D211	114.5	16.00	12.8	8.4	8.3	8.3	8.4	8.3	14.6	8.7	8.7	8.7	8.7	8.7	15.5	8.9	8.8	8.9	8.9	8.8	16.2	9.0	9.0	9.0	9.0	9.0
	E211	126.2	15.90	12.1	10.8	10.7	10.8	10.8	10.7	14.3	11.3	11.2	11.3	11.3	11.2	14.8	11.4	11.3	11.4	11.4	11.3	15.7	11.6	11.5	11.6	11.6	11.5

TABLE 17 Summary-Rigorous Methods

5.5 Influence of parameters on short-term deflections

5.5.1 Longitudinal reinforcement

The load-deflection responses of all the beams appear to be linear. That is, the deflection increases linearly with loading. A comparison of load-deflection behavior for different beams using Method 24 (Fig. 32) shows that the beams with higher values of tensile reinforcement have stiffer responses to loading due to a high moment of inertia. Beam E211 (f'_c =126.2 MPa, A_{st}=2200mm^2), is found to be less rigid than beams C511 (f'_c =88.1 MPa, A_{st}=5310 mm^2), and D211 (f'_c =114.5 MPa, A_{st}=3220 mm^2). This shows that the beam with higher steel content has a stiffer response in terms of load-deflection behavior. This tendency is mainly due to the larger effective moment of inertia due to the larger amount of tensile reinforcement. Similar observation was made by Ahmad & Barker (1991), in their study on high strength concrete beams.

In Method 20, a similar observation has been made (Fig. 33). The beams with higher reinforcement steel display a more rigid response compared to beams with lower reinforcement ratio.

Beam E211 had the highest concrete compressive strength (f'_c=126.2 MPa), compared to Beams C511 and D211. But both the beams C511 and D211 had more tensile reinforcement compared to the beam E211. These beams show greater deformation compared to the other beams. Hence, it may be noted that the tensile reinforcement significantly affects the deflection of a beam. This is the most important parameter in controlling deflections of HSC beams.

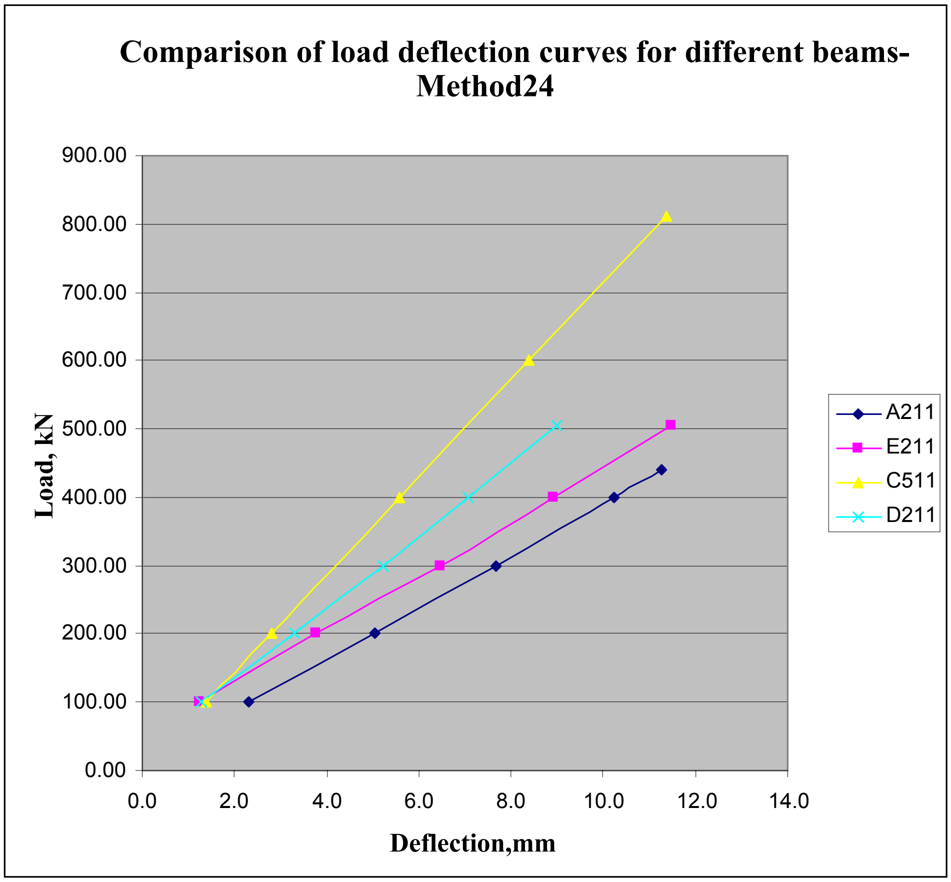


Figure 32. Load-Deflection Curves for different beams in Method 24

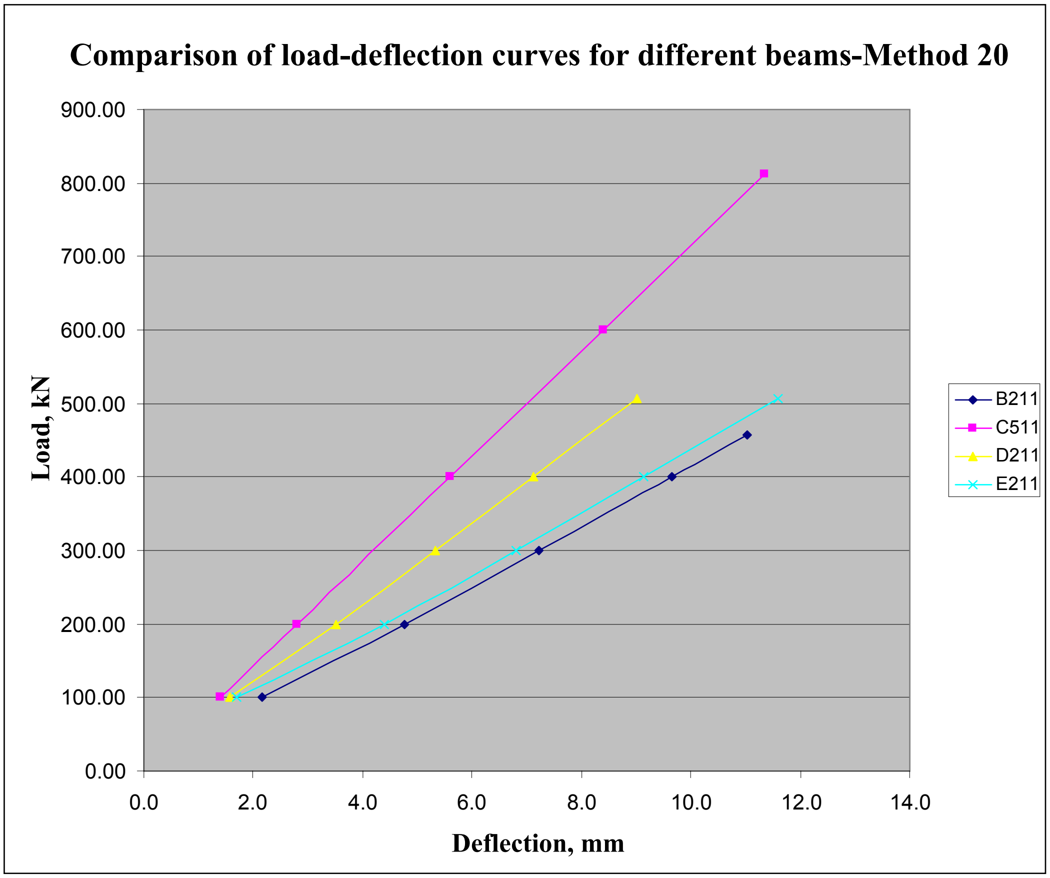


Figure 33. Load-Deflection curves for different beams in Method 20

5.5.2 Concrete compressive strength

Figs.34 and 35 present load (P) versus deflection curves obtained for different beams for Methods 23 and 18. Each figure, corresponding to three beams, chosen with varying compressive strength, but similar tensile reinforcement, includes the curves obtained for these beams belonging to that particular method of deflection calculation.

The load-deflection curves have been plotted from the predicted values of deflection corresponding to different values of increasing loads. From Fig. 34, it is observed that for a given range of tensile steel, and maintaining constant load level, the deformation capacity of the beams tends to increase as the concrete compressive strength increases. Hence, it seemed that with the longitudinal reinforcement remaining almost constant, the deflection capacity of beam increases with the increase in the compressive strength of concrete. However, Fig. 35 does not give a clear representation of the deflection capacity of the beams with the change in the concrete compressive strength. The capacity of beam C311 (f'_c =88.1 MPa) is found to increase, when compared to the capacity of the beam B311 (f'_c =72.8 MPa). The beam D211 (f'_c =114.5 MPa) did not show a similar response. The reason for this could not be identified. Nonetheless, it is suggested that more beam data should be incorporated to carry out the analysis.

It is found that based on limited data considered, deflection calculation of HSC beams requires further research.

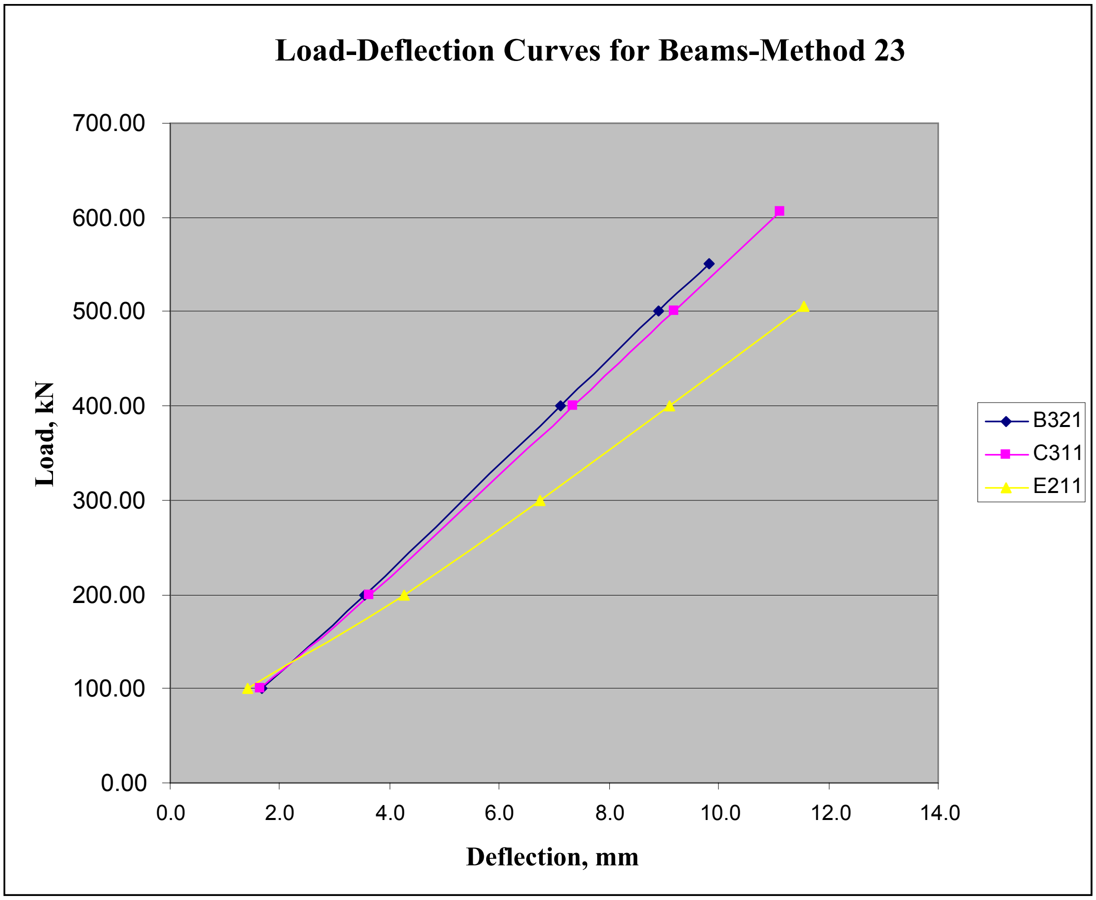


Figure 34. Load-Deflection curves for different beams in Method 23

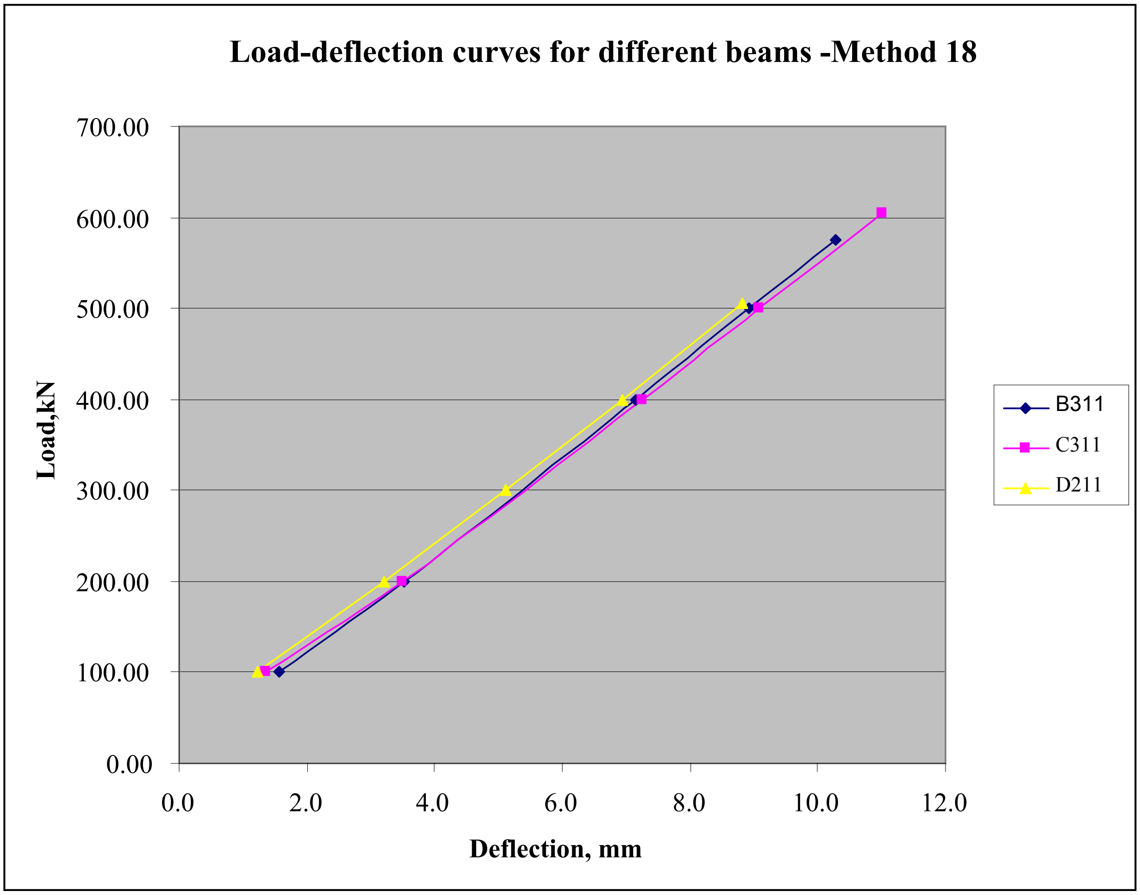


Figure 35. Load-Deflection curves for different beam as per Method 18

5.6 Conclusion

Based on the work presented in this chapter, it is concluded that the deem-to-comply method is not suitable for the calculation of deflection of HSC beams. However, further investigation is required to substantiate this, as only a limited data was considered for this study.

From all of the E_c methods, it is found that the Mendis (2001) formula produced the most accurate and conservative deflections, only when analyzing HSC beams with a tensile steel reinforcement ratio greater than 2% and f'_c less than 80MPa. The use of Carrasquillo et al (1981) formula for E_c gave conservative deflections for all beams with f'_c less than 100 MPa. From this investigation, it can be seen that there are no formulae currently used which can be applied to all high strength concrete beams and obtain accurate and conservative results.

Extensive analysis presented in this chapter clearly indicates that the deemed-to-comply method should not be used to estimate the deflection of HSC beams. Method 13, which uses Carrasquillo et al (1981) formula given in Eqn. 5.5, can be used to estimate the deflection of HSC beams with a reasonable accuracy.

$$E_c = (3320\sqrt{f'_c} + 6990)\left(\frac{\rho_c}{2320}\right)^{1.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(5.5)$$

Where, E_c =mean value of modulus of elasticity of concrete at 28 days,

f'_c = the characteristic compressive strength of concrete at 28 days,

ρ_c = density of concrete, taken as 2400kg/m^3

CHAPTER 6
SHEAR

6.1 INTRODUCTION

Researchers from their experimental and analytical studies have found that while many provisions of the code are applicable to high strength concrete materials and members, certain code provisions must be reexamined, modified, or limited to ensure structural safety and serviceability (Nilson, 1985). There is a concern about the shear strength of HSC beams due to the reduction in aggregate interlock compared to NSC beams. In high strength concrete, cracks progress through coarse aggregate, thus reducing the effect of shear resistance available in normal strength concrete due to aggregate interlock. Unavailability of exact guidelines for the calculation of shear strength of high strength concrete beams makes it complex for engineers to fully exploit high strength concrete as a structural material. The present study aims to analyse the available formulae in the Australian Code of Practice for calculating the shear strength with or without web reinforcement for its applicability to high strength concrete.

A comparison of test shear strength to predicted strengths have been carried out to investigate the precision of the available design guidelines for calculation of shear strength .The test shear strengths are obtained from beam tests on shear carried out in the past by various authors. The predicted shear strength is calculated as per AS3600-01 code provisions.

6.2. Existing recommendations for calculating shear strength in reinforced concrete beams

6.2.1 Australian Standards AS3600-2001

AS3600 gives two alternative methods for the design of beams in shear. The first method is based on truss analogy and is analyzed by the method of sections. The second method referred to as the SAA method is based on the equations provided in the Standard. The truss analogy method takes no account of the shear resisted by the concrete compressive zone, and therefore is over-conservative in this respect (Warner et al, 1998). The resistance to shear is a contribution by both concrete and steel. The ultimate shear strength V_u is given as:

$$V_u = V_{uc} + V_{us}$$

Where V_{uc} is the shear resisted by the concrete at ultimate and V_{us} is the shear resisted by the stirrups and the web concrete.

The value of V_{uc} for a reinforced concrete beam is given by

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_o \left[\frac{A_{st} f'_c}{b_v d_o} \right]^{1/3}$$

where,

$$\beta_1 = 1.1 \left[1.6 - \frac{d_o}{1000} \right] \geq 1.1$$

$\beta_2 = 1$; or

$= 1 - \left(N^* / 3.5 A_g \right) \geq 0$ for members subjected to significant axial tension; or

$= 1 + \left(N^*/14A_g\right)$ for members subject to significant axial compression

N^* is taken as the value of the axial force.

$\beta_3 = 1$;

A_{st} = cross-sectional area of longitudinal reinforcement provided in the tension zone and fully anchored at the cross-section under consideration.

6.3 Experimental Studies on Shear Strength of Normal and High Strength Concrete Beams

A number of beams with and without shear reinforcement from tests carried by other researchers are presented in the following sections. The details and shear capacities of these beams are also considered for the purpose of investigation and analysis.

6.3.1 REINFORCED CONCRETE BEAMS WITH SHEAR REINFORCEMENT

6.3.1.1 Kong and Rangan (1997)

An analytical and experimental investigation on the shear strength of High Performance Concrete (HPC) beams with vertical shear reinforcement or stirrups was carried out at the Curtin University of Technology, Perth. The analytical work involved developing a theory based on the truss analogy, capable of predicting the response and shear strength of such beams subjected to combined bending moment and shear force.

The experimental work comprised forty-eight beam specimens in eight series of tests. Most of the beams were 250 mm wide, 350 mm deep and had a clear span of approximately 2 meters. The largest beam was 250 mm wide, 600 mm deep and had a clear span of 3.1 meters. Test parameters included the concrete cover to the shear reinforcement cage, shear reinforcement ratio, longitudinal tensile steel ratio, overall beam depth, shear span-to-depth ratio and concrete compressive strength. The loading configurations varied and included using one, two or four symmetrically placed concentrated loads on simply supported spans.

The experimental results were correlated with the code predictions to check the applicability of the design provisions for high performance concrete beams. It was observed that the predictions by the shear design provisions available in the Australian Standard AS3600 (1994), the American Concrete Code ACI318-95, Canadian Standard A23.3-94 (1994), and the Eurocode-2 (1991) gave a large coefficient of variation, the reason attributed to the considerable scatter in measured shear strengths of the beams.

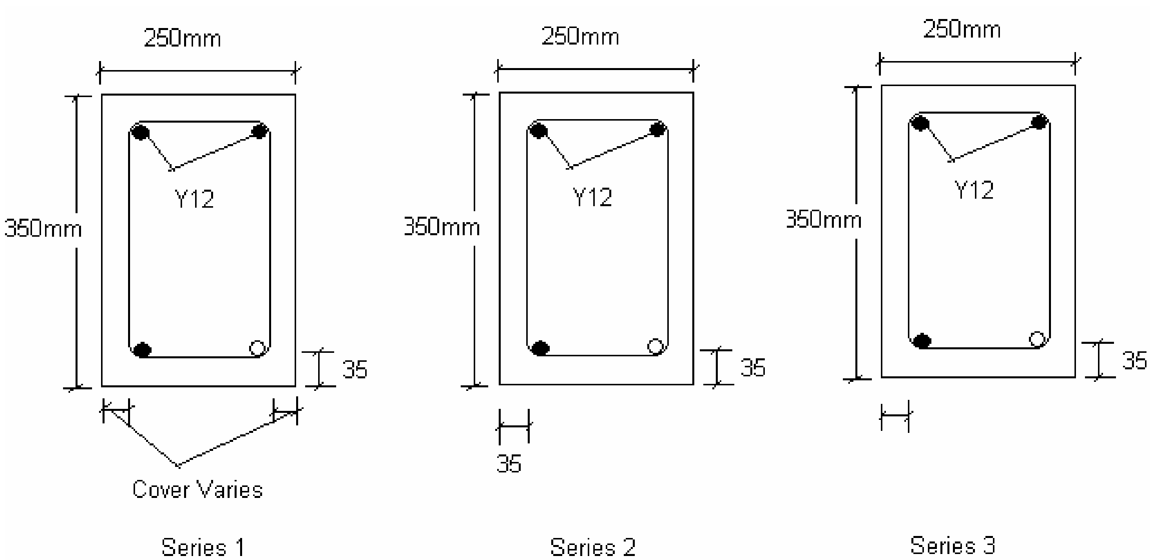


Figure 36. Cross-sections of reinforced concrete beams tested by Kong and Rangan (1998) (continued on next page)

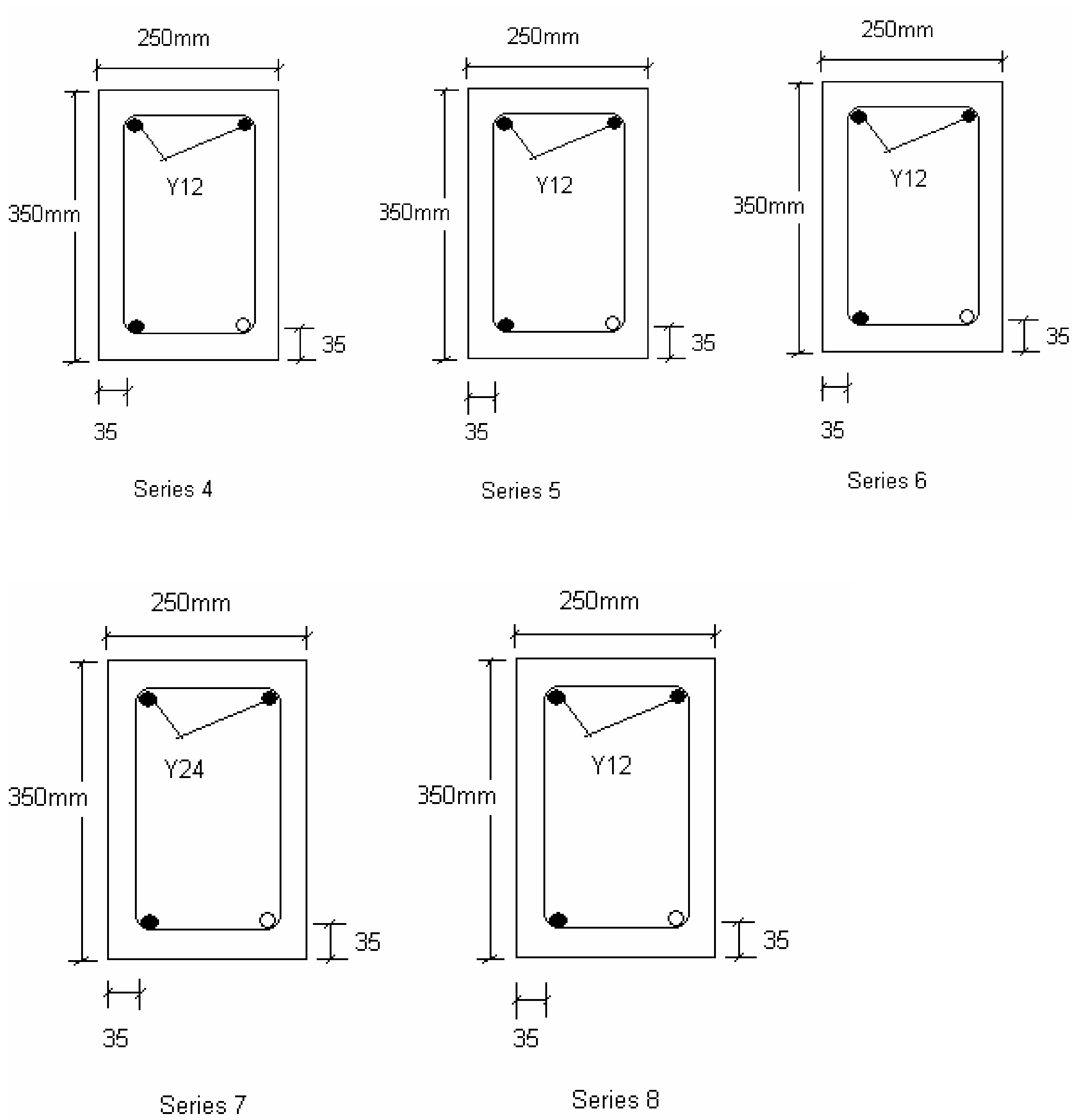


Figure 36 Cross-sections of reinforced concrete beams tested by Kong and Rangan (1998)

Specimen	*b*	f'_c	*D*	*d*	*a/d*	A_{st}	f_y	$f_{sty.}$	A_{sv}.	V_e
	(mm)	*(MPa)*	*(mm)*	*(mm)*		*(mm^2)*	*(MPa)*	*(MPa)*	*(mm^2)*	*(kN)*
S1-1	250	63.6	350	292	2.5	2046	452	569	19.63	228.3
S1-2	250	63.6	350	292	2.5	2046	452	569	19.63	208.3
S1-3	250	63.6	350	292	2.5	2046	452	569	19.63	206.1
S1-4	250	63.6	350	292	2.5	2046	452	569	19.63	277.9
S1-5	250	63.6	350	292	2.5	2046	452	569	19.63	253.3
S1- 6	250	63.6	350	292	2.5	2046	452	569	19.63	224.1
S2-1	250	72.5	350	292	2.5	2046	452	569	19.63	260.3
S2-2	250	72.5	350	292	2.5	2046	452	569	19.63	232.5
S2-3	250	72.5	350	292	2.5	2046	452	569	19.63	253.3
S2-4	250	72.5	350	292	2.5	2046	452	569	19.63	219.4
S2-5	250	72.5	350	292	2.5	2046	452	569	19.63	282.1
S3-1	250	67.4	350	297	2.49	1232	450	632	12.56	209.2
S3-2	250	67.4	350	297	2.49	1232	450	632	12.56	178
S3-3	250	67.4	350	293	2.49	2046	452	632	12.56	228.6
S3-4	250	67.4	350	293	2.49	2046	452	632	12.56	174.9
S3-5	250	67.4	350	299	2.41	2760	442	632	12.56	296.6
S3-6	250	67.4	350	299	2.41	2760	442	632	12.56	282.9
S4-1	250	87.3	600	542	2.4	4092	452	569	19.63	354
S4-2	250	87.3	500	444	2.41	3284	433	569	19.63	572.8
S4-3	250	87.3	400	346	2.4	2464	450	569	19.63	243.4
S4-4	250	87.3	350	292	2.5	2046	452	569	19.63	258.1
S4-6	250	87.3	250	198	2.53	1380	442	569	19.63	202.9
S5-1	250	89.4	350	292	3.01	2046	452	569	19.63	241.7
S5-2	250	89.4	350	292	2.74	2046	452	569	19.63	259.9
S5-3	250	89.4	350	292	2.5	2046	452	569	19.63	243.8
S5-4	250	89.4	350	292	1.99	2046	452	569	19.63	476.7
S5-5	250	89.4	350	292	1.75	2046	452	569	19.63	573.4
S6-3	250	68.9	350	292	2.73	2046	452	632	12.56	178.4
S6-4	250	68.9	350	292	2.73	2046	452	632	12.56	214.4
S6-5	250	68.9	350	292	2.64	2760	442	632	12.56	297
S6-6	250	68.9	350	292	2.64	2760	442	632	12.56	287.2
S7-1	250	74.8	350	292	3.3	3284	433	569	19.63	217.2
S7-2	250	74.8	350	292	3.3	3284	433	569	19.63	205.4
S7-3	250	74.8	350	292	3.3	3284	433	569	19.63	246.5
S7-4	250	74.8	350	292	3.3	3284	433	569	19.63	273.6
S7-5	250	74.8	350	292	3.3	3284	433	569	19.63	304.4
S7-6	250	74.8	350	292	3.3	3284	433	569	19.63	310.6
S8-1	250	74.6	350	292	2.5	2046	452	569	19.63	272.1
S8-2	250	74.6	350	292	2.5	2046	452	569	19.63	250.9
S8-3	250	74.6	350	292	2.5	2046	452	569	19.63	309.6
S8-4	250	74.6	350	292	2.5	2046	452	569	19.63	265.8
S8-5	250	74.6	350	292	2.5	2046	452	569	19.63	289.2
S8-6	250	74.6	350	292	2.5	2046	452	569	19.63	283.9

Table 18. Details of beam specimens tested by Kong & Rangan (1998)

6.3.1.2 Sarsam and Al-Musawi (1992)

Both normal and high- strength concrete beams failing in shear were tested by Sarsam and Al-Musawi with compressive strength f'_c > 41.4 N/mm^2 for high strength concrete beams and lower f'_c values for normal strength concrete beams. The concrete compressive strength ranged from 39.0 to 80.1MPa. Overall dimensions of the beams were 180mm x 270 mm. Details of the beams are given in Table 19. The shear spans varied from 580 to 940mm with shear span-to-effective depth ratios of 1.5 and 3.0. The beams were subjected to a symmetrically placed two point loading 400 mm apart.

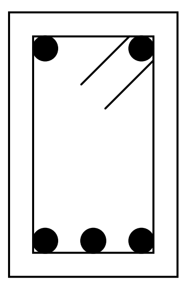
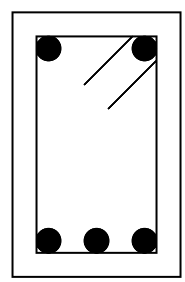
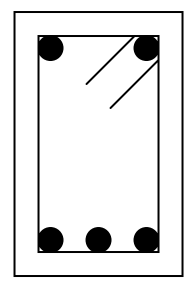
	Series A	Series B	Series C
			
Dimensions (mm)	180x270	180x270	180x270
Side Cover(mm) Top Cover(mm) Bottom Cover(mm)	25 25 25	25 25 25	25 25 25
Stirrups Yield Stress (MPa)	4mm ϕ 820	4mm ϕ 820	4mm ϕ 820
Longitudinal Steel Yield Stress(MPa)	2-10mm ϕ (Top) 3-20mm ϕ (Bottom) 450(T);495(B)	2-10mm ϕ (T) 2-25mm ϕ +1- 16mm ϕ (B) 450(T) 543,525(B)	2-10mm ϕ (T) 3-25mm ϕ (B) 450(T);543(B)

Table 19. Cross-sections of Reinforced Concrete Beams tested by Sarsam & Al-Musawi (1992)

Beam	f'_c	b	D	d	a	a/d	A_{st}	f_y	ρ_l	ρ_t	s	f_{sty}	V_e
	MPa	*mm*	*mm*	*mm*	*mm*		*mm*^{2}	*N/mm*^{2}			*mm*		*kN*
AL2-N	40.4	180	270	235	940	4	943	495	0.0223	0.00093	150	820	114.7
AL2-H	75.3	180	270	235	940	4	943	495	0.0223	0.00093	150	820	122.6
AS2-N	39.0	180	270	235	588	2.5	943	495	0.0223	0.00093	150	820	189.3
AS2-H	75.5	180	270	232	580	2.5	943	495	0.0226	0.00093	150	820	201.0
AS3-N	40.2	180	270	235	588	2.5	943	495	0.0223	0.00140	100	820	199.1
AS3-H	71.8	180	270	235	588	2.5	943	495	0.0223	0.00140	100	820	199.1
BL2-H	75.7	180	270	233	932	4	1181	540	0.0282	0.00093	150	820	138.3
BS2-H	73.9	180	270	233	583	2.5	1181	540	0.0282	0.00093	150	820	223.5
BS3-H	73.4	180	270	233	583	2.5	1181	540	0.0282	0.00140	100	820	228.1
BS4-H	80.1	180	270	233	583	2.5	1181	540	0.0282	0.00186	75	820	206.9
CL2-H	70.1	180	270	233	932	4	1470	543	0.0351	0.00093	150	820	147.2
CS2-H	70.2	180	270	233	583	2.5	1470	543	0.0351	0.00093	150	820	247.2
CS3-H	74.2	180	270	233	583	2.5	1470	543	0.0351	0.00140	100	820	247.2
CS4-H	75.7	180	270	233	583	2.5	1470	543	0.0351	0.00186	75	820	220.7

Table 20. Details of Reinforced Concrete Beams tested by Sarsam & Al-Musawi (1992)

The three series of beams A, B and C corresponded to members with longitudinal steel reinforcement of 3-20mm (series A), 2-25mm plus 1-16mm (series B), and 3-25mm (series C) diameter deformed bars respectively. The details of the test beams are given in Table 20.

The aim of this study was to

- Examine beams with stirrups made of high strength concrete and normal strength concrete failing in shear.
- To study the effect of variables like the concrete compressive strength, stirrup nominal strength ($\rho_v f_y$), longitudinal steel ratio (ρ_l), and shear span-to-depth ratio (a/d) on the shear strength.

- To compare test shear strengths with that predicted by the ACI 318-89, Canadian (CAN3-A23.3-M84), New Zealand (NZS 3101-1982) and British Codes of Practice (BS 8110-1985) and also with Zsutty's (1968) equation.

The following conclusions were drawn from the research by Sarsam and Al-Musawi (1992)

- Both the ACI 318-89 code design and the proposed method by the researchers were found to be conservative for HSC and NSC beams. Canadian code (CAN3-A23.3-M84) was also found to be conservative.
- The size effect is insignificant for beams with web reinforcement.
- Increasing the concrete compressive strength upto 82.9MPa did not lower the safety factor (ratio of test shear strength to predicted shear strength) of the ACI 318-89 code or the proposed methods.

6.3.1.3 Rahal and Al-Shaleh (2004)

Two series of beams A and B consisting of seven beams of 65 MPa concrete strength were tested by Rahal and Al-Shaleh. Each beam was tested with a four-point loading arrangement. Four of the seven beams included two test regions resulting in a total of 11 test results. All the beams were 200mm x 370mm and were 2.75m long. The beams had a shear span of 900mm giving a shear span-to-depth ratio between 2.7 and 3.0.The details of the beams are given in Table 22.

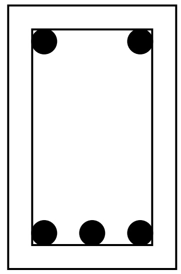
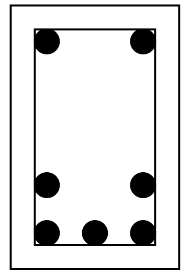
	Series A 	Series B 
Dimensions	200mm x 370mm	200mm x 370mm
Longitudinal Steel Area Yield Stress, f_y (MPa) Ultimate Strength, f_u (MPa)	3 ϕ 25mm 440 681	5 ϕ 25mm 440 681
Stirrups Area (A_{sv}) Yield Stress , f_{sty} (MPa) Ultimate Strength, f_u (MPa)	ϕ 6mm 28.3 mm^2 240 388	ϕ 6mm 24 mm^2 305 388
Target Compressive Strength(MPa)	65MPa	65MPa
Shear span-to-depth (a/d)	2.7	3

Table 21. Cross-sections of Reinforced Concrete Beams tested by Rahal & Al-Shaleh (2004)

	Beam Specimen	f'_c ,MPa	b	D	d	s	$\rho_v f_y$	V_e
		MPa	*mm*	*mm*	*mm*	*mm*	*MPa*	*kN*
SeriesA	A65-NTR	61.3	200	370	330	∞	0	125
	A65-200	60.9	200	370	325	200	1.339	175
	A65-140	62.1	200	370	325	140	0.485	150
	A65-110	60.9	200	370	325	110	0.617	188
	A65-95	62.1	200	370	325	95	0.714	220
Series	B65-NTR	61.9	200	370	305	∞	0	170
	B65-200	64.3	200	370	300	200	0.366	195
	B65-160	65.1	200	370	300	160	0.458	208
	B65-140	65.1	200	370	300	140	0.523	235
	B65-125	66.4	200	370	300	125	0.586	242
	B65-110	66.4	200	370	300	110	0.665	270

Table 22. Details of Reinforced Concrete Beams tested by Rahal & Al-Shaleh (2004)

The main objective of this research was to study the behavior of reinforced concrete beams with 65 MPa concrete to evaluate the adequacy of the current provisions for minimum transverse reinforcement in the ACI Code (ACI318-89 & ACI 318-02), CSA Standard A23.3-M94 (1994), and AASHTO LRFD Specifications (1998).

The experimental work by Rahal and Al-Shaleh resulted in the following conclusions:

- The performance of the transverse steel was found to depend on the amount and use of the co-existing longitudinal steel in the beam. The behavior of members with larger longitudinal steel was found to be more favourable.
- The ACI Codes and CSA Standard provided adequate performance in members with more longitudinal reinforcement.
- Transverse steel in members with smaller amounts of longitudinal reinforcement showed improved behaviour compared to members without transverse steel as per the ACI 318-83.

- The shear capacity equations in the ACI Code and CSA Standard, and AASHTO LFRD Specifications were found to be conservative.

6.3.1.4 Roller and Russell (1990)

Two series of concrete beams with web reinforcement were tested by Roller and Russell in their experimental investigation of the shear strength of high strength concrete beams. Each series consisted of five beams with a concrete compressive strength of 69, 117 and 124MPa respectively. All the beams were designed with the provisions of ACI 318-83. The cross-sectional details of the test beams are given in Table 23.

The beams 1 to 5 of the first series had a rectangular cross-section ranging from 356mm x 635mm to 356mm x 743mm with a concrete compressive strength of approximately 117MPa for all the beams. The beams in the second series (6-10) had the dimensions 457mm x 870mm. Two of the five beams had a concrete compressive strength of approximately 69MPa. The remaining three beams had a concrete compressive strength of approximately 124MPa. Steel reinforcement used in each beam specimen consisted of hot-rolled deformed bars. With the exception of the shear stirrups in beam specimen No. 1 of the first test series, all steel reinforcing bars conformed to ASTM A 615, Grade 60. The 6mm diameter reinforcing bars from Sweden were used as stirrups in beam specimen No. 1. No top steel was used in any of the beams.

All the beams were simply supported and loaded with a central point load. Both ends of the beams were free to rotate and translate under load. The shear spans were 1397mm in the first series and 2286 mm in the second series. The details of these beams are provided in Table 24.

The main objectives of these tests were:

- To study the effect of variables like the concrete compressive strength and the quantity of shear reinforcement on the shear strength of high-strength concrete beams.
- To verify the applicability of the ACI 318-83 code provisions for the minimum amount of shear reinforcement extended to high strength concrete beams.

The following conclusions were drawn by Roller and Russell from the results of shear tests conducted on beam specimens:

- The ACI 318-83 code provisions overestimated the nominal shear strength of the beams.
- The minimum quantity of shear reinforcement specified in the ACI 318-83 code $v_s = \frac{0.35 b_w s}{f_y}$ should be increased as the concrete compressive strength increases.

(a) Test Series I

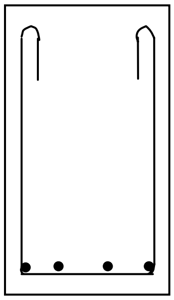
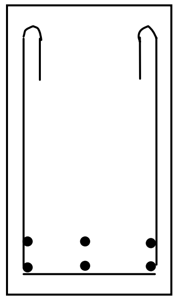
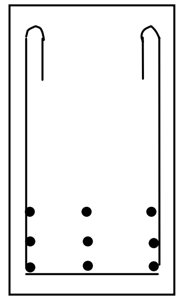
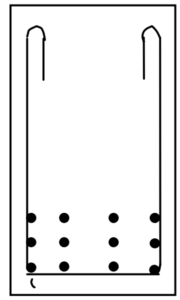
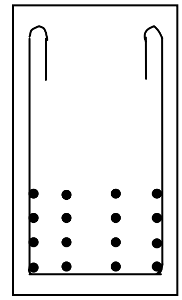
					
Dimensions (mm)	356x635	356x679	356x718	356x718	356x743
Stirrups Spacing (mm) Yield Stress (MPa)	6.4mm ϕ 216 407	12.7mm ϕ 165 448	15.9mm ϕ 127 458	15.9mm ϕ 89 458	15.9mm ϕ 64 458
Long. Steel Yield Stress (MPa)	4-31.8mm ϕ 472	3-34.9mm ϕ 3-34.9mm ϕ 431	3-34.9mm ϕ 3-34.9mm ϕ 3-34.9mm ϕ 431	4-34.9mm ϕ 4-34.9mm ϕ 4-34.9mm ϕ 431	4-31.8mm ϕ 4-31.8mm ϕ 4-31.8mm ϕ 4-34.9mm ϕ 472;431

Table 23. Cross-sections of Reinforced Concrete Beams tested by Roller & Russell (1990)

(b) Test Series II

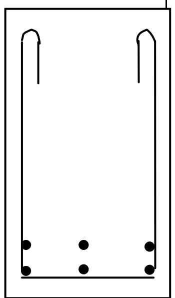
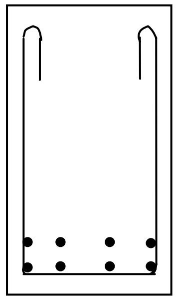
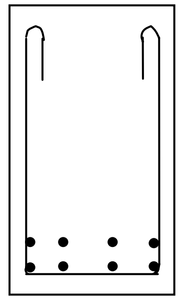
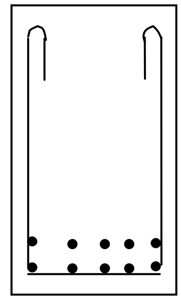
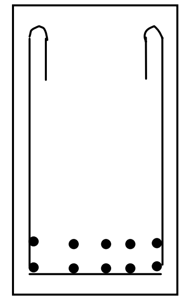
					
Dimensions (mm)	457x870	457x870	457x870	457x870	457x870
Stirrups Spacing (mm) Yield Stress (MPa)	9.5mm ϕ 381 445	9.5mm ϕ 197 445	9.5mm ϕ 381 445	9.5mm ϕ 197 445	9.5mm ϕ 133 445
Longitudina-l Steel Yield Stress(MPa)	3-34.9mm ϕ ; 3-34.9mm ϕ 464	4-31.8mm ϕ ; 4-31.8mm ϕ 483	4-31.8mm ϕ 4-31.8mm ϕ 483	5-31.8mm ϕ ; 5-31.8mm ϕ 483	5-34.9mm ϕ ; 5-34.9mm ϕ 464

Table 23. Cross-sections of Reinforced Concrete Beams Tested By Roller and Russell (1990)

Beam	f'_c, MPa	b	D	d	d_o	a	a/d_o	A_{st}	f_y	ρ_t	s	f_{sty}	V_e
	MPa	*mm*	*mm*	*mm*	*mm*	*mm*		mm^2	*MPa*		*mm*	*MPa*	*kN*
1	120.1	356	635	559	559	1397	2.50	3180	472	0.00074	216	407	297.8
2	120.1	356	679	559	599	1397	2.33	5740	431	0.00431	165	448	1099.1
3	120.1	356	718	559	635	1397	2.20	8610	431	0.00878	127	458	1657.5
4	120.1	356	718	559	635	1397	2.20	11490	431	0.01255	89	458	1942.9
5	120.1	356	743	559	660	1397	2.12	13370	460*	0.01757	64	458	2237.9
6	72.4	457	870	762	793	2286	2.88	5740	464	0.00081	381	445	665.1
7	72.4	457	870	762	795	2286	2.88	6360	483	0.00157	197	445	787.6
8	125.3	457	870	762	795	2286	2.88	6360	483	0.00081	381	445	482.6
9	125.3	457	870	762	795	2286	2.88	7940	483	0.00157	197	445	749.1
10	125.3	457	870	762	793	2286	2.88	9560	464	0.00233	133	445	1171.7

Note: * refers to the average yield stress representative of 12-31.8mmϕ bars (472MPa) and 4-34.9mmϕ bars (431 MPa).

Table 24. Details of Reinforced Concrete Beams tested by Roller & Russell (1990)

6.3.1.5 Cladera and Mari (2005)

Four series consisting of a total of eighteen beams failing in shear with and without reinforcement were tested by Cladera and Mari. The concrete compressive strength of the beams ranged from 50 to 87MPa. The primary design variables were the amount of shear and longitudinal reinforcement. The test program consisted of four series of which the series H50 beams were designed to have a concrete strength of 50MPa, the H60 series was designed to have a concrete strength of 60MPa, the H75 series, designed to have a concrete strength of 75MPa with silica fume and the H100 series, was designed to have a concrete strength of 100MPa. The details of the beam specimens are given in Table 26.

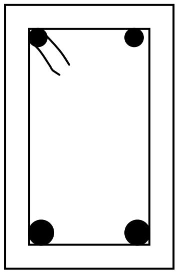
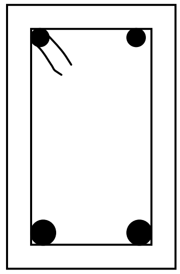
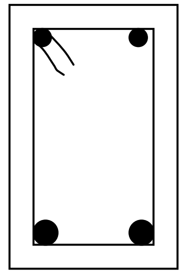
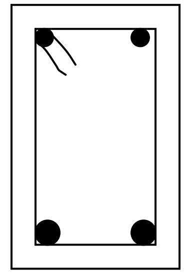
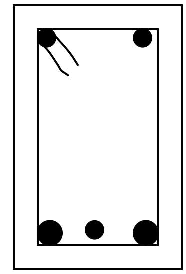
	H50/2	H60/2 H75/2	H100/2	H50/3 H60/3 H75/3 H100/3	H50/4 H60/4 H75/4 H100/4
					
Dimensions (mm)	200x400	200x400	200x400	200x400	200x400
Effective depth d,mm	353	353	353	351	351
a/d	3.06	3.06	3.06	3.08	3.08
ρ_l	2.24%	2.24% 2.24%	2.99%	2.24% 2.24% 2.24% 2.99%	2.24% 2.24% 2.24% 2.99%
Stirrups Spacing Yield Stress, MPa	6mm ϕ 260mm 530	6mm ϕ 200mm 530	6mm ϕ 165mm 530	8mm ϕ 210mm 540 (H50/3) 530 (H60/3) 530 (H75/3) 540 (H100/3)	8mm ϕ 210mm 540 (H50/4) 530 (H60/4) 530 (H75/4) 540 (H100/4)
Longitudinal Steel Yield Strength, MPa	2-8mm ϕ (T); 2-32mm ϕ (B) 500	2-8mm ϕ (T); 2-32mm ϕ (B) 500	2-8mm ϕ (T); 2-32mm ϕ (B) 500	2-8mm ϕ (T); 2-32mm ϕ (B) 500	2-8mm ϕ (T); 2-32mm ϕ + 1-25mm ϕ (B) 500

Table 25. Cross-sections of Reinforced Concrete Beams tested by Cladera & Mari (2005)

Beam	f'_c	b_v	D	d	d_o	a/d_o	ρ_l	$\rho v f_y$	s	f_{sty}	V_e
	MPa	*mm*	*mm*	*mm*	*mm*		*(%)*		*mm*	*MPa*	*kN*
H50/2	49.9	200	400	353	353	3.06	2.28	0.577	260	530	177.64
H50/3	49.9	200	400	351	351	3.08	2.29	1.291	210	540	242.07
H50/4	49.9	200	400	351	351	3.08	2.99	1.291	210	540	246.34
H60/2	60.8	200	400	353	353	3.06	2.28	0.747	200	530	179.74
H60/3	60.8	200	400	351	351	3.08	2.29	1.267	210	530	258.78
H60/4	60.8	200	400	351	351	3.08	2.99	1.267	210	530	308.71
H75/2	68.9	200	400	353	353	3.06	2.28	0.747	200	530	203.94
H75/3	68.9	200	400	353	353	3.08	2.29	1.267	210	530	269.35
H75/4	68.9	200	400	351	351	3.08	2.99	1.267	210	530	255.23
H100/2	87.0	200	400	353	353	3.06	2.28	0.906	165	530	225.55
H100/3	87.0	200	400	351	351	3.08	2.29	1.291	210	540	253.64
H100/4	87.0	200	400	351	351	3.08	2.99	1.291	210	540	266.53

Table 26. Details of Reinforced Concrete Beams tested by Cladera & Mari (2005)

The main objectives were:

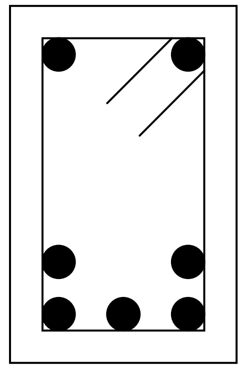
- To study the effect of the concrete compressive strength, f'_c on shear strength in beams with and without shear reinforcement.
- To propose and verify minimum shear reinforcement for HSC beams.
- To evaluate the amount of shear reinforcement as a function of f'_c
- To evaluate the influence of amount of longitudinal reinforcement on the shear strength.
- To study the influence of web-distributed longitudinal reinforcement for HSC members without stirrups.

Conclusions that were drawn from this study were:

- Beams without web reinforcement showed a brittle behavior. For such beams the failure shear strength increased with the increase in concrete compressive strength.
- HSC beams with web reinforcement were found to be less fragile compared to similar beams without web reinforcement.
- For beams with same amount of web reinforcement, the influence of the stirrups increased with the increase in concrete strength.

6.3.1.6 Johnson and Ramirez (1989)

Eight rectangular beams with concrete compressive strength in the range of 34 to 73MPa were tested for shear strength using truss model. The beams had a cross-section of 305 x 610mm with shear reinforcement consisting of 6.4mm diameter deformed bars having yield strength of 479MPa. Two symmetrically placed concentrated loads were applied on each beam which had a clear span of 4254mm. The longitudinal steel reinforcement was maintained same for all the beams. Table 27 gives the other details of the beams.

	
Dimensions (mm)	305 x 610
Side Cover (mm) Top Cover (mm) Bottom Cover(mm)	38 25 25
Stirrups Yield stress (MPa)	6.4mm ϕ 479
Longitudinal Steel Yield Stress (MPa)	2#9 (T);5#10(B) 540(T);525(B)

Note: #9: 29mm diameter bar.
#10: 32mm diameter bar.

Table 27. Cross-sections of Reinforced Concrete Beams tested by Johnson & Ramirez (1989)

Beam	f'_c	b	D	d	d_o	a	a/d_o	A_{sl}	f_y	ρ_t	f_{sty}	s	V_e
	MPa	mm	mm	mm	mm	mm		mm^2	MPa		MPa	mm	kN
1	36.4	305	610	539	562	1670	2.97	3960	525	0.00156	479	1333	338.9
2	36.4	305	610	539	562	1670	2.97	3960	525	0.00078	479	262	222.16
3	72.3	305	610	539	562	1670	2.97	3960	525	0.00078	479	267	262.97
4	72.3	305	610	539	562	1670	2.97	3960	525	0.00078	479	267	316.24
5	55.8	305	610	539	562	1670	2.97	3960	525	0.00156	479	133	383.12
6	55.8	305	610	539	562	1670	2.97	3960	525	0	0	0	191.56
7	51.3	305	610	539	562	1670	2.97	3960	525	0.00078	479	267	281.1
8	51.3	305	610	539	562	1670	2.97	3960	525	0.00078	479	267	258.43

Table 28. Details of Reinforced Concrete Beams tested by Johnson & Ramirez (1989)

The main objective of this study was to evaluate the adequacy of the minimum amount of shear reinforcement in beams with higher strength concrete according to the ACI 318-83 code provisions.

The conclusions of this study were:

- The number of inclined cracks increased with an increase in the amount of web reinforcement which indicated an improved redistribution of internal forces in the beams.
- The reserve capacity increased significantly with the increase in the amount of web reinforcement.
- In beams with minimum amount of shear reinforcement, as the concrete compressive strength increased, the transfer of forces at first inclined cracking caused the stirrups to yield and rupture thus, reducing the redistribution of internal forces and load-carrying capacity. The failure of the beam 3 confirmed this observation.
- For beams with minimum amount of web reinforcement, an evaluation of the current design provision indicated that the overall reserve shear strength after diagonal tension cracking diminished with increased compressive strength of concrete. The solution to the reduction in reserve shear strength was to increase the minimum amount of shear reinforcement with an increase in f_c'.

6.3.2 EVALUATION OF CURRENT CODE PROVISIONS

Influence of Parameters

The current AS3600-2001 code provisions and recommendations for determining the shear strength of beams are the results from experiments conducted on normal strength concrete. Recent studies on high strength concrete have shown that the shear strength varies with different parameters, including concrete compressive strength, longitudinal steel ratio, total depth of the section, and shear span-to-effective depth ratio. Although these studies provided valuable information on the variation of shear strength with respect to different parameters, the variability is still unknown. Thus, in view of the results of recent studies which have shown that the shear strength of HSC beams may be considerably different from that of NSC depending on various parameters, the applicability of the existing design rules and recommendations to HSC may be questioned.

For this purpose, the results from shear test on beams with shear reinforcement have been compiled to study the influence of major parameters on the ultimate shear strength. The ratio V_e/V_p is plotted versus the concrete compressive strength, longitudinal steel ratio, and shear span-to-effective depth ratio. The actual shear strength, V_e of each beam specimen is compared to the predicted shear strength, V_p which has been calculated using the actual material properties and the AS3600-01 code provisions.

The horizontal line in the Figs. 37 to 39 represents a reference point where the actual shear strength V_e equals the predicted shear strength, V_p. The points that fall below this line represent beams that had a measured shear strength that was less than the predicted. A minimum ratio $V_e/V_p \geq 1$ is considered as a measure of conservativeness.

It is seen that the extent of conservativeness obtained by the use of AS 3600 design provisions for shear is variable. This variability is to be expected due to the numerous variables that have an effect on shear strength. Figures 37, 38 & 39 display the results from shear tests performed on approximately 100 beams with stirrups. The AS3600 code provisions were not found conservative for most of the beams with different compressive strengths of concrete. For a majority of the beams tested by Cladera & Mari (2005), Roller & Russell (1990) and Johnson & Ramirez (1989) and for some of the beams tested by Kong and Rangan (1998), the AS3600-01 design provisions are found to be unconservative with $V_e/V_p < 1$.

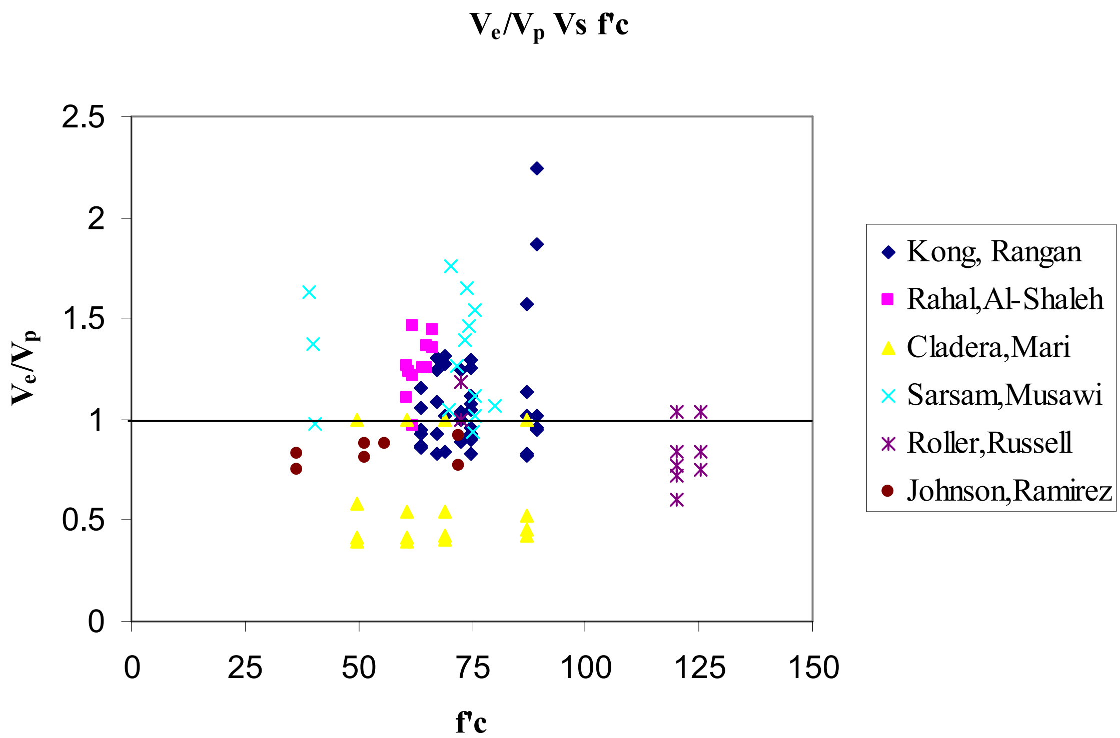


Figure 37. Influence of concrete compressive strength on beams failing in shear

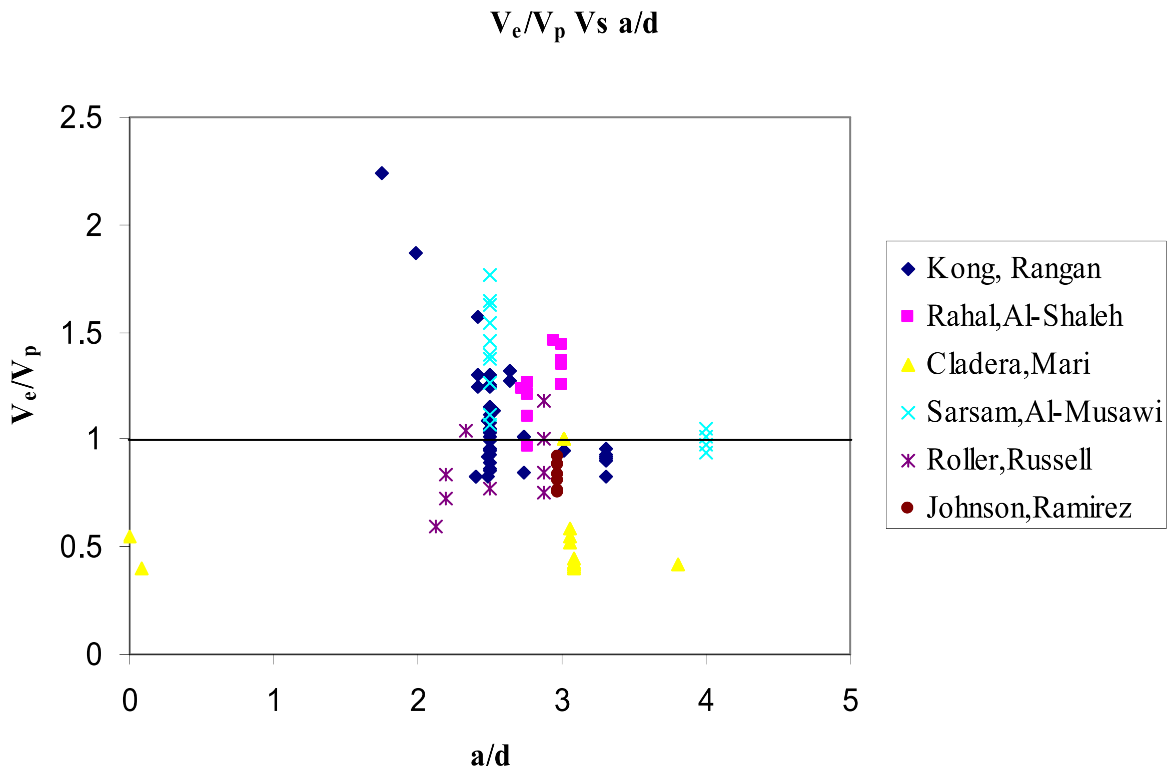


Figure 38. Influence of shear span-to-effective depth ratio on beams failing in shear

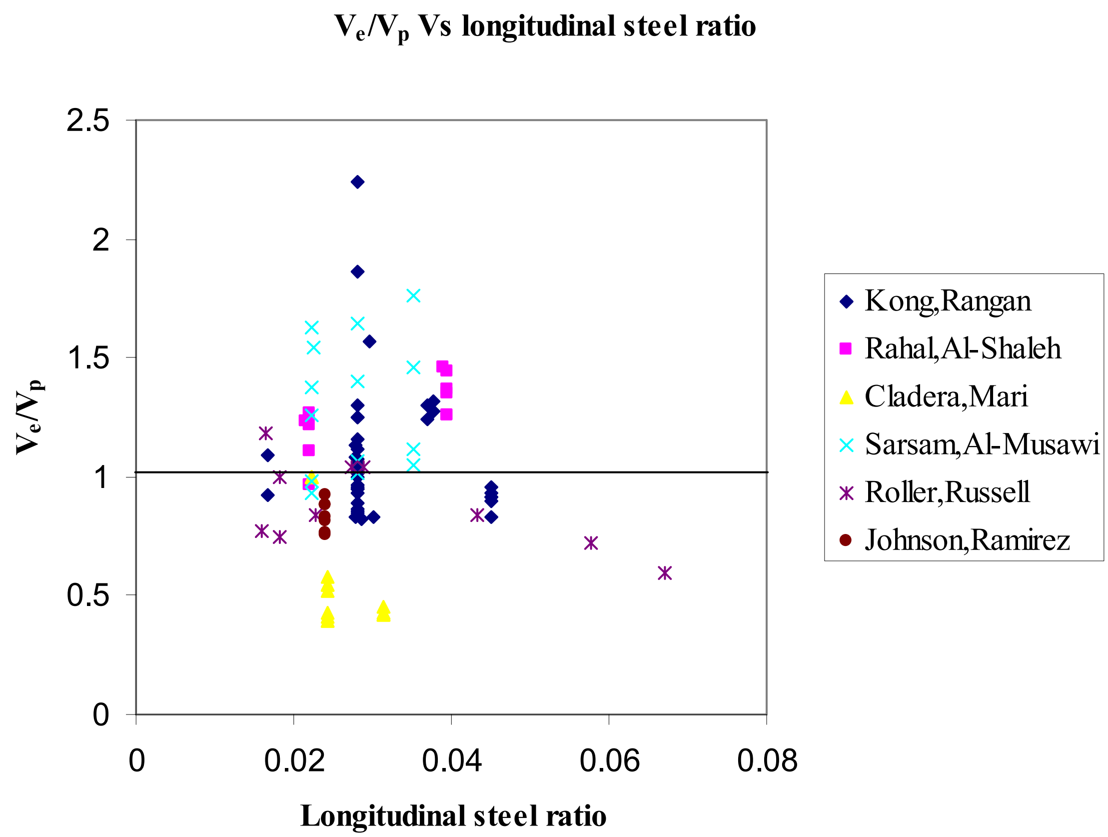


Figure 39. Influence of longitudinal steel ratio on beams failing in shear

6.4 REGRESSION ANALYSIS

6.4.1 Factors influencing Ultimate Shear Strength

There are different approaches for analysing the behavior of beams in shear. In this study, a semi-empirical approach is adopted in which shear data collected from the past literature is analyzed and an equation is generated which can be used to predict the ultimate shear strength of a high strength concrete beam. This is done by using regression analysis using a variety of parameters and beam properties.

6.4.2 Data Sources

The use of regression analysis requires a large amount of data to accurately evaluate the significance of every variable in any process. The studies on the shear strength of high strength concrete beams that were considered included those by Kong & Rangan, (1998), Rahal & Al-Shaleh (2004), Cladera & Mari (2005), Roller & Russell (1990), Sarsam & Al-Musawi (1992) and Johnson & Ramirez (1989). Together, those papers formed a shear database of 101 beams with stirrups and 47 beams without stirrups.

6.4.3 Need for Statistical Analysis

Extensive ongoing research since many years has lead to the generation of a large body of knowledge. Whilst each author has analyzed the data comparing them with existing relationships, the whole body of information has not been analysed to establish statistical significance.

Analyzing data containing a number of variables that are related to one another and establishing links between the variables can be vital to making decisions in different areas of society including engineering, medicine, business and politics. Statistical analysis is one such significant tool in scrutinizing data with a number of variables and study the relationship between the variables.

One of the statistical methods for modeling the association among two or more random variables and the predictors using a linear equation *is regression analysis*. The effect of one variable upon another can be investigated using regression analysis. A linear relationship between two random variables refers to simple linear regression and the linear relationship between more than two random variables is called multiple linear regression. For this research, multiple linear regression is adopted.

In the study of HSC beams in shear, experimental data collected from previous studies involving different parameters affecting the ultimate shear strength is collected. To this data regression analysis is applied to estimate the quantitative effect of the variables (f'_c, *a/d,* ρ_l, ρ_t) upon the shear strength of beams with and without stirrups. This analysis also gives an assessment of "statistical significance" of the estimated relationships, that is, the degree of confidence that the true relationship is close to the estimated relationship.

6.4.4 Introduction to Regression Analysis

Regression analysis is described as the study of relationship between variables. Regression analysis is used to predict a continuous dependent variable from a number of independent variables.

Linear regression investigates and models the linear relationship between a response (Y) and predictor(s) (X). Both the response and predictors are continuous variables.

In particular, linear regression analysis is often used to:

- determine how the response variable changes as a particular predictor variable changes.
- predict the value of the response variable for any value of the predictor variable, or combination of values of the predictor variables.

A relationship of the form $y = \beta 0 + \beta_1 x$, is sought using n pairs of observations (x_i, y_i), i=1,2,3,…….,n

The variable x is called the independent or the predictor variable and the variable y the dependent variable or the response variable.

The estimated relationship is then used to:

- test the strength of the relationship between the predictor and the response variables;
- predict y from a new measured x;
- check the adequacy of the fit by performing diagnostic tests on the underlying assumptions.

Software called MINITAB provides a simple and useful set of routines for analysing simple and multiple regressions. Four residual plots are produced by the Regression routine in

MINITAB in order to test assumptions underlying a regression model. The three most common residual plots used to verify the assumptions are:

1. Stem and leaf plot of the residuals
2. Standardized residuals vs. x
3. Normal plot of residuals.

6.4.5 Simple Linear Regression and Multiple Linear Regressions

A linear relationship between two random variables refers to simple linear regression and the linear relationship between more than two random variables is called multiple linear regression.

When the relationship is sought between a single predictor and the response variable, it is called simple linear regression. Relationships involving more than one variable are of the form.

$$y = (\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots\ldots\ldots\ldots \beta_k x_k) + \varepsilon$$

For this research, multiple linear regression is adopted

6.4.6 Assumptions in Regression Analysis

In Regression analysis, several assumptions are made before analysis of the data.

In regression analysis it is assumed that the relationship between y (response or dependent variable) and x (predictor or independent variable) is that of a straight line and can be expressed in the form

$$y = (\beta_0 + \beta_1 x) + \varepsilon$$

Or Response = Model + Residual

ε represents the unpredictable element in 'y' due to random variation or measurement error. This random element explains why different values of y are obtained for the same value of 'x'. Due to the variability of the response, it is unlikely that the pairs of data values will lie exactly along a line and each pair is more likely to be perturbed from the line by a "small" random amount (2). Without this random element there would be no need for regression analysis since the value of 'y' would be perfectly predictable from the value of 'x'. Because of the random element the values of β_0 and $\beta 1$ can be estimated by the intercept "a" and slope "b" of the line of best fit. Hence any predictions made based on the line are also only estimates. To be able to quantify the reliability of these estimates, the residual term ε must satisfy the following key assumptions:

- the assumption of *linearity* $E(\varepsilon)=0$, i.e. its expected value is zero. This means on an average, the errors balance out.

- The assumption of *constant variance*. $Var(\varepsilon) = \sigma^2$, i.e. its variance is a fixed constant (it is homoscedastic, to use an econometric term).

- The assumption of *independence.* The covariance between two residuals corresponding to different pairs is zero, i.e. this means disturbances associated with different observations are not correlated, and

- The assumption of *Normality*, i.e. ε has a normal distribution with mean 0 and variance σ^2.

6.4.7 Errors of Prediction and Least-Squares Estimation

From a statistical point of view, the goal of simple regression analysis is to find the slope coefficient and the intercept for the linear function that best describes the relationship between two variables. The slope coefficient, which is also known as the regression coefficient, is important because it measures the amount of change in the dependent variable associated with a one unit change in the independent variable.

A multiple regression equation involves more than one regression coefficient to describe the data. The best way to choose between alternative regression coefficients is to compare the errors of prediction associated with different linear regression equations. Errors of prediction are defined as the differences between the observed values of the independent variable and the predicted values for that variable obtained using a given regression equation and the observed values of the independent variable.

6.5 MINITAB

Minitab is powerful software that solves many statistical problems with ease. This software is frequently used in area of mathematics, statistics, economics, sports, and engineering. This is a general purpose statistical computing system, designed specially for students and researchers. Minitab is designed to be used interactively. This means that a command is immediately carried out and the results are shown. For the purpose of this research, regression analysis has been carried out using this software.

Engineers use models, which express the relationships among various characteristics, to predict a characteristic of interest, called the response or dependent variable. The characteristic used to predict the response variable are called the independent or predictor variables. Minitab can assist engineers to

1. build models;
2. examine the relationship between the response and predictor variables;
3. determine the accuracy of the model;
4. make predictions.

Minitab performs simple linear correlation, linear regression and multiple linear regressions. Both numerical and graphical presentations are available.

6.5.1 Regression Equation

A straight-line relationship can be useful in summarizing the observed dependence of one variable upon another. An equation of such a straight line can be obtained by the method of least squares for available data.

The regression equation is an algebraic representation of the regression line and is used to describe the relationship between the response and predictor variables. The regression equation takes the form of:

Response = constant + coefficient (predictor) + ……. + coefficient (predictor)

or $y = (\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots\ldots\ldots \beta_k x_k)$

Where:

- Response (Y) is the value of the response.

- Constant (β_0) is the value of the response variable when the predictor variable(s) is zero. The constant is also called the intercept because it determines where the regression line intercepts (meets) the Y-axis.
- Predictor(s) (X) is the value of the predictor variable(s).
- Coefficients (β_1, β_2… β_k) represent the estimated change in mean response for each unit change in the predictor value. In other words, it is the change in Y that occurs when X increases by one unit.

For the shear data, i.e. for beams with stirrups, the response variable is the ultimate shear strength predicted by AS3600-2001 and the predictors are the concrete compressive strength, f_c', the total depth of the section D, the percentage of longitudinal reinforcement (ρ_l) and the percentage of transverse reinforcement (ρ_t) . Regression analysis yielded the following:

V_p= -173 + 1.27 f_c' - 2.70 a/d + 0.466 D + 15.2 %ρ_l + 25.3 %ρ_t

Or

M_p = - 173 + 1.27 f_c' - 2.72 a/d + 1518 ρ_l + 2522 ρt + 0.465 D

ρ_l – longitudinal steel ratio

ρt- transverse steel ratio

Predictor	Coefficient	SE Coef	T	P
Constant	-172.87	12.11	-14.27	0.000
f'c	1.2743	0.1047	12.17	0.000
a/d	-2.719	2.978	-0.91	0.363
ρ_l	1518.3	259.3	5.86	0.000
ρ_t	2522	1112	2.27	0.026
D	0.46549	0.01299	35.84	0.000

Table 29. Table of Coefficients

S = 15.8082   R-Sq = 97.0%   R-Sq (adj) = 96.9%

Source	DF	SS	MS	F	P
Regression	5	779035	155807	623.48	0.000
Residual Error	95	23740	250		
Total	100	802776			

Table 30.(a) Analysis of Variance

Source	DF	Seq SS
f'c	1	390605
a/d	1	10
ρ_l	1	1223
ρ_t	1	66288
D	1	320910

Table 30. (b) Analysis of Variance

Obsv	*f'c*	V_p	Fit	SE Fit	Residual	St Resid
18	87	217.85	260.96	3.21	-43.11	-2.79R
57	50	102.91	119.69	8.22	-16.78	-1.24X
60	61	110.16	131.11	8.24	-20.96	-1.55X
68	87	124.13	157.05	3.00	-32.91	-2.12R
69	87	123.86	159.11	3.36	-35.25	-2.28R
70	87	134.74	169.72	2.49	-34.98	-2.24R
71	40	61.34	29.62	5.49	31.71	2.14R
88	120	438.55	427.73	8.08	10.83	0.80X
89	120	461.04	466.59	11.74	-5.54	-0.52X
92	125	418.00	413.72	6.78	4.28	0.30X
94	125	479.21	431.51	6.18	47.70	3.28R

Table 31. Table of unusual observations

R denotes an observation with a large standardized residual.

X denotes an observation whose X value gives it large influence.

6.5.2 Interpretation

The interpretation of the regression equation is as given below:

- For the shear data, the response variable is the shear predicted using AS3600-2001 and the predictors are concrete compressive strength *f'c*, percentage of longitudinal steel, percentage of transverse steel, depth of the section and shear-span-to depth ratio (a/d).
- Coefficients of the variables in the equation represent the estimated change in mean value of the shear strength V_p for each unit change in the value of the predictor variables. In other words, it is the change in the shear strength that occurs when the parameters increase by one unit.
- The constant (intercept) value (b_0 = - 173) is the predicted value of shear strength when each predictor (*f'c* , a/d, D, $\%\rho_l$, $\%\rho_t$) is zero. That is, when the predictors are zero the Rating is - 173.

The session window output lists each factor in the model and the number of levels in each factor. It then lists the analysis of variance table (Table 30), and then the unusual observations (Table 31).

Linear regression examines the relationship between a response and predictors. In order to determine whether or not the observed relationship between the response and predictors is statistically significant, the following steps are involved:

- Identifying the coefficient p-values: The coefficient value for P (p-value) is used to determine whether or not the association between the response and predictor(s) is statistically significant.
- Comparing the coefficient p-values to α-level: If the p-value is smaller than the selected α-level, then it implies that the association is statistically significant. A commonly used α -level is 0.05 and this α-level of 0.05 is adopted in the analysis.

From the regression, it was noted that the p-values for f_c', D, ρ_l and ρ_t are small when compared to 0.05. It is therefore, concluded that the relationships between the shear strength, and the predictors, f_c' (p=0.00), D (p=0.00), ρ_l (p=0.00) and ρ_t (p=0.026) are significant.

The relationship between the response, Predicted Shear Strength, and the predictor, a/d (P= 0.366), is not significant because the p-value is higher than the pre-selected - α level.

6.5.4 R-Sq and R-Sq (adj) values

The R and adjusted R values represent the proportion of variation in the response data explained by the predictors. For the shear data, the predictors show 97.0% of variation in the observations.

R (R-Sq) describes the amount of variation in the observed response values that is explained by the predictors.

Adjusted R is a modified R that has been adjusted for the number of terms in the model. A high value of R gives an indication of redundant terms in the data. Unlike R, adjusted R gets smaller when more terms are added to the model.

For the shear data, the adjusted R is 96.9%, which is a decrease of 0.1% (97.0% - 96.9%). The ***Analysis of variance*** table shows the amount of variation in the response data explained by the predictors and the amount of variation left unexplained.

The p-value obtained in the analysis explains if the regression coefficients are significantly different from zero. In this case, p-value is 0.00, which is smaller than the pre-selected α-value 0.05. This indicated that atleast one of the regression coefficients is significantly different from zero.

The unusual observation table displays cases with standardized residuals with absolute values greater than 2. These cases do not follow the proposed regression equation well.

From the analysis, six unusual observations were observed. These observations are the cases with standardized residuals having absolute values greater than 2. These are denoted by R. It was noted that the reason for these unusual observations could be the error involved in recording the data accurately and also in the data collection process which could have affected by any other factors.

6.5.5 Graphical Representation

Three graphs are generated from the regression analysis using Minitab- Histogram of the Residuals, Normal Probability Plot of the Residuals, and Residuals Versus Vp. These plots reveal that the residuals are normally distributed and are depicted in Appendix E.

1. Histogram –This graph indicates whether the data are skewed or outliers exist in the data
2. Normal probability plot – This graph indicates whether the data are normally distributed, other variables are influencing the response, or outliers exist in the data
3. Residuals versus fitted values – This graph indicates whether the variance is constant, a nonlinear relationship exists, or outliers exist in the data

From the output of the shear data, a high t-ratio and a low p-value (0.00) for the parameters f_c' (p=0.00), D (p=0.00), ρ_l (p=0.00) and ρ_t (p=0.026) in the table of coefficients indicate that a strong relationship exists between these parameters and the shear strength.

A large F-value and a low p-value in the analysis of variance table specify this relationship in a different way. The R-Sq values and adjusted R-Sq of 97% further support the confirmation that a strong relationship exists between the Shear strength of a high strength concrete beam and the parameters f_c', D, ρ_l and ρ_t. Except for the variable a/d, the other variables showed a significant relationship. It is noted that this parameter should be examined more closely.

6.5.6 Applicability of the proposed equation

To verify the applicability of the proposed regression equation, test results were collected from a paper by Rahal (2006). The ratio of V_e/V_p vs. the concrete compressive strength was plotted to determine the accuracy of these equations. From Fig. 40, it is seen that the values obtained with the proposed equation gave conservative values compared to those obtained by using the Australian Standards. The mean value obtained using the proposed equation is 1.003 while the mean for *Ve/Vp* values using the AS3600-2001 is found to be 1.85. Although, it seems that the code provisions gives conservative values, design of beams using this equation might prove inefficient for high strength concrete beams. It follows that the proposed expression is applicable to high strength concrete beams with stirrups.

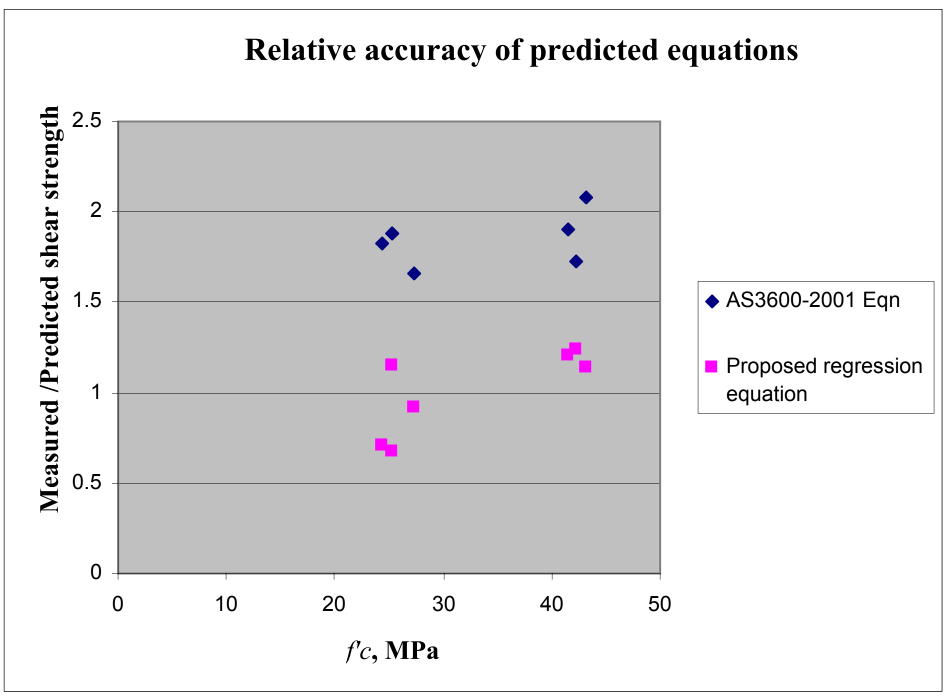


Figure 40. Relative accuracy of proposed equation (beams with stirrups)

6.6 BEAMS WITHOUT WEB REINFORCEMENT

6.6.1 Experimental Studies

6.6.1.1 Ahmad, Khaloo, and Poveda (1986)

An experimental investigation on shear strength of high-strength concrete beams without stirrups was carried out by Ahmad et al (1986). The concrete compressive strength, $f'c$ ranged from 63 to 70MPa. The beams were tested for shear span-to-depth ratios and six percentages of longitudinal steel content, $\rho_{l.}$ All the beams were loaded with two symmetrical point loads. The applicability of ACI 318-83 Building code provisions to high strength concrete beams in shear was investigated.

From the test results, the following conclusions were drawn:

- The ACI 318-83 Equation for estimating the shear contribution of concrete was found to be conservative for beams with low shear span ratios (a/d less than 2.5), and unconservative for HSC beams with high shear span ratios and relatively low longitudinal steel ratios.
- The equation available in ACI 318-83 was found to overestimate the effect of concrete strength and underestimate the effect of longitudinal steel content and a/d ratio.
- An equation was proposed to predict the ultimate shear stress which depends on the concrete compressive strength, amount of longitudinal steel and the shear span-to-depth ratio.

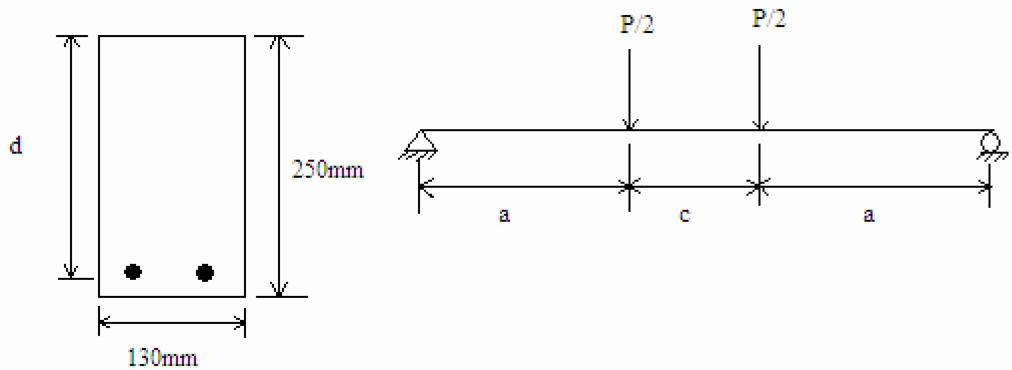


Figure 41. Beam section and loading arrangement (Ahmad et al, 1986)

Specimen	*b*	*f'c,*	*D*	*d*	*a/d*	*Steel content*	*Ult.Sh.Strength*
	mm	*MPa*	*mm*	*mm*		ρ_l	*(measured)(kN)*
A1	130	66.12	250	203.2	4	3.93	72.83
A2	130	66.12	250	203.2	3	3.93	86.83
A3	130	66.12	250	203.2	2.7	3.93	86.83
A4	130	66.12	250	203.2	2.3	3.93	117.65
A5	130	66.12	250	203.2	2	3.93	210.08
A6	130	66.12	250	203.2	1	3.93	504.20
A7	130	66.12	250	208.026	4	1.77	57.46
A8	130	66.12	250	208.026	3	1.77	60.19
A9	130	66.12	250	208.026	2.7	1.77	98.51
A10	130	66.12	250	208.026	2.3	1.77	101.24
A11	130	66.12	250	208.026	2	1.77	68.41
A12	130	66.12	250	208.026	1	1.77	273.61
B1	130	72.81	250	201.676	4	5.04	64.99
B2	130	72.81	250	201.676	3	5.04	87.48
B3	130	72.81	250	201.676	2.7	5.04	127.01
B4	130	72.81	250	201.676	2.3	5.04	181.76
B5	130	72.81	250	201.676	2	5.04	135.46
B6	130	72.81	250	201.676	1	5.04	260.77
B7	130	72.81	250	208.026	4	2.25	54.90
B8	130	72.81	250	208.026	3	2.25	57.46
B9	130	72.81	250	208.026	2.7	2.25	98.51
B10	130	72.81	250	208.026	2.3	2.25	78.79
B11	130	72.81	250	208.026	2	2.25	150.50
B12	130	72.81	250	208.026	1	2.25	262.68
C1	130	69.91	250	184.15	4	6.64	75.43
C2	130	69.91	250	184.15	3	6.64	105.10
C3	130	69.91	250	184.15	2.7	6.64	95.82
C4	130	69.91	250	184.15	2.3	6.64	123.63
C5	130	69.91	250	184.15	2	6.64	343.08
C6	130	69.91	250	184.15	1	6.64	0.00
C7	130	69.91	250	206.502	4	3.26	56.25
C8	130	69.91	250	206.502	3	3.26	55.13
C9	130	69.91	250	206.502	2.7	3.26	56.25
C10	130	69.91	250	206.502	2.3	3.26	70.57
C11	130	69.91	250	206.502	2	3.26	132.30
C12	130	69.91	250	206.502	1	3.26	303.19

Table 32. Details of Reinforced Concrete Beams tested by Ahmad et al (1986)

6.6.1.2 Mphonde and Frantz (1984)

An experimental investigation involving three series of reinforced concrete beams without shear reinforcement was carried out to determine their diagonal cracking strengths and ultimate shear capacities. Within each series, the shear span-to-depth ratio was held constant at 3.6, 2.5, or 1.5, while concrete compressive strength was varied from 21 to 103MPa. For the present study, only beams with compressive strength greater than 50MPa were taken into consideration. The researchers also determined the accuracy of the ACI code provisions for the shear strength of beams. A regression equation was also presented to predict the shear capacity of beams for a range of concrete strengths.

The beams were tested in a hydraulic testing machine and were loaded at midspan. Table 33 gives the details of the beam specimens. The following conclusions were drawn from the study by Mphonde and Frantz (1984):

- The ACI 318-77 code equations were found to be conservative at a/d ratio of 3.6. However, the ratio of measured/predicted shear capacities were found to decrease from 1.64 to 1.20 as the concrete strength increased from 21 to 103MPa.
- A considerable effect of concrete strength on shear capacity was noted as the a/d ratio decreased.
- Sudden and explosive failures were noted with the increase in concrete compressive strength at lower a/d ratios.

Specimen	b	f'_c	D	d	a/d	steel content	Ult.Sh.Strength
	mm	MPa	mm	mm		ρ_l	(measured)kN
AO-11-3a	152	74.93	337	299	3.6	3.36	101.01
AO-11-3b	152	74.64	337	299	3.6	3.36	100.66
AO-15-3a	152	81.34	337	299	3.6	3.36	105.25
AO-15-3b	152	93.68	337	299	3.6	3.36	112.67
AO-15-3c	152	91.83	337	299	3.6	3.36	110.20
AO-11-2	152	79.28	337	299	2.5	3.36	125.38
AO-15-2a	152	83.76	337	299	2.5	3.36	200.26
AO-15-2b	152	69.40	337	299	2.5	3.36	231.69
AO-11-1	152	65.74	337	299	1.5	3.36	487.05
AO-15-1a	152	79.46	337	299	1.5	3.36	310.45
AO-15-1b	152	81.26	337	299	1.5	3.36	557.33

Table 33. Details of Reinforced Concrete Beams tested by Mphonde and Frantz (1984)

6.6.2 Regression Equation

For the shear data, i.e for beams without stirrups, the response variable is the Ultimate shear strength predicted by AS3600-2001 and the predictors are the concrete compressive strength, f_c', the total depth of the section D, the percentage of longitudinal reinforcement (ρ_l) . Regression analysis yielded the following:

Vp = - 82.8 + 0.390 f'_c + 0.385 D - 0.011 a/d + 3.66 % ρ_l

Predictor	Coefficient	SE Coef	T	P
Constant	-82.834	3.249	-25.49	0.000
$f'c$	0.38967	0.06101	6.39	0.000
D	0.38494	0.01026	37.53	0.000
a/d	-0.0109	0.3043	-0.04	0.972
ρ_l	3.6611	0.1900	19.27	0.000

Table 34. Table of Coefficients

S= 1.87235 R-Seq = 98.8% R-Sq(adj) = 98.7%

Source	DF	SS	MS	F	P
Regression	4	12319.0	3079.8	878.50	0.000
Residual Error	42	417.2	3.5		
Total	46	12466.3			

Table 35. (a)Analysis of Variance

Source	DF	Seq SS
$f'c$	1	6803.0
D	1	4213.9
a/d	1	0.7
ρ_l	1	1301.4

Table 35. (b) Analysis of Variance

<u>Unusual Observations</u>

No unusual observations were noted in the analysis.

6.6.3 Interpretation

The interpretation of the regression equation follows:

- For the shear data without stirrups, the response variable is the shear predicted using AS3600-2001 and the predictors are concrete compressive strength (f'_c), Depth of the section (D), shear span-to-depth ratio (a/d) and percentage of longitudinal reinforcement (ρ_l).
- Coefficients of the variables in the equation represent the estimated change in mean value of the shear strength V_p for each unit change in the value of the

predictor variables. In other words, it is the change in the shear strength that occurs when the parameters increase by one unit.

- The constant (intercept) value (b_0 = - 82.8) is the predicted value of shear strength when each predictor (f'_c , D, a/d, ρ_l) is zero. That is, when the predictors are zero the Rating is – 82.8.

From the regression, it was noted that the p-values for f'_c , D and ρ_l are small when compared to 0.05. It is therefore, concluded that the relationships between the shear strength, and the predictors, f'_c (p=0.00), D (p=0.00) and ρ_l (p=0.00) are significant. The p-value for a/d was found to be 0.972 which is greater than 0.05. Therefore, it is considered that no significant relationship exists between the response and predictor variable.

6.6.4 R-Sq and R-Sq (adj) values

The R and adjusted R values represent the proportion of variation in the response data explained by the predictors. For the shear data, the predictors show 98.8% of variation in the observations.

For the shear data, the adjusted R is 98.7%, which is a decrease of 0.1% (98.8 - 98.7%). The ***Analysis of variance*** Table 35 shows the amount of variation in the response data explained by the predictors and the amount of variation left unexplained.

The p-value obtained in the analysis explains if the regression coefficients are significantly different from zero. In this case, p-value is 0.00, which is smaller than the pre-selected α-value 0.05. This indicated that atleast one of the regression coefficients is significantly different from zero.

The unusual observation table displays cases with standardized residuals with absolute values greater than 2. These cases do not follow the proposed regression equation well. From the analysis, no unusual observations were observed and it was noted that data was recorded accurately in the data collection process.

6.6.5 Graphical Representation

Three graphs are generated from the regression analysis using Minitab- Normal Probability Plot of the Residuals, Histogram of the Residuals, and Residuals Versus *Vp*, and are depicted in Appendix E.

The shape of a histogram should be generally bell-shaped, however, most of these histograms probably do not look bell-shaped, and hence a histogram should not be used to judge the normality of data. Instead a normal probability plot should be used to judge whether data are normally distributed.

For the shear data without stirrups, no unusual observations were noted. But the shape of the histogram is not bell –shaped. In the probability plot, the residuals appear to follow a straight line, although one or two outliers can be noted. The reason for this might be the fewer observations considered for the analysis. As the number of observations decrease, the probability plot shows greater variation and nonlinearity.

6.6.6 Applicability of the proposed Equation

The applicability of the regression equation generated from the statistical study for high strength concrete beams without stirrups has been verified with the test results of beams collected from a publication by Cladera and Mari (2005) which were not used in developing the equation. It was clear that the regression equation developed for shear capacity of HSC beams without stirrups is not generally applicable to all beams without stirrups. Further work is required in this area to develop a generic expression.

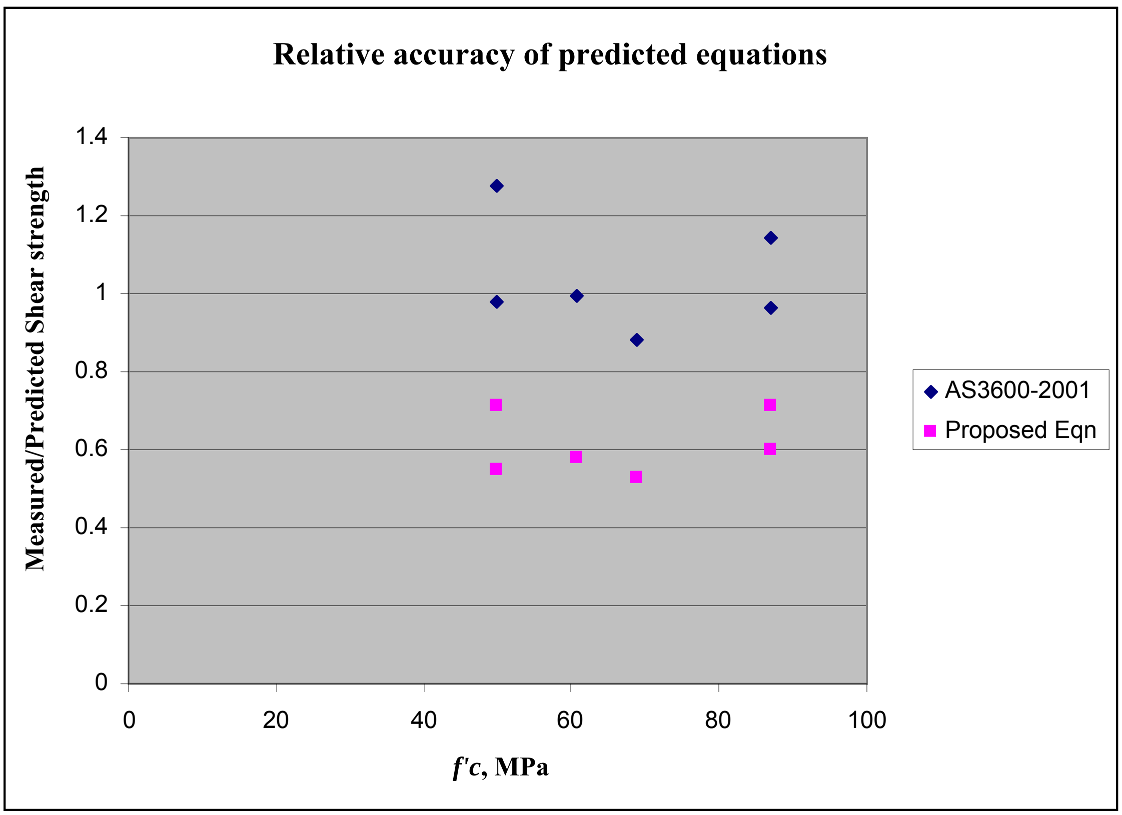


Figure 42. Relative accuracy of proposed equation (beams without stirrups)

6.7 Conclusion

Since the effect of different parameters on shear strength of HSC beams was not clearly evident from the trends observed with published work, a statistical analysis of data was conducted.

Two regression equations have been developed using published work to predict the shear capacity of HSC beams. However, in verifying the applicability of the two equations using additional data, it was concluded that only the equations developed for beams with shear reinforcement can be considered as suitable. The expression developed for beams without stirrups was observed to be unsafe for the additional set of data used for verification.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

The primary aim of this study was to gain a better understanding into the flexural behavior of high strength concrete beams to provide a more rational approach to their design. The available formula for short-term deflection of simply supported beams has been analysed for their use with high strength concrete. A statistical association between different parameters has been developed using regression analysis on shear data of beams with and without stirrups. The current code provisions have been found inadequate for its use with high strength concrete members. The deficiencies have been discussed and demonstrated in different aspects of beam behavior including flexure, ductility, deflection and shear.

7.1.1 Stress Block Parameters for Design of High Strength Concrete Beams

Various spreadsheets have been developed which calculate the ultimate moment capacity of beams using various formulae from different codes and reports. Investigation into different stress block parameters shows that the stress block parameters are unconservative for their applicability to high strength concrete. It is found that the moment capacity of high strength concrete beams cannot be predicted with reasonable accuracy using the current code provisions. This does not ensure a satisfied usage of code provisions for calculating the ultimate strength for high strength concrete beams subjected to flexure. Moreover, the stress block parameters do not seem to have a significant effect on the ultimate strength of high strength concrete beams. The applicability of the rectangular stress block for the computation of flexural strength of high strength concrete members does not seem to be acceptable. A reduction factor of 0.8 is recommended until further data are available.

7.1.2 Ductility of Beams

A computer program FRMPHI which takes into account various stress-strain models was considered to generate moment curvature curves for both NSC and HSC beams. This included Setunge et al model (1994) for high strength concrete beams and Mander et al (1988) model for normal strength concrete beams. The ductility factors were calculated from the values obtained from moment curvature curves. The influence of parameters on the ductility of beams was also determined. The influence of ductility on the depth of neutral axis was studied and it is concluded that the value of k_u for high strength concrete beams follows similar limitation already observed and accepted for normal strength concrete. It is further concluded that k_u can be used to ascertain the ductility level of HSC beams as well. However, it is recommended that further research is required to establish the range of the value of neutral axis depth parameter.

7.1.3 Short-term deflection of simply supported HSC Beams

From the analysis of the deem-to-comply method and rigorous method for calculating the deflection on simply supported high strength concrete beams, it is found that the deem-to-comply methods is ineffective in predicting the deflection of concrete beams with a concrete compressive strength greater than 50MPa. It is found that of the current formulae available to calculate deflection, only the rigorous method produces substantial results depending on method used. From all the methods generated in this investigation, the one incorporating Carrasquillo et al (1981) formula for E_c produces the most accurate and conservative results for deflection. It is recommended that this method be used for deflection calculation of HSC beams.

7.1.4 Shear strength – Statistical significance

In an effort to examine the effects of significant parameters on the shear strength of high strength concrete beams, two regression equations were developed by statistical evaluation of the 101 beams included in the study. The accuracy of these equations is determined and conclusions are drawn. It is observed that the values obtained with the proposed regression equation developed for beams with stirrups, gave conservative values compared to those obtained by using the design equations in the Australian Standards. However, the proposed equation is found to be unconservative for estimation of shear capacity of beams without stirrups. It is recommended that the proposed equation be used to estimate the shear capacity of beams with stirrups.

7.2 RECOMMENDATIONS

Work presented here clearly indicated that further work is needed to finalise design provisions for concrete with strengths over 50MPa. A general upper limit of concrete strength for the application of the work presented would be 80MPA. Interim recommendations from this research can be further developed into firm design recommendations based on more work in following areas:

1. Applicability of rectangular stress block in calculation of M_u,
2. Calculation of deflection,
3. Calculation of shear capacity.

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APPENDIX A

SPREADSHEET- ULTIMATE MOMENT CAPACITY.XLS

An example of the Excel spreadsheet moment capacity.xls is shown below which calculates the ultimate moment capacity of a rectangular beam cross-section using different stress block parameters from various codes and different publications.

Ultimate Moment Capacity of Normal and High Strength Concrete Beams

Introduction

Input Data

RECTANGULAR CROSS-SECTION

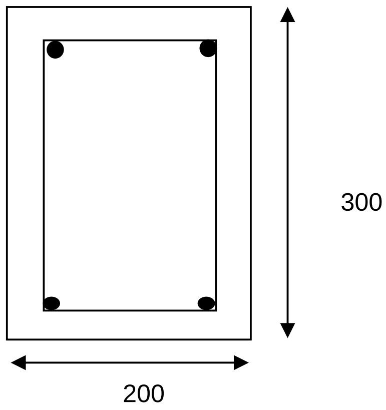
This spreadsheet calculates the ultimate moment capacity of a rectangular normal and high strength concrete beam with different stress block parameters as per the following provisions.

1. Australian Standards AS3600-2001
2. American Concrete Institute Building Code ACI318-2005
3. CAN3-A23.3-M94
4. New Zealand Standards NZS3102-1995
5. CEB-FIP Model Code 1990
6. Eurocode-2-1999
7. Azizinamini et al(1994)
8. Ibrahim and MacGregor(1997)
9. Pendyala and Mendis(1998)
10. Attard and Stewart(1998)
11. Bae and Bayrak(2003)

To Input Data

RECTANGULAR BEAM SECTION
(Input data in boxes)

Dimensions



Width(b)=	200	mm
Depth(D)=	300	mm
Effective depth(d)=	264	mm

Reinforcement

Area of longitudinal steel (Ast)	402	mm^2
Yield Strength(fy)=	579	MPa
ε_{sy}=	0.002	

Concrete

f'_c =	90	MPa
ε_{cu} =	0.003	

Next

FORMULA DETAILS RETRIEVAL

1 Australian Standards AS3600-2001

2 American ConcreteCode ACI318-2005

3 CAN3-A23.3-M94

4 New Zealand Standards NZS3102-1995

5 CEB-FIP Model Code 1990

6 Eurocode-2-1999

7 Azizinamini et al(1994)

8 Ibrahim and MacGregor(1997)

9 Pendyala and Mendis(1998)

10 Attard and Stewart(1998)

11 Bae and Bayrak(2003)

AUSTRALIAN STANDARDS AS3600-2001

Stress Block Parameters, Clause

$k_1 k_3$=0.85

k_2=0.85-0.007($f^1{}_c$-28)   , 0.65 $\leq k_2 \leq$ 0.85

k_2 = 0.416

$C = (k_1 k_3) b k_2 d_n$		(Compressive force in the section)
$T = f_{sy} A_s$		(Tensile force)
$C = T$		(For Equilibrium)
T =	232758 N	(Tensile force)
C =	15300 x a N	(compressive force)
$a = T/C$	15.2129 mm	(Stress block depth)
d_n =	36.5696 mm	(Neutral axis depth)
z =	256.394 mm	(Lever Arm distance)
M_u =	59.6776 kNm	(Ultimate Moment Capacity)

Back

AMERICAN CONCRETE CODE ACI318-2005

$K_1 K_3 = 0.85$

$K_2 = 1.09 - 0.008f'_c \quad (0.65 \le k_2 \le 0.85)$

k_2 =	0.37		
T =	232758	N	(Tensile force)
C =	15300 x a	N	(compressive force)
$a = T/C$ =	15.2129	mm	(Stress block depth)
d_n =	41.1161	mm	(Neutral axis depth)
z =	256.394	mm	(Lever Arm distance)
M_u =	59.6776	kNm	(Ultimate Moment Capacity)

Back

CANADIAN STANDARDS CAN3-A23.3-M94

$k_1 k_3 = 0.85 - 0.0015f'_c \geq 0.67$

$k_2 = 0.97 - 0.0025f'_c \geq 0.67$

$k_1 k_3 =$	0.715	
$k_2 =$	0.745	
$T =$	232758 N	(Tensile force)
$C =$	12870 x a N	(compressive force)
$a = T/C =$	18.0853 mm	(Stress block depth)
$d_n =$	24.2756 mm	(Neutral axis depth)
$z =$	254.957 mm	(Lever Arm distance)
$M_u =$	59.3434 kNm	(Ultimate Moment Capacity)

Back

NEW ZEALAND STANDARDS NZS3101-1995

$k_1 k_3$=1.07-0.004f'$_c$; 0.75 $\leq k_1 k_3 \leq$ 0.85

k_2=1.09-0.008f'$_c$; 0.65 $\leq k_2 \leq$ 0.85

$k_1 k_3$ =	0.71	
k_2 =	0.37	
T =	232758 N	(Tensile force)
C =	12780 x a N	(compressive force)
$a = T/C$ =	18.2127 mm	(Stress block depth)
d_n =	49.2234 mm	(Neutral axis depth)
z =	254.894 mm	(Lever Arm distance)
M_u =	59.3285 kNm	(Ultimate Moment Capacity)

Back

CEB-FIP MODEL CODE 1990

$\boldsymbol{k_1 k_3 = 0.85(1 - f'_c/250)}$

$\boldsymbol{k_2 = 1}$

$\varepsilon_{cu} = 0.004 - 0.002 f'_c/100$

$k_1 k_3 =$	0.544		
$k_2 =$	1		
$\varepsilon_{cu} =$	0.0022		
$T =$	232758	N	(Tensile force)
$C =$	9792 x a	N	(compressive force)
$a = T/C =$	23.7702	mm	(Stress block depth)
$d_n =$	23.7702	mm	(Neutral axis depth)
$\varepsilon_s =$	0.02223		
$\varepsilon_s > \varepsilon_{sy}$	OK		
$z =$	252.115	mm	(Lever Arm distance)
$M_u =$	58.6818	kNm	(Ultimate Moment Capacity)

Back

EUROCODE-2

$k_1 k_3$=0.85

k_2=0.9-f'_c/500

ε_{cu} =0.0035

$k_1 k_3$ =	0.85		
k_2 =	0.72		
ε_{cu} =	0.0035		
T =	232758	N	(Tensile force)
C =	15300 x a	N	(compressive force)
a=T/C=	15.2129	mm	(Stress block depth)
d_n =	21.1291	mm	(Neutral axis depth)
ε_s =	0.04023		
$\varepsilon_s > \varepsilon_{sy}$	OK		
z =	256.394	mm	(Lever Arm distance)
M_u =	59.6776	kNm	(Ultimate Moment Capacity)

Back

AZIZINAMINI ET AL (1994)

$\boldsymbol{k_1 k_3 = 0.85 - 0.00725(f'_c - 69);\quad 0.60 \le k_1 k_3 \le 0.85}$

$\boldsymbol{k_2 = 1.09 - 0.008 f'_c;\quad 0.65 \le k_2 \le 0.85}$

$\varepsilon_{cu} = 0.003$

$k_1 k_3 =$	0.85		
$k_2 =$	0.85		
$\varepsilon_{cu} =$	0.003		
$T =$	232758	N	(Tensile force)
$C =$	15300 x a	N	(compressive force)
$a = T/C =$	15.2129	mm	(Stress block depth)
$d_n =$	17.8976	mm	(Neutral axis depth)
$\varepsilon_s =$	0.04125		
$\varepsilon_s > \varepsilon_{sy}$	OK		
$z =$	256.394	mm	(Lever Arm distance)
$M_u =$	59.6776	kNm	(Ultimate Moment Capacity)

Back

IBRAHIM AND MACGREGOR (1997)

$\boldsymbol{k_1 k_3 = 0.85 - (f'_c/800) \geq 0.725}$

$\boldsymbol{k_2 = 0.95 - (f'_c/400) \geq 0.7}$

$\varepsilon_{cu} = 0.003$

$k_1 k_3 =$	0.7375	
$k_2 =$	0.725	
$T =$	232758 N	(Tensile force)
$C =$	13275 x a N	(compressive force)
$a = T/C =$	17.5336 mm	(Stress block depth)
$d_n =$	24.1842 mm	(Neutral axis depth)
$z =$	255.233 mm	(Lever Arm distance)
$M_u =$	59.4076 kNm	(Ultimate Moment Capacity)

Back

PENDYALA AND MENDIS (1997)

$\boldsymbol{k_1 k_3 = 0.85 - 0.0025(f'_c - 57)\ ;\ \ 57 < f'_c \leq 100MPa}$

$\boldsymbol{k_2 = 0.65 - 0.00125(f'_c - 57)\ \ ;\ 57 < f'_c \leq 100MPa}$

$\varepsilon_{cu} = 0.003$

$k_1 k_3 =$	0.7675	
$k_2 =$	0.60875	
$T =$	232758 N	(Tensile force)
$C =$	13815 x a N	(compressive force)
$a = T/C =$	16.8482 mm	(Stress block depth)
$d_n =$	27.6767 mm	(Neutral axis depth)
$z =$	255.576 mm	(Lever Arm distance)
$M_u =$	59.4873 kNm	(Ultimate Moment Capacity)

Back

MARIO.M ATTARD AND MARK G. STEWART (1998)

$$k_1 k_3 = 1.2932(f'_c)^{(-0.0998)} \geq 0.71 \quad (DB)$$

$$k_1 k_3 = 0.6470(f'_c)^{(0.0324)} \geq 0.58 \quad (SL)$$

$$k_2 = 1.0948(f'c)^{(-0.091)} \geq 0.67$$

$$fcyl = f'c + 7.5 \text{ MPa}$$

$\varepsilon_{cu} = 0.003$

$k_1 k_3 =$	0.82534	
$k_2 =$	0.72694	
$T =$	232758 N	(Tensile force)
$C =$	14856.1 x a N	(compressive force)
$a = T/C =$	15.6675 mm	(Stress block depth)
$d_n =$	21.5527 mm	(Neutral axis depth)
$z =$	256.166 mm	(Lever Arm distance)
$M_u =$	59.6247 kNm	(Ultimate Moment Capacity)

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SUNGJIN BAE AND OGUZHAN BAYRAK (2003)

$k_1 k_3$=0.85-0.004(f'_c-70) ;   0.67 $\leq k_1 k_3 \leq$ 0.85

k_2=0.85-0.004(f'_c-30);   0.67 $\leq k_2 \leq$ 0.85

Ultimate concrete strain ε_{cu}=0.0025 for f'_c > 55 MPa
otherwise ε_{cu}=0.003

ε_{cu} =	0.0025	
$k_1 k_3$ =	0.85	
k_2 =	0.85	
T =	232758 N	(Tensile force)
C =	15300 x a N	(compressive force)
$a = T/C$ =	15.2129 mm	(Stress block depth)
d_n =	17.8976 mm	(Neutral axis depth)
z =	256.394 mm	(Lever Arm distance)
M_u =	59.6776 kNm	(Ultimate Moment Capacity)

Back

AUSTRALIAN STANDARDS AS3600-2001

$k_1 k_3$=0.85

k_2=0.85-0.007(f'_c-28) , 0.65≤k_2≤0.85

Reference	*Beam no*	*f'_c*	*ε_{cu}*	*$k_1 k_3$*	*k_2*	*b*	*D*	*d*	*A_{st}*	*f_y*	*Tension(T)*	*C*a*	*a=T/C*	*d_n=a/g*	*0.003(d-d_n)/d_n*	*z*	*M_p=T*z*	*M_e*	*M_e/M_p*
		MPa				*mm*	*mm*	*mm*	*mm^2*	*N/mm^2*	*N*		*mm*	*mm*		*mm*	*kNm*	*kNm*	
Pam et al (2001)	1	37.4	0.003	0.85	0.7842	200	300	264	402	579	232758	6358	36.608682	46.682839	0.013965549	245.69566	57.18763	77.6	1.36
	2	36.8	0.003	0.85	0.7884	200	300	264	603	579	349137	6256	55.808344	70.786839	0.00818852	236.09583	82.429789	103.5	1.26
	3	36.4	0.003	0.85	0.7912	200	300	260	982	578	567596	6188	91.725275	115.93184	0.003728091	214.13736	121.54351	126.5	1.04
	4	42.3	0.003	0.85	0.7499	200	300	260	982	536	526352	7191	73.195939	97.6076	0.004991181	223.40203	117.58811	129	1.10
	5	46.4	0.003	0.85	0.7212	200	300	260	1183	546	645918	7888	81.886156	113.54154	0.003869733	219.05692	141.49281	142.8	1.01
	7	58.6	0.003	0.85	0.65	200	300	260	1296	520	673920	9962	67.649066	104.07549	0.00449456	226.17547	152.42417	164.6	1.08
	8	57.1	0.003	0.85	0.65	200	300	260	1473	520	765960	9707	78.908005	121.39693	0.003425204	220.546	168.92941	166.2	0.98
	9	58.6	0.003	0.85	0.65	200	300	256	1809	520	940680	9962	94.426822	145.27203	0.002286633	208.78659	196.40137	171.6	0.87
	14	95.5	0.003	0.85	0.65	200	300	260	982	578	567596	16235	34.961257	53.786549	0.011501767	242.51937	137.65303	138	1.00
	15	98	0.003	0.85	0.65	200	300	260	1473	578	851394	16660	51.104082	78.621664	0.00692093	234.44796	199.60759	200.7	1.01
	16	102.5	0.003	0.85	0.65	200	300	260	1473	578	851394	17425	48.860488	75.169981	0.007376483	235.56976	200.56268	181.7	0.91
	17	87	0.003	0.85	0.65	200	300	256	1608	546	877968	14790	59.362272	91.326572	0.005409382	226.31886	198.70072	172	0.87
Sarkar (1997)	HSC1-1	107	0.003	0.85	0.7947	150	250	220	339	470	159330	13642.5	11.678944	14.696042	0.041910052	214.16053	34.122197	38.94	1.14
	HSC1-2	97	0.003	0.85	0.8017	150	250	220	339	470	159330	12367.5	12.882959	16.069551	0.038071464	213.55852	34.026279	35.64	1.05
	HSC1-3	85	0.003	0.85	0.8101	150	250	220	339	442	149838	10837.5	13.825882	17.066884	0.035671384	213.08706	31.928539	37.62	1.18
	HSC2-1	105	0.003	0.85	0.7961	150	250	212.5	452	470	212440	13387.5	15.868534	19.93284	0.028982397	204.56573	43.457944	46.33	1.07
	HSC2-2	100	0.003	0.85	0.7996	150	250	212.5	452	470	212440	12750	16.661961	20.83787	0.027593338	204.16902	43.373667	46.86	1.08
	HSC2-3	77	0.003	0.85	0.8157	150	250	212.5	452	442	199784	9817.5	20.349784	24.947632	0.022553527	202.32511	40.421319	43.56	1.08
	HSC2-4	90	0.003	0.85	0.8066	150	250	212.5	452	442	199784	11475	17.41037	21.584888	0.026534553	203.79481	40.714943	48.84	1.20
	HSC3-1	107	0.003	0.85	0.7947	150	250	215	628	470	295160	13642.5	21.635331	27.224526	0.020691873	204.18233	60.266458	67.32	1.12
	HSC3-2	85	0.003	0.85	0.8101	150	250	215	628	470	295160	10837.5	27.235063	33.619385	0.01618536	201.38247	59.440049	66	1.11
	HSC3-3	78	0.003	0.85	0.815	150	250	215	628	442	277576	9945	27.911111	34.246762	0.015833897	201.04444	55.805113	64.68	1.16
	HSC4-1	101	0.003	0.85	0.7989	150	250	207.5	1257	470	590790	12877.5	45.877694	57.426078	0.007840023	184.56115	109.03688	92.42	0.85
	HSC4-2	87	0.003	0.85	0.8087	150	250	207.5	1257	470	590790	11092.5	53.260311	65.85917	0.006451987	180.86984	106.8561	89.6	0.84
	HSC4-3	82	0.003	0.85	0.8122	150	250	207.5	1257	442	555594	10455	53.141463	65.429036	0.006514125	180.92927	100.52322	111.63	1.11
Bernardo & Lopes (2004)	A1	62.9	0.003	0.85	0.82557	125	270	238	452	534	241368	6683.125	36.116039	43.746791	0.013321197	219.94198	53.086956	50.275	0.95
	A2	64.9	0.003	0.85	0.82417	130	270	237	628	575	361100	7171.45	50.352439	61.094725	0.008637666	211.82378	76.489567	64.94	0.85
	A3	64.1	0.003	0.85	0.82473	120	270	237	628	575	361100	6538.2	55.229268	66.966484	0.007617252	209.38537	75.609056	64.55	0.85
	A4	63.2	0.003	0.85	0.82536	120	270	234	804	575	462300	6446.4	71.714445	86.888685	0.005079303	198.14278	91.601406	78.48	0.86
	A5	65.1	0.003	0.85	0.82403	120	270	234	804	575	462300	6640.2	69.621397	84.488911	0.005308783	199.1893	92.085214	72.545	0.79
	B1	79.2	0.003	0.85	0.81416	120	264	237	452	534	241368	8078.4	29.878194	36.698184	0.016374256	222.0609	53.598396	47.995	0.90
	B2	78.9	0.003	0.85	0.81437	124	270	242	628	575	361100	8316.06	43.422005	53.31975	0.010615968	220.289	79.546357	63.665	0.80
	B3	78.5	0.003	0.85	0.81465	120	270	242	628	575	361100	8007	45.098039	55.358791	0.010114448	219.45098	79.243749	65.95	0.83
	C1	82.9	0.003	0.85	0.81157	123	270	242	628	575	361100	8667.195	41.662845	51.336108	0.011142093	221.16858	79.863973	62.785	0.79
	C2	83.9	0.003	0.85	0.81087	120	270	242	628	575	361100	8557.8	42.195424	52.037224	0.010951551	220.90229	79.767816	67.96	0.85
	C3	83.6	0.003	0.85	0.81108	125	270	239	804	575	462300	8882.5	52.046158	64.168958	0.008173627	212.97692	98.459231	72.445	0.74
	C4	83.4	0.003	0.85	0.81122	122	275	244	804	575	462300	8648.58	53.453862	65.893175	0.008108889	217.27307	100.44534	77.94	0.78
	D1	88	0.003	0.85	0.808	120	270	247	402	575	231150	8976	25.752005	31.871294	0.020249762	234.124	54.117762	44.22	0.82
	D2	85.8	0.003	0.85	0.80954	120	270	238	1030	572	589160	8751.6	67.320261	83.15866	0.005585997	204.33987	120.38888	93.34	0.78
	D3	86	0.003	0.85	0.8094	120	270	238	1030	572	589160	8772	67.163703	82.979618	0.005604523	204.41815	120.435	97.61	0.81
	E1	94.6	0.003	0.85	0.80338	123	270	239	804	575	462300	9890.43	46.742154	58.181874	0.009323426	215.62892	99.685251	78.995	0.79
	E2	90.2	0.003	0.85	0.80646	120	270	239	804	575	462300	9200.4	50.247815	62.306643	0.008507601	213.87609	98.874917	78.895	0.80
	F1	100.3	0.003	0.85	0.79939	139	263	230	628	575	361100	11850.445	30.47143	38.118352	0.015101517	214.76429	77.551383	64.23	0.83
	F2	105.2	0.003	0.85	0.79596	129	270	234	804	575	462300	11535.18	40.077398	50.35102	0.010942121	213.9613	98.914309	79.07	0.80
S.A.Ashour (2000)	B-N2	48.61	0.003	0.85	0.835573	200	250	215	509	530	269770	8263.7	32.645183	39.069217	0.013509161	198.67741	53.597204	58.17	1.09
	B-N3	48.61	0.003	0.85	0.835573	200	250	215	763	530	404390	8263.7	48.935707	58.565448	0.00801332	190.53215	77.049295	80.6	1.05
	B-N4	48.61	0.003	0.85	0.835573	200	250	215	1018	530	539540	8263.7	65.290366	78.138435	0.005254581	182.35482	98.387718	99.55	1.01
	B-M2	78.5	0.003	0.85	0.81465	200	250	215	509	530	269770	13345	20.215062	24.814413	0.022992958	204.89247	55.273841	57.95	1.05
	B-M3	78.5	0.003	0.85	0.81465	200	250	215	763	530	404390	13345	30.302735	37.197244	0.014339994	199.84863	80.816788	79.91	0.99
	B-M4	78.5	0.003	0.85	0.81465	200	250	215	1018	530	539540	13345	40.430124	49.628827	0.009996479	194.78494	105.09427	103.77	0.99
	B-H2	102.4	0.003	0.85	0.79792	200	250	215	509	530	269770	17408	15.496898	19.421619	0.030210414	207.25155	55.910251	56.8	1.02
	B-H3	102.4	0.003	0.85	0.79792	200	250	215	763	530	404390	17408	23.230124	29.11335	0.019154785	203.38494	82.246835	82.76	1.01
	B-H4	102.4	0.003	0.85	0.79792	200	250	215	1018	530	539540	17408	30.993796	38.843237	0.013605207	199.5031	107.6399	108.1	1.00

Table A.1 Calculation of Ultimate Moment Capacity for beams as per AS3600-2001

AMERICAN CONCRETE CODE ACI 318-2005

$k_1 k_3 = 0.85$
$k_2 = 1.09 - 0.008 f'_c$ ($0.65 \le k_2 \le 0.85$)

Reference	*Beam no*	*f'_c*	*ε_{cu}*	*$k_1 k_3$*	*k_2*	*b*	*D*	*d*	*A_{st}*	*f_y*	*Tension(T)*	*C*a*	*a=T/C*	*$d_n = a/g$*	*$0.003(d-d_n)/d_n$*	*z*	*$M_p = T*z$*	*M_e*	*M_e/M_p*
		MPa				*mm*	*mm*	*mm*	*mm^2*	*N/mm^2*	*N*		*mm*	*mm*		*mm*	*kNm*	*kNm*	
Pam et al (2001)	1	37.4	0.003	0.85	0.7908	200	300	264	402	579	232758	6358	36.608682	46.293225	0.014108335	245.69566	57.18763	77.6	1.36
	2	36.8	0.003	0.85	0.7956	200	300	264	603	579	349137	6256	55.808344	70.146234	0.008290699	236.09583	82.429789	103.5	1.26
	3	36.4	0.003	0.85	0.7988	200	300	260	982	578	567596	6188	91.725275	114.82884	0.003792719	214.13736	121.54351	126.5	1.04
	4	42.3	0.003	0.85	0.7516	200	300	260	982	536	526352	7191	73.195939	97.386827	0.005009297	223.40203	117.58811	129	1.10
	5	46.4	0.003	0.85	0.7188	200	300	260	1183	546	645918	7888	81.886156	113.92064	0.003846872	219.05692	141.49281	164.6	1.16
	7	58.6	0.003	0.85	0.65	200	300	260	1296	520	673920	9962	67.649066	104.07549	0.00449456	226.17547	152.42417	166.2	1.09
	8	57.1	0.003	0.85	0.65	200	300	260	1473	520	765960	9707	78.908005	121.39693	0.003425204	220.546	168.92941	171.6	1.02
	9	58.6	0.003	0.85	0.65	200	300	256	1809	520	940680	9962	94.426822	145.27203	0.002286633	208.78659	196.40137	138	0.70
	14	95.5	0.003	0.85	0.65	200	300	260	982	578	567596	16235	34.961257	53.786549	0.011501767	242.51937	137.65303	200.7	1.46
	15	98	0.003	0.85	0.65	200	300	260	1473	578	851394	16660	51.104082	78.621664	0.00692093	234.44796	199.60759	181.7	0.91
	16	102.5	0.003	0.85	0.65	200	300	260	1473	578	851394	17425	48.860488	75.169981	0.007376483	235.56976	200.56268	172	0.86
	17	87	0.003	0.85	0.65	200	300	256	1608	546	877968	14790	59.362272	91.326572	0.005409382	226.31886	198.70072	189.9	0.96
Sarkar (1997)	HSC1-1	107	0.003	0.85	0.65	150	250	220	339	470	159330	13642.5	11.678944	17.967607	0.033732772	214.16053	34.122197	38.94	1.14
	HSC1-2	97	0.003	0.85	0.65	150	250	220	339	470	159330	12367.5	12.882959	19.819937	0.030299802	213.55852	34.026279	35.64	1.05
	HSC1-3	85	0.003	0.85	0.65	150	250	220	339	442	149838	10837.5	13.825882	21.270588	0.028028761	213.08706	31.928539	37.62	1.18
	HSC2-1	105	0.003	0.85	0.65	150	250	212.5	452	470	212440	13387.5	15.868534	24.413129	0.023112998	204.56573	43.457944	46.33	1.07
	HSC2-2	100	0.003	0.85	0.65	150	250	212.5	452	470	212440	12750	16.661961	25.633786	0.021869522	204.16902	43.373667	46.86	1.08
	HSC2-3	77	0.003	0.85	0.65	150	250	212.5	452	442	199784	9817.5	20.349784	31.307359	0.017362624	202.32511	40.421319	43.56	1.08
	HSC2-4	90	0.003	0.85	0.65	150	250	212.5	452	442	199784	11475	17.41037	26.785185	0.02080047	203.79481	40.714943	48.84	1.20
	HSC3-1	107	0.003	0.85	0.65	150	250	215	628	470	295160	13642.5	21.635331	33.285124	0.016378026	204.18233	60.266458	67.32	1.12
	HSC3-2	85	0.003	0.85	0.65	150	250	215	628	470	295160	10837.5	27.235063	41.900098	0.012393759	201.38247	59.440049	66	1.11
	HSC3-3	78	0.003	0.85	0.65	150	250	215	628	442	277576	9945	27.911111	42.940171	0.0120209	201.04444	55.805113	64.68	1.16
	HSC4-1	101	0.003	0.85	0.65	150	250	207.5	1257	470	590790	12877.5	45.877694	70.581067	0.005819646	184.56115	109.03688	92.42	0.85
	HSC4-2	87	0.003	0.85	0.65	150	250	207.5	1257	470	590790	11092.5	53.260311	81.93894	0.00459712	180.86984	106.8561	89.6	0.84
	HSC4-3	82	0.003	0.85	0.65	150	250	207.5	1257	442	555594	10455	53.141463	81.756098	0.004614111	180.92927	100.52322	111.63	1.11
Bernardo & Lopes (2004)	A1	62.9	0.003	0.85	0.65	125	270	238	452	534	241368	6683.125	36.116039	55.563136	0.009850247	219.94198	53.086956	50.275	0.95
	A2	64.9	0.003	0.85	0.65	130	270	237	628	575	361100	7171.45	50.352439	77.465291	0.006178304	211.82378	76.489567	64.94	0.85
	A3	64.1	0.003	0.85	0.65	120	270	237	628	575	361100	6538.2	55.229268	84.968105	0.005367846	209.38537	75.609056	64.55	0.85
	A4	63.2	0.003	0.85	0.65	120	270	234	804	575	462300	6446.4	71.714445	110.32992	0.003362735	198.14278	91.601406	78.48	0.86
	A5	65.1	0.003	0.85	0.65	120	270	234	804	575	462300	6640.2	69.621397	107.10984	0.00355402	199.1893	92.085214	72.545	0.79
	B1	79.2	0.003	0.85	0.65	120	264	237	452	534	241368	8078.4	29.878194	45.966452	0.012467803	222.0609	53.598396	47.995	0.90
	B2	78.9	0.003	0.85	0.65	124	270	242	628	575	361100	8316.06	43.422005	66.803085	0.007867762	220.289	79.546357	63.665	0.80
	B3	78.5	0.003	0.85	0.65	120	270	242	628	575	361100	8007	45.098039	69.381599	0.00746387	219.45098	79.243749	65.95	0.83
	C1	82.9	0.003	0.85	0.65	123	270	242	628	575	361100	8667.195	41.662845	64.096684	0.008326639	221.16858	79.863973	62.785	0.79
	C2	83.9	0.003	0.85	0.65	120	270	242	628	575	361100	8557.8	42.195424	64.916037	0.008183677	220.90229	79.767816	67.96	0.85
	C3	83.6	0.003	0.85	0.65	125	270	239	804	575	462300	8882.5	52.046158	80.071013	0.005954551	212.97692	98.459231	72.445	0.74
	C4	83.4	0.003	0.85	0.65	122	275	244	804	575	462300	8648.58	53.453862	82.23671	0.005901134	217.27307	100.44534	77.94	0.78
	D1	88	0.003	0.85	0.65	120	270	247	402	575	231150	8976	25.752005	39.61847	0.015703398	234.124	54.117762	44.22	0.82
	D2	85.8	0.003	0.85	0.65	120	270	238	1030	572	589160	8751.6	67.320261	103.56963	0.003893913	204.33987	120.38888	93.34	0.78
	D3	86	0.003	0.85	0.65	120	270	238	1030	572	589160	8772	67.163703	103.32877	0.003909982	204.41815	120.435	97.61	0.81
	E1	94.6	0.003	0.85	0.65	123	270	239	804	575	462300	9890.43	46.742154	71.911006	0.006970657	215.62892	99.685251	78.995	0.79
	E2	90.2	0.003	0.85	0.65	120	270	239	804	575	462300	9200.4	50.247815	77.304331	0.00627503	213.87609	98.874917	78.895	0.80
	F1	100.3	0.003	0.85	0.65	139	263	230	628	575	361100	11850.445	30.47143	46.879122	0.011718706	214.76429	77.551383	64.23	0.83
	F2	105.2	0.003	0.85	0.65	129	270	234	804	575	462300	11535.18	40.077398	61.657535	0.00838547	213.9613	98.914309	79.07	0.80
S.A.Ashour (2000)	B-N2	48.61	0.003	0.85	0.70112	200	250	215	509	530	269770	8263.7	32.645183	46.561478	0.010852653	198.67741	53.597204	58.17	1.09
	B-N3	48.61	0.003	0.85	0.70112	200	250	215	763	530	404390	8263.7	48.935707	69.796478	0.006241154	190.53215	77.049295	80.6	1.05
	B-N4	48.61	0.003	0.85	0.70112	200	250	215	1018	530	539540	8263.7	65.290366	93.122955	0.003926327	182.35482	98.387718	99.55	1.01
	B-M2	78.5	0.003	0.85	0.65	200	250	215	509	530	269770	13345	20.215062	31.100095	0.017739486	204.89247	55.273841	57.95	1.05
	B-M3	78.5	0.003	0.85	0.65	200	250	215	763	530	404390	13345	30.302735	46.619592	0.010835385	199.84863	80.816788	79.91	0.99
	B-M4	78.5	0.003	0.85	0.65	200	250	215	1018	530	539540	13345	40.430124	62.20019	0.007369743	194.78494	105.09427	103.77	0.99
	B-H2	102.4	0.003	0.85	0.65	200	250	215	509	530	269770	17408	15.496898	23.841382	0.024053801	207.25155	55.910251	56.8	1.02
	B-H3	102.4	0.003	0.85	0.65	200	250	215	763	530	404390	17408	23.230124	35.738652	0.015047687	203.38494	82.246835	82.76	1.01
	B-H4	102.4	0.003	0.85	0.65	200	250	215	1018	530	539540	17408	30.993796	47.682763	0.010526901	199.5031	107.6399	108.1	1.00

Table A.2 Calculation of Ultimate Moment Capacity for beams as per ACI 318-2005

CANADIAN STANDARDS CAN3-A23.3-M94

$k_1 k_3$=0.85-0.0015f'_c≥0.67
k_2=0.97-0.0025f'_c≥0.67

Reference	*Beam no*	*f'_c*	*ε_{cu}*	*$k_1 k_3$*	*k_2*	*b*	*D*	*d*	*A_{st}*	*f_y*	*Tension(T)*	*C*a*	*a=T/C*	*d_n=a/g*	*0.003(d-d_n)/d_n*	*z*	*M_p=T*z*	*M_e*	*M_e/M_p*
		MPa				*mm*	*mm*	*mm*	*mm^2*	*N/mm^2*	*N*		*mm*	*mm*		*mm*	*kNm*	*kNm*	
Pam et al (2001)	1	37.4	0.003	0.7939	0.8765	200	300	264	402	579	232758	5938.372	39.195591	44.718301	0.01471087	244.4022	56.886568	77.6	1.36
	2	36.8	0.003	0.7948	0.878	200	300	264	603	579	349137	5849.728	59.684314	67.977578	0.008650901	234.15784	81.753167	103.5	1.27
	3	36.4	0.003	0.7954	0.879	200	300	260	982	578	567596	5790.512	98.021729	111.51505	0.003994572	210.98914	119.75659	126.5	1.06
	4	42.3	0.003	0.78655	0.86425	200	300	260	982	536	526352	6654.213	79.100564	91.525096	0.005522253	220.44972	116.03415	129	1.11
	5	46.4	0.003	0.7804	0.854	200	300	260	1183	546	645918	7242.112	89.189176	104.43697	0.004468619	215.40541	139.13423	142.8	1.03
	7	58.6	0.003	0.7621	0.8235	200	300	260	1296	520	673920	8931.812	75.451655	91.623139	0.005513133	222.27417	149.79501	164.6	1.10
	8	57.1	0.003	0.76435	0.82725	200	300	260	1473	520	765960	8728.877	87.75012	106.07449	0.004353323	216.12494	165.54306	166.2	1.00
	9	58.6	0.003	0.7621	0.8235	200	300	256	1809	520	940680	8931.812	105.31794	127.89063	0.003005131	203.34103	191.27884	171.6	0.90
	14	95.5	0.003	0.70675	0.73125	200	300	260	982	578	567596	13498.925	42.047496	57.50085	0.010565017	238.97625	135.64196	138	1.02
	15	98	0.003	0.703	0.725	200	300	260	1473	578	851394	13778.8	61.790141	85.227781	0.006151945	229.10493	195.05856	200.7	1.03
	16	102.5	0.003	0.69625	0.71375	200	300	260	1473	578	851394	14273.125	59.650147	83.572885	0.006333171	230.17493	195.96955	181.7	0.93
	17	87	0.003	0.7195	0.7525	200	300	256	1608	546	877968	12519.3	70.129161	93.194898	0.005240794	220.93542	193.97423	172	0.89
Sarkar (1997)	HSC1-1	107	0.003	0.6895	0.7025	150	250	220	339	470	159330	11066.475	14.397539	20.494717	0.029203421	212.80123	33.90562	38.94	1.15
	HSC1-2	97	0.003	0.7045	0.7275	150	250	220	339	470	159330	10250.475	15.54367	21.365869	0.027890388	212.22817	33.814314	35.64	1.05
	HSC1-3	85	0.003	0.7225	0.7575	150	250	220	339	442	149838	9211.875	16.265744	21.472929	0.027736375	211.86713	31.745747	37.62	1.19
	HSC2-1	105	0.003	0.6925	0.7075	150	250	212.5	452	470	212440	10906.875	19.477623	27.530209	0.02015638	202.76119	43.074587	46.33	1.08
	HSC2-2	100	0.003	0.7	0.72	150	250	212.5	452	470	212440	10500	20.232381	28.100529	0.019686406	202.38381	42.994416	46.86	1.09
	HSC2-3	77	0.003	0.7345	0.7775	150	250	212.5	452	442	199784	8483.475	23.549784	30.289111	0.018047168	200.72511	40.101665	43.56	1.09
	HSC2-4	90	0.003	0.715	0.745	150	250	212.5	452	442	199784	9652.5	20.697643	27.782071	0.019946453	202.15118	40.386571	48.84	1.21
	HSC3-1	107	0.003	0.6895	0.7025	150	250	215	628	470	295160	11066.475	26.671546	37.966614	0.01398861	201.66423	59.523213	67.32	1.13
	HSC3-2	85	0.003	0.7225	0.7575	150	250	215	628	470	295160	9211.875	32.041251	42.298681	0.012248702	198.97937	58.730752	66	1.12
	HSC3-3	78	0.003	0.733	0.775	150	250	215	628	442	277576	8576.1	32.366227	41.762874	0.01244434	198.81689	55.186796	64.68	1.17
	HSC4-1	101	0.003	0.6985	0.7175	150	250	207.5	1257	470	590790	10582.275	55.82826	77.809422	0.005000317	179.58587	106.09754	92.42	0.87
	HSC4-2	87	0.003	0.7195	0.7525	150	250	207.5	1257	470	590790	9389.475	62.920451	83.615217	0.004444817	176.03977	104.00254	89.6	0.86
	HSC4-3	82	0.003	0.727	0.765	150	250	207.5	1257	442	555594	8942.1	62.132385	81.218804	0.004664481	176.43381	98.025565	111.63	1.14
Bernardo & Lopes (2004)	A1	62.9	0.003	0.75565	0.81275	125	270	238	452	534	241368	5941.2981	40.625465	49.985192	0.01128423	217.68727	52.54274	50.275	0.96
	A2	64.9	0.003	0.75265	0.80775	130	270	237	628	575	361100	6350.1081	56.865174	70.399473	0.007099507	208.56741	75.313693	64.94	0.86
	A3	64.1	0.003	0.75385	0.80975	120	270	237	628	575	361100	5798.6142	62.2735	76.9046	0.006245221	205.86325	74.33722	64.55	0.87
	A4	63.2	0.003	0.7552	0.812	120	270	234	804	575	462300	5727.4368	80.716735	99.404846	0.00406203	193.64163	89.520527	78.48	0.88
	A5	65.1	0.003	0.75235	0.80725	120	270	234	804	575	462300	5877.3582	78.657789	97.439193	0.004204493	194.67111	89.996452	72.545	0.81
	B1	79.2	0.003	0.7312	0.772	120	264	237	452	534	241368	6949.3248	34.732583	44.990392	0.012803374	219.63371	53.012549	47.995	0.91
	B2	78.9	0.003	0.73165	0.77275	124	270	242	628	575	361100	7158.1709	50.445848	65.280942	0.008121163	216.77708	78.278202	63.665	0.81
	B3	78.5	0.003	0.73225	0.77375	120	270	242	628	575	361100	6897.795	52.350063	67.657593	0.007730503	215.82497	77.934396	65.95	0.85
	C1	82.9	0.003	0.72565	0.76275	123	270	242	628	575	361100	7399.2354	48.80234	63.982091	0.008346925	217.59883	78.574938	62.785	0.80
	C2	83.9	0.003	0.72415	0.76025	120	270	242	628	575	361100	7290.7422	49.528565	65.147734	0.008143902	217.23572	78.443818	67.96	0.87
	C3	83.6	0.003	0.7246	0.761	125	270	239	804	575	462300	7572.07	61.053318	80.227751	0.005937057	208.47334	96.377225	72.445	0.75
	C4	83.4	0.003	0.7249	0.7615	122	275	244	804	575	462300	7375.7125	62.67869	82.309508	0.005893262	212.66065	98.313021	77.94	0.79
	D1	88	0.003	0.718	0.75	120	270	247	402	575	231150	7582.08	30.486357	40.648476	0.015229466	231.75682	53.570589	44.22	0.83
	D2	85.8	0.003	0.7213	0.7555	120	270	238	1030	572	589160	7426.5048	79.33207	105.00605	0.003799608	198.33396	116.85044	93.34	0.80
	D3	86	0.003	0.721	0.755	120	270	238	1030	572	589160	7440.72	79.180509	104.87485	0.003808115	198.40975	116.89509	97.61	0.84
	E1	94.6	0.003	0.7081	0.7335	123	270	239	804	575	462300	8239.31	56.109068	76.49498	0.006373164	210.94547	97.520089	78.995	0.81
	E2	90.2	0.003	0.7147	0.7445	120	270	239	804	575	462300	7735.9128	59.760239	80.268958	0.005932469	209.11988	96.676121	78.895	0.82
	F1	100.3	0.003	0.69955	0.71925	139	263	230	628	575	361100	9752.9162	37.024823	51.476988	0.010404048	211.48759	76.368168	64.23	0.84
	F2	105.2	0.003	0.6922	0.707	129	270	234	804	575	462300	9393.7078	49.213794	69.609327	0.007084855	209.3931	96.802431	79.07	0.82
S.A.Ashour (2000)	B-N2	48.61	0.003	0.777085	0.848475	200	250	215	509	530	269770	7554.8204	35.708328	42.085303	0.012326015	197.14584	53.184032	58.17	1.09
	B-N3	48.61	0.003	0.777085	0.848475	200	250	215	763	530	404390	7554.8204	53.527414	63.086613	0.007224039	188.23629	76.120874	80.6	1.06
	B-N4	48.61	0.003	0.777085	0.848475	200	250	215	1018	530	539540	7554.8204	71.416655	84.170606	0.004663008	179.29167	96.735029	99.55	1.03
	B-M2	78.5	0.003	0.73225	0.77375	200	250	215	509	530	269770	11496.325	23.46576	30.327315	0.018267956	203.26712	54.835371	57.95	1.06
	B-M3	78.5	0.003	0.73225	0.77375	200	250	215	763	530	404390	11496.325	35.175589	45.461181	0.011187929	197.41221	79.831522	79.91	1.00
	B-M4	78.5	0.003	0.73225	0.77375	200	250	215	1018	530	539540	11496.325	46.931519	60.654629	0.007633978	191.53424	103.34038	103.77	1.00
	B-H2	102.4	0.003	0.6964	0.714	200	250	215	509	530	269770	14262.272	18.914939	26.491511	0.021347422	205.54253	55.449208	56.8	1.02
	B-H3	102.4	0.003	0.6964	0.714	200	250	215	763	530	404390	14262.272	28.353827	39.711243	0.013242252	200.82309	81.210848	82.76	1.02
	B-H4	102.4	0.003	0.6964	0.714	200	250	215	1018	530	539540	14262.272	37.829877	52.983021	0.009173711	196.08506	105.79573	108.1	1.02

Table A.3 Calculation of Ultimate Moment Capacity for beams as per CAN3-A23.3-M94

NEW ZEALAND STANDARDS NZS3101-1995

$k_1 k_3 = 1.07 - 0.004f'_c$; $0.75 \le k_1 k_3 \le 0.85$
$k_2 = 1.09 - 0.008f'_c$; $0.65 \le k_2 \le 0.85$

Reference	Beam no	f'_c	ε_{cu}	$k_1 k_3$	k_2	b	D	d	A_{st}	f_y	Tension(T)	C*a	a=T/C	d_n=a/g	0.003(d-d_n)/	z	M_p=T*z	M_e	M_e/M_p
		MPa				mm	mm	mm	mm^2	N/mm^2	N		mm	mm		mm	kNm	kNm	
Pam et al (2001)	1	37.4	0.003	0.85	0.7908	200	300	264	402	579	232758	6358	36.608682	46.293225	0.0141083	245.69566	57.18763	77.6	1.36
	2	36.8	0.003	0.85	0.7956	200	300	264	603	579	349137	6256	55.808344	70.146234	0.0082907	236.09583	82.429789	103.5	1.26
	3	36.4	0.003	0.85	0.7988	200	300	260	982	578	567596	6188	91.725275	114.82884	0.0037927	214.13736	121.54351	126.5	1.04
	4	42.3	0.003	0.85	0.7516	200	300	260	982	536	526352	7191	73.195939	97.386827	0.0050093	223.40203	117.58811	129	1.10
	5	46.4	0.003	0.85	0.7188	200	300	260	1183	546	645918	7754.368	83.297311	115.88385	0.0037309	218.35134	141.03706	142.8	1.01
	7	58.6	0.003	0.8356	0.65	200	300	260	1296	520	673920	9863.552	68.324271	105.11426	0.0044205	225.83786	152.19665	164.6	1.08
	8	57.1	0.003	0.8416	0.65	200	300	260	1473	520	765960	9542.552	80.267836	123.48898	0.0033164	219.86608	168.40862	166.2	0.99
	9	58.6	0.003	0.8356	0.65	200	300	256	1809	520	940680	8790	107.01706	164.64164	0.0016647	202.49147	190.47967	171.6	0.90
	14	95.5	0.003	0.75	0.65	200	300	260	982	578	567596	14325	39.622757	60.958088	0.0097957	240.18862	136.3301	138	1.01
	15	98	0.003	0.75	0.65	200	300	260	1473	578	851394	14700	57.917959	89.104553	0.0057538	231.04102	196.70694	200.7	1.02
	16	102.5	0.003	0.75	0.65	200	300	260	1473	578	851394	15375	55.37522	85.192645	0.0061557	232.31239	197.78938	181.7	0.92
	17	87	0.003	0.75	0.65	200	300	256	1608	546	877968	13050	67.277241	103.50345	0.00442	222.36138	195.22618	172	0.88
Sarkar (1997)	HSC1-1	107	0.003	0.75	0.65	150	250	220	339	470	159330	12037.5	13.236137	20.363288	0.0294113	213.38193	33.998143	38.94	1.15
	HSC1-2	97	0.003	0.75	0.65	150	250	220	339	470	159330	10912.5	14.600687	22.462596	0.0263822	212.69966	33.889436	35.64	1.05
	HSC1-3	85	0.003	0.75	0.65	150	250	220	339	442	149838	9562.5	15.669333	24.106667	0.0243783	212.16533	31.790429	37.62	1.18
	HSC2-1	105	0.003	0.75	0.65	150	250	212.5	452	470	212440	11812.5	17.984339	27.668213	0.0200409	203.50783	43.233204	46.33	1.07
	HSC2-2	100	0.003	0.75	0.65	150	250	212.5	452	470	212440	11430	18.586177	28.594118	0.0192948	203.20691	43.169276	46.86	1.09
	HSC2-3	77	0.003	0.762	0.65	150	250	212.5	452	442	199784	8662.5	23.063088	35.481674	0.014967	200.96846	40.150282	43.56	1.08
	HSC2-4	90	0.003	0.75	0.65	150	250	212.5	452	442	199784	10125	19.731753	30.356543	0.0180004	202.63412	40.483056	48.84	1.21
	HSC3-1	107	0.003	0.75	0.65	150	250	215	628	470	295160	12037.5	24.520042	37.723141	0.0140983	202.73998	59.840732	67.32	1.12
	HSC3-2	85	0.003	0.75	0.65	150	250	215	628	470	295160	9664.5	30.540638	46.985598	0.0107276	199.72968	58.952213	66	1.12
	HSC3-3	78	0.003	0.758	0.65	150	250	215	628	442	277576	8775	31.632593	48.665527	0.0102537	199.1837	55.288616	64.68	1.17
	HSC4-1	101	0.003	0.75	0.65	150	250	207.5	1257	470	590790	11362.5	51.994719	79.991876	0.004782	181.50264	107.22994	92.42	0.86
	HSC4-2	87	0.003	0.75	0.65	150	250	207.5	1257	470	590790	9787.5	60.361686	92.864132	0.0037033	177.31916	104.75838	89.6	0.86
	HSC4-3	82	0.003	0.75	0.65	150	250	207.5	1257	442	555594	10066.32	55.193358	84.912858	0.004331	179.90332	99.953206	111.63	1.12
Bernardo & Lopes (2004)	A1	62.9	0.003	0.8184	0.65	125	270	238	452	534	241368	6371.77	37.88084	58.278215	0.0092516	219.05958	52.873973	50.275	0.95
	A2	64.9	0.003	0.8104	0.65	130	270	237	628	575	361100	6864.3432	52.605179	80.931044	0.0057853	210.69741	76.082835	64.94	0.85
	A3	64.1	0.003	0.8136	0.65	120	270	237	628	575	361100	6285.9024	57.446008	88.378474	0.0050449	208.277	75.208823	64.55	0.86
	A4	63.2	0.003	0.8172	0.65	120	270	234	804	575	462300	6140.0064	75.293081	115.83551	0.0030603	196.35346	90.774204	78.48	0.86
	A5	65.1	0.003	0.8096	0.65	120	270	234	804	575	462300	5883.9984	78.569022	120.87542	0.0028076	194.71549	90.016971	72.545	0.81
	B1	79.2	0.003	0.7532	0.65	120	264	237	452	534	241368	7169.8176	33.664455	51.791469	0.0107281	220.16777	53.141455	47.995	0.90
	B2	78.9	0.003	0.7544	0.65	124	270	242	628	575	361100	7396.4016	48.821038	75.109288	0.0066659	217.58948	78.571562	63.665	0.81
	B3	78.5	0.003	0.756	0.65	120	270	242	628	575	361100	7065	51.111111	78.632479	0.0062328	216.44444	78.158089	65.95	0.84
	C1	82.9	0.003	0.75	0.65	123	270	242	628	575	361100	7647.525	47.217891	72.642909	0.0069941	218.39105	78.86101	62.785	0.80
	C2	83.9	0.003	0.75	0.65	120	270	242	628	575	361100	7551	47.821481	73.571509	0.006868	218.08926	78.752032	67.96	0.86
	C3	83.6	0.003	0.75	0.65	125	270	239	804	575	462300	7837.5	58.985646	90.747148	0.0049011	209.50718	96.855168	72.445	0.75
	C4	83.4	0.003	0.75	0.65	122	275	244	804	575	462300	7631.1	60.581043	93.201605	0.0048539	213.70948	98.797892	77.94	0.79
	D1	88	0.003	0.75	0.65	120	270	247	402	575	231150	7920	29.185606	44.900932	0.013503	232.4072	53.720924	44.22	0.82
	D2	85.8	0.003	0.75	0.65	120	270	238	1030	572	589160	7722	76.296296	117.37892	0.0030829	199.85185	117.74472	93.34	0.79
	D3	86	0.003	0.75	0.65	120	270	238	1030	572	589160	7740	76.118863	117.10594	0.003097	199.94057	117.79699	97.61	0.83
	E1	94.6	0.003	0.75	0.65	123	270	239	804	575	462300	8726.85	52.974441	81.49914	0.0057976	212.51278	98.244658	78.995	0.80
	E2	90.2	0.003	0.75	0.65	120	270	239	804	575	462300	8118	56.947524	87.611575	0.0051839	210.52624	97.32628	78.895	0.81
	F1	100.3	0.003	0.75	0.65	139	263	230	628	575	361100	10456.275	34.534287	53.129672	0.0099871	212.73286	76.817835	64.23	0.84
	F2	105.2	0.003	0.75	0.65	129	270	234	804	575	462300	10178.1	45.421051	69.87854	0.007046	211.28947	97.679124	79.07	0.81
S.A.Ashour (2000)	B-N2	48.61	0.003	0.85	0.70112	200	250	215	509	530	269770	8263.7	32.645183	46.561478	0.0108527	198.67741	53.597204	58.17	1.09
	B-N3	48.61	0.003	0.85	0.70112	200	250	215	763	530	404390	8263.7	48.935707	69.796478	0.0062412	190.53215	77.049295	80.6	1.05
	B-N4	48.61	0.003	0.756	0.70112	200	250	215	1018	530	539540	8263.7	65.290366	93.122955	0.0039263	182.35482	98.387718	99.55	1.01
	B-M2	78.5	0.003	0.756	0.462	200	250	215	509	530	269770	13345	20.215062	43.755545	0.011741	204.89247	55.273841	57.95	1.05
	B-M3	78.5	0.003	0.756	0.462	200	250	215	763	530	404390	13345	30.302735	65.590336	0.0068338	199.84863	80.816788	79.91	0.99
	B-M4	78.5	0.003	0.75	0.462	200	250	215	1018	530	539540	13345	40.430124	87.51109	0.0043705	194.78494	105.09427	103.77	0.99
	B-H2	102.4	0.003	0.75	0.2708	200	250	215	509	530	269770	17408	15.496898	57.226359	0.008271	207.25155	55.910251	56.8	1.02
	B-H3	102.4	0.003	0.75	0.2708	200	250	215	763	530	404390	17408	23.230124	85.783324	0.0045189	203.38494	82.246835	82.76	1.01
	B-H4	102.4	0.003	0.75	0.2708	200	250	215	1018	530	539540	17408	30.993796	114.45272	0.0026355	199.5031	107.6399	108.1	1.00

Table A.4 Calculation of Ultimate Moment Capacity for beams as per NZS3101-1995

CEB-FIP MODEL CODE 1990

$k_1 k_3 = 0.85(1 - f'_c/250)$
$k_2 = 1$
$\varepsilon_{cu} = 0.004 - 0.002 f'_c/100$

Reference	Beam no	f'_c	ε_{cu}	$k_1 k_3$	k_2	b	D	d	A_{st}	f_y	Tension(T)	C*a	a=T/C	$d_n=a/g$	$0.003(d-d_n)/d_n$	z	$M_p=T*z$	M_e	M_e/M_p
		MPa				mm	mm	mm	mm^2	N/mm^2	N		mm	mm		mm	kNm	kNm	
Pam et al (2001)	1	37.4	0.003252	0.72284	1	200	300	264	402	579	232758	5406.8432	43.048779	43.048779	0.015397734	242.47561	56.438138	77.6	1.37
	2	36.8	0.003264	0.72488	1	200	300	264	603	579	349137	5335.1168	65.441304	65.441304	0.009102448	231.27935	80.748178	103.5	1.28
	3	36.4	0.003272	0.72624	1	200	300	260	982	578	567596	5287.0272	107.35636	107.35636	0.004265522	206.32182	117.10744	126.5	1.08
	4	42.3	0.003154	0.70618	1	200	300	260	982	536	526352	5974.2828	88.10296	88.10296	0.005853278	215.94852	113.66494	129	1.13
	5	46.4	0.003072	0.69224	1	200	300	260	1183	546	645918	6423.9872	100.54783	100.54783	0.004757502	209.72608	135.46585	142.8	1.05
	7	58.6	0.002828	0.65076	1	200	300	260	1296	520	673920	7626.9072	88.36085	88.36085	0.005827439	215.81958	145.44513	164.6	1.13
	8	57.1	0.002858	0.65586	1	200	300	260	1473	520	765960	7489.9212	102.26543	102.26543	0.004627211	208.86729	159.98399	166.2	1.04
	9	58.6	0.002828	0.65076	1	200	300	256	1809	520	940680	7626.9072	123.33702	123.33702	0.003226841	194.33149	182.80375	171.6	0.94
	14	95.5	0.00209	0.5253	1	200	300	260	982	578	567596	10033.23	56.571613	56.571613	0.010787834	231.71419	131.52005	138	1.05
	15	98	0.00204	0.5168	1	200	300	260	1473	578	851394	10129.28	84.052766	84.052766	0.006279885	217.97362	185.58143	200.7	1.08
	16	102.5	0.00195	0.5015	1	200	300	260	1473	578	851394	10280.75	82.814386	82.814386	0.006418653	218.59281	186.1086	181.7	0.98
	17	87	0.00226	0.5542	1	200	300	256	1608	546	877968	9643.08	91.046429	91.046429	0.005435257	210.47679	184.79188	172	0.93
Sarkar (1997)	HSC1-1	107	0.00186	0.4862	1	150	250	220	339	470	159330	7803.51	20.417735	20.417735	0.029324839	209.79113	33.426021	38.94	1.16
	HSC1-2	97	0.00206	0.5202	1	150	250	220	339	470	159330	7568.91	21.050587	21.050587	0.028353045	209.47471	33.375605	35.64	1.07
	HSC1-3	85	0.0023	0.561	1	150	250	220	339	442	149838	7152.75	20.948307	20.948307	0.028506127	209.52585	31.394934	37.62	1.20
	HSC2-1	105	0.0019	0.493	1	150	250	212.5	452	470	212440	7764.75	27.359542	27.359542	0.020300829	198.82023	42.237369	46.33	1.10
	HSC2-2	100	0.002	0.51	1	150	250	212.5	452	470	212440	7650	27.769935	27.769935	0.019956482	198.61503	42.193778	46.86	1.11
	HSC2-3	77	0.00246	0.5882	1	150	250	212.5	452	442	199784	6793.71	29.407202	29.407202	0.018678363	197.7964	39.516556	43.56	1.10
	HSC2-4	90	0.0022	0.544	1	150	250	212.5	452	442	199784	7344	27.203704	27.203704	0.020434309	198.89815	39.736668	48.84	1.23
	HSC3-1	107	0.00186	0.4862	1	150	250	215	628	470	295160	7803.51	37.824005	37.824005	0.014052663	196.088	57.877333	67.32	1.16
	HSC3-2	85	0.0023	0.561	1	150	250	215	628	470	295160	7152.75	41.265248	41.265248	0.012630586	194.36738	57.369475	66	1.15
	HSC3-3	78	0.00244	0.5848	1	150	250	215	628	442	277576	6842.16	40.568475	40.568475	0.012899045	194.71576	54.048422	64.68	1.20
	HSC4-1	101	0.00198	0.5066	1	150	250	207.5	1257	470	590790	7674.99	76.975996	76.975996	0.005086937	169.012	99.850601	92.42	0.93
	HSC4-2	87	0.00226	0.5542	1	150	250	207.5	1257	470	590790	7232.31	81.687594	81.687594	0.004620496	166.6562	98.458818	89.6	0.91
	HSC4-3	82	0.00236	0.5712	1	150	250	207.5	1257	442	555594	7025.76	79.079559	79.079559	0.004871819	167.96022	93.317691	111.63	1.20
Bernardo & Lopes (2004)	A1	62.9	0.002742	0.63614	1	125	270	238	452	534	241368	5001.6508	48.257668	48.257668	0.011795576	213.87117	51.621656	50.275	0.97
	A2	64.9	0.002702	0.62934	1	130	270	237	628	575	361100	5309.7416	68.007076	68.007076	0.007454794	202.99646	73.302022	64.94	0.89
	A3	64.1	0.002718	0.63206	1	120	270	237	628	575	361100	4861.8055	74.272819	74.272819	0.006572816	199.86359	72.170743	64.55	0.89
	A4	63.2	0.002736	0.63512	1	120	270	234	804	575	462300	4816.7501	95.977577	95.977577	0.004314208	186.01121	85.992983	78.48	0.91
	A5	65.1	0.002698	0.62866	1	120	270	234	804	575	462300	4911.0919	94.133852	94.133852	0.004457466	186.93307	86.41916	72.545	0.84
	B1	79.2	0.002416	0.58072	1	120	264	237	452	534	241368	5519.1629	43.732719	43.732719	0.01325785	215.13364	51.926377	47.995	0.92
	B2	78.9	0.002422	0.58174	1	124	270	242	628	575	361100	5691.5115	63.445361	63.445361	0.008442917	210.27732	75.93114	63.665	0.84
	B3	78.5	0.00243	0.5831	1	120	270	242	628	575	361100	5492.802	65.740582	65.740582	0.008043407	209.12971	75.516738	65.95	0.87
	C1	82.9	0.002342	0.56814	1	123	270	242	628	575	361100	5793.1531	62.332203	62.332203	0.00864727	210.8339	76.132121	62.785	0.82
	C2	83.9	0.002322	0.56474	1	120	270	242	628	575	361100	5685.8023	63.509067	63.509067	0.008431439	210.24547	75.919638	67.96	0.90
	C3	83.6	0.002328	0.56576	1	125	270	239	804	575	462300	5912.192	78.194348	78.194348	0.006169461	199.90283	92.415076	72.445	0.78
	C4	83.4	0.002332	0.56644	1	122	275	244	804	575	462300	5763.4137	80.212878	80.212878	0.006125717	203.89356	94.259993	77.94	0.83
	D1	88	0.00224	0.5508	1	120	270	247	402	575	231150	5816.448	39.740749	39.740749	0.015645849	227.12963	52.501013	44.22	0.84
	D2	85.8	0.002284	0.55828	1	120	270	238	1030	572	589160	5748.0509	102.49735	102.49735	0.003966034	186.75132	110.02641	93.34	0.85
	D3	86	0.00228	0.5576	1	120	270	238	1030	572	589160	5754.432	102.38369	102.38369	0.003973767	186.80815	110.05989	97.61	0.89
	E1	94.6	0.002108	0.52836	1	123	270	239	804	575	462300	6147.8913	75.196515	75.196515	0.006535016	201.40174	93.108026	78.995	0.85
	E2	90.2	0.002196	0.54332	1	120	270	239	804	575	462300	5880.8957	78.610475	78.610475	0.006120922	199.69476	92.318889	78.895	0.85
	F1	100.3	0.001994	0.50898	1	139	263	230	628	575	361100	7096.0465	50.887491	50.887491	0.010559324	204.55625	73.865264	64.23	0.87
	F2	105.2	0.001896	0.49232	1	129	270	234	804	575	462300	6681.1763	69.194403	69.194403	0.007145329	199.4028	92.183914	79.07	0.86
S.A.Ashour (2000)	B-N2	48.61	0.0030278	0.684726	1	200	250	215	509	530	269770	6656.9062	40.524831	40.524831	0.012916167	194.73758	52.534358	58.17	1.11
	B-N3	48.61	0.0030278	0.684726	1	200	250	215	763	530	404390	6656.9062	60.747439	60.747439	0.007617732	184.62628	74.661022	80.6	1.08
	B-N4	48.61	0.0030278	0.684726	1	200	250	215	1018	530	539540	6656.9062	81.049663	81.049663	0.004958084	174.47517	94.136332	99.55	1.06
	B-M2	78.5	0.00243	0.5831	1	200	250	215	509	530	269770	9154.67	29.46802	29.46802	0.018888135	200.26599	54.025756	57.95	1.07
	B-M3	78.5	0.00243	0.5831	1	200	250	215	763	530	404390	9154.67	44.173083	44.173083	0.011601652	192.91346	78.012273	79.91	1.02
	B-M4	78.5	0.00243	0.5831	1	200	250	215	1018	530	539540	9154.67	58.93604	58.93604	0.007944067	185.53198	100.10192	103.77	1.04
	B-H2	102.4	0.001952	0.50184	1	200	250	215	509	530	269770	10277.683	26.248133	26.248133	0.021573176	201.87593	54.460071	56.8	1.04
	B-H3	102.4	0.001952	0.50184	1	200	250	215	763	530	404390	10277.683	39.346416	39.346416	0.013392853	195.32679	78.988201	82.76	1.05
	B-H4	102.4	0.001952	0.50184	1	200	250	215	1018	530	539540	10277.683	52.496267	52.496267	0.009286588	188.75187	101.83918	108.1	1.06

Table A.5 Calculation of Ultimate Moment Capacity for beams as per CEB-FIP 1990

EUROCODE-2

$k_1 k_3$=0.85
k_2=0.9-f'_c/100
ε_{cu}=0.0035

Reference	*Beam no*	*f'_c*	*ε_{cu}*	*$k_1 k_3$*	*k_2*	*b*	*D*	*d*	*A_{st}*	*f_y*	*Tension(T)*	*C*a*	*a=T/C*	*d_n=a/g*	*0.003(d-d_n)/d_n*	*z*	*M_p=T*z*	*M_e*	*M_e/M_p*
		MPa				*mm*	*mm*	*mm*	*mm^2*	*N/mm^2*	*N*		*mm*	*mm*		*mm*	*kNm*	*kNm*	
Pam et al (2001)	1	37.4	0.0035	0.85	0.8252	200	300	264	402	579	232758	6358	36.608682	44.363405	0.014852552	245.69566	57.18763	77.6	1.36
	2	36.8	0.0035	0.85	0.8264	200	300	264	603	579	349137	6256	55.808344	67.531878	0.008727795	236.09583	82.429789	103.5	1.26
	3	36.4	0.0035	0.85	0.8272	200	300	260	982	578	567596	6188	91.725275	110.88645	0.004034223	214.13736	121.54351	126.5	1.04
	4	42.3	0.003	0.85	0.8154	200	300	260	982	536	526352	7191	73.195939	89.766911	0.005689171	223.40203	117.58811	129	1.10
	5	46.4	0.003	0.85	0.8072	200	300	260	1183	546	645918	7888	81.886156	101.44469	0.004688919	219.05692	141.49281	142.8	1.01
	7	58.6	0.003	0.85	0.7828	200	300	260	1296	520	673920	9962	67.649066	86.419349	0.006025756	226.17547	152.42417	164.6	1.08
	8	57.1	0.003	0.85	0.7858	200	300	260	1473	520	765960	9707	78.908005	100.41741	0.004767577	220.546	168.92941	166.2	0.98
	9	58.6	0.003	0.85	0.7828	200	300	256	1809	520	940680	9962	94.426822	120.62701	0.003366733	208.78659	196.40137	171.6	0.87
	14	95.5	0.003	0.85	0.709	200	300	260	982	578	567596	16235	34.961257	49.310658	0.012818081	242.51937	137.65303	138	1.00
	15	98	0.003	0.85	0.704	200	300	260	1473	578	851394	16660	51.104082	72.591025	0.00774513	234.44796	199.60759	200.7	1.01
	16	102.5	0.003	0.85	0.695	200	300	260	1473	578	851394	17425	48.860488	70.30286	0.008094854	235.56976	200.56268	181.7	0.91
	17	87	0.003	0.85	0.726	200	300	256	1608	546	877968	14790	59.362272	81.766215	0.006392632	226.31886	198.70072	172	0.87
Sarkar (1997)	HSC1-1	107	0.0035	0.85	0.686	150	250	220	339	470	159330	13642.5	11.678944	17.0247	0.035767202	214.16053	34.122197	38.94	1.14
	HSC1-2	97	0.0035	0.85	0.706	150	250	220	339	470	159330	12367.5	12.882959	18.247818	0.033168708	213.55852	34.026279	35.64	1.05
	HSC1-3	85	0.0035	0.85	0.73	150	250	220	339	442	149838	10837.5	13.825882	18.939565	0.031847686	213.08706	31.928539	37.62	1.18
	HSC2-1	105	0.0035	0.85	0.69	150	250	212.5	452	470	212440	13387.5	15.868534	22.997875	0.024719952	204.56573	43.457944	46.33	1.07
	HSC2-2	100	0.0035	0.85	0.7	150	250	212.5	452	470	212440	12750	16.661961	23.802801	0.023782562	204.16902	43.373667	46.86	1.08
	HSC2-3	77	0.0035	0.85	0.746	150	250	212.5	452	442	199784	9817.5	20.349784	27.27853	0.020370027	202.32511	40.421319	43.56	1.08
	HSC2-4	90	0.0035	0.85	0.72	150	250	212.5	452	442	199784	11475	17.41037	24.18107	0.023363598	203.79481	40.714943	48.84	1.20
	HSC3-1	107	0.0035	0.85	0.686	150	250	215	628	470	295160	13642.5	21.635331	31.538383	0.01745127	204.18233	60.266458	67.32	1.12
	HSC3-2	85	0.0035	0.85	0.73	150	250	215	628	470	295160	10837.5	27.235063	37.308306	0.014288375	201.38247	59.440049	66	1.11
	HSC3-3	78	0.0035	0.85	0.744	150	250	215	628	442	277576	9945	27.911111	37.514934	0.014193153	201.04444	55.805113	64.68	1.16
	HSC4-1	101	0.0035	0.85	0.698	150	250	207.5	1257	470	590790	12877.5	45.877694	65.727355	0.006470943	184.56115	109.03688	92.42	0.85
	HSC4-2	87	0.0035	0.85	0.726	150	250	207.5	1257	470	590790	11092.5	53.260311	73.36131	0.005485399	180.86984	106.8561	89.6	0.84
	HSC4-3	82	0.0035	0.85	0.736	150	250	207.5	1257	442	555594	10455	53.141463	72.203075	0.005621516	180.92927	100.52322	111.63	1.11
Bernardo & Lopes (2004)	A1	62.9	0.0035	0.85	0.7742	125	270	238	452	534	241368	6683.125	36.116039	46.649494	0.012305632	219.94198	53.086956	50.275	0.95
	A2	64.9	0.0035	0.85	0.7702	130	270	237	628	575	361100	7171.45	50.352439	65.375797	0.007875584	211.82378	76.489567	64.94	0.85
	A3	64.1	0.0035	0.85	0.7718	120	270	237	628	575	361100	6538.2	55.229268	71.559041	0.006935851	209.38537	75.609056	64.55	0.85
	A4	63.2	0.0035	0.85	0.7736	120	270	234	804	575	462300	6446.4	71.714445	92.70223	0.004572633	198.14278	91.601406	78.48	0.86
	A5	65.1	0.0035	0.85	0.7698	120	270	234	804	575	462300	6640.2	69.621397	90.44089	0.004761976	199.1893	92.085214	72.545	0.79
	B1	79.2	0.0035	0.85	0.7416	120	264	237	452	534	241368	8078.4	29.878194	40.288826	0.014647573	222.0609	53.598396	47.995	0.90
	B2	78.9	0.0035	0.85	0.7422	124	270	242	628	575	361100	8316.06	43.422005	58.504453	0.009409312	220.289	79.546357	63.665	0.80
	B3	78.5	0.0035	0.85	0.743	120	270	242	628	575	361100	8007	45.098039	60.697226	0.008961008	219.45098	79.243749	65.95	0.83
	C1	82.9	0.0035	0.85	0.7342	123	270	242	628	575	361100	8667.195	41.662845	56.745907	0.009793874	221.16858	79.863973	62.785	0.79
	C2	83.9	0.0035	0.85	0.7322	120	270	242	628	575	361100	8557.8	42.195424	57.628277	0.009597982	220.90229	79.767816	67.96	0.85
	C3	83.6	0.0035	0.85	0.7328	125	270	239	804	575	462300	8882.5	52.046158	71.023687	0.007095224	212.97692	98.459231	72.445	0.74
	C4	83.4	0.0035	0.85	0.7332	122	275	244	804	575	462300	8648.58	53.453862	72.904885	0.007040479	217.27307	100.44534	77.94	0.78
	D1	88	0.0035	0.85	0.724	120	270	247	402	575	231150	8976	25.752005	35.569068	0.017832708	234.124	54.117762	44.22	0.82
	D2	85.8	0.0035	0.85	0.7284	120	270	238	1030	572	589160	8751.6	67.320261	92.422105	0.004725425	204.33987	120.38888	93.34	0.78
	D3	86	0.0035	0.85	0.728	120	270	238	1030	572	589160	8772	67.163703	92.257833	0.00473918	204.41815	120.435	97.61	0.81
	E1	94.6	0.0035	0.85	0.7108	123	270	239	804	575	462300	9890.43	46.742154	65.759924	0.007903297	215.62892	99.685251	78.995	0.79
	E2	90.2	0.0035	0.85	0.7196	120	270	239	804	575	462300	9200.4	50.247815	69.827425	0.007268172	213.87609	98.874917	78.895	0.80
	F1	100.3	0.0035	0.85	0.6994	139	263	230	628	575	361100	11850.445	30.47143	43.567958	0.012837327	214.76429	77.551383	64.23	0.83
	F2	105.2	0.0035	0.85	0.6896	129	270	234	804	575	462300	11535.18	40.077398	58.116876	0.009079108	213.9613	98.914309	79.07	0.80
S.A.Ashour (2000)	B-N2	48.61	0.0035	0.85	0.80278	200	250	215	509	530	269770	8263.7	32.645183	40.665167	0.01286124	198.67741	53.597204	58.17	1.09
	B-N3	48.61	0.0035	0.85	0.80278	200	250	215	763	530	404390	8263.7	48.935707	60.957805	0.00758109	190.53215	77.049295	80.6	1.05
	B-N4	48.61	0.0035	0.85	0.80278	200	250	215	1018	530	539540	8263.7	65.290366	81.330335	0.00493062	182.35482	98.387718	99.55	1.01
	B-M2	78.5	0.0035	0.85	0.743	200	250	215	509	530	269770	13345	20.215062	27.207351	0.020706828	204.89247	55.273841	57.95	1.05
	B-M3	78.5	0.0035	0.85	0.743	200	250	215	763	530	404390	13345	30.302735	40.7843	0.012814909	199.84863	80.816788	79.91	0.99
	B-M4	78.5	0.0035	0.85	0.743	200	250	215	1018	530	539540	13345	40.430124	54.414702	0.008853414	194.78494	105.09427	103.77	0.99
	B-H2	102.4	0.0035	0.85	0.6952	200	250	215	509	530	269770	17408	15.496898	22.29128	0.025935081	207.25155	55.910251	56.8	1.02
	B-H3	102.4	0.0035	0.85	0.6952	200	250	215	763	530	404390	17408	23.230124	33.415023	0.016302695	203.38494	82.246835	82.76	1.01
	B-H4	102.4	0.0035	0.85	0.6952	200	250	215	1018	530	539540	17408	30.993796	44.58256	0.011467541	199.5031	107.6399	108.1	1.00

Table A.6 Calculation of Ultimate Moment Capacity for beams as per Eurocode-2

PENDYALA AND MENDIS (1997)

$k_1 k_3 = 0.85 - 0.0025(f'_c - 57)$; $57 < f'_c \leq 100MPa$
$k_2 = 0.65 - 0.00125(f'_c - 57)$; $57 < f'_c \leq 100MPa$
Beams with f'c in the range 57<f'c<=100MPa only were considered in this case

Reference	*Beam no*	*f'_c*	*ε_{cu}*	*$k_1 k_3$*	*k_2*	*b*	*D*	*d*	*A_{st}*	*f_y*	*Tension(T)*	*C*a*	*a=T/C*	*d_n=a/g*	*0.003(d-d_n)/d_n*	*z*	*M_p=T*z*	*M_e*	*M_e/M_p*
		MPa				*mm*	*mm*	*mm*	*mm2*	*N/mm_2*	*N*		*mm*	*mm*		*mm*	*kNm*	*kNm*	
Pam et al (2001)	1	37.4	0.003			200	300	264	402	579	232758								
	2	36.8	0.003			200	300	264	603	579	349137								
	3	36.4	0.003	N/A		200	300	260	982	578	567596					N/A			
	4	42.3	0.003			200	300	260	982	536	526352								
	5	46.4	0.003			200	300	260	1183	546	645918								
	7	58.6	0.003	0.846	0.648	200	300	260	1296	520	673920	9915.12	67.96892	104.89031	0.00443634	226.0155399	152.31639	164.6	1.08
	8	57.1	0.003	0.84975	0.649875	200	300	260	1473	520	765960	9704.145	78.93122	121.456	0.003422079	220.5343902	168.92052	166.2	0.98
	9	58.6	0.003	0.846	0.648	200	300	256	1809	520	940680	9915.12	94.873284	146.40939	0.002245565	208.5633578	196.19138	171.6	0.87
	14	95.5	0.003	0.75375	0.601875	200	300	260	982	578	567596	14396.625	39.425629	65.50468	0.008907546	240.2871854	136.38605	138	1.01
	15	98	0.003	0.7475	0.59875	200	300	260	1473	578	851394	14651	58.111665	97.054972	0.005036683	230.9441676	196.62448	200.7	1.02
	16	102.5	0.003	0.73625	0.593125	200	300	260	1473	578	851394	15093.125	56.409392	95.105402	0.005201427	231.7953042	197.34913	181.7	0.92
	17	87	0.003	0.775	0.6125	200	300	256	1608	546	877968	13485	65.107008	106.29716	0.004225029	223.4464961	196.17887	172	0.88
Sarkar (1997)	HSC1-1	107	0.003	0.725	0.7875	150	250	220	339	470	159330	11636.25	13.692556	17.387372	0.034958582	213.1537222	33.961783	38.94	1.15
	HSC1-2	97	0.003	0.75	0.8	150	250	220	339	470	159330	10912.5	14.600687	18.250859	0.033162681	212.6996564	33.889436	35.64	1.05
	HSC1-3	85	0.003	0.78	0.815	150	250	220	339	442	149838	9945	15.066667	18.486708	0.032701327	212.4666667	31.83558	37.62	1.18
	HSC2-1	105	0.003	0.73	0.79	150	250	212.5	452	470	212440	11497.5	18.47706	23.388684	0.024256771	203.2614699	43.180867	46.33	1.07
	HSC2-2	100	0.003	0.7425	0.79625	150	250	212.5	452	470	212440	11137.5	19.074299	23.955163	0.023612217	202.9628507	43.117428	46.86	1.09
	HSC2-3	77	0.003	0.8	0.825	150	250	212.5	452	442	199784	9240	21.621645	26.208055	0.021324583	201.6891775	40.294271	43.56	1.08
	HSC2-4	90	0.003	0.7675	0.80875	150	250	212.5	452	442	199784	10361.25	19.281843	23.841537	0.023739047	202.8590783	40.527998	48.84	1.21
	HSC3-1	107	0.003	0.725	0.7875	150	250	215	628	470	295160	11636.25	25.36556	32.210235	0.017024691	202.3172199	59.715951	67.32	1.13
	HSC3-2	85	0.003	0.78	0.815	150	250	215	628	470	295160	9945	29.679236	36.41624	0.014711878	200.1603821	59.079338	66	1.12
	HSC3-3	78	0.003	0.7975	0.82375	150	250	215	628	442	277576	9330.75	29.74852	36.113529	0.014860342	200.1257402	55.550102	64.68	1.16
	HSC4-1	101	0.003	0.74	0.795	150	250	207.5	1257	470	590790	11211	52.697351	66.285976	0.006391127	181.1513246	107.02239	92.42	0.86
	HSC4-2	87	0.003	0.775	0.8125	150	250	207.5	1257	470	590790	10113.75	58.414535	71.894812	0.005658483	178.2927327	105.33356	89.6	0.85
	HSC4-3	82	0.003	0.7875	0.81875	150	250	207.5	1257	442	555594	9686.25	57.35904	70.056843	0.005885642	178.8204801	99.351586	111.63	1.12
Bernardo & Lopes (2004)	A1	62.9	0.003	0.83525	0.842625	125	270	238	452	534	241368	6567.1531	36.753826	43.618247	0.013369296	219.6230872	53.009985	50.275	0.95
	A2	64.9	0.003	0.83025	0.840125	130	270	237	628	575	361100	7004.8193	51.550224	61.360183	0.008587319	211.2248881	76.273307	64.94	0.85
	A3	64.1	0.003	0.83225	0.841125	120	270	237	628	575	361100	6401.667	56.407183	67.061593	0.007602194	208.7964087	75.396383	64.55	0.86
	A4	63.2	0.003	0.8345	0.84225	120	270	234	804	575	462300	6328.848	73.046469	86.727776	0.005094293	197.4767654	91.293509	78.48	0.86
	A5	65.1	0.003	0.82975	0.839875	120	270	234	804	575	462300	6482.007	71.320503	84.917997	0.005266799	198.3397485	91.692466	72.545	0.79
	B1	79.2	0.003	0.7945	0.82225	120	264	237	452	534	241368	7550.928	31.965343	38.875455	0.015289175	221.0173287	53.346511	47.995	0.90
	B2	78.9	0.003	0.79525	0.822625	124	270	242	628	575	361100	7780.4079	46.411448	56.418719	0.00986807	218.7942758	79.006613	63.665	0.81
	B3	78.5	0.003	0.79625	0.823125	120	270	242	628	575	361100	7500.675	48.142334	58.48727	0.009412958	217.9288331	78.694102	65.95	0.84
	C1	82.9	0.003	0.78525	0.817625	123	270	242	628	575	361100	8006.9587	45.098272	55.157648	0.010162273	219.450864	79.243707	62.785	0.79
	C2	83.9	0.003	0.78275	0.816375	120	270	242	628	575	361100	7880.727	45.820646	56.126958	0.009934961	219.0896771	79.113282	67.96	0.86
	C3	83.6	0.003	0.7835	0.81675	125	270	239	804	575	462300	8187.575	56.463605	69.132054	0.007371455	210.7681975	97.438138	72.445	0.74
	C4	83.4	0.003	0.784	0.817	122	275	244	804	575	462300	7977.0432	57.953804	70.934889	0.007319323	215.0230979	99.405178	77.94	0.78
	D1	88	0.003	0.7725	0.81125	120	270	247	402	575	231150	8157.6	28.33554	34.928246	0.018214921	232.8322301	53.81917	44.22	0.82
	D2	85.8	0.003	0.778	0.814	120	270	238	1030	572	589160	8010.288	73.550414	90.356774	0.004902009	201.2247929	118.5536	93.34	0.79
	D3	86	0.003	0.7775	0.81375	120	270	238	1030	572	589160	8023.8	73.426556	90.232327	0.004912907	201.286722	118.59009	97.61	0.82
	E1	94.6	0.003	0.756	0.803	123	270	239	804	575	462300	8796.6648	52.554009	65.447085	0.007955415	212.7229956	98.341841	78.995	0.80
	E2	90.2	0.003	0.767	0.8085	120	270	239	804	575	462300	8302.008	55.685323	68.874859	0.007410185	211.1573383	97.618038	78.895	0.81
	F1	100.3	0.003	0.74175	0.795875	139	263	230	628	575	361100	10341.256	34.918389	43.874213	0.012726778	212.5408054	76.748485	64.23	0.84
	F2	105.2	0.003	0.7295	0.78975	129	270	234	804	575	462300	9899.8986	46.697448	59.129405	0.008872265	210.651276	97.384085	79.07	0.81
S.A.Ashour (2000)	~~B-N2~~	~~48.61~~	~~0.003~~	~~0.870975~~	~~0.8604875~~	~~200~~	~~250~~	~~215~~	~~509~~	~~530~~	~~269770~~	~~8467.619~~	~~31.859015~~	~~37.024379~~	~~0.014420954~~	~~199.0704924~~	~~53.703247~~	~~58.17~~	~~1.08~~
	~~B-N3~~	~~48.61~~	~~0.003~~	~~0.870975~~	~~0.8604875~~	~~200~~	~~250~~	~~215~~	~~763~~	~~530~~	~~404390~~	~~8467.619~~	~~47.757227~~	~~55.500198~~	~~0.00862158~~	~~191.1213865~~	~~77.287577~~	~~80.6~~	~~1.04~~
	~~B-N4~~	~~48.61~~	~~0.003~~	~~0.870975~~	~~0.8604875~~	~~200~~	~~250~~	~~215~~	~~1018~~	~~530~~	~~539540~~	~~8467.619~~	~~63.71803~~	~~74.048757~~	~~0.005710477~~	~~183.1409849~~	~~98.811887~~	~~99.55~~	~~1.01~~
	B-M2	78.5	0.003	0.79625	0.823125	200	250	215	509	530	269770	12501.125	21.579658	26.216745	0.021602597	204.2101711	55.089778	57.95	1.05
	B-M3	78.5	0.003	0.79625	0.823125	200	250	215	763	530	404390	12501.125	32.348289	39.299364	0.01341248	198.8258557	80.403188	79.91	0.99
	B-M4	78.5	0.003	0.79625	0.823125	200	250	215	1018	530	539540	12501.125	43.159316	52.433489	0.009301298	193.4203422	104.35801	103.77	0.99
	B-H2	102.4	0.003	0.7365	0.79325	200	250	215	509	530	269770	15083.52	17.885083	22.54659	0.02560743	206.0574587	55.588121	56.8	1.02
	B-H3	102.4	0.003	0.7365	0.79325	200	250	215	763	530	404390	15083.52	26.810055	33.797737	0.016084118	201.5949725	81.522991	82.76	1.02
	B-H4	102.4	0.003	0.7365	0.79325	200	250	215	1018	530	539540	15083.52	35.770165	45.09318	0.011303715	197.1149175	106.35138	108.1	1.02

Table A.7 Calculation of Ultimate Moment Capacity for beams as per Mendis & Pendyala (1997)

IBRAHIM AND MACGREGOR (1997)

$k_1 k_3 = 0.85 - (f'_c/800) \geq 0.725$
$k_2 = 0.95 - (f'_c/400) \geq 0.7$

Reference	Beam no	f'_c	ε_{cu}	$k_1 k_3$	k_2	b	D	d	A_{st}	f_y	Tension(T)	C*a	a=T/C	d_n=a/g	$0.003(d-d_n)/d_n$	z	M_p=T*z	M_e	M_e/M_p
		MPa				mm	mm	mm	mm^2	N/mm^2	N		mm	mm		mm	kNm	kNm	
Pam et al (2001)	1	37.4	0.003	0.80325	0.8565	200	300	264	402	579	232758	6008.31	38.739346	45.229826	0.014510569	244.630327	56.93966565	77.6	1.36
	2	36.8	0.003	0.804	0.858	200	300	264	603	579	349137	5917.44	59.001359	68.766152	0.008517294	234.4993207	81.87238931	103.5	1.26
	3	36.4	0.003	0.8045	0.859	200	300	260	982	578	567596	5856.76	96.912969	112.82069	0.003913626	211.5435155	120.0712532	126.5	1.05
	4	42.3	0.003	0.797125	0.84425	200	300	260	982	536	526352	6743.6775	78.051182	92.450319	0.005436964	220.974409	116.3103221	129	1.11
	5	46.4	0.003	0.792	0.834	200	300	260	1183	546	645918	7888	81.886156	98.18484	0.0049442	219.0569219	141.4928089	142.8	1.01
	7	58.6	0.003	0.77675	0.8035	200	300	260	1296	520	673920	9962	67.649066	84.192989	0.006264429	226.1754668	152.4241706	164.6	1.08
	8	57.1	0.003	0.778625	0.80725	200	300	260	1473	520	765960	9707	78.908005	97.749154	0.004979609	220.5459977	168.9294124	166.2	0.98
	9	58.6	0.003	0.77675	0.8035	200	300	256	1809	520	940680	9962	94.426822	117.51938	0.003535092	208.786589	196.4013686	171.6	0.87
	14	95.5	0.003	0.730625	0.71125	200	300	260	982	578	567596	16235	34.961257	49.154666	0.01286828	242.5193717	137.6530253	138	1.00
	15	98	0.003	0.7275	0.705	200	300	260	1473	578	851394	16660	51.104082	72.488059	0.007760393	234.4479592	199.6075858	200.7	1.01
	16	102.5	0.003	0.725	0.69375	200	300	260	1473	578	851394	17425	48.860488	70.429532	0.0080749	235.5697561	200.5626769	181.7	0.91
	17	87	0.003	0.74125	0.7325	200	300	256	1608	546	877968	14790	59.362272	81.040644	0.006476726	226.3188641	198.7007205	172	0.87
Sarkar (1997)	HSC1-1	107	0.003	0.71625	0.6825	150	250	220	339	470	159330	11495.813	13.859829	20.307442	0.0295004	213.0700853	33.94845669	38.94	1.15
	HSC1-2	97	0.003	0.72875	0.7075	150	250	220	339	470	159330	10603.313	15.026436	21.238779	0.028075232	212.4867818	33.85551895	35.64	1.05
	HSC1-3	85	0.003	0.74375	0.7375	150	250	220	339	442	149838	9482.8125	15.801008	21.425096	0.027804996	212.0994958	31.78056425	37.62	1.18
	HSC2-1	105	0.003	0.71875	0.6875	150	250	212.5	452	470	212440	11320.313	18.766266	27.296387	0.020354739	203.1168668	43.15014718	46.33	1.07
	HSC2-2	100	0.003	0.725	0.7	150	250	212.5	452	470	212440	10875	19.534713	27.906732	0.01984395	202.7326437	43.06852282	46.86	1.09
	HSC2-3	77	0.003	0.75375	0.7575	150	250	212.5	452	442	199784	8705.8125	22.948346	30.294847	0.018043183	201.0258269	40.16174379	43.56	1.08
	HSC2-4	90	0.003	0.7375	0.725	150	250	212.5	452	442	199784	9956.25	20.06619	27.677503	0.020033147	202.4669052	40.44964819	48.84	1.21
	HSC3-1	107	0.003	0.71625	0.7	150	250	215	628	470	295160	11495.813	25.675436	36.679195	0.014584901	202.1622819	59.67021913	67.32	1.13
	HSC3-2	85	0.003	0.74375	0.7375	150	250	215	628	470	295160	9482.8125	31.125787	42.204457	0.012282746	199.4371066	58.86585639	66	1.12
	HSC3-3	78	0.003	0.7525	0.755	150	250	215	628	442	277576	8804.25	31.527501	41.758279	0.012446039	199.2362495	55.3032012	64.68	1.17
	HSC4-1	101	0.003	0.72375	0.7	150	250	207.5	1257	470	590790	10964.813	53.880538	76.972198	0.005087336	180.5597308	106.6728834	92.42	0.87
	HSC4-2	87	0.003	0.74125	0.7325	150	250	207.5	1257	470	590790	9673.3125	61.074218	83.377773	0.004466019	176.9628908	104.5479063	89.6	0.86
	HSC4-3	82	0.003	0.7475	0.745	150	250	207.5	1257	442	555594	9194.25	60.42842	81.111973	0.004674576	177.28579	98.49892123	111.63	1.13
Bernardo & Lopes (2004)	A1	62.9	0.003	0.771375	0.79275	125	270	238	452	534	241368	6064.9359	39.797288	50.201561	0.011222665	218.1013562	52.64268814	50.275	0.96
	A2	64.9	0.003	0.768875	0.78775	130	270	237	628	575	361100	6486.9984	55.66519	70.663523	0.007061768	209.1674048	75.53034986	64.94	0.86
	A3	64.1	0.003	0.769875	0.78975	120	270	237	628	575	361100	5921.8785	60.977273	77.210855	0.00620855	206.5113637	74.57125342	64.55	0.87
	A4	63.2	0.003	0.771	0.792	120	270	234	804	575	462300	5847.264	79.062618	99.826538	0.004032198	194.468691	89.90287585	78.48	0.87
	A5	65.1	0.003	0.768625	0.78725	120	270	234	804	575	462300	6004.4985	76.992275	97.799016	0.004177986	195.5038625	90.38143562	72.545	0.80
	B1	79.2	0.003	0.751	0.752	120	264	237	452	534	241368	7137.504	33.816864	44.969234	0.01281081	220.0915681	53.12306162	47.995	0.90
	B2	78.9	0.003	0.751375	0.75275	124	270	242	628	575	361100	7351.1525	49.12155	65.256127	0.008125392	217.4392252	78.51730421	63.665	0.81
	B3	78.5	0.003	0.751875	0.75375	120	270	242	628	575	361100	7082.6625	50.983652	67.640003	0.007733293	216.508174	78.18110163	65.95	0.84
	C1	82.9	0.003	0.746375	0.74275	123	270	242	628	575	361100	7610.562	47.447219	63.88047	0.008364976	218.2763905	78.81960461	62.785	0.80
	C2	83.9	0.003	0.745125	0.74025	120	270	242	628	575	361100	7501.9185	48.134354	65.024456	0.00816503	217.9328231	78.6955424	67.96	0.86
	C3	83.6	0.003	0.7455	0.741	125	270	239	804	575	462300	7790.475	59.341696	80.083261	0.005953182	209.3291519	96.77286694	72.445	0.75
	C4	83.4	0.003	0.74575	0.7415	122	275	244	804	575	462300	7587.8571	60.926292	82.166274	0.005908765	213.5368538	98.71808752	77.94	0.79
	D1	88	0.003	0.74	0.73	120	270	247	402	575	231150	7814.4	29.580006	40.520556	0.015287014	232.2099969	53.67534079	44.22	0.82
	D2	85.8	0.003	0.74275	0.7355	120	270	238	1030	572	589160	7647.354	77.041026	104.74647	0.003816459	199.4794869	117.5253345	93.34	0.79
	D3	86	0.003	0.7425	0.735	120	270	238	1030	572	589160	7662.6	76.88774	104.60917	0.003825405	199.5561298	117.5704894	97.61	0.83
	E1	94.6	0.003	0.73175	0.7135	123	270	239	804	575	462300	8514.4967	54.295635	76.097596	0.006422111	211.8521826	97.93926403	78.995	0.81
	E2	90.2	0.003	0.73725	0.7245	120	270	239	804	575	462300	7979.994	57.932374	79.961869	0.005966774	210.0338128	97.09863166	78.895	0.81
	F1	100.3	0.003	0.724625	0.69925	139	263	230	628	575	361100	10102.504	35.743612	51.117072	0.010498426	212.1281938	76.59949078	64.23	0.84
	F2	105.2	0.003	0.7185	0.687	129	270	234	804	575	462300	9750.6198	47.412371	69.01364	0.007171902	210.2938147	97.21883052	79.07	0.81
S.A.Ashour (2000)	B-N2	48.61	0.003	0.7892375	0.828475	200	250	215	509	530	269770	8263.7	32.645183	39.403945	0.013368919	198.6774084	53.59720447	58.17	1.09
	B-N3	48.61	0.003	0.7892375	0.828475	200	250	215	763	530	404390	8263.7	48.935707	59.06721	0.007919764	190.5321466	77.04929477	80.6	1.05
	B-N4	48.61	0.003	0.7892375	0.828475	200	250	215	1018	530	539540	8263.7	65.290366	78.80789	0.00518446	182.3548168	98.38771788	99.55	1.01
	B-M2	78.5	0.003	0.751875	0.75375	200	250	215	509	530	269770	13345	20.215062	26.819319	0.021049828	204.8924691	55.27384139	57.95	1.05
	B-M3	78.5	0.003	0.751875	0.75375	200	250	215	763	530	404390	13345	30.302735	40.202634	0.013043725	199.8486324	80.81678848	79.91	0.99
	B-M4	78.5	0.003	0.751875	0.75375	200	250	215	1018	530	539540	13345	40.430124	53.638638	0.009024914	194.7849382	105.0942655	103.77	0.99
	B-H2	102.4	0.003	0.725	0.7	200	250	215	509	530	269770	17408	15.496898	22.138426	0.026134863	207.251551	55.91025092	56.8	1.02
	B-H3	102.4	0.003	0.725	0.7	200	250	215	763	530	404390	17408	23.230124	33.185892	0.01643597	203.384938	82.24683506	82.76	1.01
	B-H4	102.4	0.003	0.725	0.7	200	250	215	1018	530	539540	17408	30.993796	44.276851	0.011567432	199.503102	107.6399037	108.1	1.00

Table A.8 Calculation of Ultimate Moment Capacity for beams as per Ibrahim & MacGregor (1997)

MARIO.M ATTARD AND MARK G. STEWART (1998)

$k_1 k_3 = 1.2932(f'_c)^{(-0.0998)} \geq 0.71$ (DB)
$k_1 k_3 = 0.6470(f'_c)^{(0.0324)} \geq 0.58$ (SL)
$k_2 = 1.0948(f'c)^{(-0.091)} \geq 0.67$
$fcyl = f'c + 7.5$ MPa

Reference	Beam no	f'_c	ε_{cu}	$k_1 k_3$	k_2	b	D	d	A_{st}	f_y	Tension(T)	C*a	a=T/C	$d_n=a/g$	$0.003(d-d_n)/d_n$	z	$M_p=T*z$	M_e	M_e/M_p
		MPa				mm	mm	mm	mm^2	N/mm^2	N		mm	mm		mm	kNm	kNm	
Pam et al (2001)	1	37.4	0.003	0.7275541	0.7874146	200	300	264	402	579	232758	5442.105	42.769847	54.316806	0.011581122	242.61508	56.4706	77.6	1.37
	2	36.8	0.003	0.727173	0.7885743	200	300	264	603	579	349137	5351.9933	65.234947	82.725171	0.00657387	231.38253	80.784201	103.5	1.28
	3	36.4	0.003	0.7269156	0.789359	200	300	260	982	578	567596	5291.9453	107.25659	135.87809	0.00274044	206.37171	117.13575	126.5	1.08
	4	42.3	0.003	0.7304621	0.778642	200	300	260	982	536	526352	6179.7097	85.174228	109.38818	0.004130569	217.41289	114.43571	129	1.13
	5	46.4	0.003	0.7326549	0.7721144	200	300	260	1183	546	645918	6799.0375	95.001387	123.04056	0.003339373	212.49931	137.25713	142.8	1.04
	7	58.6	0.003	0.7382172	0.7558857	200	300	260	1296	520	673920	8651.9057	77.892666	103.04821	0.004569272	221.05367	148.97249	164.6	1.10
	8	57.1	0.003	0.7375973	0.7576714	200	300	260	1473	520	765960	8423.3606	90.932828	120.01618	0.003499124	214.53359	164.32415	166.2	1.01
	9	58.6	0.003	0.7382172	0.7558857	200	300	256	1809	520	940680	8651.9057	108.72518	143.83813	0.002339335	201.63741	189.67628	171.6	0.90
	14	95.5	0.003	0.7499916	0.7230269	200	300	260	982	578	567596	14324.839	39.623202	54.801833	0.0112331	240.1884	136.32997	138	1.01
	15	98	0.003	0.7506198	0.7213287	200	300	260	1473	578	851394	14712.148	57.870137	80.227137	0.006722396	231.06493	196.7273	200.7	1.02
	16	102.5	0.003	0.7517124	0.7183877	200	300	260	1473	578	851394	15410.105	55.249072	76.907036	0.007142115	232.37546	197.84308	181.7	0.92
	17	87	0.003	0.7477298	0.7291864	200	300	256	1608	546	877968	13010.499	67.481501	92.543559	0.005298795	222.25925	195.13651	172	0.88
Sarkar (1997)	HSC1-1	107	0.003	0.7527596	0.7155844	150	250	220	339	470	159330	12081.792	13.187613	18.429151	0.032812826	213.40619	34.002009	38.94	1.15
	HSC1-2	97	0.003	0.7503704	0.7220023	150	250	220	339	470	159330	10917.889	14.59348	20.212513	0.02965304	212.70326	33.89001	35.64	1.05
	HSC1-3	85	0.003	0.7471666	0.7307312	150	250	220	339	442	149838	9526.3742	15.728754	21.524678	0.02766248	212.13562	31.785977	37.62	1.18
	HSC2-1	105	0.003	0.7522996	0.7168141	150	250	212.5	452	470	212440	11848.718	17.929366	25.012572	0.022487183	203.53532	43.239043	46.33	1.07
	HSC2-2	100	0.003	0.7511113	0.7200038	150	250	212.5	452	470	212440	11266.669	18.855617	26.188219	0.021343007	203.07219	43.140656	46.86	1.09
	HSC2-3	77	0.003	0.7447776	0.7373338	150	250	212.5	452	442	199784	8602.1808	23.224808	31.498365	0.017239146	200.8876	40.134127	43.56	1.09
	HSC2-4	90	0.003	0.7485516	0.7269403	150	250	212.5	452	442	199784	10105.446	19.769933	27.19609	0.020440869	202.61503	40.479242	48.84	1.21
	HSC3-1	107	0.003	0.7527596	0.7155844	150	250	215	628	470	295160	12081.792	24.430151	34.140139	0.015892717	202.78492	59.853998	67.32	1.12
	HSC3-2	85	0.003	0.7471666	0.7307312	150	250	215	628	470	295160	9526.3742	30.983456	42.400619	0.012212042	199.50827	58.886862	66	1.12
	HSC3-3	78	0.003	0.745089	0.7364685	150	250	215	628	442	277576	8717.5412	31.841088	43.234827	0.011918529	199.07946	55.259679	64.68	1.17
	HSC4-1	101	0.003	0.7513535	0.7193521	150	250	207.5	1257	470	590790	11383.005	51.901058	72.149723	0.005627892	181.54947	107.25761	92.42	0.86
	HSC4-2	87	0.003	0.7477298	0.7291864	150	250	207.5	1257	470	590790	9757.8742	60.544949	83.030831	0.004497215	177.22753	104.70425	89.6	0.86
	HSC4-3	82	0.003	0.7462973	0.7331245	150	250	207.5	1257	442	555594	9179.4563	60.525807	82.558702	0.004540089	177.2371	98.471867	111.63	1.13
Bernardo & Lopes (2004)	A1	62.9	0.003	0.7399128	0.7510305	125	270	238	452	534	241368	5817.5647	41.489526	55.243463	0.009924606	217.25524	52.438462	50.275	0.96
	A2	64.9	0.003	0.7406636	0.7488943	130	270	237	628	575	361100	6248.9789	57.785441	77.161011	0.006214498	208.10728	75.147539	64.94	0.86
	A3	64.1	0.003	0.740366	0.74974	120	270	237	628	575	361100	5694.8955	63.407661	84.572861	0.005406952	205.29617	74.132447	64.55	0.87
	A4	63.2	0.003	0.7400269	0.7507054	120	270	234	804	575	462300	5612.3641	82.371705	109.72574	0.00339777	192.81415	89.13798	78.48	0.88
	A5	65.1	0.003	0.7407375	0.7486846	120	270	234	804	575	462300	5786.641	79.890907	106.70836	0.003578679	194.05455	89.711417	72.545	0.81
	B1	79.2	0.003	0.7454577	0.735446	120	264	237	452	534	241368	7084.8295	34.068286	46.3233	0.012348647	219.96586	53.092719	47.995	0.90
	B2	78.9	0.003	0.745366	0.7357	124	270	242	628	575	361100	7292.3627	49.517559	67.306722	0.007786441	217.24122	78.445805	63.665	0.81
	B3	78.5	0.003	0.7452433	0.7360404	120	270	242	628	575	361100	7020.1915	51.437343	69.883859	0.007388665	216.28133	78.099188	65.95	0.84
	C1	82.9	0.003	0.7465613	0.7323966	123	270	242	628	575	361100	7612.4612	47.435382	64.767342	0.008209353	218.28231	78.821742	62.785	0.80
	C2	83.9	0.003	0.7468513	0.7315979	120	270	242	628	575	361100	7519.2994	48.023091	65.641374	0.008060098	217.98845	78.715631	67.96	0.86
	C3	83.6	0.003	0.7467647	0.7318364	125	270	239	804	575	462300	7803.6908	59.241199	80.948689	0.005857463	209.3794	96.796097	72.445	0.75
	C4	83.4	0.003	0.7467067	0.7319959	122	275	244	804	575	462300	7597.5916	60.84823	83.126458	0.005805861	213.57588	98.736132	77.94	0.79
	D1	88	0.003	0.7480068	0.7284284	120	270	247	402	575	231150	7898.9513	29.263378	40.173308	0.015445083	232.36831	53.711935	44.22	0.82
	D2	85.8	0.003	0.7473934	0.7301086	120	270	238	1030	572	589160	7695.1626	76.562384	104.86438	0.003808794	199.71881	117.66633	93.34	0.79
	D3	86	0.003	0.7474498	0.7299539	120	270	238	1030	572	589160	7713.6819	76.37857	104.63479	0.003823734	199.81071	117.72048	97.61	0.83
	E1	94.6	0.003	0.7497615	0.7236502	123	270	239	804	575	462300	8724.0752	52.99129	73.227768	0.006791368	212.50435	98.240763	78.995	0.80
	E2	90.2	0.003	0.7486054	0.7267934	120	270	239	804	575	462300	8102.9052	57.053611	78.500449	0.006133706	210.47319	97.301758	78.895	0.81
	F1	100.3	0.003	0.7511842	0.7198076	139	263	230	628	575	361100	10472.784	34.479847	47.901479	0.011404566	212.76008	76.827664	64.23	0.84
	F2	105.2	0.003	0.752346	0.71669	129	270	234	804	575	462300	10209.936	45.27942	63.178527	0.00811137	211.36029	97.711862	79.07	0.81
S.A.Ashour (2000)	B-N2	48.61	0.003	0.7337603	0.768852	200	250	215	509	530	269770	7133.6173	37.816719	49.185953	0.0101135	196.09164	52.899642	58.17	1.10
	B-N3	48.61	0.003	0.7337603	0.768852	200	250	215	763	530	404390	7133.6173	56.68793	73.730613	0.005748062	186.65603	75.481834	80.6	1.07
	B-N4	48.61	0.003	0.7337603	0.768852	200	250	215	1018	530	539540	7133.6173	75.633438	98.371905	0.00355675	177.18328	95.597468	99.55	1.04
	B-M2	78.5	0.003	0.7452433	0.7360404	200	250	215	509	530	269770	11700.319	23.056636	31.325232	0.01759043	203.47168	54.890556	57.95	1.06
	B-M3	78.5	0.003	0.7452433	0.7360404	200	250	215	763	530	404390	11700.319	34.562305	46.957076	0.010735949	197.71885	79.955525	79.91	1.00
	B-M4	78.5	0.003	0.7452433	0.7360404	200	250	215	1018	530	539540	11700.319	46.113272	62.650463	0.007295215	191.94336	103.56112	103.77	1.00
	B-H2	102.4	0.003	0.7516887	0.7184516	200	250	215	509	530	269770	15394.584	17.523696	24.390921	0.023444265	206.23815	55.636866	56.8	1.02
	B-H3	102.4	0.003	0.7516887	0.7184516	200	250	215	763	530	404390	15394.584	26.26833	36.562423	0.014641063	201.86584	81.632525	82.76	1.01
	B-H4	102.4	0.003	0.7516887	0.7184516	200	250	215	1018	530	539540	15394.584	35.047391	48.781843	0.010222133	197.4763	106.54637	108.1	1.01

Table A.9 Calculation of Ultimate Moment Capacity for beams as per Attard & Stewart (1998)

AZIZINAMINI ET AL (1994)

$k_1 k_3 = 0.85 - 0.00725(f'_c - 69)$; $0.60 \le k_1 k_3 \le 0.85$
$k_2 = 1.09 - 0.008 f'_c$; $0.65 \le k_2 \le 0.85$
$\varepsilon_{cu} = 0.003$

Reference	*Beam no*	*f'_c*	*ε_{cu}*	*$k_1 k_3$*	*k_2*	*b*	*D*	*d*	*A_{st}*	*f_y*	*Tension(T)*	*C*a*	*a=T/C*	*d_n=a/g*	*$0.003(d-d_n)/d_n$*	*z*	*M_p=T*z*	*M_e*	*M_e/M_p*
		MPa				*mm*	*mm*	*mm*	*mm^2*	*N/mm^2*	*N*		*mm*	*mm*		*mm*	*kNm*	*kNm*	
Pam et al (2001)	1	37.4	0.003	0.85	0.7908	200	300	264	402	579	232758	6358	36.608682	46.293225	0.014108335	245.69566	57.18763	77.6	1.36
	2	36.8	0.003	0.85	0.7956	200	300	264	603	579	349137	6256	55.808344	70.146234	0.008290699	236.09583	82.429789	103.5	1.26
	3	36.4	0.003	0.85	0.7988	200	300	260	982	578	567596	6188	91.725275	114.82884	0.003792719	214.13736	121.54351	126.5	1.04
	4	42.3	0.003	0.85	0.7516	200	300	260	982	536	526352	7191	73.195939	97.386827	0.005009297	223.40203	117.58811	129	1.10
	5	46.4	0.003	0.85	0.7188	200	300	260	1183	546	645918	7888	81.886156	113.92064	0.003846872	219.05692	141.49281	142.8	1.01
	7	58.6	0.003	0.85	0.65	200	300	260	1296	520	673920	9962	67.649066	104.07549	0.00449456	226.17547	152.42417	164.6	1.08
	8	57.1	0.003	0.85	0.65	200	300	260	1473	520	765960	9707	78.908005	121.39693	0.003425204	220.546	168.92941	166.2	0.98
	9	58.6	0.003	0.85	0.65	200	300	256	1809	520	940680	9962	94.426822	145.27203	0.002286633	208.78659	196.40137	171.6	0.87
	14	95.5	0.003	0.657875	0.65	200	300	260	982	578	567596	16235	34.961257	53.786549	0.011501767	242.51937	137.65303	138	1.00
	15	98	0.003	0.63975	0.65	200	300	260	1473	578	851394	16660	51.104082	78.621664	0.00692093	234.44796	199.60759	200.7	1.01
	16	102.5	0.003	0.607125	0.65	200	300	260	1473	578	851394	17425	48.860488	75.169981	0.007376483	235.56976	200.56268	181.7	0.91
	17	87	0.003	0.7195	0.65	200	300	256	1608	546	877968	14790	59.362272	91.326572	0.005409382	226.31886	198.70072	172	0.87
Sarkar (1997)	HSC1-1	107	0.003	0.6	0.65	150	250	220	339	470	159330	9630	16.545171	25.45411	0.022929015	211.72741	33.734529	38.94	1.15
	HSC1-2	97	0.003	0.647	0.65	150	250	220	339	470	159330	9413.85	16.925063	26.038558	0.022347026	211.53747	33.704265	35.64	1.06
	HSC1-3	85	0.003	0.734	0.65	150	250	220	339	442	149838	9358.5	16.010899	24.632153	0.023794248	211.99455	31.764839	37.62	1.18
	HSC2-1	105	0.003	0.6	0.65	150	250	212.5	452	470	212440	9450	22.480423	34.585267	0.015432705	201.25979	42.755629	46.33	1.08
	HSC2-2	100	0.003	0.62525	0.65	150	250	212.5	452	470	212440	9378.75	22.651206	34.84801	0.015293728	201.1744	42.737489	46.86	1.10
	HSC2-3	77	0.003	0.792	0.65	150	250	212.5	452	442	199784	9147.6	21.840045	33.60007	0.015973175	201.57998	40.272454	43.56	1.08
	HSC2-4	90	0.003	0.69775	0.65	150	250	212.5	452	442	199784	9419.625	21.209337	32.629749	0.016537386	201.89533	40.335457	48.84	1.21
	HSC3-1	107	0.003	0.6	0.65	150	250	215	628	470	295160	9630	30.650052	47.153926	0.010678607	199.67497	58.936065	67.32	1.14
	HSC3-2	85	0.003	0.734	0.65	150	250	215	628	470	295160	9358.5	31.539242	48.521911	0.010292964	199.23038	58.804839	66	1.12
	HSC3-3	78	0.003	0.78475	0.65	150	250	215	628	442	277576	9181.575	30.23185	46.510539	0.010867825	199.88407	55.483022	64.68	1.17
	HSC4-1	101	0.003	0.618	0.65	150	250	207.5	1257	470	590790	9362.7	63.100388	97.07752	0.003412401	175.94981	103.94939	92.42	0.89
	HSC4-2	87	0.003	0.7195	0.65	150	250	207.5	1257	470	590790	9389.475	62.920451	96.800694	0.003430739	176.03977	104.00254	89.6	0.86
	HSC4-3	82	0.003	0.75575	0.65	150	250	207.5	1257	442	555594	9295.725	59.768765	91.951946	0.00376984	177.61562	98.682171	111.63	1.13
Bernardo & Lopes (2004)	A1	62.9	0.003	0.85	0.65	125	270	238	452	534	241368	6683.125	36.116039	55.563136	0.009850247	219.94198	53.086956	50.275	0.95
	A2	64.9	0.003	0.85	0.65	130	270	237	628	575	361100	7171.45	50.352439	77.465291	0.006178304	211.82378	76.489567	64.94	0.85
	A3	64.1	0.003	0.85	0.65	120	270	237	628	575	361100	6538.2	55.229268	84.968105	0.005367846	209.38537	75.609056	64.55	0.85
	A4	63.2	0.003	0.85	0.65	120	270	234	804	575	462300	6446.4	71.714445	110.32992	0.003362735	198.14278	91.601406	78.48	0.86
	A5	65.1	0.003	0.85	0.65	120	270	234	804	575	462300	6640.2	69.621397	107.10984	0.00355402	199.1893	92.085214	72.545	0.79
	B1	79.2	0.003	0.77605	0.65	120	264	237	452	534	241368	7375.5792	32.725294	50.346607	0.011122104	220.63735	53.254797	47.995	0.90
	B2	78.9	0.003	0.778225	0.65	124	270	242	628	575	361100	7613.8421	47.426778	72.964274	0.006950075	218.28661	78.823295	63.665	0.81
	B3	78.5	0.003	0.781125	0.65	120	270	242	628	575	361100	7358.1975	49.074519	75.499259	0.006615988	217.46274	78.525796	65.95	0.84
	C1	82.9	0.003	0.749225	0.65	123	270	242	628	575	361100	7639.6226	47.266733	72.718051	0.006983766	218.36663	78.852191	62.785	0.80
	C2	83.9	0.003	0.741975	0.65	120	270	242	628	575	361100	7470.2043	48.338705	74.367238	0.006762363	217.83065	78.658647	67.96	0.86
	C3	83.6	0.003	0.74415	0.65	125	270	239	804	575	462300	7776.3675	59.449351	91.46054	0.004839446	209.27532	96.747983	72.445	0.75
	C4	83.4	0.003	0.7456	0.65	122	275	244	804	575	462300	7586.3309	60.93855	93.751615	0.004807866	213.53073	98.715254	77.94	0.79
	D1	88	0.003	0.71225	0.65	120	270	247	402	575	231150	7521.36	30.732474	47.280729	0.012672347	231.63376	53.542144	44.22	0.83
	D2	85.8	0.003	0.7282	0.65	120	270	238	1030	572	589160	7497.5472	78.580366	120.89287	0.002906055	198.70982	117.07188	93.34	0.80
	D3	86	0.003	0.72675	0.65	120	270	238	1030	572	589160	7500.06	78.554038	120.85237	0.002908035	198.72298	117.07963	97.61	0.83
	E1	94.6	0.003	0.6644	0.65	123	270	239	804	575	462300	7730.8255	59.799565	91.99933	0.004793535	209.10022	96.667031	78.995	0.82
	E2	90.2	0.003	0.6963	0.65	120	270	239	804	575	462300	7536.7512	61.339427	94.368349	0.004597886	208.33029	96.311091	78.895	0.82
	F1	100.3	0.003	0.623075	0.65	139	263	230	628	575	361100	8686.7247	41.569177	63.95258	0.007789244	209.21541	75.547685	64.23	0.85
	F2	105.2	0.003	0.6	0.65	129	270	234	804	575	462300	8142.48	56.776314	87.348175	0.005036802	205.61184	95.054355	79.07	0.83
S.A.Ashour (2000)	B-N2	48.61	0.003	0.85	0.70112	200	250	215	509	530	269770	8263.7	32.645183	46.561478	0.010852653	198.67741	53.597204	58.17	1.09
	B-N3	48.61	0.003	0.85	0.70112	200	250	215	763	530	404390	8263.7	48.935707	69.796478	0.006241154	190.53215	77.049295	80.6	1.05
	B-N4	48.61	0.003	0.85	0.70112	200	250	215	1018	530	539540	8263.7	65.290366	93.122955	0.003926327	182.35482	98.387718	99.55	1.01
	B-M2	78.5	0.003	0.781125	0.65	200	250	215	509	530	269770	12263.663	21.997507	33.842318	0.016058978	204.00125	55.033416	57.95	1.05
	B-M3	78.5	0.003	0.781125	0.65	200	250	215	763	530	404390	12263.663	32.974652	50.730233	0.009714312	198.51267	80.27654	79.91	1.00
	B-M4	78.5	0.003	0.781125	0.65	200	250	215	1018	530	539540	12263.663	43.995014	67.684636	0.006529489	193.00249	104.13257	103.77	1.00
	B-H2	102.4	0.003	0.60785	0.65	200	250	215	509	530	269770	12448.768	21.670418	33.339104	0.016346651	204.16479	55.077536	56.8	1.03
	B-H3	102.4	0.003	0.60785	0.65	200	250	215	763	530	404390	12448.768	32.484339	49.975906	0.009906219	198.75783	80.375679	82.76	1.03
	B-H4	102.4	0.003	0.60785	0.65	200	250	215	1018	530	539540	12448.768	43.340835	66.678208	0.006673325	193.32958	104.30904	108.1	1.04

Table A.10 Calculation of Ultimate Moment Capacity for beams as per Azizinamini et al (1994)

SUNGJIN BAE AND OGUZHAN BAYRAK (2003)

$k_1 k_3$=0.85-0.004(f'$_c$-70) ; 0.67 ≤ $k_1 k_3$ ≤ 0.85
k_2=0.85-0.004(f'$_c$-30); 0.67 ≤ k_2 ≤ 0.85

Ultimate concrete strain ε_{cu}=0.0025 for f'$_c$ >55 MPa
otherwise ε_{cu}=0.003

Reference	Beam no	f'_c	ε_{cu}	$k_1 k_3$	k_2	b	D	d	A_{st}	f_y	Tension(T)	C*a	a=T/C	d_n=a/g	0.003(d-d_n)/d_n	z	M_p=T*z	M_e	M_e/M_p
		MPa				mm	mm	mm	mm^2	N/mm^2	N		mm	mm		mm	kNm	kNm	
Pam et al (2001)	1	37.4	0.003	0.85	0.8204	200	300	264	402	579	232758	6358	36.608682	44.622967	0.014748708	245.69566	57.18763	77.6	1.36
	2	36.8	0.003	0.85	0.8228	200	300	264	603	579	349137	6256	55.808344	67.82735	0.008676706	236.09583	82.429789	103.5	1.26
	3	36.4	0.003	0.85	0.8244	200	300	260	982	578	567596	6188	91.725275	111.26307	0.004010412	214.13736	121.54351	126.5	1.04
	4	42.3	0.003	0.85	0.8008	200	300	260	982	536	526352	7191	73.195939	91.403521	0.005533588	223.40203	117.58811	129	1.10
	5	46.4	0.003	0.85	0.7212	200	300	260	1183	546	645918	7888	81.886156	113.54154	0.003869733	219.05692	141.49281	142.8	1.01
	7	58.6	0.0025	0.85	0.65	200	300	260	1296	520	673920	9962	67.649066	104.07549	0.00449456	226.17547	152.42417	164.6	1.08
	8	57.1	0.0025	0.85	0.65	200	300	260	1473	520	765960	9707	78.908005	121.39693	0.003425204	220.546	168.92941	166.2	0.98
	9	58.6	0.0025	0.85	0.65	200	300	256	1809	520	940680	9962	94.426822	145.27203	0.002286633	208.78659	196.40137	171.6	0.87
	14	95.5	0.0025	0.748	0.65	200	300	260	982	578	567596	16235	34.961257	53.786549	0.011501767	242.51937	137.65303	138	1.00
	15	98	0.0025	0.738	0.65	200	300	260	1473	578	851394	16660	51.104082	78.621664	0.00692093	234.44796	199.60759	200.7	1.01
	16	102.5	0.0025	0.72	0.65	200	300	260	1473	578	851394	17425	48.860488	75.169981	0.007376483	235.56976	200.56268	181.7	0.91
	17	87	0.0025	0.782	0.65	200	300	256	1608	546	877968	14790	59.362272	91.326572	0.005409382	226.31886	198.70072	172	0.87
Sarkar (1997)	HSC1-1	107	0.0025	0.702	0.67	150	250	220	339	470	159330	11267.1	14.141172	21.106227	0.02355866	212.92941	33.926044	38.94	1.15
	HSC1-2	97	0.0025	0.742	0.67	150	250	220	339	470	159330	10796.1	14.758107	22.027026	0.022469327	212.62095	33.876895	35.64	1.05
	HSC1-3	85	0.0025	0.79	0.67	150	250	220	339	442	149838	10072.5	14.875949	22.20291	0.022271528	212.56203	31.849869	37.62	1.18
	HSC2-1	105	0.0025	0.71	0.67	150	250	212.5	452	470	212440	11182.5	18.997541	28.354539	0.016235978	203.00123	43.125581	46.33	1.07
	HSC2-2	100	0.0025	0.73	0.67	150	250	212.5	452	470	212440	10950	19.400913	28.956587	0.01584643	202.79954	43.082735	46.86	1.09
	HSC2-3	77	0.0025	0.822	0.67	150	250	212.5	452	442	199784	9494.1	21.042964	31.407408	0.014414799	201.97852	40.352076	43.56	1.08
	HSC2-4	90	0.0025	0.77	0.67	150	250	212.5	452	442	199784	10395	19.21924	28.685433	0.016019853	202.89038	40.534252	48.84	1.20
	HSC3-1	107	0.0025	0.702	0.67	150	250	215	628	470	295160	11267.1	26.196626	39.099441	0.011247	201.90169	59.593302	67.32	1.13
	HSC3-2	85	0.0025	0.79	0.67	150	250	215	628	470	295160	10072.5	29.303549	43.736641	0.009789467	200.34823	59.134782	66	1.12
	HSC3-3	78	0.0025	0.818	0.67	150	250	215	628	442	277576	9570.6	29.002988	43.288042	0.009916824	200.49851	55.653573	64.68	1.16
	HSC4-1	101	0.0025	0.726	0.67	150	250	207.5	1257	470	590790	10998.9	53.713553	80.169482	0.003970667	180.64322	106.72221	92.42	0.87
	HSC4-2	87	0.0025	0.782	0.67	150	250	207.5	1257	470	590790	10205.1	57.891642	86.405436	0.003503673	178.55418	105.48802	89.6	0.85
	HSC4-3	82	0.0025	0.802	0.67	150	250	207.5	1257	442	555594	9864.6	56.322	84.062686	0.00367099	179.339	99.639672	111.63	1.12
Bernardo & Lopes (2004)	A1	62.9	0.0025	0.85	0.7184	125	270	238	452	534	241368	6683.125	36.116039	50.272882	0.009335407	219.94198	53.086956	50.275	0.95
	A2	64.9	0.0025	0.85	0.7104	130	270	237	628	575	361100	7171.45	50.352439	70.878997	0.005859317	211.82378	76.489567	64.94	0.85
	A3	64.1	0.0025	0.85	0.7136	120	270	237	628	575	361100	6538.2	55.229268	77.395275	0.005155506	209.38537	75.609056	64.55	0.85
	A4	63.2	0.0025	0.85	0.7172	120	270	234	804	575	462300	6446.4	71.714445	99.992255	0.003350453	198.14278	91.601406	78.48	0.86
	A5	65.1	0.0025	0.85	0.7096	120	270	234	804	575	462300	6640.2	69.621397	98.113581	0.003462477	199.1893	92.085214	72.545	0.79
	B1	79.2	0.0025	0.8132	0.67	120	264	237	452	534	241368	7728.6528	31.230281	46.61236	0.010211221	221.38486	53.435221	47.995	0.90
	B2	78.9	0.0025	0.8144	0.67	124	270	242	628	575	361100	7967.7638	45.320118	67.641968	0.006444151	219.33994	79.203653	63.665	0.80
	B3	78.5	0.0025	0.816	0.67	120	270	242	628	575	361100	7686.72	46.977124	70.115111	0.006128668	218.51144	78.90448	65.95	0.84
	C1	82.9	0.0025	0.7984	0.67	123	270	242	628	575	361100	8141.0453	44.355484	66.202214	0.006638667	219.82226	79.377817	62.785	0.79
	C2	83.9	0.0025	0.7944	0.67	120	270	242	628	575	361100	7998.0192	45.148679	67.386088	0.006478114	219.42566	79.234606	67.96	0.86
	C3	83.6	0.0025	0.7956	0.67	125	270	239	804	575	462300	8314.02	55.60487	82.992343	0.004699459	211.19757	97.636634	72.445	0.74
	C4	83.4	0.0025	0.7964	0.67	122	275	244	804	575	462300	8103.2107	57.05146	85.151432	0.004663708	215.47427	99.613755	77.94	0.78
	D1	88	0.0025	0.778	0.67	120	270	247	402	575	231150	8215.68	28.135224	41.992872	0.012204877	232.93239	53.842321	44.22	0.82
	D2	85.8	0.0025	0.7868	0.67	120	270	238	1030	572	589160	8100.8928	72.727786	108.54893	0.002981399	201.63611	118.79593	93.34	0.79
	D3	86	0.0025	0.786	0.67	120	270	238	1030	572	589160	8111.52	72.632503	108.40672	0.00298859	201.68375	118.824	97.61	0.82
	E1	94.6	0.0025	0.7516	0.67	123	270	239	804	575	462300	8745.4673	52.861669	78.898014	0.005073068	212.56917	98.270725	78.995	0.80
	E2	90.2	0.0025	0.7692	0.67	120	270	239	804	575	462300	8325.8208	55.526057	82.874712	0.004709678	211.23697	97.654852	78.895	0.81
	F1	100.3	0.0025	0.7288	0.67	139	263	230	628	575	361100	10160.711	35.538852	53.043062	0.008340249	212.23057	76.63646	64.23	0.84
	F2	105.2	0.0025	0.7092	0.67	129	270	234	804	575	462300	9624.4114	48.034106	71.692696	0.005659827	209.98295	97.075116	79.07	0.81
S.A.Ashour (2000)	B-N2	48.61	0.003	0.85	0.77556	200	250	215	509	530	269770	8263.7	32.645183	42.092402	0.012323431	198.67741	53.597204	58.17	1.09
	B-N3	48.61	0.003	0.85	0.77556	200	250	215	763	530	404390	8263.7	48.935707	63.097255	0.007222315	190.53215	77.049295	80.6	1.05
	B-N4	48.61	0.003	0.85	0.77556	200	250	215	1018	530	539540	8263.7	65.290366	84.184804	0.004661715	182.35482	98.387718	99.55	1.01
	B-M2	78.5	0.0025	0.816	0.656	200	250	215	509	530	269770	12811.2	21.057356	32.099628	0.017093691	204.47132	55.160229	57.95	1.05
	B-M3	78.5	0.0025	0.816	0.656	200	250	215	763	530	404390	12811.2	31.565349	48.11791	0.010404572	199.21733	80.561494	79.91	0.99
	B-M4	78.5	0.0025	0.816	0.656	200	250	215	1018	530	539540	12811.2	42.114712	64.199256	0.007046845	193.94264	104.63981	103.77	0.99
	B-H2	102.4	0.0025	0.7204	0.5604	200	250	215	509	530	269770	14753.792	18.284791	32.628106	0.016768233	205.8576	55.534206	56.8	1.02
	B-H3	102.4	0.0025	0.7204	0.5604	200	250	215	763	530	404390	14753.792	27.409225	48.910108	0.010187458	201.29539	81.401842	82.76	1.02
	B-H4	102.4	0.0025	0.7204	0.5604	200	250	215	1018	530	539540	14753.792	36.569582	65.256213	0.006884116	196.71521	106.13572	108.1	1.02

Table A.11 Calculation of Ultimate Moment Capacity for beams as per Bae & Bayrak (2003)

	M_e/M_p ratio	
Method	Mean	Coefficient of variation
AS3600-2001	0.966	14.93%
ACI318-05	0.974	16.98%
CEB-FIP1990	1.011	13.73%
CAN-A23.3-M94	0.981	14.475
NZS3101-1995	0.974	14.45%
Eurocode-2	0.966	14.93%
Mendis & Pendyala	0.95	13.86%
Ibrahim & MacGregor	0.974	14.57%
M.Attard &Mark.G.Stewart	0.967	14.78%
Azizinamini et al	0.976	14.57%
S.Bae & O.Bayrak	0.971	14.74%

Table A.12 Summary of Correlation

APPENDIX B

MOMENT CURVATURE CURVES FOR BEAMS TESTED BY PAM ET AL (2001)

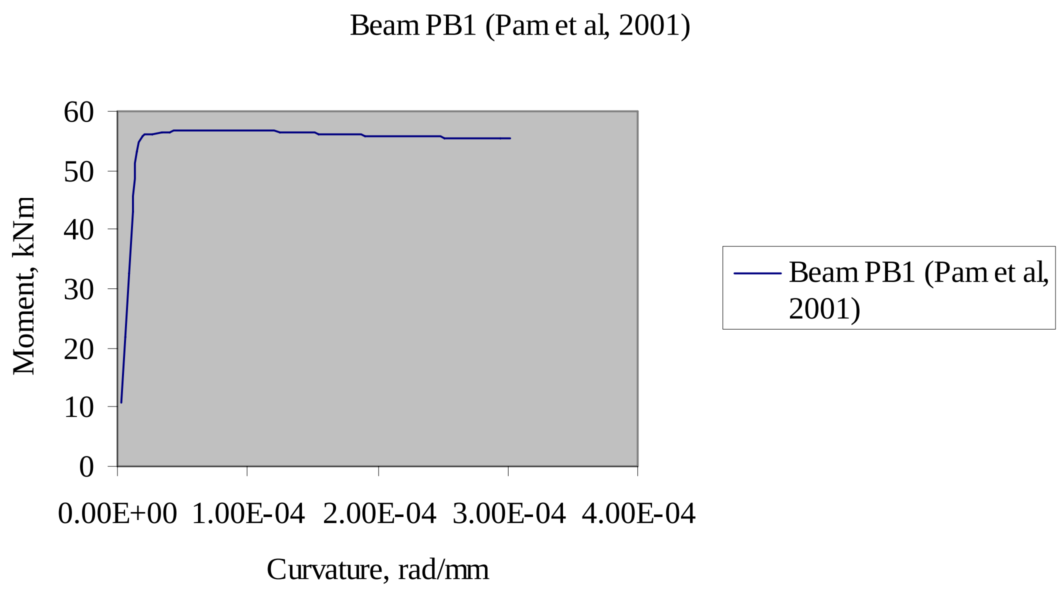


Fig. C.1 Moment curvature curve for Beam PB1 tested by Pam et al (2001)

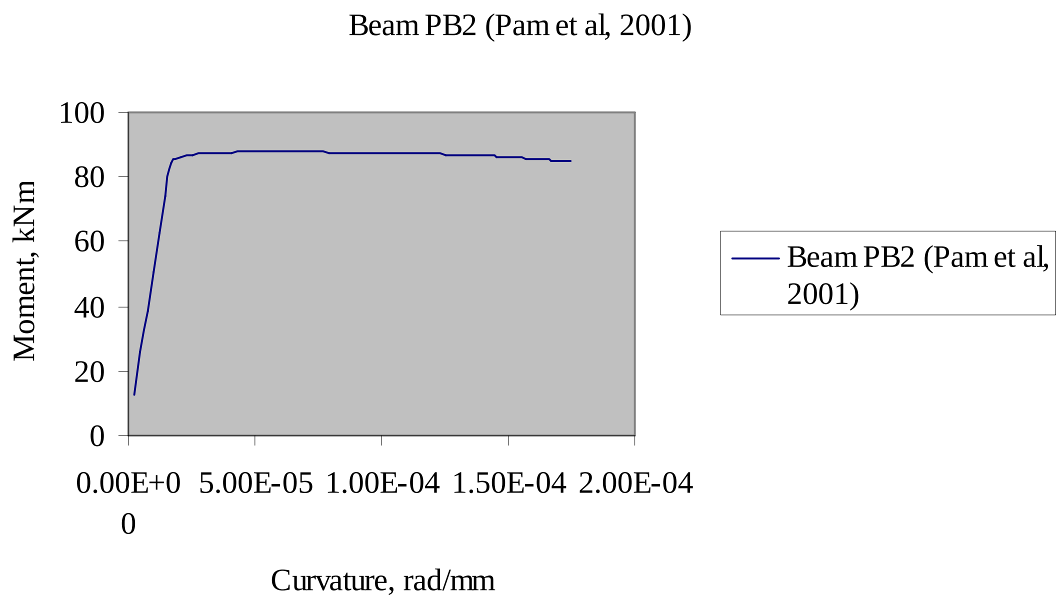


Fig. C.2 Moment curvature curve for Beam PB2 tested by Pam et al (2001)

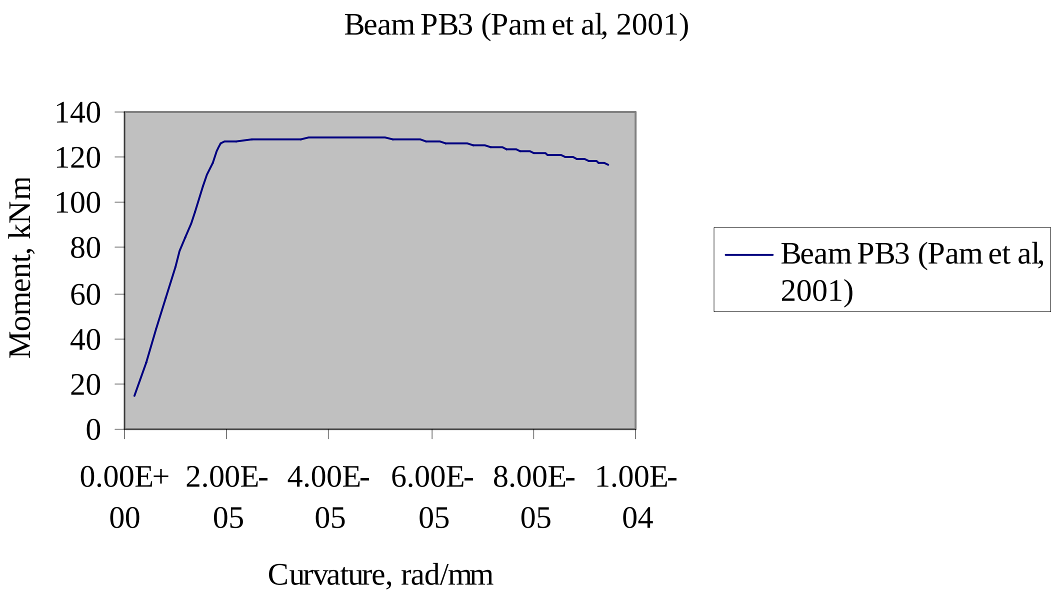


Fig. C.3 Moment curvature curve for Beam PB3 tested by Pam et al (2001)

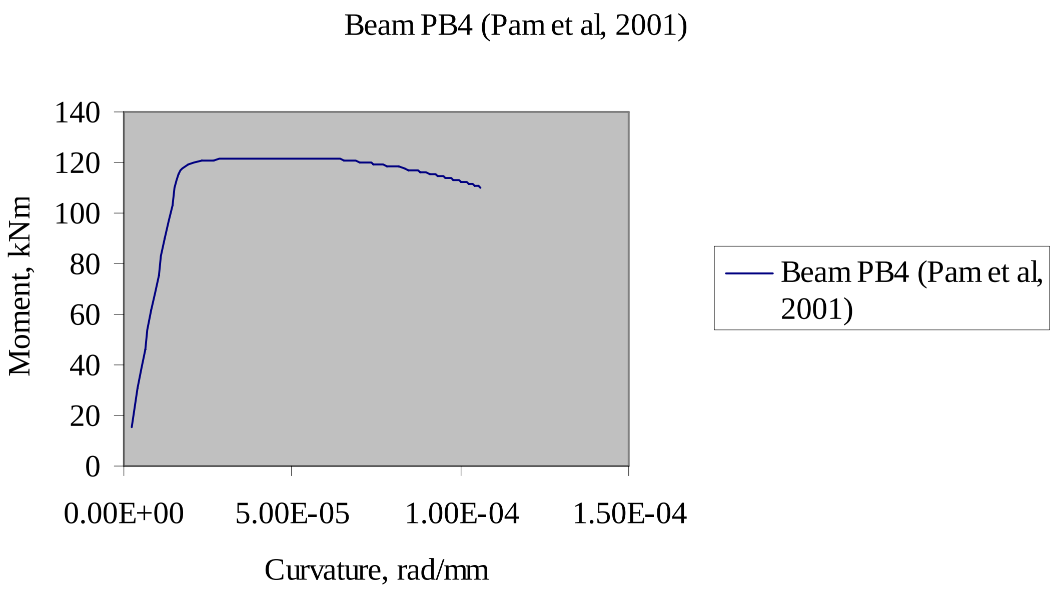


Fig. C.4 Moment curvature curve for Beam PB4 tested by Pam et al (2001)

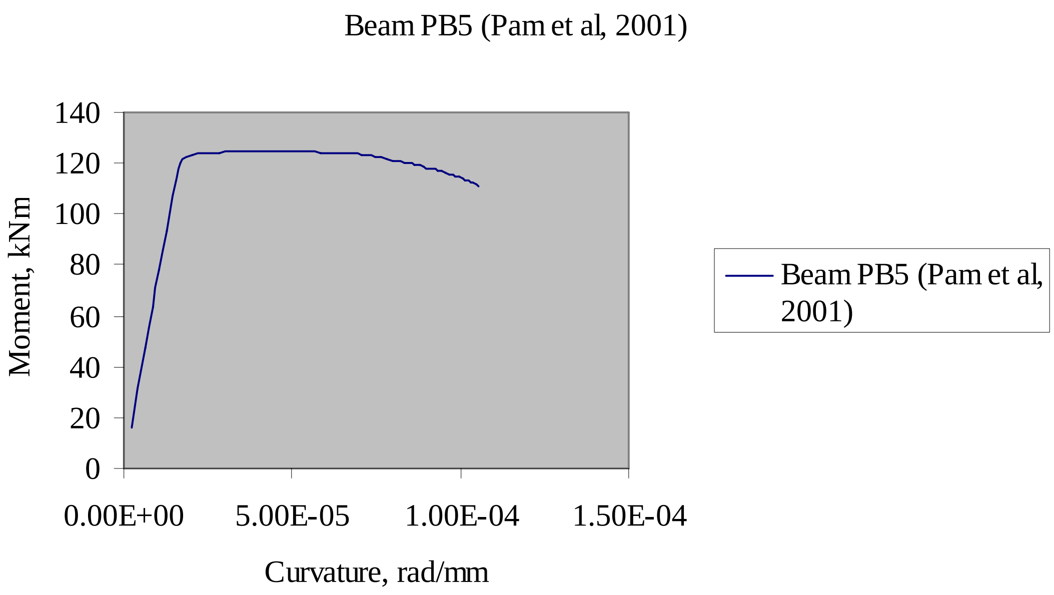


Fig. C.5 Moment curvature curve for Beam PB5 tested by Pam et al (2001)

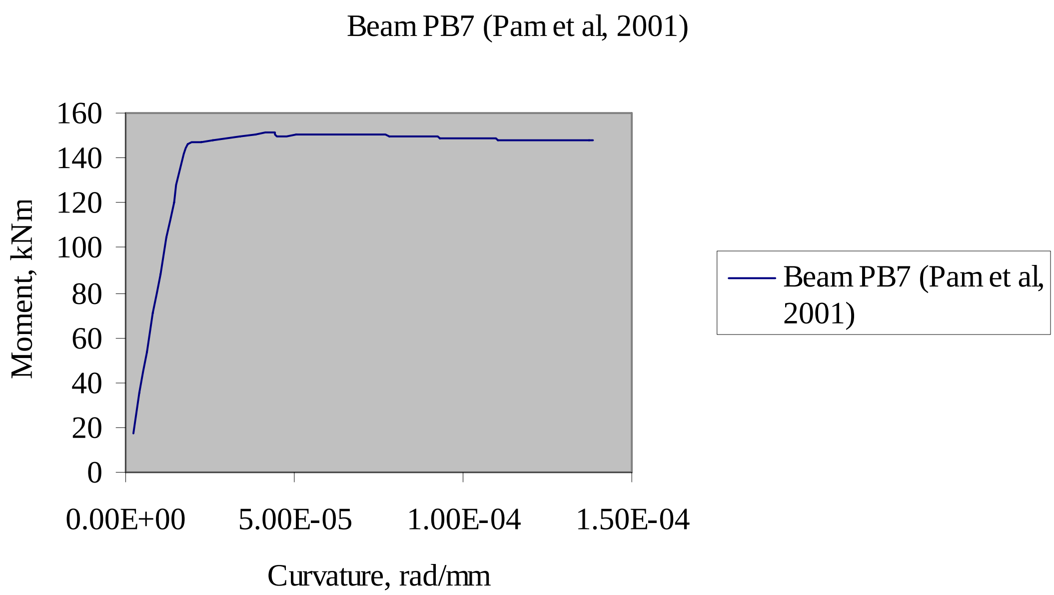


Fig. C.6 Moment curvature curve for Beam PB7 tested by Pam et al (2001)

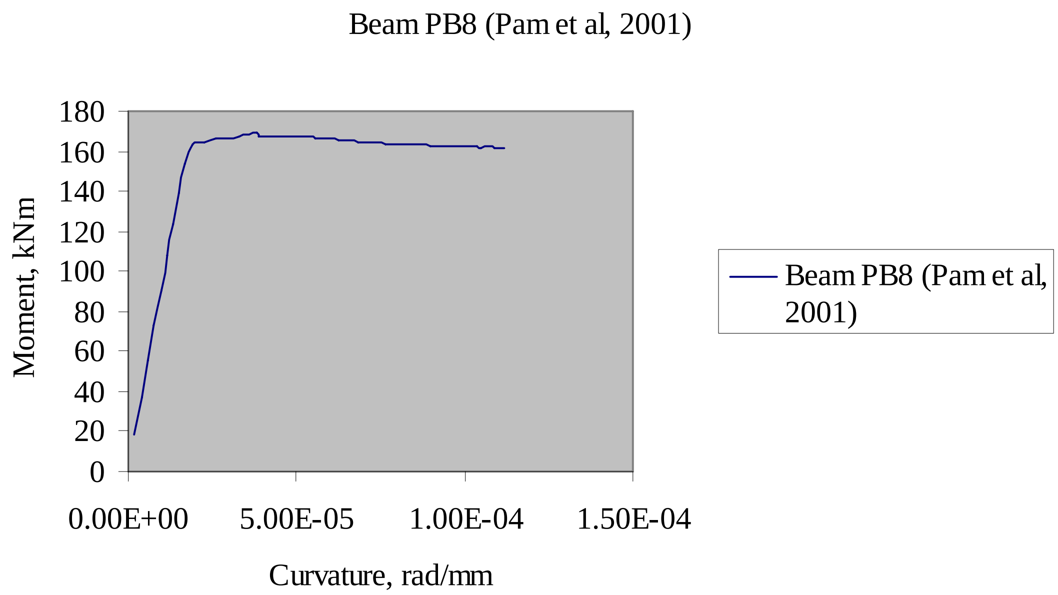


Fig. C.7 Moment curvature curve for Beam PB8 tested by Pam et al (2001)

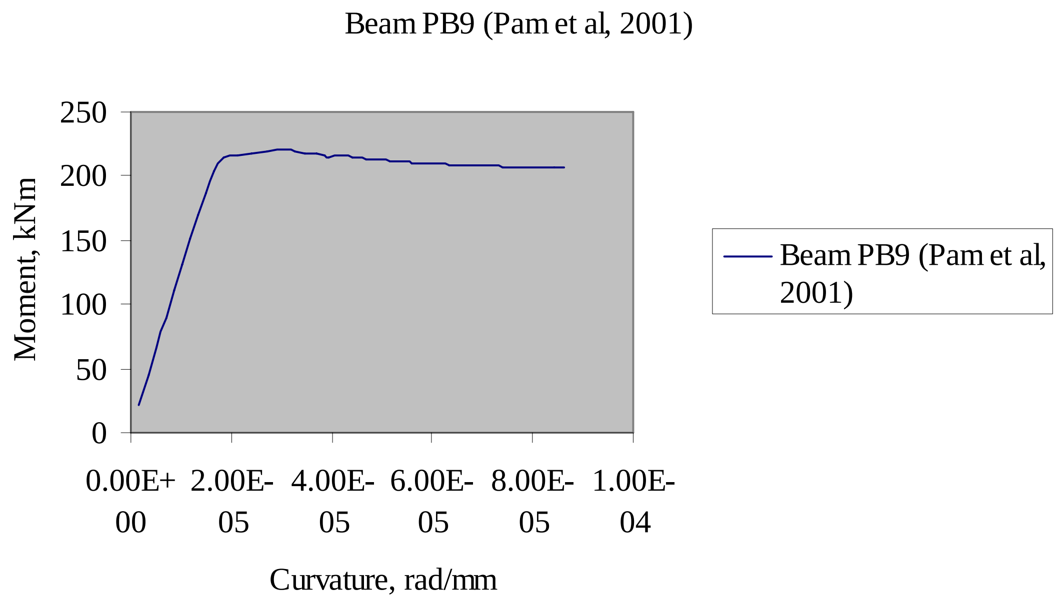


Fig. C.8 Moment curvature curve for Beam PB9 tested by Pam et al (2001)

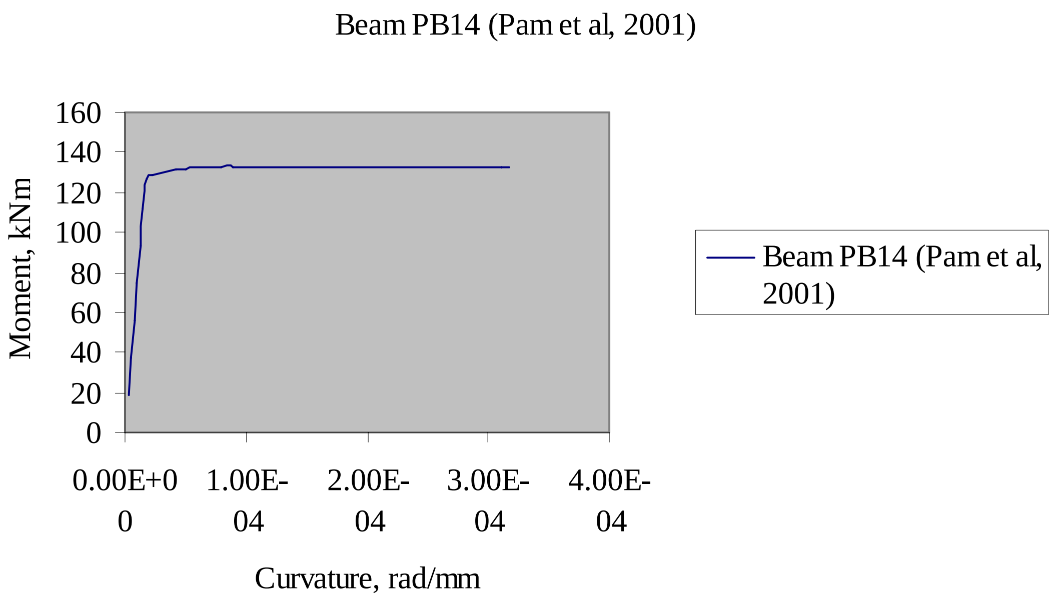


Fig. C.9 Moment curvature curve for Beam PB14 tested by Pam et al (2001)

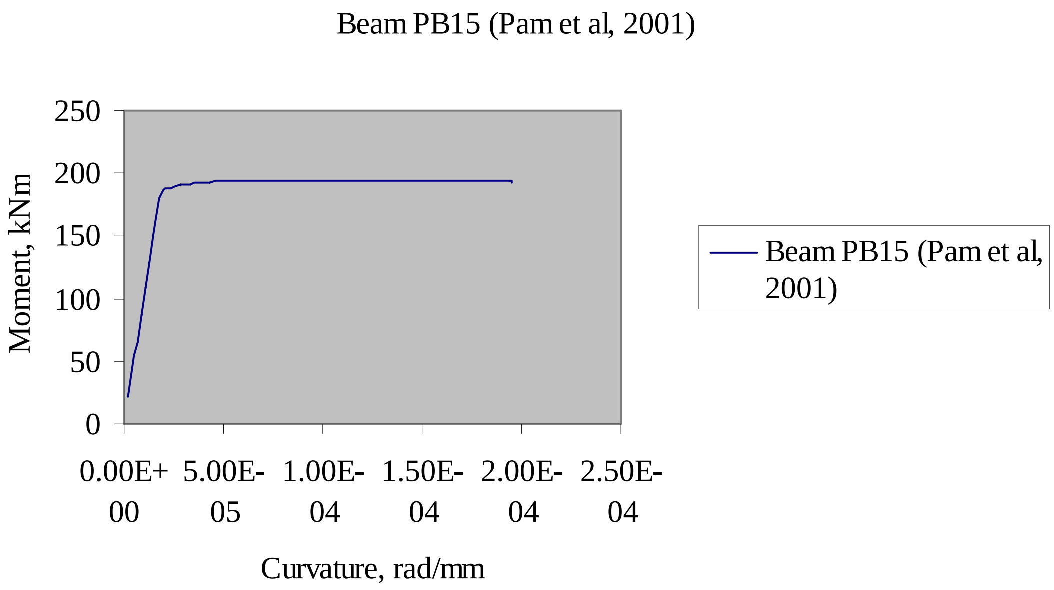


Fig. C.10 Moment curvature curve for Beam PB15 tested by Pam et al (2001)

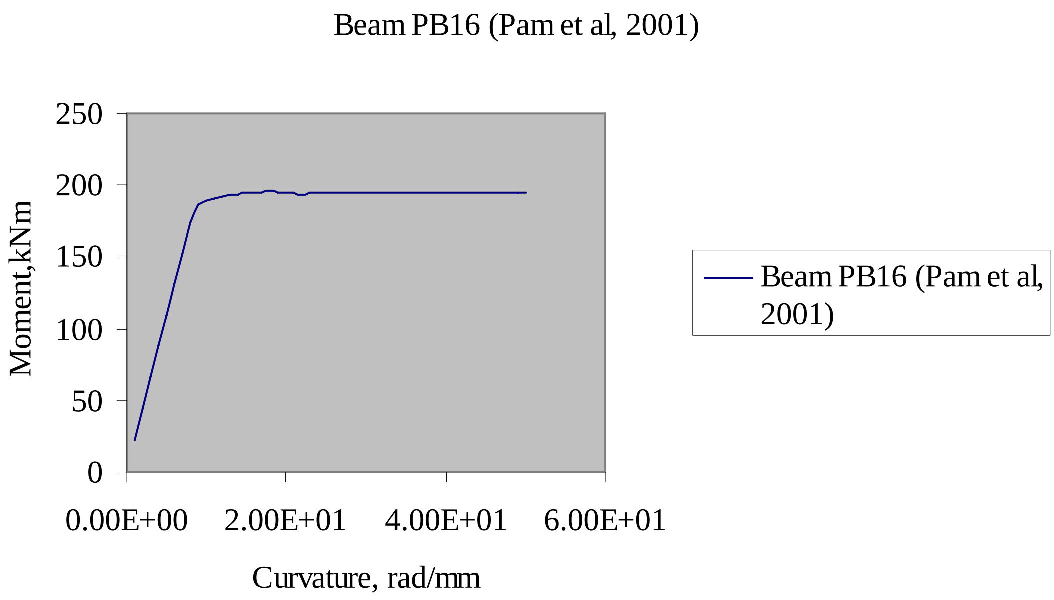


Fig. C.11 Moment curvature curve for Beam PB16 tested by Pam et al (2001)

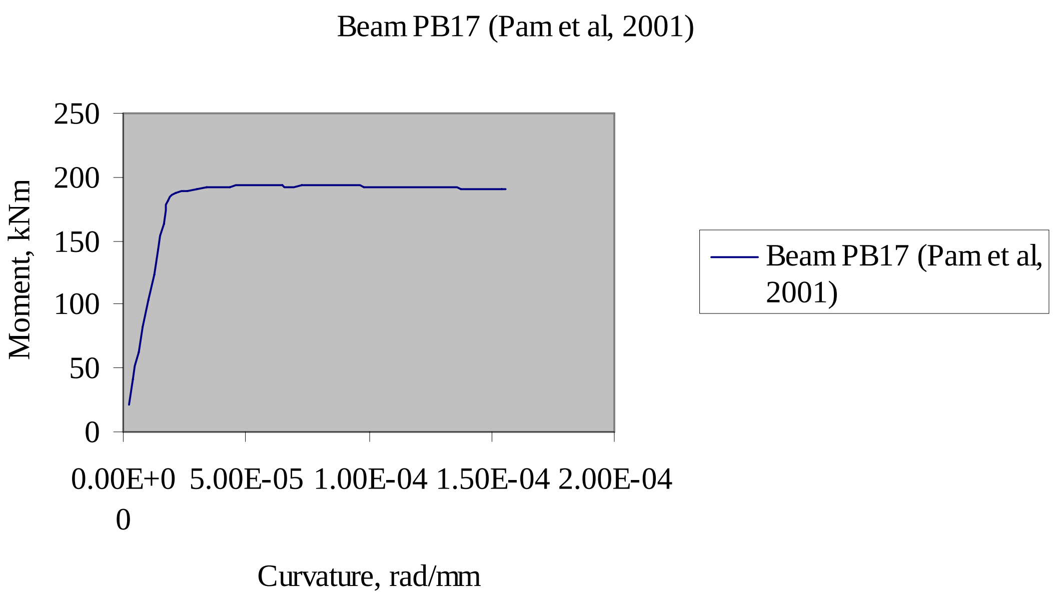


Fig. C.12 Moment curvature curve for Beam PB17 tested by Pam et al (2001)

MOMENT CURVATURE CURVES FOR BEAMS TESTED BY SARKAR ET AL (1997)

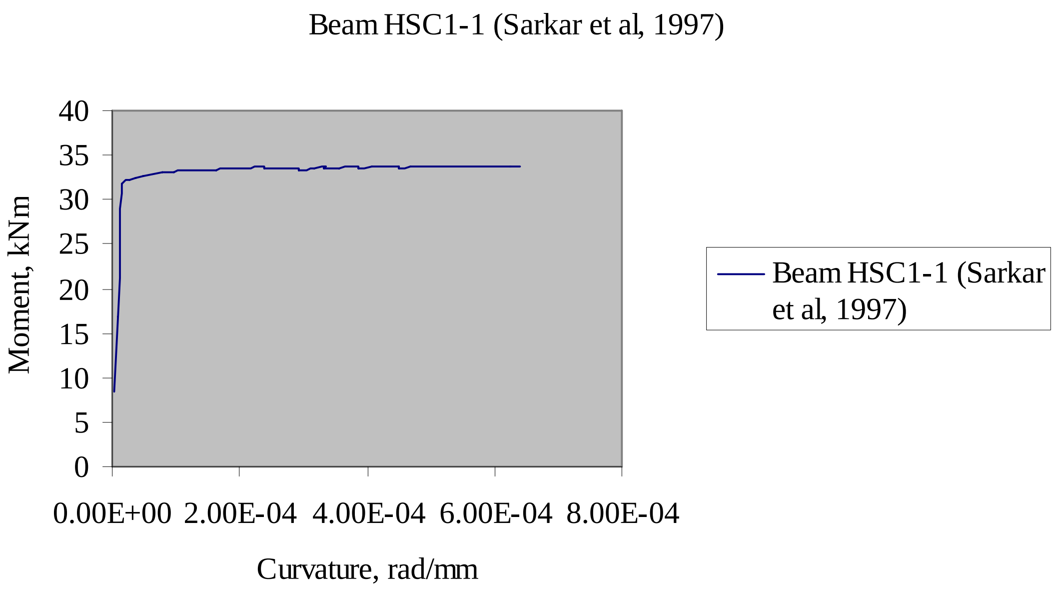


Fig. C.13 Moment curvature curve for Beam HSC 1-1 tested by Sarkar et al (1997)

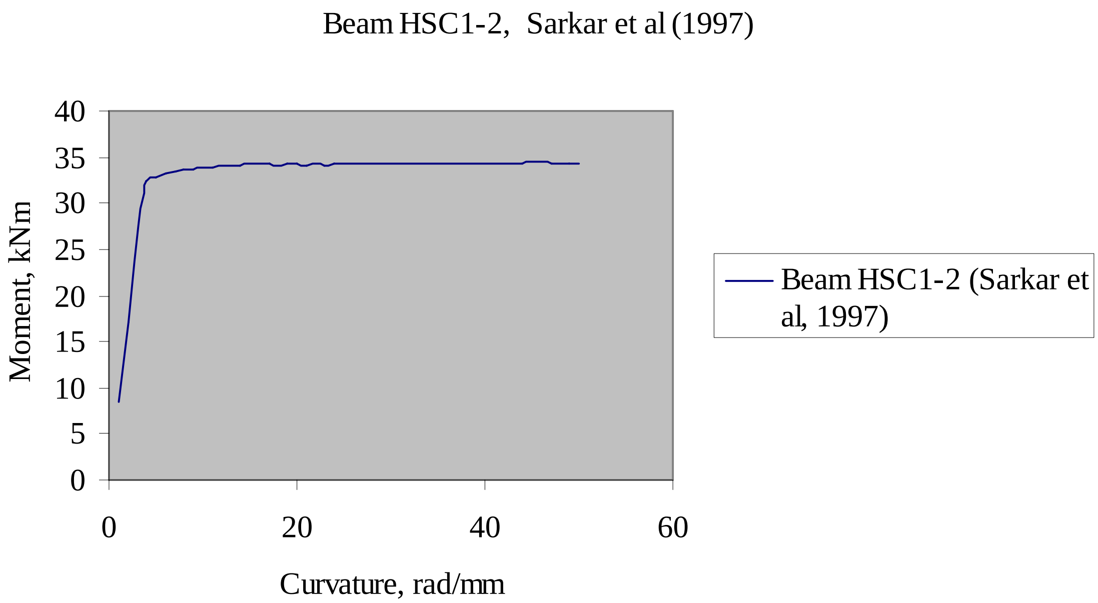


Fig. C.14 Moment curvature curve for Beam HSC 1-2 tested by Sarkar et al (1997)

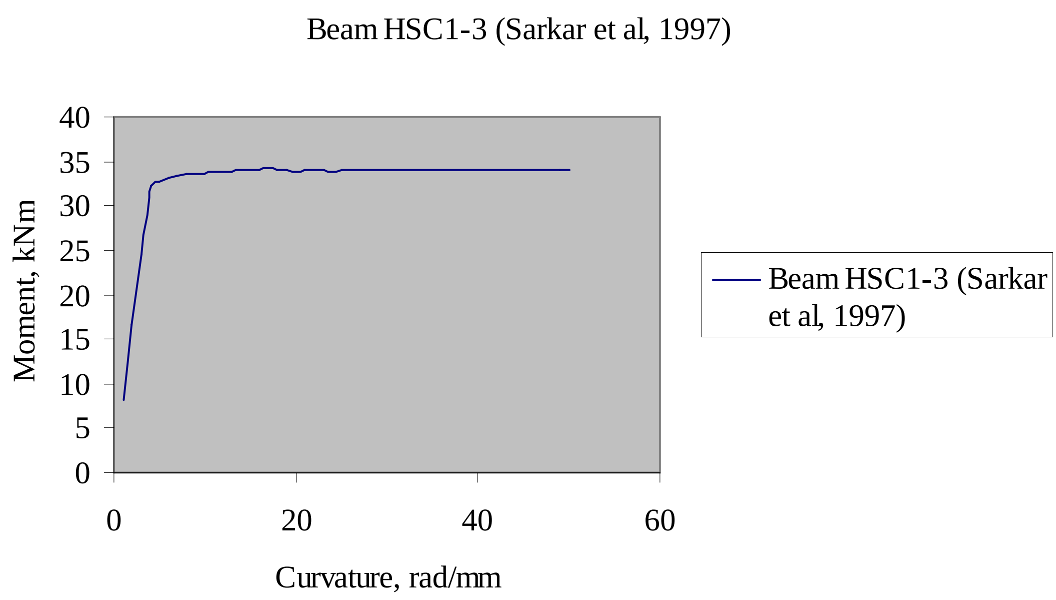


Fig. C.15 Moment curvature curve for Beam HSC 1-3 tested by Sarkar et al (1997)

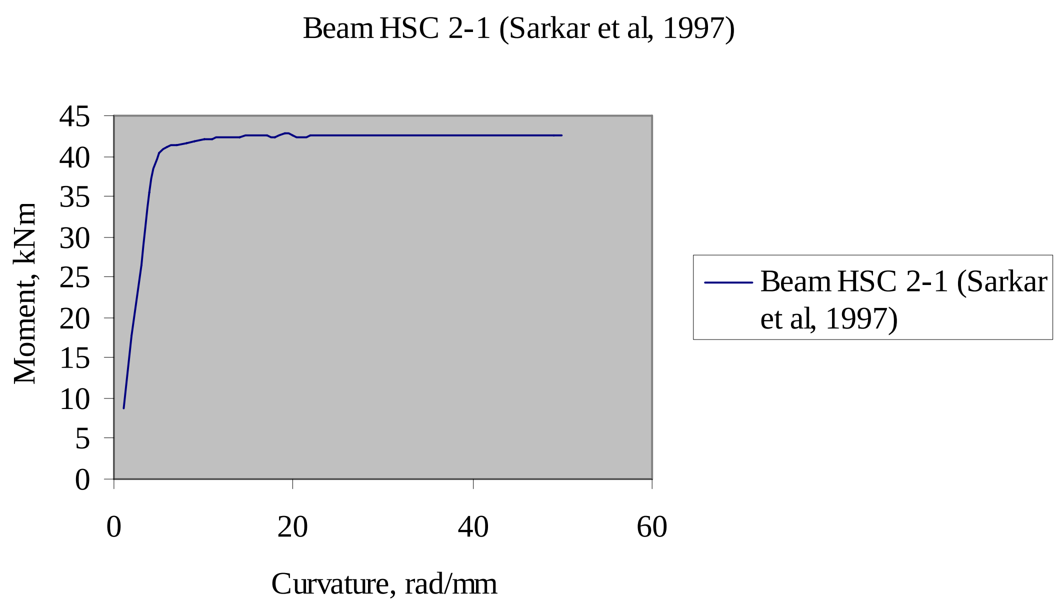


Fig. C.16 Moment curvature curve for Beam HSC 2-1 tested by Sarkar et al (1997)

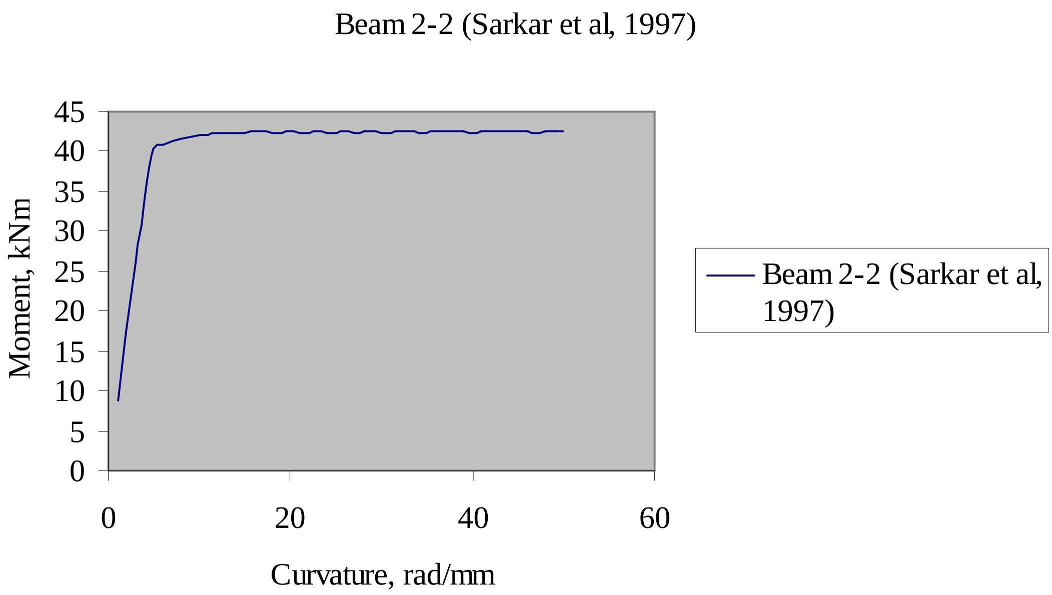
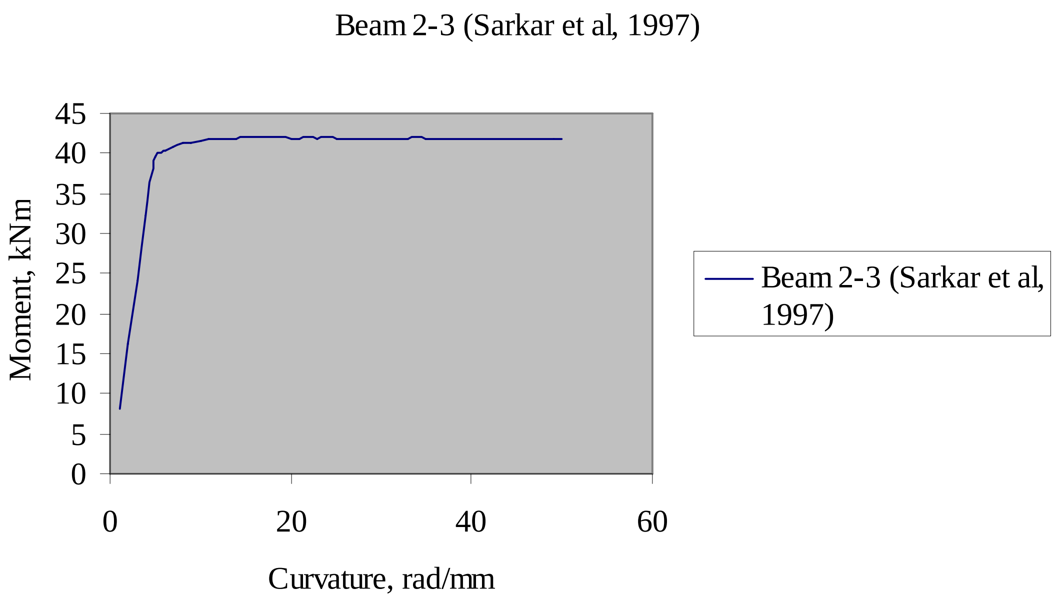
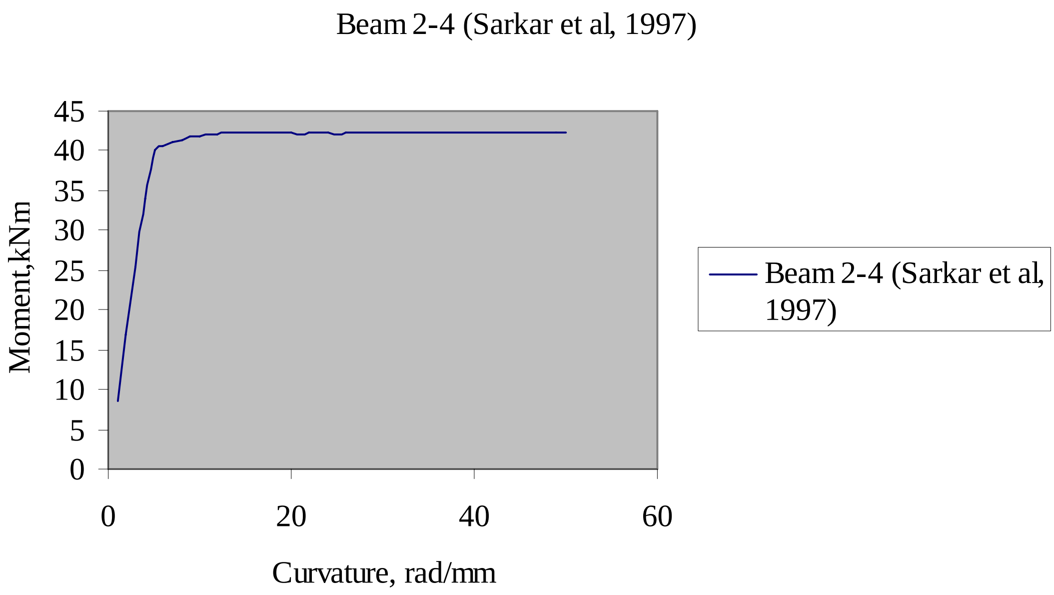


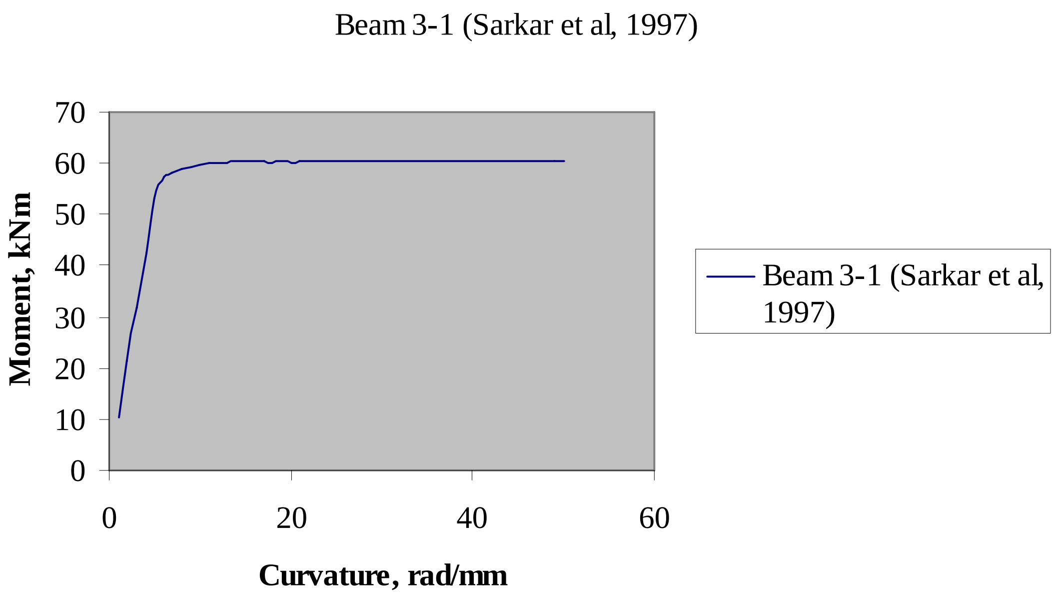
Fig. C.17 Moment curvature curve for Beam HSC 2-2 tested by Sarkar et al (1997)



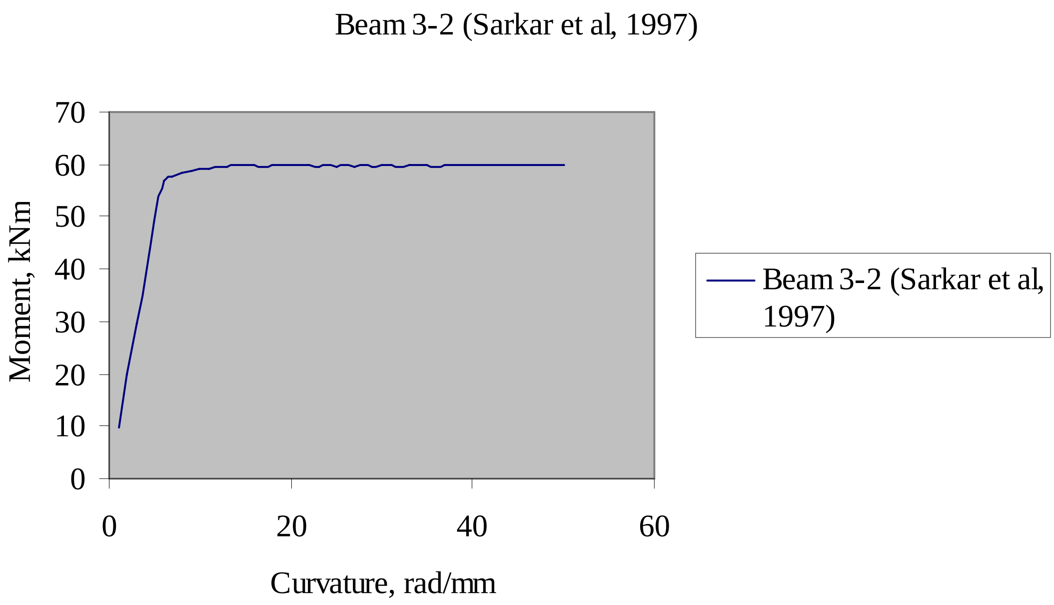
C.18 Moment curvature curve for Beam HSC 2-3 tested by Sarkar et al (1997)



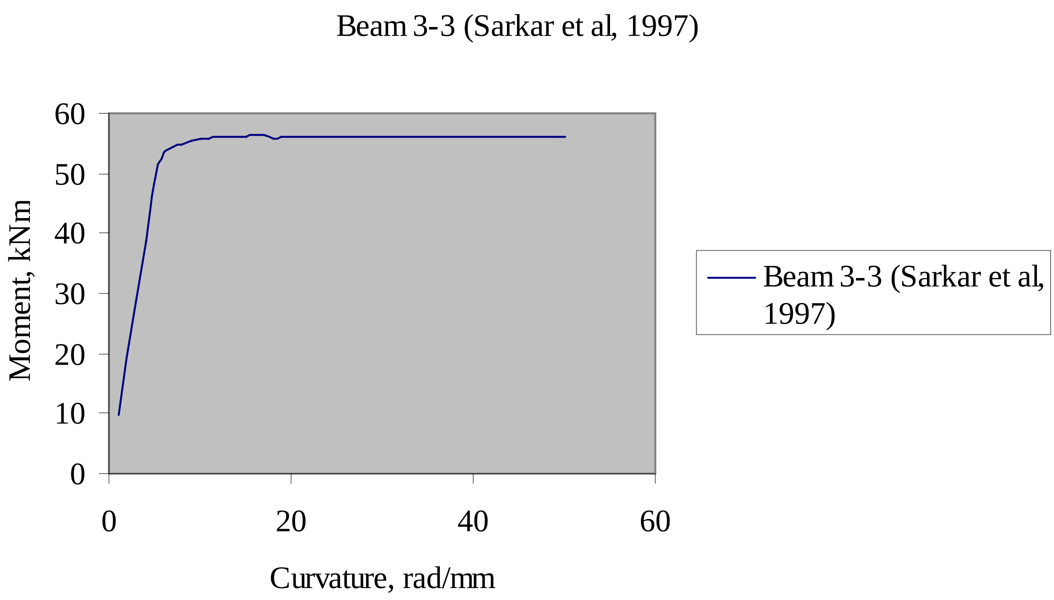
C.19 Moment curvature curve for Beam HSC 2-4 tested by Sarkar et al (1997)



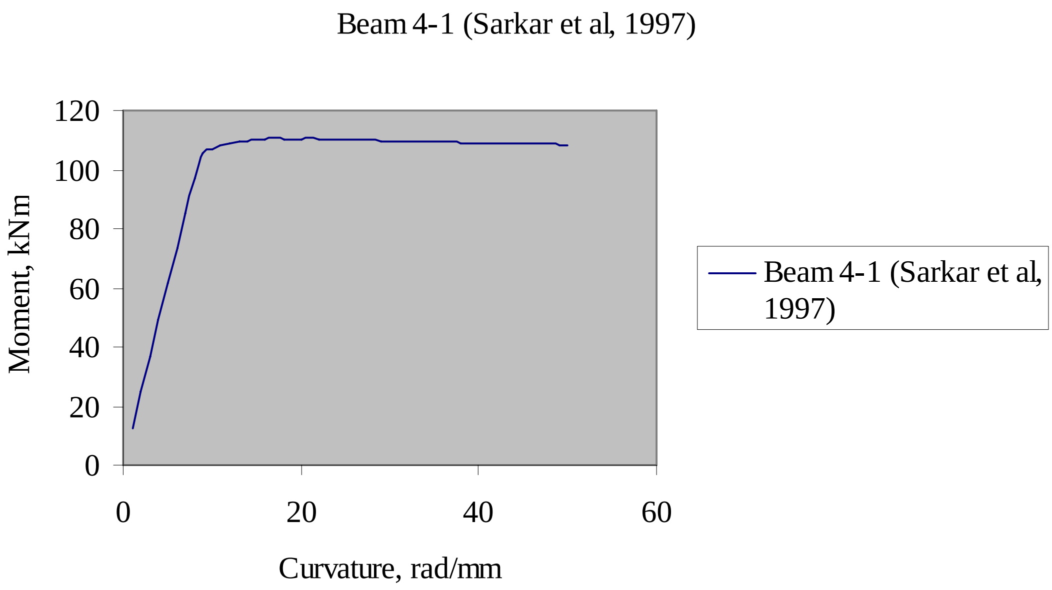
C.20 Moment curvature curve for Beam HSC 3-1 tested by Sarkar et al (1997)



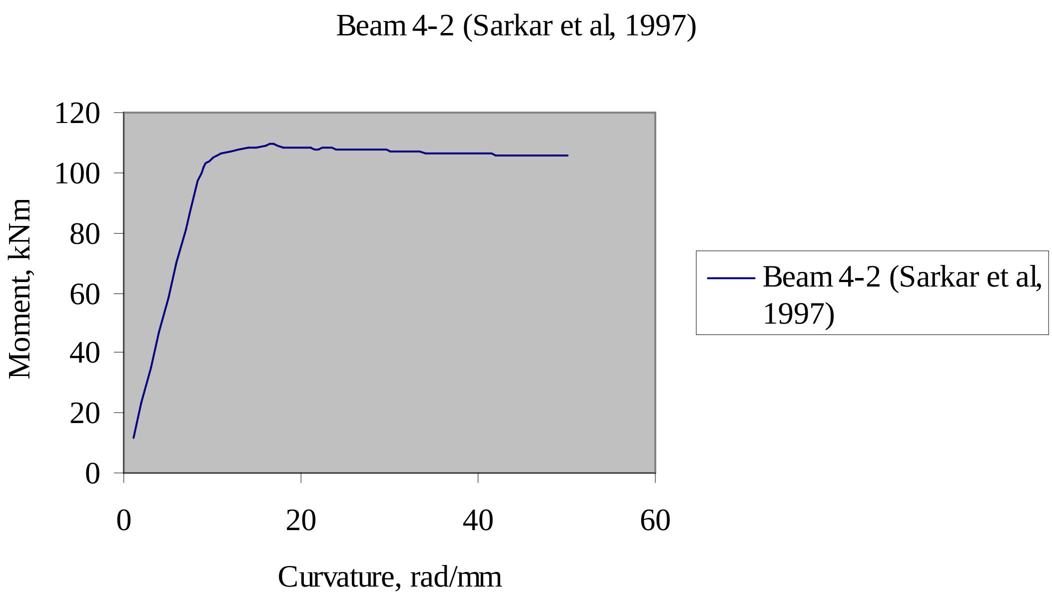
C.21 Moment curvature curve for Beam HSC 3-2 tested by Sarkar et al (1997)



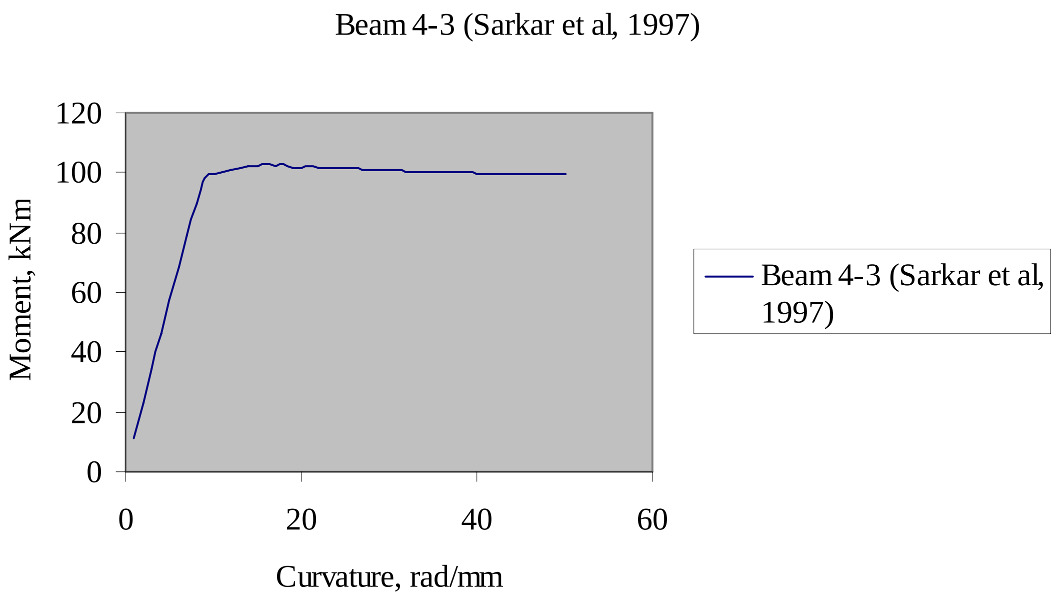
C.22 Moment curvature curve for Beam HSC 3-3 tested by Sarkar et al (1997)



C.23Moment curvature curve for Beam HSC 4-1 tested by Sarkar et al (1997)

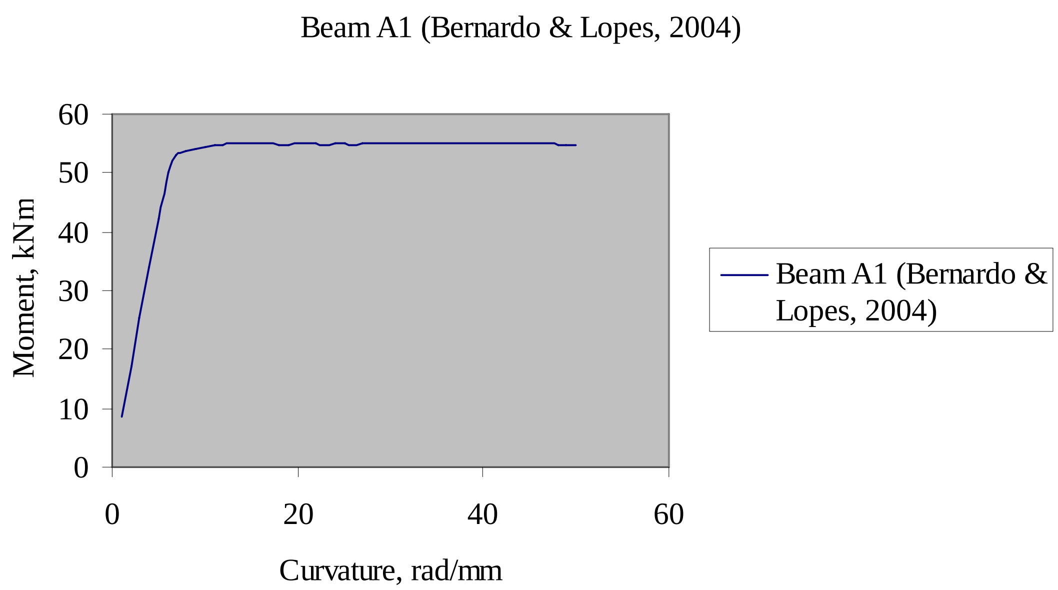


C.24 Moment curvature curve for Beam HSC 4-2 tested by Sarkar et al (1997)

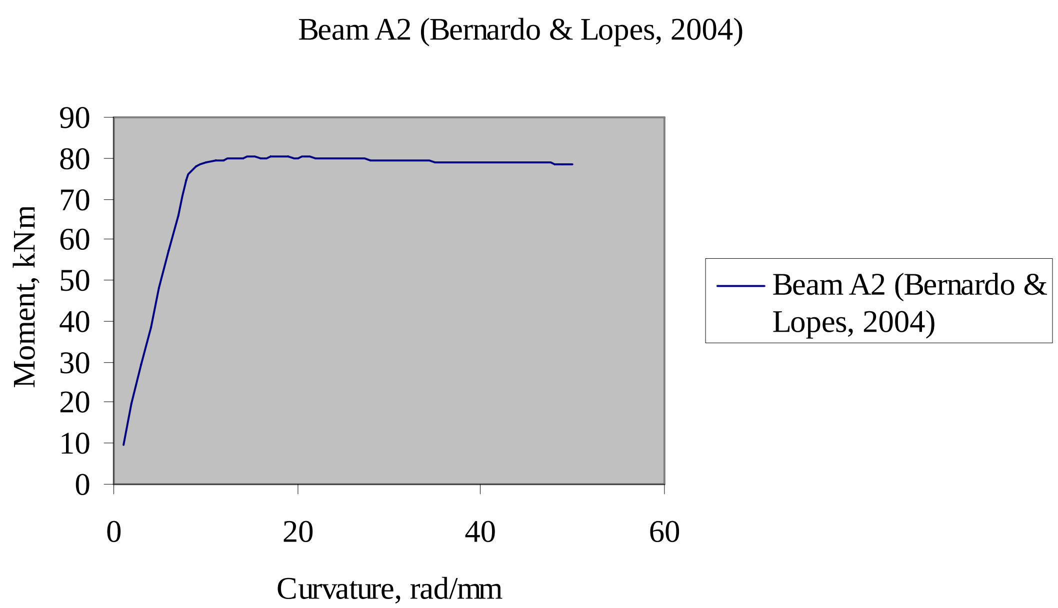


C.25 Moment curvature curve for Beam HSC 4-3 tested by Sarkar et al (1997)

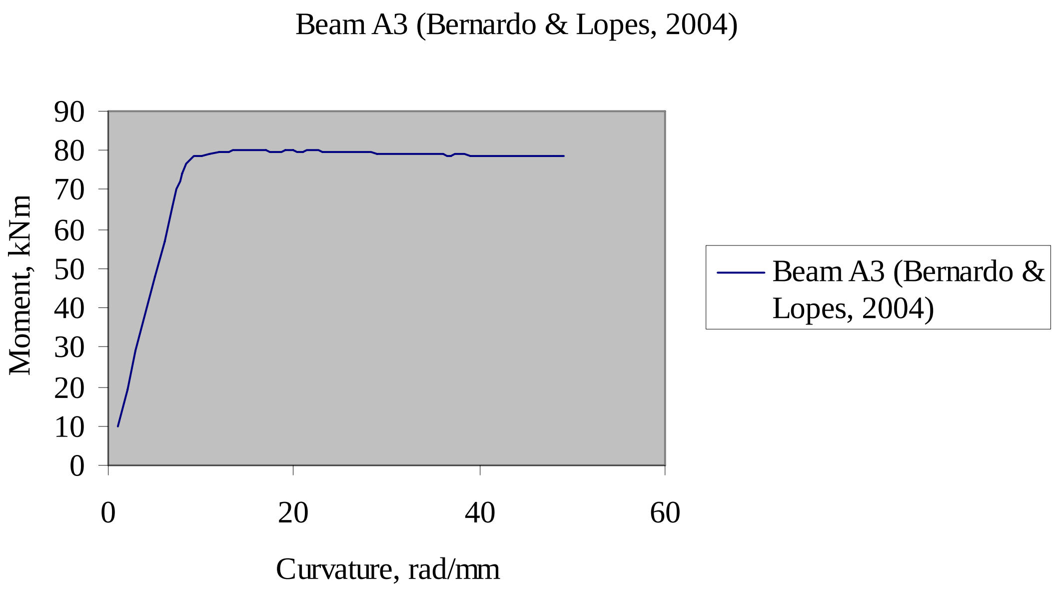
MOMENT CURVATURE CURVES FOR BEAMS TESTED BY BERNARDO & LOPES (2004)



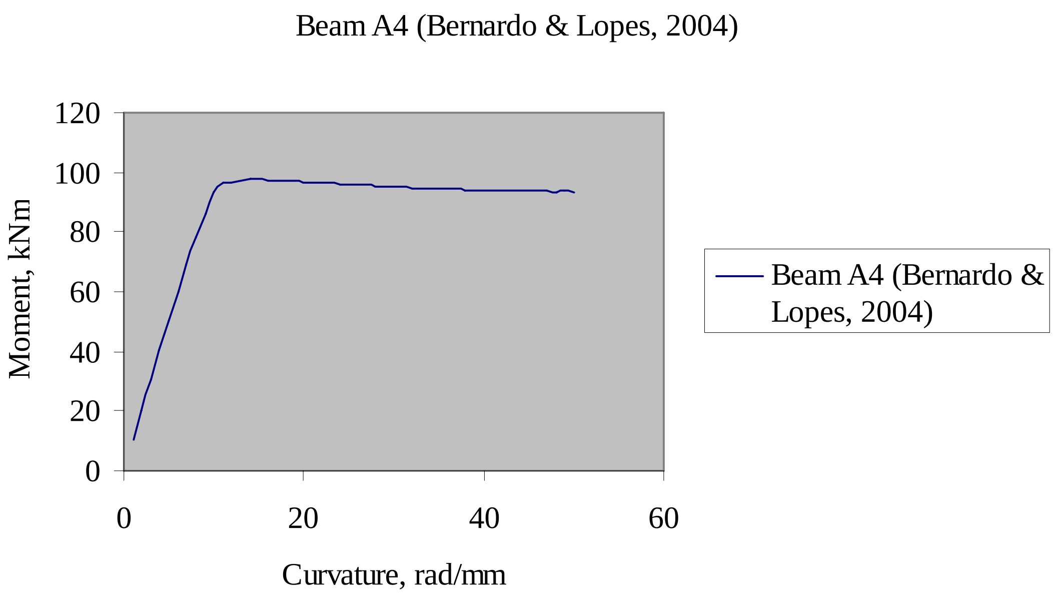
C.26 Moment curvature curve for Beam A1 tested by Bernardo & Lopes (2004)



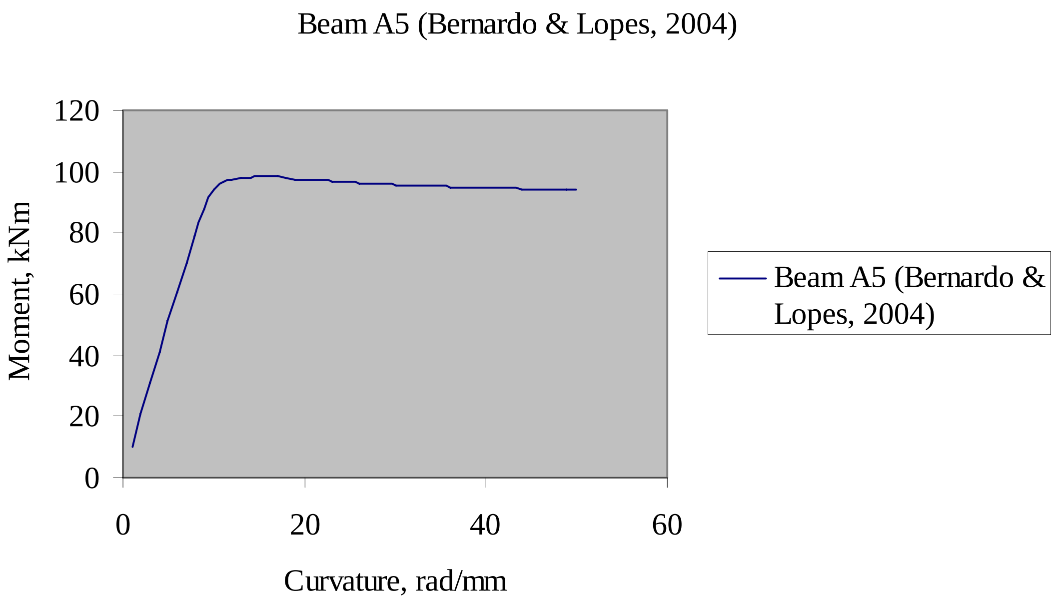
C.27 Moment curvature curve for Beam A2 tested by Bernardo & Lopes (2004)



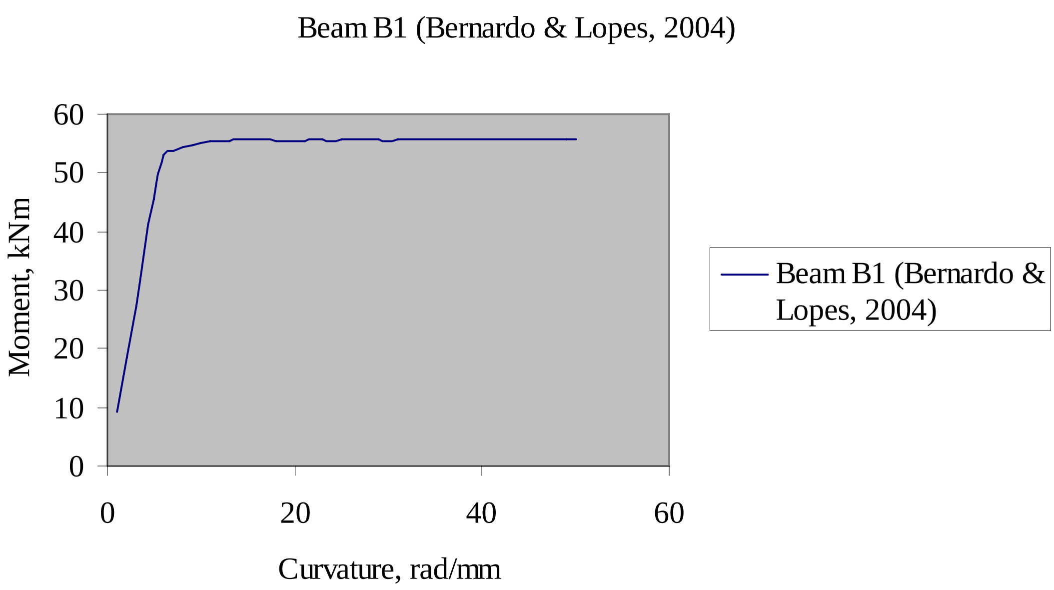
C.28 Moment curvature curve for Beam A3 tested by Bernardo & Lopes (2004)



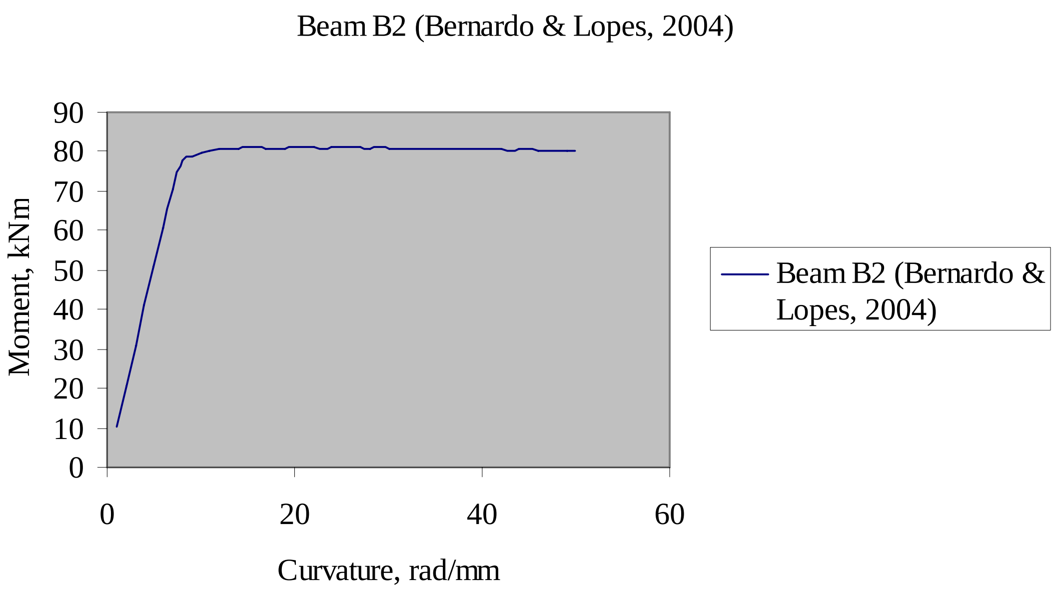
C.29 Moment curvature curve for Beam A4 tested by Bernardo & Lopes (2004)



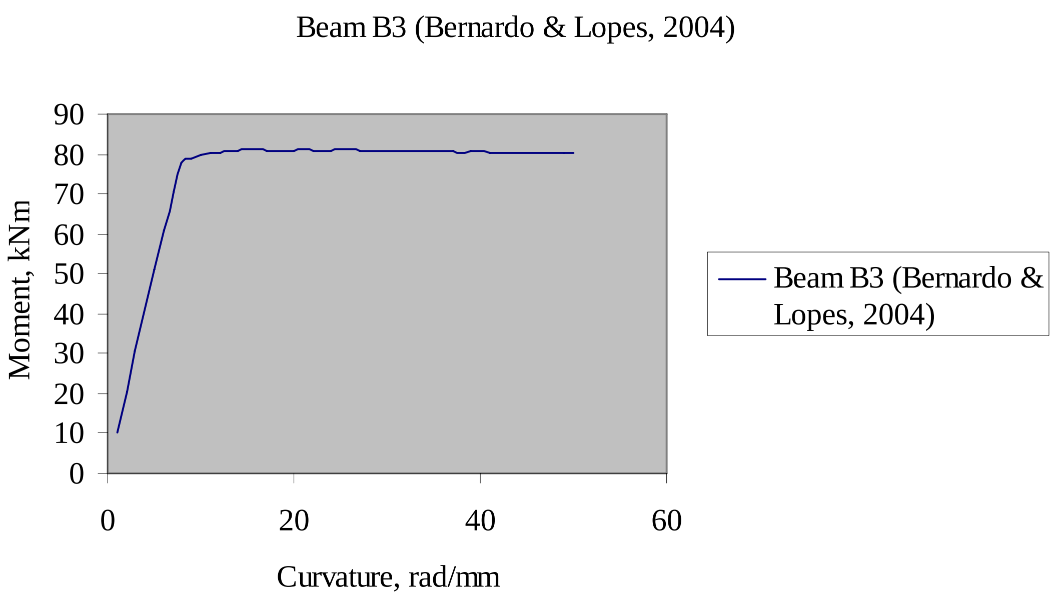
C.30 Moment curvature curve for Beam A5 tested by Bernardo & Lopes (2004)



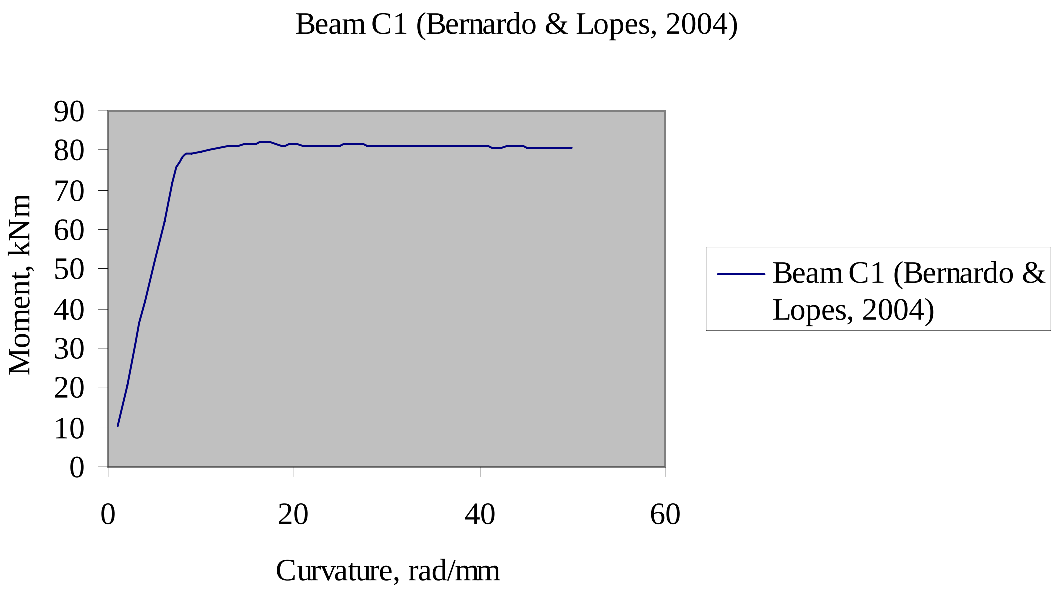
C.31 Moment curvature curve for Beam B1 tested by Bernardo & Lopes (2004)



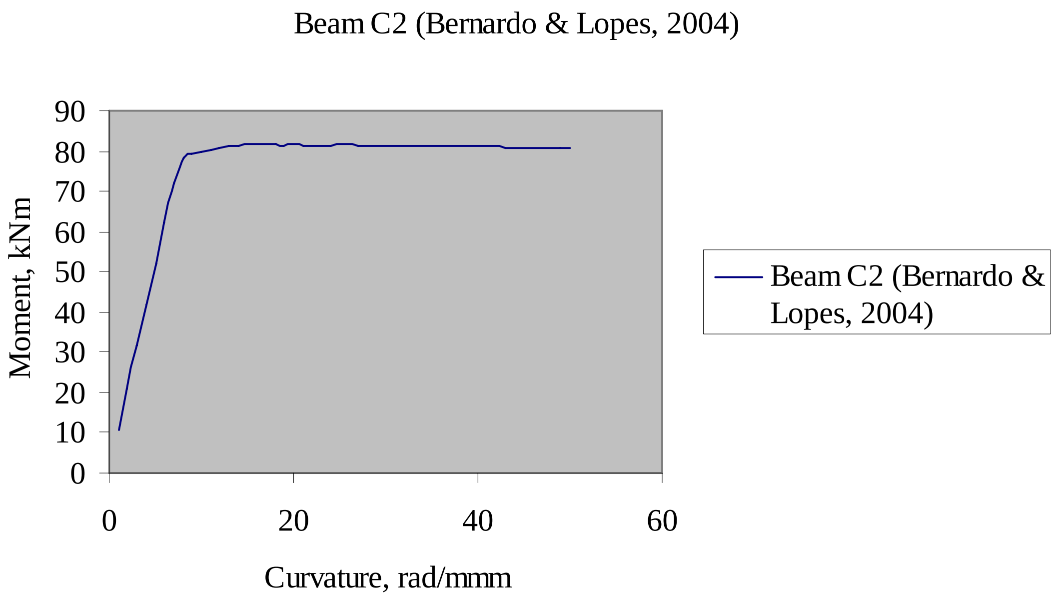
C.32 Moment curvature curve for Beam B2 tested by Bernardo & Lopes (2004)



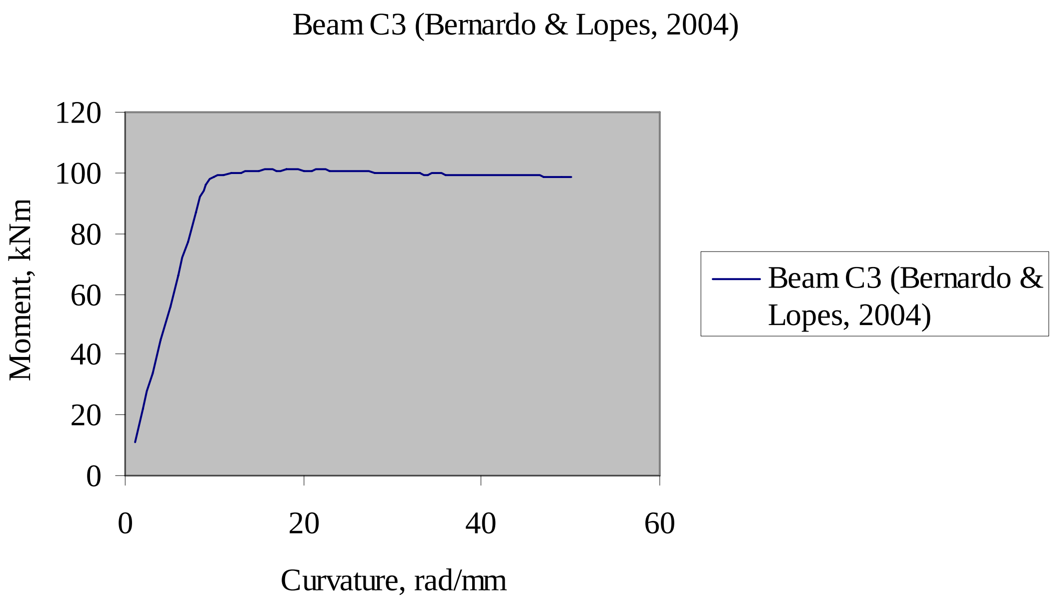
C.33 Moment curvature curve for Beam B3 tested by Bernardo & Lopes (2004)



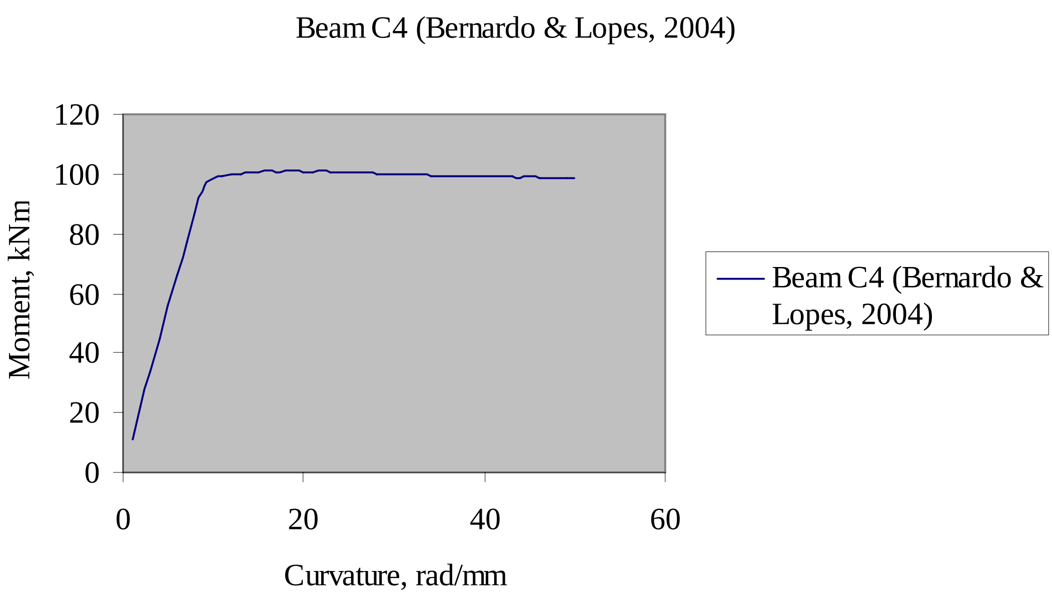
C.34 Moment curvature curve for BeamC1 tested by Bernardo & Lopes (2004)



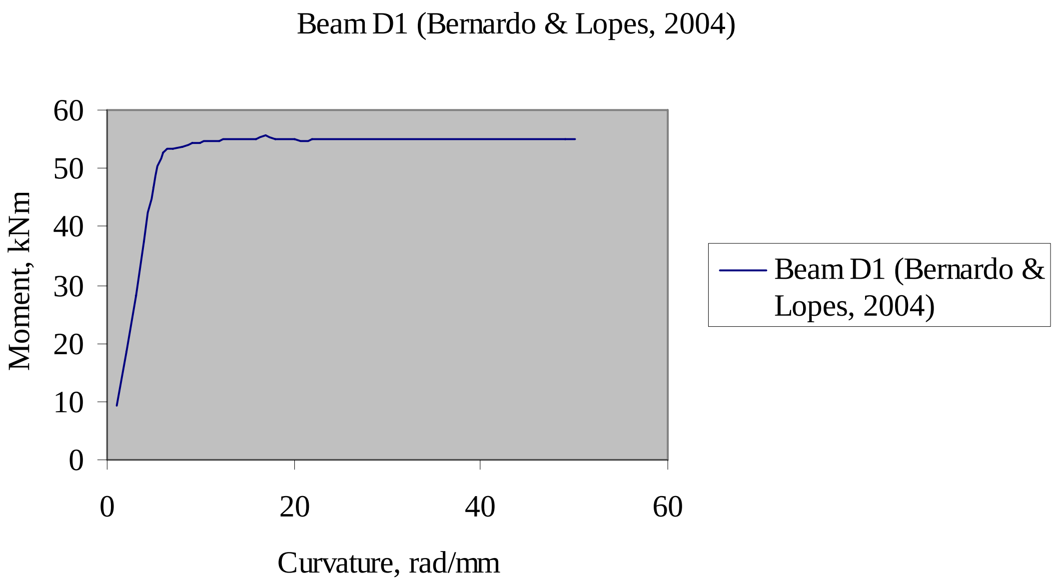
C.35 Moment curvature curve for BeamC2 tested by Bernardo & Lopes (2004)



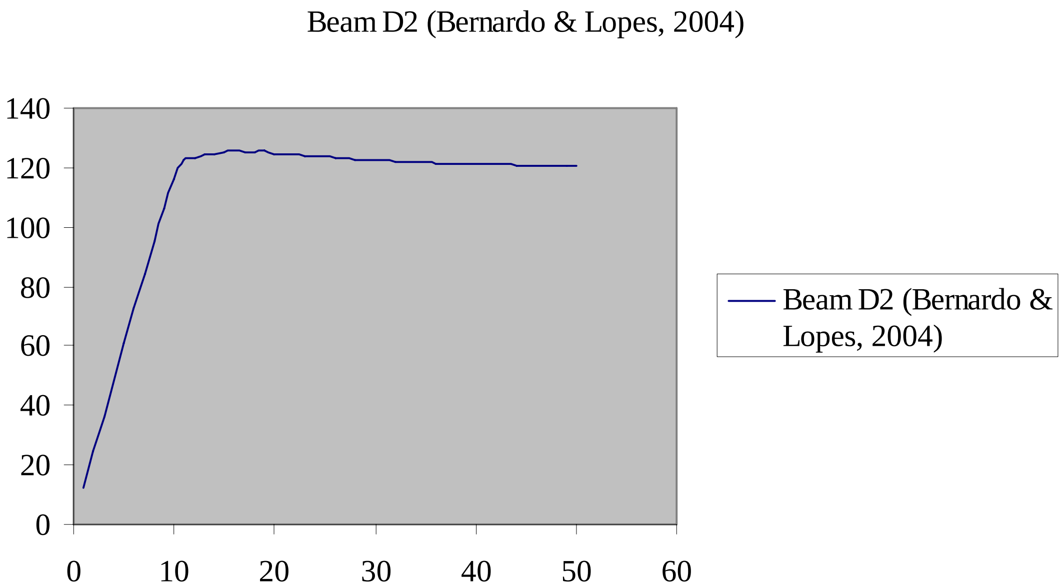
C.36 Moment curvature curve for BeamC3 tested by Bernardo & Lopes (2004)



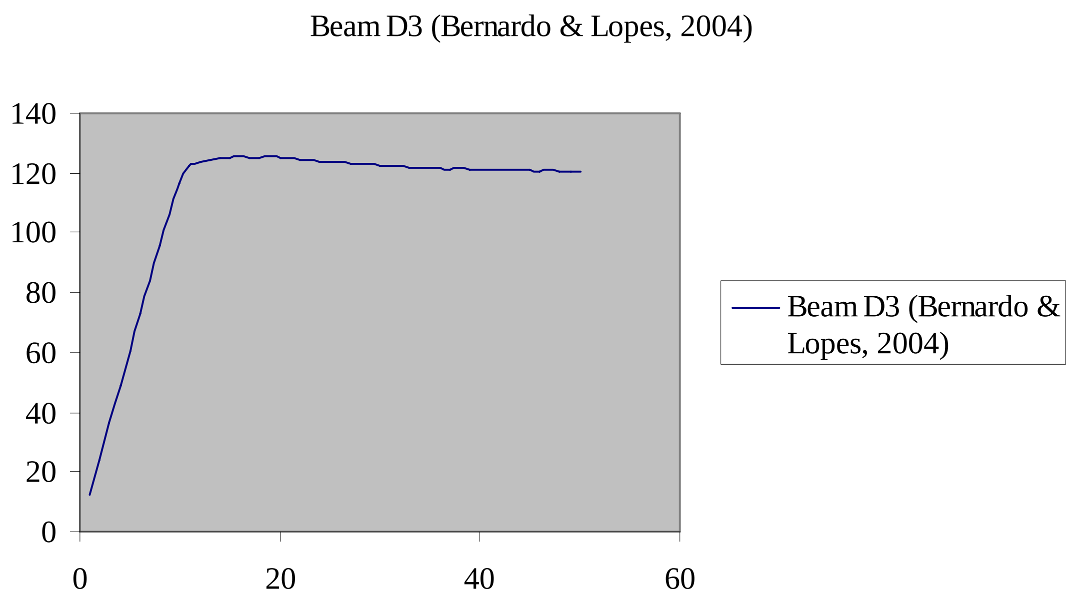
C.37 Moment curvature curve for BeamC4 tested by Bernardo & Lopes (2004)



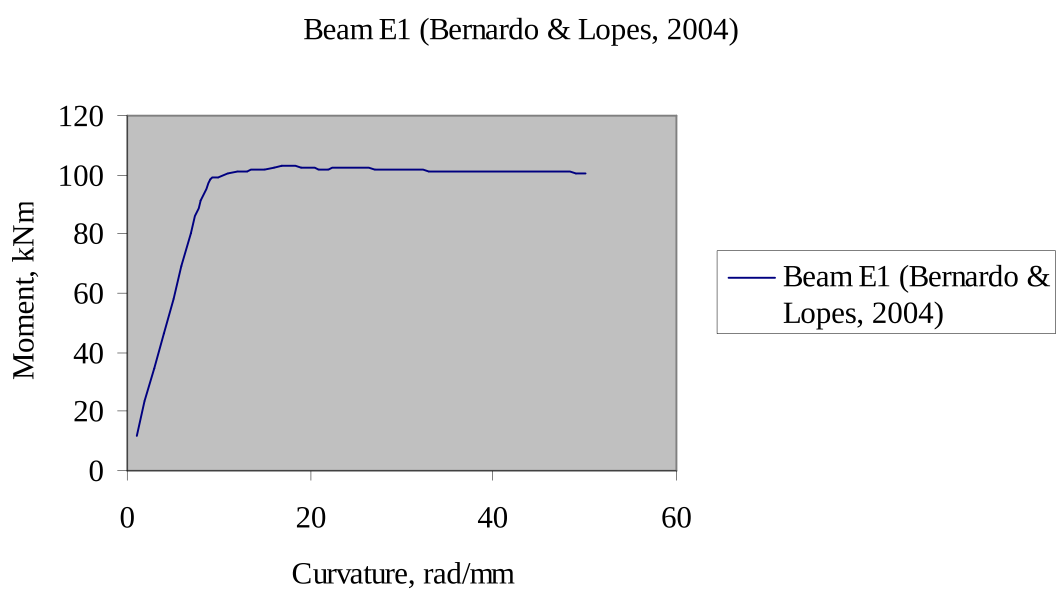
C.38 Moment curvature curve for Beam D1 tested by Bernardo & Lopes (2004)



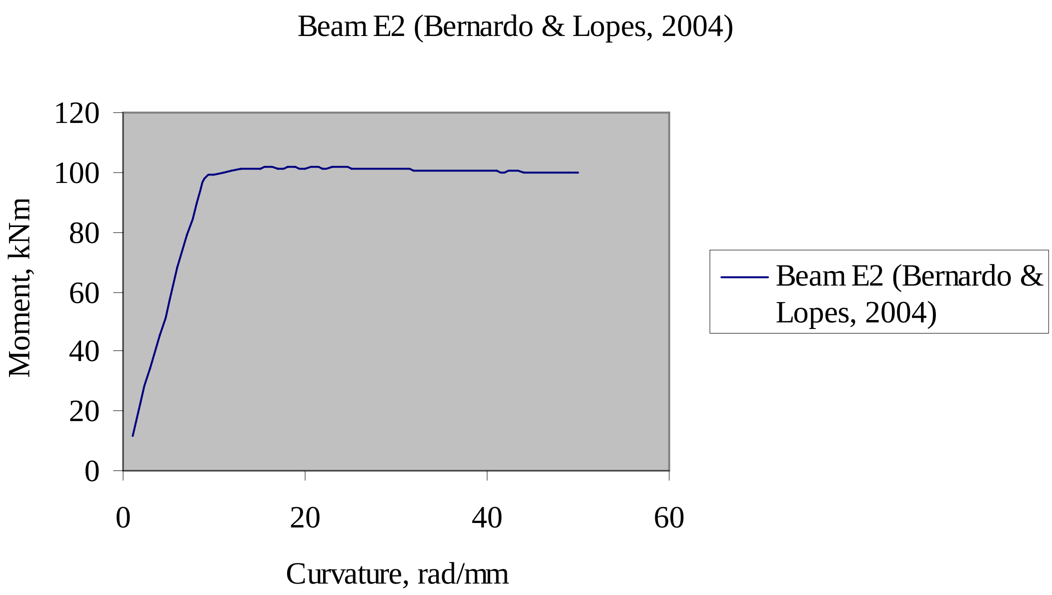
C.39 Moment curvature curve for Beam D2 tested by Bernardo & Lopes (2004)



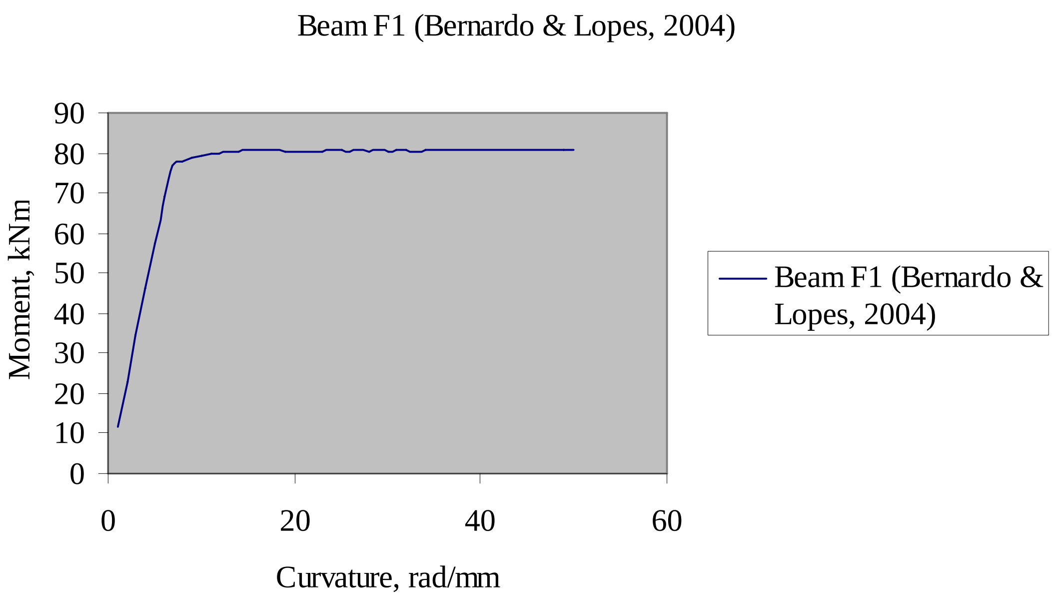
C.40 Moment curvature curve for Beam D2 tested by Bernardo & Lopes (2004)



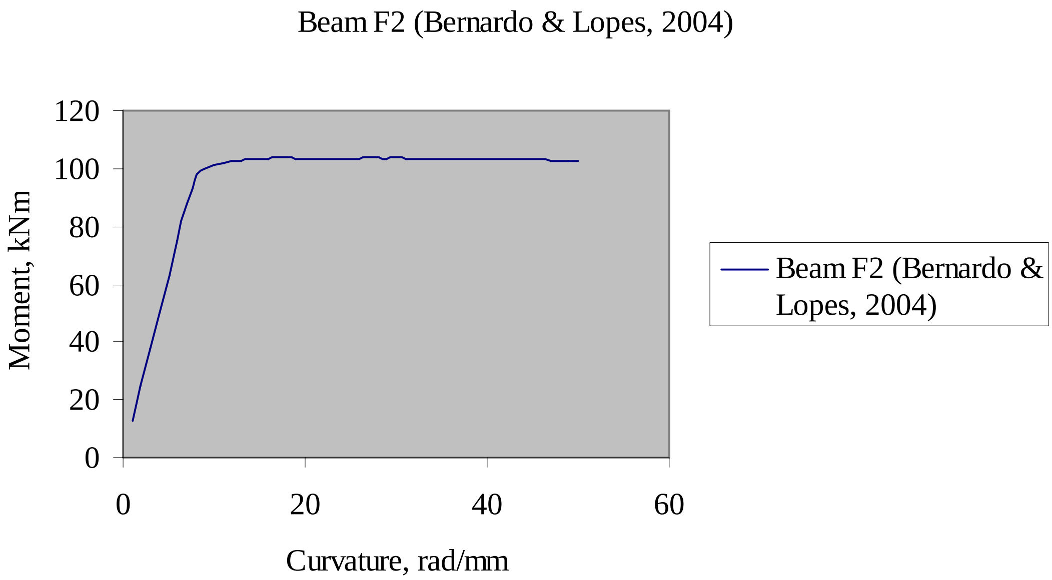
C.41 Moment curvature curve for Beam E1 tested by Bernardo & Lopes (2004)



C.42 Moment curvature curve for Beam E2 tested by Bernardo & Lopes (2004)

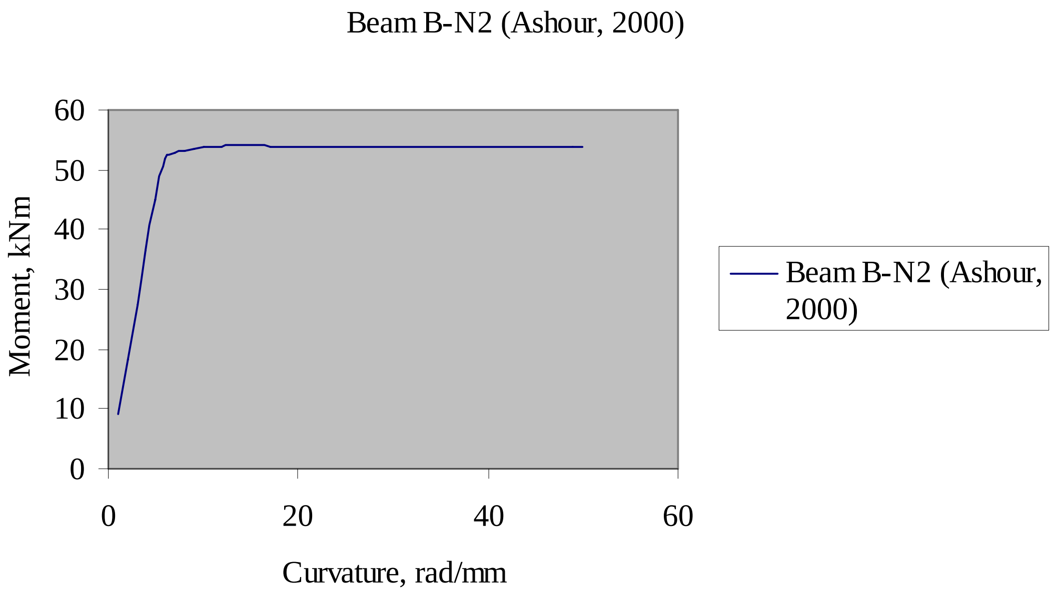


C.43 Moment curvature curve for Beam F1 tested by Bernardo & Lopes (2004)

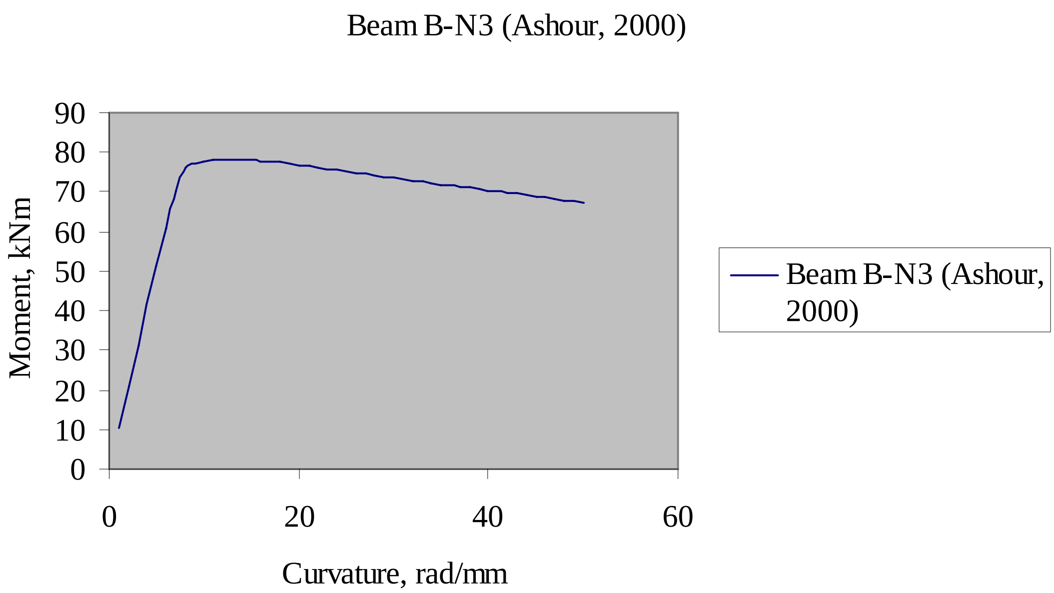


C.44 Moment curvature curve for Beam F2 tested by Bernardo & Lopes (2004)

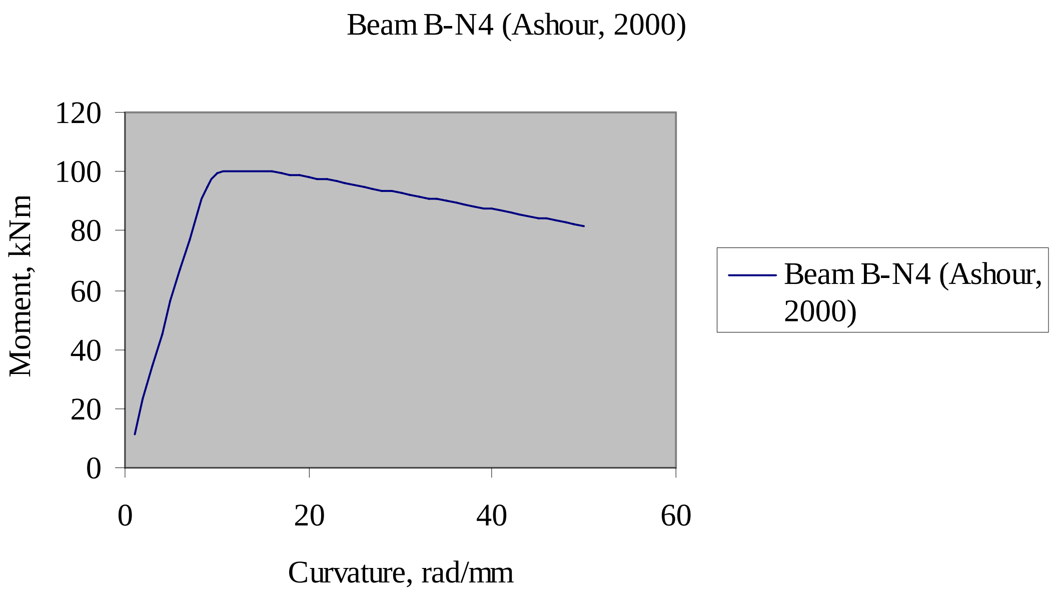
MOMENT CURVATURE CURVES FOR BEAMS TESTED BY ASHOUR (2000)



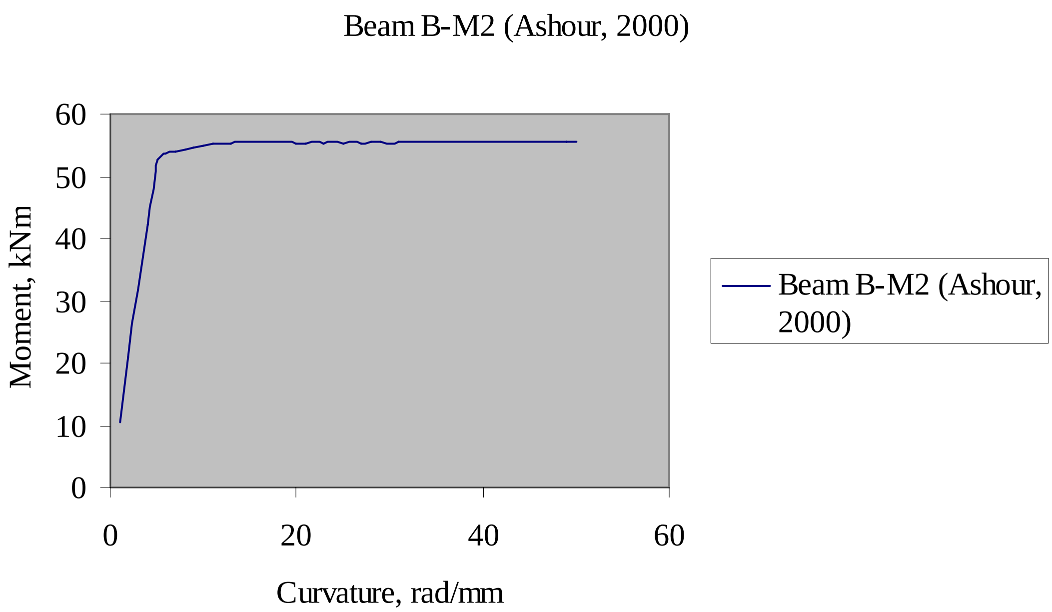
C.45 Moment curvature curve for Beam B-N2 tested by Ashour (2000)



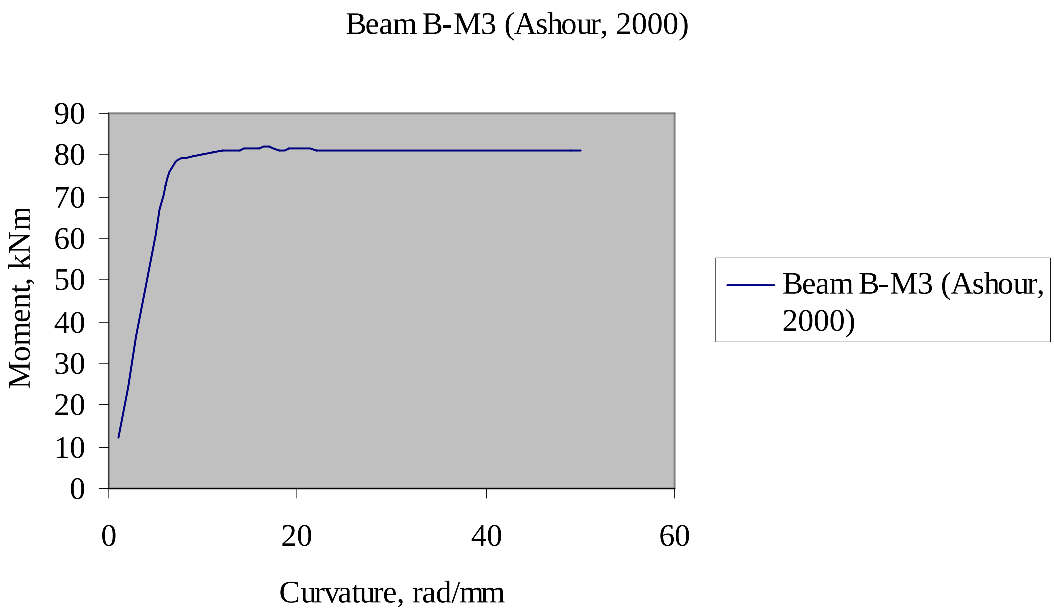
C.46 Moment curvature curve for Beam B-N3 tested by Ashour (2000)



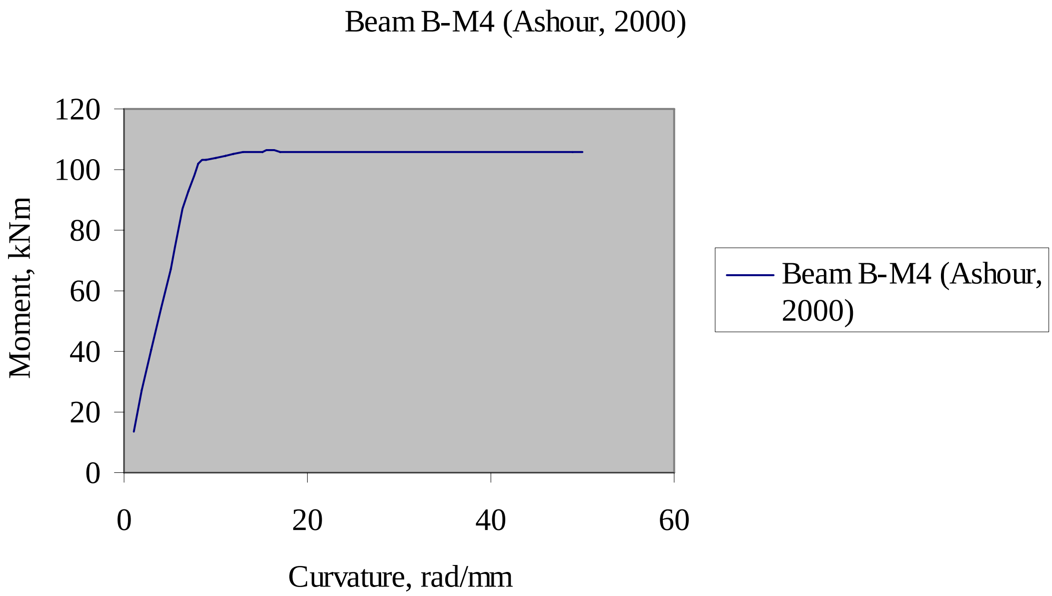
C.47 Moment curvature curve for Beam B-N3 tested by Ashour (2000)



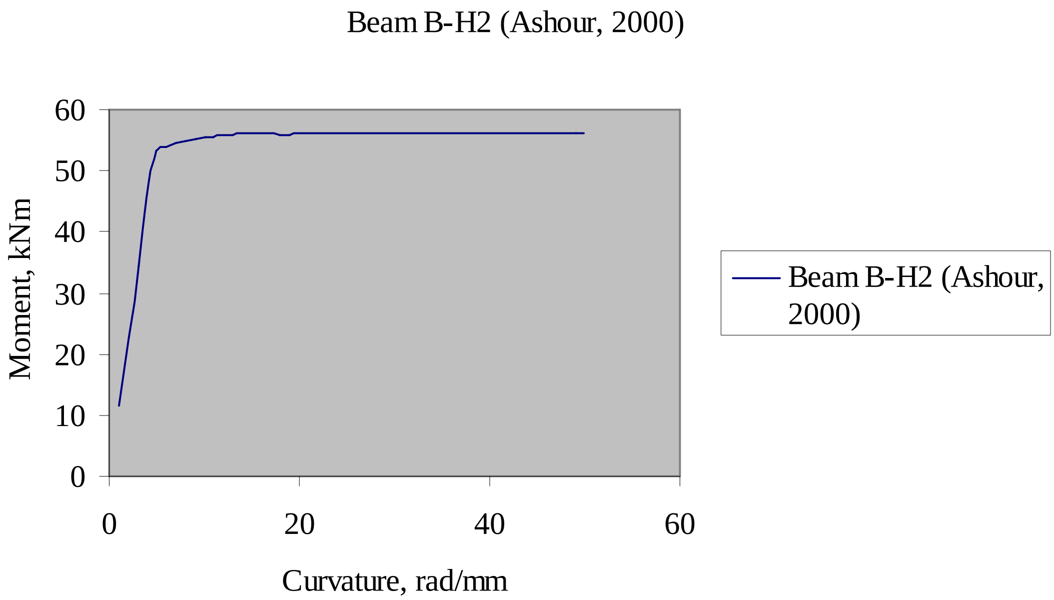
C.48 Moment curvature curve for Beam B-M2 tested by Ashour (2000)



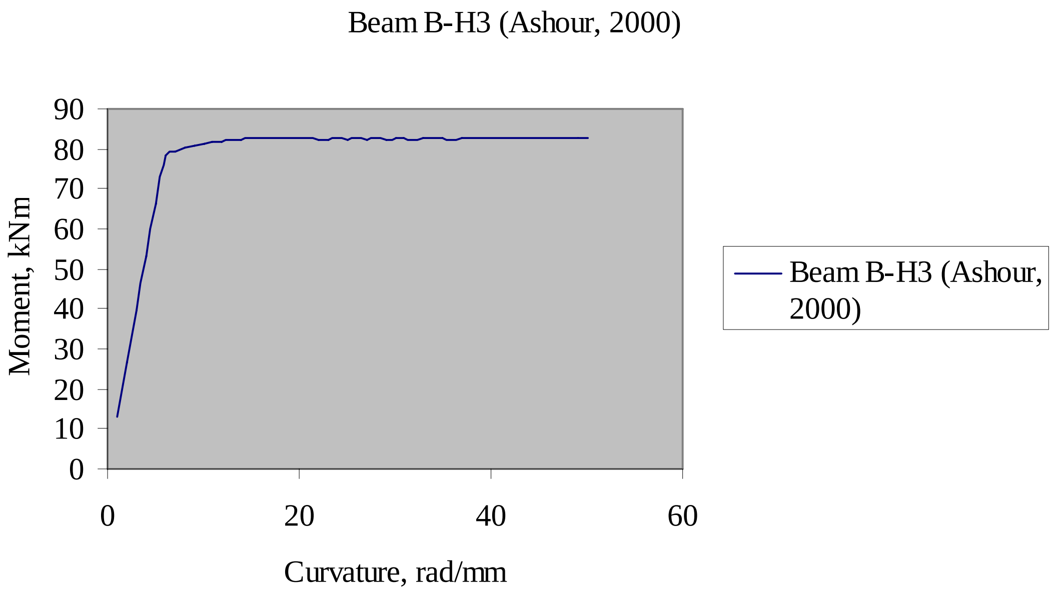
C.49 Moment curvature curve for Beam B-M3 tested by Ashour (2000)



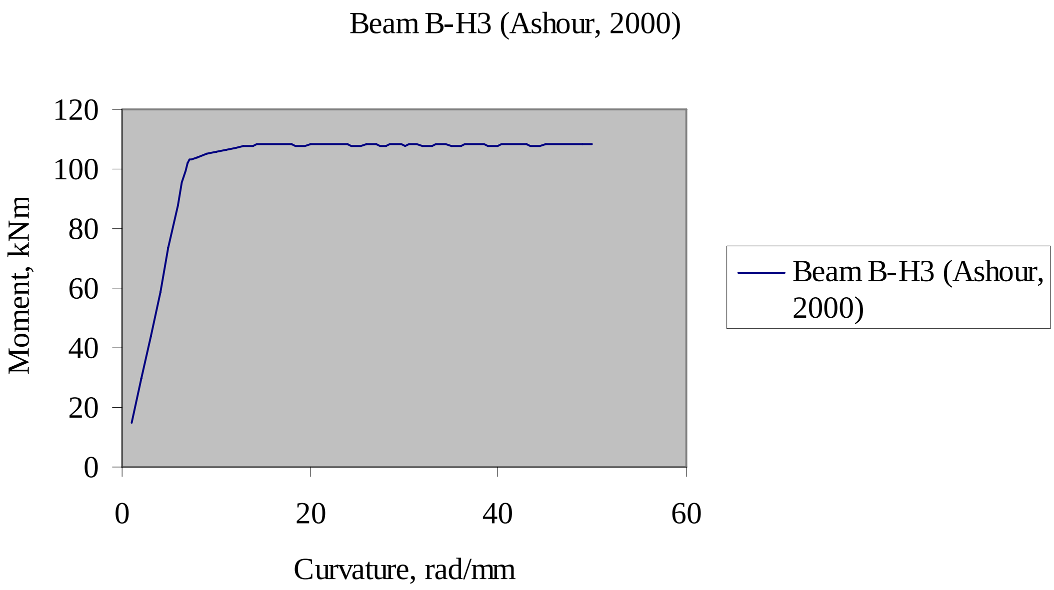
C.50 Moment curvature curve for Beam B-M4 tested by Ashour (2000)



C.51 Moment curvature curve for Beam B-H2 tested by Ashour (2000)



C.52 Moment curvature curve for Beam B-H2 tested by Ashour (2000)



C.53 Moment curvature curve for Beam B-H2 tested by Ashour (2000)

COMPARISON OF ULTIMATE MOMENT CAPACITY OBTAINED USING THE CODE PROVISIONS AND THAT OBTAINED USING PROGRAM FRMPHI

				AS-3600-2001	ACI318-2005	CAN-A23.3-M94	NZS3101-1995	CEB-FIP-1990	AFREM-95	EUROCODE-2	MENDIS & PENDYALA (1997)	IBRAHIM & McGREGOR (1997)	ATTARD & STEWART (1998)	AZIZINAMINI ET AL (1994)	BAE & BAYRAK (2003)	**(FRMPHI)**	
Reference	***Beam no***	***f'_c***	***M_e***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p=T*z***	***M_p***	***M_e/M_p***
		MPa	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	
Pam et al (2000)	1	37.4	77.6	57.19	57.19	56.89	57.19	56.44	57.19	57.19	N/A*	56.94	57.43	57.19	57.19	56.80	1.37
	2	36.8	103.5	82.43	82.43	81.75	82.43	80.75	82.43	82.43	N/A*	81.87	83.00	82.43	82.43	87.80	1.18
	3	36.4	126.5	121.54	121.54	119.76	121.54	117.11	121.54	121.54	N/A*	120.07	123.08	121.54	121.54	128.40	0.99
	4	42.3	129	117.59	117.59	116.03	117.59	113.66	117.59	117.59	N/A*	116.31	118.45	117.59	117.59	121.90	1.06
	5	46.4	142.8	141.49	141.49	139.13	141.04	135.47	141.49	141.49	N/A*	141.49	142.45	141.49	141.49	124.90	1.14
	7	58.6	164.6	152.42	152.42	149.80	152.20	145.45	152.42	152.42	152.32	152.42	152.73	152.42	152.42	150.70	1.09
	8	57.1	166.2	168.93	168.93	165.54	168.41	159.98	168.93	168.93	168.92	168.93	169.41	168.93	168.93	167.70	0.99
	9	58.6	171.6	196.40	196.40	191.28	190.48	182.80	196.40	196.40	196.19	196.40	196.99	196.40	196.40	220.50	0.78
	14	95.5	138	137.65	137.65	135.64	136.33	131.52	137.65	137.65	136.39	137.65	137.30	137.65	137.65	133.40	1.03
	15	98	200.7	199.61	199.61	195.06	196.71	185.58	199.61	199.61	196.62	199.61	198.77	199.61	199.61	194.80	1.03
	16	102.5	181.7	200.56	200.56	195.97	197.79	186.11	200.56	200.56	197.35	200.56	199.66	200.56	200.56	195.00	0.93
	17	87	172	198.70	198.70	193.97	195.23	184.79	198.70	198.70	196.18	198.70	198.01	198.70	198.70	194.20	0.89
Sarkar et al (1997)	HSC1-1	107	38.94	34.12	34.12	33.91	34.00	33.43	34.12	34.12	33.96	33.95	34.08	33.73	33.93	33.80	1.15
	HSC1-2	97	35.64	34.03	34.03	33.81	33.89	33.38	34.03	34.03	33.89	33.86	33.99	33.70	33.88	34.40	1.04
	HSC1-3	85	37.62	31.93	31.93	31.75	31.79	31.39	31.93	31.93	31.84	31.78	31.90	31.76	31.85	34.10	1.10
	HSC2-1	105	46.33	43.46	43.46	43.07	43.23	42.24	43.46	43.46	43.18	43.15	43.38	42.76	43.13	42.70	1.09
	HSC2-2	100	46.86	43.37	43.37	42.99	43.17	42.19	43.37	43.37	43.12	43.07	43.30	42.74	43.08	42.50	1.10
	HSC2-3	77	43.56	40.42	40.42	40.10	40.15	39.52	40.42	40.42	40.29	40.16	40.39	40.27	40.35	42.00	1.04
	HSC2-4	90	48.84	40.71	40.71	40.39	40.48	39.74	40.71	40.71	40.53	40.45	40.66	40.34	40.53	42.40	1.15
	HSC3-1	107	67.32	60.27	60.27	59.52	59.84	57.88	60.27	60.27	59.72	59.67	60.11	58.94	59.59	60.60	1.11
	HSC3-2	85	66	59.44	59.44	58.73	58.95	57.37	59.44	59.44	59.08	58.87	59.34	58.80	59.13	59.90	1.10
	HSC3-3	78	64.68	55.81	55.81	55.19	55.29	54.05	55.81	55.81	55.55	55.30	55.75	55.48	55.65	56.40	1.15
	HSC4-1	101	92.42	109.04	109.04	106.10	107.23	99.85	109.04	109.04	107.02	106.67	108.47	103.95	106.72	110.80	0.83
	HSC4-2	87	89.6	106.86	106.86	104.00	104.76	98.46	106.86	106.86	105.33	104.55	106.44	104.00	105.49	108.90	0.82
	HSC4-3	82	111.63	100.52	100.52	98.03	99.95	93.32	100.52	100.52	99.35	98.50	100.22	98.68	99.64	102.80	1.09
Bernardo & Lopes(2004)	A1	62.9	50.275	53.09	53.09	52.54	52.87	51.62	53.09	53.09	53.01	52.64	53.11	53.09	53.09	55.20	0.91
	A2	64.9	64.94	76.49	76.49	75.31	76.08	73.30	76.49	76.49	76.27	75.53	76.52	76.49	76.49	80.30	0.81
	A3	64.1	64.55	75.61	75.61	74.34	75.21	72.17	75.61	75.61	75.40	74.57	75.65	75.61	75.61	80.30	0.80
	A4	63.2	78.48	91.60	91.60	89.52	90.77	85.99	91.60	91.60	91.29	89.90	91.70	91.60	91.60	98.10	0.80
	A5	65.1	72.545	92.09	92.09	90.00	90.02	86.42	92.09	92.09	91.69	90.38	92.13	92.09	92.09	98.50	0.74
	B1	79.2	47.995	53.60	53.60	53.01	53.14	51.93	53.60	53.60	53.35	53.12	53.54	53.25	53.44	55.80	0.86
	B2	78.9	63.665	79.55	79.55	78.28	78.57	75.93	79.55	79.55	79.01	78.52	79.42	78.82	79.20	81.20	0.78
	B3	78.5	65.95	79.24	79.24	77.93	78.16	75.52	79.24	79.24	78.69	78.18	79.11	78.53	78.90	81.30	0.81
	C1	82.9	62.785	79.86	79.86	78.57	78.86	76.13	79.86	79.86	79.24	78.82	79.70	78.85	79.38	81.90	0.77
	C2	83.9	67.96	79.77	79.77	78.44	78.75	75.92	79.77	79.77	79.11	78.70	79.59	78.66	79.23	81.80	0.83
	C3	83.6	72.445	98.46	98.46	96.38	96.86	92.42	98.46	98.46	97.44	96.77	98.19	96.75	97.64	101.20	0.72

COMPARISON OF ULTIMATE MOMENT CAPACITY OBTAINED USING THE CODE PROVISIONS AND THAT OBTAINED USING PROGRAM FRMPHI

				AS-3600-2001	ACI318-2005	CAN-A23.3-M94	NZS3101-1995	CEB-FIP-1990	AFREM-95	EUROCODE-2	MENDIS & PENDYALA (1997)	IBRAHIM & McGREGOR (1997)	ATTARD & STEWART (1998)	AZIZINAMINI ET AL (1994)	BAE & BAYRAK (2003)	**(FRMPHI)**	
Reference	***Beam no***	***f'_c***	***M_e***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***$M_p=T*z$***	***M_p***	***M_e/M_p***
		MPa	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	***kNm***	
	C4	83.4	77.94	100.45	100.45	98.31	98.80	94.26	100.45	100.45	99.41	98.72	100.17	98.72	99.61	101.20	0.77
	D1	88	44.22	54.12	54.12	53.57	53.72	52.50	54.12	54.12	53.82	53.68	54.04	53.54	53.84	55.10	0.80
	D2	85.8	93.34	120.39	120.39	116.85	117.74	110.03	120.39	120.39	118.55	117.53	119.89	117.07	118.80	125.70	0.74
	D3	86	97.61	120.43	120.43	116.90	117.80	110.06	120.43	120.43	118.59	117.57	119.94	117.08	118.82	125.70	0.78
	E1	94.6	78.995	99.69	99.69	97.52	98.24	93.11	99.69	99.69	98.34	97.94	99.31	96.67	98.27	102.80	0.77
	E2	90.2	78.895	98.87	98.87	96.68	97.33	92.32	98.87	98.87	97.62	97.10	98.53	96.31	97.65	101.80	0.78
	F1	100.3	64.23	77.55	77.55	76.37	76.82	73.87	77.55	77.55	76.75	76.60	77.33	75.55	76.64	80.70	0.80
	F2	105.2	79.07	98.91	98.91	96.80	97.68	92.18	98.91	98.91	97.38	97.22	98.49	95.05	97.08	103.90	0.76
S.A.Ashour (2000)	B-N2	48.61	58.17	53.60	53.60	53.18	53.60	52.53	53.60	53.60	N/A*	53.60	53.74	53.60	53.60	54.20	1.07
	B-N3	48.61	80.6	77.05	77.05	76.12	77.05	74.66	77.05	77.05	N/A*	77.05	77.36	77.05	77.05	78.10	1.03
	B-N4	48.61	99.55	98.39	98.39	96.74	98.39	94.14	98.39	98.39	N/A*	98.39	98.94	98.39	98.39	100.20	0.99
	B-M2	78.5	57.95	55.27	55.27	54.84	55.27	54.03	55.27	55.27	55.09	55.27	55.23	55.03	55.16	55.60	1.04
	B-M3	78.5	79.91	80.82	80.82	79.83	80.82	78.01	80.82	80.82	80.40	80.82	80.72	80.28	80.56	55.60	1.44
	B-M4	78.5	103.77	105.09	105.09	103.34	105.09	100.10	105.09	105.09	104.36	105.09	104.92	104.13	104.64	55.60	1.87
	B-H2	102.4	56.8	55.91	55.91	55.45	55.91	54.46	55.91	55.91	55.59	55.91	55.82	55.08	55.53	56.30	1.01
	B-H3	102.4	82.76	82.25	82.25	81.21	82.25	78.99	82.25	82.25	81.52	82.25	82.04	80.38	81.40	82.70	1.00
	B-H4	102.4	108.1	107.64	107.64	105.80	107.64	101.84	107.64	107.64	106.35	107.64	107.28	104.31	106.14	108.60	1.00

Table B.1 Comparison of ultimate moment capacities

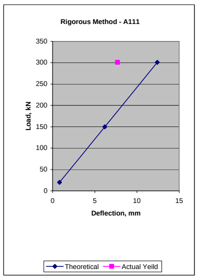
N/A*: Stress block parameters proposed by Mendis and Pendyala (1997) are applicable for concrete strengths greater than 50MPa

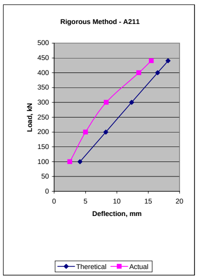
APPENDIX C

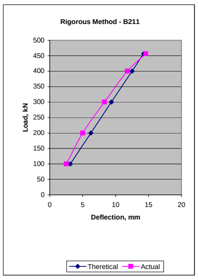
METHOD 1 - AS3600 - SIMPLIFIED

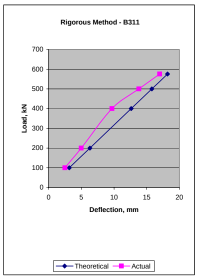
E_c =	$0.043\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$0.045bd^3$
f'_{cf} =	NA

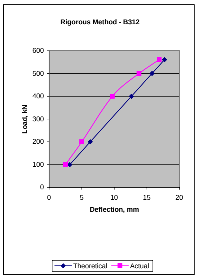
			Dimensions			Reinforcement Ratio					At Yield						Deem-to-Comply						Rigorous Method
SOURCE	BEAM	f'_c, MPa	b, mm	D, mm	d, mm	Tensile Steel, %	Area of Steel, A_{st}, mm²	Compression Steel, %	Area of Steel, A_{sc}, mm²	L_{ef}, m	Load, kN	Deflection, mm	Equivalent Point Loads, kN	E_c	I_{eff}, 10^6mm	Load, kN/m	Ratio	≤	k_1	k_{cs}	F_{def}	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	250	400	362	1.25	1250	0.30	300	3.4	300.78	7.70	150.39	33076	533.7	124.9	9.4	≤	0.0450	1.71	338.7	9.71	12.4
		42.8	250	400	362	1.25	1250	0.30	300	3.4	20.00		10.00	33076	533.7	8.3	9.4	≤	0.0450	1.71	22.5	17.19	0.8
		42.8	250	400	362	1.25	1250	0.30	300	3.4	150.00		75.00	33076	533.7	62.3	9.4	≤	0.0450	1.71	168.9	8.78	6.2
		42.8	250	400	362	1.25	1250	0.30	300	3.4	300.78		150.39	33076	533.7	124.9	9.4	≤	0.0450	1.71	338.7	6.96	12.4
	A211	42.8	250	400	362	2.2	2200	0.30	300	3.4	440.48	15.50	220.24	33076	533.7	182.9	9.4	≤	0.0450	1.84	518.8	6.04	18.2
		42.8	250	400	362	2.2	2200	0.30	300	3.4	100.00	2.50	50.00	33076	533.7	41.5	9.4	≤	0.0450	1.84	117.8	9.90	4.1
		42.8	250	400	362	2.2	2200	0.30	300	3.4	200.00	5.00	100.00	33076	533.7	83.0	9.4	≤	0.0450	1.84	235.5	7.86	8.2
		42.8	250	400	362	2.2	2200	0.30	300	3.4	300.00	8.30	150.00	33076	533.7	124.6	9.4	≤	0.0450	1.84	353.3	6.86	12.4
		42.8	250	400	362	2.2	2200	0.30	300	3.4	400.00	13.50	200.00	33076	533.7	166.1	9.4	≤	0.0450	1.84	471.1	6.24	16.5
		42.8	250	400	362	2.2	2200	0.30	300	3.4	440.48	15.50	220.24	33076	533.7	182.9	9.4	≤	0.0450	1.84	518.8	6.04	18.2
	B211	74.6	250	400	362	2.2	2200	0.30	300	3.4	456.70	14.60	228.35	43667	533.7	189.6	9.4	≤	0.0450	1.84	537.9	6.55	14.3
		74.6	250	400	362	2.2	2200	0.30	300	3.4	100.00	2.50	50.00	43667	533.7	41.5	9.4	≤	0.0450	1.84	117.8	10.86	3.1
		74.6	250	400	362	2.2	2200	0.30	300	3.4	200.00	5.00	100.00	43667	533.7	83.0	9.4	≤	0.0450	1.84	235.5	8.62	6.2
		74.6	250	400	362	2.2	2200	0.30	300	3.4	300.00	8.30	150.00	43667	533.7	124.6	9.4	≤	0.0450	1.84	353.3	7.53	9.4
		74.6	250	400	362	2.2	2200	0.30	300	3.4	400.00	11.80	200.00	43667	533.7	166.1	9.4	≤	0.0450	1.84	471.1	6.84	12.5
		74.6	250	400	362	2.2	2200	0.30	300	3.4	456.70	14.60	228.35	43667	533.7	189.6	9.4	≤	0.0450	1.84	537.9	6.55	14.3
	B311	72.8	250	400	362	3.46	3460	0.31	310	3.4	575.68	17.00	287.84	43137	533.7	239.0	9.4	≤	0.0450	1.89	691.4	6.00	18.2
		72.8	250	400	362	3.46	3460	0.31	310	3.4	100.00	2.50	50.00	43137	533.7	41.5	9.4	≤	0.0450	1.89	120.1	10.75	3.2
		72.8	250	400	362	3.46	3460	0.31	310	3.4	200.00	5.00	100.00	43137	533.7	83.0	9.4	≤	0.0450	1.89	240.2	8.53	6.3
		72.8	250	400	362	3.46	3460	0.31	310	3.4	400.00	9.70	200.00	43137	533.7	166.1	9.4	≤	0.0450	1.89	480.4	6.77	12.6
		72.8	250	400	362	3.46	3460	0.31	310	3.4	500.00	13.80	250.00	43137	533.7	207.6	9.4	≤	0.0450	1.89	600.5	6.28	15.8
		72.8	250	400	362	3.46	3460	0.31	310	3.4	575.68	17.00	287.84	43137	533.7	239.0	9.4	≤	0.0450	1.89	691.4	6.00	18.2
	B312	72.8	250	400	362	3.46	3460	0.31	310	3.4	560.36	16.90	280.18	43137	533.7	232.7	9.4	≤	0.0450	1.89	673.0	6.05	17.7
		72.8	250	400	362	3.46	3460	0.31	310	3.4	100.00	2.50	50.00	43137	533.7	41.5	9.4	≤	0.0450	1.89	120.1	10.75	3.2
		72.8	250	400	362	3.46	3460	0.31	310	3.4	200.00	5.00	100.00	43137	533.7	83.0	9.4	≤	0.0450	1.89	240.2	8.53	6.3
		72.8	250	400	362	3.46	3460	0.31	310	3.4	400.00	9.70	200.00	43137	533.7	166.1	9.4	≤	0.0450	1.89	480.4	6.77	12.6
		72.8	250	400	362	3.46	3460	0.31	310	3.4	500.00	13.80	250.00	43137	533.7	207.6	9.4	≤	0.0450	1.89	600.5	6.28	15.8
		72.8	250	400	362	3.46	3460	0.31	310	3.4	560.36	16.90	280.18	43137	533.7	232.7	9.4	≤	0.0450	1.89	673.0	6.05	17.7
	B313	72.8	250	400	362	3.46	3460	0.31	310	3.4	580.26	16.10	290.13	43137	533.7	240.9	9.4	≤	0.0450	1.89	696.9	5.98	18.3
		72.8	250	400	362	3.46	3460	0.31	310	3.4	100.00	2.50	50.00	43137	533.7	41.5	9.4	≤	0.0450	1.89	120.1	10.75	3.2
		72.8	250	400	362	3.46	3460	0.31	310	3.4	200.00	5.00	100.00	43137	533.7	83.0	9.4	≤	0.0450	1.89	240.2	8.53	6.3
		72.8	250	400	362	3.46	3460	0.31	310	3.4	400.00	9.70	200.00	43137	533.7	166.1	9.4	≤	0.0450	1.89	480.4	6.77	12.6
		72.8	250	400	362	3.46	3460	0.31	310	3.4	500.00	13.80	250.00	43137	533.7	207.6	9.4	≤	0.0450	1.89	600.5	6.28	15.8
		72.8	250	400	362	3.46	3460	0.31	310	3.4	580.26	16.10	290.13	43137	533.7	240.9	9.4	≤	0.0450	1.89	696.9	5.98	18.3
	B321	77	250	400	362	3.46	3460	0.62	620	3.4	551.16	15.50	275.58	44364	533.7	228.9	9.4	≤	0.0450	1.78	637.4	6.22	16.9
		77	250	400	362	3.46	3460	0.62	620	3.4	100.00	1.70	50.00	44364	533.7	41.5	9.4	≤	0.0450	1.78	115.6	10.99	3.1
		77	250	400	362	3.46	3460	0.62	620	3.4	200.00	4.20	100.00	44364	533.7	83.0	9.4	≤	0.0450	1.78	231.3	8.72	6.1
		77	250	400	362	3.46	3460	0.62	620	3.4	400.00	10.00	200.00	44364	533.7	166.1	9.4	≤	0.0450	1.78	462.6	6.92	12.3
		77	250	400	362	3.46	3460	0.62	620	3.4	500.00	13.30	250.00	44364	533.7	207.6	9.4	≤	0.0450	1.78	578.2	6.42	15.4
		77	250	400	362	3.46	3460	0.62	620	3.4	551.16	15.50	275.58	44364	533.7	228.9	9.4	≤	0.0450	1.78	637.4	6.22	16.9
	B331	72.8	250	400	362	3.46	3460	0.94	940	3.4	590.38	16.50	295.19	43137	533.7	245.1	9.4	≤	0.0450	1.67	655.5	6.10	18.7
		72.8	250	400	362	3.46	3460	0.94	940	3.4	100.00	1.70	50.00	43137	533.7	41.5	9.4	≤	0.0450	1.67	111.0	11.03	3.2
		72.8	250	400	362	3.46	3460	0.94	940	3.4	200.00	4.20	100.00	43137	533.7	83.0	9.4	≤	0.0450	1.67	222.1	8.76	6.3
		72.8	250	400	362	3.46	3460	0.94	940	3.4	400.00	10.00	200.00	43137	533.7	166.1	9.4	≤	0.0450	1.67	444.1	6.95	12.6
		72.8	250	400	362	3.46	3460	0.94	940	3.4	500.00	13.30	250.00	43137	533.7	207.6	9.4	≤	0.0450	1.67	555.2	6.45	15.8
		72.8	250	400	362	3.46	3460	0.94	940	3.4	590.38	16.50	295.19	43137	533.7	245.1	9.4	≤	0.0450	1.67	655.5	6.10	18.7
	B411	77	250	400	362	4.73	4730	0.32	320	3.4	621.34	15.50	310.67	44364	533.7	258.0	9.4	≤	0.0450	1.92	753.0	5.88	19.1
		77	250	400	362	4.73	4730	0.32	320	3.4	100.00		50.00	44364	533.7	41.5	9.4	≤	0.0450	1.92	121.2	10.82	3.1
		77	250	400	362	4.73	4730	0.32	320	3.4	200.00		100.00	44364	533.7	83.0	9.4	≤	0.0450	1.92	242.4	8.58	6.1
		77	250	400	362	4.73	4730	0.32	320	3.4	300.00		150.00	44364	533.7	124.6	9.4	≤	0.0450	1.92	363.6	7.50	9.2
		77	250	400	362	4.73	4730	0.32	320	3.4	400.00		200.00	44364	533.7	166.1	9.4	≤	0.0450	1.92	484.8	6.81	12.3
		77	250	400	362	4.73	4730	0.32	320	3.4	500.00		250.00	44364	533.7	207.6	9.4	≤	0.0450	1.92	606.0	6.32	15.4
		77	250	400	362	4.73	4730	0.32	320	3.4	621.34		310.67	44364	533.7	258.0	9.4	≤	0.0450	1.92	753.0	5.88	19.1
	C211	85.6	250	400	362	2.71	2710	0.30	300	3.4	560.94	18.20	280.47	46776	533.7	232.9	9.4	≤	0.0450	1.87	667.8	6.23	16.3
		85.6	250	400	362	2.71	2710	0.3	300	3.4	100.00	2.50	50.00	46776	533.7	41.5	9.4	≤	0.0450	1.87	119.1	11.07	2.9
		85.6	250	400	362	2.71	2710	0.3	300	3.4	200.00	5.20	100.00	46776	533.7	83.0	9.4	≤	0.0450	1.87	238.1	8.79	5.8
		85.6	250	400	362	2.71	2710	0.3	300	3.4	400.00	11.70	200.00	46776	533.7	166.1	9.4	≤	0.0450	1.87	476.2	6.98	11.7
		85.6	250	400	362	2.71	2710	0.3	300	3.4	500.00	15.10	250.00	46776	533.7	207.6	9.4	≤	0.0450	1.87	595.3	6.48	14.6
		85.6	250	400	362	2.71	2710	0.3	300	3.4	560.94	18.20	280.47	46776	533.7	232.9	9.4	≤	0.0450	1.87	667.8	6.23	16.3
	C311	88.1	250	400	362	3.22	3220	0.31	310	3.4	605.46	18.00	302.73	47454	533.7	251.4	9.4	≤	0.0450	1.88	725.2	6.09	17.4
		88.1	250	400	362	3.22	3220	0.31	310	3.4	100.00	1.70	50.00	47454	533.7	41.5	9.4	≤	0.0450	1.88	119.8	11.10	2.9
		88.1	250	400	362	3.22	3220	0.31	310	3.4	200.00	5.30	100.00	47454	533.7	83.0	9.4	≤	0.0450	1.88	239.5	8.81	5.7
		88.1	250	400	362	3.22	3220	0.31	310	3.4	400.00	12.10	200.00	47454	533.7	166.1	9.4	≤	0.0450	1.88	479.1	7.00	11.5
		88.1	250	400	362	3.22	3220	0.31	310	3.4	500.00	15.40	250.00	47454	533.7	207.6	9.4	≤	0.0450	1.88	598.9	6.49	14.4
		88.1	250	400	362	3.22	3220	0.31	310	3.4	605.46	18.00	302.73	47454	533.7	251.4	9.4	≤	0.0450	1.88	725.2	6.09	17.4
	C411	85.6	250	400	362	4.26	4260	0.32	320	3.4	722.56	19.30	361.28	46776	533.7	300.0	9.4	≤	0.0450	1.91	873.0	5.70	21.1
		85.6	250	400	362	4.26	4260	0.32	320	3.4	100.00	1.70	50.00	46776	533.7	41.5	9.4	≤	0.0450	1.91	120.8	11.02	2.9
		85.6	250	400	362	4.26	4260	0.32	320	3.4	200.00	4.70	100.00	46776	533.7	83.0	9.4	≤	0.0450	1.91	241.6	8.75	5.8
		85.6	250	400	362	4.26	4260	0.32	320	3.4	400.00	10.00	200.00	46776	533.7	166.1	9.4	≤	0.0450	1.91	483.3	6.94	11.7
		85.6	250	400	362	4.26	4260	0.32	320	3.4	600.00	15.80	300.00	46776	533.7	249.1	9.4	≤	0.0450	1.91	724.9	6.06	17.5
		85.6	250	400	362	4.26	4260	0.32	320	3.4	722.56	19.30	361.28	46776	533.7	300.0	9.4	≤	0.0450	1.91	873.0	5.70	21.1
	C511	88.1	250	400	362	5.31	5310	0.33	330	3.4	811.82	20.80	405.91	47454	533.7	337.1	9.4	≤	0.0450	1.93	986.1	5.50	23.3
		88.1	250	400	362	5.31	5310	0.33	330	3.4	100.00	1.70	50.00	47454	533.7	41.5	9.4	≤	0.0450	1.93	121.5	11.05	2.9
		88.1	250	400	362	5.31	5310	0.33	330	3.4	200.00	3.30	100.00	47454	533.7	83.0	9.4	≤	0.0450	1.93	242.9	8.77	5.7
		88.1	250	400	362	5.31	5310	0.33	330	3.4	400.00	8.70	200.00	47454	533.7	166.1	9.4	≤	0.0450	1.93	485.9	6.96	11.5
		88.1	250	400	362	5.31	5310	0.33	330	3.4	600.00	14.20	300.00	47454	533.7	249.1	9.4	≤	0.0450	1.93	728.8	6.08	17.2
		88.1	250	400	362	5.31	5310	0.33	330	3.4	811.82	20.80	405.91	47454	533.7	337.1	9.4	≤	0.0450	1.93	986.1	5.50	23.3
	D211	114.5	250	400	362	3.22	3220	0.30	300	3.4	506.00	16.00	253.00	54099	533.7	210.1	9.4	≤	0.0450	1.89	606.8	6.75	12.8
		114.5	250	400	362	3.22	3220	0.30	300	3.4	100.00	1.60	50.00	54099	533.7	41.5	9.4	≤	0.0450	1.89	119.9	11.60	2.5
		114.5	250	400	362	3.22	3220	0.30	300	3.4	200.00	5.00	100.00	54099	533.7	83.0	9.4	≤	0.0450	1.89	239.9	9.20	5.0
		114.5	250	400	362	3.22	3220	0.30	300	3.4	300.00	8.60	150.00	54099	533.7	124.6	9.4	≤	0.0450	1.89	359.8	8.04	7.6
		114.5	250	400	362	3.22	3220	0.30	300	3.4	400.00	12.10	200.00	54099	533.7	166.1	9.4	≤	0.0450	1.89	479.7	7.30	10.1
		114.5	250	400	362	3.22	3220	0.30	300	3.4	506.00	16.00	253.00	54099	533.7	210.1	9.4	≤	0.0450	1.89	606.8	6.75	12.8
	E211	126.2	250	400	362	2.2	2200	0.30	300	3.4	506.00	15.90	253.00	56796	533.7	210.1	9.4	≤	0.0450	1.84	595.9	6.91	12.1
		126.2	250	400	362	2.2	2200	0.30	300	3.4	100.00	1.70	50.00	56796	533.7	41.5	9.4	≤	0.0450	1.84	117.8	11.86	2.4
		126.2	250	400	362	2.2	2200	0.30	300	3.4	200.00	5.00	100.00	56796	533.7	83.0	9.4	≤	0.0450	1.84	235.5	9.41	4.8
		126.2	250	400	362	2.2	2200	0.30	300	3.4	300.00	8.60	150.00	56796	533.7	124.6	9.4	≤	0.0450	1.84	353.3	8.22	7.2
		126.2	250	400	362	2.2	2200	0.30	300	3.4	400.00	12.10	200.00	56796	533.7	166.1	9.4	≤	0.0450	1.84	471.1	7.47	9.6
		126.2	250	400	362	2.2	2200	0.30	300	3.4	506.00	15.90	253.00	56796	533.7	210.1	9.4	≤	0.0450	1.84	595.9	6.91	12.1

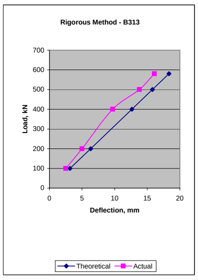

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

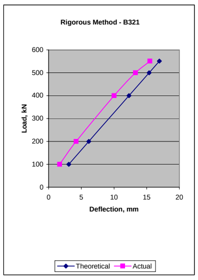

Rigorous Method - A211
Load, kN
Deflection, mm
Theoretical
Actual

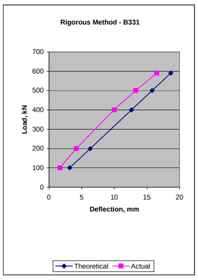

Rigorous Method - B211
Load, kN
Deflection, mm
Theoretical
Actual

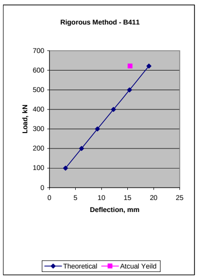

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

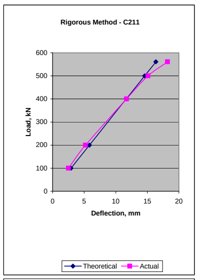

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

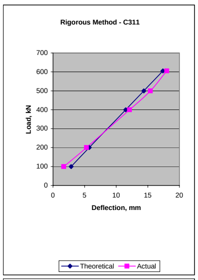

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

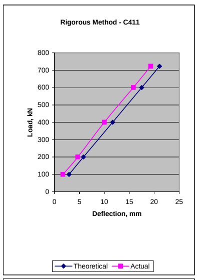

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

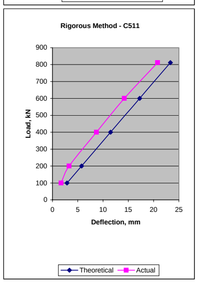

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

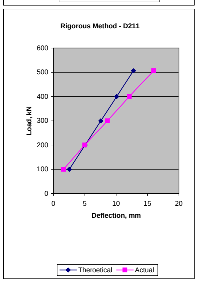

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Atcual Yeild

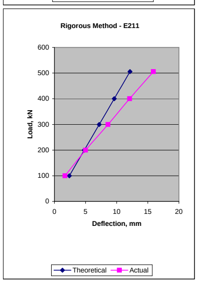

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

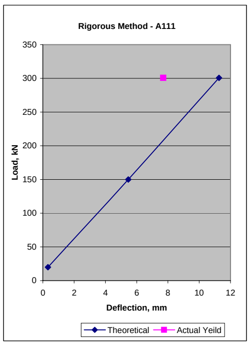

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

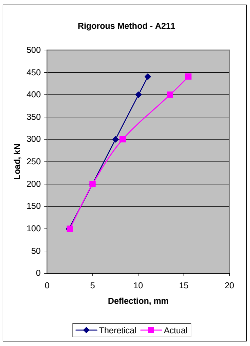

Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

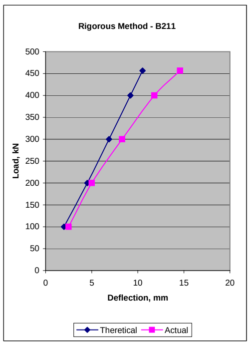
METHOD 2 - AS3600

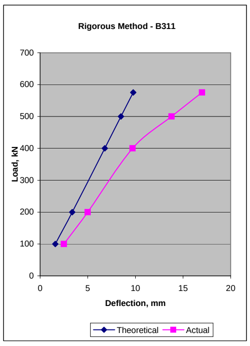
$E_c =$	$0.043\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.6\sqrt{f'_c}$

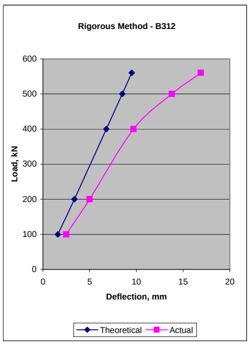
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	3.93	588.9	9.4	≤	10.03	11.3
		42.8	1250	300	3.4	20.00		33076	3.93	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	3.93	605.0	9.4	≤	9.15	5.5
		42.8	1250	300	3.4	300.78		33076	3.93	588.9	9.4	≤	7.19	11.3
	A211	42.8	2200	300	3.4	440.48	15.50	33076	3.93	879.6	9.4	≤	7.13	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	3.93	916.8	9.4	≤	11.86	2.4
		42.8	2200	300	3.4	200.00	5.00	33076	3.93	883.8	9.4	≤	9.30	5.0
		42.8	2200	300	3.4	300.00	8.30	33076	3.93	880.5	9.4	≤	8.11	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	3.93	879.7	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	3.93	879.6	9.4	≤	7.13	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	43667	5.18	725.5	9.4	≤	7.25	10.5
		74.6	2200	300	3.4	100.00	2.50	43667	5.18	840.5	9.4	≤	12.64	2.0
		74.6	2200	300	3.4	200.00	5.00	43667	5.18	738.8	9.4	≤	9.61	4.5
		74.6	2200	300	3.4	300.00	8.30	43667	5.18	728.6	9.4	≤	8.35	6.9
		74.6	2200	300	3.4	400.00	11.80	43667	5.18	726.1	9.4	≤	7.58	9.2
		74.6	2200	300	3.4	456.70	14.60	43667	5.18	725.5	9.4	≤	7.25	10.5
	B311	72.8	3460	310	3.4	575.68	17.00	43137	5.12	996.2	9.4	≤	7.38	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	5.12	1058.0	9.4	≤	13.50	1.6
		72.8	3460	310	3.4	200.00	5.00	43137	5.12	1003.6	9.4	≤	10.53	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.12	996.8	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.12	996.4	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	575.68	17.00	43137	5.12	996.2	9.4	≤	7.38	9.8
	B312	72.8	3460	310	3.4	560.36	16.90	43137	5.12	996.2	9.4	≤	7.45	9.5
		72.8	3460	310	3.4	100.00	2.50	43137	5.12	1058.0	9.4	≤	13.50	1.6
		72.8	3460	310	3.4	200.00	5.00	43137	5.12	1003.6	9.4	≤	10.53	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.12	996.8	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.12	996.4	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	560.36	16.90	43137	5.12	996.2	9.4	≤	7.45	9.5
	B313	72.8	3460	310	3.4	580.26	16.10	43137	5.12	996.2	9.4	≤	7.36	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	5.12	1058.0	9.4	≤	13.50	1.6
		72.8	3460	310	3.4	200.00	5.00	43137	5.12	1003.6	9.4	≤	10.53	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.12	996.8	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.12	996.4	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	580.26	16.10	43137	5.12	996.2	9.4	≤	7.36	9.8
	B321	77	3460	620	3.4	551.16	15.50	44364	5.26	978.2	9.4	≤	7.61	9.3
		77	3460	620	3.4	100.00	1.70	44364	5.26	1048.9	9.4	≤	13.76	1.6
		77	3460	620	3.4	200.00	4.20	44364	5.26	986.7	9.4	≤	10.70	3.3
		77	3460	620	3.4	400.00	10.00	44364	5.26	978.9	9.4	≤	8.47	6.7
		77	3460	620	3.4	500.00	13.30	44364	5.26	978.3	9.4	≤	7.86	8.4
		77	3460	620	3.4	551.16	15.50	44364	5.26	978.2	9.4	≤	7.61	9.3
	B331	72.8	3460	940	3.4	590.38	16.50	43137	5.12	996.2	9.4	≤	7.52	10.0
		72.8	3460	940	3.4	100.00	1.70	43137	5.12	1058.0	9.4	≤	13.86	1.6
		72.8	3460	940	3.4	200.00	4.20	43137	5.12	1003.6	9.4	≤	10.81	3.4
		72.8	3460	940	3.4	400.00	10.00	43137	5.12	996.8	9.4	≤	8.56	6.8
		72.8	3460	940	3.4	500.00	13.30	43137	5.12	996.4	9.4	≤	7.94	8.5
		72.8	3460	940	3.4	590.38	16.50	43137	5.12	996.2	9.4	≤	7.52	10.0
	B411	77	4730	320	3.4	621.34	15.50	44364	5.26	1192.0	9.4	≤	7.69	8.6
		77	4730	320	3.4	100.00		44364	5.26	1220.2	9.4	≤	14.25	1.3
		77	4730	320	3.4	200.00		44364	5.26	1195.4	9.4	≤	11.23	2.8
		77	4730	320	3.4	300.00		44364	5.26	1192.9	9.4	≤	9.80	4.1
		77	4730	320	3.4	400.00		44364	5.26	1192.3	9.4	≤	8.91	5.5
		77	4730	320	3.4	500.00		44364	5.26	1192.1	9.4	≤	8.27	6.9
		77	4730	320	3.4	621.34		44364	5.26	1192.0	9.4	≤	7.69	8.6
	C211	85.6	2710	300	3.4	560.94	18.20	46776	5.55	800.1	9.4	≤	7.13	10.9
		85.6	2710	300	3.4	100.00	2.50	46776	5.55	924.7	9.4	≤	13.30	1.7
		85.6	2710	300	3.4	200.00	5.20	46776	5.55	815.1	9.4	≤	10.12	3.8
		85.6	2710	300	3.4	400.00	11.70	46776	5.55	801.4	9.4	≤	7.99	7.8
		85.6	2710	300	3.4	500.00	15.10	46776	5.55	800.4	9.4	≤	7.41	9.7
		85.6	2710	300	3.4	560.94	18.20	46776	5.55	800.1	9.4	≤	7.13	10.9
	C311	88.1	3220	310	3.4	605.46	18.00	47454	5.63	891.6	9.4	≤	7.23	10.4
		88.1	3220	310	3.4	100.00	1.70	47454	5.63	999.5	9.4	≤	13.69	1.5
		88.1	3220	310	3.4	200.00	5.30	47454	5.63	904.6	9.4	≤	10.51	3.4
		88.1	3220	310	3.4	400.00	12.10	47454	5.63	892.8	9.4	≤	8.30	6.9
		88.1	3220	310	3.4	500.00	15.40	47454	5.63	892.0	9.4	≤	7.71	8.6
		88.1	3220	310	3.4	605.46	18.00	47454	5.63	891.6	9.4	≤	7.23	10.4
	C411	85.6	4260	320	3.4	722.56	19.30	46776	5.55	1080.8	9.4	≤	7.21	10.4
		85.6	4260	320	3.4	100.00	1.70	46776	5.55	1139.9	9.4	≤	14.19	1.4
		85.6	4260	320	3.4	200.00	4.70	46776	5.55	1088.1	9.4	≤	11.09	2.9
		85.6	4260	320	3.4	400.00	10.00	46776	5.55	1081.6	9.4	≤	8.78	5.8
		85.6	4260	320	3.4	600.00	15.80	46776	5.55	1080.9	9.4	≤	7.67	8.7
		85.6	4260	320	3.4	722.56	19.30	46776	5.55	1080.8	9.4	≤	7.21	10.4
	C511	88.1	5310	330	3.4	811.82	20.80	47454	5.63	1227.3	9.4	≤	7.26	10.2
		88.1	5310	330	3.4	100.00	1.70	47454	5.63	1253.3	9.4	≤	14.69	1.2
		88.1	5310	330	3.4	200.00	3.30	47454	5.63	1230.5	9.4	≤	11.59	2.5
		88.1	5310	330	3.4	400.00	8.70	47454	5.63	1227.7	9.4	≤	9.19	5.0
		88.1	5310	330	3.4	600.00	14.20	47454	5.63	1227.4	9.4	≤	8.03	7.5
		88.1	5310	330	3.4	811.82	20.80	47454	5.63	1227.3	9.4	≤	7.26	10.2
	D211	114.5	3220	300	3.4	506.00	16.00	54099	6.42	816.0	9.4	≤	7.78	8.4
		114.5	3220	300	3.4	100.00	1.60	54099	6.42	1002.9	9.4	≤	14.31	1.3
		114.5	3220	300	3.4	200.00	5.00	54099	6.42	838.1	9.4	≤	10.70	3.2
		114.5	3220	300	3.4	300.00	8.60	54099	6.42	821.5	9.4	≤	9.28	4.9
		114.5	3220	300	3.4	400.00	12.10	54099	6.42	817.5	9.4	≤	8.42	6.6
		114.5	3220	300	3.4	506.00	16.00	54099	6.42	816.0	9.4	≤	7.78	8.4
	E211	126.2	2200	300	3.4	506.00	15.90	56796	6.74	599.9	9.4	≤	7.18	10.8
		126.2	2200	300	3.4	100.00	1.70	56796	6.74	906.6	9.4	≤	14.15	1.4
		126.2	2200	300	3.4	200.00	5.00	56796	6.74	636.2	9.4	≤	9.98	4.0
		126.2	2200	300	3.4	300.00	8.60	56796	6.74	609.0	9.4	≤	8.59	6.3
		126.2	2200	300	3.4	400.00	12.10	56796	6.74	602.4	9.4	≤	7.78	8.5
		126.2	2200	300	3.4	506.00	15.90	56796	6.74	599.9	9.4	≤	7.18	10.8

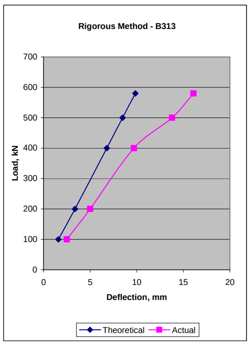

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

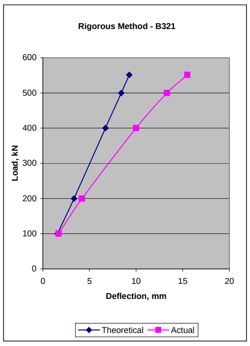

Rigorous Method - A211
Load, kN
Deflection, mm
Theoretical
Actual

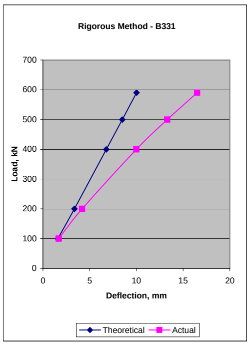

Rigorous Method - B211
Load, kN
Deflection, mm
Theoretical
Actual

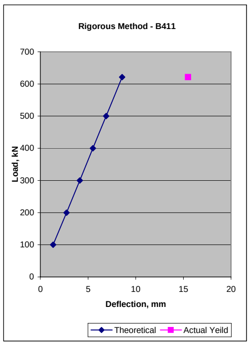

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

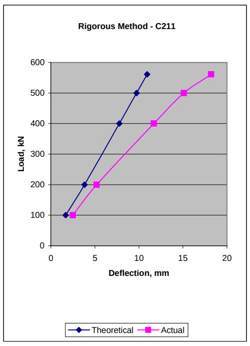

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

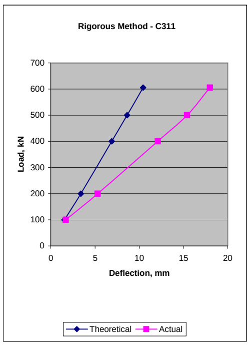

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

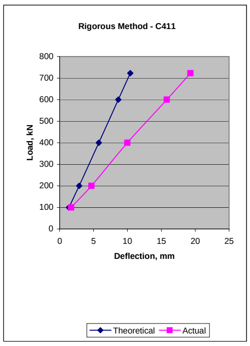

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

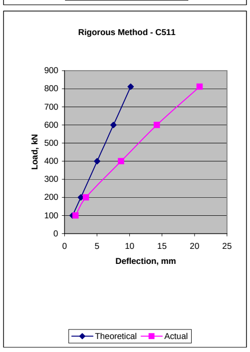

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

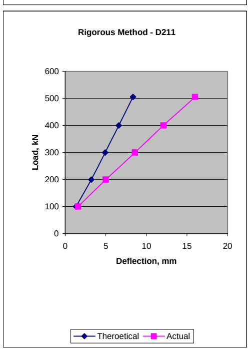

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

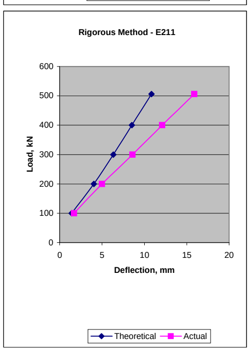

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

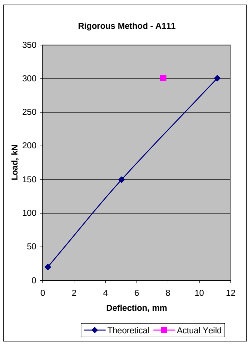
METHOD 3

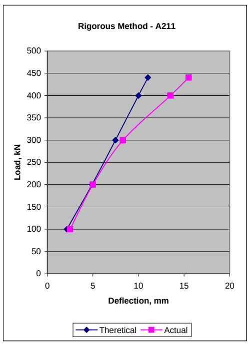
$E_c = 0.043\rho^{1.5}\sqrt{f'_c}$

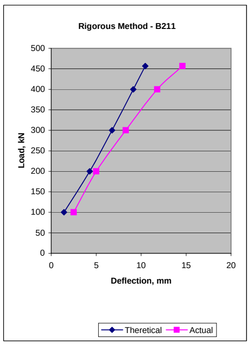
$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_a)^3$

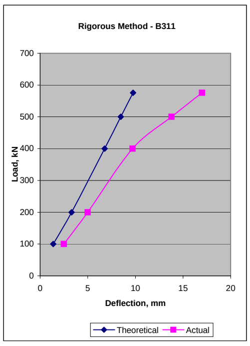
$f'_{cf} = 0.94\sqrt{f'_c}$

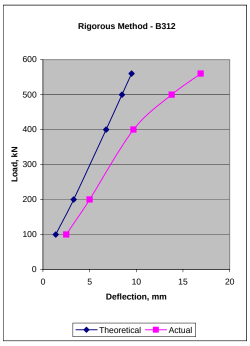
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	6.15	595.4	9.4	≤	10.07	11.1
		42.8	1250	300	3.4	20.00		33076	6.15	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	6.15	657.2	9.4	≤	9.41	5.0
		42.8	1250	300	3.4	300.78		33076	6.15	595.4	9.4	≤	7.22	11.1
	A211	42.8	2200	300	3.4	440.48	15.50	33076	6.15	880.8	9.4	≤	7.14	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	6.15	1024.0	9.4	≤	12.30	2.2
		42.8	2200	300	3.4	200.00	5.00	33076	6.15	897.2	9.4	≤	9.34	4.9
		42.8	2200	300	3.4	300.00	8.30	33076	6.15	884.5	9.4	≤	8.12	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	6.15	881.4	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	6.15	880.8	9.4	≤	7.14	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	43667	8.12	728.9	9.4	≤	7.26	10.5
		74.6	2200	300	3.4	100.00	2.50	43667	8.12	1171.4	9.4	≤	14.12	1.4
		74.6	2200	300	3.4	200.00	5.00	43667	8.12	780.1	9.4	≤	9.78	4.3
		74.6	2200	300	3.4	300.00	8.30	43667	8.12	740.8	9.4	≤	8.40	6.8
		74.6	2200	300	3.4	400.00	11.80	43667	8.12	731.2	9.4	≤	7.60	9.1
		74.6	2200	300	3.4	456.70	14.60	43667	8.12	728.9	9.4	≤	7.26	10.5
	B311	72.8	3460	310	3.4	575.68	17.00	43137	8.02	997.1	9.4	≤	7.39	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	8.02	1234.7	9.4	≤	14.21	1.4
		72.8	3460	310	3.4	200.00	5.00	43137	8.02	1025.7	9.4	≤	10.61	3.3
		72.8	3460	310	3.4	400.00	9.70	43137	8.02	999.6	9.4	≤	8.35	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	8.02	997.8	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	575.68	17.00	43137	8.02	997.1	9.4	≤	7.39	9.8
	B312	72.8	3460	310	3.4	560.36	16.90	43137	8.02	997.2	9.4	≤	7.45	9.5
		72.8	3460	310	3.4	100.00	2.50	43137	8.02	1234.7	9.4	≤	14.21	1.4
		72.8	3460	310	3.4	200.00	5.00	43137	8.02	1025.7	9.4	≤	10.61	3.3
		72.8	3460	310	3.4	400.00	9.70	43137	8.02	999.6	9.4	≤	8.35	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	8.02	997.8	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	560.36	16.90	43137	8.02	997.2	9.4	≤	7.45	9.5
	B313	72.8	3460	310	3.4	580.26	16.10	43137	8.02	997.1	9.4	≤	7.37	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	8.02	1234.7	9.4	≤	14.21	1.4
		72.8	3460	310	3.4	200.00	5.00	43137	8.02	1025.7	9.4	≤	10.61	3.3
		72.8	3460	310	3.4	400.00	9.70	43137	8.02	999.6	9.4	≤	8.35	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	8.02	997.8	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	580.26	16.10	43137	8.02	997.1	9.4	≤	7.37	9.8
	B321	77	3460	620	3.4	551.16	15.50	44364	8.25	979.4	9.4	≤	7.61	9.3
		77	3460	620	3.4	100.00	1.70	44364	8.25	1251.5	9.4	≤	14.60	1.3
		77	3460	620	3.4	200.00	4.20	44364	8.25	1012.0	9.4	≤	10.79	3.2
		77	3460	620	3.4	400.00	10.00	44364	8.25	982.0	9.4	≤	8.48	6.7
		77	3460	620	3.4	500.00	13.30	44364	8.25	979.9	9.4	≤	7.87	8.4
		77	3460	620	3.4	551.16	15.50	44364	8.25	979.4	9.4	≤	7.61	9.3
	B331	72.8	3460	940	3.4	590.38	16.50	43137	8.02	997.0	9.4	≤	7.52	10.0
		72.8	3460	940	3.4	100.00	1.70	43137	8.02	1234.7	9.4	≤	14.59	1.4
		72.8	3460	940	3.4	200.00	4.20	43137	8.02	1025.7	9.4	≤	10.89	3.3
		72.8	3460	940	3.4	400.00	10.00	43137	8.02	999.6	9.4	≤	8.57	6.8
		72.8	3460	940	3.4	500.00	13.30	43137	8.02	997.8	9.4	≤	7.95	8.5
		72.8	3460	940	3.4	590.38	16.50	43137	8.02	997.0	9.4	≤	7.52	10.0
	B411	77	4730	320	3.4	621.34	15.50	44364	8.25	1192.3	9.4	≤	7.69	8.6
		77	4730	320	3.4	100.00		44364	8.25	1300.8	9.4	≤	14.56	1.3
		77	4730	320	3.4	200.00		44364	8.25	1205.5	9.4	≤	11.26	2.7
		77	4730	320	3.4	300.00		44364	8.25	1195.9	9.4	≤	9.81	4.1
		77	4730	320	3.4	400.00		44364	8.25	1193.6	9.4	≤	8.91	5.5
		77	4730	320	3.4	500.00		44364	8.25	1192.8	9.4	≤	8.27	6.9
		77	4730	320	3.4	621.34		44364	8.25	1192.3	9.4	≤	7.69	8.6
	C211	85.6	2710	300	3.4	560.94	18.20	46776	8.70	802.1	9.4	≤	7.14	10.9
		85.6	2710	300	3.4	100.00	2.50	46776	8.70	1281.2	9.4	≤	14.83	1.2
		85.6	2710	300	3.4	200.00	5.20	46776	8.70	859.6	9.4	≤	10.30	3.6
		85.6	2710	300	3.4	400.00	11.70	46776	8.70	806.9	9.4	≤	8.01	7.7
		85.6	2710	300	3.4	500.00	15.10	46776	8.70	803.3	9.4	≤	7.42	9.7
		85.6	2710	300	3.4	560.94	18.20	46776	8.70	802.1	9.4	≤	7.14	10.9
	C311	88.1	3220	310	3.4	605.46	18.00	47454	8.82	893.0	9.4	≤	7.23	10.4
		88.1	3220	310	3.4	100.00	1.70	47454	8.82	1307.8	9.4	≤	14.97	1.2
		88.1	3220	310	3.4	200.00	5.30	47454	8.82	943.2	9.4	≤	10.66	3.3
		88.1	3220	310	3.4	400.00	12.10	47454	8.82	897.6	9.4	≤	8.32	6.9
		88.1	3220	310	3.4	500.00	15.40	47454	8.82	894.4	9.4	≤	7.71	8.6
		88.1	3220	310	3.4	605.46	18.00	47454	8.82	893.0	9.4	≤	7.23	10.4
	C411	85.6	4260	320	3.4	722.56	19.30	46776	8.70	1081.3	9.4	≤	7.21	10.4
		85.6	4260	320	3.4	100.00	1.70	46776	8.70	1308.7	9.4	≤	14.86	1.2
		85.6	4260	320	3.4	200.00	4.70	46776	8.70	1109.2	9.4	≤	11.16	2.8
		85.6	4260	320	3.4	400.00	10.00	46776	8.70	1084.2	9.4	≤	8.79	5.8
		85.6	4260	320	3.4	600.00	15.80	46776	8.70	1081.7	9.4	≤	7.67	8.6
		85.6	4260	320	3.4	722.56	19.30	46776	8.70	1081.3	9.4	≤	7.21	10.4
	C511	88.1	5310	330	3.4	811.82	20.80	47454	8.82	1227.5	9.4	≤	7.26	10.2
		88.1	5310	330	3.4	100.00	1.70	47454	8.82	1327.2	9.4	≤	14.97	1.2
		88.1	5310	330	3.4	200.00	3.30	47454	8.82	1239.8	9.4	≤	11.62	2.5
		88.1	5310	330	3.4	400.00	8.70	47454	8.82	1228.9	9.4	≤	9.19	5.0
		88.1	5310	330	3.4	600.00	14.20	47454	8.82	1227.8	9.4	≤	8.03	7.5
		88.1	5310	330	3.4	811.82	20.80	47454	8.82	1227.5	9.4	≤	7.26	10.2
	D211	114.5	3220	300	3.4	506.00	16.00	54099	10.06	820.2	9.4	≤	7.79	8.3
		114.5	3220	300	3.4	100.00	1.60	54099	10.06	1333.3	9.4	≤	15.73	1.0
		114.5	3220	300	3.4	200.00	5.00	54099	10.06	905.1	9.4	≤	10.98	3.0
		114.5	3220	300	3.4	300.00	8.60	54099	10.06	841.4	9.4	≤	9.36	4.8
		114.5	3220	300	3.4	400.00	12.10	54099	10.06	825.9	9.4	≤	8.45	6.5
		114.5	3220	300	3.4	506.00	16.00	54099	10.06	820.2	9.4	≤	7.79	8.3
	E211	126.2	2200	300	3.4	506.00	15.90	56796	10.56	606.7	9.4	≤	7.21	10.7
		126.2	2200	300	3.4	100.00	1.70	56796	10.56	1333.3	9.4	≤	16.09	1.0
		126.2	2200	300	3.4	200.00	5.00	56796	10.56	746.1	9.4	≤	10.52	3.4
		126.2	2200	300	3.4	300.00	8.60	56796	10.56	641.6	9.4	≤	8.74	6.0
		126.2	2200	300	3.4	400.00	12.10	56796	10.56	616.1	9.4	≤	7.84	8.3
		126.2	2200	300	3.4	506.00	15.90	56796	10.56	606.7	9.4	≤	7.21	10.7

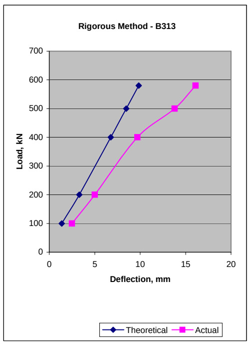

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

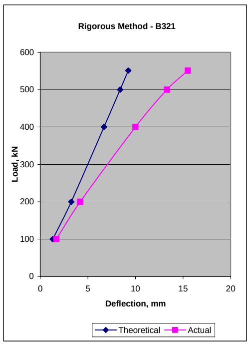

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

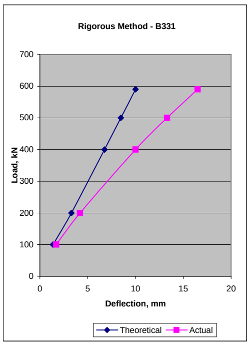

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

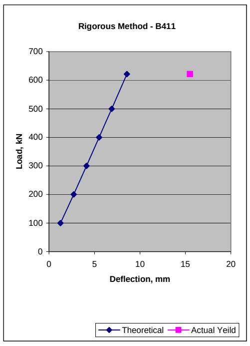

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

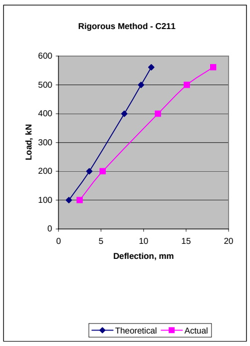

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

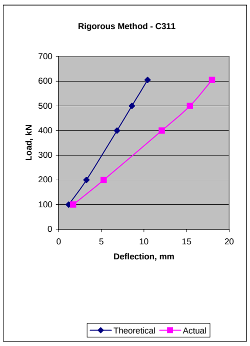

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

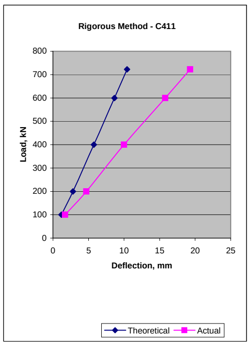

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

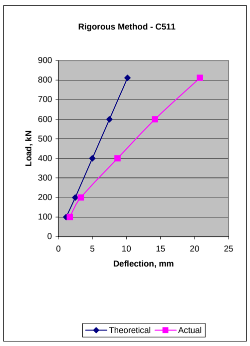

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

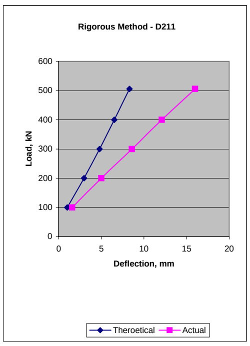

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

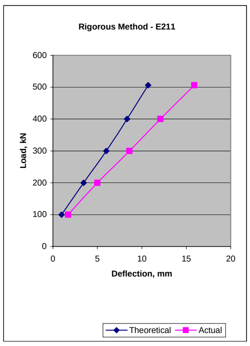

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

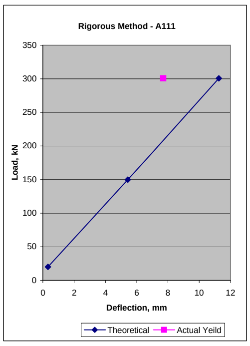
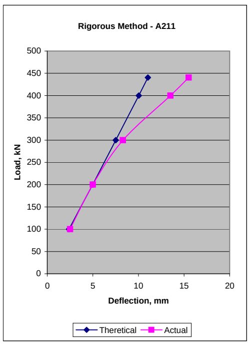

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

METHOD 4

E_c =	$0.043\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$I_{cr} + (I - I_{cr})(M_{cr}/M_a)^3$
f'_{cf} =	$0.342(f'_c)^{2/3}$

SOURCE	BEAM	f'_c, MPa	Reinforcement Ratio: Area of Steel, A_{st}, mm^2	Reinforcement Ratio: Area of Steel, A_{sc}, mm^2	L_{ef}, m	At Yield: Load, kN	At Yield: Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Deem-to-Comply: Ratio	≤	Deem-to-Comply: Allowed Ratio	Rigorous Method: Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	4.18	589.4	9.4	≤	10.04	11.3
		42.8	1250	300	3.4	20.00		33076	4.18	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	4.18	608.9	9.4	≤	9.17	5.4
		42.8	1250	300	3.4	300.78		33076	4.18	589.4	9.4	≤	7.20	11.3
	A211	42.8	2200	300	3.4	440.48	15.50	33076	4.18	879.7	9.4	≤	7.13	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	4.18	924.8	9.4	≤	11.89	2.4
		42.8	2200	300	3.4	200.00	5.00	33076	4.18	884.8	9.4	≤	9.30	5.0
		42.8	2200	300	3.4	300.00	8.30	33076	4.18	880.8	9.4	≤	8.11	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	4.18	879.8	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	4.18	879.7	9.4	≤	7.13	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	43667	6.06	726.2	9.4	≤	7.25	10.5
		74.6	2200	300	3.4	100.00	2.50	43667	6.06	910.2	9.4	≤	12.98	1.8
		74.6	2200	300	3.4	200.00	5.00	43667	6.06	747.5	9.4	≤	9.65	4.5
		74.6	2200	300	3.4	300.00	8.30	43667	6.06	731.1	9.4	≤	8.36	6.9
		74.6	2200	300	3.4	400.00	11.80	43667	6.06	727.2	9.4	≤	7.59	9.2
		74.6	2200	300	3.4	456.70	14.60	43667	6.06	726.2	9.4	≤	7.25	10.5
	B311	72.8	3460	310	3.4	575.68	17.00	43137	5.96	996.4	9.4	≤	7.38	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	5.96	1094.0	9.4	≤	13.65	1.5
		72.8	3460	310	3.4	200.00	5.00	43137	5.96	1008.1	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.96	997.4	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.96	996.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	575.68	17.00	43137	5.96	996.4	9.4	≤	7.38	9.8
	B312	72.8	3460	310	3.4	560.36	16.90	43137	5.96	996.4	9.4	≤	7.45	9.5
		72.8	3460	310	3.4	100.00	2.50	43137	5.96	1094.0	9.4	≤	13.65	1.5
		72.8	3460	310	3.4	200.00	5.00	43137	5.96	1008.1	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.96	997.4	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.96	996.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	560.36	16.90	43137	5.96	996.4	9.4	≤	7.45	9.5
	B313	72.8	3460	310	3.4	580.26	16.10	43137	5.96	996.4	9.4	≤	7.36	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	5.96	1094.0	9.4	≤	13.65	1.5
		72.8	3460	310	3.4	200.00	5.00	43137	5.96	1008.1	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.96	997.4	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.96	996.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	580.26	16.10	43137	5.96	996.4	9.4	≤	7.36	9.8
	B321	77	3460	620	3.4	551.16	15.50	44364	6.19	978.5	9.4	≤	7.61	9.3
		77	3460	620	3.4	100.00	1.70	44364	6.19	1093.4	9.4	≤	13.95	1.5
		77	3460	620	3.4	200.00	4.20	44364	6.19	992.2	9.4	≤	10.72	3.3
		77	3460	620	3.4	400.00	10.00	44364	6.19	979.6	9.4	≤	8.47	6.7
		77	3460	620	3.4	500.00	13.30	44364	6.19	978.7	9.4	≤	7.86	8.4
		77	3460	620	3.4	551.16	15.50	44364	6.19	978.5	9.4	≤	7.61	9.3
	B331	72.8	3460	940	3.4	590.38	16.50	43137	5.96	996.4	9.4	≤	7.52	10.0
		72.8	3460	940	3.4	100.00	1.70	43137	5.96	1094.0	9.4	≤	14.01	1.5
		72.8	3460	940	3.4	200.00	4.20	43137	5.96	1008.1	9.4	≤	10.82	3.4
		72.8	3460	940	3.4	400.00	10.00	43137	5.96	997.4	9.4	≤	8.56	6.8
		72.8	3460	940	3.4	500.00	13.30	43137	5.96	996.7	9.4	≤	7.94	8.5
		72.8	3460	940	3.4	590.38	16.50	43137	5.96	996.4	9.4	≤	7.52	10.0
	B411	77	4730	320	3.4	621.34	15.50	44364	6.19	1192.1	9.4	≤	7.69	8.6
		77	4730	320	3.4	100.00		44364	6.19	1237.9	9.4	≤	14.32	1.3
		77	4730	320	3.4	200.00		44364	6.19	1197.6	9.4	≤	11.24	2.7
		77	4730	320	3.4	300.00		44364	6.19	1193.6	9.4	≤	9.81	4.1
		77	4730	320	3.4	400.00		44364	6.19	1192.6	9.4	≤	8.91	5.5
		77	4730	320	3.4	500.00		44364	6.19	1192.3	9.4	≤	8.27	6.9
		77	4730	320	3.4	621.34		44364	6.19	1192.1	9.4	≤	7.69	8.6
	C211	85.6	2710	300	3.4	560.94	18.20	46776	6.64	800.6	9.4	≤	7.13	10.9
		85.6	2710	300	3.4	100.00	2.50	46776	6.64	1014.1	9.4	≤	13.72	1.5
		85.6	2710	300	3.4	200.00	5.20	46776	6.64	826.2	9.4	≤	10.17	3.8
		85.6	2710	300	3.4	400.00	11.70	46776	6.64	802.8	9.4	≤	7.99	7.8
		85.6	2710	300	3.4	500.00	15.10	46776	6.64	801.1	9.4	≤	7.41	9.7
		85.6	2710	300	3.4	560.94	18.20	46776	6.64	800.6	9.4	≤	7.13	10.9
	C311	88.1	3220	310	3.4	605.46	18.00	47454	6.77	892.0	9.4	≤	7.23	10.4
		88.1	3220	310	3.4	100.00	1.70	47454	6.77	1079.4	9.4	≤	14.04	1.4
		88.1	3220	310	3.4	200.00	5.30	47454	6.77	914.6	9.4	≤	10.55	3.4
		88.1	3220	310	3.4	400.00	12.10	47454	6.77	894.0	9.4	≤	8.31	6.9
		88.1	3220	310	3.4	500.00	15.40	47454	6.77	892.6	9.4	≤	7.71	8.6
		88.1	3220	310	3.4	605.46	18.00	47454	6.77	892.0	9.4	≤	7.23	10.4
	C411	85.6	4260	320	3.4	722.56	19.30	46776	6.64	1080.9	9.4	≤	7.21	10.4
		85.6	4260	320	3.4	100.00	1.70	46776	6.64	1182.2	9.4	≤	14.36	1.3
		85.6	4260	320	3.4	200.00	4.70	46776	6.64	1093.4	9.4	≤	11.11	2.9
		85.6	4260	320	3.4	400.00	10.00	46776	6.64	1082.2	9.4	≤	8.79	5.8
		85.6	4260	320	3.4	600.00	15.80	46776	6.64	1081.1	9.4	≤	7.67	8.7
		85.6	4260	320	3.4	722.56	19.30	46776	6.64	1080.9	9.4	≤	7.21	10.4
	C511	88.1	5310	330	3.4	811.82	20.80	47454	6.77	1227.4	9.4	≤	7.26	10.2
		88.1	5310	330	3.4	100.00	1.70	47454	6.77	1272.5	9.4	≤	14.77	1.2
		88.1	5310	330	3.4	200.00	3.30	47454	6.77	1232.9	9.4	≤	11.60	2.5
		88.1	5310	330	3.4	400.00	8.70	47454	6.77	1228.0	9.4	≤	9.19	5.0
		88.1	5310	330	3.4	600.00	14.20	47454	6.77	1227.5	9.4	≤	8.03	7.5
		88.1	5310	330	3.4	811.82	20.80	47454	6.77	1227.4	9.4	≤	7.26	10.2
	D211	114.5	3220	300	3.4	506.00	16.00	54099	8.06	817.4	9.4	≤	7.79	8.3
		114.5	3220	300	3.4	100.00	1.60	54099	8.06	1187.8	9.4	≤	15.14	1.1
		114.5	3220	300	3.4	200.00	5.00	54099	8.06	861.2	9.4	≤	10.79	3.1
		114.5	3220	300	3.4	300.00	8.60	54099	8.06	828.4	9.4	≤	9.31	4.9
		114.5	3220	300	3.4	400.00	12.10	54099	8.06	820.4	9.4	≤	8.43	6.6
		114.5	3220	300	3.4	506.00	16.00	54099	8.06	817.4	9.4	≤	7.79	8.3
	E211	126.2	2200	300	3.4	506.00	15.90	56796	8.60	602.5	9.4	≤	7.19	10.8
		126.2	2200	300	3.4	100.00	1.70	56796	8.60	1240.6	9.4	≤	15.71	1.0
		126.2	2200	300	3.4	200.00	5.00	56796	8.60	677.9	9.4	≤	10.19	3.8
		126.2	2200	300	3.4	300.00	8.60	56796	8.60	621.4	9.4	≤	8.65	6.2
		126.2	2200	300	3.4	400.00	12.10	56796	8.60	607.6	9.4	≤	7.80	8.5
		126.2	2200	300	3.4	506.00	15.90	56796	8.60	602.5	9.4	≤	7.19	10.8


Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

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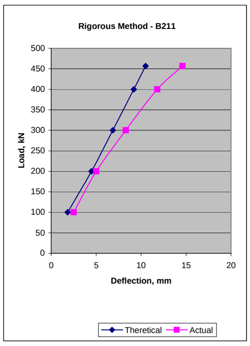
Load, kN
Deflection, mm
Theretical
Actual

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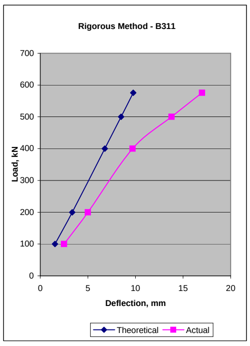
Load, kN
Deflection, mm
Theretical
Actual

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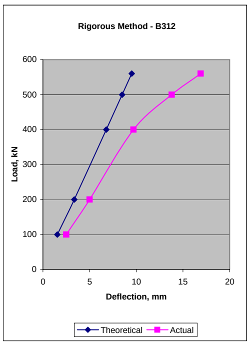
Load, kN
Deflection, mm
Theoretical
Actual

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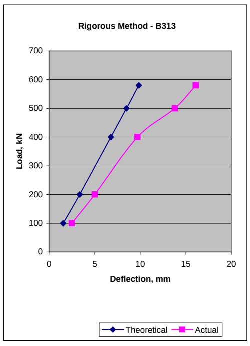
Load, kN
Deflection, mm
Theoretical
Actual

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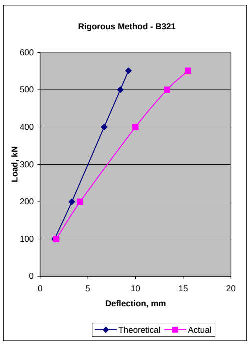
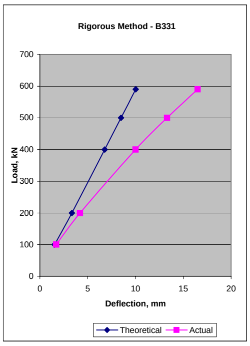
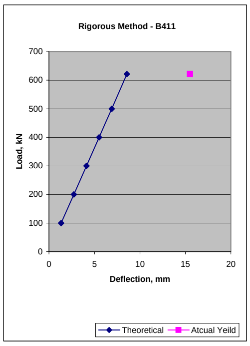
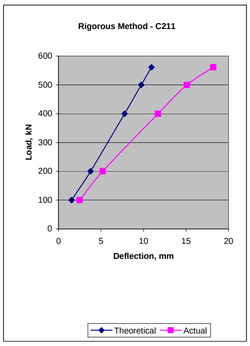
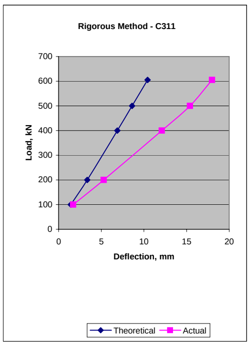
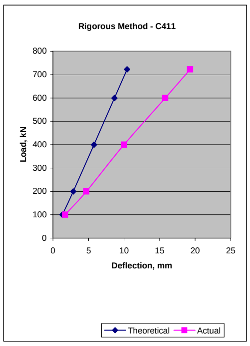
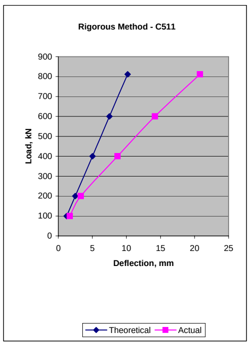
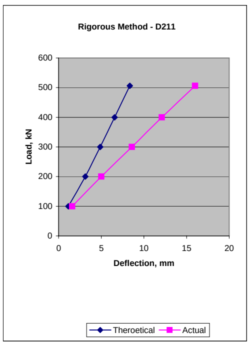
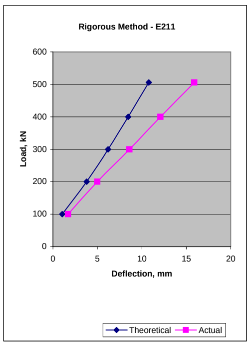
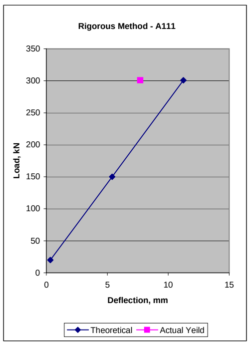
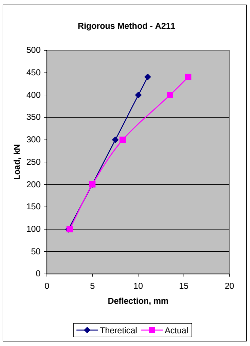
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Atcual Yeild

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

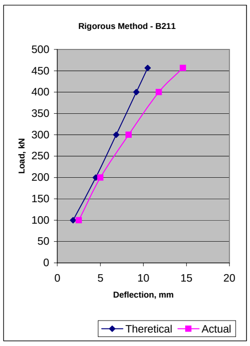
METHOD 5

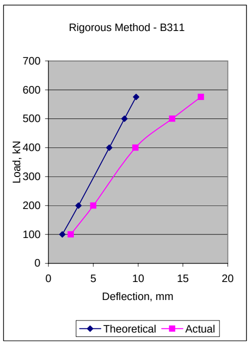
$E_c =$	$0.043\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.564(f'_c)^{0.55}$

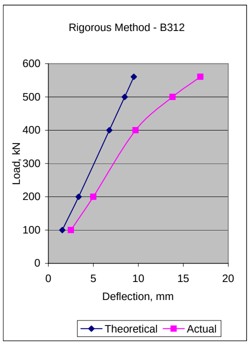
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm²	Area of Steel, A_{sc}, mm²	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	4.45	590.0	9.4	≤	10.04	11.2
		42.8	1250	300	3.4	20.00		33076	4.45	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	4.45	613.4	9.4	≤	9.20	5.4
		42.8	1250	300	3.4	300.78		33076	4.45	590.0	9.4	≤	7.20	11.2
	A211	42.8	2200	300	3.4	440.48	15.50	33076	4.45	879.8	9.4	≤	7.14	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	4.45	934.1	9.4	≤	11.93	2.4
		42.8	2200	300	3.4	200.00	5.00	33076	4.45	886.0	9.4	≤	9.31	5.0
		42.8	2200	300	3.4	300.00	8.30	33076	4.45	881.2	9.4	≤	8.11	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	4.45	880.0	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	4.45	879.8	9.4	≤	7.14	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	43667	6.04	726.2	9.4	≤	7.25	10.5
		74.6	2200	300	3.4	100.00	2.50	43667	6.04	908.7	9.4	≤	12.97	1.8
		74.6	2200	300	3.4	200.00	5.00	43667	6.04	747.3	9.4	≤	9.64	4.5
		74.6	2200	300	3.4	300.00	8.30	43667	6.04	731.1	9.4	≤	8.36	6.9
		74.6	2200	300	3.4	400.00	11.80	43667	6.04	727.1	9.4	≤	7.59	9.2
		74.6	2200	300	3.4	456.70	14.60	43667	6.04	726.2	9.4	≤	7.25	10.5
	B311	72.8	3460	310	3.4	575.68	17.00	43137	5.96	996.4	9.4	≤	7.38	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	5.96	1094.0	9.4	≤	13.65	1.5
		72.8	3460	310	3.4	200.00	5.00	43137	5.96	1008.1	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.96	997.4	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.96	996.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	575.68	17.00	43137	5.96	996.4	9.4	≤	7.38	9.8
	B312	72.8	3460	310	3.4	560.36	16.90	43137	5.96	996.4	9.4	≤	7.45	9.5
		72.8	3460	310	3.4	100.00	2.50	43137	5.96	1094.0	9.4	≤	13.65	1.5
		72.8	3460	310	3.4	200.00	5.00	43137	5.96	1008.1	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.96	997.4	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.96	996.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	560.36	16.90	43137	5.96	996.4	9.4	≤	7.45	9.5
	B313	72.8	3460	310	3.4	580.26	16.10	43137	5.96	996.4	9.4	≤	7.36	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	5.96	1094.0	9.4	≤	13.65	1.5
		72.8	3460	310	3.4	200.00	5.00	43137	5.96	1008.1	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	43137	5.96	997.4	9.4	≤	8.34	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	5.96	996.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	580.26	16.10	43137	5.96	996.4	9.4	≤	7.36	9.8
	B321	77	3460	620	3.4	551.16	15.50	44364	6.15	978.4	9.4	≤	7.61	9.3
		77	3460	620	3.4	100.00	1.70	44364	6.15	1091.2	9.4	≤	13.94	1.5
		77	3460	620	3.4	200.00	4.20	44364	6.15	991.9	9.4	≤	10.72	3.3
		77	3460	620	3.4	400.00	10.00	44364	6.15	979.5	9.4	≤	8.47	6.7
		77	3460	620	3.4	500.00	13.30	44364	6.15	978.7	9.4	≤	7.86	8.4
		77	3460	620	3.4	551.16	15.50	44364	6.15	978.4	9.4	≤	7.61	9.3
	B331	72.8	3460	940	3.4	590.38	16.50	43137	5.96	996.4	9.4	≤	7.52	10.0
		72.8	3460	940	3.4	100.00	1.70	43137	5.96	1094.0	9.4	≤	14.01	1.5
		72.8	3460	940	3.4	200.00	4.20	43137	5.96	1008.1	9.4	≤	10.82	3.4
		72.8	3460	940	3.4	400.00	10.00	43137	5.96	997.4	9.4	≤	8.56	6.8
		72.8	3460	940	3.4	500.00	13.30	43137	5.96	996.7	9.4	≤	7.94	8.5
		72.8	3460	940	3.4	590.38	16.50	43137	5.96	996.4	9.4	≤	7.52	10.0
	B411	77	4730	320	3.4	621.34	15.50	44364	6.15	1192.1	9.4	≤	7.69	8.6
		77	4730	320	3.4	100.00		44364	6.15	1237.0	9.4	≤	14.31	1.3
		77	4730	320	3.4	200.00		44364	6.15	1197.5	9.4	≤	11.24	2.7
		77	4730	320	3.4	300.00		44364	6.15	1193.6	9.4	≤	9.81	4.1
		77	4730	320	3.4	400.00		44364	6.15	1192.6	9.4	≤	8.91	5.5
		77	4730	320	3.4	500.00		44364	6.15	1192.3	9.4	≤	8.27	6.9
		77	4730	320	3.4	621.34		44364	6.15	1192.1	9.4	≤	7.69	8.6
	C211	85.6	2710	300	3.4	560.94	18.20	46776	6.52	800.6	9.4	≤	7.13	10.9
		85.6	2710	300	3.4	100.00	2.50	46776	6.52	1002.3	9.4	≤	13.66	1.6
		85.6	2710	300	3.4	200.00	5.20	46776	6.52	824.8	9.4	≤	10.16	3.8
		85.6	2710	300	3.4	400.00	11.70	46776	6.52	802.6	9.4	≤	7.99	7.8
		85.6	2710	300	3.4	500.00	15.10	46776	6.52	801.0	9.4	≤	7.41	9.7
		85.6	2710	300	3.4	560.94	18.20	46776	6.52	800.6	9.4	≤	7.13	10.9
	C311	88.1	3220	310	3.4	605.46	18.00	47454	6.62	891.9	9.4	≤	7.23	10.4
		88.1	3220	310	3.4	100.00	1.70	47454	6.62	1067.3	9.4	≤	13.99	1.4
		88.1	3220	310	3.4	200.00	5.30	47454	6.62	913.1	9.4	≤	10.54	3.4
		88.1	3220	310	3.4	400.00	12.10	47454	6.62	893.9	9.4	≤	8.31	6.9
		88.1	3220	310	3.4	500.00	15.40	47454	6.62	892.5	9.4	≤	7.71	8.6
		88.1	3220	310	3.4	605.46	18.00	47454	6.62	891.9	9.4	≤	7.23	10.4
	C411	85.6	4260	320	3.4	722.56	19.30	46776	6.52	1080.9	9.4	≤	7.21	10.4
		85.6	4260	320	3.4	100.00	1.70	46776	6.52	1176.7	9.4	≤	14.34	1.3
		85.6	4260	320	3.4	200.00	4.70	46776	6.52	1092.7	9.4	≤	11.11	2.9
		85.6	4260	320	3.4	400.00	10.00	46776	6.52	1082.2	9.4	≤	8.79	5.8
		85.6	4260	320	3.4	600.00	15.80	46776	6.52	1081.1	9.4	≤	7.67	8.7
		85.6	4260	320	3.4	722.56	19.30	46776	6.52	1080.9	9.4	≤	7.21	10.4
	C511	88.1	5310	330	3.4	811.82	20.80	47454	6.62	1227.4	9.4	≤	7.26	10.2
		88.1	5310	330	3.4	100.00	1.70	47454	6.62	1269.5	9.4	≤	14.75	1.2
		88.1	5310	330	3.4	200.00	3.30	47454	6.62	1232.6	9.4	≤	11.60	2.5
		88.1	5310	330	3.4	400.00	8.70	47454	6.62	1228.0	9.4	≤	9.19	5.0
		88.1	5310	330	3.4	600.00	14.20	47454	6.62	1227.5	9.4	≤	8.03	7.5
		88.1	5310	330	3.4	811.82	20.80	47454	6.62	1227.4	9.4	≤	7.26	10.2
	D211	114.5	3220	300	3.4	506.00	16.00	54099	7.65	817.0	9.4	≤	7.78	8.4
		114.5	3220	300	3.4	100.00	1.60	54099	7.65	1133.1	9.4	≤	14.90	1.2
		114.5	3220	300	3.4	200.00	5.00	54099	7.65	854.4	9.4	≤	10.77	3.2
		114.5	3220	300	3.4	300.00	8.60	54099	7.65	826.4	9.4	≤	9.30	4.9
		114.5	3220	300	3.4	400.00	12.10	54099	7.65	819.5	9.4	≤	8.43	6.6
		114.5	3220	300	3.4	506.00	16.00	54099	7.65	817.0	9.4	≤	7.78	8.4
	E211	126.2	2200	300	3.4	506.00	15.90	56796	8.07	601.6	9.4	≤	7.19	10.8
		126.2	2200	300	3.4	100.00	1.70	56796	8.07	1128.0	9.4	≤	15.22	1.1
		126.2	2200	300	3.4	200.00	5.00	56796	8.07	663.9	9.4	≤	10.12	3.9
		126.2	2200	300	3.4	300.00	8.60	56796	8.07	617.2	9.4	≤	8.63	6.2
		126.2	2200	300	3.4	400.00	12.10	56796	8.07	605.8	9.4	≤	7.79	8.5
		126.2	2200	300	3.4	506.00	15.90	56796	8.07	601.6	9.4	≤	7.19	10.8

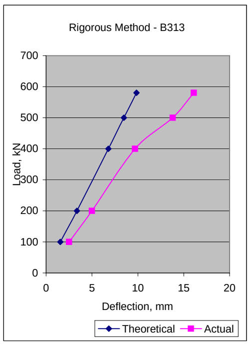

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

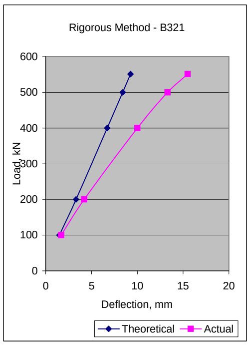

Rigorous Method - A211
Load, kN
Deflection, mm
Theoretical
Actual

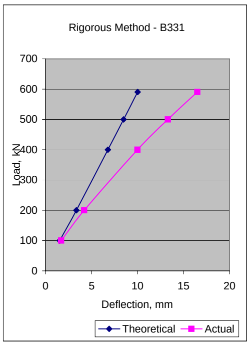

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

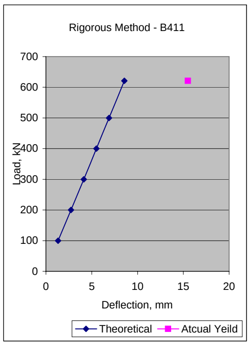

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

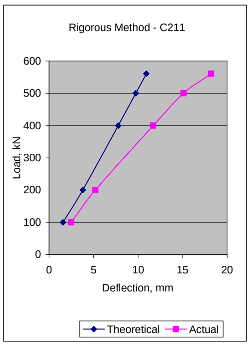

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

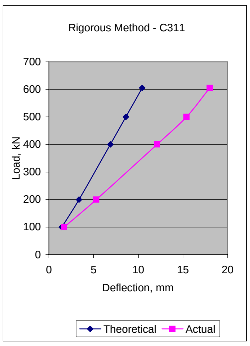

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

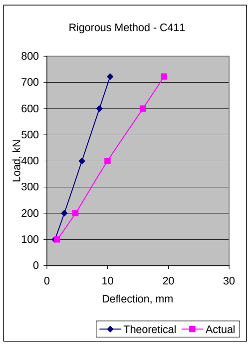

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

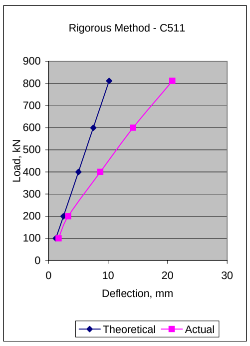

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

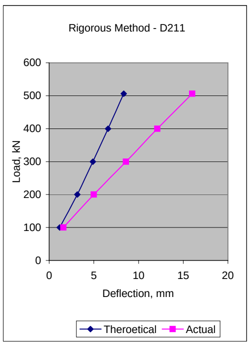

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Atcual Yeild

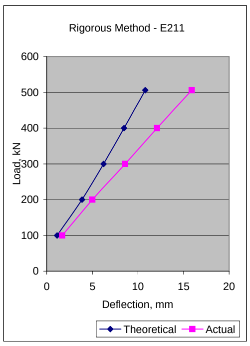

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

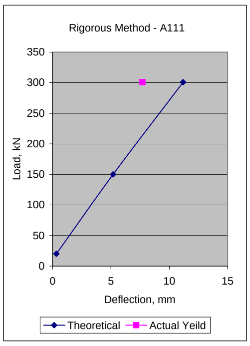

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

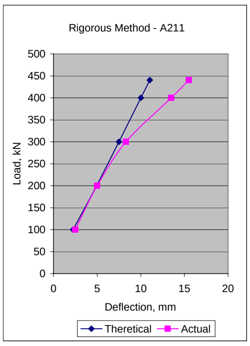

Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

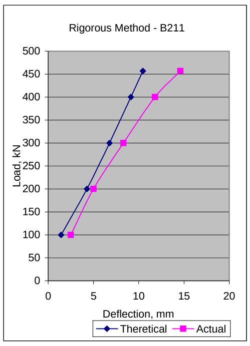
METHOD 6

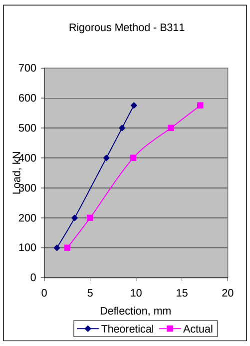
E_c =	$0.043\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
f'_{cf} =	$0.43(f'_c)^{0.68}$

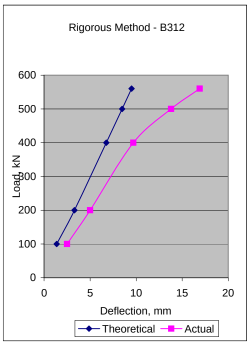
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	5.53	593.0	9.4	≤	10.06	11.2
		42.8	1250	300	3.4	20.00		33076	5.53	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	5.53	638.0	9.4	≤	9.32	5.2
		42.8	1250	300	3.4	300.78		33076	5.53	593.0	9.4	≤	7.21	11.2
	A211	42.8	2200	300	3.4	440.48	15.50	33076	5.53	880.4	9.4	≤	7.14	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	5.53	984.6	9.4	≤	12.14	2.2
		42.8	2200	300	3.4	200.00	5.00	33076	5.53	892.3	9.4	≤	9.33	4.9
		42.8	2200	300	3.4	300.00	8.30	33076	5.53	883.0	9.4	≤	8.12	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	5.53	880.8	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	5.53	880.4	9.4	≤	7.14	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	43667	8.07	728.9	9.4	≤	7.26	10.5
		74.6	2200	300	3.4	100.00	2.50	43667	8.07	1163.5	9.4	≤	14.08	1.4
		74.6	2200	300	3.4	200.00	5.00	43667	8.07	779.2	9.4	≤	9.78	4.3
		74.6	2200	300	3.4	300.00	8.30	43667	8.07	740.5	9.4	≤	8.40	6.8
		74.6	2200	300	3.4	400.00	11.80	43667	8.07	731.1	9.4	≤	7.60	9.1
		74.6	2200	300	3.4	456.70	14.60	43667	8.07	728.9	9.4	≤	7.26	10.5
	B311	72.8	3460	310	3.4	575.68	17.00	43137	7.94	997.1	9.4	≤	7.39	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	7.94	1227.4	9.4	≤	14.19	1.4
		72.8	3460	310	3.4	200.00	5.00	43137	7.94	1024.8	9.4	≤	10.60	3.3
		72.8	3460	310	3.4	400.00	9.70	43137	7.94	999.5	9.4	≤	8.35	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	7.94	997.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	575.68	17.00	43137	7.94	997.1	9.4	≤	7.39	9.8
	B312	72.8	3460	310	3.4	560.36	16.90	43137	7.94	997.2	9.4	≤	7.45	9.5
		72.8	3460	310	3.4	100.00	2.50	43137	7.94	1227.4	9.4	≤	14.19	1.4
		72.8	3460	310	3.4	200.00	5.00	43137	7.94	1024.8	9.4	≤	10.60	3.3
		72.8	3460	310	3.4	400.00	9.70	43137	7.94	999.5	9.4	≤	8.35	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	7.94	997.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	560.36	16.90	43137	7.94	997.2	9.4	≤	7.45	9.5
	B313	72.8	3460	310	3.4	580.26	16.10	43137	7.94	997.1	9.4	≤	7.37	9.8
		72.8	3460	310	3.4	100.00	2.50	43137	7.94	1227.4	9.4	≤	14.19	1.4
		72.8	3460	310	3.4	200.00	5.00	43137	7.94	1024.8	9.4	≤	10.60	3.3
		72.8	3460	310	3.4	400.00	9.70	43137	7.94	999.5	9.4	≤	8.35	6.8
		72.8	3460	310	3.4	500.00	13.80	43137	7.94	997.7	9.4	≤	7.74	8.5
		72.8	3460	310	3.4	580.26	16.10	43137	7.94	997.1	9.4	≤	7.37	9.8
	B321	77	3460	620	3.4	551.16	15.50	44364	8.25	979.4	9.4	≤	7.61	9.3
		77	3460	620	3.4	100.00	1.70	44364	8.25	1251.3	9.4	≤	14.59	1.3
		77	3460	620	3.4	200.00	4.20	44364	8.25	1012.0	9.4	≤	10.79	3.2
		77	3460	620	3.4	400.00	10.00	44364	8.25	982.0	9.4	≤	8.48	6.7
		77	3460	620	3.4	500.00	13.30	44364	8.25	979.9	9.4	≤	7.87	8.4
		77	3460	620	3.4	551.16	15.50	44364	8.25	979.4	9.4	≤	7.61	9.3
	B331	72.8	3460	940	3.4	590.38	16.50	43137	7.94	997.0	9.4	≤	7.52	10.0
		72.8	3460	940	3.4	100.00	1.70	43137	7.94	1227.4	9.4	≤	14.56	1.4
		72.8	3460	940	3.4	200.00	4.20	43137	7.94	1024.8	9.4	≤	10.88	3.3
		72.8	3460	940	3.4	400.00	10.00	43137	7.94	999.5	9.4	≤	8.57	6.8
		72.8	3460	940	3.4	500.00	13.30	43137	7.94	997.7	9.4	≤	7.95	8.5
		72.8	3460	940	3.4	590.38	16.50	43137	7.94	997.0	9.4	≤	7.52	10.0
	B411	77	4730	320	3.4	621.34	15.50	44364	8.25	1192.3	9.4	≤	7.69	8.6
		77	4730	320	3.4	100.00		44364	8.25	1300.7	9.4	≤	14.55	1.3
		77	4730	320	3.4	200.00		44364	8.25	1205.5	9.4	≤	11.26	2.7
		77	4730	320	3.4	300.00		44364	8.25	1195.9	9.4	≤	9.81	4.1
		77	4730	320	3.4	400.00		44364	8.25	1193.6	9.4	≤	8.91	5.5
		77	4730	320	3.4	500.00		44364	8.25	1192.8	9.4	≤	8.27	6.9
		77	4730	320	3.4	621.34		44364	8.25	1192.3	9.4	≤	7.69	8.6
	C211	85.6	2710	300	3.4	560.94	18.20	46776	8.86	802.3	9.4	≤	7.14	10.9
		85.6	2710	300	3.4	100.00	2.50	46776	8.86	1309.2	9.4	≤	14.93	1.2
		85.6	2710	300	3.4	200.00	5.20	46776	8.86	863.1	9.4	≤	10.32	3.6
		85.6	2710	300	3.4	400.00	11.70	46776	8.86	807.4	9.4	≤	8.01	7.7
		85.6	2710	300	3.4	500.00	15.10	46776	8.86	803.5	9.4	≤	7.42	9.7
		85.6	2710	300	3.4	560.94	18.20	46776	8.86	802.3	9.4	≤	7.14	10.9
	C311	88.1	3220	310	3.4	605.46	18.00	47454	9.04	893.1	9.4	≤	7.23	10.4
		88.1	3220	310	3.4	100.00	1.70	47454	9.04	1333.3	9.4	≤	15.07	1.2
		88.1	3220	310	3.4	200.00	5.30	47454	9.04	947.1	9.4	≤	10.67	3.2
		88.1	3220	310	3.4	400.00	12.10	47454	9.04	898.1	9.4	≤	8.32	6.8
		88.1	3220	310	3.4	500.00	15.40	47454	9.04	894.7	9.4	≤	7.71	8.6
		88.1	3220	310	3.4	605.46	18.00	47454	9.04	893.1	9.4	≤	7.23	10.4
	C411	85.6	4260	320	3.4	722.56	19.30	46776	8.86	1081.3	9.4	≤	7.21	10.4
		85.6	4260	320	3.4	100.00	1.70	46776	8.86	1321.9	9.4	≤	14.91	1.2
		85.6	4260	320	3.4	200.00	4.70	46776	8.86	1110.8	9.4	≤	11.17	2.8
		85.6	4260	320	3.4	400.00	10.00	46776	8.86	1084.4	9.4	≤	8.79	5.8
		85.6	4260	320	3.4	600.00	15.80	46776	8.86	1081.8	9.4	≤	7.67	8.6
		85.6	4260	320	3.4	722.56	19.30	46776	8.86	1081.3	9.4	≤	7.21	10.4
	C511	88.1	5310	330	3.4	811.82	20.80	47454	9.04	1227.5	9.4	≤	7.26	10.2
		88.1	5310	330	3.4	100.00	1.70	47454	9.04	1333.3	9.4	≤	15.00	1.2
		88.1	5310	330	3.4	200.00	3.30	47454	9.04	1240.7	9.4	≤	11.62	2.5
		88.1	5310	330	3.4	400.00	8.70	47454	9.04	1229.0	9.4	≤	9.19	5.0
		88.1	5310	330	3.4	600.00	14.20	47454	9.04	1227.8	9.4	≤	8.03	7.5
		88.1	5310	330	3.4	811.82	20.80	47454	9.04	1227.5	9.4	≤	7.26	10.2
	D211	114.5	3220	300	3.4	506.00	16.00	54099	10.80	821.5	9.4	≤	7.80	8.3
		114.5	3220	300	3.4	100.00	1.60	54099	10.80	1333.3	9.4	≤	15.73	1.0
		114.5	3220	300	3.4	200.00	5.00	54099	10.80	926.6	9.4	≤	11.06	2.9
		114.5	3220	300	3.4	300.00	8.60	54099	10.80	847.8	9.4	≤	9.38	4.8
		114.5	3220	300	3.4	400.00	12.10	54099	10.80	828.6	9.4	≤	8.46	6.5
		114.5	3220	300	3.4	506.00	16.00	54099	10.80	821.5	9.4	≤	7.80	8.3
	E211	126.2	2200	300	3.4	506.00	15.90	56796	11.54	609.5	9.4	≤	7.22	10.7
		126.2	2200	300	3.4	100.00	1.70	56796	11.54	1333.3	9.4	≤	16.09	1.0
		126.2	2200	300	3.4	200.00	5.00	56796	11.54	791.4	9.4	≤	10.73	3.2
		126.2	2200	300	3.4	300.00	8.60	56796	11.54	655.0	9.4	≤	8.80	5.9
		126.2	2200	300	3.4	400.00	12.10	56796	11.54	621.8	9.4	≤	7.86	8.3
		126.2	2200	300	3.4	506.00	15.90	56796	11.54	609.5	9.4	≤	7.22	10.7

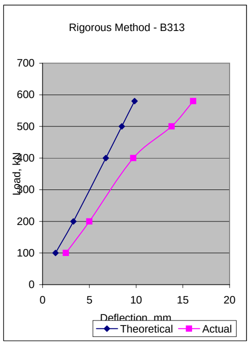

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

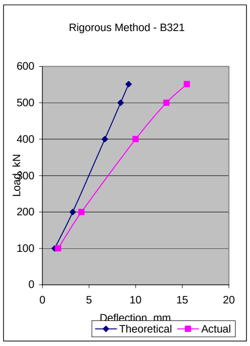

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

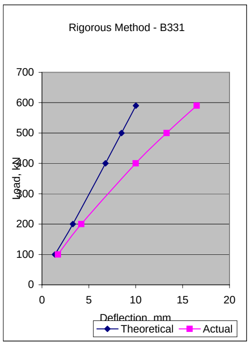

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

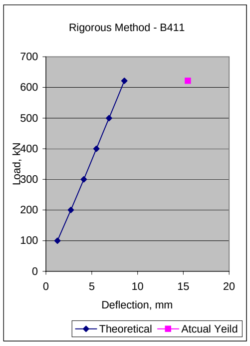

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

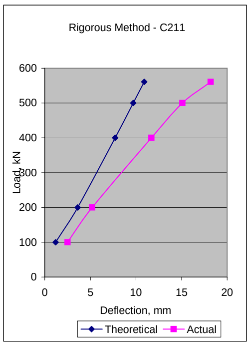

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

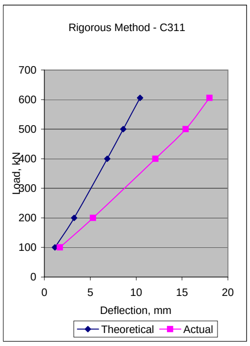

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

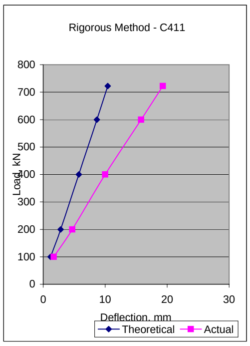

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

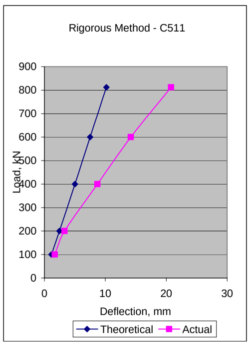

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

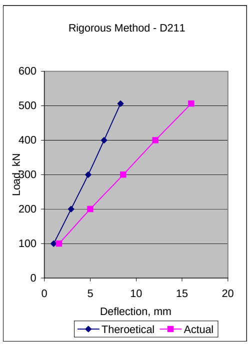

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Atual Yeild

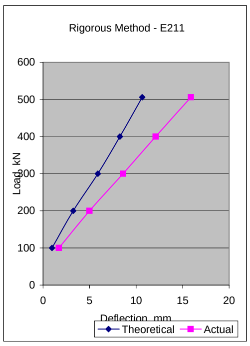

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

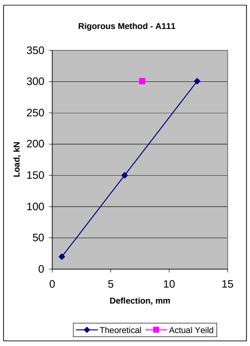

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

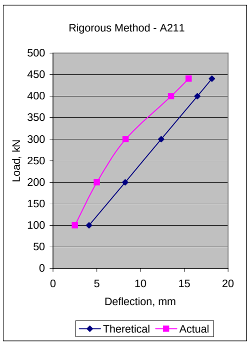

Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

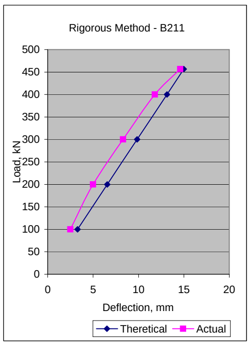
METHOD 7

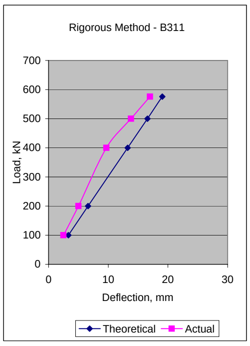
E_c =	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$0.045bd^3$
f'_{cf} =	NA

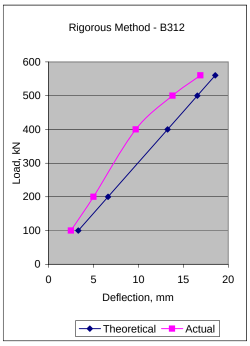
			Reinforcement Ratio			At Yield				Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	533.7	9.4	≤	9.71	12.4
		42.8	1250	300	3.4	20.00		33076	533.7	9.4	≤	17.19	0.8
		42.8	1250	300	3.4	150.00		33076	533.7	9.4	≤	8.78	6.2
		42.8	1250	300	3.4	300.78		33076	533.7	9.4	≤	6.96	12.4
	A211	42.8	2200	300	3.4	440.48	15.50	33076	533.7	9.4	≤	6.04	18.2
		42.8	2200	300	3.4	100.00	2.50	33076	533.7	9.4	≤	9.90	4.1
		42.8	2200	300	3.4	200.00	5.00	33076	533.7	9.4	≤	7.86	8.2
		42.8	2200	300	3.4	300.00	8.30	33076	533.7	9.4	≤	6.86	12.4
		42.8	2200	300	3.4	400.00	13.50	33076	533.7	9.4	≤	6.24	16.5
		42.8	2200	300	3.4	440.48	15.50	33076	533.7	9.4	≤	6.04	18.2
	B211	74.6	2200	300	3.4	456.70	14.60	41519	533.7	9.4	≤	6.44	15.0
		74.6	2200	300	3.4	100.00	2.50	41519	533.7	9.4	≤	10.68	3.3
		74.6	2200	300	3.4	200.00	5.00	41519	533.7	9.4	≤	8.48	6.6
		74.6	2200	300	3.4	300.00	8.30	41519	533.7	9.4	≤	7.41	9.8
		74.6	2200	300	3.4	400.00	11.80	41519	533.7	9.4	≤	6.73	13.1
		74.6	2200	300	3.4	456.70	14.60	41519	533.7	9.4	≤	6.44	15.0
	B311	72.8	3460	310	3.4	575.68	17.00	41170	533.7	9.4	≤	5.90	19.1
		72.8	3460	310	3.4	100.00	2.50	41170	533.7	9.4	≤	10.58	3.3
		72.8	3460	310	3.4	200.00	5.00	41170	533.7	9.4	≤	8.40	6.6
		72.8	3460	310	3.4	400.00	9.70	41170	533.7	9.4	≤	6.67	13.2
		72.8	3460	310	3.4	500.00	13.80	41170	533.7	9.4	≤	6.19	16.6
		72.8	3460	310	3.4	575.68	17.00	41170	533.7	9.4	≤	5.90	19.1
	B312	72.8	3460	310	3.4	560.36	16.90	41170	533.7	9.4	≤	5.96	18.6
		72.8	3460	310	3.4	100.00	2.50	41170	533.7	9.4	≤	10.58	3.3
		72.8	3460	310	3.4	200.00	5.00	41170	533.7	9.4	≤	8.40	6.6
		72.8	3460	310	3.4	400.00	9.70	41170	533.7	9.4	≤	6.67	13.2
		72.8	3460	310	3.4	500.00	13.80	41170	533.7	9.4	≤	6.19	16.6
		72.8	3460	310	3.4	560.36	16.90	41170	533.7	9.4	≤	5.96	18.6
	B313	72.8	3460	310	3.4	580.26	16.10	41170	533.7	9.4	≤	5.89	19.2
		72.8	3460	310	3.4	100.00	2.50	41170	533.7	9.4	≤	10.58	3.3
		72.8	3460	310	3.4	200.00	5.00	41170	533.7	9.4	≤	8.40	6.6
		72.8	3460	310	3.4	400.00	9.70	41170	533.7	9.4	≤	6.67	13.2
		72.8	3460	310	3.4	500.00	13.80	41170	533.7	9.4	≤	6.19	16.6
		72.8	3460	310	3.4	580.26	16.10	41170	533.7	9.4	≤	5.89	19.2
	B321	77	3460	620	3.4	551.16	15.50	41968	533.7	9.4	≤	6.11	17.9
		77	3460	620	3.4	100.00	1.70	41968	533.7	9.4	≤	10.78	3.2
		77	3460	620	3.4	200.00	4.20	41968	533.7	9.4	≤	8.56	6.5
		77	3460	620	3.4	400.00	10.00	41968	533.7	9.4	≤	6.79	13.0
		77	3460	620	3.4	500.00	13.30	41968	533.7	9.4	≤	6.31	16.2
		77	3460	620	3.4	551.16	15.50	41968	533.7	9.4	≤	6.11	17.9
	B331	72.8	3460	940	3.4	590.38	16.50	41170	533.7	9.4	≤	6.01	19.5
		72.8	3460	940	3.4	100.00	1.70	41170	533.7	9.4	≤	10.86	3.3
		72.8	3460	940	3.4	200.00	4.20	41170	533.7	9.4	≤	8.62	6.6
		72.8	3460	940	3.4	400.00	10.00	41170	533.7	9.4	≤	6.84	13.2
		72.8	3460	940	3.4	500.00	13.30	41170	533.7	9.4	≤	6.35	16.6
		72.8	3460	940	3.4	590.38	16.50	41170	533.7	9.4	≤	6.01	19.5
	B411	77	4730	320	3.4	621.34	15.50	41968	533.7	9.4	≤	5.78	20.2
		77	4730	320	3.4	100.00		41968	533.7	9.4	≤	10.62	3.2
		77	4730	320	3.4	200.00		41968	533.7	9.4	≤	8.43	6.5
		77	4730	320	3.4	300.00		41968	533.7	9.4	≤	7.36	9.7
		77	4730	320	3.4	400.00		41968	533.7	9.4	≤	6.69	13.0
		77	4730	320	3.4	500.00		41968	533.7	9.4	≤	6.21	16.2
		77	4730	320	3.4	621.34		41968	533.7	9.4	≤	5.78	20.2
	C211	85.6	2710	300	3.4	560.94	18.20	43445	533.7	9.4	≤	6.08	17.6
		85.6	2710	300	3.4	100.00	2.50	43445	533.7	9.4	≤	10.80	3.1
		85.6	2710	300	3.4	200.00	5.20	43445	533.7	9.4	≤	8.58	6.3
		85.6	2710	300	3.4	400.00	11.70	43445	533.7	9.4	≤	6.81	12.6
		85.6	2710	300	3.4	500.00	15.10	43445	533.7	9.4	≤	6.32	15.7
		85.6	2710	300	3.4	560.94	18.20	43445	533.7	9.4	≤	6.08	17.6
	C311	88.1	3220	310	3.4	605.46	18.00	43838	533.7	9.4	≤	5.93	18.8
		88.1	3220	310	3.4	100.00	1.70	43838	533.7	9.4	≤	10.81	3.1
		88.1	3220	310	3.4	200.00	5.30	43838	533.7	9.4	≤	8.58	6.2
		88.1	3220	310	3.4	400.00	12.10	43838	533.7	9.4	≤	6.81	12.4
		88.1	3220	310	3.4	500.00	15.40	43838	533.7	9.4	≤	6.32	15.5
		88.1	3220	310	3.4	605.46	18.00	43838	533.7	9.4	≤	5.93	18.8
	C411	85.6	4260	320	3.4	722.56	19.30	43445	533.7	9.4	≤	5.56	22.7
		85.6	4260	320	3.4	100.00	1.70	43445	533.7	9.4	≤	10.75	3.1
		85.6	4260	320	3.4	200.00	4.70	43445	533.7	9.4	≤	8.53	6.3
		85.6	4260	320	3.4	400.00	10.00	43445	533.7	9.4	≤	6.77	12.6
		85.6	4260	320	3.4	600.00	15.80	43445	533.7	9.4	≤	5.92	18.8
		85.6	4260	320	3.4	722.56	19.30	43445	533.7	9.4	≤	5.56	22.7
	C511	88.1	5310	330	3.4	811.82	20.80	43838	533.7	9.4	≤	5.36	25.2
		88.1	5310	330	3.4	100.00	1.70	43838	533.7	9.4	≤	10.76	3.1
		88.1	5310	330	3.4	200.00	3.30	43838	533.7	9.4	≤	8.54	6.2
		88.1	5310	330	3.4	400.00	8.70	43838	533.7	9.4	≤	6.78	12.4
		88.1	5310	330	3.4	600.00	14.20	43838	533.7	9.4	≤	5.92	18.7
		88.1	5310	330	3.4	811.82	20.80	43838	533.7	9.4	≤	5.36	25.2
	D211	114.5	3220	300	3.4	506.00	16.00	47120	533.7	9.4	≤	6.45	14.6
		114.5	3220	300	3.4	100.00	1.60	47120	533.7	9.4	≤	11.07	2.9
		114.5	3220	300	3.4	200.00	5.00	47120	533.7	9.4	≤	8.79	5.8
		114.5	3220	300	3.4	300.00	8.60	47120	533.7	9.4	≤	7.68	8.7
		114.5	3220	300	3.4	400.00	12.10	47120	533.7	9.4	≤	6.98	11.6
		114.5	3220	300	3.4	506.00	16.00	47120	533.7	9.4	≤	6.45	14.6
	E211	126.2	2200	300	3.4	506.00	15.90	48140	533.7	9.4	≤	6.54	14.3
		126.2	2200	300	3.4	100.00	1.70	48140	533.7	9.4	≤	11.22	2.8
		126.2	2200	300	3.4	200.00	5.00	48140	533.7	9.4	≤	8.91	5.7
		126.2	2200	300	3.4	300.00	8.60	48140	533.7	9.4	≤	7.78	8.5
		126.2	2200	300	3.4	400.00	12.10	48140	533.7	9.4	≤	7.07	11.3
		126.2	2200	300	3.4	506.00	15.90	48140	533.7	9.4	≤	6.54	14.3

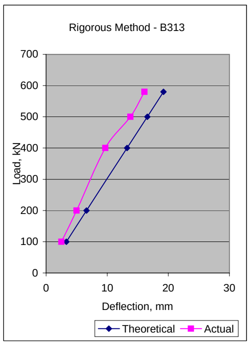

Rigorous Method - A111
350 300 250 200 150 100 50 0
0 5 10 15
Load, kN
Deflection, mm
Theoretical
Actual Yeild

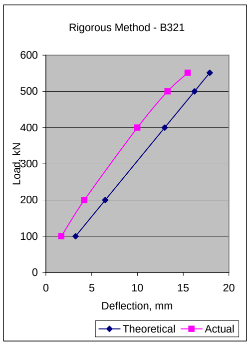

Rigorous Method - A211
500 450 400 350 300 250 200 150 100 50 0
0 5 10 15 20
Load, kN
Deflection, mm
Theretical
Actual

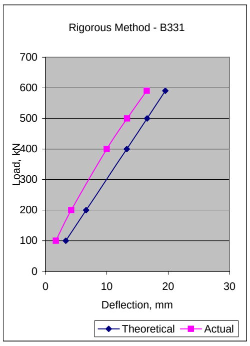

Rigorous Method - B211
500 450 400 350 300 250 200 150 100 50 0
0 5 10 15 20
Load, kN
Deflection, mm
Theretical
Actual

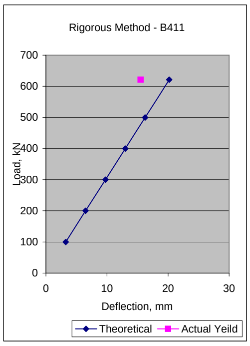

Rigorous Method - B311
700 600 500 400 300 200 100 0
0 10 20 30
Load, kN
Deflection, mm
Theoretical
Actual

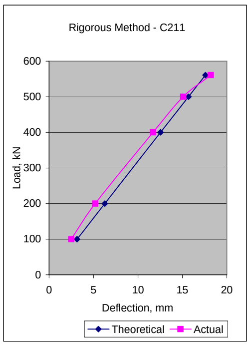

Rigorous Method - B312
600 500 400 300 200 100 0
0 5 10 15 20
Load, kN
Deflection, mm
Theoretical
Actual

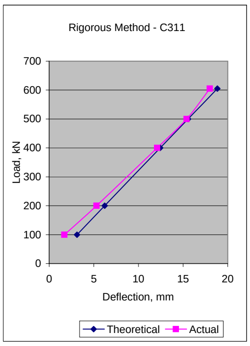

Rigorous Method - B313
700 600 500 400 300 200 100 0
0 10 20 30
Load, kN
Deflection, mm
Theoretical
Actual

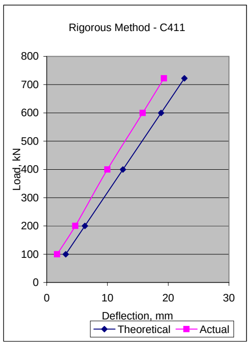

Rigorous Method - B321
600 500 400 300 200 100 0
0 5 10 15 20
Load, kN
Deflection, mm
Theoretical
Actual

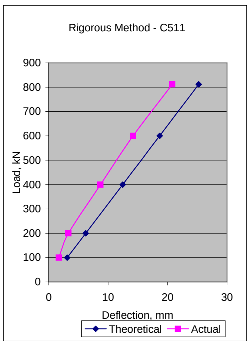

Rigorous Method - B331
700 600 500 400 300 200 100 0
0 10 20 30
Load, kN
Deflection, mm
Theoretical
Actual

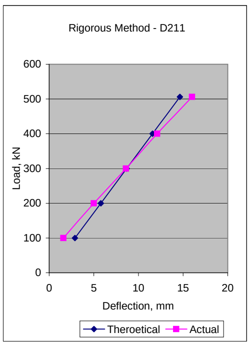

Rigorous Method - B411
700 600 500 400 300 200 100 0
0 10 20 30
Load, kN
Deflection, mm
Theoretical
Actual Yeild

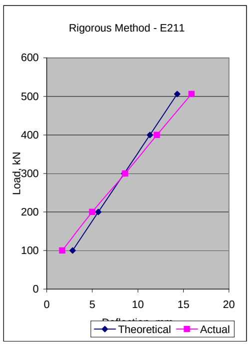

Rigorous Method - C211
600 500 400 300 200 100 0
0 5 10 15 20
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
700 600 500 400 300 200 100 0
0 5 10 15 20
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
800 700 600 500 400 300 200 100 0
0 10 20 30
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
900 800 700 600 500 400 300 200 100 0
0 10 20 30
Load, kN
Deflection, mm
Theoretical
Actual

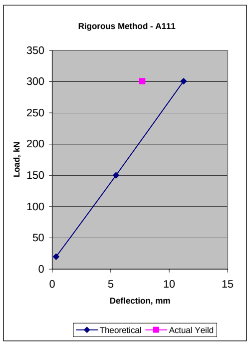

Rigorous Method - D211
600 500 400 300 200 100 0
0 5 10 15 20
Load, kN
Deflection, mm
Thereotical
Actual

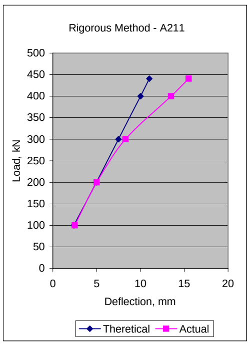

Rigorous Method - E211
600 500 400 300 200 100 0
0 5 10 15 20
Load, kN
Theoretical
Actual

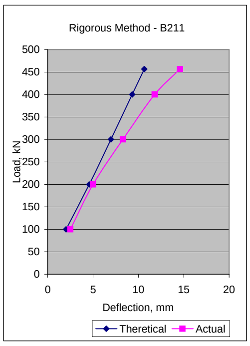
METHOD 8

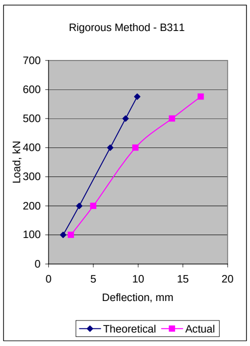
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.6\sqrt{f'_c}$

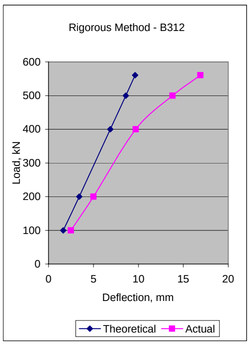
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	3.93	588.9	9.4	≤	10.03	11.2
		42.8	1250	300	3.4	20.00		33076	3.93	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	3.93	605.0	9.4	≤	9.15	5.5
		42.8	1250	300	3.4	300.78		33076	3.93	588.9	9.4	≤	7.19	11.2
	A211	42.8	2200	300	3.4	440.48	15.50	33076	3.93	879.6	9.4	≤	7.13	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	3.93	916.8	9.4	≤	11.86	2.4
		42.8	2200	300	3.4	200.00	5.00	33076	3.93	883.8	9.4	≤	9.30	5.0
		42.8	2200	300	3.4	300.00	8.30	33076	3.93	880.5	9.4	≤	8.11	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	3.93	879.7	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	3.93	879.6	9.4	≤	7.13	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	41519	5.18	751.9	9.4	≤	7.22	10.6
		74.6	2200	300	3.4	100.00	2.50	41519	5.18	862.0	9.4	≤	12.53	2.0
		74.6	2200	300	3.4	200.00	5.00	41519	5.18	764.7	9.4	≤	9.56	4.6
		74.6	2200	300	3.4	300.00	8.30	41519	5.18	754.9	9.4	≤	8.31	7.0
		74.6	2200	300	3.4	400.00	11.80	41519	5.18	752.5	9.4	≤	7.54	9.3
		74.6	2200	300	3.4	456.70	14.60	41519	5.18	751.9	9.4	≤	7.22	10.6
	B311	72.8	3460	310	3.4	575.68	17.00	41170	5.12	1026.8	9.4	≤	7.34	9.9
		72.8	3460	310	3.4	100.00	2.50	41170	5.12	1083.0	9.4	≤	13.40	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.12	1033.6	9.4	≤	10.47	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.12	1027.4	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.12	1027.0	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	575.68	17.00	41170	5.12	1026.8	9.4	≤	7.34	9.9
	B312	72.8	3460	310	3.4	560.36	16.90	41170	5.12	1026.8	9.4	≤	7.41	9.6
		72.8	3460	310	3.4	100.00	2.50	41170	5.12	1083.0	9.4	≤	13.40	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.12	1033.6	9.4	≤	10.47	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.12	1027.4	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.12	1027.0	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	560.36	16.90	41170	5.12	1026.8	9.4	≤	7.41	9.6
	B313	72.8	3460	310	3.4	580.26	16.10	41170	5.12	1026.8	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	41170	5.12	1083.0	9.4	≤	13.40	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.12	1033.6	9.4	≤	10.47	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.12	1027.4	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.12	1027.0	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	580.26	16.10	41170	5.12	1026.8	9.4	≤	7.32	10.0
	B321	77	3460	620	3.4	551.16	15.50	41968	5.26	1014.2	9.4	≤	7.56	9.4
		77	3460	620	3.4	100.00	1.70	41968	5.26	1077.8	9.4	≤	13.63	1.6
		77	3460	620	3.4	200.00	4.20	41968	5.26	1021.8	9.4	≤	10.63	3.4
		77	3460	620	3.4	400.00	10.00	41968	5.26	1014.8	9.4	≤	8.42	6.8
		77	3460	620	3.4	500.00	13.30	41968	5.26	1014.3	9.4	≤	7.81	8.5
		77	3460	620	3.4	551.16	15.50	41968	5.26	1014.2	9.4	≤	7.56	9.4
	B331	72.8	3460	940	3.4	590.38	16.50	41170	5.12	1026.8	9.4	≤	7.47	10.2
		72.8	3460	940	3.4	100.00	1.70	41170	5.12	1083.0	9.4	≤	13.75	1.6
		72.8	3460	940	3.4	200.00	4.20	41170	5.12	1033.6	9.4	≤	10.75	3.4
		72.8	3460	940	3.4	400.00	10.00	41170	5.12	1027.4	9.4	≤	8.51	6.9
		72.8	3460	940	3.4	500.00	13.30	41170	5.12	1027.0	9.4	≤	7.90	8.6
		72.8	3460	940	3.4	590.38	16.50	41170	5.12	1026.8	9.4	≤	7.47	10.2
	B411	77	4730	320	3.4	621.34	15.50	41968	5.26	1232.7	9.4	≤	7.63	8.7
		77	4730	320	3.4	100.00		41968	5.26	1252.8	9.4	≤	14.11	1.4
		77	4730	320	3.4	200.00		41968	5.26	1235.1	9.4	≤	11.15	2.8
		77	4730	320	3.4	300.00		41968	5.26	1233.3	9.4	≤	9.73	4.2
		77	4730	320	3.4	400.00		41968	5.26	1232.9	9.4	≤	8.84	5.6
		77	4730	320	3.4	500.00		41968	5.26	1232.8	9.4	≤	8.21	7.0
		77	4730	320	3.4	621.34		41968	5.26	1232.7	9.4	≤	7.63	8.7
	C211	85.6	2710	300	3.4	560.94	18.20	43445	5.55	842.0	9.4	≤	7.08	11.2
		85.6	2710	300	3.4	100.00	2.50	43445	5.55	956.8	9.4	≤	13.13	1.8
		85.6	2710	300	3.4	200.00	5.20	43445	5.55	855.8	9.4	≤	10.04	3.9
		85.6	2710	300	3.4	400.00	11.70	43445	5.55	843.2	9.4	≤	7.93	7.9
		85.6	2710	300	3.4	500.00	15.10	43445	5.55	842.3	9.4	≤	7.36	9.9
		85.6	2710	300	3.4	560.94	18.20	43445	5.55	842.0	9.4	≤	7.08	11.2
	C311	88.1	3220	310	3.4	605.46	18.00	43838	5.63	940.2	9.4	≤	7.17	10.7
		88.1	3220	310	3.4	100.00	1.70	43838	5.63	1036.2	9.4	≤	13.49	1.6
		88.1	3220	310	3.4	200.00	5.30	43838	5.63	951.8	9.4	≤	10.41	3.5
		88.1	3220	310	3.4	400.00	12.10	43838	5.63	941.3	9.4	≤	8.23	7.1
		88.1	3220	310	3.4	500.00	15.40	43838	5.63	940.5	9.4	≤	7.64	8.8
		88.1	3220	310	3.4	605.46	18.00	43838	5.63	940.2	9.4	≤	7.17	10.7
	C411	85.6	4260	320	3.4	722.56	19.30	43445	5.55	1132.1	9.4	≤	7.15	10.7
		85.6	4260	320	3.4	100.00	1.70	43445	5.55	1179.2	9.4	≤	14.00	1.4
		85.6	4260	320	3.4	200.00	4.70	43445	5.55	1137.9	9.4	≤	10.98	2.9
		85.6	4260	320	3.4	400.00	10.00	43445	5.55	1132.7	9.4	≤	8.70	5.9
		85.6	4260	320	3.4	600.00	15.80	43445	5.55	1132.2	9.4	≤	7.60	8.9
		85.6	4260	320	3.4	722.56	19.30	43445	5.55	1132.1	9.4	≤	7.15	10.7
	C511	88.1	5310	330	3.4	811.82	20.80	43838	5.63	1286.6	9.4	≤	7.18	10.5
		88.1	5310	330	3.4	100.00	1.70	43838	5.63	1298.0	9.4	≤	14.48	1.3
		88.1	5310	330	3.4	200.00	3.30	43838	5.63	1288.0	9.4	≤	11.46	2.6
		88.1	5310	330	3.4	400.00	8.70	43838	5.63	1286.8	9.4	≤	9.09	5.2
		88.1	5310	330	3.4	600.00	14.20	43838	5.63	1286.6	9.4	≤	7.94	7.7
		88.1	5310	330	3.4	811.82	20.80	43838	5.63	1286.6	9.4	≤	7.18	10.5
	D211	114.5	3220	300	3.4	506.00	16.00	47120	6.42	896.6	9.4	≤	7.67	8.7
		114.5	3220	300	3.4	100.00	1.60	47120	6.42	1054.4	9.4	≤	13.90	1.5
		114.5	3220	300	3.4	200.00	5.00	47120	6.42	915.2	9.4	≤	10.52	3.4
		114.5	3220	300	3.4	300.00	8.60	47120	6.42	901.3	9.4	≤	9.14	5.1
		114.5	3220	300	3.4	400.00	12.10	47120	6.42	897.9	9.4	≤	8.30	6.9
		114.5	3220	300	3.4	506.00	16.00	47120	6.42	896.6	9.4	≤	7.67	8.7
	E211	126.2	2200	300	3.4	506.00	15.90	48140	6.74	677.2	9.4	≤	7.08	11.3
		126.2	2200	300	3.4	100.00	1.70	48140	6.74	951.6	9.4	≤	13.61	1.6
		126.2	2200	300	3.4	200.00	5.00	48140	6.74	709.6	9.4	≤	9.79	4.3
		126.2	2200	300	3.4	300.00	8.60	48140	6.74	685.3	9.4	≤	8.46	6.6
		126.2	2200	300	3.4	400.00	12.10	48140	6.74	679.4	9.4	≤	7.66	8.9
		126.2	2200	300	3.4	506.00	15.90	48140	6.74	677.2	9.4	≤	7.08	11.3

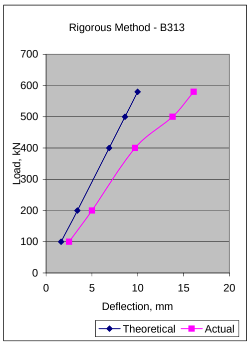

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

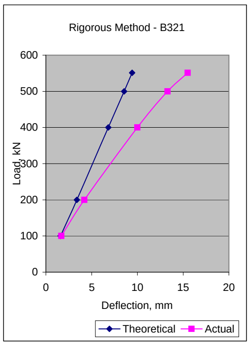

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

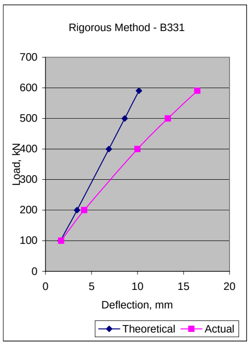

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

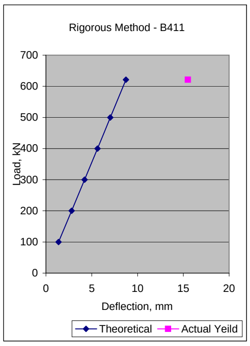

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

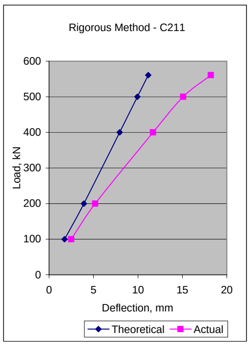

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

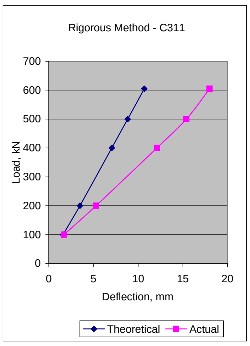

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

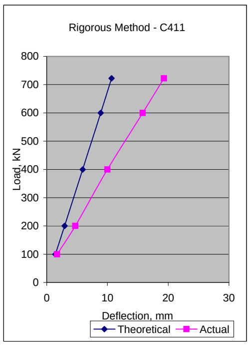

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

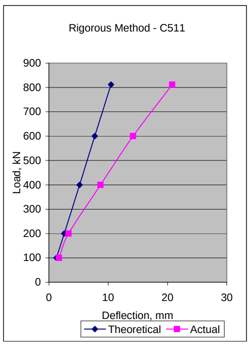

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

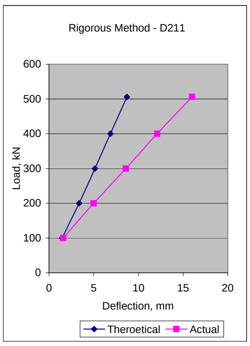

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

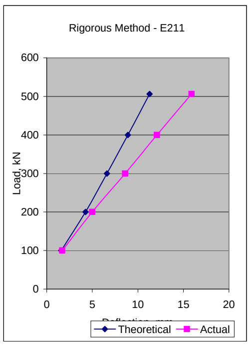

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

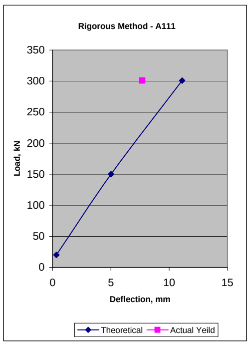

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

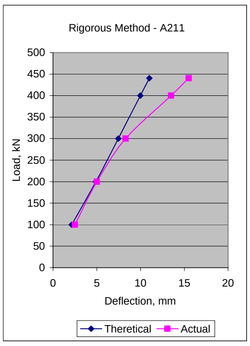

Rigorous Method - E211
Load, kN
Theoretical
Actual

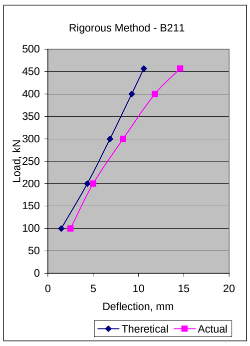
METHOD 9

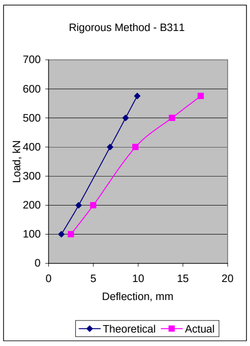
E_c =	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
f'_{cf} =	$0.94\sqrt{f'_c}$

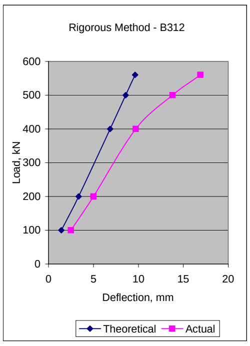
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, $10^6 mm$	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	6.15	595.4	9.4	≤	10.07	11.1
		42.8	1250	300	3.4	20.00		33076	6.15	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	6.15	657.2	9.4	≤	9.41	5.0
		42.8	1250	300	3.4	300.78		33076	6.15	595.4	9.4	≤	7.22	11.1
	A211	42.8	2200	300	3.4	440.48	15.50	33076	6.15	880.8	9.4	≤	7.14	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	6.15	1024.0	9.4	≤	12.30	2.1
		42.8	2200	300	3.4	200.00	5.00	33076	6.15	897.2	9.4	≤	9.34	4.9
		42.8	2200	300	3.4	300.00	8.30	33076	6.15	884.5	9.4	≤	8.12	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	6.15	881.4	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	6.15	880.8	9.4	≤	7.14	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	41519	8.12	755.2	9.4	≤	7.23	10.6
		74.6	2200	300	3.4	100.00	2.50	41519	8.12	1178.4	9.4	≤	13.91	1.5
		74.6	2200	300	3.4	200.00	5.00	41519	8.12	804.2	9.4	≤	9.72	4.4
		74.6	2200	300	3.4	300.00	8.30	41519	8.12	766.6	9.4	≤	8.36	6.9
		74.6	2200	300	3.4	400.00	11.80	41519	8.12	757.4	9.4	≤	7.56	9.3
		74.6	2200	300	3.4	456.70	14.60	41519	8.12	755.2	9.4	≤	7.23	10.6
	B311	72.8	3460	310	3.4	575.68	17.00	41170	8.02	1027.7	9.4	≤	7.35	9.9
		72.8	3460	310	3.4	100.00	2.50	41170	8.02	1243.7	9.4	≤	14.03	1.4
		72.8	3460	310	3.4	200.00	5.00	41170	8.02	1053.7	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	8.02	1029.9	9.4	≤	8.30	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	8.02	1028.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	575.68	17.00	41170	8.02	1027.7	9.4	≤	7.35	9.9
	B312	72.8	3460	310	3.4	560.36	16.90	41170	8.02	1027.7	9.4	≤	7.41	9.6
		72.8	3460	310	3.4	100.00	2.50	41170	8.02	1243.7	9.4	≤	14.03	1.4
		72.8	3460	310	3.4	200.00	5.00	41170	8.02	1053.7	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	8.02	1029.9	9.4	≤	8.30	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	8.02	1028.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	560.36	16.90	41170	8.02	1027.7	9.4	≤	7.41	9.6
	B313	72.8	3460	310	3.4	580.26	16.10	41170	8.02	1027.6	9.4	≤	7.33	10.0
		72.8	3460	310	3.4	100.00	2.50	41170	8.02	1243.7	9.4	≤	14.03	1.4
		72.8	3460	310	3.4	200.00	5.00	41170	8.02	1053.7	9.4	≤	10.54	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	8.02	1029.9	9.4	≤	8.30	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	8.02	1028.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	580.26	16.10	41170	8.02	1027.6	9.4	≤	7.33	10.0
	B321	77	3460	620	3.4	551.16	15.50	41968	8.25	1015.3	9.4	≤	7.57	9.4
		77	3460	620	3.4	100.00	1.70	41968	8.25	1259.8	9.4	≤	14.36	1.4
		77	3460	620	3.4	200.00	4.20	41968	8.25	1044.6	9.4	≤	10.71	3.3
		77	3460	620	3.4	400.00	10.00	41968	8.25	1017.7	9.4	≤	8.42	6.8
		77	3460	620	3.4	500.00	13.30	41968	8.25	1015.8	9.4	≤	7.82	8.5
		77	3460	620	3.4	551.16	15.50	41968	8.25	1015.3	9.4	≤	7.57	9.4
	B331	72.8	3460	940	3.4	590.38	16.50	41170	8.02	1027.6	9.4	≤	7.48	10.2
		72.8	3460	940	3.4	100.00	1.70	41170	8.02	1243.7	9.4	≤	14.40	1.4
		72.8	3460	940	3.4	200.00	4.20	41170	8.02	1053.7	9.4	≤	10.82	3.4
		72.8	3460	940	3.4	400.00	10.00	41170	8.02	1029.9	9.4	≤	8.52	6.9
		72.8	3460	940	3.4	500.00	13.30	41170	8.02	1028.2	9.4	≤	7.90	8.6
		72.8	3460	940	3.4	590.38	16.50	41170	8.02	1027.6	9.4	≤	7.48	10.2
	B411	77	4730	320	3.4	621.34	15.50	41968	8.25	1232.9	9.4	≤	7.63	8.7
		77	4730	320	3.4	100.00		41968	8.25	1310.1	9.4	≤	14.32	1.3
		77	4730	320	3.4	200.00		41968	8.25	1242.3	9.4	≤	11.17	2.8
		77	4730	320	3.4	300.00		41968	8.25	1235.5	9.4	≤	9.74	4.2
		77	4730	320	3.4	400.00		41968	8.25	1233.8	9.4	≤	8.84	5.6
		77	4730	320	3.4	500.00		41968	8.25	1233.2	9.4	≤	8.21	7.0
		77	4730	320	3.4	621.34		41968	8.25	1232.9	9.4	≤	7.63	8.7
	C211	85.6	2710	300	3.4	560.94	18.20	43445	8.70	843.9	9.4	≤	7.08	11.1
		85.6	2710	300	3.4	100.00	2.50	43445	8.70	1285.3	9.4	≤	14.48	1.3
		85.6	2710	300	3.4	200.00	5.20	43445	8.70	896.9	9.4	≤	10.20	3.7
		85.6	2710	300	3.4	400.00	11.70	43445	8.70	848.3	9.4	≤	7.94	7.9
		85.6	2710	300	3.4	500.00	15.10	43445	8.70	844.9	9.4	≤	7.36	9.9
		85.6	2710	300	3.4	560.94	18.20	43445	8.70	843.9	9.4	≤	7.08	11.1
	C311	88.1	3220	310	3.4	605.46	18.00	43838	8.82	941.4	9.4	≤	7.17	10.7
		88.1	3220	310	3.4	100.00	1.70	43838	8.82	1310.6	9.4	≤	14.59	1.3
		88.1	3220	310	3.4	200.00	5.30	43838	8.82	986.1	9.4	≤	10.53	3.4
		88.1	3220	310	3.4	400.00	12.10	43838	8.82	945.6	9.4	≤	8.24	7.0
		88.1	3220	310	3.4	500.00	15.40	43838	8.82	942.7	9.4	≤	7.65	8.8
		88.1	3220	310	3.4	605.46	18.00	43838	8.82	941.4	9.4	≤	7.17	10.7
	C411	85.6	4260	320	3.4	722.56	19.30	43445	8.70	1132.5	9.4	≤	7.15	10.7
		85.6	4260	320	3.4	100.00	1.70	43445	8.70	1313.7	9.4	≤	14.52	1.3
		85.6	4260	320	3.4	200.00	4.70	43445	8.70	1154.7	9.4	≤	11.04	2.9
		85.6	4260	320	3.4	400.00	10.00	43445	8.70	1134.8	9.4	≤	8.71	5.9
		85.6	4260	320	3.4	600.00	15.80	43445	8.70	1132.8	9.4	≤	7.60	8.9
		85.6	4260	320	3.4	722.56	19.30	43445	8.70	1132.5	9.4	≤	7.15	10.7
	C511	88.1	5310	330	3.4	811.82	20.80	43838	8.82	1286.7	9.4	≤	7.18	10.5
		88.1	5310	330	3.4	100.00	1.70	43838	8.82	1330.6	9.4	≤	14.60	1.2
		88.1	5310	330	3.4	200.00	3.30	43838	8.82	1292.1	9.4	≤	11.47	2.6
		88.1	5310	330	3.4	400.00	8.70	43838	8.82	1287.3	9.4	≤	9.09	5.2
		88.1	5310	330	3.4	600.00	14.20	43838	8.82	1286.8	9.4	≤	7.94	7.7
		88.1	5310	330	3.4	811.82	20.80	43838	8.82	1286.7	9.4	≤	7.18	10.5
	D211	114.5	3220	300	3.4	506.00	16.00	47120	10.06	900.1	9.4	≤	7.68	8.7
		114.5	3220	300	3.4	100.00	1.60	47120	10.06	1333.3	9.4	≤	15.03	1.2
		114.5	3220	300	3.4	200.00	5.00	47120	10.06	971.8	9.4	≤	10.73	3.2
		114.5	3220	300	3.4	300.00	8.60	47120	10.06	918.0	9.4	≤	9.20	5.0
		114.5	3220	300	3.4	400.00	12.10	47120	10.06	904.9	9.4	≤	8.32	6.8
		114.5	3220	300	3.4	506.00	16.00	47120	10.06	900.1	9.4	≤	7.68	8.7
	E211	126.2	2200	300	3.4	506.00	15.90	48140	10.56	683.2	9.4	≤	7.10	11.2
		126.2	2200	300	3.4	100.00	1.70	48140	10.56	1333.3	9.4	≤	15.23	1.1
		126.2	2200	300	3.4	200.00	5.00	48140	10.56	808.0	9.4	≤	10.23	3.7
		126.2	2200	300	3.4	300.00	8.60	48140	10.56	714.4	9.4	≤	8.57	6.3
		126.2	2200	300	3.4	400.00	12.10	48140	10.56	691.6	9.4	≤	7.71	8.7
		126.2	2200	300	3.4	506.00	15.90	48140	10.56	683.2	9.4	≤	7.10	11.2

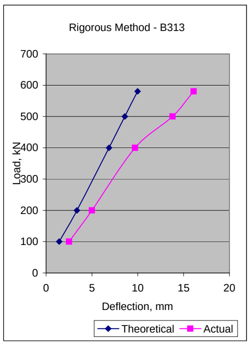

Rigorous Method - A111
Load, kN
350 300 250 200 150 100 50 0
0 5 10 15
Deflection, mm
Theoretical
Actual Yeild

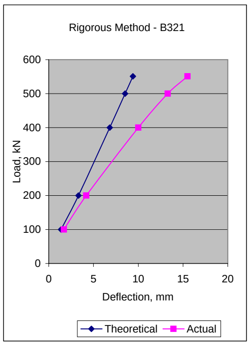

Rigorous Method - A211
Load, kN
500 450 400 350 300 250 200 150 100 50 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

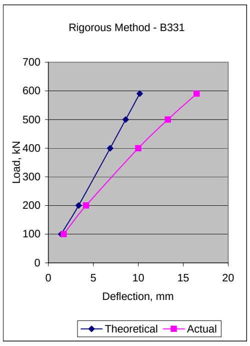

Rigorous Method - B211
Load, kN
500 450 400 350 300 250 200 150 100 50 0
0 5 10 15 20
Deflection, mm
Theretical
Actual

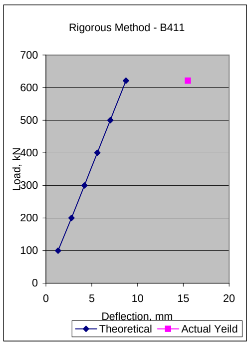

Rigorous Method - B311
Load, kN
700 600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

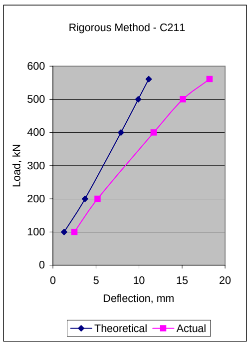

Rigorous Method - B312
Load, kN
600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

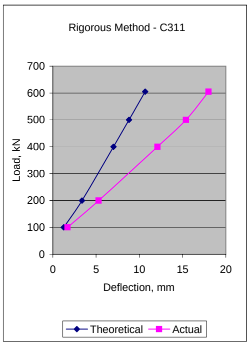

Rigorous Method - B313
Load, kN
700 600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

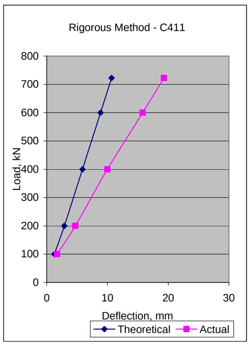

Rigorous Method - B321
Load, kN
600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

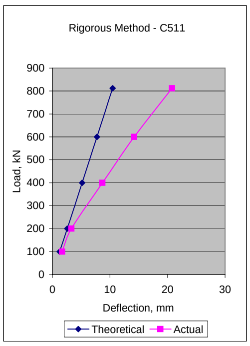

Rigorous Method - B331
Load, kN
700 600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

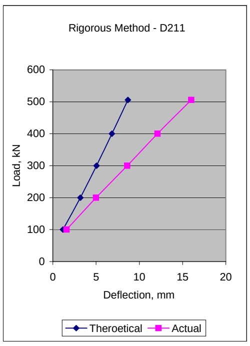

Rigorous Method - B411
Load, kN
700 600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual Yeild

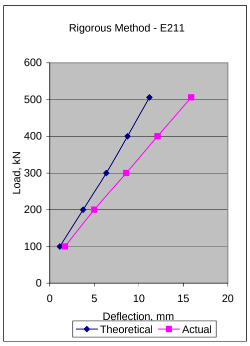

Rigorous Method - C211
Load, kN
600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
700 600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
800 700 600 500 400 300 200 100 0
0 10 20 30
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
900 800 700 600 500 400 300 200 100 0
0 10 20 30
Deflection, mm
Theoretical
Actual

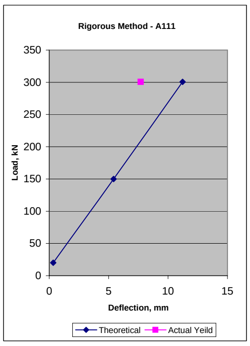

Rigorous Method - D211
Load, kN
600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

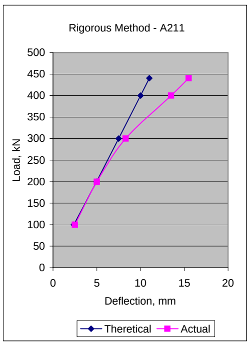

Rigorous Method - E211
Load, kN
600 500 400 300 200 100 0
0 5 10 15 20
Deflection, mm
Theoretical
Actual

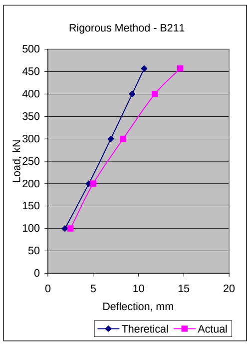
METHOD 10

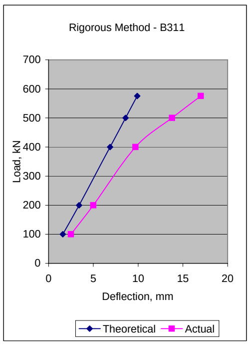
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.342(f'_c)^{2/3}$

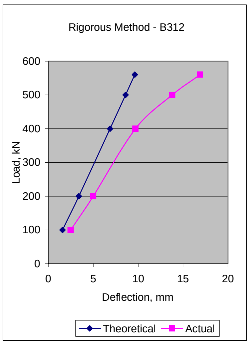
SOURCE	BEAM	f'_c, MPa	Reinforcement Ratio: Area of Steel, A_{st}, mm^2	Reinforcement Ratio: Area of Steel, A_{sc}, mm^2	L_{eff}, m	At Yield: Load, kN	At Yield: Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Deem-to-Comply: Ratio	≤	Deem-to-Comply: Allowed Ratio	Rigorous Method: Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	4.18	589.4	9.4	≤	10.04	11.2
		42.8	1250	300	3.4	20.00		33076	4.18	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	4.18	608.9	9.4	≤	9.17	5.4
		42.8	1250	300	3.4	300.78		33076	4.18	589.4	9.4	≤	7.20	11.2
	A211	42.8	2200	300	3.4	440.48	15.50	33076	4.18	879.7	9.4	≤	7.13	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	4.18	924.8	9.4	≤	11.89	2.4
		42.8	2200	300	3.4	200.00	5.00	33076	4.18	884.8	9.4	≤	9.30	5.0
		42.8	2200	300	3.4	300.00	8.30	33076	4.18	880.8	9.4	≤	8.11	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	4.18	879.8	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	4.18	879.7	9.4	≤	7.13	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	41519	6.06	752.6	9.4	≤	7.22	10.6
		74.6	2200	300	3.4	100.00	2.50	41519	6.06	928.7	9.4	≤	12.85	1.9
		74.6	2200	300	3.4	200.00	5.00	41519	6.06	773.0	9.4	≤	9.59	4.5
		74.6	2200	300	3.4	300.00	8.30	41519	6.06	757.3	9.4	≤	8.32	6.9
		74.6	2200	300	3.4	400.00	11.80	41519	6.06	753.5	9.4	≤	7.55	9.3
		74.6	2200	300	3.4	456.70	14.60	41519	6.06	752.6	9.4	≤	7.22	10.6
	B311	72.8	3460	310	3.4	575.68	17.00	41170	5.96	1027.0	9.4	≤	7.34	9.9
		72.8	3460	310	3.4	100.00	2.50	41170	5.96	1115.7	9.4	≤	13.53	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.96	1037.7	9.4	≤	10.48	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.96	1027.9	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.96	1027.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	575.68	17.00	41170	5.96	1027.0	9.4	≤	7.34	9.9
	B312	72.8	3460	310	3.4	560.36	16.90	41170	5.96	1027.0	9.4	≤	7.41	9.6
		72.8	3460	310	3.4	100.00	2.50	41170	5.96	1115.7	9.4	≤	13.53	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.96	1037.7	9.4	≤	10.48	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.96	1027.9	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.96	1027.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	560.36	16.90	41170	5.96	1027.0	9.4	≤	7.41	9.6
	B313	72.8	3460	310	3.4	580.26	16.10	41170	5.96	1027.0	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	41170	5.96	1115.7	9.4	≤	13.53	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.96	1037.7	9.4	≤	10.48	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.96	1027.9	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.96	1027.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	580.26	16.10	41170	5.96	1027.0	9.4	≤	7.32	10.0
	B321	77	3460	620	3.4	551.16	15.50	41968	6.19	1014.5	9.4	≤	7.56	9.4
		77	3460	620	3.4	100.00	1.70	41968	6.19	1117.8	9.4	≤	13.80	1.6
		77	3460	620	3.4	200.00	4.20	41968	6.19	1026.8	9.4	≤	10.65	3.4
		77	3460	620	3.4	400.00	10.00	41968	6.19	1015.5	9.4	≤	8.42	6.8
		77	3460	620	3.4	500.00	13.30	41968	6.19	1014.7	9.4	≤	7.81	8.5
		77	3460	620	3.4	551.16	15.50	41968	6.19	1014.5	9.4	≤	7.56	9.4
	B331	72.8	3460	940	3.4	590.38	16.50	41170	5.96	1026.9	9.4	≤	7.48	10.2
		72.8	3460	940	3.4	100.00	1.70	41170	5.96	1115.7	9.4	≤	13.89	1.6
		72.8	3460	940	3.4	200.00	4.20	41170	5.96	1037.7	9.4	≤	10.76	3.4
		72.8	3460	940	3.4	400.00	10.00	41170	5.96	1027.9	9.4	≤	8.51	6.9
		72.8	3460	940	3.4	500.00	13.30	41170	5.96	1027.2	9.4	≤	7.90	8.6
		72.8	3460	940	3.4	590.38	16.50	41170	5.96	1026.9	9.4	≤	7.48	10.2
	B411	77	4730	320	3.4	621.34	15.50	41968	6.19	1232.7	9.4	≤	7.63	8.7
		77	4730	320	3.4	100.00		41968	6.19	1265.4	9.4	≤	14.16	1.4
		77	4730	320	3.4	200.00		41968	6.19	1236.7	9.4	≤	11.15	2.8
		77	4730	320	3.4	300.00		41968	6.19	1233.8	9.4	≤	9.73	4.2
		77	4730	320	3.4	400.00		41968	6.19	1233.1	9.4	≤	8.84	5.6
		77	4730	320	3.4	500.00		41968	6.19	1232.9	9.4	≤	8.21	7.0
		77	4730	320	3.4	621.34		41968	6.19	1232.7	9.4	≤	7.63	8.7
	C211	85.6	2710	300	3.4	560.94	18.20	43445	6.64	842.5	9.4	≤	7.08	11.1
		85.6	2710	300	3.4	100.00	2.50	43445	6.64	1039.2	9.4	≤	13.49	1.6
		85.6	2710	300	3.4	200.00	5.20	43445	6.64	866.1	9.4	≤	10.08	3.9
		85.6	2710	300	3.4	400.00	11.70	43445	6.64	844.5	9.4	≤	7.93	7.9
		85.6	2710	300	3.4	500.00	15.10	43445	6.64	843.0	9.4	≤	7.36	9.9
		85.6	2710	300	3.4	560.94	18.20	43445	6.64	842.5	9.4	≤	7.08	11.1
	C311	88.1	3220	310	3.4	605.46	18.00	43838	6.77	940.5	9.4	≤	7.17	10.7
		88.1	3220	310	3.4	100.00	1.70	43838	6.77	1107.4	9.4	≤	13.79	1.5
		88.1	3220	310	3.4	200.00	5.30	43838	6.77	960.7	9.4	≤	10.44	3.5
		88.1	3220	310	3.4	400.00	12.10	43838	6.77	942.4	9.4	≤	8.23	7.0
		88.1	3220	310	3.4	500.00	15.40	43838	6.77	941.1	9.4	≤	7.64	8.8
		88.1	3220	310	3.4	605.46	18.00	43838	6.77	940.5	9.4	≤	7.17	10.7
	C411	85.6	4260	320	3.4	722.56	19.30	43445	6.64	1132.2	9.4	≤	7.15	10.7
		85.6	4260	320	3.4	100.00	1.70	43445	6.64	1212.9	9.4	≤	14.14	1.4
		85.6	4260	320	3.4	200.00	4.70	43445	6.64	1142.1	9.4	≤	11.00	2.9
		85.6	4260	320	3.4	400.00	10.00	43445	6.64	1133.2	9.4	≤	8.71	5.9
		85.6	4260	320	3.4	600.00	15.80	43445	6.64	1132.3	9.4	≤	7.60	8.9
		85.6	4260	320	3.4	722.56	19.30	43445	6.64	1132.2	9.4	≤	7.15	10.7
	C511	88.1	5310	330	3.4	811.82	20.80	43838	6.77	1286.6	9.4	≤	7.18	10.5
		88.1	5310	330	3.4	100.00	1.70	43838	6.77	1306.5	9.4	≤	14.51	1.3
		88.1	5310	330	3.4	200.00	3.30	43838	6.77	1289.1	9.4	≤	11.46	2.6
		88.1	5310	330	3.4	400.00	8.70	43838	6.77	1286.9	9.4	≤	9.09	5.2
		88.1	5310	330	3.4	600.00	14.20	43838	6.77	1286.7	9.4	≤	7.94	7.7
		88.1	5310	330	3.4	811.82	20.80	43838	6.77	1286.6	9.4	≤	7.18	10.5
	D211	114.5	3220	300	3.4	506.00	16.00	47120	8.06	897.8	9.4	≤	7.67	8.7
		114.5	3220	300	3.4	100.00	1.60	47120	8.06	1210.4	9.4	≤	14.55	1.3
		114.5	3220	300	3.4	200.00	5.00	47120	8.06	934.8	9.4	≤	10.59	3.3
		114.5	3220	300	3.4	300.00	8.60	47120	8.06	907.0	9.4	≤	9.16	5.1
		114.5	3220	300	3.4	400.00	12.10	47120	8.06	900.3	9.4	≤	8.30	6.9
		114.5	3220	300	3.4	506.00	16.00	47120	8.06	897.8	9.4	≤	7.67	8.7
	E211	126.2	2200	300	3.4	506.00	15.90	48140	8.60	679.5	9.4	≤	7.08	11.3
		126.2	2200	300	3.4	100.00	1.70	48140	8.60	1250.3	9.4	≤	14.90	1.2
		126.2	2200	300	3.4	200.00	5.00	48140	8.60	746.9	9.4	≤	9.96	4.0
		126.2	2200	300	3.4	300.00	8.60	48140	8.60	696.3	9.4	≤	8.50	6.5
		126.2	2200	300	3.4	400.00	12.10	48140	8.60	684.0	9.4	≤	7.68	8.8
		126.2	2200	300	3.4	506.00	15.90	48140	8.60	679.5	9.4	≤	7.08	11.3

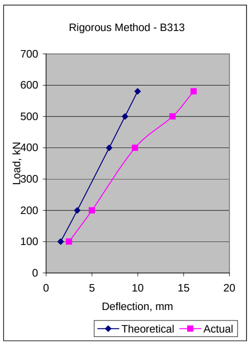

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

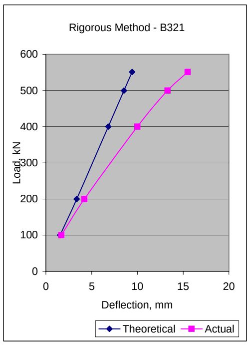

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

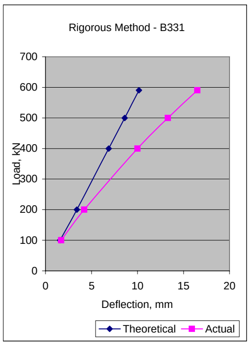

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

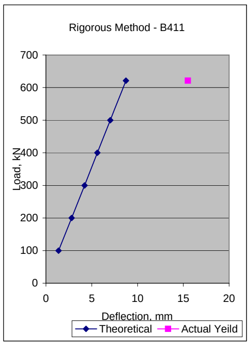

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

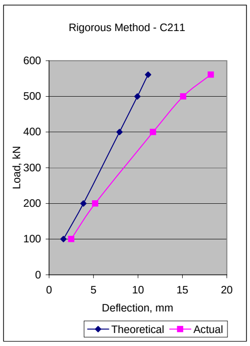

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

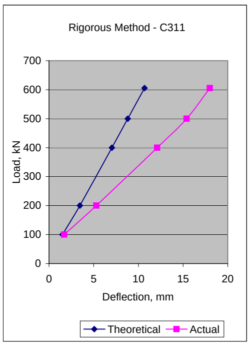

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

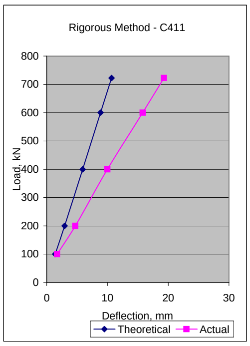

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

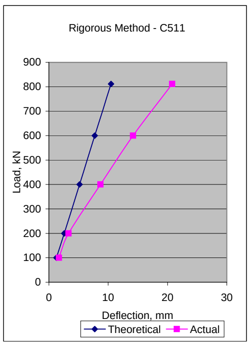

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

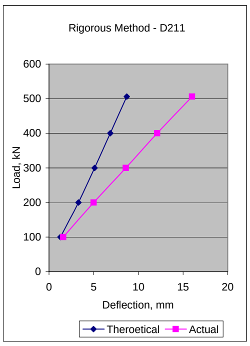

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

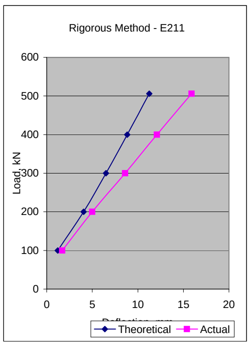

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

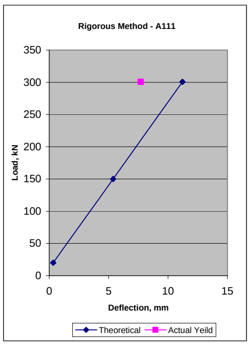

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

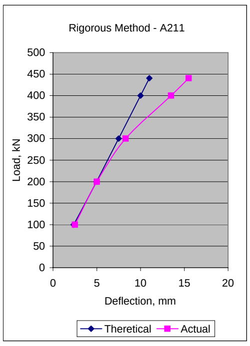

Rigorous Method - E211
Load, kN
Theoretical
Actual

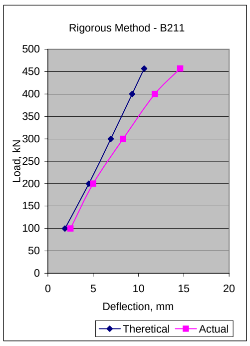
METHOD 11

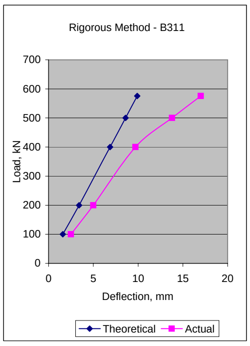
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.564(f'_c)^{0.55}$

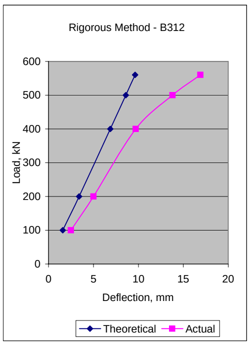
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	**BEAM**	**f'_c, MPa**	**Area of Steel, A_{st}, mm^2**	**Area of Steel, A_{sc}, mm^2**	**L_{ef}, m**	**Load, kN**	**Deflection, mm**	**E_c**	**f'_{cf}, MPa**	**I_{eff}, $10^6 mm$**	**Ratio**	**≤**	**Allowed Ratio**	**Deflection, mm**
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	4.45	590.0	9.4	≤	10.04	11.2
		42.8	1250	300	3.4	20.00		33076	4.45	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	4.45	613.4	9.4	≤	9.20	5.4
		42.8	1250	300	3.4	300.78		33076	4.45	590.0	9.4	≤	7.20	11.2
	A211	42.8	2200	300	3.4	440.48	15.50	33076	4.45	879.8	9.4	≤	7.14	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	4.45	934.1	9.4	≤	11.93	2.4
		42.8	2200	300	3.4	200.00	5.00	33076	4.45	886.0	9.4	≤	9.31	5.0
		42.8	2200	300	3.4	300.00	8.30	33076	4.45	881.2	9.4	≤	8.11	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	4.45	880.0	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	4.45	879.8	9.4	≤	7.14	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	41519	6.04	752.6	9.4	≤	7.22	10.6
		74.6	2200	300	3.4	100.00	2.50	41519	6.04	927.1	9.4	≤	12.84	1.9
		74.6	2200	300	3.4	200.00	5.00	41519	6.04	772.8	9.4	≤	9.59	4.5
		74.6	2200	300	3.4	300.00	8.30	41519	6.04	757.3	9.4	≤	8.32	6.9
		74.6	2200	300	3.4	400.00	11.80	41519	6.04	753.5	9.4	≤	7.55	9.3
		74.6	2200	300	3.4	456.70	14.60	41519	6.04	752.6	9.4	≤	7.22	10.6
	B311	72.8	3460	310	3.4	575.68	17.00	41170	5.96	1027.0	9.4	≤	7.34	9.9
		72.8	3460	310	3.4	100.00	2.50	41170	5.96	1115.7	9.4	≤	13.53	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.96	1037.7	9.4	≤	10.48	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.96	1027.9	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.96	1027.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	575.68	17.00	41170	5.96	1027.0	9.4	≤	7.34	9.9
	B312	72.8	3460	310	3.4	560.36	16.90	41170	5.96	1027.0	9.4	≤	7.41	9.6
		72.8	3460	310	3.4	100.00	2.50	41170	5.96	1115.7	9.4	≤	13.53	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.96	1037.7	9.4	≤	10.48	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.96	1027.9	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.96	1027.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	560.36	16.90	41170	5.96	1027.0	9.4	≤	7.41	9.6
	B313	72.8	3460	310	3.4	580.26	16.10	41170	5.96	1027.0	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	41170	5.96	1115.7	9.4	≤	13.53	1.6
		72.8	3460	310	3.4	200.00	5.00	41170	5.96	1037.7	9.4	≤	10.48	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	5.96	1027.9	9.4	≤	8.29	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	5.96	1027.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	580.26	16.10	41170	5.96	1027.0	9.4	≤	7.32	10.0
	B321	77	3460	620	3.4	551.16	15.50	41968	6.15	1014.4	9.4	≤	7.56	9.4
		77	3460	620	3.4	100.00	1.70	41968	6.15	1115.8	9.4	≤	13.79	1.6
		77	3460	620	3.4	200.00	4.20	41968	6.15	1026.6	9.4	≤	10.65	3.4
		77	3460	620	3.4	400.00	10.00	41968	6.15	1015.4	9.4	≤	8.42	6.8
		77	3460	620	3.4	500.00	13.30	41968	6.15	1014.6	9.4	≤	7.81	8.5
		77	3460	620	3.4	551.16	15.50	41968	6.15	1014.4	9.4	≤	7.56	9.4
	B331	72.8	3460	940	3.4	590.38	16.50	41170	5.96	1026.9	9.4	≤	7.48	10.2
		72.8	3460	940	3.4	100.00	1.70	41170	5.96	1115.7	9.4	≤	13.89	1.6
		72.8	3460	940	3.4	200.00	4.20	41170	5.96	1037.7	9.4	≤	10.76	3.4
		72.8	3460	940	3.4	400.00	10.00	41170	5.96	1027.9	9.4	≤	8.51	6.9
		72.8	3460	940	3.4	500.00	13.30	41170	5.96	1027.2	9.4	≤	7.90	8.6
		72.8	3460	940	3.4	590.38	16.50	41170	5.96	1026.9	9.4	≤	7.48	10.2
	B411	77	4730	320	3.4	621.34	15.50	41968	6.15	1232.7	9.4	≤	7.63	8.7
		77	4730	320	3.4	100.00		41968	6.15	1264.7	9.4	≤	14.15	1.4
		77	4730	320	3.4	200.00		41968	6.15	1236.6	9.4	≤	11.15	2.8
		77	4730	320	3.4	300.00		41968	6.15	1233.8	9.4	≤	9.73	4.2
		77	4730	320	3.4	400.00		41968	6.15	1233.1	9.4	≤	8.84	5.6
		77	4730	320	3.4	500.00		41968	6.15	1232.9	9.4	≤	8.21	7.0
		77	4730	320	3.4	621.34		41968	6.15	1232.7	9.4	≤	7.63	8.7
	C211	85.6	2710	300	3.4	560.94	18.20	43445	6.52	842.4	9.4	≤	7.08	11.1
		85.6	2710	300	3.4	100.00	2.50	43445	6.52	1028.3	9.4	≤	13.44	1.6
		85.6	2710	300	3.4	200.00	5.20	43445	6.52	864.7	9.4	≤	10.07	3.9
		85.6	2710	300	3.4	400.00	11.70	43445	6.52	844.3	9.4	≤	7.93	7.9
		85.6	2710	300	3.4	500.00	15.10	43445	6.52	842.9	9.4	≤	7.36	9.9
		85.6	2710	300	3.4	560.94	18.20	43445	6.52	842.4	9.4	≤	7.08	11.1
	C311	88.1	3220	310	3.4	605.46	18.00	43838	6.62	940.5	9.4	≤	7.17	10.7
		88.1	3220	310	3.4	100.00	1.70	43838	6.62	1096.6	9.4	≤	13.75	1.5
		88.1	3220	310	3.4	200.00	5.30	43838	6.62	959.4	9.4	≤	10.44	3.5
		88.1	3220	310	3.4	400.00	12.10	43838	6.62	942.2	9.4	≤	8.23	7.0
		88.1	3220	310	3.4	500.00	15.40	43838	6.62	941.0	9.4	≤	7.64	8.8
		88.1	3220	310	3.4	605.46	18.00	43838	6.62	940.5	9.4	≤	7.17	10.7
	C411	85.6	4260	320	3.4	722.56	19.30	43445	6.52	1132.2	9.4	≤	7.15	10.7
		85.6	4260	320	3.4	100.00	1.70	43445	6.52	1208.5	9.4	≤	14.12	1.4
		85.6	4260	320	3.4	200.00	4.70	43445	6.52	1141.5	9.4	≤	10.99	2.9
		85.6	4260	320	3.4	400.00	10.00	43445	6.52	1133.2	9.4	≤	8.71	5.9
		85.6	4260	320	3.4	600.00	15.80	43445	6.52	1132.3	9.4	≤	7.60	8.9
		85.6	4260	320	3.4	722.56	19.30	43445	6.52	1132.2	9.4	≤	7.15	10.7
	C511	88.1	5310	330	3.4	811.82	20.80	43838	6.62	1286.6	9.4	≤	7.18	10.5
		88.1	5310	330	3.4	100.00	1.70	43838	6.62	1305.2	9.4	≤	14.50	1.3
		88.1	5310	330	3.4	200.00	3.30	43838	6.62	1288.9	9.4	≤	11.46	2.6
		88.1	5310	330	3.4	400.00	8.70	43838	6.62	1286.9	9.4	≤	9.09	5.2
		88.1	5310	330	3.4	600.00	14.20	43838	6.62	1286.7	9.4	≤	7.94	7.7
		88.1	5310	330	3.4	811.82	20.80	43838	6.62	1286.6	9.4	≤	7.18	10.5
	D211	114.5	3220	300	3.4	506.00	16.00	47120	7.65	897.4	9.4	≤	7.67	8.7
		114.5	3220	300	3.4	100.00	1.60	47120	7.65	1164.3	9.4	≤	14.36	1.3
		114.5	3220	300	3.4	200.00	5.00	47120	7.65	929.0	9.4	≤	10.57	3.3
		114.5	3220	300	3.4	300.00	8.60	47120	7.65	905.3	9.4	≤	9.16	5.1
		114.5	3220	300	3.4	400.00	12.10	47120	7.65	899.6	9.4	≤	8.30	6.9
		114.5	3220	300	3.4	506.00	16.00	47120	7.65	897.4	9.4	≤	7.67	8.7
	E211	126.2	2200	300	3.4	506.00	15.90	48140	8.07	678.7	9.4	≤	7.08	11.3
		126.2	2200	300	3.4	100.00	1.70	48140	8.07	1149.6	9.4	≤	14.49	1.3
		126.2	2200	300	3.4	200.00	5.00	48140	8.07	734.4	9.4	≤	9.91	4.1
		126.2	2200	300	3.4	300.00	8.60	48140	8.07	692.6	9.4	≤	8.49	6.5
		126.2	2200	300	3.4	400.00	12.10	48140	8.07	682.4	9.4	≤	7.67	8.9
		126.2	2200	300	3.4	506.00	15.90	48140	8.07	678.7	9.4	≤	7.08	11.3

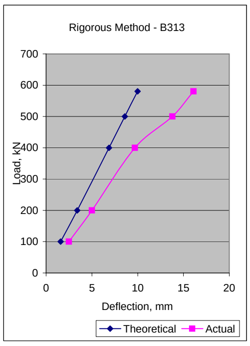

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

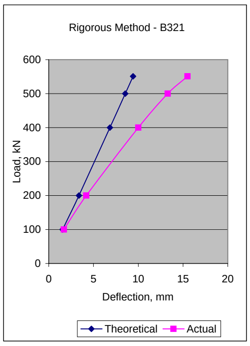

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

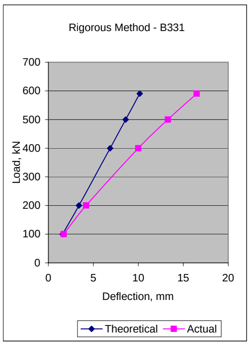

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

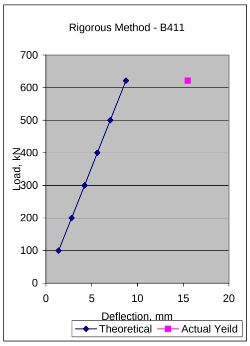

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

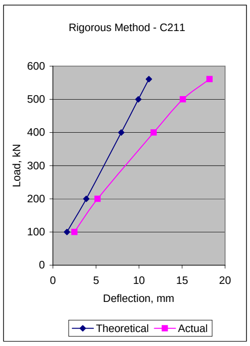

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

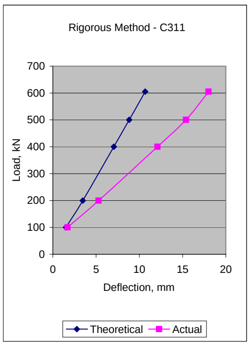

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

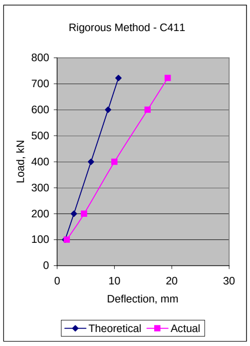

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

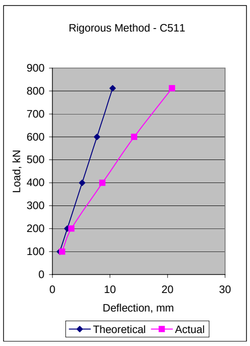

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

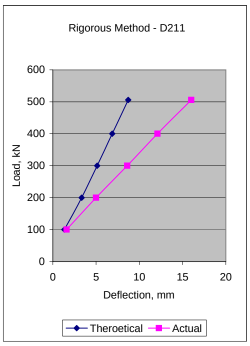

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

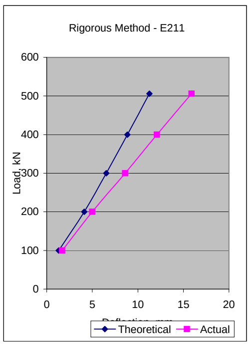

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

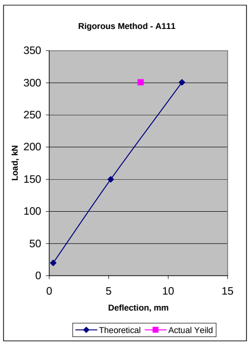

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

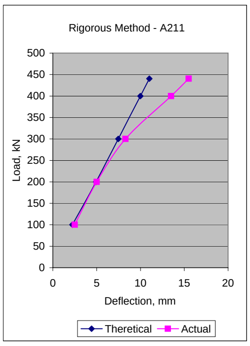

Rigorous Method - E211
Load, kN
Theoretical
Actual

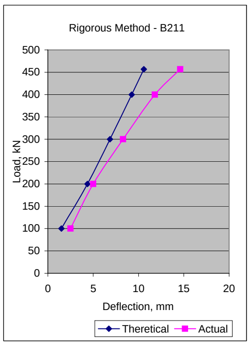
METHOD 12

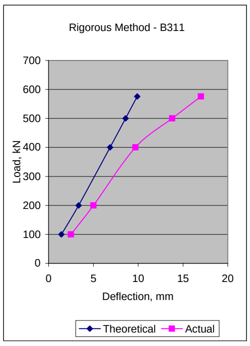
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.43(f'_c)^{0.68}$

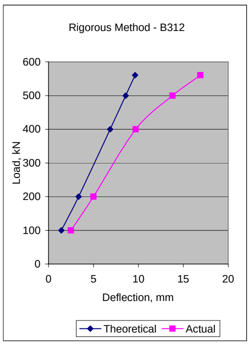
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	33076	5.53	593.0	9.4	≤	10.06	11.2
		42.8	1250	300	3.4	20.00		33076	5.53	1333.3	9.4	≤	23.32	0.3
		42.8	1250	300	3.4	150.00		33076	5.53	638.0	9.4	≤	9.32	5.2
		42.8	1250	300	3.4	300.78		33076	5.53	593.0	9.4	≤	7.21	11.2
	A211	42.8	2200	300	3.4	440.48	15.50	33076	5.53	880.4	9.4	≤	7.14	11.0
		42.8	2200	300	3.4	100.00	2.50	33076	5.53	984.6	9.4	≤	12.14	2.2
		42.8	2200	300	3.4	200.00	5.00	33076	5.53	892.3	9.4	≤	9.33	4.9
		42.8	2200	300	3.4	300.00	8.30	33076	5.53	883.0	9.4	≤	8.12	7.5
		42.8	2200	300	3.4	400.00	13.50	33076	5.53	880.8	9.4	≤	7.37	10.0
		42.8	2200	300	3.4	440.48	15.50	33076	5.53	880.4	9.4	≤	7.14	11.0
	B211	74.6	2200	300	3.4	456.70	14.60	41519	8.07	755.2	9.4	≤	7.23	10.6
		74.6	2200	300	3.4	100.00	2.50	41519	8.07	1170.9	9.4	≤	13.88	1.5
		74.6	2200	300	3.4	200.00	5.00	41519	8.07	803.3	9.4	≤	9.71	4.4
		74.6	2200	300	3.4	300.00	8.30	41519	8.07	766.3	9.4	≤	8.35	6.9
		74.6	2200	300	3.4	400.00	11.80	41519	8.07	757.3	9.4	≤	7.56	9.3
		74.6	2200	300	3.4	456.70	14.60	41519	8.07	755.2	9.4	≤	7.23	10.6
	B311	72.8	3460	310	3.4	575.68	17.00	41170	7.94	1027.6	9.4	≤	7.34	9.9
		72.8	3460	310	3.4	100.00	2.50	41170	7.94	1237.0	9.4	≤	14.00	1.4
		72.8	3460	310	3.4	200.00	5.00	41170	7.94	1052.8	9.4	≤	10.53	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	7.94	1029.8	9.4	≤	8.30	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	7.94	1028.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	575.68	17.00	41170	7.94	1027.6	9.4	≤	7.34	9.9
	B312	72.8	3460	310	3.4	560.36	16.90	41170	7.94	1027.7	9.4	≤	7.41	9.6
		72.8	3460	310	3.4	100.00	2.50	41170	7.94	1237.0	9.4	≤	14.00	1.4
		72.8	3460	310	3.4	200.00	5.00	41170	7.94	1052.8	9.4	≤	10.53	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	7.94	1029.8	9.4	≤	8.30	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	7.94	1028.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	560.36	16.90	41170	7.94	1027.7	9.4	≤	7.41	9.6
	B313	72.8	3460	310	3.4	580.26	16.10	41170	7.94	1027.6	9.4	≤	7.33	10.0
		72.8	3460	310	3.4	100.00	2.50	41170	7.94	1237.0	9.4	≤	14.00	1.4
		72.8	3460	310	3.4	200.00	5.00	41170	7.94	1052.8	9.4	≤	10.53	3.4
		72.8	3460	310	3.4	400.00	9.70	41170	7.94	1029.8	9.4	≤	8.30	6.9
		72.8	3460	310	3.4	500.00	13.80	41170	7.94	1028.2	9.4	≤	7.70	8.6
		72.8	3460	310	3.4	580.26	16.10	41170	7.94	1027.6	9.4	≤	7.33	10.0
	B321	77	3460	620	3.4	551.16	15.50	41968	8.25	1015.3	9.4	≤	7.57	9.4
		77	3460	620	3.4	100.00	1.70	41968	8.25	1259.6	9.4	≤	14.36	1.4
		77	3460	620	3.4	200.00	4.20	41968	8.25	1044.6	9.4	≤	10.71	3.3
		77	3460	620	3.4	400.00	10.00	41968	8.25	1017.7	9.4	≤	8.42	6.8
		77	3460	620	3.4	500.00	13.30	41968	8.25	1015.8	9.4	≤	7.82	8.5
		77	3460	620	3.4	551.16	15.50	41968	8.25	1015.3	9.4	≤	7.57	9.4
	B331	72.8	3460	940	3.4	590.38	16.50	41170	7.94	1027.5	9.4	≤	7.48	10.2
		72.8	3460	940	3.4	100.00	1.70	41170	7.94	1237.0	9.4	≤	14.37	1.4
		72.8	3460	940	3.4	200.00	4.20	41170	7.94	1052.8	9.4	≤	10.81	3.4
		72.8	3460	940	3.4	400.00	10.00	41170	7.94	1029.8	9.4	≤	8.52	6.9
		72.8	3460	940	3.4	500.00	13.30	41170	7.94	1028.2	9.4	≤	7.90	8.6
		72.8	3460	940	3.4	590.38	16.50	41170	7.94	1027.5	9.4	≤	7.48	10.2
	B411	77	4730	320	3.4	621.34	15.50	41968	8.25	1232.9	9.4	≤	7.63	8.7
		77	4730	320	3.4	100.00		41968	8.25	1310.1	9.4	≤	14.32	1.3
		77	4730	320	3.4	200.00		41968	8.25	1242.3	9.4	≤	11.17	2.8
		77	4730	320	3.4	300.00		41968	8.25	1235.5	9.4	≤	9.74	4.2
		77	4730	320	3.4	400.00		41968	8.25	1233.8	9.4	≤	8.84	5.6
		77	4730	320	3.4	500.00		41968	8.25	1233.2	9.4	≤	8.21	7.0
		77	4730	320	3.4	621.34		41968	8.25	1232.9	9.4	≤	7.63	8.7
	C211	85.6	2710	300	3.4	560.94	18.20	43445	8.86	844.0	9.4	≤	7.08	11.1
		85.6	2710	300	3.4	100.00	2.50	43445	8.86	1311.1	9.4	≤	14.58	1.3
		85.6	2710	300	3.4	200.00	5.20	43445	8.86	900.1	9.4	≤	10.21	3.7
		85.6	2710	300	3.4	400.00	11.70	43445	8.86	848.7	9.4	≤	7.94	7.9
		85.6	2710	300	3.4	500.00	15.10	43445	8.86	845.1	9.4	≤	7.36	9.9
		85.6	2710	300	3.4	560.94	18.20	43445	8.86	844.0	9.4	≤	7.08	11.1
	C311	88.1	3220	310	3.4	605.46	18.00	43838	9.04	941.6	9.4	≤	7.17	10.7
		88.1	3220	310	3.4	100.00	1.70	43838	9.04	1333.3	9.4	≤	14.68	1.2
		88.1	3220	310	3.4	200.00	5.30	43838	9.04	989.6	9.4	≤	10.55	3.4
		88.1	3220	310	3.4	400.00	12.10	43838	9.04	946.0	9.4	≤	8.25	7.0
		88.1	3220	310	3.4	500.00	15.40	43838	9.04	943.0	9.4	≤	7.65	8.8
		88.1	3220	310	3.4	605.46	18.00	43838	9.04	941.6	9.4	≤	7.17	10.7
	C411	85.6	4260	320	3.4	722.56	19.30	43445	8.86	1132.5	9.4	≤	7.15	10.7
		85.6	4260	320	3.4	100.00	1.70	43445	8.86	1324.2	9.4	≤	14.56	1.3
		85.6	4260	320	3.4	200.00	4.70	43445	8.86	1156.0	9.4	≤	11.04	2.9
		85.6	4260	320	3.4	400.00	10.00	43445	8.86	1135.0	9.4	≤	8.71	5.9
		85.6	4260	320	3.4	600.00	15.80	43445	8.86	1132.9	9.4	≤	7.60	8.9
		85.6	4260	320	3.4	722.56	19.30	43445	8.86	1132.5	9.4	≤	7.15	10.7
	C511	88.1	5310	330	3.4	811.82	20.80	43838	9.04	1286.7	9.4	≤	7.18	10.5
		88.1	5310	330	3.4	100.00	1.70	43838	9.04	1333.3	9.4	≤	14.61	1.2
		88.1	5310	330	3.4	200.00	3.30	43838	9.04	1292.5	9.4	≤	11.47	2.6
		88.1	5310	330	3.4	400.00	8.70	43838	9.04	1287.3	9.4	≤	9.09	5.2
		88.1	5310	330	3.4	600.00	14.20	43838	9.04	1286.8	9.4	≤	7.94	7.7
		88.1	5310	330	3.4	811.82	20.80	43838	9.04	1286.7	9.4	≤	7.18	10.5
	D211	114.5	3220	300	3.4	506.00	16.00	47120	10.80	901.2	9.4	≤	7.68	8.7
		114.5	3220	300	3.4	100.00	1.60	47120	10.80	1333.3	9.4	≤	15.03	1.2
		114.5	3220	300	3.4	200.00	5.00	47120	10.80	990.0	9.4	≤	10.80	3.1
		114.5	3220	300	3.4	300.00	8.60	47120	10.80	923.4	9.4	≤	9.22	5.0
		114.5	3220	300	3.4	400.00	12.10	47120	10.80	907.2	9.4	≤	8.33	6.8
		114.5	3220	300	3.4	506.00	16.00	47120	10.80	901.2	9.4	≤	7.68	8.7
	E211	126.2	2200	300	3.4	506.00	15.90	48140	11.54	685.7	9.4	≤	7.11	11.2
		126.2	2200	300	3.4	100.00	1.70	48140	11.54	1333.3	9.4	≤	15.23	1.1
		126.2	2200	300	3.4	200.00	5.00	48140	11.54	848.5	9.4	≤	10.39	3.6
		126.2	2200	300	3.4	300.00	8.60	48140	11.54	726.4	9.4	≤	8.62	6.2
		126.2	2200	300	3.4	400.00	12.10	48140	11.54	696.7	9.4	≤	7.73	8.7
		126.2	2200	300	3.4	506.00	15.90	48140	11.54	685.7	9.4	≤	7.11	11.2

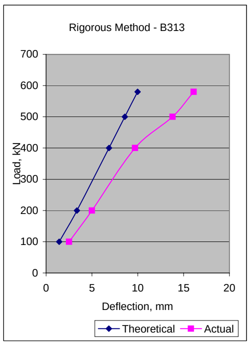

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

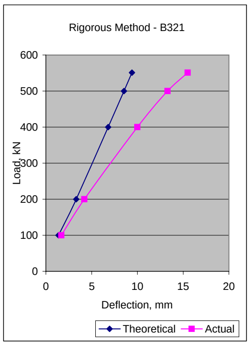

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

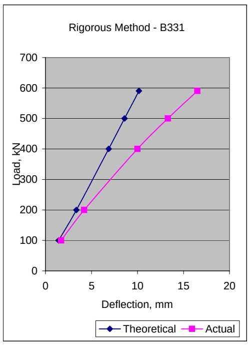

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

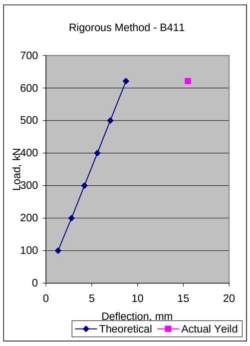

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

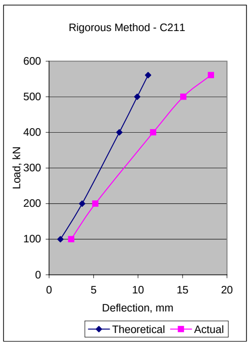

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

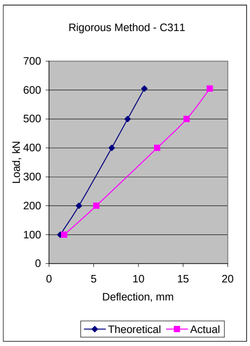

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

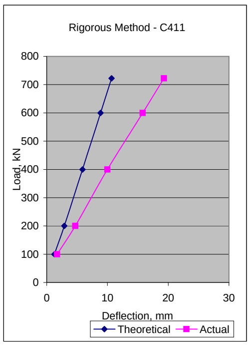

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

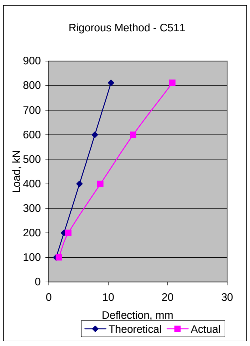

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

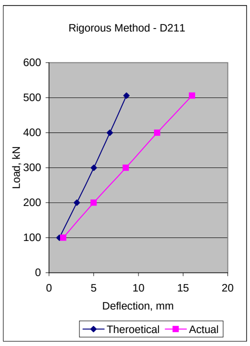

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

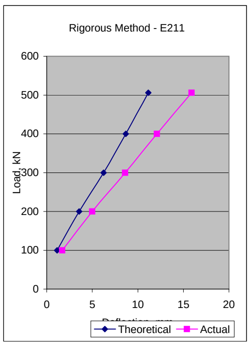

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

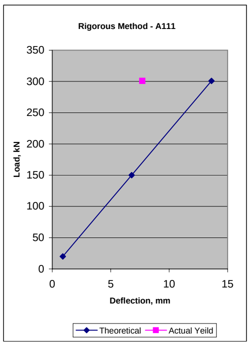

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

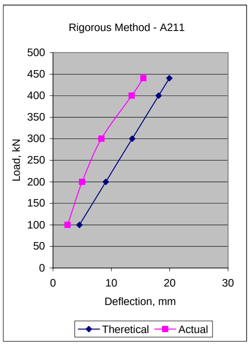

Rigorous Method - E211
Load, kN
Theoretical
Actual

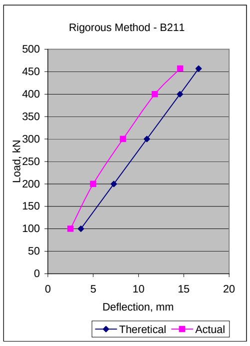
METHOD 13

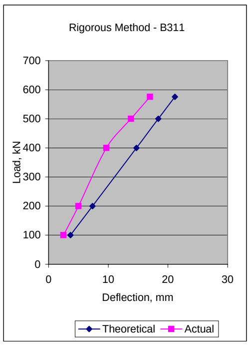
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$0.045bd^3$
$f'_{cf} =$	NA

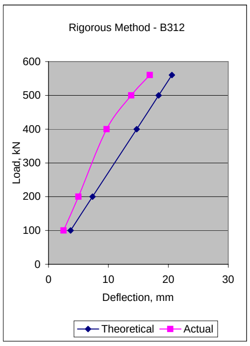
			Reinforcement Ratio			At Yield				Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30113	533.7	9.4	≤	9.41	13.6
		42.8	1250	300	3.4	20.00		30113	533.7	9.4	≤	16.66	0.9
		42.8	1250	300	3.4	150.00		30113	533.7	9.4	≤	8.51	6.8
		42.8	1250	300	3.4	300.78		30113	533.7	9.4	≤	6.75	13.6
	A211	42.8	2200	300	3.4	440.48	15.50	30113	533.7	9.4	≤	5.85	19.9
		42.8	2200	300	3.4	100.00	2.50	30113	533.7	9.4	≤	9.60	4.5
		42.8	2200	300	3.4	200.00	5.00	30113	533.7	9.4	≤	7.62	9.1
		42.8	2200	300	3.4	300.00	8.30	30113	533.7	9.4	≤	6.65	13.6
		42.8	2200	300	3.4	400.00	13.50	30113	533.7	9.4	≤	6.05	18.1
		42.8	2200	300	3.4	440.48	15.50	30113	533.7	9.4	≤	5.85	19.9
	B211	74.6	2200	300	3.4	456.70	14.60	37431	533.7	9.4	≤	6.22	16.6
		74.6	2200	300	3.4	100.00	2.50	37431	533.7	9.4	≤	10.32	3.6
		74.6	2200	300	3.4	200.00	5.00	37431	533.7	9.4	≤	8.19	7.3
		74.6	2200	300	3.4	300.00	8.30	37431	533.7	9.4	≤	7.15	10.9
		74.6	2200	300	3.4	400.00	11.80	37431	533.7	9.4	≤	6.50	14.6
		74.6	2200	300	3.4	456.70	14.60	37431	533.7	9.4	≤	6.22	16.6
	B311	72.8	3460	310	3.4	575.68	17.00	37065	533.7	9.4	≤	5.70	21.2
		72.8	3460	310	3.4	100.00	2.50	37065	533.7	9.4	≤	10.22	3.7
		72.8	3460	310	3.4	200.00	5.00	37065	533.7	9.4	≤	8.11	7.4
		72.8	3460	310	3.4	400.00	9.70	37065	533.7	9.4	≤	6.44	14.7
		72.8	3460	310	3.4	500.00	13.80	37065	533.7	9.4	≤	5.97	18.4
		72.8	3460	310	3.4	575.68	17.00	37065	533.7	9.4	≤	5.70	21.2
	B312	72.8	3460	310	3.4	560.36	16.90	37065	533.7	9.4	≤	5.75	20.6
		72.8	3460	310	3.4	100.00	2.50	37065	533.7	9.4	≤	10.22	3.7
		72.8	3460	310	3.4	200.00	5.00	37065	533.7	9.4	≤	8.11	7.4
		72.8	3460	310	3.4	400.00	9.70	37065	533.7	9.4	≤	6.44	14.7
		72.8	3460	310	3.4	500.00	13.80	37065	533.7	9.4	≤	5.97	18.4
		72.8	3460	310	3.4	560.36	16.90	37065	533.7	9.4	≤	5.75	20.6
	B313	72.8	3460	310	3.4	580.26	16.10	37065	533.7	9.4	≤	5.69	21.3
		72.8	3460	310	3.4	100.00	2.50	37065	533.7	9.4	≤	10.22	3.7
		72.8	3460	310	3.4	200.00	5.00	37065	533.7	9.4	≤	8.11	7.4
		72.8	3460	310	3.4	400.00	9.70	37065	533.7	9.4	≤	6.44	14.7
		72.8	3460	310	3.4	500.00	13.80	37065	533.7	9.4	≤	5.97	18.4
		72.8	3460	310	3.4	580.26	16.10	37065	533.7	9.4	≤	5.69	21.3
	B321	77	3460	620	3.4	551.16	15.50	37913	533.7	9.4	≤	5.90	19.8
		77	3460	620	3.4	100.00	1.70	37913	533.7	9.4	≤	10.43	3.6
		77	3460	620	3.4	200.00	4.20	37913	533.7	9.4	≤	8.27	7.2
		77	3460	620	3.4	400.00	10.00	37913	533.7	9.4	≤	6.57	14.4
		77	3460	620	3.4	500.00	13.30	37913	533.7	9.4	≤	6.10	18.0
		77	3460	620	3.4	551.16	15.50	37913	533.7	9.4	≤	5.90	19.8
	B331	72.8	3460	940	3.4	590.38	16.50	37065	533.7	9.4	≤	5.80	21.7
		72.8	3460	940	3.4	100.00	1.70	37065	533.7	9.4	≤	10.49	3.7
		72.8	3460	940	3.4	200.00	4.20	37065	533.7	9.4	≤	8.32	7.4
		72.8	3460	940	3.4	400.00	10.00	37065	533.7	9.4	≤	6.61	14.7
		72.8	3460	940	3.4	500.00	13.30	37065	533.7	9.4	≤	6.13	18.4
		72.8	3460	940	3.4	590.38	16.50	37065	533.7	9.4	≤	5.80	21.7
	B411	77	4730	320	3.4	621.34	15.50	37913	533.7	9.4	≤	5.58	22.3
		77	4730	320	3.4	100.00		37913	533.7	9.4	≤	10.26	3.6
		77	4730	320	3.4	200.00		37913	533.7	9.4	≤	8.15	7.2
		77	4730	320	3.4	300.00		37913	533.7	9.4	≤	7.12	10.8
		77	4730	320	3.4	400.00		37913	533.7	9.4	≤	6.47	14.4
		77	4730	320	3.4	500.00		37913	533.7	9.4	≤	6.00	18.0
		77	4730	320	3.4	621.34		37913	533.7	9.4	≤	5.58	22.3
	C211	85.6	2710	300	3.4	560.94	18.20	39579	533.7	9.4	≤	5.89	19.3
		85.6	2710	300	3.4	100.00	2.50	39579	533.7	9.4	≤	10.47	3.4
		85.6	2710	300	3.4	200.00	5.20	39579	533.7	9.4	≤	8.31	6.9
		85.6	2710	300	3.4	400.00	11.70	39579	533.7	9.4	≤	6.60	13.8
		85.6	2710	300	3.4	500.00	15.10	39579	533.7	9.4	≤	6.13	17.2
		85.6	2710	300	3.4	560.94	18.20	39579	533.7	9.4	≤	5.89	19.3
	C311	88.1	3220	310	3.4	605.46	18.00	40048	533.7	9.4	≤	5.76	20.6
		88.1	3220	310	3.4	100.00	1.70	40048	533.7	9.4	≤	10.49	3.4
		88.1	3220	310	3.4	200.00	5.30	40048	533.7	9.4	≤	8.33	6.8
		88.1	3220	310	3.4	400.00	12.10	40048	533.7	9.4	≤	6.61	13.6
		88.1	3220	310	3.4	500.00	15.40	40048	533.7	9.4	≤	6.14	17.0
		88.1	3220	310	3.4	605.46	18.00	40048	533.7	9.4	≤	5.76	20.6
	C411	85.6	4260	320	3.4	722.56	19.30	39579	533.7	9.4	≤	5.39	24.9
		85.6	4260	320	3.4	100.00	1.70	39579	533.7	9.4	≤	10.42	3.4
		85.6	4260	320	3.4	200.00	4.70	39579	533.7	9.4	≤	8.27	6.9
		85.6	4260	320	3.4	400.00	10.00	39579	533.7	9.4	≤	6.57	13.8
		85.6	4260	320	3.4	600.00	15.80	39579	533.7	9.4	≤	5.74	20.7
		85.6	4260	320	3.4	722.56	19.30	39579	533.7	9.4	≤	5.39	24.9
	C511	88.1	5310	330	3.4	811.82	20.80	40048	533.7	9.4	≤	5.20	27.6
		88.1	5310	330	3.4	100.00	1.70	40048	533.7	9.4	≤	10.44	3.4
		88.1	5310	330	3.4	200.00	3.30	40048	533.7	9.4	≤	8.29	6.8
		88.1	5310	330	3.4	400.00	8.70	40048	533.7	9.4	≤	6.58	13.6
		88.1	5310	330	3.4	600.00	14.20	40048	533.7	9.4	≤	5.75	20.4
		88.1	5310	330	3.4	811.82	20.80	40048	533.7	9.4	≤	5.20	27.6
	D211	114.5	3220	300	3.4	506.00	16.00	44639	533.7	9.4	≤	6.33	15.5
		114.5	3220	300	3.4	100.00	1.60	44639	533.7	9.4	≤	10.88	3.1
		114.5	3220	300	3.4	200.00	5.00	44639	533.7	9.4	≤	8.63	6.1
		114.5	3220	300	3.4	300.00	8.60	44639	533.7	9.4	≤	7.54	9.2
		114.5	3220	300	3.4	400.00	12.10	44639	533.7	9.4	≤	6.85	12.2
		114.5	3220	300	3.4	506.00	16.00	44639	533.7	9.4	≤	6.33	15.5
	E211	126.2	2200	300	3.4	506.00	15.90	46502	533.7	9.4	≤	6.46	14.8
		126.2	2200	300	3.4	100.00	1.70	46502	533.7	9.4	≤	11.09	2.9
		126.2	2200	300	3.4	200.00	5.00	46502	533.7	9.4	≤	8.80	5.9
		126.2	2200	300	3.4	300.00	8.60	46502	533.7	9.4	≤	7.69	8.8
		126.2	2200	300	3.4	400.00	12.10	46502	533.7	9.4	≤	6.99	11.7
		126.2	2200	300	3.4	506.00	15.90	46502	533.7	9.4	≤	6.46	14.8

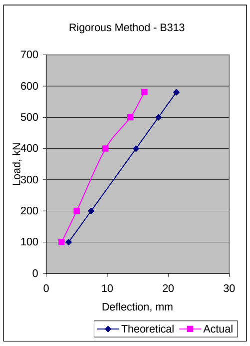

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

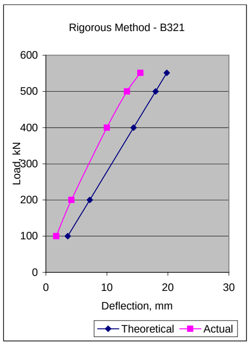

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

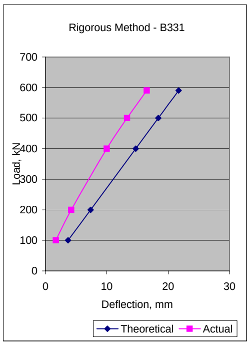

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

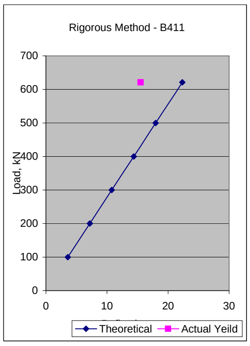

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

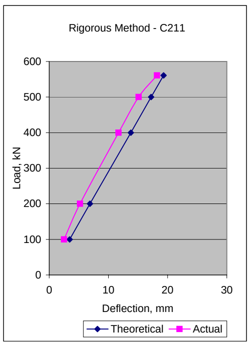

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

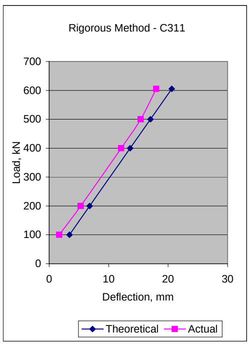

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

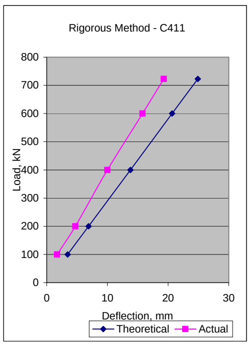

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

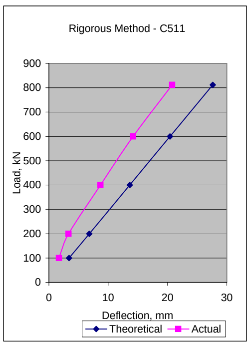

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

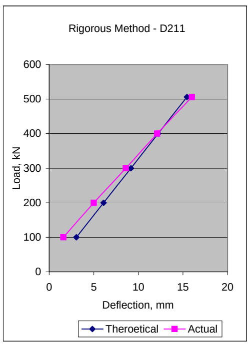

Rigorous Method - B411
Load, kN
Theoretical
Actual Yeild

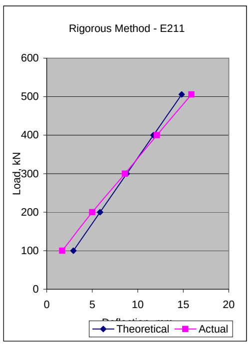

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

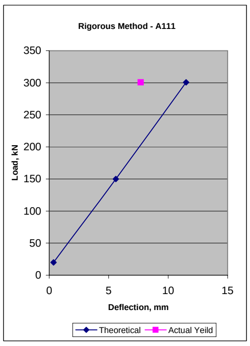

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual


Rigorous Method - E211
Load, kN
Theoretical
Actual

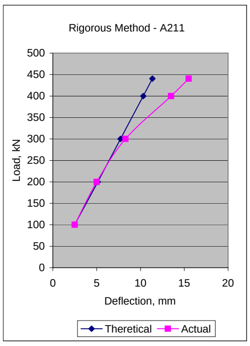
METHOD 14

$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.6\sqrt{f'_c}$

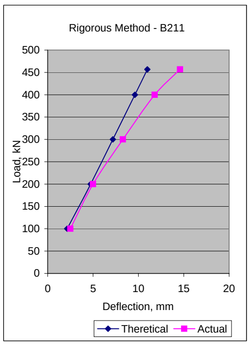
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	**BEAM**	**f'_c, MPa**	**Area of Steel, A_{st}, mm^2**	**Area of Steel, A_{sc}, mm^2**	**L_{ef}, m**	**Load, kN**	**Deflection, mm**	**E_c**	**f'_{cf}, MPa**	**I_{eff}, 10^6mm**	**Ratio**	**≤**	**Allowed Ratio**	**Deflection, mm**
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30113	3.93	631.2	9.4	≤	9.95	11.5
		42.8	1250	300	3.4	20.00		30113	3.93	1333.3	9.4	≤	22.60	0.4
		42.8	1250	300	3.4	150.00		30113	3.93	646.4	9.4	≤	9.07	5.6
		42.8	1250	300	3.4	300.78		30113	3.93	631.2	9.4	≤	7.14	11.5
	A211	42.8	2200	300	3.4	440.48	15.50	30113	3.93	936.8	9.4	≤	7.06	11.4
		42.8	2200	300	3.4	100.00	2.50	30113	3.93	969.3	9.4	≤	11.71	2.5
		42.8	2200	300	3.4	200.00	5.00	30113	3.93	940.5	9.4	≤	9.20	5.1
		42.8	2200	300	3.4	300.00	8.30	30113	3.93	937.6	9.4	≤	8.03	7.7
		42.8	2200	300	3.4	400.00	13.50	30113	3.93	936.9	9.4	≤	7.29	10.3
		42.8	2200	300	3.4	440.48	15.50	30113	3.93	936.8	9.4	≤	7.06	11.4
	B211	74.6	2200	300	3.4	456.70	14.60	37431	5.18	808.5	9.4	≤	7.14	11.0
		74.6	2200	300	3.4	100.00	2.50	37431	5.18	907.9	9.4	≤	12.32	2.1
		74.6	2200	300	3.4	200.00	5.00	37431	5.18	820.0	9.4	≤	9.45	4.7
		74.6	2200	300	3.4	300.00	8.30	37431	5.18	811.2	9.4	≤	8.23	7.2
		74.6	2200	300	3.4	400.00	11.80	37431	5.18	809.0	9.4	≤	7.47	9.6
		74.6	2200	300	3.4	456.70	14.60	37431	5.18	808.5	9.4	≤	7.14	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	37065	5.12	1097.9	9.4	≤	7.25	10.3
		72.8	3460	310	3.4	100.00	2.50	37065	5.12	1141.0	9.4	≤	13.16	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.12	1103.1	9.4	≤	10.33	3.6
		72.8	3460	310	3.4	400.00	9.70	37065	5.12	1098.3	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.12	1098.0	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	575.68	17.00	37065	5.12	1097.9	9.4	≤	7.25	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	37065	5.12	1097.9	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	37065	5.12	1141.0	9.4	≤	13.16	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.12	1103.1	9.4	≤	10.33	3.6
		72.8	3460	310	3.4	400.00	9.70	37065	5.12	1098.3	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.12	1098.0	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	560.36	16.90	37065	5.12	1097.9	9.4	≤	7.32	10.0
	B313	72.8	3460	310	3.4	580.26	16.10	37065	5.12	1097.9	9.4	≤	7.23	10.4
		72.8	3460	310	3.4	100.00	2.50	37065	5.12	1141.0	9.4	≤	13.16	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.12	1103.1	9.4	≤	10.33	3.6
		72.8	3460	310	3.4	400.00	9.70	37065	5.12	1098.3	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.12	1098.0	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	580.26	16.10	37065	5.12	1097.9	9.4	≤	7.23	10.4
	B321	77	3460	620	3.4	551.16	15.50	37913	5.26	1082.4	9.4	≤	7.47	9.8
		77	3460	620	3.4	100.00	1.70	37913	5.26	1132.4	9.4	≤	13.40	1.7
		77	3460	620	3.4	200.00	4.20	37913	5.26	1088.4	9.4	≤	10.49	3.5
		77	3460	620	3.4	400.00	10.00	37913	5.26	1082.9	9.4	≤	8.31	7.1
		77	3460	620	3.4	500.00	13.30	37913	5.26	1082.5	9.4	≤	7.72	8.9
		77	3460	620	3.4	551.16	15.50	37913	5.26	1082.4	9.4	≤	7.47	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	37065	5.12	1097.9	9.4	≤	7.38	10.6
		72.8	3460	940	3.4	100.00	1.70	37065	5.12	1141.0	9.4	≤	13.51	1.7
		72.8	3460	940	3.4	200.00	4.20	37065	5.12	1103.1	9.4	≤	10.60	3.6
		72.8	3460	940	3.4	400.00	10.00	37065	5.12	1098.3	9.4	≤	8.40	7.1
		72.8	3460	940	3.4	500.00	13.30	37065	5.12	1098.0	9.4	≤	7.80	8.9
		72.8	3460	940	3.4	590.38	16.50	37065	5.12	1097.9	9.4	≤	7.38	10.6
	B411	77	4730	320	3.4	621.34	15.50	37913	5.26	1309.1	9.4	≤	7.53	9.1
		77	4730	320	3.4	100.00		37913	5.26	1313.9	9.4	≤	13.86	1.5
		77	4730	320	3.4	200.00		37913	5.26	1309.7	9.4	≤	10.99	2.9
		77	4730	320	3.4	300.00		37913	5.26	1309.3	9.4	≤	9.60	4.4
		77	4730	320	3.4	400.00		37913	5.26	1309.2	9.4	≤	8.72	5.9
		77	4730	320	3.4	500.00		37913	5.26	1309.1	9.4	≤	8.09	7.3
		77	4730	320	3.4	621.34		37913	5.26	1309.1	9.4	≤	7.53	9.1
	C211	85.6	2710	300	3.4	560.94	18.20	39579	5.55	897.1	9.4	≤	7.01	11.5
		85.6	2710	300	3.4	100.00	2.50	39579	5.55	999.1	9.4	≤	12.91	1.8
		85.6	2710	300	3.4	200.00	5.20	39579	5.55	909.4	9.4	≤	9.93	4.0
		85.6	2710	300	3.4	400.00	11.70	39579	5.55	898.2	9.4	≤	7.85	8.2
		85.6	2710	300	3.4	500.00	15.10	39579	5.55	897.4	9.4	≤	7.28	10.2
		85.6	2710	300	3.4	560.94	18.20	39579	5.55	897.1	9.4	≤	7.01	11.5
	C311	88.1	3220	310	3.4	605.46	18.00	40048	5.63	997.8	9.4	≤	7.09	11.0
		88.1	3220	310	3.4	100.00	1.70	40048	5.63	1079.7	9.4	≤	13.27	1.7
		88.1	3220	310	3.4	200.00	5.30	40048	5.63	1007.7	9.4	≤	10.29	3.6
		88.1	3220	310	3.4	400.00	12.10	40048	5.63	998.7	9.4	≤	8.15	7.3
		88.1	3220	310	3.4	500.00	15.40	40048	5.63	998.1	9.4	≤	7.56	9.1
		88.1	3220	310	3.4	605.46	18.00	40048	5.63	997.8	9.4	≤	7.09	11.0
	C411	85.6	4260	320	3.4	722.56	19.30	39579	5.55	1198.9	9.4	≤	7.06	11.1
		85.6	4260	320	3.4	100.00	1.70	39579	5.55	1230.4	9.4	≤	13.77	1.5
		85.6	4260	320	3.4	200.00	4.70	39579	5.55	1202.7	9.4	≤	10.85	3.1
		85.6	4260	320	3.4	400.00	10.00	39579	5.55	1199.3	9.4	≤	8.60	6.1
		85.6	4260	320	3.4	600.00	15.80	39579	5.55	1198.9	9.4	≤	7.51	9.2
		85.6	4260	320	3.4	722.56	19.30	39579	5.55	1198.9	9.4	≤	7.06	11.1
	C511	88.1	5310	330	3.4	811.82	20.80	40048	5.63	1333.3	9.4	≤	7.05	11.1
		88.1	5310	330	3.4	100.00	1.70	40048	5.63	1333.3	9.4	≤	14.17	1.4
		88.1	5310	330	3.4	200.00	3.30	40048	5.63	1333.3	9.4	≤	11.25	2.7
		88.1	5310	330	3.4	400.00	8.70	40048	5.63	1333.3	9.4	≤	8.93	5.4
		88.1	5310	330	3.4	600.00	14.20	40048	5.63	1333.3	9.4	≤	7.80	8.2
		88.1	5310	330	3.4	811.82	20.80	40048	5.63	1333.3	9.4	≤	7.05	11.1
	D211	114.5	3220	300	3.4	506.00	16.00	44639	6.42	929.6	9.4	≤	7.62	8.9
		114.5	3220	300	3.4	100.00	1.60	44639	6.42	1075.5	9.4	≤	13.74	1.5
		114.5	3220	300	3.4	200.00	5.00	44639	6.42	946.9	9.4	≤	10.45	3.4
		114.5	3220	300	3.4	300.00	8.60	44639	6.42	933.9	9.4	≤	9.09	5.2
		114.5	3220	300	3.4	400.00	12.10	44639	6.42	930.8	9.4	≤	8.25	7.0
		114.5	3220	300	3.4	506.00	16.00	44639	6.42	929.6	9.4	≤	7.62	8.9
	E211	126.2	2200	300	3.4	506.00	15.90	46502	6.74	694.3	9.4	≤	7.05	11.4
		126.2	2200	300	3.4	100.00	1.70	46502	6.74	961.5	9.4	≤	13.50	1.6
		126.2	2200	300	3.4	200.00	5.00	46502	6.74	725.9	9.4	≤	9.75	4.3
		126.2	2200	300	3.4	300.00	8.60	46502	6.74	702.2	9.4	≤	8.43	6.7
		126.2	2200	300	3.4	400.00	12.10	46502	6.74	696.4	9.4	≤	7.64	9.0
		126.2	2200	300	3.4	506.00	15.90	46502	6.74	694.3	9.4	≤	7.05	11.4



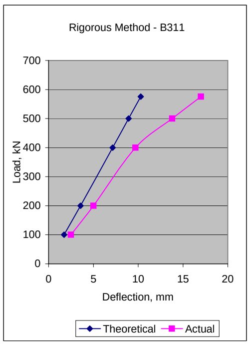
Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild



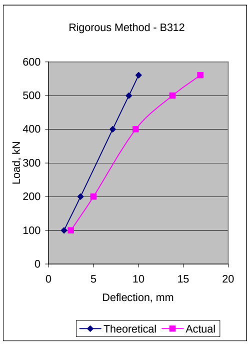
Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual



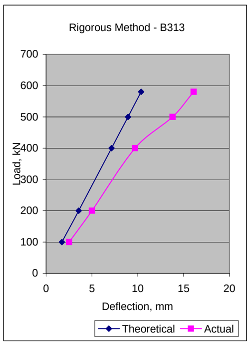
Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual



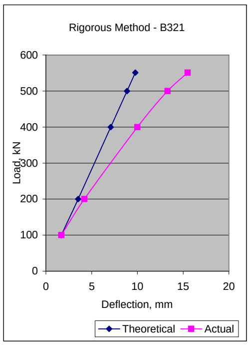
Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual



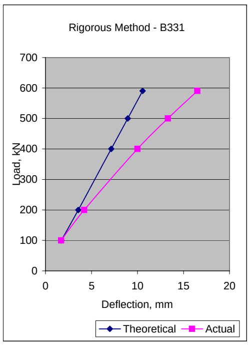
Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual



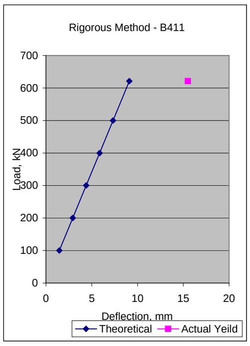
Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual



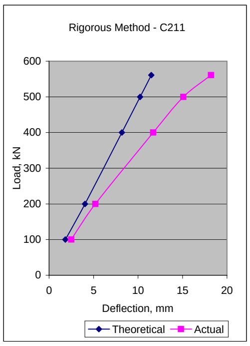
Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual



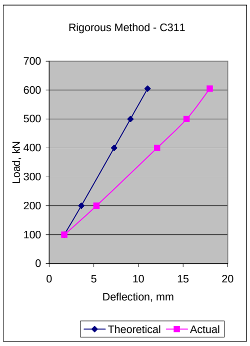
Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual



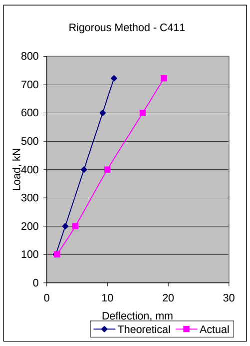
Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild



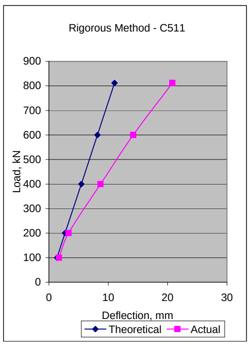
Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual



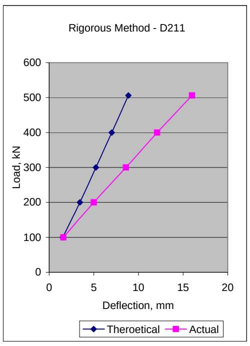
Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual



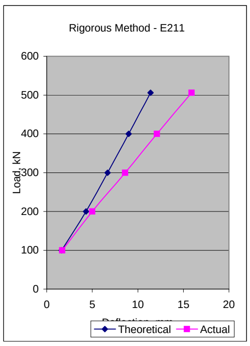
Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual



Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual



Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

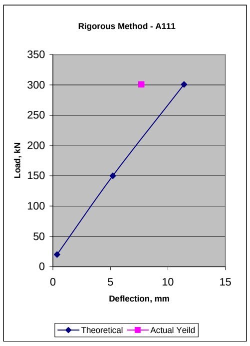
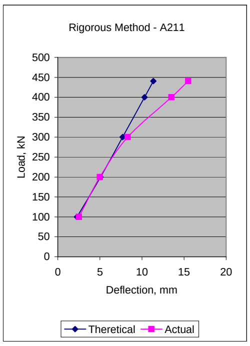
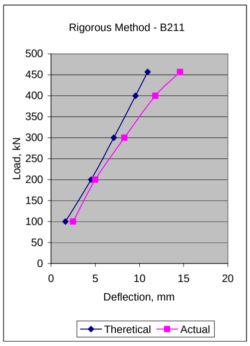
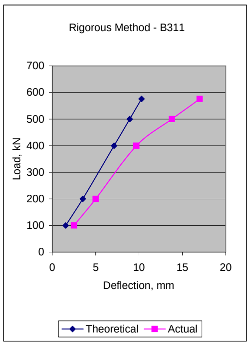


Rigorous Method - E211
Load, kN
Theoretical
Actual

METHOD 15

E_c =	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$I_{cr} + (I - I_{cr})(M_{cr}/M_a)^3$
f'_{cf} =	$0.94\sqrt{f'_c}$

SOURCE	BEAM	f'_c, MPa	Reinforcement Ratio: Area of Steel, A_{st}, mm^2	Reinforcement Ratio: Area of Steel, A_{sc}, mm^2	L_{eff}, m	At Yield: Load, kN	At Yield: Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Deem-to-Comply: Ratio	≤	Deem-to-Comply: Allowed Ratio	Rigorous Method: Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30113	6.15	637.3	9.4	≤	9.98	11.4
		42.8	1250	300	3.4	20.00		30113	6.15	1333.3	9.4	≤	22.60	0.4
		42.8	1250	300	3.4	150.00		30113	6.15	695.6	9.4	≤	9.30	5.2
		42.8	1250	300	3.4	300.78		30113	6.15	637.3	9.4	≤	7.16	11.4
	A211	42.8	2200	300	3.4	440.48	15.50	30113	6.15	937.9	9.4	≤	7.06	11.3
		42.8	2200	300	3.4	100.00	2.50	30113	6.15	1063.0	9.4	≤	12.07	2.3
		42.8	2200	300	3.4	200.00	5.00	30113	6.15	952.2	9.4	≤	9.24	5.1
		42.8	2200	300	3.4	300.00	8.30	30113	6.15	941.1	9.4	≤	8.04	7.7
		42.8	2200	300	3.4	400.00	13.50	30113	6.15	938.4	9.4	≤	7.30	10.3
		42.8	2200	300	3.4	440.48	15.50	30113	6.15	937.9	9.4	≤	7.06	11.3
	B211	74.6	2200	300	3.4	456.70	14.60	37431	8.12	811.5	9.4	≤	7.15	10.9
		74.6	2200	300	3.4	100.00	2.50	37431	8.12	1193.5	9.4	≤	13.49	1.6
		74.6	2200	300	3.4	200.00	5.00	37431	8.12	855.7	9.4	≤	9.59	4.5
		74.6	2200	300	3.4	300.00	8.30	37431	8.12	821.8	9.4	≤	8.26	7.1
		74.6	2200	300	3.4	400.00	11.80	37431	8.12	813.5	9.4	≤	7.48	9.6
		74.6	2200	300	3.4	456.70	14.60	37431	8.12	811.5	9.4	≤	7.15	10.9
	B311	72.8	3460	310	3.4	575.68	17.00	37065	8.02	1098.5	9.4	≤	7.25	10.3
		72.8	3460	310	3.4	100.00	2.50	37065	8.02	1264.4	9.4	≤	13.62	1.6
		72.8	3460	310	3.4	200.00	5.00	37065	8.02	1118.5	9.4	≤	10.38	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	8.02	1100.3	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	8.02	1099.0	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	575.68	17.00	37065	8.02	1098.5	9.4	≤	7.25	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	37065	8.02	1098.6	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	37065	8.02	1264.4	9.4	≤	13.62	1.6
		72.8	3460	310	3.4	200.00	5.00	37065	8.02	1118.5	9.4	≤	10.38	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	8.02	1100.3	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	8.02	1099.0	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	560.36	16.90	37065	8.02	1098.6	9.4	≤	7.32	10.0
	B313	72.8	3460	310	3.4	580.26	16.10	37065	8.02	1098.5	9.4	≤	7.23	10.4
		72.8	3460	310	3.4	100.00	2.50	37065	8.02	1264.4	9.4	≤	13.62	1.6
		72.8	3460	310	3.4	200.00	5.00	37065	8.02	1118.5	9.4	≤	10.38	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	8.02	1100.3	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	8.02	1099.0	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	580.26	16.10	37065	8.02	1098.5	9.4	≤	7.23	10.4
	B321	77	3460	620	3.4	551.16	15.50	37913	8.25	1083.2	9.4	≤	7.47	9.8
		77	3460	620	3.4	100.00	1.70	37913	8.25	1275.5	9.4	≤	13.94	1.5
		77	3460	620	3.4	200.00	4.20	37913	8.25	1106.3	9.4	≤	10.55	3.5
		77	3460	620	3.4	400.00	10.00	37913	8.25	1085.1	9.4	≤	8.32	7.1
		77	3460	620	3.4	500.00	13.30	37913	8.25	1083.6	9.4	≤	7.72	8.9
		77	3460	620	3.4	551.16	15.50	37913	8.25	1083.2	9.4	≤	7.47	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	37065	8.02	1098.5	9.4	≤	7.38	10.5
		72.8	3460	940	3.4	100.00	1.70	37065	8.02	1264.4	9.4	≤	13.98	1.6
		72.8	3460	940	3.4	200.00	4.20	37065	8.02	1118.5	9.4	≤	10.65	3.5
		72.8	3460	940	3.4	400.00	10.00	37065	8.02	1100.3	9.4	≤	8.41	7.1
		72.8	3460	940	3.4	500.00	13.30	37065	8.02	1099.0	9.4	≤	7.80	8.9
		72.8	3460	940	3.4	590.38	16.50	37065	8.02	1098.5	9.4	≤	7.38	10.5
	B411	77	4730	320	3.4	621.34	15.50	37913	8.25	1309.2	9.4	≤	7.53	9.1
		77	4730	320	3.4	100.00		37913	8.25	1327.8	9.4	≤	13.91	1.4
		77	4730	320	3.4	200.00		37913	8.25	1311.4	9.4	≤	10.99	2.9
		77	4730	320	3.4	300.00		37913	8.25	1309.8	9.4	≤	9.60	4.4
		77	4730	320	3.4	400.00		37913	8.25	1309.4	9.4	≤	8.72	5.9
		77	4730	320	3.4	500.00		37913	8.25	1309.2	9.4	≤	8.09	7.3
		77	4730	320	3.4	621.34		37913	8.25	1309.2	9.4	≤	7.53	9.1
	C211	85.6	2710	300	3.4	560.94	18.20	39579	8.70	898.8	9.4	≤	7.01	11.5
		85.6	2710	300	3.4	100.00	2.50	39579	8.70	1290.7	9.4	≤	14.06	1.4
		85.6	2710	300	3.4	200.00	5.20	39579	8.70	945.8	9.4	≤	10.06	3.9
		85.6	2710	300	3.4	400.00	11.70	39579	8.70	902.7	9.4	≤	7.86	8.1
		85.6	2710	300	3.4	500.00	15.10	39579	8.70	899.7	9.4	≤	7.29	10.2
		85.6	2710	300	3.4	560.94	18.20	39579	8.70	898.8	9.4	≤	7.01	11.5
	C311	88.1	3220	310	3.4	605.46	18.00	40048	8.82	998.9	9.4	≤	7.10	11.0
		88.1	3220	310	3.4	100.00	1.70	40048	8.82	1313.9	9.4	≤	14.17	1.4
		88.1	3220	310	3.4	200.00	5.30	40048	8.82	1037.0	9.4	≤	10.39	3.5
		88.1	3220	310	3.4	400.00	12.10	40048	8.82	1002.4	9.4	≤	8.16	7.2
		88.1	3220	310	3.4	500.00	15.40	40048	8.82	1000.0	9.4	≤	7.57	9.1
		88.1	3220	310	3.4	605.46	18.00	40048	8.82	998.9	9.4	≤	7.10	11.0
	C411	85.6	4260	320	3.4	722.56	19.30	39579	8.70	1199.1	9.4	≤	7.06	11.1
		85.6	4260	320	3.4	100.00	1.70	39579	8.70	1320.2	9.4	≤	14.10	1.4
		85.6	4260	320	3.4	200.00	4.70	39579	8.70	1214.0	9.4	≤	10.88	3.0
		85.6	4260	320	3.4	400.00	10.00	39579	8.70	1200.7	9.4	≤	8.60	6.1
		85.6	4260	320	3.4	600.00	15.80	39579	8.70	1199.3	9.4	≤	7.51	9.2
		85.6	4260	320	3.4	722.56	19.30	39579	8.70	1199.1	9.4	≤	7.06	11.1
	C511	88.1	5310	330	3.4	811.82	20.80	40048	8.82	1333.3	9.4	≤	7.05	11.1
		88.1	5310	330	3.4	100.00	1.70	40048	8.82	1333.3	9.4	≤	14.17	1.4
		88.1	5310	330	3.4	200.00	3.30	40048	8.82	1333.3	9.4	≤	11.25	2.7
		88.1	5310	330	3.4	400.00	8.70	40048	8.82	1333.3	9.4	≤	8.93	5.4
		88.1	5310	330	3.4	600.00	14.20	40048	8.82	1333.3	9.4	≤	7.80	8.2
		88.1	5310	330	3.4	811.82	20.80	40048	8.82	1333.3	9.4	≤	7.05	11.1
	D211	114.5	3220	300	3.4	506.00	16.00	44639	10.06	932.9	9.4	≤	7.63	8.8
		114.5	3220	300	3.4	100.00	1.60	44639	10.06	1333.3	9.4	≤	14.76	1.2
		114.5	3220	300	3.4	200.00	5.00	44639	10.06	999.1	9.4	≤	10.64	3.3
		114.5	3220	300	3.4	300.00	8.60	44639	10.06	949.4	9.4	≤	9.14	5.1
		114.5	3220	300	3.4	400.00	12.10	44639	10.06	937.3	9.4	≤	8.27	7.0
		114.5	3220	300	3.4	506.00	16.00	44639	10.06	932.9	9.4	≤	7.63	8.8
	E211	126.2	2200	300	3.4	506.00	15.90	46502	10.56	700.2	9.4	≤	7.07	11.3
		126.2	2200	300	3.4	100.00	1.70	46502	10.56	1333.3	9.4	≤	15.05	1.2
		126.2	2200	300	3.4	200.00	5.00	46502	10.56	821.7	9.4	≤	10.17	3.8
		126.2	2200	300	3.4	300.00	8.60	46502	10.56	730.6	9.4	≤	8.54	6.4
		126.2	2200	300	3.4	400.00	12.10	46502	10.56	708.4	9.4	≤	7.68	8.8
		126.2	2200	300	3.4	506.00	15.90	46502	10.56	700.2	9.4	≤	7.07	11.3


Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

Rigorous Method - B211
Load, kN
Deflection, mm
Thertical
Actual

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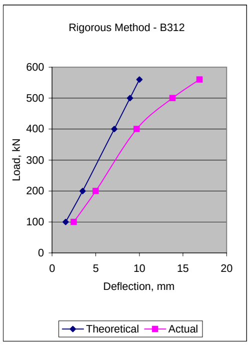
Load, kN
Deflection, mm
Theoretical
Actual

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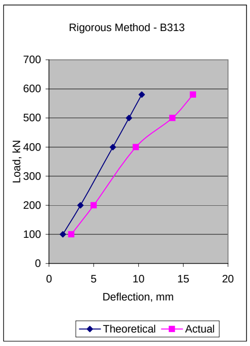
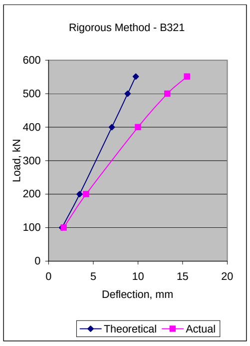
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

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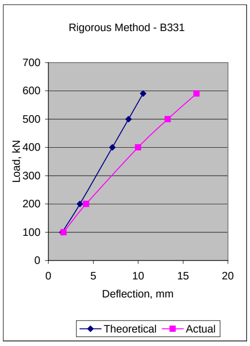
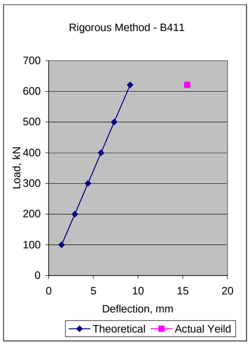
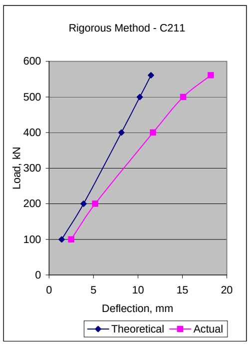
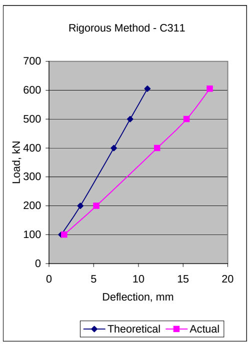
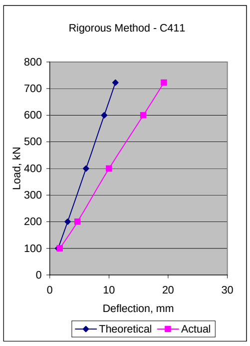
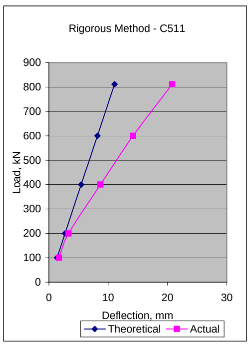
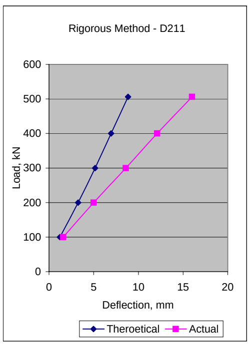
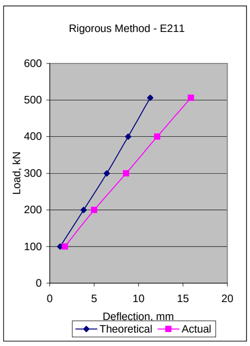
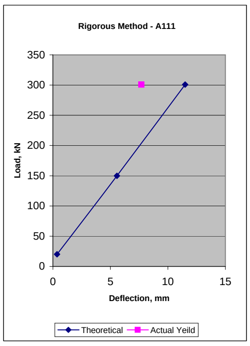
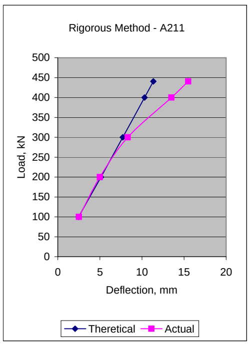
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

Rigorous Method - D211
Load, kN
Deflection, mm
Threoretical
Actual

Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

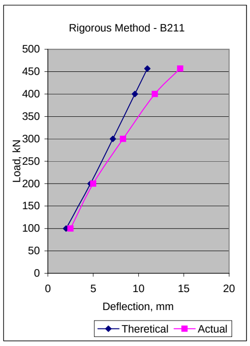
METHOD 16

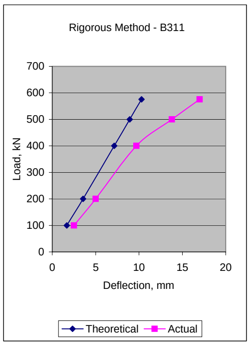
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.342(f'_c)^{2/3}$

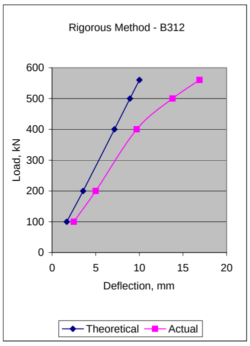
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30113	4.18	631.7	9.4	≤	9.95	11.5
		42.8	1250	300	3.4	20.00		30113	4.18	1333.3	9.4	≤	22.60	0.4
		42.8	1250	300	3.4	150.00		30113	4.18	650.0	9.4	≤	9.09	5.6
		42.8	1250	300	3.4	300.78		30113	4.18	631.7	9.4	≤	7.14	11.5
	A211	42.8	2200	300	3.4	440.48	15.50	30113	4.18	936.9	9.4	≤	7.06	11.4
		42.8	2200	300	3.4	100.00	2.50	30113	4.18	976.3	9.4	≤	11.74	2.5
		42.8	2200	300	3.4	200.00	5.00	30113	4.18	941.4	9.4	≤	9.20	5.1
		42.8	2200	300	3.4	300.00	8.30	30113	4.18	937.9	9.4	≤	8.03	7.7
		42.8	2200	300	3.4	400.00	13.50	30113	4.18	937.0	9.4	≤	7.29	10.3
		42.8	2200	300	3.4	440.48	15.50	30113	4.18	936.9	9.4	≤	7.06	11.4
	B211	74.6	2200	300	3.4	456.70	14.60	37431	6.06	809.1	9.4	≤	7.14	11.0
		74.6	2200	300	3.4	100.00	2.50	37431	6.06	968.0	9.4	≤	12.58	2.0
		74.6	2200	300	3.4	200.00	5.00	37431	6.06	827.5	9.4	≤	9.48	4.7
		74.6	2200	300	3.4	300.00	8.30	37431	6.06	813.4	9.4	≤	8.23	7.2
		74.6	2200	300	3.4	400.00	11.80	37431	6.06	810.0	9.4	≤	7.47	9.6
		74.6	2200	300	3.4	456.70	14.60	37431	6.06	809.1	9.4	≤	7.14	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	37065	5.96	1098.0	9.4	≤	7.25	10.3
		72.8	3460	310	3.4	100.00	2.50	37065	5.96	1166.2	9.4	≤	13.26	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.96	1106.2	9.4	≤	10.34	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	5.96	1098.7	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.96	1098.2	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	575.68	17.00	37065	5.96	1098.0	9.4	≤	7.25	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	37065	5.96	1098.0	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	37065	5.96	1166.2	9.4	≤	13.26	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.96	1106.2	9.4	≤	10.34	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	5.96	1098.7	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.96	1098.2	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	560.36	16.90	37065	5.96	1098.0	9.4	≤	7.32	10.0
	B313	72.8	3460	310	3.4	580.26	16.10	37065	5.96	1098.0	9.4	≤	7.23	10.4
		72.8	3460	310	3.4	100.00	2.50	37065	5.96	1166.2	9.4	≤	13.26	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.96	1106.2	9.4	≤	10.34	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	5.96	1098.7	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.96	1098.2	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	580.26	16.10	37065	5.96	1098.0	9.4	≤	7.23	10.4
	B321	77	3460	620	3.4	551.16	15.50	37913	6.19	1082.6	9.4	≤	7.47	9.8
		77	3460	620	3.4	100.00	1.70	37913	6.19	1163.8	9.4	≤	13.52	1.6
		77	3460	620	3.4	200.00	4.20	37913	6.19	1092.3	9.4	≤	10.51	3.5
		77	3460	620	3.4	400.00	10.00	37913	6.19	1083.4	9.4	≤	8.32	7.1
		77	3460	620	3.4	500.00	13.30	37913	6.19	1082.7	9.4	≤	7.72	8.9
		77	3460	620	3.4	551.16	15.50	37913	6.19	1082.6	9.4	≤	7.47	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	37065	5.96	1098.0	9.4	≤	7.38	10.6
		72.8	3460	940	3.4	100.00	1.70	37065	5.96	1166.2	9.4	≤	13.61	1.7
		72.8	3460	940	3.4	200.00	4.20	37065	5.96	1106.2	9.4	≤	10.61	3.5
		72.8	3460	940	3.4	400.00	10.00	37065	5.96	1098.7	9.4	≤	8.41	7.1
		72.8	3460	940	3.4	500.00	13.30	37065	5.96	1098.2	9.4	≤	7.80	8.9
		72.8	3460	940	3.4	590.38	16.50	37065	5.96	1098.0	9.4	≤	7.38	10.6
	B411	77	4730	320	3.4	621.34	15.50	37913	6.19	1309.1	9.4	≤	7.53	9.1
		77	4730	320	3.4	100.00		37913	6.19	1317.0	9.4	≤	13.87	1.5
		77	4730	320	3.4	200.00		37913	6.19	1310.1	9.4	≤	10.99	2.9
		77	4730	320	3.4	300.00		37913	6.19	1309.4	9.4	≤	9.60	4.4
		77	4730	320	3.4	400.00		37913	6.19	1309.2	9.4	≤	8.72	5.9
		77	4730	320	3.4	500.00		37913	6.19	1309.2	9.4	≤	8.09	7.3
		77	4730	320	3.4	621.34		37913	6.19	1309.1	9.4	≤	7.53	9.1
	C211	85.6	2710	300	3.4	560.94	18.20	39579	6.64	897.6	9.4	≤	7.01	11.5
		85.6	2710	300	3.4	100.00	2.50	39579	6.64	1072.2	9.4	≤	13.22	1.7
		85.6	2710	300	3.4	200.00	5.20	39579	6.64	918.5	9.4	≤	9.96	4.0
		85.6	2710	300	3.4	400.00	11.70	39579	6.64	899.3	9.4	≤	7.85	8.2
		85.6	2710	300	3.4	500.00	15.10	39579	6.64	898.0	9.4	≤	7.29	10.2
		85.6	2710	300	3.4	560.94	18.20	39579	6.64	897.6	9.4	≤	7.01	11.5
	C311	88.1	3220	310	3.4	605.46	18.00	40048	6.77	998.1	9.4	≤	7.09	11.0
		88.1	3220	310	3.4	100.00	1.70	40048	6.77	1140.5	9.4	≤	13.52	1.6
		88.1	3220	310	3.4	200.00	5.30	40048	6.77	1015.3	9.4	≤	10.32	3.6
		88.1	3220	310	3.4	400.00	12.10	40048	6.77	999.7	9.4	≤	8.15	7.3
		88.1	3220	310	3.4	500.00	15.40	40048	6.77	998.6	9.4	≤	7.56	9.1
		88.1	3220	310	3.4	605.46	18.00	40048	6.77	998.1	9.4	≤	7.09	11.0
	C411	85.6	4260	320	3.4	722.56	19.30	39579	6.64	1198.9	9.4	≤	7.06	11.1
		85.6	4260	320	3.4	100.00	1.70	39579	6.64	1252.9	9.4	≤	13.85	1.5
		85.6	4260	320	3.4	200.00	4.70	39579	6.64	1205.5	9.4	≤	10.85	3.0
		85.6	4260	320	3.4	400.00	10.00	39579	6.64	1199.6	9.4	≤	8.60	6.1
		85.6	4260	320	3.4	600.00	15.80	39579	6.64	1199.0	9.4	≤	7.51	9.2
		85.6	4260	320	3.4	722.56	19.30	39579	6.64	1198.9	9.4	≤	7.06	11.1
	C511	88.1	5310	330	3.4	811.82	20.80	40048	6.77	1333.3	9.4	≤	7.05	11.1
		88.1	5310	330	3.4	100.00	1.70	40048	6.77	1333.3	9.4	≤	14.17	1.4
		88.1	5310	330	3.4	200.00	3.30	40048	6.77	1333.3	9.4	≤	11.25	2.7
		88.1	5310	330	3.4	400.00	8.70	40048	6.77	1333.3	9.4	≤	8.93	5.4
		88.1	5310	330	3.4	600.00	14.20	40048	6.77	1333.3	9.4	≤	7.80	8.2
		88.1	5310	330	3.4	811.82	20.80	40048	6.77	1333.3	9.4	≤	7.05	11.1
	D211	114.5	3220	300	3.4	506.00	16.00	44639	8.06	930.7	9.4	≤	7.63	8.9
		114.5	3220	300	3.4	100.00	1.60	44639	8.06	1219.7	9.4	≤	14.33	1.3
		114.5	3220	300	3.4	200.00	5.00	44639	8.06	964.9	9.4	≤	10.52	3.4
		114.5	3220	300	3.4	300.00	8.60	44639	8.06	939.3	9.4	≤	9.10	5.2
		114.5	3220	300	3.4	400.00	12.10	44639	8.06	933.0	9.4	≤	8.25	7.0
		114.5	3220	300	3.4	506.00	16.00	44639	8.06	930.7	9.4	≤	7.63	8.9
	E211	126.2	2200	300	3.4	506.00	15.90	46502	8.60	696.5	9.4	≤	7.06	11.4
		126.2	2200	300	3.4	100.00	1.70	46502	8.60	1252.5	9.4	≤	14.74	1.2
		126.2	2200	300	3.4	200.00	5.00	46502	8.60	762.2	9.4	≤	9.91	4.1
		126.2	2200	300	3.4	300.00	8.60	46502	8.60	713.0	9.4	≤	8.47	6.6
		126.2	2200	300	3.4	400.00	12.10	46502	8.60	701.0	9.4	≤	7.65	8.9
		126.2	2200	300	3.4	506.00	15.90	46502	8.60	696.5	9.4	≤	7.06	11.4

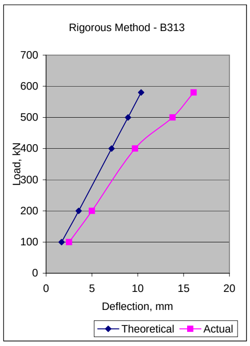

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

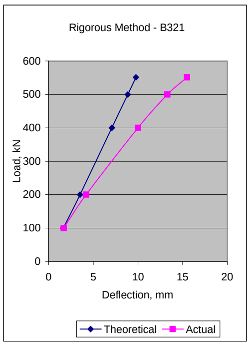

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

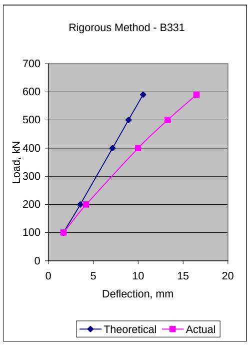

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

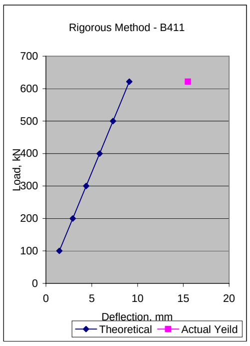

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

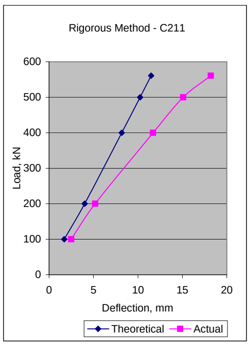

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

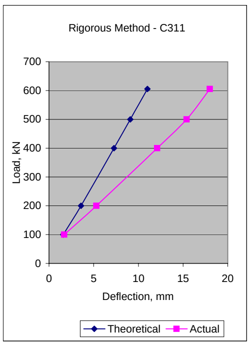

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

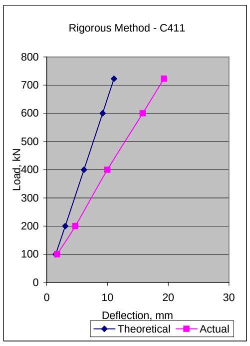

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

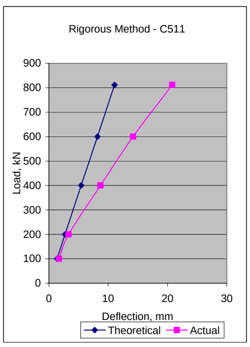

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

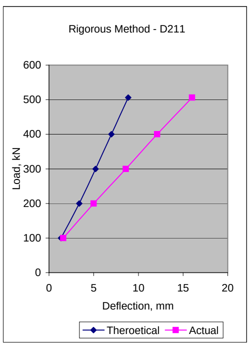

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

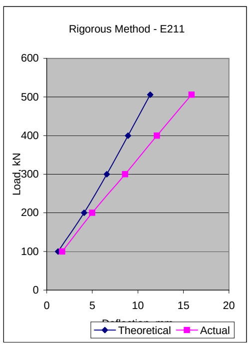

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

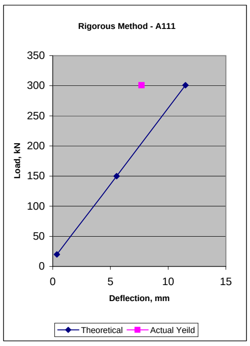

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

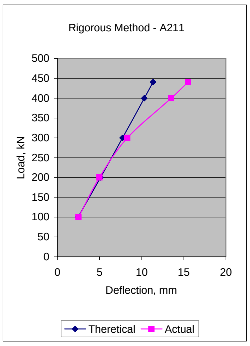

Rigorous Method - E211
Load, kN
Theoretical
Actual

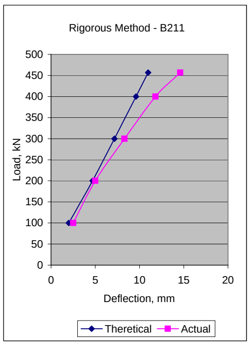
METHOD 17

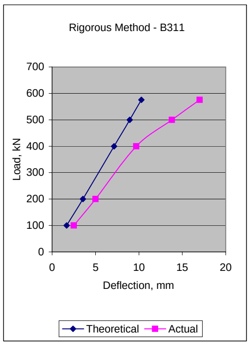
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.564(f'_c)^{0.55}$

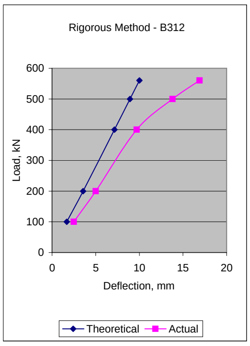
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30113	4.45	632.2	9.4	≤	9.96	11.5
		42.8	1250	300	3.4	20.00		30113	4.45	1333.3	9.4	≤	22.60	0.4
		42.8	1250	300	3.4	150.00		30113	4.45	654.3	9.4	≤	9.11	5.5
		42.8	1250	300	3.4	300.78		30113	4.45	632.2	9.4	≤	7.14	11.5
	A211	42.8	2200	300	3.4	440.48	15.50	30113	4.45	937.0	9.4	≤	7.06	11.4
		42.8	2200	300	3.4	100.00	2.50	30113	4.45	984.5	9.4	≤	11.77	2.5
		42.8	2200	300	3.4	200.00	5.00	30113	4.45	942.4	9.4	≤	9.21	5.1
		42.8	2200	300	3.4	300.00	8.30	30113	4.45	938.2	9.4	≤	8.03	7.7
		42.8	2200	300	3.4	400.00	13.50	30113	4.45	937.2	9.4	≤	7.29	10.3
		42.8	2200	300	3.4	440.48	15.50	30113	4.45	937.0	9.4	≤	7.06	11.4
	B211	74.6	2200	300	3.4	456.70	14.60	37431	6.04	809.1	9.4	≤	7.14	11.0
		74.6	2200	300	3.4	100.00	2.50	37431	6.04	966.7	9.4	≤	12.58	2.0
		74.6	2200	300	3.4	200.00	5.00	37431	6.04	827.4	9.4	≤	9.48	4.7
		74.6	2200	300	3.4	300.00	8.30	37431	6.04	813.4	9.4	≤	8.23	7.2
		74.6	2200	300	3.4	400.00	11.80	37431	6.04	809.9	9.4	≤	7.47	9.6
		74.6	2200	300	3.4	456.70	14.60	37431	6.04	809.1	9.4	≤	7.14	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	37065	5.96	1098.0	9.4	≤	7.25	10.3
		72.8	3460	310	3.4	100.00	2.50	37065	5.96	1166.2	9.4	≤	13.26	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.96	1106.2	9.4	≤	10.34	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	5.96	1098.7	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.96	1098.2	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	575.68	17.00	37065	5.96	1098.0	9.4	≤	7.25	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	37065	5.96	1098.0	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	37065	5.96	1166.2	9.4	≤	13.26	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.96	1106.2	9.4	≤	10.34	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	5.96	1098.7	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.96	1098.2	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	560.36	16.90	37065	5.96	1098.0	9.4	≤	7.32	10.0
	B313	72.8	3460	310	3.4	580.26	16.10	37065	5.96	1098.0	9.4	≤	7.23	10.4
		72.8	3460	310	3.4	100.00	2.50	37065	5.96	1166.2	9.4	≤	13.26	1.7
		72.8	3460	310	3.4	200.00	5.00	37065	5.96	1106.2	9.4	≤	10.34	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	5.96	1098.7	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	5.96	1098.2	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	580.26	16.10	37065	5.96	1098.0	9.4	≤	7.23	10.4
	B321	77	3460	620	3.4	551.16	15.50	37913	6.15	1082.6	9.4	≤	7.47	9.8
		77	3460	620	3.4	100.00	1.70	37913	6.15	1162.2	9.4	≤	13.51	1.7
		77	3460	620	3.4	200.00	4.20	37913	6.15	1092.1	9.4	≤	10.51	3.5
		77	3460	620	3.4	400.00	10.00	37913	6.15	1083.3	9.4	≤	8.32	7.1
		77	3460	620	3.4	500.00	13.30	37913	6.15	1082.7	9.4	≤	7.72	8.9
		77	3460	620	3.4	551.16	15.50	37913	6.15	1082.6	9.4	≤	7.47	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	37065	5.96	1098.0	9.4	≤	7.38	10.6
		72.8	3460	940	3.4	100.00	1.70	37065	5.96	1166.2	9.4	≤	13.61	1.7
		72.8	3460	940	3.4	200.00	4.20	37065	5.96	1106.2	9.4	≤	10.61	3.5
		72.8	3460	940	3.4	400.00	10.00	37065	5.96	1098.7	9.4	≤	8.41	7.1
		72.8	3460	940	3.4	500.00	13.30	37065	5.96	1098.2	9.4	≤	7.80	8.9
		72.8	3460	940	3.4	590.38	16.50	37065	5.96	1098.0	9.4	≤	7.38	10.6
	B411	77	4730	320	3.4	621.34	15.50	37913	6.15	1309.1	9.4	≤	7.53	9.1
		77	4730	320	3.4	100.00		37913	6.15	1316.8	9.4	≤	13.87	1.5
		77	4730	320	3.4	200.00		37913	6.15	1310.1	9.4	≤	10.99	2.9
		77	4730	320	3.4	300.00		37913	6.15	1309.4	9.4	≤	9.60	4.4
		77	4730	320	3.4	400.00		37913	6.15	1309.2	9.4	≤	8.72	5.9
		77	4730	320	3.4	500.00		37913	6.15	1309.2	9.4	≤	8.09	7.3
		77	4730	320	3.4	621.34		37913	6.15	1309.1	9.4	≤	7.53	9.1
	C211	85.6	2710	300	3.4	560.94	18.20	39579	6.52	897.5	9.4	≤	7.01	11.5
		85.6	2710	300	3.4	100.00	2.50	39579	6.52	1062.5	9.4	≤	13.18	1.7
		85.6	2710	300	3.4	200.00	5.20	39579	6.52	917.3	9.4	≤	9.96	4.0
		85.6	2710	300	3.4	400.00	11.70	39579	6.52	899.2	9.4	≤	7.85	8.2
		85.6	2710	300	3.4	500.00	15.10	39579	6.52	897.9	9.4	≤	7.28	10.2
		85.6	2710	300	3.4	560.94	18.20	39579	6.52	897.5	9.4	≤	7.01	11.5
	C311	88.1	3220	310	3.4	605.46	18.00	40048	6.62	998.1	9.4	≤	7.09	11.0
		88.1	3220	310	3.4	100.00	1.70	40048	6.62	1131.3	9.4	≤	13.48	1.6
		88.1	3220	310	3.4	200.00	5.30	40048	6.62	1014.2	9.4	≤	10.32	3.6
		88.1	3220	310	3.4	400.00	12.10	40048	6.62	999.5	9.4	≤	8.15	7.3
		88.1	3220	310	3.4	500.00	15.40	40048	6.62	998.5	9.4	≤	7.56	9.1
		88.1	3220	310	3.4	605.46	18.00	40048	6.62	998.1	9.4	≤	7.09	11.0
	C411	85.6	4260	320	3.4	722.56	19.30	39579	6.52	1198.9	9.4	≤	7.06	11.1
		85.6	4260	320	3.4	100.00	1.70	39579	6.52	1249.9	9.4	≤	13.84	1.5
		85.6	4260	320	3.4	200.00	4.70	39579	6.52	1205.2	9.4	≤	10.85	3.1
		85.6	4260	320	3.4	400.00	10.00	39579	6.52	1199.6	9.4	≤	8.60	6.1
		85.6	4260	320	3.4	600.00	15.80	39579	6.52	1199.0	9.4	≤	7.51	9.2
		85.6	4260	320	3.4	722.56	19.30	39579	6.52	1198.9	9.4	≤	7.06	11.1
	C511	88.1	5310	330	3.4	811.82	20.80	40048	6.62	1333.3	9.4	≤	7.05	11.1
		88.1	5310	330	3.4	100.00	1.70	40048	6.62	1333.3	9.4	≤	14.17	1.4
		88.1	5310	330	3.4	200.00	3.30	40048	6.62	1333.3	9.4	≤	11.25	2.7
		88.1	5310	330	3.4	400.00	8.70	40048	6.62	1333.3	9.4	≤	8.93	5.4
		88.1	5310	330	3.4	600.00	14.20	40048	6.62	1333.3	9.4	≤	7.80	8.2
		88.1	5310	330	3.4	811.82	20.80	40048	6.62	1333.3	9.4	≤	7.05	11.1
	D211	114.5	3220	300	3.4	506.00	16.00	44639	7.65	930.4	9.4	≤	7.62	8.9
		114.5	3220	300	3.4	100.00	1.60	44639	7.65	1177.0	9.4	≤	14.16	1.4
		114.5	3220	300	3.4	200.00	5.00	44639	7.65	959.6	9.4	≤	10.50	3.4
		114.5	3220	300	3.4	300.00	8.60	44639	7.65	937.7	9.4	≤	9.10	5.2
		114.5	3220	300	3.4	400.00	12.10	44639	7.65	932.4	9.4	≤	8.25	7.0
		114.5	3220	300	3.4	506.00	16.00	44639	7.65	930.4	9.4	≤	7.62	8.9
	E211	126.2	2200	300	3.4	506.00	15.90	46502	8.07	695.8	9.4	≤	7.06	11.4
		126.2	2200	300	3.4	100.00	1.70	46502	8.07	1154.4	9.4	≤	14.34	1.4
		126.2	2200	300	3.4	200.00	5.00	46502	8.07	750.0	9.4	≤	9.86	4.2
		126.2	2200	300	3.4	300.00	8.60	46502	8.07	709.3	9.4	≤	8.46	6.6
		126.2	2200	300	3.4	400.00	12.10	46502	8.07	699.4	9.4	≤	7.65	8.9
		126.2	2200	300	3.4	506.00	15.90	46502	8.07	695.8	9.4	≤	7.06	11.4

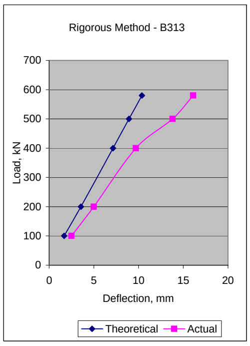

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

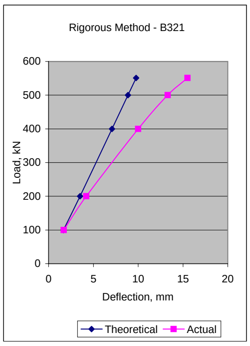

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

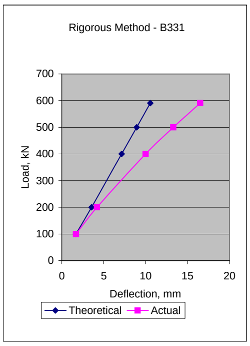

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

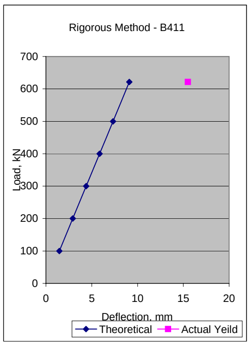

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

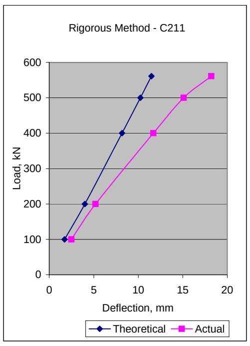

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

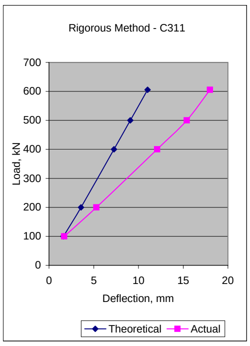

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

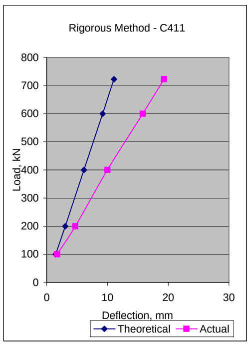

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

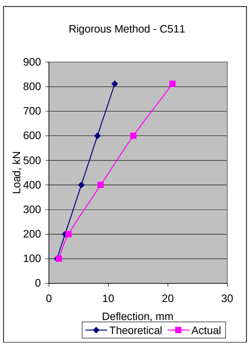

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

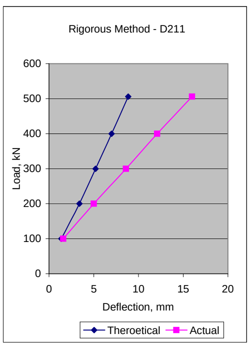

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

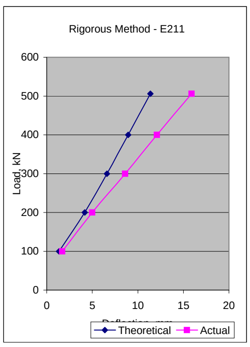

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

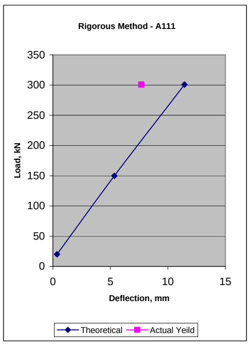

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

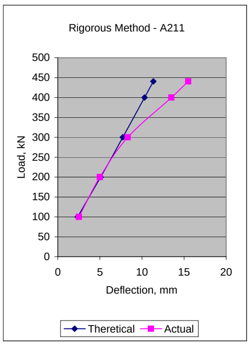

Rigorous Method - E211
Load, kN
Theoretical
Actual

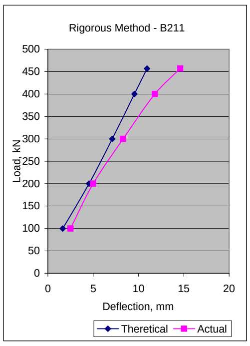
METHOD 18

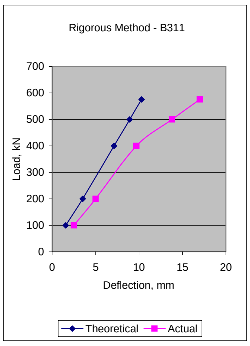
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.43(f'_c)^{0.68}$

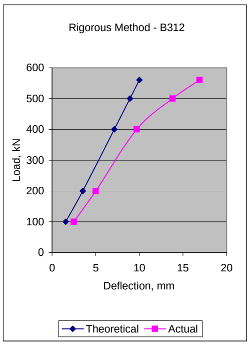
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, $10^6 mm$	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30113	5.53	635.1	9.4	≤	9.97	11.4
		42.8	1250	300	3.4	20.00		30113	5.53	1333.3	9.4	≤	22.60	0.4
		42.8	1250	300	3.4	150.00		30113	5.53	677.5	9.4	≤	9.21	5.3
		42.8	1250	300	3.4	300.78		30113	5.53	635.1	9.4	≤	7.15	11.4
	A211	42.8	2200	300	3.4	440.48	15.50	30113	5.53	937.5	9.4	≤	7.06	11.4
		42.8	2200	300	3.4	100.00	2.50	30113	5.53	1028.6	9.4	≤	11.94	2.3
		42.8	2200	300	3.4	200.00	5.00	30113	5.53	947.9	9.4	≤	9.22	5.1
		42.8	2200	300	3.4	300.00	8.30	30113	5.53	939.8	9.4	≤	8.03	7.7
		42.8	2200	300	3.4	400.00	13.50	30113	5.53	937.8	9.4	≤	7.30	10.3
		42.8	2200	300	3.4	440.48	15.50	30113	5.53	937.5	9.4	≤	7.06	11.4
	B211	74.6	2200	300	3.4	456.70	14.60	37431	8.07	811.4	9.4	≤	7.15	10.9
		74.6	2200	300	3.4	100.00	2.50	37431	8.07	1186.7	9.4	≤	13.47	1.6
		74.6	2200	300	3.4	200.00	5.00	37431	8.07	854.9	9.4	≤	9.58	4.5
		74.6	2200	300	3.4	300.00	8.30	37431	8.07	821.5	9.4	≤	8.26	7.1
		74.6	2200	300	3.4	400.00	11.80	37431	8.07	813.4	9.4	≤	7.48	9.6
		74.6	2200	300	3.4	456.70	14.60	37431	8.07	811.4	9.4	≤	7.15	10.9
	B311	72.8	3460	310	3.4	575.68	17.00	37065	7.94	1098.5	9.4	≤	7.25	10.3
		72.8	3460	310	3.4	100.00	2.50	37065	7.94	1259.4	9.4	≤	13.60	1.6
		72.8	3460	310	3.4	200.00	5.00	37065	7.94	1117.9	9.4	≤	10.38	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	7.94	1100.2	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	7.94	1098.9	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	575.68	17.00	37065	7.94	1098.5	9.4	≤	7.25	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	37065	7.94	1098.6	9.4	≤	7.32	10.0
		72.8	3460	310	3.4	100.00	2.50	37065	7.94	1259.4	9.4	≤	13.60	1.6
		72.8	3460	310	3.4	200.00	5.00	37065	7.94	1117.9	9.4	≤	10.38	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	7.94	1100.2	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	7.94	1098.9	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	560.36	16.90	37065	7.94	1098.6	9.4	≤	7.32	10.0
	B313	72.8	3460	310	3.4	580.26	16.10	37065	7.94	1098.5	9.4	≤	7.23	10.4
		72.8	3460	310	3.4	100.00	2.50	37065	7.94	1259.4	9.4	≤	13.60	1.6
		72.8	3460	310	3.4	200.00	5.00	37065	7.94	1117.9	9.4	≤	10.38	3.5
		72.8	3460	310	3.4	400.00	9.70	37065	7.94	1100.2	9.4	≤	8.19	7.1
		72.8	3460	310	3.4	500.00	13.80	37065	7.94	1098.9	9.4	≤	7.60	8.9
		72.8	3460	310	3.4	580.26	16.10	37065	7.94	1098.5	9.4	≤	7.23	10.4
	B321	77	3460	620	3.4	551.16	15.50	37913	8.25	1083.2	9.4	≤	7.47	9.8
		77	3460	620	3.4	100.00	1.70	37913	8.25	1275.4	9.4	≤	13.94	1.5
		77	3460	620	3.4	200.00	4.20	37913	8.25	1106.2	9.4	≤	10.55	3.5
		77	3460	620	3.4	400.00	10.00	37913	8.25	1085.1	9.4	≤	8.32	7.1
		77	3460	620	3.4	500.00	13.30	37913	8.25	1083.6	9.4	≤	7.72	8.9
		77	3460	620	3.4	551.16	15.50	37913	8.25	1083.2	9.4	≤	7.47	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	37065	7.94	1098.4	9.4	≤	7.38	10.5
		72.8	3460	940	3.4	100.00	1.70	37065	7.94	1259.4	9.4	≤	13.96	1.6
		72.8	3460	940	3.4	200.00	4.20	37065	7.94	1117.9	9.4	≤	10.65	3.5
		72.8	3460	940	3.4	400.00	10.00	37065	7.94	1100.2	9.4	≤	8.41	7.1
		72.8	3460	940	3.4	500.00	13.30	37065	7.94	1098.9	9.4	≤	7.80	8.9
		72.8	3460	940	3.4	590.38	16.50	37065	7.94	1098.4	9.4	≤	7.38	10.5
	B411	77	4730	320	3.4	621.34	15.50	37913	8.25	1309.2	9.4	≤	7.53	9.1
		77	4730	320	3.4	100.00		37913	8.25	1327.7	9.4	≤	13.91	1.4
		77	4730	320	3.4	200.00		37913	8.25	1311.4	9.4	≤	10.99	2.9
		77	4730	320	3.4	300.00		37913	8.25	1309.8	9.4	≤	9.60	4.4
		77	4730	320	3.4	400.00		37913	8.25	1309.4	9.4	≤	8.72	5.9
		77	4730	320	3.4	500.00		37913	8.25	1309.2	9.4	≤	8.09	7.3
		77	4730	320	3.4	621.34		37913	8.25	1309.2	9.4	≤	7.53	9.1
	C211	85.6	2710	300	3.4	560.94	18.20	39579	8.86	898.9	9.4	≤	7.01	11.5
		85.6	2710	300	3.4	100.00	2.50	39579	8.86	1313.6	9.4	≤	14.14	1.4
		85.6	2710	300	3.4	200.00	5.20	39579	8.86	948.7	9.4	≤	10.07	3.9
		85.6	2710	300	3.4	400.00	11.70	39579	8.86	903.1	9.4	≤	7.86	8.1
		85.6	2710	300	3.4	500.00	15.10	39579	8.86	899.9	9.4	≤	7.29	10.2
		85.6	2710	300	3.4	560.94	18.20	39579	8.86	898.9	9.4	≤	7.01	11.5
	C311	88.1	3220	310	3.4	605.46	18.00	40048	9.04	999.0	9.4	≤	7.10	11.0
		88.1	3220	310	3.4	100.00	1.70	40048	9.04	1333.3	9.4	≤	14.24	1.4
		88.1	3220	310	3.4	200.00	5.30	40048	9.04	1040.0	9.4	≤	10.40	3.5
		88.1	3220	310	3.4	400.00	12.10	40048	9.04	1002.8	9.4	≤	8.16	7.2
		88.1	3220	310	3.4	500.00	15.40	40048	9.04	1000.2	9.4	≤	7.57	9.1
		88.1	3220	310	3.4	605.46	18.00	40048	9.04	999.0	9.4	≤	7.10	11.0
	C411	85.6	4260	320	3.4	722.56	19.30	39579	8.86	1199.1	9.4	≤	7.06	11.1
		85.6	4260	320	3.4	100.00	1.70	39579	8.86	1327.3	9.4	≤	14.12	1.4
		85.6	4260	320	3.4	200.00	4.70	39579	8.86	1214.8	9.4	≤	10.88	3.0
		85.6	4260	320	3.4	400.00	10.00	39579	8.86	1200.8	9.4	≤	8.60	6.1
		85.6	4260	320	3.4	600.00	15.80	39579	8.86	1199.4	9.4	≤	7.51	9.2
		85.6	4260	320	3.4	722.56	19.30	39579	8.86	1199.1	9.4	≤	7.06	11.1
	C511	88.1	5310	330	3.4	811.82	20.80	40048	9.04	1333.3	9.4	≤	7.05	11.1
		88.1	5310	330	3.4	100.00	1.70	40048	9.04	1333.0	9.4	≤	14.17	1.4
		88.1	5310	330	3.4	200.00	3.30	40048	9.04	1333.3	9.4	≤	11.25	2.7
		88.1	5310	330	3.4	400.00	8.70	40048	9.04	1333.3	9.4	≤	8.93	5.4
		88.1	5310	330	3.4	600.00	14.20	40048	9.04	1333.3	9.4	≤	7.80	8.2
		88.1	5310	330	3.4	811.82	20.80	40048	9.04	1333.3	9.4	≤	7.05	11.1
	D211	114.5	3220	300	3.4	506.00	16.00	44639	10.80	933.9	9.4	≤	7.63	8.8
		114.5	3220	300	3.4	100.00	1.60	44639	10.80	1333.3	9.4	≤	14.76	1.2
		114.5	3220	300	3.4	200.00	5.00	44639	10.80	1016.0	9.4	≤	10.70	3.2
		114.5	3220	300	3.4	300.00	8.60	44639	10.80	954.4	9.4	≤	9.15	5.1
		114.5	3220	300	3.4	400.00	12.10	44639	10.80	939.4	9.4	≤	8.27	6.9
		114.5	3220	300	3.4	506.00	16.00	44639	10.80	933.9	9.4	≤	7.63	8.8
	E211	126.2	2200	300	3.4	506.00	15.90	46502	11.54	702.6	9.4	≤	7.08	11.3
		126.2	2200	300	3.4	100.00	1.70	46502	11.54	1333.3	9.4	≤	15.05	1.2
		126.2	2200	300	3.4	200.00	5.00	46502	11.54	861.1	9.4	≤	10.33	3.6
		126.2	2200	300	3.4	300.00	8.60	46502	11.54	742.3	9.4	≤	8.58	6.3
		126.2	2200	300	3.4	400.00	12.10	46502	11.54	713.3	9.4	≤	7.70	8.8
		126.2	2200	300	3.4	506.00	15.90	46502	11.54	702.6	9.4	≤	7.08	11.3

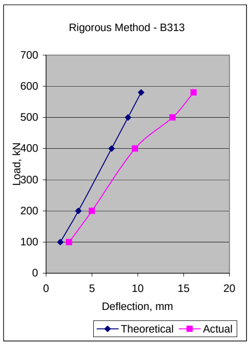

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

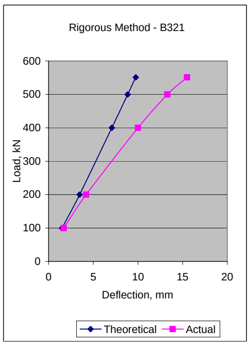

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

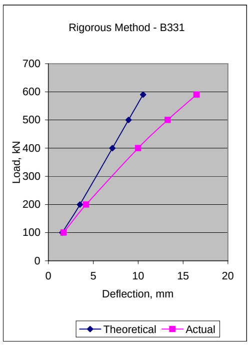

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

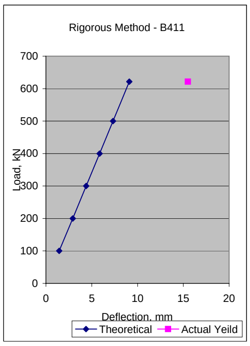

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

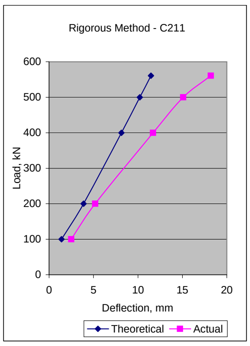

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

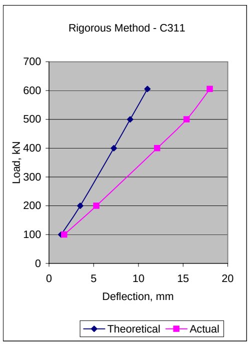

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

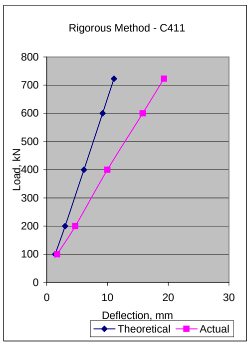

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

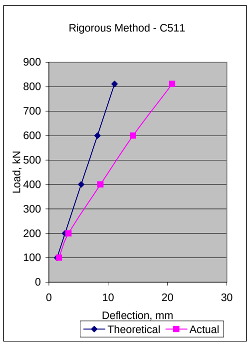

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

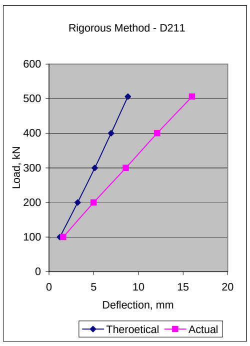

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

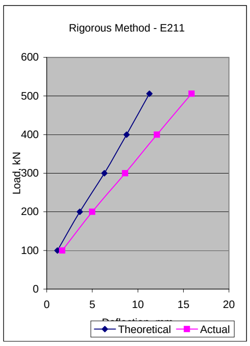

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

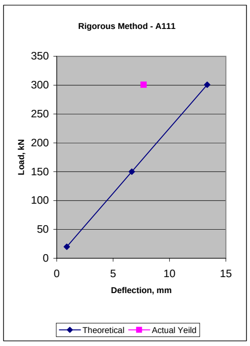

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

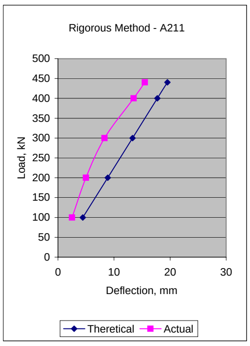

Rigorous Method - E211
Load, kN
Theoretical
Actual

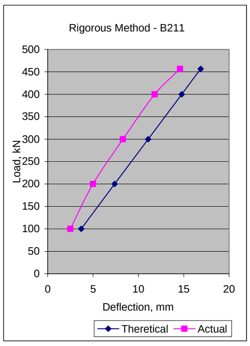
METHOD 19

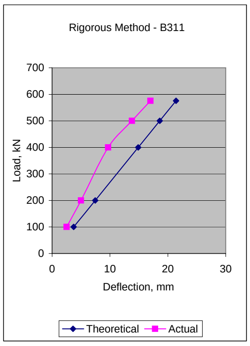
E_c =	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$0.045bd^3$
f'_{cf} =	NA

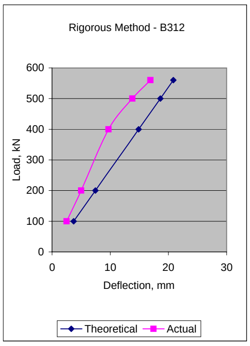
SOURCE	BEAM	f'_c, MPa	Reinforcement Ratio: Area of Steel, A_{st}, mm²	Reinforcement Ratio: Area of Steel, A_{sc}, mm²	L_{eff}, m	At Yield: Load, kN	At Yield: Deflection, mm	E_c	I_{eff}, 10^6mm	Deem-to-Comply: Ratio	≤	Deem-to-Comply: Allowed Ratio	Rigorous Method: Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30744	533.7	9.4	≤	9.48	13.3
		42.8	1250	300	3.4	20.00		30744	533.7	9.4	≤	16.77	0.9
		42.8	1250	300	3.4	150.00		30744	533.7	9.4	≤	8.57	6.7
		42.8	1250	300	3.4	300.78		30744	533.7	9.4	≤	6.79	13.3
	A211	42.8	2200	300	3.4	440.48	15.50	30744	533.7	9.4	≤	5.89	19.5
		42.8	2200	300	3.4	100.00	2.50	30744	533.7	9.4	≤	9.66	4.4
		42.8	2200	300	3.4	200.00	5.00	30744	533.7	9.4	≤	7.67	8.9
		42.8	2200	300	3.4	300.00	8.30	30744	533.7	9.4	≤	6.70	13.3
		42.8	2200	300	3.4	400.00	13.50	30744	533.7	9.4	≤	6.09	17.7
		42.8	2200	300	3.4	440.48	15.50	30744	533.7	9.4	≤	5.89	19.5
	B211	74.6	2200	300	3.4	456.70	14.60	36931	533.7	9.4	≤	6.19	16.9
		74.6	2200	300	3.4	100.00	2.50	36931	533.7	9.4	≤	10.27	3.7
		74.6	2200	300	3.4	200.00	5.00	36931	533.7	9.4	≤	8.15	7.4
		74.6	2200	300	3.4	300.00	8.30	36931	533.7	9.4	≤	7.12	11.1
		74.6	2200	300	3.4	400.00	11.80	36931	533.7	9.4	≤	6.47	14.8
		74.6	2200	300	3.4	456.70	14.60	36931	533.7	9.4	≤	6.19	16.9
	B311	72.8	3460	310	3.4	575.68	17.00	36635	533.7	9.4	≤	5.68	21.4
		72.8	3460	310	3.4	100.00	2.50	36635	533.7	9.4	≤	10.18	3.7
		72.8	3460	310	3.4	200.00	5.00	36635	533.7	9.4	≤	8.08	7.4
		72.8	3460	310	3.4	400.00	9.70	36635	533.7	9.4	≤	6.41	14.9
		72.8	3460	310	3.4	500.00	13.80	36635	533.7	9.4	≤	5.95	18.6
		72.8	3460	310	3.4	575.68	17.00	36635	533.7	9.4	≤	5.68	21.4
	B312	72.8	3460	310	3.4	560.36	16.90	36635	533.7	9.4	≤	5.73	20.9
		72.8	3460	310	3.4	100.00	2.50	36635	533.7	9.4	≤	10.18	3.7
		72.8	3460	310	3.4	200.00	5.00	36635	533.7	9.4	≤	8.08	7.4
		72.8	3460	310	3.4	400.00	9.70	36635	533.7	9.4	≤	6.41	14.9
		72.8	3460	310	3.4	500.00	13.80	36635	533.7	9.4	≤	5.95	18.6
		72.8	3460	310	3.4	560.36	16.90	36635	533.7	9.4	≤	5.73	20.9
	B313	72.8	3460	310	3.4	580.26	16.10	36635	533.7	9.4	≤	5.66	21.6
		72.8	3460	310	3.4	100.00	2.50	36635	533.7	9.4	≤	10.18	3.7
		72.8	3460	310	3.4	200.00	5.00	36635	533.7	9.4	≤	8.08	7.4
		72.8	3460	310	3.4	400.00	9.70	36635	533.7	9.4	≤	6.41	14.9
		72.8	3460	310	3.4	500.00	13.80	36635	533.7	9.4	≤	5.95	18.6
		72.8	3460	310	3.4	580.26	16.10	36635	533.7	9.4	≤	5.66	21.6
	B321	77	3460	620	3.4	551.16	15.50	37319	533.7	9.4	≤	5.87	20.1
		77	3460	620	3.4	100.00	1.70	37319	533.7	9.4	≤	10.37	3.7
		77	3460	620	3.4	200.00	4.20	37319	533.7	9.4	≤	8.23	7.3
		77	3460	620	3.4	400.00	10.00	37319	533.7	9.4	≤	6.53	14.6
		77	3460	620	3.4	500.00	13.30	37319	533.7	9.4	≤	6.06	18.3
		77	3460	620	3.4	551.16	15.50	37319	533.7	9.4	≤	5.87	20.1
	B331	72.8	3460	940	3.4	590.38	16.50	36635	533.7	9.4	≤	5.78	22.0
		72.8	3460	940	3.4	100.00	1.70	36635	533.7	9.4	≤	10.45	3.7
		72.8	3460	940	3.4	200.00	4.20	36635	533.7	9.4	≤	8.29	7.4
		72.8	3460	940	3.4	400.00	10.00	36635	533.7	9.4	≤	6.58	14.9
		72.8	3460	940	3.4	500.00	13.30	36635	533.7	9.4	≤	6.11	18.6
		72.8	3460	940	3.4	590.38	16.50	36635	533.7	9.4	≤	5.78	22.0
	B411	77	4730	320	3.4	621.34	15.50	37319	533.7	9.4	≤	5.55	22.7
		77	4730	320	3.4	100.00		37319	533.7	9.4	≤	10.21	3.7
		77	4730	320	3.4	200.00		37319	533.7	9.4	≤	8.10	7.3
		77	4730	320	3.4	300.00		37319	533.7	9.4	≤	7.08	11.0
		77	4730	320	3.4	400.00		37319	533.7	9.4	≤	6.43	14.6
		77	4730	320	3.4	500.00		37319	533.7	9.4	≤	5.97	18.3
		77	4730	320	3.4	621.34		37319	533.7	9.4	≤	5.55	22.7
	C211	85.6	2710	300	3.4	560.94	18.20	38646	533.7	9.4	≤	5.85	19.8
		85.6	2710	300	3.4	100.00	2.50	38646	533.7	9.4	≤	10.39	3.5
		85.6	2710	300	3.4	200.00	5.20	38646	533.7	9.4	≤	8.25	7.1
		85.6	2710	300	3.4	400.00	11.70	38646	533.7	9.4	≤	6.55	14.1
		85.6	2710	300	3.4	500.00	15.10	38646	533.7	9.4	≤	6.08	17.6
		85.6	2710	300	3.4	560.94	18.20	38646	533.7	9.4	≤	5.85	19.8
	C311	88.1	3220	310	3.4	605.46	18.00	39015	533.7	9.4	≤	5.71	21.2
		88.1	3220	310	3.4	100.00	1.70	39015	533.7	9.4	≤	10.40	3.5
		88.1	3220	310	3.4	200.00	5.30	39015	533.7	9.4	≤	8.26	7.0
		88.1	3220	310	3.4	400.00	12.10	39015	533.7	9.4	≤	6.55	14.0
		88.1	3220	310	3.4	500.00	15.40	39015	533.7	9.4	≤	6.08	17.5
		88.1	3220	310	3.4	605.46	18.00	39015	533.7	9.4	≤	5.71	21.2
	C411	85.6	4260	320	3.4	722.56	19.30	38646	533.7	9.4	≤	5.35	25.5
		85.6	4260	320	3.4	100.00	1.70	38646	533.7	9.4	≤	10.34	3.5
		85.6	4260	320	3.4	200.00	4.70	38646	533.7	9.4	≤	8.21	7.1
		85.6	4260	320	3.4	400.00	10.00	38646	533.7	9.4	≤	6.51	14.1
		85.6	4260	320	3.4	600.00	15.80	38646	533.7	9.4	≤	5.69	21.2
		85.6	4260	320	3.4	722.56	19.30	38646	533.7	9.4	≤	5.35	25.5
	C511	88.1	5310	330	3.4	811.82	20.80	39015	533.7	9.4	≤	5.15	28.4
		88.1	5310	330	3.4	100.00	1.70	39015	533.7	9.4	≤	10.35	3.5
		88.1	5310	330	3.4	200.00	3.30	39015	533.7	9.4	≤	8.22	7.0
		88.1	5310	330	3.4	400.00	8.70	39015	533.7	9.4	≤	6.52	14.0
		88.1	5310	330	3.4	600.00	14.20	39015	533.7	9.4	≤	5.70	21.0
		88.1	5310	330	3.4	811.82	20.80	39015	533.7	9.4	≤	5.15	28.4
	D211	114.5	3220	300	3.4	506.00	16.00	42540	533.7	9.4	≤	6.23	16.2
		114.5	3220	300	3.4	100.00	1.60	42540	533.7	9.4	≤	10.70	3.2
		114.5	3220	300	3.4	200.00	5.00	42540	533.7	9.4	≤	8.49	6.4
		114.5	3220	300	3.4	300.00	8.60	42540	533.7	9.4	≤	7.42	9.6
		114.5	3220	300	3.4	400.00	12.10	42540	533.7	9.4	≤	6.74	12.8
		114.5	3220	300	3.4	506.00	16.00	42540	533.7	9.4	≤	6.23	16.2
	E211	126.2	2200	300	3.4	506.00	15.90	43928	533.7	9.4	≤	6.34	15.7
		126.2	2200	300	3.4	100.00	1.70	43928	533.7	9.4	≤	10.88	3.1
		126.2	2200	300	3.4	200.00	5.00	43928	533.7	9.4	≤	8.64	6.2
		126.2	2200	300	3.4	300.00	8.60	43928	533.7	9.4	≤	7.55	9.3
		126.2	2200	300	3.4	400.00	12.10	43928	533.7	9.4	≤	6.86	12.4
		126.2	2200	300	3.4	506.00	15.90	43928	533.7	9.4	≤	6.34	15.7

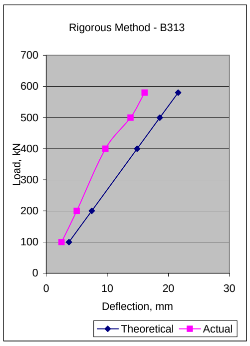

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

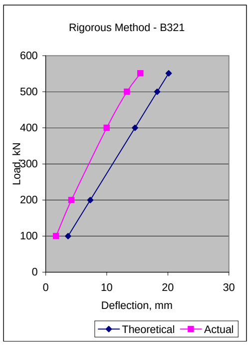

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

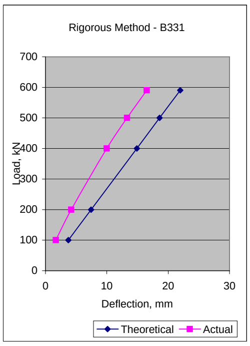

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

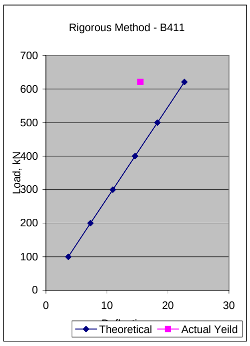

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

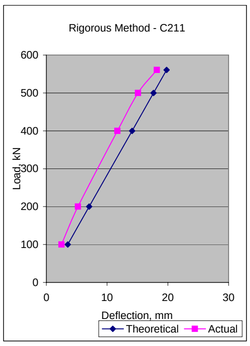

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

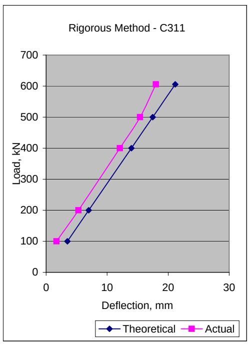

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

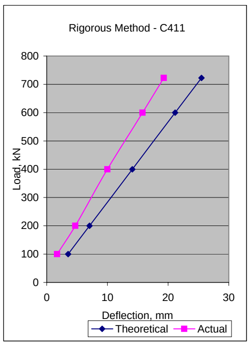

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

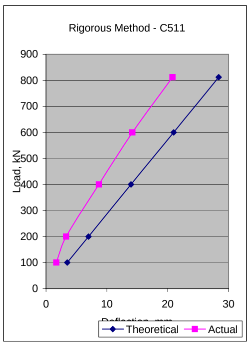

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

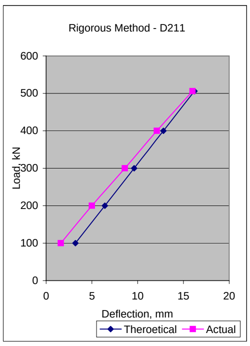

Rigorous Method - B411
Load, kN
Theoretical
Actual Yeild

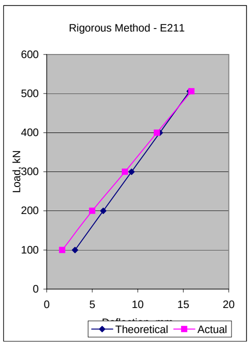

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Theoretical
Actual

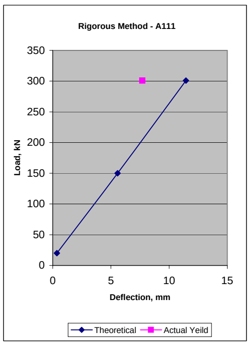

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

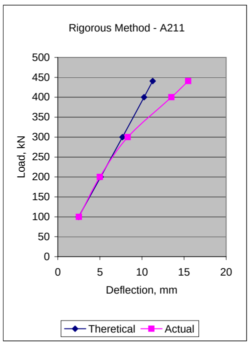

Rigorous Method - E211
Load, kN
Theoretical
Actual

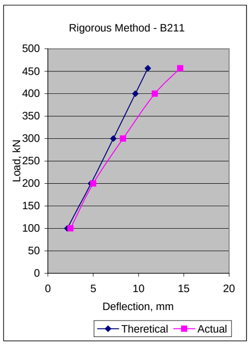
METHOD 20

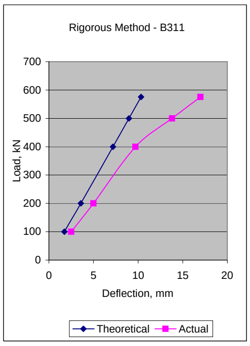
E_c =	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
I_{eff} =	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
f'_{cf} =	$0.6\sqrt{f'_c}$

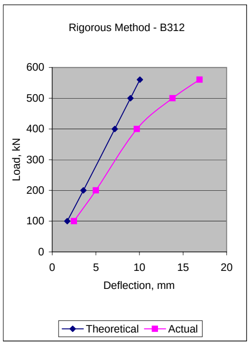
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30744	3.93	621.7	9.4	≤	9.97	11.4
		42.8	1250	300	3.4	20.00		30744	3.93	1333.3	9.4	≤	22.76	0.4
		42.8	1250	300	3.4	150.00		30744	3.93	637.0	9.4	≤	9.09	5.6
		42.8	1250	300	3.4	300.78		30744	3.93	621.7	9.4	≤	7.15	11.4
	A211	42.8	2200	300	3.4	440.48	15.50	30744	3.93	923.9	9.4	≤	7.08	11.3
		42.8	2200	300	3.4	100.00	2.50	30744	3.93	957.5	9.4	≤	11.74	2.5
		42.8	2200	300	3.4	200.00	5.00	30744	3.93	927.8	9.4	≤	9.22	5.1
		42.8	2200	300	3.4	300.00	8.30	30744	3.93	924.8	9.4	≤	8.05	7.7
		42.8	2200	300	3.4	400.00	13.50	30744	3.93	924.1	9.4	≤	7.31	10.2
		42.8	2200	300	3.4	440.48	15.50	30744	3.93	923.9	9.4	≤	7.08	11.3
	B211	74.6	2200	300	3.4	456.70	14.60	36931	5.18	816.1	9.4	≤	7.13	11.0
		74.6	2200	300	3.4	100.00	2.50	36931	5.18	914.0	9.4	≤	12.29	2.2
		74.6	2200	300	3.4	200.00	5.00	36931	5.18	827.4	9.4	≤	9.44	4.8
		74.6	2200	300	3.4	300.00	8.30	36931	5.18	818.7	9.4	≤	8.21	7.2
		74.6	2200	300	3.4	400.00	11.80	36931	5.18	816.6	9.4	≤	7.46	9.6
		74.6	2200	300	3.4	456.70	14.60	36931	5.18	816.1	9.4	≤	7.13	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	36635	5.12	1106.0	9.4	≤	7.24	10.3
		72.8	3460	310	3.4	100.00	2.50	36635	5.12	1147.6	9.4	≤	13.14	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.12	1111.0	9.4	≤	10.31	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.12	1106.4	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.12	1106.1	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	575.68	17.00	36635	5.12	1106.0	9.4	≤	7.24	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	36635	5.12	1106.0	9.4	≤	7.31	10.1
		72.8	3460	310	3.4	100.00	2.50	36635	5.12	1147.6	9.4	≤	13.14	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.12	1111.0	9.4	≤	10.31	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.12	1106.4	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.12	1106.1	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	560.36	16.90	36635	5.12	1106.0	9.4	≤	7.31	10.1
	B313	72.8	3460	310	3.4	580.26	16.10	36635	5.12	1106.0	9.4	≤	7.22	10.4
		72.8	3460	310	3.4	100.00	2.50	36635	5.12	1147.6	9.4	≤	13.14	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.12	1111.0	9.4	≤	10.31	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.12	1106.4	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.12	1106.1	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	580.26	16.10	36635	5.12	1106.0	9.4	≤	7.22	10.4
	B321	77	3460	620	3.4	551.16	15.50	37319	5.26	1093.2	9.4	≤	7.46	9.8
		77	3460	620	3.4	100.00	1.70	37319	5.26	1141.1	9.4	≤	13.36	1.7
		77	3460	620	3.4	200.00	4.20	37319	5.26	1098.9	9.4	≤	10.47	3.5
		77	3460	620	3.4	400.00	10.00	37319	5.26	1093.7	9.4	≤	8.30	7.1
		77	3460	620	3.4	500.00	13.30	37319	5.26	1093.3	9.4	≤	7.70	8.9
		77	3460	620	3.4	551.16	15.50	37319	5.26	1093.2	9.4	≤	7.46	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	36635	5.12	1105.9	9.4	≤	7.37	10.6
		72.8	3460	940	3.4	100.00	1.70	36635	5.12	1147.6	9.4	≤	13.48	1.7
		72.8	3460	940	3.4	200.00	4.20	36635	5.12	1111.0	9.4	≤	10.59	3.6
		72.8	3460	940	3.4	400.00	10.00	36635	5.12	1106.4	9.4	≤	8.39	7.2
		72.8	3460	940	3.4	500.00	13.30	36635	5.12	1106.1	9.4	≤	7.79	9.0
		72.8	3460	940	3.4	590.38	16.50	36635	5.12	1105.9	9.4	≤	7.37	10.6
	B411	77	4730	320	3.4	621.34	15.50	37319	5.26	1321.2	9.4	≤	7.51	9.2
		77	4730	320	3.4	100.00		37319	5.26	1323.6	9.4	≤	13.82	1.5
		77	4730	320	3.4	200.00		37319	5.26	1321.5	9.4	≤	10.96	3.0
		77	4730	320	3.4	300.00		37319	5.26	1321.3	9.4	≤	9.58	4.4
		77	4730	320	3.4	400.00		37319	5.26	1321.2	9.4	≤	8.70	5.9
		77	4730	320	3.4	500.00		37319	5.26	1321.2	9.4	≤	8.08	7.4
		77	4730	320	3.4	621.34		37319	5.26	1321.2	9.4	≤	7.51	9.2
	C211	85.6	2710	300	3.4	560.94	18.20	38646	5.55	911.6	9.4	≤	6.99	11.6
		85.6	2710	300	3.4	100.00	2.50	38646	5.55	1010.2	9.4	≤	12.85	1.9
		85.6	2710	300	3.4	200.00	5.20	38646	5.55	923.5	9.4	≤	9.90	4.1
		85.6	2710	300	3.4	400.00	11.70	38646	5.55	912.6	9.4	≤	7.83	8.3
		85.6	2710	300	3.4	500.00	15.10	38646	5.55	911.9	9.4	≤	7.26	10.3
		85.6	2710	300	3.4	560.94	18.20	38646	5.55	911.6	9.4	≤	6.99	11.6
	C311	88.1	3220	310	3.4	605.46	18.00	39015	5.63	1014.9	9.4	≤	7.07	11.1
		88.1	3220	310	3.4	100.00	1.70	39015	5.63	1092.7	9.4	≤	13.21	1.7
		88.1	3220	310	3.4	200.00	5.30	39015	5.63	1024.3	9.4	≤	10.26	3.6
		88.1	3220	310	3.4	400.00	12.10	39015	5.63	1015.8	9.4	≤	8.12	7.3
		88.1	3220	310	3.4	500.00	15.40	39015	5.63	1015.2	9.4	≤	7.54	9.2
		88.1	3220	310	3.4	605.46	18.00	39015	5.63	1014.9	9.4	≤	7.07	11.1
	C411	85.6	4260	320	3.4	722.56	19.30	38646	5.55	1216.3	9.4	≤	7.04	11.2
		85.6	4260	320	3.4	100.00	1.70	38646	5.55	1243.7	9.4	≤	13.71	1.5
		85.6	4260	320	3.4	200.00	4.70	38646	5.55	1219.7	9.4	≤	10.81	3.1
		85.6	4260	320	3.4	400.00	10.00	38646	5.55	1216.7	9.4	≤	8.57	6.2
		85.6	4260	320	3.4	600.00	15.80	38646	5.55	1216.4	9.4	≤	7.49	9.3
		85.6	4260	320	3.4	722.56	19.30	38646	5.55	1216.3	9.4	≤	7.04	11.2
	C511	88.1	5310	330	3.4	811.82	20.80	39015	5.63	1333.3	9.4	≤	6.99	11.4
		88.1	5310	330	3.4	100.00	1.70	39015	5.63	1333.3	9.4	≤	14.05	1.4
		88.1	5310	330	3.4	200.00	3.30	39015	5.63	1333.3	9.4	≤	11.15	2.8
		88.1	5310	330	3.4	400.00	8.70	39015	5.63	1333.3	9.4	≤	8.85	5.6
		88.1	5310	330	3.4	600.00	14.20	39015	5.63	1333.3	9.4	≤	7.73	8.4
		88.1	5310	330	3.4	811.82	20.80	39015	5.63	1333.3	9.4	≤	6.99	11.4
	D211	114.5	3220	300	3.4	506.00	16.00	42540	6.42	959.7	9.4	≤	7.58	9.0
		114.5	3220	300	3.4	100.00	1.60	42540	6.42	1094.7	9.4	≤	13.60	1.6
		114.5	3220	300	3.4	200.00	5.00	42540	6.42	975.7	9.4	≤	10.39	3.5
		114.5	3220	300	3.4	300.00	8.60	42540	6.42	963.7	9.4	≤	9.04	5.3
		114.5	3220	300	3.4	400.00	12.10	42540	6.42	960.8	9.4	≤	8.20	7.1
		114.5	3220	300	3.4	506.00	16.00	42540	6.42	959.7	9.4	≤	7.58	9.0
	E211	126.2	2200	300	3.4	506.00	15.90	43928	6.74	723.2	9.4	≤	7.01	11.6
		126.2	2200	300	3.4	100.00	1.70	43928	6.74	978.3	9.4	≤	13.32	1.7
		126.2	2200	300	3.4	200.00	5.00	43928	6.74	753.3	9.4	≤	9.69	4.4
		126.2	2200	300	3.4	300.00	8.60	43928	6.74	730.7	9.4	≤	8.38	6.8
		126.2	2200	300	3.4	400.00	12.10	43928	6.74	725.2	9.4	≤	7.59	9.1
		126.2	2200	300	3.4	506.00	15.90	43928	6.74	723.2	9.4	≤	7.01	11.6

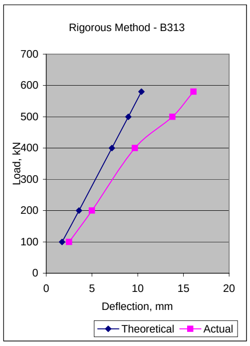

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

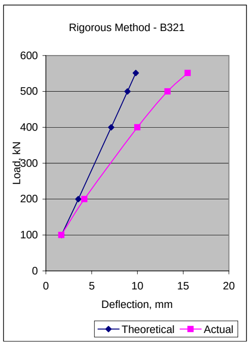

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

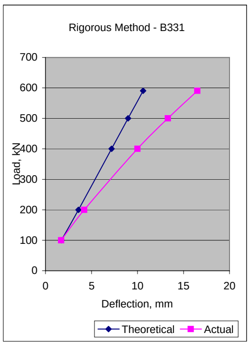

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

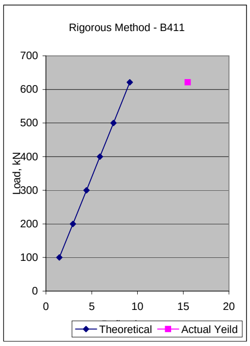

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

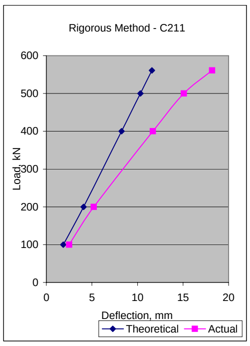

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

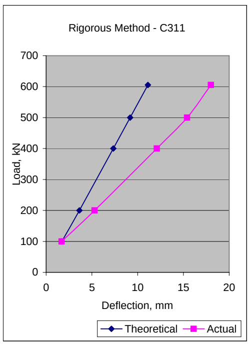

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

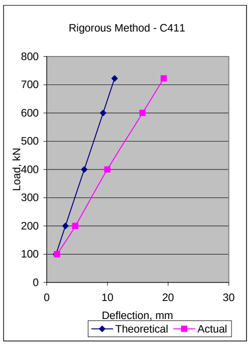

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

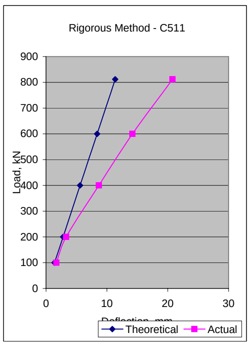

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

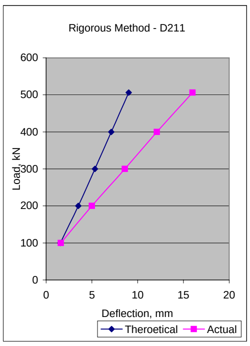

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

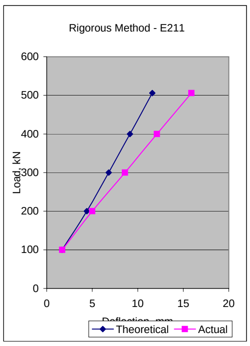

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Theoretical
Actual

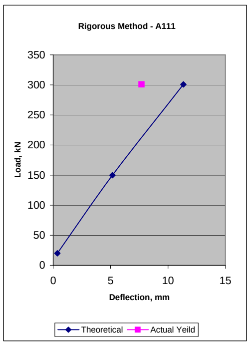

Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual

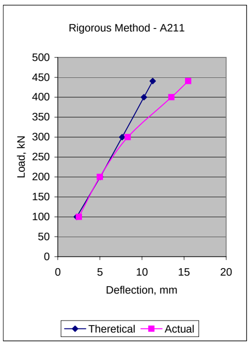

Rigorous Method - E211
Load, kN
Theoretical
Actual

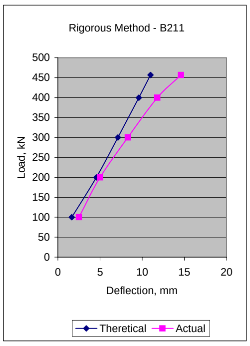
METHOD 21

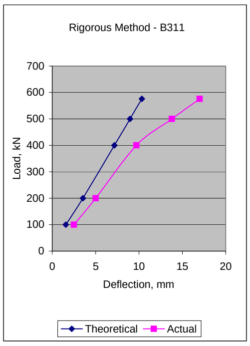
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_a)^3$
$f'_{cf} =$	$0.94\sqrt{f'_c}$

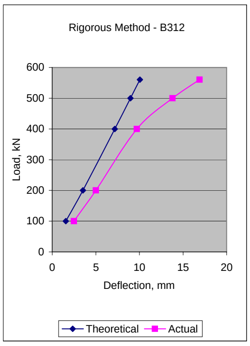
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	**BEAM**	**f'_c, MPa**	**Area of Steel, A_{st}, mm^2**	**Area of Steel, A_{sc}, mm^2**	**L_{eff}, m**	**Load, kN**	**Deflection, mm**	**E_c**	**f'_{cf}, MPa**	**I_{eff}, 10^6mm**	**Ratio**	**≤**	**Allowed Ratio**	**Deflection, mm**
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30744	6.15	627.9	9.4	≤	10.00	11.3
		42.8	1250	300	3.4	20.00		30744	6.15	1333.3	9.4	≤	22.76	0.4
		42.8	1250	300	3.4	150.00		30744	6.15	687.0	9.4	≤	9.32	5.2
		42.8	1250	300	3.4	300.78		30744	6.15	627.9	9.4	≤	7.17	11.3
	A211	42.8	2200	300	3.4	440.48	15.50	30744	6.15	925.1	9.4	≤	7.08	11.3
		42.8	2200	300	3.4	100.00	2.50	30744	6.15	1054.3	9.4	≤	12.12	2.2
		42.8	2200	300	3.4	200.00	5.00	30744	6.15	939.9	9.4	≤	9.26	5.0
		42.8	2200	300	3.4	300.00	8.30	30744	6.15	928.4	9.4	≤	8.06	7.6
		42.8	2200	300	3.4	400.00	13.50	30744	6.15	925.6	9.4	≤	7.31	10.2
		42.8	2200	300	3.4	440.48	15.50	30744	6.15	925.1	9.4	≤	7.08	11.3
	B211	74.6	2200	300	3.4	456.70	14.60	36931	8.12	819.0	9.4	≤	7.14	11.0
		74.6	2200	300	3.4	100.00	2.50	36931	8.12	1195.5	9.4	≤	13.44	1.6
		74.6	2200	300	3.4	200.00	5.00	36931	8.12	862.6	9.4	≤	9.57	4.6
		74.6	2200	300	3.4	300.00	8.30	36931	8.12	829.1	9.4	≤	8.25	7.1
		74.6	2200	300	3.4	400.00	11.80	36931	8.12	821.0	9.4	≤	7.47	9.6
		74.6	2200	300	3.4	456.70	14.60	36931	8.12	819.0	9.4	≤	7.14	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	36635	8.02	1106.6	9.4	≤	7.24	10.3
		72.8	3460	310	3.4	100.00	2.50	36635	8.02	1266.8	9.4	≤	13.58	1.6
		72.8	3460	310	3.4	200.00	5.00	36635	8.02	1125.9	9.4	≤	10.36	3.5
		72.8	3460	310	3.4	400.00	9.70	36635	8.02	1108.3	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	8.02	1107.0	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	575.68	17.00	36635	8.02	1106.6	9.4	≤	7.24	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	36635	8.02	1106.7	9.4	≤	7.31	10.1
		72.8	3460	310	3.4	100.00	2.50	36635	8.02	1266.8	9.4	≤	13.58	1.6
		72.8	3460	310	3.4	200.00	5.00	36635	8.02	1125.9	9.4	≤	10.36	3.5
		72.8	3460	310	3.4	400.00	9.70	36635	8.02	1108.3	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	8.02	1107.0	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	560.36	16.90	36635	8.02	1106.7	9.4	≤	7.31	10.1
	B313	72.8	3460	310	3.4	580.26	16.10	36635	8.02	1106.6	9.4	≤	7.22	10.4
		72.8	3460	310	3.4	100.00	2.50	36635	8.02	1266.8	9.4	≤	13.58	1.6
		72.8	3460	310	3.4	200.00	5.00	36635	8.02	1125.9	9.4	≤	10.36	3.5
		72.8	3460	310	3.4	400.00	9.70	36635	8.02	1108.3	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	8.02	1107.0	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	580.26	16.10	36635	8.02	1106.6	9.4	≤	7.22	10.4
	B321	77	3460	620	3.4	551.16	15.50	37319	8.25	1094.0	9.4	≤	7.46	9.8
		77	3460	620	3.4	100.00	1.70	37319	8.25	1278.0	9.4	≤	13.87	1.5
		77	3460	620	3.4	200.00	4.20	37319	8.25	1116.1	9.4	≤	10.53	3.5
		77	3460	620	3.4	400.00	10.00	37319	8.25	1095.8	9.4	≤	8.30	7.1
		77	3460	620	3.4	500.00	13.30	37319	8.25	1094.4	9.4	≤	7.71	8.9
		77	3460	620	3.4	551.16	15.50	37319	8.25	1094.0	9.4	≤	7.46	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	36635	8.02	1106.5	9.4	≤	7.37	10.6
		72.8	3460	940	3.4	100.00	1.70	36635	8.02	1266.8	9.4	≤	13.94	1.6
		72.8	3460	940	3.4	200.00	4.20	36635	8.02	1125.9	9.4	≤	10.63	3.5
		72.8	3460	940	3.4	400.00	10.00	36635	8.02	1108.3	9.4	≤	8.40	7.2
		72.8	3460	940	3.4	500.00	13.30	36635	8.02	1107.0	9.4	≤	7.79	9.0
		72.8	3460	940	3.4	590.38	16.50	36635	8.02	1106.5	9.4	≤	7.37	10.6
	B411	77	4730	320	3.4	621.34	15.50	37319	8.25	1321.2	9.4	≤	7.51	9.2
		77	4730	320	3.4	100.00		37319	8.25	1330.5	9.4	≤	13.84	1.5
		77	4730	320	3.4	200.00		37319	8.25	1322.4	9.4	≤	10.97	2.9
		77	4730	320	3.4	300.00		37319	8.25	1321.5	9.4	≤	9.58	4.4
		77	4730	320	3.4	400.00		37319	8.25	1321.3	9.4	≤	8.70	5.9
		77	4730	320	3.4	500.00		37319	8.25	1321.3	9.4	≤	8.08	7.4
		77	4730	320	3.4	621.34		37319	8.25	1321.2	9.4	≤	7.51	9.2
	C211	85.6	2710	300	3.4	560.94	18.20	38646	8.70	913.2	9.4	≤	6.99	11.6
		85.6	2710	300	3.4	100.00	2.50	38646	8.70	1292.1	9.4	≤	13.95	1.5
		85.6	2710	300	3.4	200.00	5.20	38646	8.70	958.7	9.4	≤	10.03	3.9
		85.6	2710	300	3.4	400.00	11.70	38646	8.70	917.0	9.4	≤	7.84	8.2
		85.6	2710	300	3.4	500.00	15.10	38646	8.70	914.1	9.4	≤	7.27	10.3
		85.6	2710	300	3.4	560.94	18.20	38646	8.70	913.2	9.4	≤	6.99	11.6
	C311	88.1	3220	310	3.4	605.46	18.00	39015	8.82	1015.9	9.4	≤	7.07	11.1
		88.1	3220	310	3.4	100.00	1.70	39015	8.82	1314.9	9.4	≤	14.05	1.4
		88.1	3220	310	3.4	200.00	5.30	39015	8.82	1052.1	9.4	≤	10.35	3.5
		88.1	3220	310	3.4	400.00	12.10	39015	8.82	1019.2	9.4	≤	8.13	7.3
		88.1	3220	310	3.4	500.00	15.40	39015	8.82	1016.9	9.4	≤	7.54	9.2
		88.1	3220	310	3.4	605.46	18.00	39015	8.82	1015.9	9.4	≤	7.07	11.1
	C411	85.6	4260	320	3.4	722.56	19.30	38646	8.70	1216.5	9.4	≤	7.04	11.2
		85.6	4260	320	3.4	100.00	1.70	38646	8.70	1321.9	9.4	≤	13.99	1.4
		85.6	4260	320	3.4	200.00	4.70	38646	8.70	1229.4	9.4	≤	10.84	3.1
		85.6	4260	320	3.4	400.00	10.00	38646	8.70	1217.9	9.4	≤	8.58	6.2
		85.6	4260	320	3.4	600.00	15.80	38646	8.70	1216.7	9.4	≤	7.49	9.3
		85.6	4260	320	3.4	722.56	19.30	38646	8.70	1216.5	9.4	≤	7.04	11.2
	C511	88.1	5310	330	3.4	811.82	20.80	39015	8.82	1333.3	9.4	≤	6.99	11.4
		88.1	5310	330	3.4	100.00	1.70	39015	8.82	1333.3	9.4	≤	14.05	1.4
		88.1	5310	330	3.4	200.00	3.30	39015	8.82	1333.3	9.4	≤	11.15	2.8
		88.1	5310	330	3.4	400.00	8.70	39015	8.82	1333.3	9.4	≤	8.85	5.6
		88.1	5310	330	3.4	600.00	14.20	39015	8.82	1333.3	9.4	≤	7.73	8.4
		88.1	5310	330	3.4	811.82	20.80	39015	8.82	1333.3	9.4	≤	6.99	11.4
	D211	114.5	3220	300	3.4	506.00	16.00	42540	10.06	962.7	9.4	≤	7.59	9.0
		114.5	3220	300	3.4	100.00	1.60	42540	10.06	1333.3	9.4	≤	14.52	1.3
		114.5	3220	300	3.4	200.00	5.00	42540	10.06	1024.1	9.4	≤	10.56	3.3
		114.5	3220	300	3.4	300.00	8.60	42540	10.06	978.1	9.4	≤	9.08	5.2
		114.5	3220	300	3.4	400.00	12.10	42540	10.06	966.9	9.4	≤	8.22	7.1
		114.5	3220	300	3.4	506.00	16.00	42540	10.06	962.7	9.4	≤	7.59	9.0
	E211	126.2	2200	300	3.4	506.00	15.90	43928	10.56	728.8	9.4	≤	7.03	11.5
		126.2	2200	300	3.4	100.00	1.70	43928	10.56	1333.3	9.4	≤	14.77	1.2
		126.2	2200	300	3.4	200.00	5.00	43928	10.56	844.8	9.4	≤	10.07	3.9
		126.2	2200	300	3.4	300.00	8.60	43928	10.56	757.8	9.4	≤	8.48	6.6
		126.2	2200	300	3.4	400.00	12.10	43928	10.56	736.6	9.4	≤	7.63	9.0
		126.2	2200	300	3.4	506.00	15.90	43928	10.56	728.8	9.4	≤	7.03	11.5

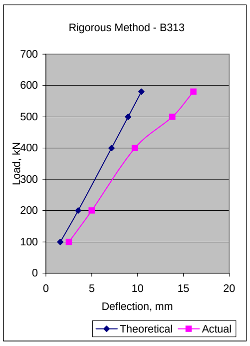

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

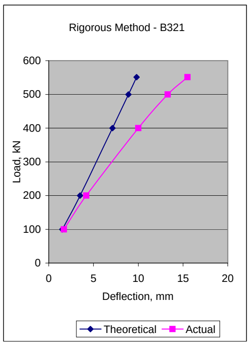

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

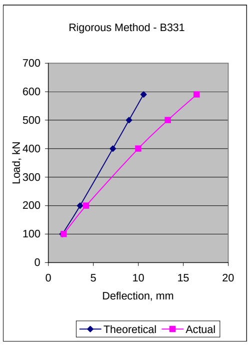

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

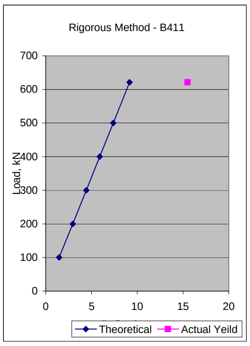

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

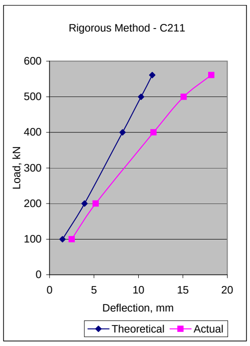

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

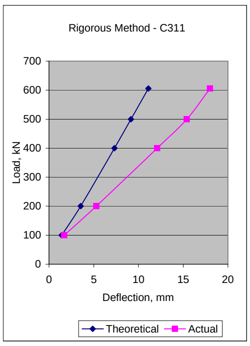

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

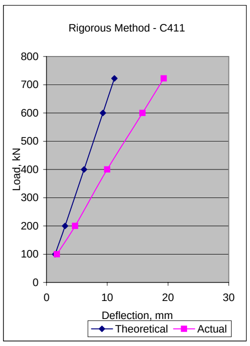

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

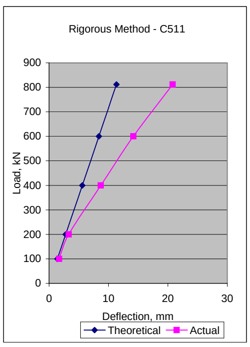

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

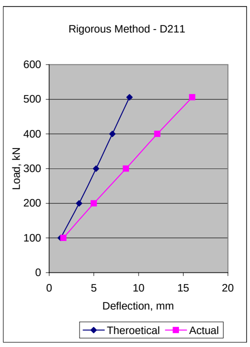

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

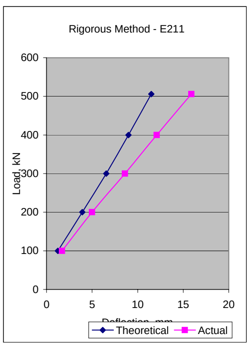

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual

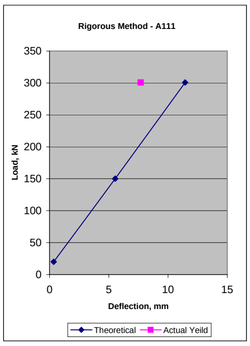

Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual

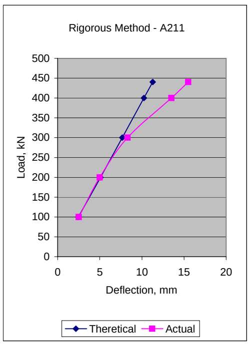

Rigorous Method - E211
Load, kN
Theoretical
Actual

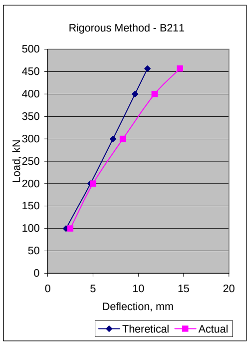
METHOD 22

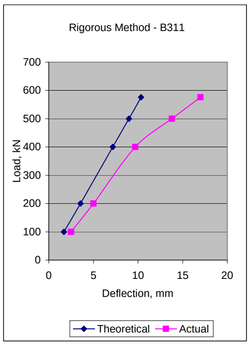
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3$
$f'_{cf} =$	$0.342(f'_c)^{2/3}$

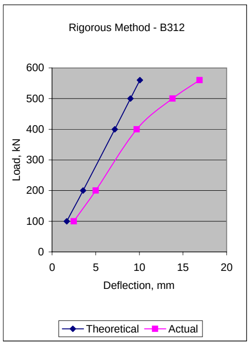
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{ef}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, $10^6 mm$	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30744	4.18	622.1	9.4	≤	9.97	11.4
		42.8	1250	300	3.4	20.00		30744	4.18	1333.3	9.4	≤	22.76	0.4
		42.8	1250	300	3.4	150.00		30744	4.18	640.7	9.4	≤	9.11	5.5
		42.8	1250	300	3.4	300.78		30744	4.18	622.1	9.4	≤	7.15	11.4
	A211	42.8	2200	300	3.4	440.48	15.50	30744	4.18	924.0	9.4	≤	7.08	11.3
		42.8	2200	300	3.4	100.00	2.50	30744	4.18	964.7	9.4	≤	11.77	2.5
		42.8	2200	300	3.4	200.00	5.00	30744	4.18	928.7	9.4	≤	9.22	5.1
		42.8	2200	300	3.4	300.00	8.30	30744	4.18	925.1	9.4	≤	8.05	7.7
		42.8	2200	300	3.4	400.00	13.50	30744	4.18	924.2	9.4	≤	7.31	10.2
		42.8	2200	300	3.4	440.48	15.50	30744	4.18	924.0	9.4	≤	7.08	11.3
	B211	74.6	2200	300	3.4	456.70	14.60	36931	6.06	816.7	9.4	≤	7.13	11.0
		74.6	2200	300	3.4	100.00	2.50	36931	6.06	973.3	9.4	≤	12.55	2.0
		74.6	2200	300	3.4	200.00	5.00	36931	6.06	834.8	9.4	≤	9.46	4.7
		74.6	2200	300	3.4	300.00	8.30	36931	6.06	820.9	9.4	≤	8.22	7.2
		74.6	2200	300	3.4	400.00	11.80	36931	6.06	817.5	9.4	≤	7.46	9.6
		74.6	2200	300	3.4	456.70	14.60	36931	6.06	816.7	9.4	≤	7.13	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	36635	5.96	1106.1	9.4	≤	7.24	10.3
		72.8	3460	310	3.4	100.00	2.50	36635	5.96	1171.9	9.4	≤	13.23	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.96	1114.0	9.4	≤	10.32	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.96	1106.8	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.96	1106.3	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	575.68	17.00	36635	5.96	1106.1	9.4	≤	7.24	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	36635	5.96	1106.1	9.4	≤	7.31	10.1
		72.8	3460	310	3.4	100.00	2.50	36635	5.96	1171.9	9.4	≤	13.23	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.96	1114.0	9.4	≤	10.32	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.96	1106.8	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.96	1106.3	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	560.36	16.90	36635	5.96	1106.1	9.4	≤	7.31	10.1
	B313	72.8	3460	310	3.4	580.26	16.10	36635	5.96	1106.1	9.4	≤	7.22	10.4
		72.8	3460	310	3.4	100.00	2.50	36635	5.96	1171.9	9.4	≤	13.23	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.96	1114.0	9.4	≤	10.32	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.96	1106.8	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.96	1106.3	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	580.26	16.10	36635	5.96	1106.1	9.4	≤	7.22	10.4
	B321	77	3460	620	3.4	551.16	15.50	37319	6.19	1093.4	9.4	≤	7.46	9.8
		77	3460	620	3.4	100.00	1.70	37319	6.19	1171.1	9.4	≤	13.48	1.7
		77	3460	620	3.4	200.00	4.20	37319	6.19	1102.7	9.4	≤	10.48	3.5
		77	3460	620	3.4	400.00	10.00	37319	6.19	1094.2	9.4	≤	8.30	7.1
		77	3460	620	3.4	500.00	13.30	37319	6.19	1093.6	9.4	≤	7.70	8.9
		77	3460	620	3.4	551.16	15.50	37319	6.19	1093.4	9.4	≤	7.46	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	36635	5.96	1106.1	9.4	≤	7.37	10.6
		72.8	3460	940	3.4	100.00	1.70	36635	5.96	1171.9	9.4	≤	13.58	1.7
		72.8	3460	940	3.4	200.00	4.20	36635	5.96	1114.0	9.4	≤	10.60	3.6
		72.8	3460	940	3.4	400.00	10.00	36635	5.96	1106.8	9.4	≤	8.39	7.2
		72.8	3460	940	3.4	500.00	13.30	36635	5.96	1106.3	9.4	≤	7.79	9.0
		72.8	3460	940	3.4	590.38	16.50	36635	5.96	1106.1	9.4	≤	7.37	10.6
	B411	77	4730	320	3.4	621.34	15.50	37319	6.19	1321.2	9.4	≤	7.51	9.2
		77	4730	320	3.4	100.00		37319	6.19	1325.1	9.4	≤	13.83	1.5
		77	4730	320	3.4	200.00		37319	6.19	1321.7	9.4	≤	10.96	2.9
		77	4730	320	3.4	300.00		37319	6.19	1321.3	9.4	≤	9.58	4.4
		77	4730	320	3.4	400.00		37319	6.19	1321.2	9.4	≤	8.70	5.9
		77	4730	320	3.4	500.00		37319	6.19	1321.2	9.4	≤	8.08	7.4
		77	4730	320	3.4	621.34		37319	6.19	1321.2	9.4	≤	7.51	9.2
	C211	85.6	2710	300	3.4	560.94	18.20	38646	6.64	912.0	9.4	≤	6.99	11.6
		85.6	2710	300	3.4	100.00	2.50	38646	6.64	1080.9	9.4	≤	13.15	1.7
		85.6	2710	300	3.4	200.00	5.20	38646	6.64	932.3	9.4	≤	9.93	4.0
		85.6	2710	300	3.4	400.00	11.70	38646	6.64	913.7	9.4	≤	7.83	8.2
		85.6	2710	300	3.4	500.00	15.10	38646	6.64	912.4	9.4	≤	7.27	10.3
		85.6	2710	300	3.4	560.94	18.20	38646	6.64	912.0	9.4	≤	6.99	11.6
	C311	88.1	3220	310	3.4	605.46	18.00	39015	6.77	1015.2	9.4	≤	7.07	11.1
		88.1	3220	310	3.4	100.00	1.70	39015	6.77	1150.3	9.4	≤	13.44	1.6
		88.1	3220	310	3.4	200.00	5.30	39015	6.77	1031.5	9.4	≤	10.29	3.6
		88.1	3220	310	3.4	400.00	12.10	39015	6.77	1016.7	9.4	≤	8.12	7.3
		88.1	3220	310	3.4	500.00	15.40	39015	6.77	1015.6	9.4	≤	7.54	9.2
		88.1	3220	310	3.4	605.46	18.00	39015	6.77	1015.2	9.4	≤	7.07	11.1
	C411	85.6	4260	320	3.4	722.56	19.30	38646	6.64	1216.4	9.4	≤	7.04	11.2
		85.6	4260	320	3.4	100.00	1.70	38646	6.64	1263.3	9.4	≤	13.78	1.5
		85.6	4260	320	3.4	200.00	4.70	38646	6.64	1222.1	9.4	≤	10.82	3.1
		85.6	4260	320	3.4	400.00	10.00	38646	6.64	1217.0	9.4	≤	8.57	6.2
		85.6	4260	320	3.4	600.00	15.80	38646	6.64	1216.5	9.4	≤	7.49	9.3
		85.6	4260	320	3.4	722.56	19.30	38646	6.64	1216.4	9.4	≤	7.04	11.2
	C511	88.1	5310	330	3.4	811.82	20.80	39015	6.77	1333.3	9.4	≤	6.99	11.4
		88.1	5310	330	3.4	100.00	1.70	39015	6.77	1333.3	9.4	≤	14.05	1.4
		88.1	5310	330	3.4	200.00	3.30	39015	6.77	1333.3	9.4	≤	11.15	2.8
		88.1	5310	330	3.4	400.00	8.70	39015	6.77	1333.3	9.4	≤	8.85	5.6
		88.1	5310	330	3.4	600.00	14.20	39015	6.77	1333.3	9.4	≤	7.73	8.4
		88.1	5310	330	3.4	811.82	20.80	39015	6.77	1333.3	9.4	≤	6.99	11.4
	D211	114.5	3220	300	3.4	506.00	16.00	42540	8.06	960.8	9.4	≤	7.58	9.0
		114.5	3220	300	3.4	100.00	1.60	42540	8.06	1228.2	9.4	≤	14.13	1.4
		114.5	3220	300	3.4	200.00	5.00	42540	8.06	992.4	9.4	≤	10.45	3.4
		114.5	3220	300	3.4	300.00	8.60	42540	8.06	968.7	9.4	≤	9.05	5.3
		114.5	3220	300	3.4	400.00	12.10	42540	8.06	962.9	9.4	≤	8.21	7.1
		114.5	3220	300	3.4	506.00	16.00	42540	8.06	960.8	9.4	≤	7.58	9.0
	E211	126.2	2200	300	3.4	506.00	15.90	43928	8.60	725.3	9.4	≤	7.02	11.6
		126.2	2200	300	3.4	100.00	1.70	43928	8.60	1256.1	9.4	≤	14.48	1.3
		126.2	2200	300	3.4	200.00	5.00	43928	8.60	788.0	9.4	≤	9.84	4.2
		126.2	2200	300	3.4	300.00	8.60	43928	8.60	741.0	9.4	≤	8.42	6.7
		126.2	2200	300	3.4	400.00	12.10	43928	8.60	729.5	9.4	≤	7.61	9.1
		126.2	2200	300	3.4	506.00	15.90	43928	8.60	725.3	9.4	≤	7.02	11.6

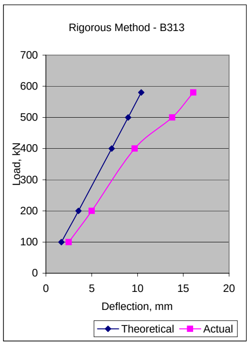

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

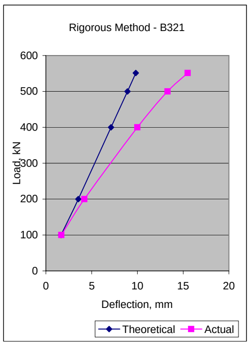

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

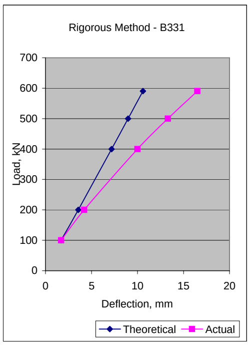

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

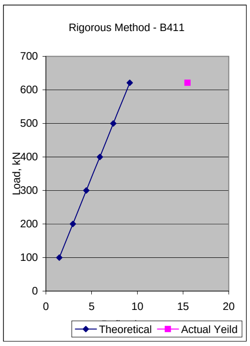

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

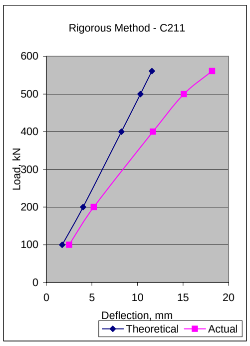

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

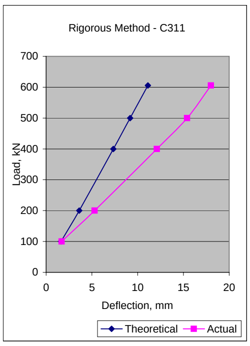

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

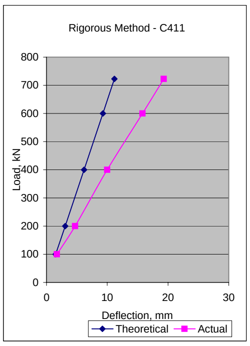

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

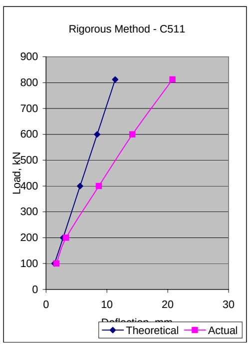

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

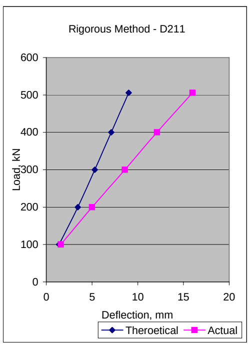

Rigorous Method - B411
Load, kN
Deflection, mm
Theoretical
Actual Yeild

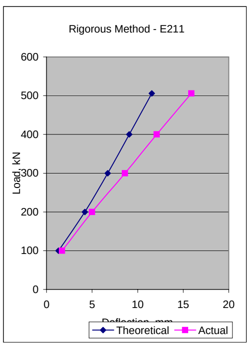

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Theoretical
Actual


Rigorous Method - D211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - E211
Load, kN
Theoretical
Actual

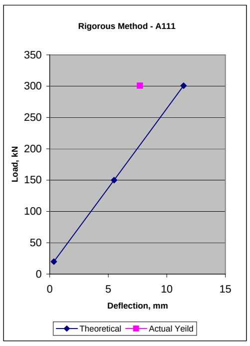
METHOD 23

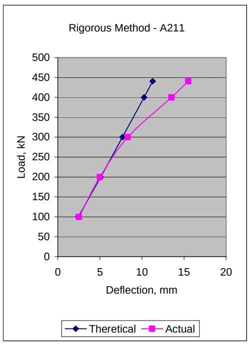
$E_c = 0.043\eta\rho^{1.5}\sqrt{f'_c}$

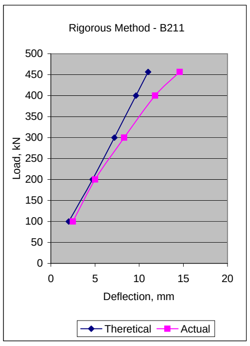
$I_{eff} = I_{cr} + (I - I_{cr})(M_{cr}/M_a)^3$

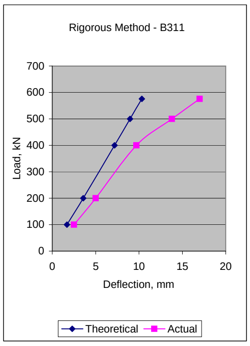
$f'_{cf} = 0.564(f'_c)^{0.55}$

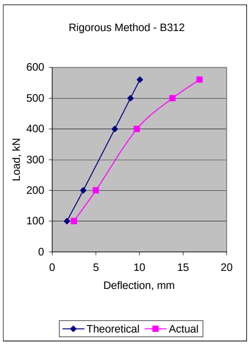
SOURCE	BEAM	f'_c, MPa	Reinforcement Ratio: Area of Steel, A_{st}, mm^2	Reinforcement Ratio: Area of Steel, A_{sc}, mm^2	L_{eff}, m	At Yield: Load, kN	At Yield: Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Deem-to-Comply: Ratio	≤	Deem-to-Comply: Allowed Ratio	Rigorous Method: Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30744	4.45	622.7	9.4	≤	9.98	11.4
		42.8	1250	300	3.4	20.00		30744	4.45	1333.3	9.4	≤	22.76	0.4
		42.8	1250	300	3.4	150.00		30744	4.45	645.1	9.4	≤	9.13	5.5
		42.8	1250	300	3.4	300.78		30744	4.45	622.7	9.4	≤	7.15	11.4
	A211	42.8	2200	300	3.4	440.48	15.50	30744	4.45	924.1	9.4	≤	7.08	11.3
		42.8	2200	300	3.4	100.00	2.50	30744	4.45	973.1	9.4	≤	11.80	2.4
		42.8	2200	300	3.4	200.00	5.00	30744	4.45	929.7	9.4	≤	9.23	5.1
		42.8	2200	300	3.4	300.00	8.30	30744	4.45	925.4	9.4	≤	8.05	7.7
		42.8	2200	300	3.4	400.00	13.50	30744	4.45	924.3	9.4	≤	7.31	10.2
		42.8	2200	300	3.4	440.48	15.50	30744	4.45	924.1	9.4	≤	7.08	11.3
	B211	74.6	2200	300	3.4	456.70	14.60	36931	6.04	816.7	9.4	≤	7.13	11.0
		74.6	2200	300	3.4	100.00	2.50	36931	6.04	972.0	9.4	≤	12.54	2.0
		74.6	2200	300	3.4	200.00	5.00	36931	6.04	834.7	9.4	≤	9.46	4.7
		74.6	2200	300	3.4	300.00	8.30	36931	6.04	820.8	9.4	≤	8.22	7.2
		74.6	2200	300	3.4	400.00	11.80	36931	6.04	817.5	9.4	≤	7.46	9.6
		74.6	2200	300	3.4	456.70	14.60	36931	6.04	816.7	9.4	≤	7.13	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	36635	5.96	1106.1	9.4	≤	7.24	10.3
		72.8	3460	310	3.4	100.00	2.50	36635	5.96	1171.9	9.4	≤	13.23	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.96	1114.0	9.4	≤	10.32	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.96	1106.8	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.96	1106.3	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	575.68	17.00	36635	5.96	1106.1	9.4	≤	7.24	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	36635	5.96	1106.1	9.4	≤	7.31	10.1
		72.8	3460	310	3.4	100.00	2.50	36635	5.96	1171.9	9.4	≤	13.23	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.96	1114.0	9.4	≤	10.32	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.96	1106.8	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.96	1106.3	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	560.36	16.90	36635	5.96	1106.1	9.4	≤	7.31	10.1
	B313	72.8	3460	310	3.4	580.26	16.10	36635	5.96	1106.1	9.4	≤	7.22	10.4
		72.8	3460	310	3.4	100.00	2.50	36635	5.96	1171.9	9.4	≤	13.23	1.7
		72.8	3460	310	3.4	200.00	5.00	36635	5.96	1114.0	9.4	≤	10.32	3.6
		72.8	3460	310	3.4	400.00	9.70	36635	5.96	1106.8	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	5.96	1106.3	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	580.26	16.10	36635	5.96	1106.1	9.4	≤	7.22	10.4
	B321	77	3460	620	3.4	551.16	15.50	37319	6.15	1093.4	9.4	≤	7.46	9.8
		77	3460	620	3.4	100.00	1.70	37319	6.15	1169.6	9.4	≤	13.47	1.7
		77	3460	620	3.4	200.00	4.20	37319	6.15	1102.5	9.4	≤	10.48	3.5
		77	3460	620	3.4	400.00	10.00	37319	6.15	1094.1	9.4	≤	8.30	7.1
		77	3460	620	3.4	500.00	13.30	37319	6.15	1093.5	9.4	≤	7.70	8.9
		77	3460	620	3.4	551.16	15.50	37319	6.15	1093.4	9.4	≤	7.46	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	36635	5.96	1106.1	9.4	≤	7.37	10.6
		72.8	3460	940	3.4	100.00	1.70	36635	5.96	1171.9	9.4	≤	13.58	1.7
		72.8	3460	940	3.4	200.00	4.20	36635	5.96	1114.0	9.4	≤	10.60	3.6
		72.8	3460	940	3.4	400.00	10.00	36635	5.96	1106.8	9.4	≤	8.39	7.2
		72.8	3460	940	3.4	500.00	13.30	36635	5.96	1106.3	9.4	≤	7.79	9.0
		72.8	3460	940	3.4	590.38	16.50	36635	5.96	1106.1	9.4	≤	7.37	10.6
	B411	77	4730	320	3.4	621.34	15.50	37319	6.15	1321.2	9.4	≤	7.51	9.2
		77	4730	320	3.4	100.00		37319	6.15	1325.1	9.4	≤	13.82	1.5
		77	4730	320	3.4	200.00		37319	6.15	1321.7	9.4	≤	10.96	2.9
		77	4730	320	3.4	300.00		37319	6.15	1321.3	9.4	≤	9.58	4.4
		77	4730	320	3.4	400.00		37319	6.15	1321.2	9.4	≤	8.70	5.9
		77	4730	320	3.4	500.00		37319	6.15	1321.2	9.4	≤	8.08	7.4
		77	4730	320	3.4	621.34		37319	6.15	1321.2	9.4	≤	7.51	9.2
	C211	85.6	2710	300	3.4	560.94	18.20	38646	6.52	912.0	9.4	≤	6.99	11.6
		85.6	2710	300	3.4	100.00	2.50	38646	6.52	1071.5	9.4	≤	13.11	1.8
		85.6	2710	300	3.4	200.00	5.20	38646	6.52	931.1	9.4	≤	9.93	4.0
		85.6	2710	300	3.4	400.00	11.70	38646	6.52	913.6	9.4	≤	7.83	8.2
		85.6	2710	300	3.4	500.00	15.10	38646	6.52	912.4	9.4	≤	7.27	10.3
		85.6	2710	300	3.4	560.94	18.20	38646	6.52	912.0	9.4	≤	6.99	11.6
	C311	88.1	3220	310	3.4	605.46	18.00	39015	6.62	1015.1	9.4	≤	7.07	11.1
		88.1	3220	310	3.4	100.00	1.70	39015	6.62	1141.6	9.4	≤	13.40	1.6
		88.1	3220	310	3.4	200.00	5.30	39015	6.62	1030.4	9.4	≤	10.28	3.6
		88.1	3220	310	3.4	400.00	12.10	39015	6.62	1016.5	9.4	≤	8.12	7.3
		88.1	3220	310	3.4	500.00	15.40	39015	6.62	1015.6	9.4	≤	7.54	9.2
		88.1	3220	310	3.4	605.46	18.00	39015	6.62	1015.1	9.4	≤	7.07	11.1
	C411	85.6	4260	320	3.4	722.56	19.30	38646	6.52	1216.4	9.4	≤	7.04	11.2
		85.6	4260	320	3.4	100.00	1.70	38646	6.52	1260.7	9.4	≤	13.77	1.5
		85.6	4260	320	3.4	200.00	4.70	38646	6.52	1221.8	9.4	≤	10.82	3.1
		85.6	4260	320	3.4	400.00	10.00	38646	6.52	1216.9	9.4	≤	8.57	6.2
		85.6	4260	320	3.4	600.00	15.80	38646	6.52	1216.4	9.4	≤	7.49	9.3
		85.6	4260	320	3.4	722.56	19.30	38646	6.52	1216.4	9.4	≤	7.04	11.2
	C511	88.1	5310	330	3.4	811.82	20.80	39015	6.62	1333.3	9.4	≤	6.99	11.4
		88.1	5310	330	3.4	100.00	1.70	39015	6.62	1333.3	9.4	≤	14.05	1.4
		88.1	5310	330	3.4	200.00	3.30	39015	6.62	1333.3	9.4	≤	11.15	2.8
		88.1	5310	330	3.4	400.00	8.70	39015	6.62	1333.3	9.4	≤	8.85	5.6
		88.1	5310	330	3.4	600.00	14.20	39015	6.62	1333.3	9.4	≤	7.73	8.4
		88.1	5310	330	3.4	811.82	20.80	39015	6.62	1333.3	9.4	≤	6.99	11.4
	D211	114.5	3220	300	3.4	506.00	16.00	42540	7.65	960.5	9.4	≤	7.58	9.0
		114.5	3220	300	3.4	100.00	1.60	42540	7.65	1188.7	9.4	≤	13.98	1.4
		114.5	3220	300	3.4	200.00	5.00	42540	7.65	987.4	9.4	≤	10.43	3.5
		114.5	3220	300	3.4	300.00	8.60	42540	7.65	967.2	9.4	≤	9.05	5.3
		114.5	3220	300	3.4	400.00	12.10	42540	7.65	962.3	9.4	≤	8.21	7.1
		114.5	3220	300	3.4	506.00	16.00	42540	7.65	960.5	9.4	≤	7.58	9.0
	E211	126.2	2200	300	3.4	506.00	15.90	43928	8.07	724.6	9.4	≤	7.02	11.6
		126.2	2200	300	3.4	100.00	1.70	43928	8.07	1162.5	9.4	≤	14.11	1.4
		126.2	2200	300	3.4	200.00	5.00	43928	8.07	776.3	9.4	≤	9.79	4.3
		126.2	2200	300	3.4	300.00	8.60	43928	8.07	737.5	9.4	≤	8.41	6.7
		126.2	2200	300	3.4	400.00	12.10	43928	8.07	728.1	9.4	≤	7.60	9.1
		126.2	2200	300	3.4	506.00	15.90	43928	8.07	724.6	9.4	≤	7.02	11.6

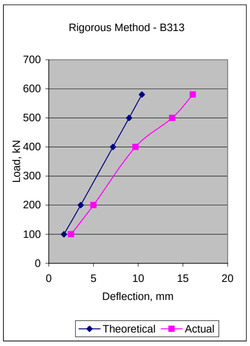

Rigorous Method - A111
Load, kN
Deflection, mm
0 50 100 150 200 250 300 350
0 5 10 15
Theoretical
Actual Yeild

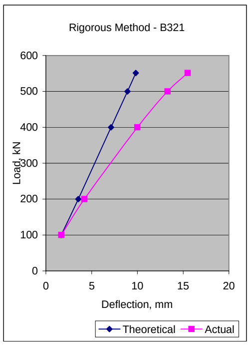

Rigorous Method - A211
Load, kN
Deflection, mm
0 50 100 150 200 250 300 350 400 450 500
0 5 10 15 20
Theretical
Actual

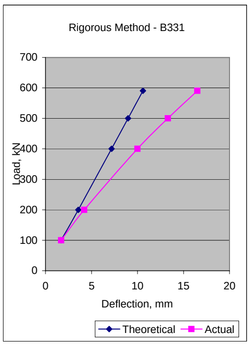

Rigorous Method - B211
Load, kN
Deflection, mm
0 50 100 150 200 250 300 350 400 450 500
0 5 10 15 20
Theretical
Actual

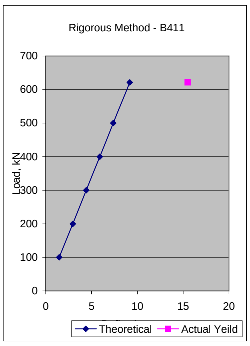

Rigorous Method - B311
Load, kN
Deflection, mm
0 100 200 300 400 500 600 700
0 5 10 15 20
Theoretical
Actual

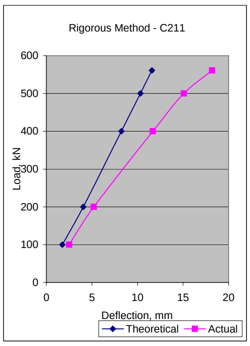

Rigorous Method - B312
Load, kN
Deflection, mm
0 100 200 300 400 500 600
0 5 10 15 20
Theoretical
Actual

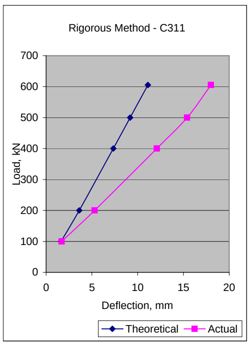

Rigorous Method - B313
Load, kN
Deflection, mm
0 100 200 300 400 500 600 700
0 5 10 15 20
Theoretical
Actual

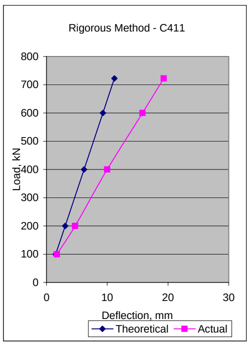

Rigorous Method - B321
Load, kN
Deflection, mm
0 100 200 300 400 500 600
0 5 10 15 20
Theoretical
Actual

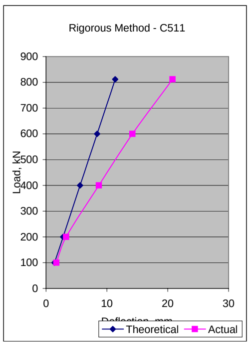

Rigorous Method - B331
Load, kN
Deflection, mm
0 100 200 300 400 500 600 700
0 5 10 15 20
Theoretical
Actual

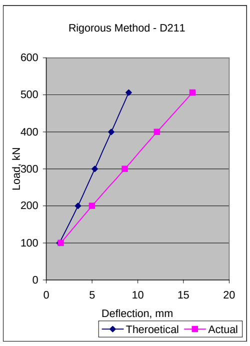

Rigorous Method - B411
Load, kN
Deflection, mm
0 100 200 300 400 500 600 700
0 5 10 15 20
Theoretical
Actual Yeild

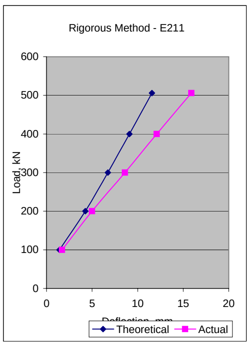

Rigorous Method - C211
Load, kN
Deflection, mm
0 100 200 300 400 500 600
0 5 10 15 20
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
0 100 200 300 400 500 600 700
0 5 10 15 20
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
0 100 200 300 400 500 600 700 800
0 10 20 30
Theoretical
Actual


Rigorous Method - C511
Load, kN
0 100 200 300 400 500 600 700 800 900
0 10 20 30
Theoretical
Actual

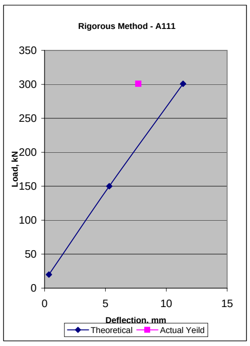

Rigorous Method - D211
Load, kN
Deflection, mm
0 100 200 300 400 500 600
0 5 10 15 20
Theoretical
Actual

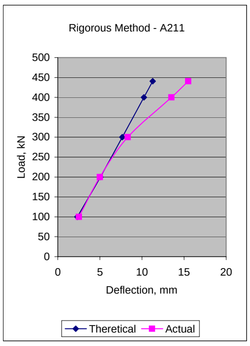

Rigorous Method - E211
Load, kN
0 100 200 300 400 500 600
0 5 10 15 20
Theoretical
Actual

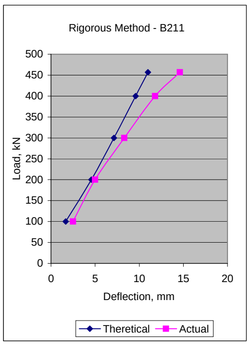
METHOD 24

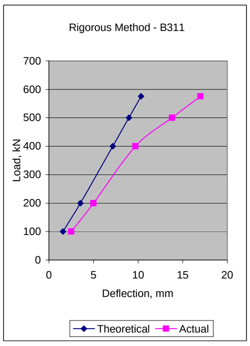
$E_c =$	$0.043\eta\rho^{1.5}\sqrt{f'_c}$
$I_{eff} =$	$I_{cr} + (I - I_{cr})(M_{cr}/M_a)^3$
$f'_{cf} =$	$0.43(f'_c)^{0.68}$

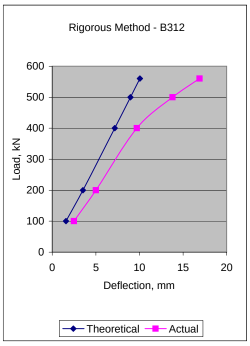
			Reinforcement Ratio			At Yield					Deem-to-Comply			Rigorous Method
SOURCE	BEAM	f'_c, MPa	Area of Steel, A_{st}, mm^2	Area of Steel, A_{sc}, mm^2	L_{eff}, m	Load, kN	Deflection, mm	E_c	f'_{cf}, MPa	I_{eff}, 10^6mm	Ratio	≤	Allowed Ratio	Deflection, mm
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	1250	300	3.4	300.78	7.70	30744	5.53	625.6	9.4	≤	9.99	11.4
		42.8	1250	300	3.4	20.00		30744	5.53	1333.3	9.4	≤	22.76	0.4
		42.8	1250	300	3.4	150.00		30744	5.53	668.6	9.4	≤	9.24	5.3
		42.8	1250	300	3.4	300.78		30744	5.53	625.6	9.4	≤	7.16	11.4
	A211	42.8	2200	300	3.4	440.48	15.50	30744	5.53	924.6	9.4	≤	7.08	11.3
		42.8	2200	300	3.4	100.00	2.50	30744	5.53	1018.7	9.4	≤	11.99	2.3
		42.8	2200	300	3.4	200.00	5.00	30744	5.53	935.4	9.4	≤	9.25	5.1
		42.8	2200	300	3.4	300.00	8.30	30744	5.53	927.1	9.4	≤	8.05	7.7
		42.8	2200	300	3.4	400.00	13.50	30744	5.53	925.0	9.4	≤	7.31	10.2
		42.8	2200	300	3.4	440.48	15.50	30744	5.53	924.6	9.4	≤	7.08	11.3
	B211	74.6	2200	300	3.4	456.70	14.60	36931	8.07	819.0	9.4	≤	7.14	11.0
		74.6	2200	300	3.4	100.00	2.50	36931	8.07	1188.8	9.4	≤	13.41	1.7
		74.6	2200	300	3.4	200.00	5.00	36931	8.07	861.8	9.4	≤	9.56	4.6
		74.6	2200	300	3.4	300.00	8.30	36931	8.07	828.9	9.4	≤	8.25	7.1
		74.6	2200	300	3.4	400.00	11.80	36931	8.07	820.9	9.4	≤	7.47	9.6
		74.6	2200	300	3.4	456.70	14.60	36931	8.07	819.0	9.4	≤	7.14	11.0
	B311	72.8	3460	310	3.4	575.68	17.00	36635	7.94	1106.6	9.4	≤	7.24	10.3
		72.8	3460	310	3.4	100.00	2.50	36635	7.94	1261.9	9.4	≤	13.56	1.6
		72.8	3460	310	3.4	200.00	5.00	36635	7.94	1125.3	9.4	≤	10.36	3.5
		72.8	3460	310	3.4	400.00	9.70	36635	7.94	1108.2	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	7.94	1107.0	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	575.68	17.00	36635	7.94	1106.6	9.4	≤	7.24	10.3
	B312	72.8	3460	310	3.4	560.36	16.90	36635	7.94	1106.6	9.4	≤	7.31	10.1
		72.8	3460	310	3.4	100.00	2.50	36635	7.94	1261.9	9.4	≤	13.56	1.6
		72.8	3460	310	3.4	200.00	5.00	36635	7.94	1125.3	9.4	≤	10.36	3.5
		72.8	3460	310	3.4	400.00	9.70	36635	7.94	1108.2	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	7.94	1107.0	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	560.36	16.90	36635	7.94	1106.6	9.4	≤	7.31	10.1
	B313	72.8	3460	310	3.4	580.26	16.10	36635	7.94	1106.5	9.4	≤	7.22	10.4
		72.8	3460	310	3.4	100.00	2.50	36635	7.94	1261.9	9.4	≤	13.56	1.6
		72.8	3460	310	3.4	200.00	5.00	36635	7.94	1125.3	9.4	≤	10.36	3.5
		72.8	3460	310	3.4	400.00	9.70	36635	7.94	1108.2	9.4	≤	8.18	7.2
		72.8	3460	310	3.4	500.00	13.80	36635	7.94	1107.0	9.4	≤	7.59	9.0
		72.8	3460	310	3.4	580.26	16.10	36635	7.94	1106.5	9.4	≤	7.22	10.4
	B321	77	3460	620	3.4	551.16	15.50	37319	8.25	1094.0	9.4	≤	7.46	9.8
		77	3460	620	3.4	100.00	1.70	37319	8.25	1277.9	9.4	≤	13.87	1.5
		77	3460	620	3.4	200.00	4.20	37319	8.25	1116.1	9.4	≤	10.53	3.5
		77	3460	620	3.4	400.00	10.00	37319	8.25	1095.8	9.4	≤	8.30	7.1
		77	3460	620	3.4	500.00	13.30	37319	8.25	1094.4	9.4	≤	7.71	8.9
		77	3460	620	3.4	551.16	15.50	37319	8.25	1094.0	9.4	≤	7.46	9.8
	B331	72.8	3460	940	3.4	590.38	16.50	36635	7.94	1106.5	9.4	≤	7.37	10.6
		72.8	3460	940	3.4	100.00	1.70	36635	7.94	1261.9	9.4	≤	13.92	1.6
		72.8	3460	940	3.4	200.00	4.20	36635	7.94	1125.3	9.4	≤	10.63	3.5
		72.8	3460	940	3.4	400.00	10.00	36635	7.94	1108.2	9.4	≤	8.40	7.2
		72.8	3460	940	3.4	500.00	13.30	36635	7.94	1107.0	9.4	≤	7.79	9.0
		72.8	3460	940	3.4	590.38	16.50	36635	7.94	1106.5	9.4	≤	7.37	10.6
	B411	77	4730	320	3.4	621.34	15.50	37319	8.25	1321.2	9.4	≤	7.51	9.2
		77	4730	320	3.4	100.00		37319	8.25	1330.5	9.4	≤	13.84	1.5
		77	4730	320	3.4	200.00		37319	8.25	1322.4	9.4	≤	10.97	2.9
		77	4730	320	3.4	300.00		37319	8.25	1321.5	9.4	≤	9.58	4.4
		77	4730	320	3.4	400.00		37319	8.25	1321.3	9.4	≤	8.70	5.9
		77	4730	320	3.4	500.00		37319	8.25	1321.3	9.4	≤	8.08	7.4
		77	4730	320	3.4	621.34		37319	8.25	1321.2	9.4	≤	7.51	9.2
	C211	85.6	2710	300	3.4	560.94	18.20	38646	8.86	913.4	9.4	≤	7.00	11.6
		85.6	2710	300	3.4	100.00	2.50	38646	8.86	1314.3	9.4	≤	14.03	1.4
		85.6	2710	300	3.4	200.00	5.20	38646	8.86	961.5	9.4	≤	10.04	3.9
		85.6	2710	300	3.4	400.00	11.70	38646	8.86	917.4	9.4	≤	7.84	8.2
		85.6	2710	300	3.4	500.00	15.10	38646	8.86	914.3	9.4	≤	7.27	10.3
		85.6	2710	300	3.4	560.94	18.20	38646	8.86	913.4	9.4	≤	7.00	11.6
	C311	88.1	3220	310	3.4	605.46	18.00	39015	9.04	1016.0	9.4	≤	7.07	11.1
		88.1	3220	310	3.4	100.00	1.70	39015	9.04	1333.3	9.4	≤	14.12	1.4
		88.1	3220	310	3.4	200.00	5.30	39015	9.04	1054.9	9.4	≤	10.36	3.5
		88.1	3220	310	3.4	400.00	12.10	39015	9.04	1019.6	9.4	≤	8.13	7.3
		88.1	3220	310	3.4	500.00	15.40	39015	9.04	1017.1	9.4	≤	7.54	9.2
		88.1	3220	310	3.4	605.46	18.00	39015	9.04	1016.0	9.4	≤	7.07	11.1
	C411	85.6	4260	320	3.4	722.56	19.30	38646	8.86	1216.5	9.4	≤	7.04	11.2
		85.6	4260	320	3.4	100.00	1.70	38646	8.86	1328.0	9.4	≤	14.01	1.4
		85.6	4260	320	3.4	200.00	4.70	38646	8.86	1230.2	9.4	≤	10.84	3.1
		85.6	4260	320	3.4	400.00	10.00	38646	8.86	1218.0	9.4	≤	8.58	6.2
		85.6	4260	320	3.4	600.00	15.80	38646	8.86	1216.8	9.4	≤	7.49	9.3
		85.6	4260	320	3.4	722.56	19.30	38646	8.86	1216.5	9.4	≤	7.04	11.2
	C511	88.1	5310	330	3.4	811.82	20.80	39015	9.04	1333.3	9.4	≤	6.99	11.4
		88.1	5310	330	3.4	100.00	1.70	39015	9.04	1332.8	9.4	≤	14.05	1.4
		88.1	5310	330	3.4	200.00	3.30	39015	9.04	1333.3	9.4	≤	11.15	2.8
		88.1	5310	330	3.4	400.00	8.70	39015	9.04	1333.3	9.4	≤	8.85	5.6
		88.1	5310	330	3.4	600.00	14.20	39015	9.04	1333.3	9.4	≤	7.73	8.4
		88.1	5310	330	3.4	811.82	20.80	39015	9.04	1333.3	9.4	≤	6.99	11.4
	D211	114.5	3220	300	3.4	506.00	16.00	42540	10.80	963.7	9.4	≤	7.59	9.0
		114.5	3220	300	3.4	100.00	1.60	42540	10.80	1333.3	9.4	≤	14.52	1.3
		114.5	3220	300	3.4	200.00	5.00	42540	10.80	1039.6	9.4	≤	10.61	3.3
		114.5	3220	300	3.4	300.00	8.60	42540	10.80	982.7	9.4	≤	9.10	5.2
		114.5	3220	300	3.4	400.00	12.10	42540	10.80	968.8	9.4	≤	8.22	7.1
		114.5	3220	300	3.4	506.00	16.00	42540	10.80	963.7	9.4	≤	7.59	9.0
	E211	126.2	2200	300	3.4	506.00	15.90	43928	11.54	731.1	9.4	≤	7.04	11.5
		126.2	2200	300	3.4	100.00	1.70	43928	11.54	1333.3	9.4	≤	14.77	1.2
		126.2	2200	300	3.4	200.00	5.00	43928	11.54	882.5	9.4	≤	10.21	3.8
		126.2	2200	300	3.4	300.00	8.60	43928	11.54	769.0	9.4	≤	8.52	6.5
		126.2	2200	300	3.4	400.00	12.10	43928	11.54	741.3	9.4	≤	7.65	8.9
		126.2	2200	300	3.4	506.00	15.90	43928	11.54	731.1	9.4	≤	7.04	11.5

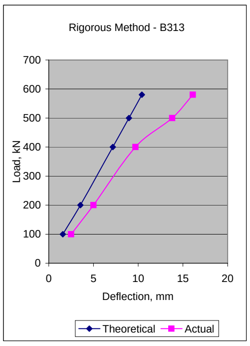

Rigorous Method - A111
Load, kN
Deflection, mm
Theoretical
Actual Yeild

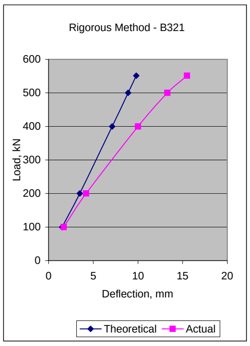

Rigorous Method - A211
Load, kN
Deflection, mm
Theretical
Actual

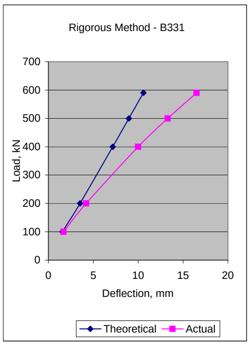

Rigorous Method - B211
Load, kN
Deflection, mm
Theretical
Actual

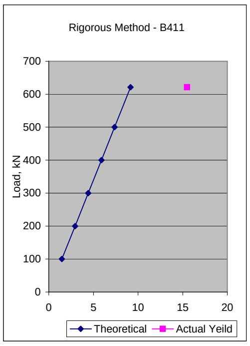

Rigorous Method - B311
Load, kN
Deflection, mm
Theoretical
Actual

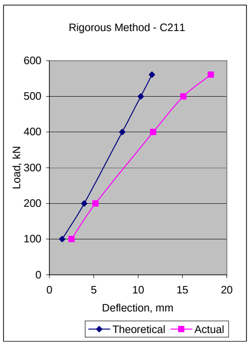

Rigorous Method - B312
Load, kN
Deflection, mm
Theoretical
Actual

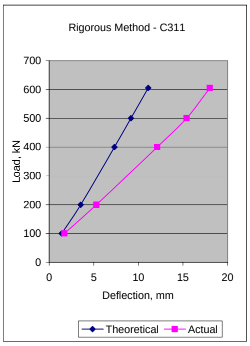

Rigorous Method - B313
Load, kN
Deflection, mm
Theoretical
Actual

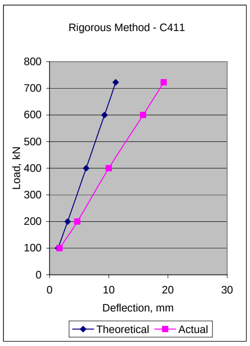

Rigorous Method - B321
Load, kN
Deflection, mm
Theoretical
Actual

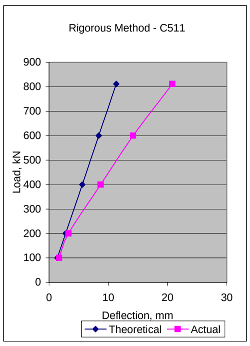

Rigorous Method - B331
Load, kN
Deflection, mm
Theoretical
Actual

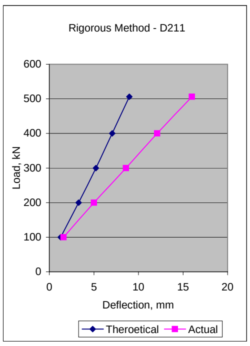

Rigorous Method - B411
Load, kN
Theoretical
Actual Yeild

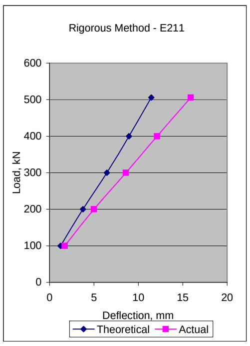

Rigorous Method - C211
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C311
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C411
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - C511
Load, kN
Deflection, mm
Theoretical
Actual


Rigorous Method - D211
Load, kN
Deflection, mm
Theroetical
Actual


Rigorous Method - E211
Load, kN
Deflection, mm
Theoretical
Actual

				RIGOROUS METHOD -Deflection, mm																							
SOURCE	BEAM	f'_c, MPa	Actual Yield Deflection, mm	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7	Method 8	Method 9	Method 10	Method 11	Method 12	Method 13	Method 14	Method 15	Method 16	Method 17	Method 18	Method 19	Method 20	Method 21	Method 22	Method 23	Method 24
ACI STRUCTURAL JOURNAL MAY/JUNE 2005	A111	42.8	7.70	12.4	11.3	11.1	11.3	11.2	11.2	12.4	11.2	11.1	11.2	11.2	11.2	13.6	11.5	11.4	11.5	11.5	11.4	13.3	11.4	11.3	11.4	11.4	11.4
		42.8		0.8	0.3	0.3	0.3	0.3	0.3	0.8	0.3	0.3	0.3	0.3	0.3	0.9	0.4	0.4	0.4	0.4	0.4	0.9	0.4	0.4	0.4	0.4	0.4
		42.8		6.2	5.5	5.0	5.4	5.4	5.2	6.2	5.5	5.0	5.4	5.4	5.2	6.8	5.6	5.2	5.6	5.5	5.3	6.7	5.6	5.2	5.5	5.5	5.3
		42.8		12.4	11.3	11.1	11.3	11.2	11.2	12.4	11.2	11.1	11.2	11.2	11.2	13.6	11.5	11.4	11.5	11.5	11.4	13.3	11.4	11.3	11.4	11.4	11.4
	A211	42.8	15.50	18.2	11.0	11.0	11.0	11.0	11.0	18.2	11.0	11.0	11.0	11.0	11.0	19.9	11.4	11.3	11.4	11.4	11.4	19.5	11.3	11.3	11.3	11.3	11.3
		42.8	2.50	18.2	2.4	2.2	2.4	2.4	2.2	4.1	2.4	2.1	2.4	2.4	2.2	4.5	2.5	2.3	2.5	2.5	2.3	4.4	2.5	2.2	2.5	2.4	2.3
		42.8	5.00	8.2	5.0	4.9	5.0	5.0	4.9	8.2	5.0	4.9	5.0	5.0	4.9	9.1	5.1	5.1	5.1	5.1	5.1	8.9	5.1	5.0	5.1	5.1	5.1
		42.8	8.30	12.4	7.5	7.5	7.5	7.5	7.5	12.4	7.5	7.5	7.5	7.5	7.5	13.6	7.7	7.7	7.7	7.7	7.7	13.3	7.7	7.6	7.7	7.7	7.7
		42.8	13.50	16.5	10.0	10.0	10.0	10.0	10.0	16.5	10.0	10.0	10.0	10.0	10.0	18.1	10.3	10.3	10.3	10.3	10.3	17.7	10.2	10.2	10.2	10.2	10.2
		42.8	15.50	18.2	11.0	11.0	11.0	11.0	11.0	18.2	11.0	11.0	11.0	11.0	11.0	19.9	11.4	11.3	11.4	11.4	11.4	19.5	11.3	11.3	11.3	11.3	11.3
	B211	74.6	14.60	14.3	10.5	10.5	10.5	10.5	10.5	15.0	10.6	10.6	10.6	10.6	10.6	16.6	11.0	10.9	11.0	11.0	10.9	16.9	11.0	11.0	11.0	11.0	11.0
		74.6	2.50	3.1	2.0	1.4	1.8	1.8	1.4	3.3	2.0	1.5	1.9	1.9	1.5	3.6	2.1	1.6	2.0	2.0	1.6	3.7	2.2	1.6	2.0	2.0	1.7
		74.6	5.00	6.2	4.5	4.3	4.5	4.5	4.3	6.6	4.6	4.4	4.5	4.5	4.4	7.3	4.7	4.5	4.7	4.7	4.5	7.4	4.8	4.6	4.7	4.7	4.6
		74.6	8.30	9.4	6.9	6.8	6.9	6.9	6.8	9.8	7.0	6.9	6.9	6.9	6.9	10.9	7.2	7.1	7.2	7.2	7.1	11.1	7.2	7.1	7.2	7.2	7.1
		74.6	11.80	12.5	9.2	9.1	9.2	9.2	9.1	13.1	9.3	9.3	9.3	9.3	9.3	14.6	9.6	9.6	9.6	9.6	9.6	14.8	9.6	9.6	9.6	9.6	9.6
		74.6	14.60	14.3	10.5	10.5	10.5	10.5	10.5	15.0	10.6	10.6	10.6	10.6	10.6	16.6	11.0	10.9	11.0	11.0	10.9	16.9	11.0	11.0	11.0	11.0	11.0
	B311	72.8	17.00	18.2	9.8	9.8	9.8	9.8	9.8	19.1	9.9	9.9	9.9	9.9	9.9	21.2	10.3	10.3	10.3	10.3	10.3	21.4	10.3	10.3	10.3	10.3	10.3
		72.8	2.50	3.2	1.6	1.4	1.5	1.5	1.4	3.3	1.6	1.4	1.6	1.6	1.4	3.7	1.7	1.6	1.7	1.7	1.6	3.7	1.7	1.6	1.7	1.7	1.6
		72.8	5.00	6.3	3.4	3.3	3.4	3.4	3.3	6.6	3.4	3.4	3.4	3.4	3.4	7.4	3.6	3.5	3.5	3.5	3.5	7.4	3.6	3.5	3.6	3.6	3.5
		72.8	9.70	12.6	6.8	6.8	6.8	6.8	6.8	13.2	6.9	6.9	6.9	6.9	6.9	14.7	7.1	7.1	7.1	7.1	7.1	14.9	7.2	7.2	7.2	7.2	7.2
		72.8	13.80	15.8	8.5	8.5	8.5	8.5	8.5	16.6	8.6	8.6	8.6	8.6	8.6	18.4	8.9	8.9	8.9	8.9	8.9	18.6	9.0	9.0	9.0	9.0	9.0
		72.8	17.00	18.2	9.8	9.8	9.8	9.8	9.8	19.1	9.9	9.9	9.9	9.9	9.9	21.2	10.3	10.3	10.3	10.3	10.3	21.4	10.3	10.3	10.3	10.3	10.3
	B312	72.8	16.90	17.7	9.5	9.5	9.5	9.5	9.5	18.6	9.6	9.6	9.6	9.6	9.6	20.6	10.0	10.0	10.0	10.0	10.0	20.9	10.1	10.1	10.1	10.1	10.1
		72.8	2.50	3.2	1.6	1.4	1.5	1.5	1.4	3.3	1.6	1.4	1.6	1.6	1.4	3.7	1.7	1.6	1.7	1.7	1.6	3.7	1.7	1.6	1.7	1.7	1.6
		72.8	5.00	6.3	3.4	3.3	3.4	3.4	3.3	6.6	3.4	3.4	3.4	3.4	3.4	7.4	3.6	3.5	3.5	3.5	3.5	7.4	3.6	3.5	3.6	3.6	3.5
		72.8	9.70	12.6	6.8	6.8	6.8	6.8	6.8	13.2	6.9	6.9	6.9	6.9	6.9	14.7	7.1	7.1	7.1	7.1	7.1	14.9	7.2	7.2	7.2	7.2	7.2
		72.8	13.80	15.8	8.5	8.5	8.5	8.5	8.5	16.6	8.6	8.6	8.6	8.6	8.6	18.4	8.9	8.9	8.9	8.9	8.9	18.6	9.0	9.0	9.0	9.0	9.0
		72.8	16.90	17.7	9.5	9.5	9.5	9.5	9.5	18.6	9.6	9.6	9.6	9.6	9.6	20.6	10.0	10.0	10.0	10.0	10.0	20.9	10.1	10.1	10.1	10.1	10.1
	B313	72.8	16.10	18.3	9.8	9.8	9.8	9.8	9.8	19.2	10.0	10.0	10.0	10.0	10.0	21.3	10.4	10.4	10.4	10.4	10.4	21.6	10.4	10.4	10.4	10.4	10.4
		72.8	2.50	3.2	1.6	1.4	1.5	1.5	1.4	3.3	1.6	1.4	1.6	1.6	1.4	3.7	1.7	1.6	1.7	1.7	1.6	3.7	1.7	1.6	1.7	1.7	1.6
		72.8	5.00	6.3	3.4	3.3	3.4	3.4	3.3	6.6	3.4	3.4	3.4	3.4	3.4	7.4	3.6	3.5	3.5	3.5	3.5	7.4	3.6	3.5	3.6	3.6	3.5
		72.8	9.70	12.6	6.8	6.8	6.8	6.8	6.8	13.2	6.9	6.9	6.9	6.9	6.9	14.7	7.1	7.1	7.1	7.1	7.1	14.9	7.2	7.2	7.2	7.2	7.2
		72.8	13.80	15.8	8.5	8.5	8.5	8.5	8.5	16.6	8.6	8.6	8.6	8.6	8.6	18.4	8.9	8.9	8.9	8.9	8.9	18.6	9.0	9.0	9.0	9.0	9.0
		72.8	16.10	18.3	9.8	9.8	9.8	9.8	9.8	19.2	10.0	10.0	10.0	10.0	10.0	21.3	10.4	10.4	10.4	10.4	10.4	21.6	10.4	10.4	10.4	10.4	10.4
	B321	77	15.50	16.9	9.3	9.3	9.3	9.3	9.3	17.9	9.4	9.4	9.4	9.4	9.4	19.8	9.8	9.8	9.8	9.8	9.8	20.1	9.8	9.8	9.8	9.8	9.8
		77	1.70	3.1	1.6	1.3	1.5	1.5	1.3	3.2	1.6	1.4	1.6	1.6	1.4	3.6	1.7	1.5	1.6	1.7	1.5	3.7	1.7	1.5	1.7	1.7	1.5
		77	4.20	6.1	3.3	3.2	3.3	3.3	3.2	6.5	3.4	3.3	3.4	3.4	3.3	7.2	3.5	3.5	3.5	3.5	3.5	7.3	3.5	3.5	3.5	3.5	3.5
		77	10.00	12.3	6.7	6.7	6.7	6.7	6.7	13.0	6.8	6.8	6.8	6.8	6.8	14.4	7.1	7.1	7.1	7.1	7.1	14.6	7.1	7.1	7.1	7.1	7.1
		77	13.30	15.4	8.4	8.4	8.4	8.4	8.4	16.2	8.5	8.5	8.5	8.5	8.5	18.0	8.9	8.9	8.9	8.9	8.9	18.3	8.9	8.9	8.9	8.9	8.9
		77	15.50	16.9	9.3	9.3	9.3	9.3	9.3	17.9	9.4	9.4	9.4	9.4	9.4	19.8	9.8	9.8	9.8	9.8	9.8	20.1	9.8	9.8	9.8	9.8	9.8
	B331	72.8	16.50	18.7	10.0	10.0	10.0	10.0	10.0	19.5	10.2	10.2	10.2	10.2	10.2	21.7	10.6	10.5	10.6	10.6	10.5	22.0	10.6	10.6	10.6	10.6	10.6
		72.8	1.70	3.2	1.6	1.4	1.5	1.5	1.4	3.3	1.6	1.4	1.6	1.6	1.4	3.7	1.7	1.6	1.7	1.7	1.6	3.7	1.7	1.6	1.7	1.7	1.6
		72.8	4.20	6.3	3.4	3.3	3.4	3.4	3.3	6.6	3.4	3.4	3.4	3.4	3.4	7.4	3.6	3.5	3.5	3.5	3.5	7.4	3.6	3.5	3.6	3.6	3.5
		72.8	10.00	12.6	6.8	6.8	6.8	6.8	6.8	13.2	6.9	6.9	6.9	6.9	6.9	14.7	7.1	7.1	7.1	7.1	7.1	14.9	7.2	7.2	7.2	7.2	7.2
		72.8	13.30	15.8	8.5	8.5	8.5	8.5	8.5	16.6	8.6	8.6	8.6	8.6	8.6	18.4	8.9	8.9	8.9	8.9	8.9	18.6	9.0	9.0	9.0	9.0	9.0
		72.8	16.50	18.7	10.0	10.0	10.0	10.0	10.0	19.5	10.2	10.2	10.2	10.2	10.2	21.7	10.6	10.5	10.6	10.6	10.5	22.0	10.6	10.6	10.6	10.6	10.6
	B411	77	15.50	19.1	8.6	8.6	8.6	8.6	8.6	20.2	8.7	8.7	8.7	8.7	8.7	22.3	9.1	9.1	9.1	9.1	9.1	22.7	9.2	9.2	9.2	9.2	9.2
		77		3.1	1.3	1.3	1.3	1.3	1.3	3.2	1.4	1.3	1.4	1.4	1.3	3.6	1.5	1.4	1.5	1.5	1.4	3.7	1.5	1.5	1.5	1.5	1.5
		77		6.1	2.8	2.7	2.7	2.7	2.7	6.5	2.8	2.8	2.8	2.8	2.8	7.2	2.9	2.9	2.9	2.9	2.9	7.3	3.0	2.9	2.9	2.9	2.9
		77		9.2	4.1	4.1	4.1	4.1	4.1	9.7	4.2	4.2	4.2	4.2	4.2	10.8	4.4	4.4	4.4	4.4	4.4	11.0	4.4	4.4	4.4	4.4	4.4
		77		12.3	5.5	5.5	5.5	5.5	5.5	13.0	5.6	5.6	5.6	5.6	5.6	14.4	5.9	5.9	5.9	5.9	5.9	14.6	5.9	5.9	5.9	5.9	5.9
		77		15.4	6.9	6.9	6.9	6.9	6.9	16.2	7.0	7.0	7.0	7.0	7.0	18.0	7.3	7.3	7.3	7.3	7.3	18.3	7.4	7.4	7.4	7.4	7.4
		77		19.1	8.6	8.6	8.6	8.6	8.6	20.2	8.7	8.7	8.7	8.7	8.7	22.3	9.1	9.1	9.1	9.1	9.1	22.7	9.2	9.2	9.2	9.2	9.2
	C211	85.6	18.20	16.3	10.9	10.9	10.9	10.9	10.9	17.6	11.2	11.1	11.1	11.1	11.1	19.3	11.5	11.5	11.5	11.5	11.5	19.8	11.6	11.6	11.6	11.6	11.6
		85.6	2.50	2.9	1.7	1.2	1.5	1.6	1.2	3.1	1.8	1.3	1.6	1.6	1.3	3.4	1.8	1.4	1.7	1.7	1.4	3.5	1.9	1.5	1.7	1.8	1.4
		85.6	5.20	5.8	3.8	3.6	3.8	3.8	3.6	6.3	3.9	3.7	3.9	3.9	3.7	6.9	4.0	3.9	4.0	4.0	3.9	7.1	4.1	3.9	4.0	4.0	3.9
		85.6	11.70	11.7	7.8	7.7	7.8	7.8	7.7	12.6	7.9	7.9	7.9	7.9	7.9	13.8	8.2	8.1	8.2	8.2	8.1	14.1	8.3	8.2	8.2	8.2	8.2
		85.6	15.10	14.6	9.7	9.7	9.7	9.7	9.7	15.7	9.9	9.9	9.9	9.9	9.9	17.2	10.2	10.2	10.2	10.2	10.2	17.6	10.3	10.3	10.3	10.3	10.3
		85.6	18.20	16.3	10.9	10.9	10.9	10.9	10.9	17.6	11.2	11.1	11.1	11.1	11.1	19.3	11.5	11.5	11.5	11.5	11.5	19.8	11.6	11.6	11.6	11.6	11.6
	C311	88.1	18.00	17.4	10.4	10.4	10.4	10.4	10.4	18.8	10.7	10.7	10.7	10.7	10.7	20.6	11.0	11.0	11.0	11.0	11.0	21.2	11.1	11.1	11.1	11.1	11.1
		88.1	1.70	2.9	1.5	1.2	1.4	1.4	1.2	3.1	1.6	1.3	1.5	1.5	1.2	3.4	1.7	1.4	1.6	1.6	1.4	3.5	1.7	1.4	1.6	1.6	1.4
		88.1	5.30	5.7	3.4	3.3	3.4	3.4	3.2	6.2	3.5	3.4	3.5	3.5	3.4	6.8	3.6	3.5	3.6	3.6	3.5	7.0	3.6	3.5	3.6	3.6	3.5
		88.1	12.10	11.5	6.9	6.9	6.9	6.9	6.8	12.4	7.1	7.0	7.0	7.0	7.0	13.6	7.3	7.2	7.3	7.3	7.2	14.0	7.3	7.3	7.3	7.3	7.3
		88.1	15.40	14.4	8.6	8.6	8.6	8.6	8.6	15.5	8.8	8.8	8.8	8.8	8.8	17.0	9.1	9.1	9.1	9.1	9.1	17.5	9.2	9.2	9.2	9.2	9.2
		88.1	18.00	17.4	10.4	10.4	10.4	10.4	10.4	18.8	10.7	10.7	10.7	10.7	10.7	20.6	11.0	11.0	11.0	11.0	11.0	21.2	11.1	11.1	11.1	11.1	11.1
	C411	85.6	19.30	21.1	10.4	10.4	10.4	10.4	10.4	22.7	10.7	10.7	10.7	10.7	10.7	24.9	11.1	11.1	11.1	11.1	11.1	25.5	11.2	11.2	11.2	11.2	11.2
		85.6	1.70	2.9	1.4	1.2	1.3	1.3	1.2	3.1	1.4	1.3	1.4	1.4	1.3	3.4	1.5	1.4	1.5	1.5	1.4	3.5	1.5	1.4	1.5	1.5	1.4
		85.6	4.70	5.8	2.9	2.8	2.9	2.9	2.8	6.3	2.9	2.9	2.9	2.9	2.9	6.9	3.1	3.0	3.0	3.1	3.0	7.1	3.1	3.1	3.1	3.1	3.1
		85.6	10.00	11.7	5.8	5.8	5.8	5.8	5.8	12.6	5.9	5.9	5.9	5.9	5.9	13.8	6.1	6.1	6.1	6.1	6.1	14.1	6.2	6.2	6.2	6.2	6.2
		85.6	15.80	17.5	8.7	8.6	8.7	8.7	8.6	18.8	8.9	8.9	8.9	8.9	8.9	20.7	9.2	9.2	9.2	9.2	9.2	21.2	9.3	9.3	9.3	9.3	9.3
		85.6	19.30	21.1	10.4	10.4	10.4	10.4	10.4	22.7	10.7	10.7	10.7	10.7	10.7	24.9	11.1	11.1	11.1	11.1	11.1	25.5	11.2	11.2	11.2	11.2	11.2
	C511	88.1	20.80	23.3	10.2	10.2	10.2	10.2	10.2	25.2	10.5	10.5	10.5	10.5	10.5	27.6	11.1	11.1	11.1	11.1	11.1	28.4	11.4	11.4	11.4	11.4	11.4
		88.1	1.70	2.9	1.2	1.2	1.2	1.2	1.2	3.1	1.3	1.2	1.3	1.3	1.2	3.4	1.4	1.4	1.4	1.4	1.4	3.5	1.4	1.4	1.4	1.4	1.4
		88.1	3.30	5.7	2.5	2.5	2.5	2.5	2.5	6.2	2.6	2.6	2.6	2.6	2.6	6.8	2.7	2.7	2.7	2.7	2.7	7.0	2.8	2.8	2.8	2.8	2.8
		88.1	8.70	11.5	5.0	5.0	5.0	5.0	5.0	12.4	5.2	5.2	5.2	5.2	5.2	13.6	5.4	5.4	5.4	5.4	5.4	14.0	5.6	5.6	5.6	5.6	5.6
		88.1	14.20	17.2	7.5	7.5	7.5	7.5	7.5	18.7	7.7	7.7	7.7	7.7	7.7	20.4	8.2	8.2	8.2	8.2	8.2	21.0	8.4	8.4	8.4	8.4	8.4
		88.1	20.80	23.3	10.2	10.2	10.2	10.2	10.2	25.2	10.5	10.5	10.5	10.5	10.5	27.6	11.1	11.1	11.1	11.1	11.1	28.4	11.4	11.4	11.4	11.4	11.4
	D211	114.5	16.00	12.8	8.4	8.3	8.3	8.4	8.3	14.6	8.7	8.7	8.7	8.7	8.7	15.5	8.9	8.8	8.9	8.9	8.8	16.2	9.0	9.0	9.0	9.0	9.0
		114.5	1.60	2.5	1.3	1.0	1.1	1.2	1.0	2.9	1.5	1.2	1.3	1.3	1.2	3.1	1.5	1.2	1.3	1.4	1.2	3.2	1.6	1.3	1.4	1.4	1.3
		114.5	5.00	5.0	3.2	3.0	3.1	3.2	2.9	5.8	3.4	3.2	3.3	3.3	3.1	6.1	3.4	3.3	3.4	3.4	3.2	6.4	3.5	3.3	3.4	3.5	3.3
		114.5	8.60	7.6	4.9	4.8	4.9	4.9	4.8	8.7	5.1	5.0	5.1	5.1	5.0	9.2	5.2	5.1	5.2	5.2	5.1	9.6	5.3	5.2	5.3	5.3	5.2
		114.5	12.10	10.1	6.6	6.5	6.6	6.6	6.5	11.6	6.9	6.8	6.9	6.9	6.8	12.2	7.0	7.0	7.0	7.0	6.9	12.8	7.1	7.1	7.1	7.1	7.1
		114.5	16.00	12.8	8.4	8.3	8.3	8.4	8.3	14.6	8.7	8.7	8.7	8.7	8.7	15.5	8.9	8.8	8.9	8.9	8.8	16.2	9.0	9.0	9.0	9.0	9.0
	E211	126.2	15.90	12.1	10.8	10.7	10.8	10.8	10.7	14.3	11.3	11.2	11.3	11.3	11.2	14.8	11.4	11.3	11.4	11.4	11.3	15.7	11.6	11.5	11.6	11.6	11.5
		126.2	1.70	2.4	1.4	1.0	1.0	1.1	1.0	2.8	1.6	1.1	1.2	1.3	1.1	2.9	1.6	1.2	1.2	1.4	1.2	3.1	1.7	1.2	1.3	1.4	1.2
		126.2	5.00	4.8	4.0	3.4	3.8	3.9	3.2	5.7	4.3	3.7	4.0	4.1	3.6	5.9	4.3	3.8	4.1	4.2	3.6	6.2	4.4	3.9	4.2	4.3	3.8
		126.2	8.60	7.2	6.3	6.0	6.2	6.2	5.9	8.5	6.6	6.3	6.5	6.5	6.2	8.8	6.7	6.4	6.6	6.6	6.3	9.3	6.8	6.6	6.7	6.7	6.5
		126.2	12.10	9.6	8.5	8.3	8.5	8.5	8.3	11.3	8.9	8.7	8.8	8.9	8.7	11.7	9.0	8.8	8.9	8.9	8.8	12.4	9.1	9.0	9.1	9.1	8.9
		126.2	15.90	12.1	10.8	10.7	10.8	10.8	10.7	14.3	11.3	11.2	11.3	11.3	11.2	14.8	11.4	11.3	11.4	11.4	11.3	15.7	11.6	11.5	11.6	11.6	11.5

Table C.1 Comparison of deflections

APPENDIX D

Paul Y.L Kong and B. Vijaya Rangan (1998) - Beams with Stirrups

Specimen	*b*	*f'c*	*D*	*d*	*a/d*	*Ast* mm^2	*fy* *MPa*	*Transverse Steel* *ρt=Asv/bv.s*	*fsty.f* *(MPa)*	*Asv.*	*Ult.Sh strength* V_e *(kN)* *(Measured)*	β_1	β_2	β_3	V_{uc} *(N)* *Predicted*	V_{uc} *(kN)* *Predicted*	V_{us} *(N)*	V_{us} *(kN)*	$V_p=V_{uc}+V_{us}$ *(kN)*	V_e/V_p
S1-1	250	63.6	350	292	2.5	2046	452	0.00157	569	19.63	228.3	1.4388	1	1	127351.26	127.35126	112949.07	112.94907	240.3003333	0.9500611
S1-2	250	63.6	350	292	2.5	2046	452	0.00157	569	19.63	208.3	1.4388	1	1	127351.26	127.35126	112949.07	112.94907	240.3003333	0.8668319
S1-3	250	63.6	350	292	2.5	2046	452	0.00157	569	19.63	206.1	1.4388	1	1	127351.26	127.35126	112949.07	112.94907	240.3003333	0.8576767
S1-4	250	63.6	350	292	2.5	2046	452	0.00157	569	19.63	277.9	1.4388	1	1	127351.26	127.35126	112949.07	112.94907	240.3003333	1.1564695
S1-5	250	63.6	350	292	2.5	2046	452	0.00157	569	19.63	253.3	1.4388	1	1	127351.26	127.35126	112949.07	112.94907	240.3003333	1.0540976
S1- 6	250	63.6	350	292	2.5	2046	452	0.00157	569	19.63	224.1	1.4388	1	1	127351.26	127.35126	112949.07	112.94907	240.3003333	0.932583
S2-1	250	72.5	350	292	2.5	2046	452	0.00105	569	19.63	260.3	1.4388	1	1	133034.28	133.03428	75539.188	75.539188	208.5734634	1.2480015
S2-2	250	72.5	350	292	2.5	2046	452	0.00126	569	19.63	232.5	1.4388	1	1	133034.28	133.03428	90647.026	90.647026	223.6813011	1.0394253
S2-3	250	72.5	350	292	2.5	2046	452	0.00157	569	19.63	253.3	1.4388	1	1	133034.28	133.03428	112949.07	112.94907	245.9833471	1.0297445
S2-4	250	72.5	350	292	2.5	2046	452	0.00157	569	19.63	219.4	1.4388	1	1	133034.28	133.03428	112949.07	112.94907	245.9833471	0.8919303
S2-5	250	72.5	350	292	2.5	2046	452	0.00209	569	19.63	282.1	1.4388	1	1	133034.28	133.03428	150358.96	150.35896	283.3932308	0.9954366
S3-1	250	67.4	350	297	2.49	1232	450	0.00101	632	12.56	209.2	1.4333	1	1	110465.06	110.46506	82088.59	82.08859	192.5536524	1.0864504
S3-2	250	67.4	350	297	2.49	1232	450	0.00101	632	12.56	178	1.4333	1	1	110465.06	110.46506	82088.59	82.08859	192.5536524	0.9244177
S3-3	250	67.4	350	293	2.49	2046	452	0.00101	632	12.56	228.6	1.4377	1	1	130035.48	130.03548	80983.02	80.98302	211.0185008	1.0833173
S3-4	250	67.4	350	293	2.49	2046	452	0.00101	632	12.56	174.9	1.4377	1	1	130035.48	130.03548	80983.02	80.98302	211.0185008	0.8288373
S3-5	250	67.4	350	299	2.41	2760	442	0.00101	632	12.56	296.6	1.4311	1	1	144966.32	144.96632	82641.375	82.641375	227.6076926	1.3031194
S3-6	250	67.4	350	299	2.41	2760	442	0.00101	632	12.56	282.9	1.4311	1	1	144966.32	144.96632	82641.375	82.641375	227.6076926	1.2429281
S4-1	250	87.3	600	542	2.4	4092	452	0.00157	569	19.63	354	1.1638	1	1	217848.47	217.84847	209652.04	209.65204	427.5005103	0.8280692
S4-2	250	87.3	500	444	2.41	3284	433	0.00157	569	19.63	572.8	1.2716	1	1	193659.28	193.65928	171744.48	171.74448	365.4037545	1.567581
S4-3	250	87.3	400	346	2.4	2464	450	0.00157	569	19.63	243.4	1.3794	1	1	161653.81	161.65381	133836.91	133.83691	295.490723	0.8237145
S4-4	250	87.3	350	292	2.5	2046	452	0.00157	569	19.63	258.1	1.4388	1	1	141532.32	141.53232	112949.07	112.94907	254.4813923	1.0142195
S4-6	250	87.3	250	198	2.53	1380	442	0.00157	569	19.63	202.9	1.5422	1	1	102685.38	102.68538	76588.754	76.588754	179.2741327	1.1317863
S5-1	250	89.4	350	292	3.01	2046	452	0.00157	569	19.63	241.7	1.4388	1	1	142658.19	142.65819	112949.07	112.94907	255.6072649	0.9455913
S5-2	250	89.4	350	292	2.74	2046	452	0.00157	569	19.63	259.9	1.4388	1	1	142658.19	142.65819	112949.07	112.94907	255.6072649	1.0167943
S5-3	250	89.4	350	292	2.5	2046	452	0.00157	569	19.63	243.8	1.4388	1	1	142658.19	142.65819	112949.07	112.94907	255.6072649	0.953807
S5-4	250	89.4	350	292	1.99	2046	452	0.00157	569	19.63	476.7	1.4388	1	1	142658.19	142.65819	112949.07	112.94907	255.6072649	1.8649705
S5-5	250	89.4	350	292	1.75	2046	452	0.00157	569	19.63	573.4	1.4388	1	1	142658.19	142.65819	112949.07	112.94907	255.6072649	2.2432852
S6-3	250	68.9	350	292	2.73	2046	452	0.00101	632	12.56	178.4	1.4388	1	1	130794.84	130.79484	80706.628	80.706628	211.5014701	0.843493
S6-4	250	68.9	350	292	2.73	2046	452	0.00101	632	12.56	214.4	1.4388	1	1	130794.84	130.79484	80706.628	80.706628	211.5014701	1.0137045
S6-5	250	68.9	350	292	2.64	2760	442	0.00101	632	12.56	297	1.4388	1	1	144519.05	144.51905	80706.628	80.706628	225.2256795	1.3186773
S6-6	250	68.9	350	292	2.64	2760	442	0.00101	632	12.56	287.2	1.4388	1	1	144519.05	144.51905	80706.628	80.706628	225.2256795	1.2751654
S7-1	250	74.8	350	292	3.3	3284	433	0.00105	569	19.63	217.2	1.4388	1	1	157392.46	157.39246	75539.188	75.539188	232.9316463	0.9324624
S7-2	250	74.8	350	292	3.3	3284	433	0.00126	569	19.63	205.4	1.4388	1	1	157392.46	157.39246	90647.026	90.647026	248.0394839	0.828094
S7-3	250	74.8	350	292	3.3	3284	433	0.00157	569	19.63	246.5	1.4388	1	1	157392.46	157.39246	112949.07	112.94907	270.3415299	0.9118096
S7-4	250	74.8	350	292	3.3	3284	433	0.00196	569	19.63	273.6	1.4388	1	1	157392.46	157.39246	141006.48	141.00648	298.3989427	0.9168933
S7-5	250	74.8	350	292	3.3	3284	433	0.00224	569	19.63	304.4	1.4388	1	1	157392.46	157.39246	161150.27	161.15027	318.5427262	0.9556018
S7-6	250	74.8	350	292	3.3	3284	433	0.00262	569	19.63	310.6	1.4388	1	1	157392.46	157.39246	188488.26	188.48826	345.8807181	0.8979974
S8-1	250	74.6	350	292	2.5	2046	452	0.00105	569	19.63	272.1	1.4388	1	1	134306.54	134.30654	75539.188	75.539188	209.8457263	1.2966669
S8-2	250	74.6	350	292	2.5	2046	452	0.00126	569	19.63	250.9	1.4388	1	1	134306.54	134.30654	90647.026	90.647026	224.9535639	1.1153413
S8-3	250	74.6	350	292	2.5	2046	452	0.00157	569	19.63	309.6	1.4388	1	1	134306.54	134.30654	112949.07	112.94907	247.25561	1.2521455
S8-4	250	74.6	350	292	2.5	2046	452	0.00157	569	19.63	265.8	1.4388	1	1	134306.54	134.30654	112949.07	112.94907	247.25561	1.0750009
S8-5	250	74.6	350	292	2.5	2046	452	0.00196	569	19.63	289.2	1.4388	1	1	134306.54	134.30654	141006.48	141.00648	275.3130227	1.0504407
S8-6	250	74.6	350	292	2.5	2046	452	0.00224	569	19.63	283.9	1.4388	1	1	134306.54	134.30654	161150.27	161.15027	295.4568063	0.960885

Table D.1 Predicted shear strength for beams tested by Kong & Rangan (1998)

Mark K. Johnson and Julio A. Ramirez (1989) - Beams with Stirrups

Beam No	*b* *mm*	*f'c* *Mpa*	*D* *mm*	*d* *mm*	d_0 *mm*	*a/d*	*Ast* mm^2	*fy*	*ρt* *(MPa)*	*fsty.f* *(MPa)*	*spacing(s)* *mm*	*Ult.Sh strength* V_e *(kN)* *(Measured)*	β_1	β_2	β_3	V_{uc} *(N)* *Predicted*	V_{uc} *(kN)* *Predicted*	V_{us} *(N)*	V_{us} *(kN)*	$V_p = V_{uc} + V_{us}$ *(kN)*	V_e / V_p
1	305	36.4	610	539	562	2.97	3960	525	0.00156	479	1333	338.9	1.1418	1	1	184733.7864	184.7337864	221848.703	221.8487	406.5824894	0.83353319
2	305	36.4	610	539	562	2.97	3960	525	0.00078	479	262	222.16	1.1418	1	1	184733.7864	184.7337864	110924.3515	110.92435	295.6581379	0.75140837
3	305	72.3	610	539	562	2.97	3960	525	0.00078	479	267	262.97	1.1418	1	1	232215.909	232.215909	110924.3515	110.92435	343.1402605	0.766363
4	305	72.3	610	539	562	2.97	3960	525	0.00078	479	267	316.24	1.1418	1	1	232215.909	232.215909	110924.3515	110.92435	343.1402605	0.92160564
5	305	55.8	610	539	562	2.97	3960	525	0.00156	479	133	383.12	1.1418	1	1	213005.3951	213.0053951	221848.703	221.8487	434.8540981	0.88103114
7	305	51.3	610	539	562	2.97	3960	525	0.00078	479	267	281.1	1.1418	1	1	207118.2298	207.1182298	110924.3515	110.92435	318.0425813	0.88384391
8	305	51.3	610	539	562	2.97	3960	525	0.00078	479	267	258.43	1.1418	1	1	207118.2298	207.1182298	110924.3515	110.92435	318.0425813	0.81256415

Table D.2 Predicted shear strength for beams tested by Johnson & Ramirez (1989)

A. Cladera & A. R. Mari (2005)- Beams with Stirrups

Specimen	*b or bv*	*f'c Mpa*	*D mm*	*d mm*	*a/d*	*ρ_l (%)*	*A_{st} mm^2*	*$\rho_w f_y$ (MPa)*	*$f_{sty.f}$ (MPa)*	*A_{sv}*	*Ult.Sh strength V_e (kN) (Measured)*	*β_1*	*β_2*	*β_3*	*V_{uc} (N) Predicted*	*V_{uc} (kN) Predicted*	*V_{us} (N)*	*V_{us} (kN)*	*$V_p = V_{uc} + V_{us}$ (kN)*	*V_e/V_p*
H50/2	200	49.9	400	353	3.06	2.28	1709.03	0.605	530	28.27	177.64	1.3717	1	1	103136.4907	103.1364907	73981.086	73.981086	177.1175768	0.5823052
H50/3	200	49.9	400	351	0.08	2.29	1709.03	1.291	540	50.27	242.07	1.3739	1	1	102911.3501	102.9113501	156972.65	156.97265	259.8839971	0.3959896
H50/4	200	49.9	400	351	3.8	2.99	2199.9	1.291	540	50.27	246.34	1.3739	1	1	111947.4899	111.9474899	156972.65	156.97265	268.9201369	0.4162853
H60/2	200	60.8	400	353	0	2.28	1709.03	0.747	530	28.27	179.74	1.3717	1	1	110157.3199	110.1573199	91345.242	91.345242	201.5025618	0.5466795
H60/3	200	60.8	400	351	3.08	2.29	1709.03	1.374	530	50.27	258.78	1.3739	1	1	109916.8533	109.9168533	167064.61	167.06461	276.9814675	0.3968383
H60/4	200	60.8	400	351	3.08	2.99	2199.9	1.374	530	50.27	308.71	1.3739	1	1	119568.1119	119.5681119	167064.61	167.06461	286.6327261	0.4171475
H75/2	200	68.9	400	353	3.06	2.28	1709.03	0.783	530	50.27	203.94	1.3717	1	1	114846.7145	114.8467145	95747.422	95.747422	210.5941367	0.5453462
H75/3	200	68.9	400	351	3.08	2.29	1709.03	1.362	530	50.27	269.35	1.3739	1	1	114596.0112	114.5960112	165605.53	165.60553	280.2015458	0.4089771
H75/4	200	68.9	400	351	3.08	2.99	2199.9	1.362	530	50.27	255.23	1.3739	1	1	124658.1236	124.6581236	165605.53	165.60553	290.2636582	0.4294651
H100/2	200	87	400	353	3.06	2.28	1709.03	0.929	530	28.27	225.55	1.3717	1	1	124132.4286	124.1324286	113600.71	113.60071	237.7331378	0.5221503
H100/3	200	87	400	351	3.08	2.29	1709.03	1.355	540	50.27	253.64	1.3739	1	1	123861.4552	123.8614552	164754.4	164.7544	288.61586	0.4291568
H100/4	200	87	400	351	3.08	2.99	2199.9	1.353	540	50.27	266.53	1.3739	1	1	134737.1207	134.7371207	164511.22	164.51122	299.2483456	0.4502518

Table D.3 Predicted shear strength for beams tested by Cladera & Mari (2005)

Khaldoun N. Rahal and Khaled S. Al-Shaleh (2004)- Beams with Stirrups

Series	*Specimen*	*b or bv*	*f'c*	*D*	*d*	*a*	*a/d*	*Ast* mm^2	*fy* *MPa*	$\rho_y . f_y$ *(MPa)*	*spacing (s)* *(mm)*	A_{sv}	$f_{sty} . f$ *(MPa)*	*Ult.Sh strength* V_e *(kN)* *(Measured)*	$\beta_1 \geq 1$	β_2	β_3	V_{uc} *(N)* *Predicted*	V_{uc} *(kN)* *Predicted*	V_{us} *(N)*	V_{us} *(kN)*	$V_p = V_{uc} + V_{us}$ *(kN)*	V_e/V_p
Series A	A65-NTR	200	61.3	370	330	900	2.72	1425	440	0	∞	28.3	240	125	1.397	1	1	101231.92	101.23192	0	0	101.2319205	1.2347884
	A65-200	200	60.9	370	325	900	2.76	1425	440	0.339	200	28.3	240	175	1.4025	1	1	100382	100.382	38164.62	38.16462	138.5466159	1.2631128
	A65-140	200	62.1	370	325	900	2.76	1425	440	0.485	140	28.3	240	150	1.4025	1	1	101037.04	101.03704	54601.3	54.6013	155.6383357	0.9637728
	A65-110	200	60.9	370	325	900	2.76	1425	440	0.617	110	28.3	240	188	1.4025	1	1	100382	100.382	69461.86	69.46186	169.8438559	1.106899
	A65-95	200	62.1	370	325	900	2.76	1425	440	0.714	95	28.3	240	220	1.4025	1	1	101037.04	101.03704	80382.12	80.38212	181.4191557	1.2126614
Series B	B65-NTR	200	61.9	370	305	900	2.95	2375	440	0	∞	24	305	170	1.4245	1	1	116502.1	116.5021	0	0	116.5021029	1.4592011
	B65-200	200	64.3	370	300	900	3	2375	440	0.366	200	24	305	195	1.43	1	1	117146.25	117.14625	38034.72	38.03472	155.1809725	1.2565974
	B65-160	200	65.1	370	300	900	3	2375	440	0.458	160	24	305	208	1.43	1	1	117630.08	117.63008	47595.36	47.59536	165.2254435	1.258886
	B65-140	200	65.1	370	300	900	3	2375	440	0.523	140	24	305	235	1.43	1	1	117630.08	117.63008	54350.16	54.35016	171.9802435	1.366436
	B65-125	200	66.4	370	300	900	3	2375	440	0.586	125	24	305	242	1.43	1	1	118407.92	118.40792	60897.12	60.89712	179.3050445	1.3496553
	B65-110	200	66.4	370	300	900	3	2375	440	0.665	110	24	305	270	1.43	1	1	118407.92	118.40792	69106.8	69.1068	187.5147245	1.4398869

Table D.4 Predicted shear strength for beams tested by Rahal & Al-Shaleh (2004)

Kaiss F. Sarsam and Janan M. S. Al-Musawi (1992)- Beams with Stirrups

Specimen	*b or bv*	*f'c Mpa*	*D mm*	*d mm*	*a/d*	*ρl*	*Ast mm^2*	*$f_{sl.y}$ MPa*	*Transverse Steel ρt=Asv/bv.s*	*fsty.f (MPa)*	*$\rho_v f_y$ N/mm^2*	*Ult.Sh strength (kN) (Measured)*	*β_1*	*β_2*	*β_3*	*V_{uc} (N) Predicted*	*V_{uc} (kN) Predicted*	*V_{us} (N)*	*V_{us} (kN)*	*$V_p = V_{uc} + V_{us}$ (kN)*	*V_e/V_p*
AL2-N	180	40.4	270	235	4	0.0223	943	495	0.00093	820	0.76	114.7	1.5015	1	1	61336.162	61.336162	55681.969	55.681969	117.0181318	0.98019
AL2-H	180	75.3	270	235	4	0.0223	943	495	0.00093	820	0.76	122.6	1.5015	1	1	75483.924	75.483924	55681.969	55.681969	131.1658938	0.9346942
AS2-N	180	39	270	235	2.5	0.0223	943	495	0.00093	820	0.76	189.3	1.5015	1	1	60619.314	60.619314	55681.969	55.681969	116.301283	1.6276691
AS2-H	180	75.5	270	232	2.5	0.0226	943	495	0.00093	820	0.76	201	1.5048	1	1	75070.964	75.070964	54971.136	54.971136	130.0420993	1.5456533
AS3-N	180	40.2	270	235	2.5	0.0223	943	495	0.0014	820	1.14	199.1	1.5015	1	1	61234.78	61.23478	83522.954	83.522954	144.7577343	1.3754015
AS3-H	180	71.8	270	235	2.5	0.0223	943	495	0.0014	820	1.14	199.1	1.5015	1	1	74295.805	74.295805	83522.954	83.522954	157.8187592	1.2615737
BL2-H	180	75.7	270	233	4	0.0282	1181	540	0.00093	820	0.76	138.3	1.5037	1	1	81163.745	81.163745	55208.08	55.20808	136.371825	1.0141391
BS2-H	180	73.9	270	233	2.5	0.0282	1181	540	0.00093	820	0.76	223.5	1.5037	1	1	80515.272	80.515272	55208.08	55.20808	135.7233519	1.6467321
BS3-H	180	73.4	270	233	2.5	0.0282	1181	540	0.0014	820	1.14	228.1	1.5037	1	1	80333.274	80.333274	82812.12	82.81212	163.1453949	1.3981394
BS4-H	180	80.1	270	233	2.5	0.0282	1181	540	0.00186	820	1.53	206.9	1.5037	1	1	82706.75	82.70675	111142.58	111.14258	193.8493326	1.0673238
CL2-H	180	70.1	270	233	4	0.0351	1470	543	0.00093	820	0.76	147.2	1.5037	1	1	85099.159	85.099159	55208.08	55.20808	140.3072392	1.0491262
CS2-H	180	70.2	270	233	2.5	0.0351	1470	543	0.00093	820	0.76	247.2	1.5037	1	1	85139.605	85.139605	55208.08	55.20808	140.3476856	1.7613401
CS3-H	180	74.2	270	233	2.5	0.0351	1470	543	0.0014	820	1.14	247.2	1.5037	1	1	86726.915	86.726915	82812.12	82.81212	169.5390352	1.4580713
CS4-H	180	75.7	270	233	2.5	0.0351	1470	543	0.00186	820	1.53	220.7	1.5037	1	1	87307.434	87.307434	111142.58	111.14258	198.4500161	1.1121188

Table D.5 Predicted shear strength for beams tested by Sarsam & Al-Musawi (1992)

John J. Roller and Henry G. Russell (1990)- Beams with Stirrups

Specimen	*b or* b_v	*f'c Mpa*	*D mm*	*d mm*	d_0 *mm*	*a/d*	*Ast* mm^2	f_y *MPa*	*Transverse Steel* $\rho t = A_{sv}/b_v.s$	$f_{sty.f}$ *(MPa)*	*spacing(s) mm*	*Ult.Sh strength* V_e *(kN) (Measured)*	β_1	β_2	β_3	V_{uc} *(N) Predicted*	V_{uc} *(kN) Predicted*	*Vus(N)*	*Vus(kN)*	$V_p = V_{uc} + V_{us}$ *(kN)*	V_e/V_p
1	356	120.1	635	559	559	2.5	3180	472	0.00074	407	216	297.8	1.1451	1	1	283187.8792	283.1878792	103812.24	103.81224	387.0001192	0.769508807
2	356	120.1	679	559	599	2.33	5740	431	0.00431	448	165	1099.1	1.1011	1	1	347186.5904	347.1865904	713169.4514	713.1694514	1060.356042	1.036538631
3	356	120.1	718	559	635	2.2	8610	431	0.00878	458	127	1657.5	1.0615	1	1	398337.5064	398.3375064	1574506.089	1574.506089	1972.843595	0.840157833
4	356	120.1	718	559	635	2.2	11490	431	0.01255	458	89	1942.9	1.0615	1	1	438554.4203	438.5544203	2250575.332	2250.575332	2689.129753	0.722501396
5	356	120.1	743	559	660	2.12	13370	460	0.01757	458	64	2237.9	1.034	1	1	461042.8586	461.0428586	3274852.925	3274.852925	3735.895783	0.599026346
6	457	72.4	870	762	793	2.88	5740	464	0.00081	445	381	665.1	0.8877	1	1	336725.4673	336.7254673	226253.3637	226.2533637	562.9788311	1.181394332
7	457	72.4	870	762	795	2.88	6360	483	0.00157	445	197	787.6	0.8855	1	1	348157.7059	348.1577059	439646.4994	439.6464994	787.8042052	0.999740792
8	457	125.3	870	762	795	2.88	6360	483	0.00081	445	381	482.6	0.8855	1	1	418003.7522	418.0037522	226823.9901	226.8239901	644.8277423	0.748416931
9	457	125.3	870	762	795	2.88	7940	483	0.00157	445	197	749.1	0.8855	1	1	450092.0159	450.0920159	439646.4994	439.6464994	889.7385153	0.841932756
10	457	125.3	870	762	793	2.88	9560	464	0.00233	445	133	1171.7	0.8877	1	1	479213.2298	479.2132298	650827.5771	650.8275771	1130.040807	1.036865211

Table D.6 Predicted shear strength for beams tested by Roller & Russell (1990)

Shuaib H.Ahmed, A.R. Khaloo, and A. Poveda (1986)---Beams without stirrups

Specimen	*b(or bv)*	*f'_c Mpa*	*D mm*	*d inches*	*d(or do) mm*	*a/d*	*steel content ρ*	*ult.sh.stress (measured) V_{uc} (psi)*	*ult.sh.stress (measured) V_{uc} (MPa)*	*Ult.Sh.Strength (measured) V_e (kN)*	*Ast mm^2*	*$\beta_1 \geq 1.1$*	*β_2*	*β_3*	*V_p predicted (N)*	*V_p predicted (kN)*
A1	130	66.12	250	8	203.2	4	3.93	325	2.24	72.83	1038.15	1.53648	1	1	55800.2	55.8002
A2	130	66.12	250	8	203.2	3	3.93	387.5	2.67	86.83	1038.15	1.53648	1	1	55800.2	55.8002
A3	130	66.12	250	8	203.2	2.7	3.93	387.5	2.67	86.83	1038.15	1.53648	1	1	55800.2	55.8002
A4	130	66.12	250	8	203.2	2.3	3.93	525	3.62	117.65	1038.15	1.53648	1	1	55800.2	55.8002
A5	130	66.12	250	8	203.2	2	3.93	937.5	6.46	210.08	1038.15	1.53648	1	1	55800.2	55.8002
A6	130	66.12	250	8	203.2	1	3.93	2250	15.51	504.20	1038.15	1.53648	1	1	55800.2	55.8002
A7	130	66.12	250	8.19	208.026	4	1.77	256.4	1.77	57.46	478.67	1.531171	1	1	43636.86	43.63686
A8	130	66.12	250	8.19	208.026	3	1.77	268.6	1.85	60.19	478.67	1.531171	1	1	43636.86	43.63686
A9	130	66.12	250	8.19	208.026	2.7	1.77	439.6	3.03	98.51	478.67	1.531171	1	1	43636.86	43.63686
A10	130	66.12	250	8.19	208.026	2.3	1.77	451.8	3.12	101.24	478.67	1.531171	1	1	43636.86	43.63686
A11	130	66.12	250	8.19	208.026	2	1.77	305.3	2.11	68.41	478.67	1.531171	1	1	43636.86	43.63686
A12	130	66.12	250	8.19	208.026	1	1.77	1221	8.42	273.61	478.67	1.531171	1	1	43636.86	43.63686
B1	130	72.81	250	7.94	201.676	4	5.04	290	2.00	64.99	1321.38	1.538156	1	1	62202.13	62.20213
B2	130	72.81	250	7.94	201.676	3	5.04	390.4	2.69	87.48	1321.38	1.538156	1	1	62202.13	62.20213
B3	130	72.81	250	7.94	201.676	2.7	5.04	566.8	3.91	127.01	1321.38	1.538156	1	1	62202.13	62.20213
B4	130	72.81	250	7.94	201.676	2.3	5.04	811.1	5.59	181.76	1321.38	1.538156	1	1	62202.13	62.20213
B5	130	72.81	250	7.94	201.676	2	5.04	604.5	4.17	135.46	1321.38	1.538156	1	1	62202.13	62.20213
B6	130	72.81	250	7.94	201.676	1	5.04	1163.7	8.02	260.77	1321.38	1.538156	1	1	62202.13	62.20213
B7	130	72.81	250	8.19	208.026	4	2.25	245	1.69	54.90	608.48	1.531171	1	1	48813.8	48.8138
B8	130	72.81	250	8.19	208.026	3	2.25	256.4	1.77	57.46	608.48	1.531171	1	1	48813.8	48.8138
B9	130	72.81	250	8.19	208.026	2.7	2.25	439.6	3.03	98.51	608.48	1.531171	1	1	48813.8	48.8138
B10	130	72.81	250	8.19	208.026	2.3	2.25	351.6	2.42	78.79	608.48	1.531171	1	1	48813.8	48.8138
B11	130	72.81	250	8.19	208.026	2	2.25	671.6	4.63	150.50	608.48	1.531171	1	1	48813.8	48.8138
B12	130	72.81	250	8.19	208.026	1	2.25	1172.2	8.08	262.68	608.48	1.531171	1	1	48813.8	48.8138
C1	130	69.91	250	7.25	184.15	4	6.64	336.6	2.32	75.43	1589.58	1.557435	1	1	62195.76	62.19576
C2	130	69.91	250	7.25	184.15	3	6.64	469	3.23	105.10	1589.58	1.557435	1	1	62195.76	62.19576
C3	130	69.91	250	7.25	184.15	2.7	6.64	427.6	2.95	95.82	1589.58	1.557435	1	1	62195.76	62.19576
C4	130	69.91	250	7.25	184.15	2.3	6.64	551.7	3.80	123.63	1589.58	1.557435	1	1	62195.76	62.19576
C5	130	69.91	250	7.25	184.15	2	6.64	1531	10.56	343.08	1589.58	1.557435	1	1	62195.76	62.19576
C6	130	69.91	250	7.25	184.15	1	6.64		0.00	0.00	1589.58	1.557435	1	1	62195.76	62.19576
C7	130	69.91	250	8.13	206.502	4	3.26	251	1.73	56.25	875.16	1.532848	1	1	54152.55	54.15255
C8	130	69.91	250	8.13	206.502	3	3.26	246	1.70	55.13	875.16	1.532848	1	1	54152.55	54.15255
C9	130	69.91	250	8.13	206.502	2.7	3.26	251	1.73	56.25	875.16	1.532848	1	1	54152.55	54.15255
C10	130	69.91	250	8.13	206.502	2.3	3.26	314.9	2.17	70.57	875.16	1.532848	1	1	54152.55	54.15255
C11	130	69.91	250	8.13	206.502	2	3.26	590.4	4.07	132.30	875.16	1.532848	1	1	54152.55	54.15255
C12	130	69.91	250	8.13	206.502	1	3.26	1353	9.33	303.19	875.16	1.532848	1	1	54152.55	54.15255

Table D.6 shear Strength predicted as per AS3600-01, for beams tested by Ahmed et al (1986)

Andrew G. Mphonde and Gregory C. Frantz (1984)-Beams without stirrups

Specimen	*b(or b_v)*		*f'_c Mpa*	*D mm*	*d mm*	*a/d*	*steel content ρ_l*	*ult.sh.stress (measured) V_{uc} (psi)*	*ult.sh.stress (measured) V_{uc} (MPa)*	*Ult.Sh.Strength (measured) V_e (kN)*	*Ast mm^2*	*$\beta_1 \geq 1.1$*	*β_2*	*β_3*	*V_p predicted (N)*	*V_p predicted (kN)*
AO-11-3a	152	10867	74.93	337	299	3.6	3.36	286	1.97	101.01	1527.05	1.4311	1	1	88480.15	88.48015
AO-11-3b	152	10825	74.64	337	299	3.6	3.36	285	1.97	100.66	1527.05	1.4311	1	1	88366.01	88.36601
AO-15-3a	152	11797	81.34	337	299	3.6	3.36	298	2.05	105.25	1527.05	1.4311	1	1	90935.43	90.93543
AO-15-3b	152	13587	93.68	337	299	3.6	3.36	319	2.20	112.67	1527.05	1.4311	1	1	95319.95	95.31995
AO-15-3c	152	13319	91.83	337	299	3.6	3.36	312	2.15	110.20	1527.05	1.4311	1	1	94689.06	94.68906
AO-11-2	152	11498	79.28	337	299	2.5	3.36	355	2.45	125.38	1527.05	1.4311	1	1	90160.58	90.16058
AO-15-2a	152	12148	83.76	337	299	2.5	3.36	567	3.91	200.26	1527.05	1.4311	1	1	91828.51	91.82851
AO-15-2b	152	10065	69.40	337	299	2.5	3.36	656	4.52	231.69	1527.05	1.4311	1	1	86247.64	86.24764
AO-11-1	152	9535	65.74	337	299	1.5	3.36	1379	9.51	487.05	1527.05	1.4311	1	1	84706.39	84.70639
AO-15-1a	152	11524	79.46	337	299	1.5	3.36	879	6.06	310.45	1527.05	1.4311	1	1	90228.49	90.22849
AO-15-1b	152	11785	81.26	337	299	1.5	3.36	1578	10.88	557.33	1527.05	1.4311	1	1	90904.59	90.90459

Only beams with concrete compressive strength higher than 50 MPa are considered.

Table D.7 Shear strength predicted as per AS3600-01, for beams tested by Mphonde and Frantz (1984)

APPENDIX E

$f'c$	$a//d$	ρ_l	ρ_t	D	V_p
63.6	2.50	0.0280274	0.0015700	350	127.351
63.6	2.50	0.0280274	0.0015700	350	127.351
63.6	2.50	0.0280274	0.0015700	350	127.351
63.6	2.50	0.0280274	0.0015700	350	127.351
63.6	2.50	0.0280274	0.0015700	350	127.351
63.6	2.50	0.0280274	0.0015700	350	127.351
72.5	2.50	0.0280274	0.0010500	350	133.034
72.5	2.50	0.0280274	0.0012600	350	133.034
72.5	2.50	0.0280274	0.0015700	350	133.034
72.5	2.50	0.0280274	0.0015700	350	133.034
72.5	2.50	0.0280274	0.0020900	350	133.034
67.4	2.49	0.0165926	0.0010100	350	110.465
67.4	2.49	0.0165926	0.0010100	350	110.465
67.4	2.49	0.0279317	0.0010100	350	130.035
67.4	2.49	0.0279317	0.0010100	350	130.035
67.4	2.41	0.0369231	0.0010100	350	144.966
67.4	2.41	0.0369231	0.0010100	350	144.966
87.3	2.40	0.0301993	0.0015700	600	217.848
87.3	2.41	0.0295856	0.0015700	500	193.659
87.3	2.40	0.0284855	0.0015700	400	161.654
87.3	2.50	0.0280274	0.0015700	350	141.532
87.3	2.53	0.0278788	0.0015700	250	102.685
89.4	3.01	0.0280274	0.0015700	350	142.658
89.4	2.74	0.0280274	0.0015700	350	142.658
89.4	2.50	0.0280274	0.0015700	350	142.658
89.4	1.99	0.0280274	0.0015700	350	142.658
89.4	1.75	0.0280274	0.0015700	350	142.658
68.9	2.73	0.0280274	0.0010100	350	130.795
68.9	2.73	0.0280274	0.0010100	350	130.795
68.9	2.64	0.0378082	0.0010100	350	144.519
68.9	2.64	0.0378082	0.0010100	350	144.519
74.8	3.30	0.0449863	0.0010500	350	157.392
74.8	3.30	0.0449863	0.0012600	350	157.392
74.8	3.30	0.0449863	0.0015700	350	157.392
74.8	3.30	0.0449863	0.0019600	350	157.392
74.8	3.30	0.0449863	0.0022400	350	157.392
74.8	3.30	0.0449863	0.0026200	350	157.392
74.6	2.50	0.0280274	0.0010500	350	134.307
74.6	2.50	0.0280274	0.0012600	350	134.307
74.6	2.50	0.0280274	0.0015700	350	134.307
74.6	2.50	0.0280274	0.0015700	350	134.307
74.6	2.50	0.0280274	0.0019600	350	134.307
74.6	2.50	0.0280274	0.0022400	350	134.307
61.3	2.72	0.0215909	0.0000000	370	101.232
60.9	2.76	0.0219231	0.0007075	370	100.382
62.1	2.76	0.0219231	0.0010107	370	101.037
60.9	2.76	0.0219231	0.0012864	370	100.382
62.1	2.76	0.0219231	0.0014895	370	101.037
61.9	2.95	0.0389344	0.0000000	370	116.502
64.3	3.00	0.0395833	0.0006000	370	117.146

Table E.1 Shear data for regression analysis (Beams with stirrups)

$f'c$	$a//d$	ρ_l	ρ_t	D	V_p
65.1	3.00	0.0395833	0.0007500	370	117.630
65.1	3.00	0.0395833	0.0008571	370	117.630
66.4	3.00	0.0395833	0.0009600	370	118.408
66.4	3.00	0.0395833	0.0010909	370	118.408
49.9	3.01	0.0224025	0.0000000	400	101.724
49.9	3.06	0.0242072	0.0011415	400	103.136
49.9	0.08	0.0243452	0.0023907	400	102.911
49.9	3.80	0.0313376	0.0023907	400	111.947
60.8	3.01	0.0224025	0.0000000	400	108.648
60.8	0.00	0.0242072	0.0014094	400	110.157
60.8	3.08	0.0243452	0.0025925	400	109.917
60.8	3.08	0.0313376	0.0025925	400	119.568
68.9	3.01	0.0224025	0.0000000	400	113.273
68.9	3.06	0.0242072	0.0014774	400	114.847
68.9	3.08	0.0243452	0.0025698	400	114.596
68.9	3.08	0.0313376	0.0025698	400	124.658
87.0	3.01	0.0224025	0.0000000	400	122.432
87.0	3.06	0.0242072	0.0017528	400	124.132
87.0	3.08	0.0243452	0.0025093	400	123.861
87.0	3.08	0.0313376	0.0025056	400	134.737
40.4	4.00	0.0223000	0.0009300	270	61.336
75.3	4.00	0.0223000	0.0009300	270	75.484
39.0	2.50	0.0223000	0.0009300	270	60.619
75.5	2.50	0.0226000	0.0009300	270	75.071
40.2	2.50	0.0223000	0.0014000	270	61.235
71.8	2.50	0.0223000	0.0014000	270	74.296
75.7	4.00	0.0282000	0.0009300	270	81.164
73.9	2.50	0.0282000	0.0009300	270	80.515
73.4	2.50	0.0282000	0.0014000	270	80.333
80.1	2.50	0.0282000	0.0018600	270	82.707
70.1	4.00	0.0351000	0.0009300	270	85.099
70.2	2.50	0.0351000	0.0009300	270	85.140
74.2	2.50	0.0351000	0.0014000	270	86.727
75.7	2.50	0.0351000	0.0018600	270	87.307
120.1	2.50	0.0159796	0.0007400	635	283.188
120.1	2.33	0.0288436	0.0043100	679	347.187
120.1	2.20	0.0432655	0.0087800	718	398.338
120.1	2.20	0.0577375	0.0125500	718	438.554
120.1	2.12	0.0671846	0.0175700	743	461.043
72.4	2.88	0.0164832	0.0008100	870	336.725
72.4	2.88	0.0182636	0.0015700	870	348.158
125.3	2.88	0.0182636	0.0008100	870	418.004
125.3	2.88	0.0228008	0.0015700	870	450.092
125.3	2.88	0.0274528	0.0023300	870	479.213
36.4	2.97	0.0240883	0.0015600	610	184.734
36.4	2.97	0.0240883	0.0007800	610	184.734
72.3	2.97	0.0240883	0.0007800	610	232.216
72.3	2.97	0.0240883	0.0007800	610	232.216
55.8	2.97	0.0240883	0.0015600	610	213.005
51.3	2.97	0.0240883	0.0007800	610	207.118
51.3	2.97	0.0240883	0.0007800	610	207.118

Table E.1 Shear data for regression analysis (Beams with stirrups), contd.

Graphs generated from regression analysis-Beams with stirrups

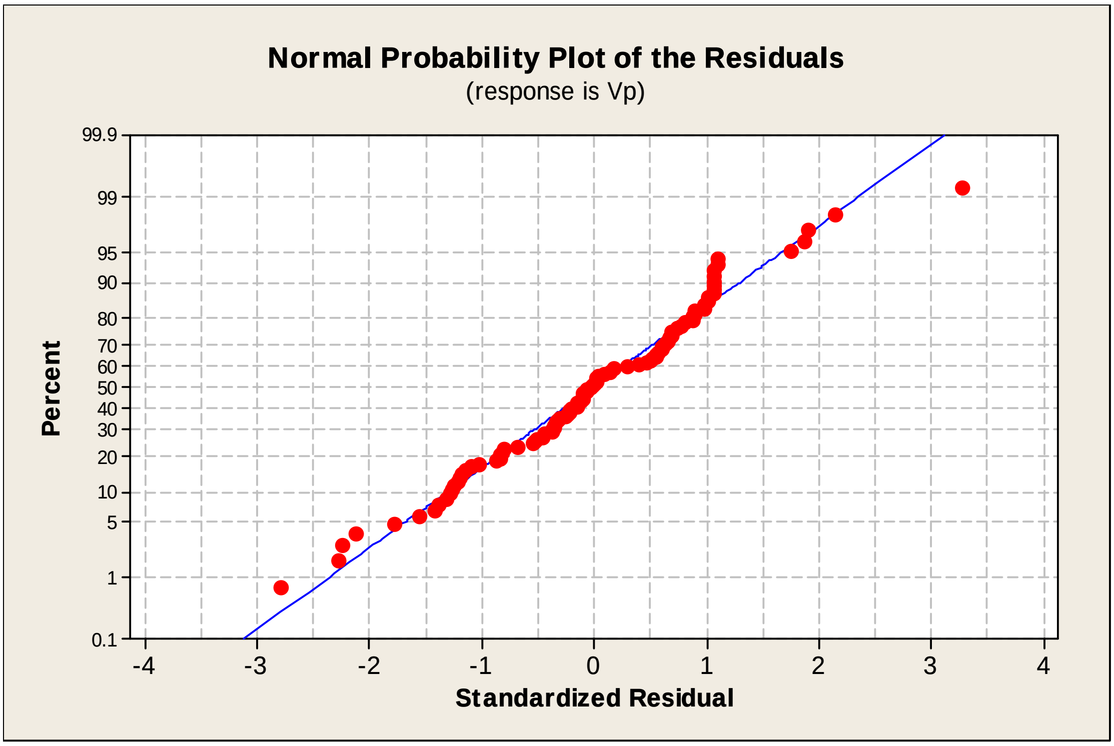


Fig. E.1 Normal Probability Plot

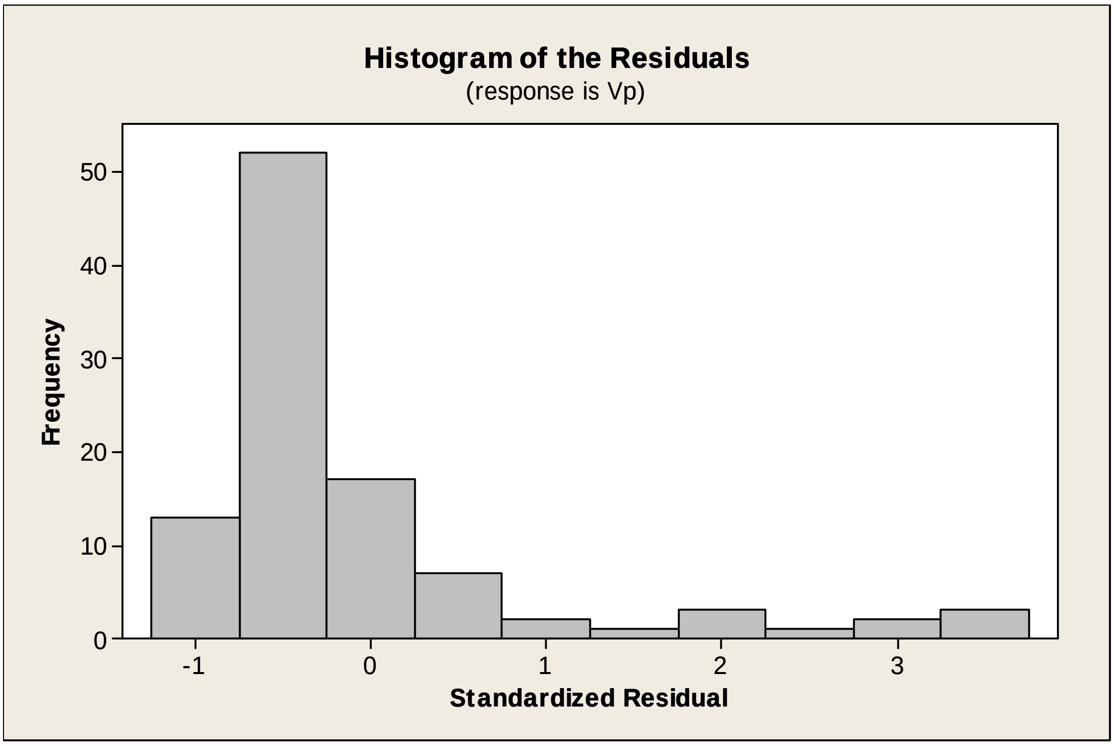


Fig. E.2 Histogram of the Residuals

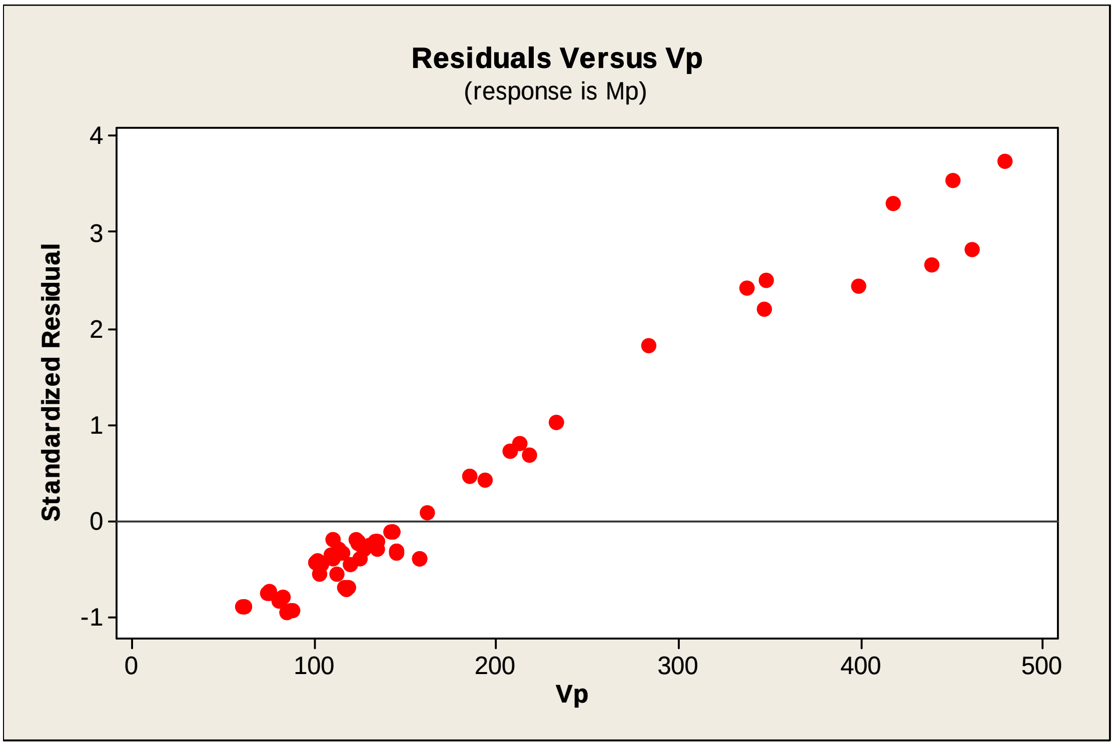


Fig. E.3 Residuals vs. V_p

f'c	***D***	***a/d***	ρ_l	V_p
66.12	250	4.0	3.93	55.8002
66.12	250	3.0	3.93	55.8002
66.12	250	2.7	3.93	55.8002
66.12	250	2.3	3.93	55.8002
66.12	250	2.0	3.93	55.8002
66.12	250	1.0	3.93	55.8002
66.12	250	4.0	1.77	43.6369
66.12	250	3.0	1.77	43.6369
66.12	250	2.7	1.77	43.6369
66.12	250	2.3	1.77	43.6369
66.12	250	2.0	1.77	43.6369
66.12	250	1.0	1.77	43.6369
72.81	250	4.0	5.04	62.2021
72.81	250	3.0	5.04	62.2021
72.81	250	2.7	5.04	62.2021
72.81	250	2.3	5.04	62.2021
72.81	250	2.0	5.04	62.2021
72.81	250	1.0	5.04	62.2021
72.81	250	4.0	2.25	48.8138
72.81	250	3.0	2.25	48.8138
72.81	250	2.7	2.25	48.8138
72.81	250	2.3	2.25	48.8138
72.81	250	2.0	2.25	48.8138
72.81	250	1.0	2.25	48.8138
69.91	250	4.0	6.64	62.1958
69.91	250	3.0	6.64	62.1958
69.91	250	2.7	6.64	62.1958
69.91	250	2.3	6.64	62.1958
69.91	250	2.0	6.64	62.1958
69.91	250	1.0	6.64	62.1958
69.91	250	4.0	3.26	54.1526
69.91	250	3.0	3.26	54.1526
69.91	250	2.7	3.26	54.1526
69.91	250	2.3	3.26	54.1526
69.91	250	2.0	3.26	54.1526
69.91	250	1.0	3.26	54.1526
74.93	337	3.6	3.36	88.4801
74.64	337	3.6	3.36	88.3660
81.34	337	3.6	3.36	90.9354
93.68	337	3.6	3.36	95.3199
91.83	337	3.6	3.36	94.6891
79.28	337	2.5	3.36	90.1606
83.76	337	2.5	3.36	91.8285
69.40	337	2.5	3.36	86.2476
65.74	337	1.5	3.36	84.7064
79.46	337	1.5	3.36	90.2285
81.26	337	1.5	3.36	90.9046

Table E.2 Shear data for regression analysis (Beams without stirrups)

Graphs generated from regression analysis-Beams without stirrups

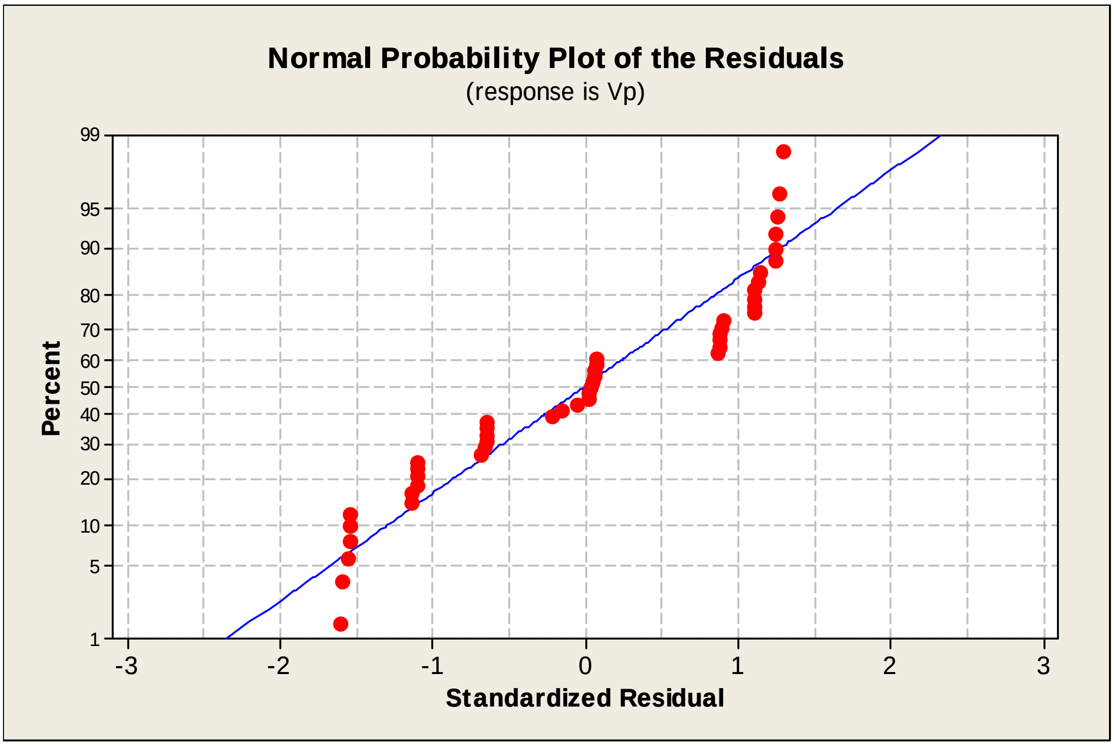


Fig. E.4 Normal Probability Plot

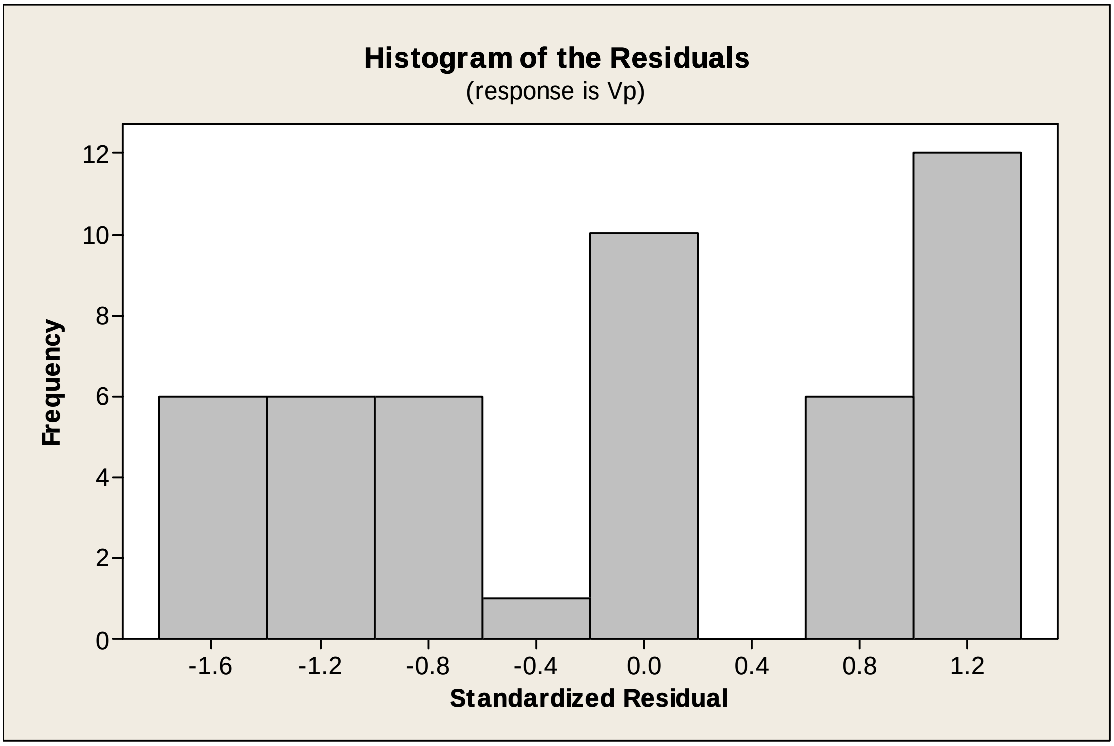


Fig. E.5 Histogram of the Residuals

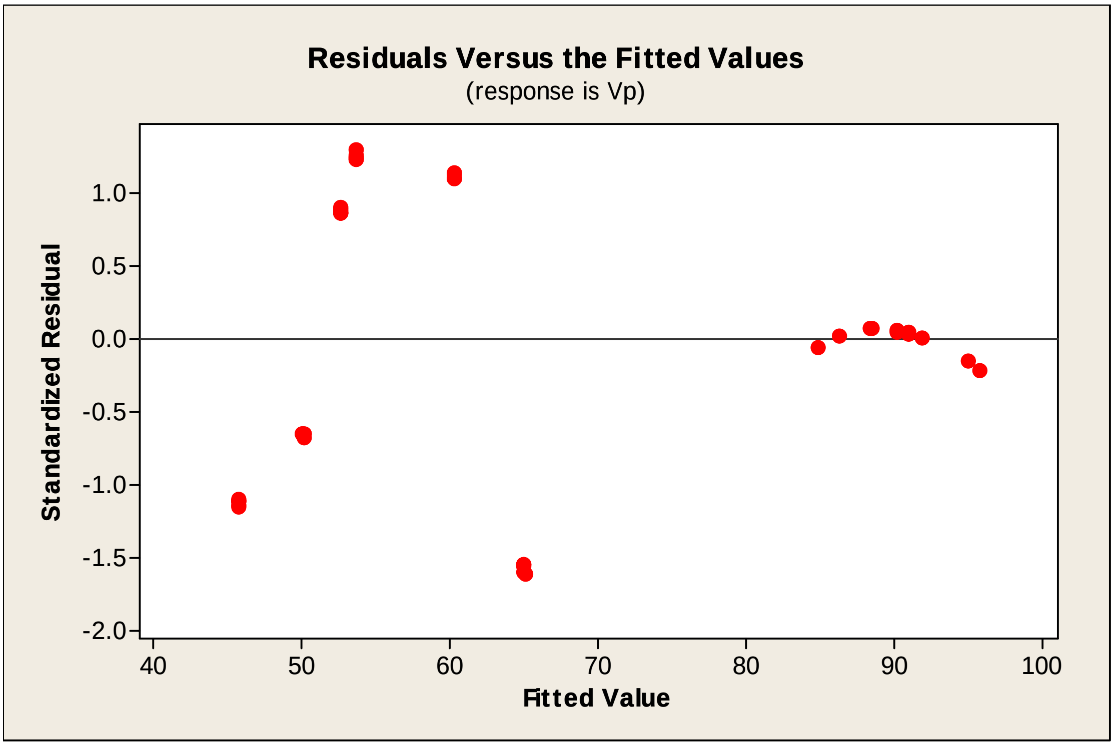


Fig. E.6 Residuals vs. V_p