# AGING STRUCTURE LIFE PREDICTION AND RELIABILITY ASSESSMENT

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## *CANDIDATE STATEMENT*

I hereby declare the genuineness of the following statement:

1. That except where due acknowledgement has been made, the work is that of mine alone;

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3. That the content of the thesis is the result of work that has been carried out since the official commencement date of the approved research proposal;

**Candidate signature**
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**20th June 2008**

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# ABSTRACT

Confront with the serious aging problem in aircraft structure field, the profession was tasked to unveil the mysterious in the mechanism of aging. In decades, many endeavours were put into different subjects such as, fatigue and crack calculation, corrosion analysis, reliability evaluation, life prediction, structure monitor and protection, structure repair, etc.

In an effort of developing a reasonable model for life prediction and reliability evaluation, the author reviewed a wide range of topics in the field of aging structure reliability. Many existing methods and tools are carefully studied to distinguish the advantages, disadvantages and the special application. With consideration of corrosion fatigue life, and based on the data obtained through investigating service status of the aging aircraft, a fuzzy reliability approach is proposed and presented.

Initially, the thesis presents the literature review in the field, introducing the well-established theories and analysis tools of reliability and points out how such these methods can be used to assess the life and reliability of aging structure. Meanwhile, some characteristic parameters and distributions, as well as some crucial calculation formulations, procedures for aging aircraft reliability/risk analysis are given.

Secondly, mathematical models are established to evaluate the initial crack size and to assess both randomness and fuzziness of the variables, which also successfully work out the probability of survival of existing structures over a time period and predict the operation time under specific reliability requirement. As a practical approach to the reliability of aging aircraft structure, example is presented and evaluated.

While conduct the calculation, a few programs based on FORTRAN code are developed to solve the none-linear equation, to work out the multi dimension integration and to simulate the survival probability. The crack life prediction software AFGROW is selected for comparison of the calculation results, which also shows the appropriate accuracy of the established model.

As conclusion, the effects of some variables including fuzzy factors on reliability and life of aging aircraft structure are finally discussed. It is apparent that the confines of the model are existing as fact because of the huge assumption of the parameters input and model uncertainties. Suggestions on further prospective research are proposed respectively.

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# CHAPTER 1 INTRODUCTION

## 1.1 *Background*

### 1.1.1 Aging problems in practice

In the field of aircraft structure, aging problems are one of the most concerned. World widely, the many old aircrafts that form the backbone of the total operational force structure are still far away from replacement and are even expected to remain in service for many years of extension. It is known that by the year 2000 about 60% of the worldwide fleet of US manufactured aircraft would be 20 or more years old[1]. In Australia, the problem was highlighted by the November 1990 failure of a Royal Australian Air Force (RAAF) Macchi aircraft which suffered a port wing failure whilst in an estimated 6 *g* manoeuvre. From ATSB TRANSPORT SAFETY report [2,3], the single-engine piston fixed-wing aircraft had an average age of 30 years in 2005.These aircraft, typically used in general aviation, and might not receive continuing airworthiness support from their manufacturer. In addition, the maintenance requirements are even not as stringent for general aviation aircraft compared with regular public transport aircraft.

The aircraft is said to be aging when the flight beyond design usage that could introduce new critical problems, corrosion, widespread fatigue damage (WFD) and multiple site damage (MSD), or repairs. The economic burden associated with the inspection and repair of aging problems can be expected to increase with age until the task of maintaining aircraft safety could become so overwhelming and the aircraft availability is so poor that the continued operation of the aircraft is no longer viable. In addition, corrosion detection, repair, and component replacement can dominate the total structural maintenance burden.

Experiences with operational aircraft have shown that there are many factors which tend to degrade the capability of the aircraft, such as additional equipments or modifications, function or mission changes because of new weapons and tactics, difference in pilot technique that results in different manoeuvre loading, and all kinds of corrosion due to pitting, fretting, exfoliation, crevice, galvanic, stress and many others. Fatigue and corrosion are worthy of the concerns from both airworthiness authorities and aircraft manufactures with the need to acquire improved understanding of the behaviour of aging aircraft structures. It is said that these two factors had led to the most significant amount of the catastrophic disasters of aircraft [1]. The mechanism of fatigue and corrosion is very similar and they can have an interacting effect on each other which damage the structure more severely. The combination effects of the two factors make it even more sophisticated to identify the mechanism of each factor. For example, when an airplane is in flight, factors such as engine vibration and flutter produce extended cyclic load while corrosion is minimal; and when it is on the ground, corrosive environment has its maximum effect while there is little or no cyclic loading.

Once the damage formed, it could coalesce rapidly and form a larger damage. The residual strength, i.e. the capability of the structure to sustain certain loads in the presence of a large damage of certain size, will decrease as a result. Since it became apparent that the MSD/WFD problem may be of the widespread significance to aging aircraft, the operator should then develop engineering tools to assess influence of MSD/WFD on the airworthiness of aircraft.

To protect aging aircraft against the risk of failure from fracture, structure integrity programs are then developed either to limit the operating life of the aircraft to a safe life, or for components that can be inspected, providing a means of ensuring cracks are detected before they become critical. Different international fora have been set up to deal with this type of issue, such as SMAAC(structural maintenance of aging aircraft) of European Union, (ASIP) Air Force Aircraft Structural Integrity Program and (ENSIP)Engine Structural Integrity Program by US Air Force, NATO Specialist's Meeting on the subject of Widespread Fatigue Damage(Rotterdam, 1995)[1].

### 1.1.2 Corrosion as one of the major causes

A study by Cooke (1990) found that corrosion damage to USAF aging aircraft causes the most significant cost burden of any structurally related item. In his study, funded by Warner Robins Air Logistics Centre, the researchers found that the costs of corrosion to the Air Force could be conservatively estimated at $700 million per year [4]. This is the largest maintenance cost of any structurally related item. Being reckoned as a century old problem, corrosion is a form of material damage caused by chemical and/or electro-chemical process from exposure to the corrosive environment such as moisture, acid or many others. There are many types of corrosion in terms of cause and effect, however, it can't be distinguished clearly from each other as it usually interact each other.

There is a common idea of sorting out corrosion by means of pitting corrosion, fretting corrosion, exfoliation corrosion, crevice corrosion, galvanic corrosion and stress corrosion. Pitting corrosion in typical aluminium alloy is resulted from anodic dissolution and cathode reduction, and initiated at or near the alloying elements with the help of moisture or other corrosion elements [4, 5]. On the other hand, fretting corrosion is resulted from rubbing action coupled with chemical or electro-chemical process. Therefore, it generally can be found at places adjacent to a rivet and its countersink surface where overlap addressed. Exfoliation is a form of corrosion that leads to macroscopic material surfaces separation-missing of the material- but may be caused from many different types of corrosion mechanism such as pitting and fretting corrosion. Galvanic corrosion happens when different materials are put next to each other and thus a galvanic cell is created, whilst the different material play the role of anode and cathode. A pair of steel and aluminium is a good example. The conventional referred stress corrosion cracking is the type of fracture phenomenon that exists only when mechanical load and chemical and/or electro-chemical corrosion process existing simultaneously.

The different types of corrosion damage exhibit different characteristics and potential consequences with respect to both detect ability and structural consequence. Although the corrosion forms are evident as surface deterioration, they may not be found if the surface is inaccessible to visual inspection, challenging the reliability of non-destructive inspection (NDI) methods, which is discussed later in this thesis. Those undetected corrosion can then progress significantly before being observed, leading to an increased risk that corrosion may cause a more significant decrease in damage tolerance than otherwise estimated.

The most important issue in understanding the mechanism of corrosion is to reveal the chemical and electro-chemical process throughout the corrosion. Since the corrosion is only related to material and environment, any approach on corrosion should identify the relationship between them. From the aging aircraft point of view, the conventional aircraft materials such as aluminium and steel are of the most interest. In terms of environment, moisture, acid and temperature are the most ineluctable of all. Nevertheless, further study on the fundamental corrosion mechanism of the aircraft materials is still necessary due to the sophisticated mechanism of environmental corrosion and its interaction. On the other hand, although material manufacturers have improved their metallic products in terms of their resistance to corrosion, but still are far from developing corrosion proof alloys.

Once the corrosion mechanisms are addressed, corrosion prevention methods, techniques and tools can then be developed. These could be material treatment/replacement, environment protection, and even design refining. As part of structural maintenance programs, the commercial aircraft industry has developed provisions to upgrade corrosion resistance through the use of substitute materials and heat treatments (e.g., more corrosion-resistant 7050, 7150, or 7055 alloy for 7075, stress corrosion-and exfoliation-resistant T-7X tempers for 7XXX-series aluminium alloys)[4, 5], improved protective finishes and corrosion-preventive compounds, and incorporation of design features such as drainage and sealing to prevent corrosion. Air Force Scientific Advisory Board concluded that a reduction of the relative humidity to 30–40 percent would significantly reduce the corrosion of stored aircraft and that existing dehumidification and storage systems appeared to be adequate (SAB, 1996). The report describes equipment and logistics relevant to dehumidified storage and discusses successful dehumidification programs in the other U.S. services (e.g., Marine Corps A-6E, Navy SH-60B, and Army CH-47D) and internationally (e.g., Swedish and Danish air forces).

In order to avoid the costly component repair and replacement, much more emphasis should be given to early detection of corrosion and implementation of effective corrosion control and mitigation practices. A practicable and more cost-efficient strategy for dealing with corrosion damage of airframe structures is needed to effectively guide prevention, control, and force management decisions for aging aircraft. The most important operational needs would include:

- environmentally compatible protective coatings to replace the hazardous materials being phased out (e.g., chromates), lasting the initial corrosion;
- using improved protective finishes and corrosion-preventive compounds that can be applied on external surfaces and that will penetrate and protect unsealed joints and around fastener heads on aging aircraft structures;

- guidance for the application of advances in alloys and processes offering improved corrosion protection;
- improved techniques to discover and roughly quantify hidden corrosion without requiring disassembly of the aircraft;
- classification of corrosion severity, similar to current commercial aircraft practice, to provide guidance to maintenance actions;
- improved understanding of the probable rates of corrosion and corrosion trends for specific operational aircraft for use in planning maintenance actions;
- dehumidified storage of aircraft or dehumidification of susceptible areas of particular aircraft;

### 1.1.3 Fatigue leading to the destruction

Fatigue is another most important issue of aging aircraft and is always increasing with the aircraft's age. Unlike corrosion, which could be controlled and thus would not physically affect the structure life, fatigue is a direct result of aircraft use and will eventually occur in all aircraft. Experience with operational aircraft has stated that fatigue accounts for more than 80% of all observed service failures in mechanical and structural systems [6]. Moreover, fatigue and fracture failures are often catastrophic; they may come without any warning or omen and may cause significant structural damage, as well as loss of life. Many cases of critical component fractures are observed in applications in which failures previously had not been encountered.

Failure comes out when the applied load exceeds the material's resistance. Resistance of the material diminishes with time due either to repeated loads which cause defect growth or a wear process which removes material. Furthermore, many aircrafts are expected to perform most challenging functions, such as unconventional air manoeuvre flight, where various combined strict loading conditions exist. Extreme care must also be taken to ensure that repairs or modifications intended to reduce dynamic effects do not cause further harm, like extra dynamic loading and the resulting high-cycle fatigue. The key technical issues include:

- identification, reduction, or elimination of sources of dynamic excitation;
- passive and active methods to reduce the response of aircraft structures;

- measurement and characterization of the threshold for fatigue propagation (Kth) values for airframe materials, including the applicability of long crack thresholds to small crack behaviour;
- in-flight monitoring of changes in dynamic loading;

Fatigue analysis theories and applications are achieved by many predecessors, including Nelson(1978), Dowling(1979), Gurney(1979), Schijve(1979), Fuchs and Stephens(1980), Collins(1981), Broek(1984), Rolfe and Balsam(1987) and Hertzberg(1989), as well as other organizations such as American Society for Testing and Materials(ASTM) and Society of Automotive Engineers(SAE). In addition, the most popular crack growth law is that of Paris (1964), but other models have been proposed and studied (Miller and Gallagher, 1981: Ortiz et al., 1988). Of particular interest is the studies by Hudak (1981), which suggest that the use of an initiation model coupled with a Paris propagation model having no threshold, will produce accurate life prediction in real structure [2,7].

Before fracture, there are two loosing continuous phases referred as crack initiation and crack propagation, which are both important in fatigue process depending on the nature of the structure and the service loads applied to it. From the engineering practice point of view, crack initiation refers to the formation of cracks that are detectable with the use of non-destructive evaluation techniques, rather than to the beginning of microstructure cracking. In most cases, welds and certain other structural details do have some kinds of unavoidable defects because of the fabrication process. That means, fatigue cracks may appear quite early and a significant portion of the service life of the structure may be spent in propagating the crack to a critical size.

Fatigue analysis is to identify the initial crack size and to find out the factors that affect the crack growth rate and finally to extend the fatigue growth time. The classic approach to fatigue has focused on the S-N diagram that relates fatigue life (cycles to failure, N, either the crack initiation period or total fatigue life) to cyclic stress S. The general strain-life model has been developed as an extension of the characteristic S-N approach (Coffin, 1954; Manson, 1954; Dowling, 1979; Fuchs and Stephens, 1980). As flaws are inherent in many components, the fracture mechanics approach to fatigue is as functional as others, which based on the stress intensity factor range,

$\Delta K$. Based on fatigue analysis, the predictions are the output of a number of procedures and Fig 1-1 presents the scenario of the various aspects involved.

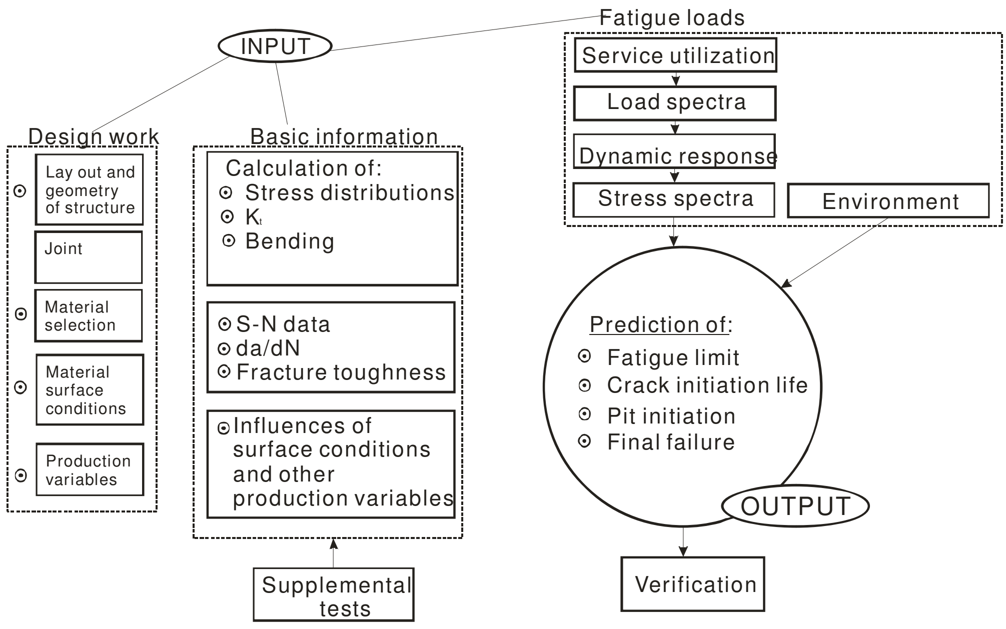


*Figure 1-1* Survey of the various aspects of fatigue of structures

The input problems occur in three categories:

(1). design work;

(2). basic information used for the predictions;

(3). fatigue load spectra to which the structure is subjected.

Each of the categories contains a number of separate problems, which again can be subdivided into specific aspects, e.g. 'joints cover welded joints, bolted joints, riveted joints, adhesively bonded joints.

Fig. 1-1 illustrates that the full problem can be very complex depending on the structural design, type of material, production variables, load spectra and environment. In general, a number of plausible assumptions are involved, which implies that the accuracy of the final result can be limited. The reliability of the prediction should be carefully evaluated, which requires a profound judgement, and also so-called engineering judgement, experience and intuition. It has been emphasized that physical understanding of the fatigue phenomena is essential for the evaluation of fatigue predictions. Without some satisfactory understanding of aspects involved, predictions on fatigue become inconceivable.

### 1.1.4 Non-destructive evaluation

The development of non-destructive evaluation (NDE) technology for aging airframe structures is driven by structural requirements and cost considerations. It protects structural safety by detecting, providing quantifiable characterization, and screening fatigue cracking, stress corrosion cracking, and corrosion conditions that are, or could become a flight-safety concern. Along with the cost, this information will assist the manager to determine the most economical and most effective maintenance decisions, regarding inspection timing and inspection methods whist maintaining high level of safety.

Many NDE techniques were developed over the decades that can be used to inspect the aging aircraft. The fundamental approach of all NDE techniques is the one that an interrogate energy source signal, such as optical, acoustic, electro-magnetic and others, is launched into the components [8]. Measurements are taken after the signal has interacted and passed through, or reflected back into signal receivers. The variation between the launched and received signal gives the characteristics inside the material or structural systems. It is from this signal variation that the defects, such as cracks, flaws, voids and corrosion can then be deduced in the components. However, it is challenging to recover and isolate the portion of the signal that is attributed directly to the defect alone, because the received signal is the total integrated effect of the defect, the surround material and the geometry. Therefore, selecting proper energy source by which its interaction with the defect can be systematically collected and quantified is an essential ingredient of an effective NDE technique [4].

Comparative Vacuum Monitoring (CVM) technology, Australia innovated aircraft Structural Health Monitoring (SHM) technology, has recently been recognized world widely in AU, UK, Europe, US, China, Singapore, etc, as it is the first time that an in-situ sensor based technology has been approved as an alternative to manual NDE inspections methods[9]. The new maintenance philosophy involves the use of sophisticated maintenance decision support IT systems and databases, utilizing quality data input on the condition of equipment and structures. CVM provides a simple and cost effective means of providing quality structural integrity data into these IT systems. CVM is very well placed with its high level of maturity, approval for use and level of readiness for in-service use to meet this increasing demand. The incorporated CVM technology avoids the labour-intensive scheduled and reactive unscheduled maintenance programs, and makes it possible to be included in the design of future new aircraft.

For the purpose of crack and corrosion measurement, some NDE techniques are relatively mature and can readily be adopted for aging aircraft applications with only minor engineering development effort. On the other hand, since defects could occur nearly anywhere on the aircraft, it is not practical to inspect every inch of the whole aircraft with very accurate, yet time consuming techniques. A combination of multi-step, multi-technique NDE approaches is necessary for the specific purpose of inspection. For example, a large area, non-contact method, such as optical surface measurement can provide large area, yet less accurate, coverage for a first cut identification of possible local problem areas. Depending upon the measurement accuracy required and the specific geometry of the identified local problem areas more accurate but more labour intensive NDE methods can then be employed for proper quantification. The low frequency eddy current probe is such a good tool for finding corrosion near and around hidden rivet holes. Meanwhile, NDE techniques can also meet the other aging aircraft engineering and logistic needs, such as determination of corrosion prevention coating integrity, location and identification of hidden flight critical subsystem components, identification of disbanding and delimitation of composites, honeycomb structures, and adhesive bonded composite repair patches, etc.

Practically, different kinds of defects need different combinations of NDE techniques with carefully consideration. For those hidden corrosion defections, the resource energy must penetrate through the material in which the corrosion is hidden. And the received signal which contains the total effect of the material must be able to recover the information related to corrosion alone. In US Navy, some success gained using ultrasonic and eddy-current techniques to assess coating integrity before large area peel-off.

In terms of capability of NDE techniques, it is neither necessary nor economically feasible to demand NDE techniques to find the smallest defects. The so-called maximum allowable defects must be determined from the structural integrity assessment [8]. The experimental evidence to date indicates that cracks of the order of two millimetres can significantly lower the fail safety capability of certain structural configurations. Specifically, the structural integrity assessment provides the answer as to what size of the allowable defects must be detected along with the inspection interval required for a given aircraft. Recently, the passive forms of NDE techniques have emerged in the forms of smart materials and smart structures. Embedded sensor such as optical fibres, give out mechanical, optical, chemical or electro-magnetically signals upon interaction with cracks or corrosion products. These passive NDE methods have the potential of becoming power tools for supporting the operation and maintenance of the aircraft structures. However, much research and development effort is needed to realize their full potential.

Reliability is one of the most important characteristics of an effective NDE. NDE inspection is a statistical process that depends on the inherent variability of many features including flaw size, orientation, distance of flaw from surface, surface roughness, and variations in material properties. The equipment and operation technical person would also play an important role and contribute the majority to the reliability. A frequently used measure of the reliability of a NDE system is the probability of detection (POD) which is a conditional probability defined as the probability that a flaw with given characteristics will be found in an inspection. Obviously, a requirement for an efficient NDE inspection is a high value of POD for the particular flaw and geometry involved in the inspection.

The most important needs include:

- detection of fatigue cracks under fasteners; the inability to detect cracks beneath fasteners can result in unmanageably short inspection intervals for fatigue-critical structures with small critical crack lengths;
- detection of small cracks associated with WFD for cracks as small as a few hundredths of an inch;
- techniques to discover and quantify hidden corrosion without disassembly of the aircraft;
- detection and characterization of cracks and corrosion in multilayer structures;
- detection of stress corrosion cracking in thick sections;

### 1.1.5 Structural Integrity /Airworthiness

Reliable structural integrity of the aging aircraft assessment is the key to safe operation, mission effectiveness and proper maintenance. Considerations must be involved initially in design to ensure the aircraft structural integrity, which is to design the structure to sustain fail-safe loads with limited damage for a period of service prior to detection and repair [10, 11]. The USAF Aircraft Structural Integrity Program (ASIP) established in 1958(Neegard, 1980) originally had the objectives to control structural failure of operational aircraft, determine methods of accurately predicting aircraft service life, and provide a design and test approach that will avoid structural fatigue problems in future.

Airworthiness is such an important concept for safe operation of aircraft that the requirements for airworthiness are now precisely defined for various classes of aircraft. There are requirements for airworthiness applicable at the time of design and initial approval of an aircraft type; covering all systems and crucial parts of the aircraft; throughout the life of each aircraft; and prior to each flight. Airworthiness Directives and its amended bulletin are such the regulatory, providing the platform for the involved parties.

In Australia, airworthiness relies on standards originating in the USA and Europe, which includes [12]:

- Joint Airworthiness Requirements for Very Light Aeroplanes (JAR-VLA).
- US Federal Aviation Regulations, Part 23: “Airworthiness Standards: Normal, Utility, Acrobatic, and Commuter Category Airplanes”.
- The European standard: Joint Airworthiness Requirements 22E.
- The USA standard: Federal Aviation Regulations, Part 33 “Airworthiness Standards – Aircraft Engines” .

For aging aircraft, continuing airworthiness has traditionally been ensured by inspection programs. In the event of known, specific fatigue cracking and/or corrosion problems, that if not detected and repaired had the potential to cause a significant degradation in airworthiness, the normal practice is to work out the inspection interval according to the accurate estimation of fatigue and corrosion progress, then to perform the repair/maintenance. Thus, continuing structural airworthiness was totally dependent on repetitive inspections and repairs.

While fatigue life is assumed to be log normally distributed (Payne, 1972), and the scatter factor is the factor which when applied to the mean of the distribution, will represent the 99.9$^{th}$ percentile of the population. Hence by applying the scatter factor to the full-scale fatigue test result the predicted safe life is expected to ensure that the probability of failure form fatigue will be less than 1 in 1000. As long as the assumed standard deviation of log life is well established, this safe life approach is conservative. It is normally applied when crack growth is so rapid, that there is little opportunity to inspect for cracking before catastrophic failure occurs. Because of this conservatism, many aircraft will be retired earlier than they need to be.

For critical locations in structures which have crack growth periods sufficiently long for cracks to be detected before failure, then a less wasteful safety-by-inspection approach can be applied. This approach requires inspection techniques that are sufficiently reliable that small defects can be found early in their life, and their growth can be monitored and cracks removed or repaired before they become critical. There is thus a need to be able to determine safe crack inspection intervals, neither too often because of the cost of inspection, nor too infrequent because of the risk of a crack reaching critical size before it is found.

However, additional complexity arises from the fact that the aging aircraft structural components or systems to be evaluated are no longer made of virgin materials. Therefore, the corroded material properties must determined first. Mechanically, the material degradation can be quantified as reduction in strength, reduction in fracture toughness and change in modules. But, the current NDE techniques can only measure the change in modules property by means of ultrasonic techniques. For aircraft structures, change in modules means change in structural vibration characterization which could lead to enhanced crack propagation or to the development of damaging flutter response.

The other complexity arises because the corrosion damage is not limited to a few local regions which are referred as multi-site damage and many of them are not damage tolerant. The integrity of the airframe that contains many non-uniform randomly distributed corrosion damage sites must be evaluated. The uncertainties in these corrosion damages resulted in the susceptivity of integrity assessment. Challenges are facing maintainers, operators, researchers and regulators.

Another important issue of structure integrity is the repair of components, inclusive of replacement. Improvements in commercial aluminium alloys, tempers, process controls, and fabrication methods could be beneficial when replacement is considered. Bonded compound material patch is of most concern because of its less concentrations, feasibility to complex shapes and more efficient load transfer. Research in this field is concerned by the material development centre and airframe manufacturer, but not the interest of this thesis.

## 1.2 *Purpose of the study*

Traditional design experience and data of virgin and undamaged materials/structures are no longer sufficient to assure safe and efficient operation of the aged aircraft. Also required is a better understanding of the materials and their structural behaviour beyond the conventional design limit. Facing this challenge, the profession have been committed to developing robust analytical methods to assure the safety of these existing structures with limited funds, which is expected to be evaluated in terms of life and reliability. This is also the motivation of this study.

General methodologies for incorporating probabilistic structural mechanics into the reliability analysis process for general aircraft structures have already been presented by vanguard. These are useful in establishing the methodology and approach needed to implement probabilistic approaches to engineering practice. Although many of these methods could be applicable to aircraft structures, the combination of randomness and fuzziness of the parameters, the uniqueness of the load spectrum, the special structure integrity and usage differences must be carefully considered.

The purpose of this research effort was to study the current well-established theories and analysis tools of reliability. After the thoroughly understand of these methods, probably little part of them, a hybrid method considering the special behaviour of aircraft structures is to be established. The sensitivity of fatigue life estimates on the parameters used in the model is then to be discussed. Subsequently, comparison between results deducted from different analysis methods could be conducted. As a result, more accurate fatigue life prediction and reliability assessment could be carried out and thus more appropriate aircraft inspection schedules are determined.

Attentions must be drawn to some of the factors that the variance and functions of some parameters was based on statistical and experiment data if available, but in most cases reliable data could not be found; therefore, engineering empirical assumption was used. Prospective research on the effects of such assumptions is then respectively suggested.

## 1.3 *Research scope and methods*

This research work was devoted to the so called corrosion fatigue model, incorporating the fuzziness and randomness of some parameters and analysing the parameter effects on the reliability and life prediction. The research will firstly introduce the well-established theories of reliability and point out how such these methods can be used to assess the life and reliability of aging structure. Secondly, it will identify the specific situation for aging structure, considering the initial design/material treatment, the structure's deterioration and degradation with time, the in-place uncertainty parameters, and so on. Then, a robust mathematical model could be established upon time-related parameters to evaluate the probability of survival of the existing structure over a time period. As a practical approach to the reliability of aging aircraft structure, the model will incorporate the field data obtained through investigating of the aging aircraft. The calculation result could then be used in management decision of airworthiness comparing to the design specification and operation requirement.

Equivalent initial crack size calculation is also the main task in the process of model construction, which could be used as the input data of the model. More reasonable input of the initial crack size comparing to the direct use of pit size makes the model more practicable. In the calculation, empirical judgement is necessary for solving the non- linear equation since multi roots may exist as such the character of the non-linear equation.

Monte Carlo simulation has played an important role in the research as the main idea of model calculation, which is defined as experimenting with the model over time. Also, fuzzy reliability method incorporating the elaborate linear transformation of the membership function into probability density function provides a simple solution for its simulation. However, many powerful convergence methodologies and variance reduction techniques, such as, importance sampling, directional sampling, can still be incorporated to improve the efficiency and even the accuracy of the calculation.

The powerful programming code FORTRAN has contributed the majority of the calculation work. Although more operation friendly program (window operation based on visual basic) not yet developed as intended because of time limitation, the current program provides the appropriate result of the model. It has to be noted that some input of the program are still need to be conducted manually while not designed in the program.

In addition, this research work would not have been completed without the USAF‘s structural life prediction program, AFGROW, which is designed, developed, and maintained by Mr. Jim Harter, AFRL/VASM. This robust software actually reduced by a million the research time required to complete all the comparison works.

# CHAPTER 2 LITERATURE REVIEW

## 2.1 *Methodologies*

### 2.1.1 Introduction

Aircraft structures have traditionally used a fail-safe or damage-tolerant design approach that uses specific factors of safety, conservative loads, and material allowable. These durability and damage-tolerant design approaches are detailed in a number of military standard and specification [13, 14]. Probabilistic structural reliability assessment are used only occasionally in the aircraft industry because the reliability of an aircraft and its components should be very large, and, therefore, detailed design procedures, sophisticated manufacturing methods and extensive quality control reduce uncertainties in all resistance quantities. Additionally, detailed inspection procedures are employed in order to reduce the probability of failure during the operation of the aircraft. All these procedures are deterministic, but since quality control and/or inspection can't be perfect, some uncertainties always remain.

Therefore, a need for more efficient, higher performance structures and the desire to have better analytical tools for predicting structural performance are encouraging more applications of probabilistic methods to these design schemes. Palmberg et al. (1987)[15] give an overview of the US Air Force damage-tolerant design approach and discuss the inherent variability in loads, initial quality, the crack growth process, inspection results, and material behaviour. Conservative loads, material allowable, and a 1.5 safety factor were typically used to allow for the variability. Whereas this approach has served the industry fairly well in the past, new analysis techniques based on probabilistic methods can provide more optimal results.

General methodologies for incorporating statistical methods in to the design and analysis process for general aerospace structures have been presented by Prof. Y. S. Feng (1982) and Walker (1989) and a discussion of an approach for treatment of aircraft engines has been presented by Roth (1991)[16]. They are useful in establishing the methodology and approach needed to implement probabilistic approach to design, certification, and maintenance. Although many methods mentioned in the following sections are applicable to aircraft structures, the uniqueness of the load spectrum and usage difference must be considered.

Aircraft structure is evaluated at a number of critical conditions or “points in the sky”, which are determined for the mission profile of the aircraft. The wing and fuselage air loads are determined for those conditions and are applied to the finite element model to determine the internal loads. The actual external loads on the structure will vary somewhat due to differences in pilot technique. For example, the coefficient of variation (COV) of the wing root bending moment (WRBM) of a fighter aircraft could vary from 0.012 to 0.04 [16].

The applications of structural reliability methods in aircraft structures primarily emphasis on the calculation of probability of fracture versus flight hours (life), probability of fracture versus the length of inspection internal, and a single flight failure probability. These applications normally involve the techniques of crack detection and the crack detective probability function as well.

The state of the art in the area of structural reliability analysis has improved significantly in the last two decades. A considerable amount of work has been conducted in the areas of element-level and system-level reliability estimations. The general area of risk-based engineering design is still growing at a rapid rate. This area is being advanced significantly by the introduction of several risk-based design codes that can be applied routinely in the design office. It needs to be pointed out that a considerable amount of research work is still being conducted in the areas of system reliability, simulation, time-dependent reliability analysis, and stochastic finite element analysis. The results of these investigations need to be synthesized and adapted to simple, practical methods for realistic engineering applications.

### 2.1.2 Stress-strength Interference Method

#### *2.1.2.1 Introduction of the method*

The “Stress-strength Interference Method” is known as the basic structural reliability method, considering only one load effect (stress) resisted by one resistance (strength). It gives the definition of structure failure as the imposed stress (Load) exceeds the strength (Capability) of structure. Failure probability or unreliability is the probability that the stress is greater than the strength. The method could be used in conjunction with a variety of failure modes such as yielding, buckling, fracture, and fatigue. The following Figure 2-1 shows the theory of the “Stress-strength Interference Method”, that is the probability density functions of stress and strength and their interference (overlap).

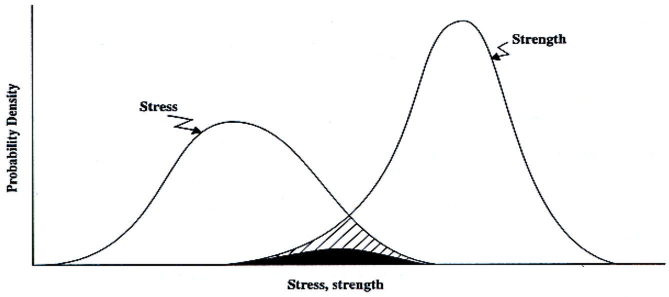


*Figure 2-2* Graphical representation of Stress-strength Interference

It should be pointed out that the overlapped area is not equal to the failure probability. However, this area is qualitatively proportional to the failure probability (the large the area, the higher the failure probability) as long as the mean value of stress is less than the mean value of strength. The failure probability is equal to the black area in Figure 2-1.

The ‘Numerical Evaluation of Reliability Function’ (NERF) computer software package is an example that utilised this probabilistic method in evaluating the failure probabilities due to structural fatigue. It is capable of addressing the variability in fatigue life, residual strength, initial flaw size distribution and inspections, which will be introduced in the later section.

#### *2.1.2.2 A summary of Some Analytical Results*

Let C and L represent the Capability (strength) and Load (stress) respectively, then variable U can be defined by:

$$U=C-L \qquad 2\text{- }1$$

The failure probability of a structure is given by:

$$P_t = P[U < 0] = \int_{-\infty}^{0} f_U(u)du = F_U(0) \qquad 2\text{- }2$$

Where $f_U(u)$ is the density function of variable U.

With respect to different density functions of the stress and strength, there would be a variety of analytical results. Table 2-1 gives a summary of some often used distribution results.

*Table 2-1* Summary of some analytical expressions for failure probability [15]

| **Probability Density Function for Stress** | **Probability Density Function for Strength** | **Failure Probability ($P_t$)** |
|---|---|---|
| 1. Gaussian: PDF formula with A=L, $a=\ell$, $\bar{a}=\bar{l}$, and $\sigma_A=\sigma_L$ | Gaussian: PDF formula with A=C, $a=c$, $\bar{a}=\bar{C}$, and $\sigma_A=\sigma_c$ | Pt= $\Phi(-\beta)$<br>Where $\beta=(\bar{C}-\bar{l})/(\sigma_C^2+\sigma_L^2)^{1/2}$ |
| 2. Lognormal: PDF formula with A=L, $a=\ell$, $\bar{a}=\bar{l}$, and $\sigma_A=\sigma_L$ | Lognormal: PDF formula with A=C, $a=c$, $\bar{a}=\bar{C}$, and $\sigma_A=\sigma_C$ | Pt= $\Phi(-\beta')$<br>Where $\beta'=(\ln\bar{C}-\ln\bar{l})/(\sigma_{lnC}^2+\sigma_{lnL}^2)^{1/2}$<br>$=\ln(\bar{c}/\bar{l})/\sigma_{\ln}(C/L)$ |
| 3. Exponential: PDF formula with A=L, $a=\ell$, and $\lambda_A=\lambda_L$ | Exponential: PDF formula with A=C, $a=c$, and $\lambda_A=\lambda_c$ | Pt= $\frac{\lambda_C}{\lambda_C+\lambda_L}$ |
| 4. Exponential: PDF formula with A=L, $a=\ell$, and $\lambda_A=\lambda_L$ | Gaussian: PDF formula with A=C, $a=c$, $\bar{a}=\bar{C}$, and $\sigma_A=\sigma_C$ | Pt= $\Phi(-\frac{\bar{C}}{\sigma_C})+\exp[1-\frac{1}{2}(2\bar{C}\lambda_L-\lambda_f^2)]$<br>$\times[1-\Phi(-\frac{\bar{C}-\lambda_L\sigma_C^2}{\sigma_C})]$ |
| 5. Exponential: PDF formula with A=L, $a=\ell$, and $\lambda_A=\lambda_L$ | Gamma: PDF formula with A=C, $a=c$, $n=j$, and $\lambda_A=\lambda_c$ | Pt= $(\frac{\lambda_C}{\lambda_C+\lambda_L})^j$ |
| 6. Gamma: PDF formula with A=L, $a=\ell$, $n=i$, and $\lambda_A=\lambda_L$ | Exponential: PDF formula with A=C, $a=c$, and $\lambda_A=\lambda_c$ | Pt= $1-(\frac{\lambda_L}{\lambda_C+\lambda_L})^i$ |
| 7. Gamma: PDF formula with A=L, $a=\ell$, $n=i$, and $\lambda_A=\lambda_L$ | Gamma: PDF formula with A=C, $a=c$, $n=j$, and $\lambda_A=\lambda_c$ | Pt= $1-[\frac{ƨ(i+j)}{ƨ(i)ƨ(j)}\,Bt(i,j)]$<br>Where $t=\lambda_L/(\lambda_L+\lambda_C)$ |

### *2.1.2.3 Numerical Solutions*

If analytical expressions or tabulated results are not available for a particular combination of stress and strength probability density functions of interest, then a numerical solution procedure has to be used to compute the failure probability. There are also cases in which the stress and strength data (derived from theoretical models, experimental measurements or field measurements) may not fit well to any standard probability density function. In such cases the probability density functions of stress and strength are present in the form of histograms. Numerical solution procedures have to be used in these cases also. There are four approaches to a numerical solution:

- Numerical integration of interference integral
- Numerical integration of the different distribution
- Numerical integration of the ratio distribution
- Integral transform procedure

The numerical integration details could be found in a lot of mathematical books. It is not necessary to list here.

#### *2.1.2.4 Comments*

The Stress-strength Interference Method is one of the oldest methods of structural reliability analysis. Although more powerful methods of reliability analysis such as the first-order/second-order reliability methods and simulation techniques (which are applicable to a broader class of problems and with less restrictive assumptions) are now available, the Stress-strength Interference Method continues to be a popular method of reliability analysis among practicing engineers in many industries, especially in the case of the strength and stress are all distributed as the Normal distribution (Gauss). The advantage of this method lies in its simplicity, ease, and economy. A major drawback is the assumption that the strength and stress are statistically independent, which may not be valid for some problems. If this assumption can be justified, then reliability can be computed relatively quickly.

Furthermore, with the development of other advanced methods, use of the Stress-strength Interference Method will definitely decline. But the generalized conditional expectation method of simulation opens a way for combining the method with simulation. The many analytical expressions for failure probability listed in Table 2-1 could be used in conjunction with simulation (Sundararajan and Gopal, 1992). A judicious merging of the analytical expressions with simulation could drastically reduce the computational effort and cost of simulation-based reliability assessment.

### 2.1.3 First Order / Second Order Reliability Methods (FORM/SORM)

#### *2.1.3.1 Introduction*

The development of the FORM can be traced historically to second-moment methods, which used the information on first and second moments of the random variables. These are first-order second-moment (FOSM) and advance first-order second-moment (AFOSM) methods. The FOSM was firstly proposed by Cornell in 1969. It was derived from the fact that it is based on a first-order Taylor Series approximation of the performance function or limit state function and uses only second moment statistics (means and covariance) of the random variables. The original formulation by Cornel (1969) uses the simple two-variable approach of the previous section. A limit state function is defined as Z=R-S. Assuming that R and S are statistically independent normally distributed random variables, the variable Z is also normally distributed. Accordingly, Its mean and variance can be determined readily. Obviously, the probability of failure depends on the ratio of the mean value of Z to its standard deviation. Cornell (1969) named this ratio the safety index (reliability index) and denoted it as $\beta = \mu_z / \sigma_z$. Thus the failure probability could be denoted as $P_t = \Phi(-\beta)$ [16]. An alternative formulation proposed by Rosentbleuth and Esteva (1972) may also be used, assuming that the variable R and S are statistically independent lognormal random variables. For physical reasons, these variables are restricted to positive values; and the limit state function in this case can be defined as Z=ln(R/S), obviously, Z is again a normal random variable. Hence the formula of the failure probability could be readily determined [17].

However, the first-order second-moment approach has more serious problems. The method does not use the distribution information about variables, even if they are available. More importantly, Cornell's safety index fails to be constant under different but "mechanically equivalent" formulations of the same performance function. The problem of the lack of invariance was observed by Ditleven (1973) and Lind (1973). It was overcome by Hasofer and Lind (1974). They firstly presented an approach to reduce and transform the variables.

For the more general case where the number of random variables may be greater than two, the limit state surface may be non-linear, and the random variables may not be normal distribution, a number of iterative solution techniques have been developed. Since then, FORM/SORM is introduced for the analysis of structural reliability.

Numerous computer programs have been developed by researchers to implement the FORM/SORM algorithms, such as NESSUS, PROBAN, CALREL, which will be compared in detail in the following section.

### *2.1.3.2 Theories of FORM/SORM*

To simplify the explanation of the theory herein, the basic variables $x_i$, are assumed to be statistically independent and therefore uncorrelated. Further, it can be show that weak correlation (i.e. ρ<0.2, where ρ is the correlation coefficient) can general be neglected and that strong correlation (i.e. ρ>0.8) can be considered to imply fully dependent variables. The additional discussion of correlated variables in FORM/SORM may be found in references[18,19]. The limit state function, expressed in terms of the basic variables, $x_i$, is first transformed to reduced variables, $u_i$, have zero mean and unit standard deviation:

$$u_i = \frac{x_i - \mu x_i}{\sigma x_i} \qquad \text{2- 3}$$

A transformed limit state function can then be expressed in terms of the reduced variable. A transformed limited state function can then be expressed in terms of the reduced variables:

$$g_1(u_1,...,u_n) = 0 \qquad \text{2- 4}$$

With failure now being defined as $g_1(u) < 0$, the space corresponding to the reduced variables can be shown to have rotational symmetry, as indicated by the concentric circles of equiprobability shown on Figure 2-2.

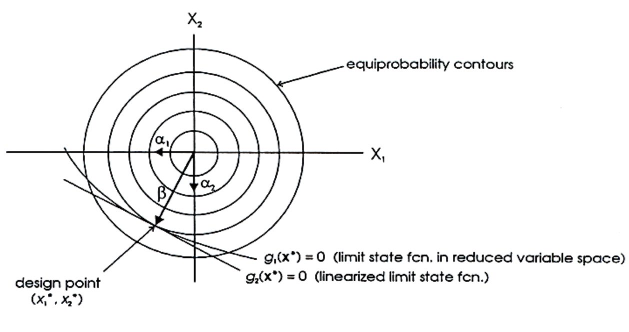


*Figure 2-2* Formulation of reliability analysis in reduced variable space

The reliability index β is now defined as the shortest distance between the limit state surface, $g_1(u)=0$ and the origin in reduced variable space. The points on the limit state surface that corresponds to this minimum distance is referred to as the checking (or design) point and can be determined by simultaneously solving the set of equations:

$$\alpha_i = \frac{\frac{\partial g_1}{\partial u_i}}{\sqrt{\sum_i \left(\frac{\partial g_1}{\partial u_i}\right)^2}} \quad 2\text{- }5$$

$$u_i^* = -\alpha_i \beta \quad 2\text{- }6$$

$$g_1(u_1^*,...,u_n^*) = 0 \quad 2\text{- }7$$

And search for the direction cosines, $\alpha_I$ , that minimize β. The partial derivatives in equation 2-5 are evaluated at the reduced space design point ($u^*_1$,..., $u^*_n$). This procedure results from linearizing the limit state surface (in reduce space) and computing the reliability as the shortest distance from the origin in reduced space to the limit state hyperplane. Once the convergent solution is obtained, it can be shown that the checking point in the original random variable space corresponds to the points:

$$X_i^* = \mu x_i (1 - \alpha_i \beta V x_i) \quad 2\text{- }8$$

Such that $g_1(X_1^*,...,X_n^*) = 0$. These variables will correspond to values in the upper tails of the probability distributions for load variables and the lower tails for resistance (or geometric) variable.

Because of the ease of working with normal variables, the objective here is to transform the non-normal random variable into equivalent normal variables, and then to perform the analysis for a solution of the reliability index, as described previously. This transformation is accomplished by approximating the true distribution by a normal distribution at the value corresponding to the design point on the failure surface. By fitting an equivalent normal distribution at this point, we are forcing the best approximation to be in the tail of interest of the particular random variable. The fitting is accompanied by determining the mean and standard deviation of the equivalent normal variable such that, at the value corresponding to the design point, the cumulative probability and the probability density of the actual (non-normal) and the equivalent normal variable are equal. (This is the basis for the so-called Rackwitz algorithm). These moments of the equivalent normal variable are given by

$$\sigma_i^N = \frac{\phi(\Phi^{-1}(F_i(X_i^*)))}{f_i(X_i^*)} \quad 2\text{- }9$$

$$\mu_i^N = X_i^* - \Phi^{-1}(F_i(X_i^*))\sigma_i^N \quad 2\text{-}10$$

Where function $F_i()$ and $f_i()$ are the non-normal cumulative distribution function(CDF) and probability density function (PDF), $\phi()$=standard normal PDF, and $\Phi^{-1}$=inverse standard normal CDF.

SORM was introduced for solving inaccuracy issue that approximates the limit state surface by a linear surface (Through a Taylor series expansion). As it may not be satisfactory if the limit state surface has significant curvature. Second moment methods to deal with the non-linearity of the limit state function have been termed “Second order” methods (Fiessler 1979; Hohenbichler 1987). The most common approach has been to attempt to fit a parabolic, quadratic or higher order surface to the actual surface, centred on the design point. This requires some decision about the extent to which the approximation is valid away from the design point. Figure 2-3 [15] shows the relationship between a first order (linear) approximation and a second order (parabolic) approximation all the design point in standard normal space (Der Kiureghian, 1987). More detail discussion may be found in a lot of relevant references.

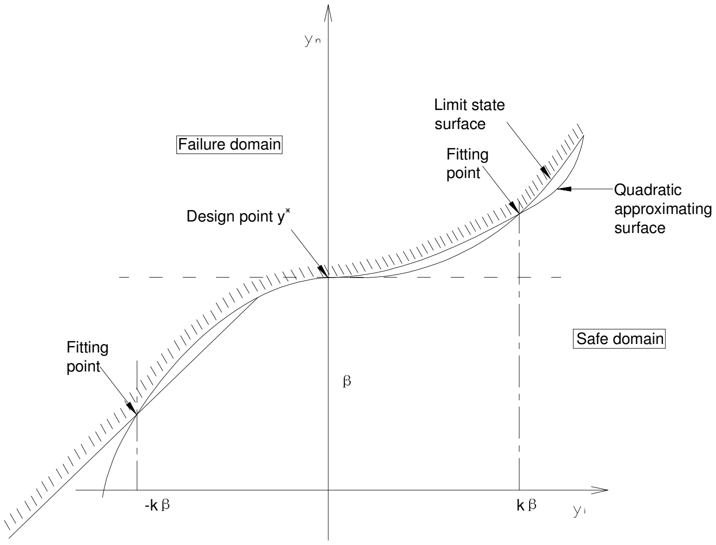


*Figure 2-3* Second Order and First Order Approximation

#### *2.1.3.3 The computational procedure of FORM/SORM*

One possible procedure of SORM for computing the reliability index β, for a limit state with non-normal basic variable is shown below.

(1).Define the appropriate limit state function

(2). Make an initial guess at the reliability index, β

(3). Set the initial checking point values, X*I = u

(4). Compute the equivalent normal mean and standard deviation for non-normal variables

(5). Compute the partial derivative evaluate at the design point X*I

(6). Compute the direction cosines, $\alpha_i$ as

$$\alpha_i = \frac{\frac{\partial g_1}{\partial X_i}\sigma_i^N}{\sqrt{\sum_i\left(\frac{\partial g_1}{\partial u_i}\sigma_i^N\right)^2}}. \qquad 2\text{-}11$$

(7). Compute the new value of design point $X_i^*$ as

$$X_i^* = \mu_i^N - \alpha_i \beta \sigma_i^N \qquad 2\text{-}12$$

(8). Repeat steps 4 through 7 until estimates of αI, stabilize (usually fast)

(9). Compute the value of β such that g($X_i^*$,…,$x^*_n$)=0

(10). Repeat steps 4 to 9 until the value for β converges. (This normally occurs within five cycles or less, depending on the non-linearity of the limit state function.)

#### *2.1.3.4 Discussion*

The advantages of SORM/FORM are they succeed in the structural reliability analysis and in the prediction of with arbitrary distribution variables and non-linear limit state function. Particularly when a series of numerical techniques were very well developed, they are easy to be formulated in computer program. SORM/FORM are becoming popular methodologies in the engineering application varied from airspace engineering to civil engineering area. The disadvantage is they might not lead to a more accurate result, although the SORM had made a progress in accuracy compared to the FORM. And it has been proven that in some extreme situations has this approximation technique been seen to fail.

### 2.1.4 Monte Carlo Simulation

#### *2.1.4.1 Introduction*

The term "Monte Carlo" was introduced by Von Neumann during World War II and the method has long been an important tool of designers. "Monte Carlo" simulation was defined as a technique of performing sampling experiment on the model of the system. Stochastic simulation is defined as experimenting with the model over time; it includes sampling stochastic variables from probability distribution. The principle behind Monte Carlo simulation is that the behaviour of a statistic in random samples can be assessed by empirical process of actually drawing many random samples and observing this behaviour. The strategy for this is to create an artificial "world", or pseudo-population, which resembles the real world in all relevant respects. This pseudo-population consists of mathematical procedures for generating sets of random numbers that resemble samples data draw from the true population.

Yang et al demonstrated this simulation technique, used to assess the probability of failure of jet engine discs. The purpose of this assessment was to determine the fatigue reliability of gas turbine engine discs under scheduled inspection maintenance in service for supporting the engine retirement for cause philosophy.

Rohrbaugh et al also utilised the Monte Carlo simulation to simulate the failure probability of fastener holes in flat panels such as those for aircraft fuselage lap joints. A computer code for risk assessment of aircraft structures developed by Cavallini et al, also utilises the Monte Carlo simulation to determine the probability of failure, which is called 'Probabilistic Investigation for Safe Aircraft, -PISA'.

#### *2.1.4.2 The Procedures*

The analytical and computational steps that are needed for performing Monte Carlo simulation are:

(1).Definition of the system

(2). Generation of input random variables

(3). Evaluation of the model

(4). Statistical analysis of the resulting behaviour

(5). Study of efficiency and convergence

The definition of the system should include its boundaries, input parameters, output (or behaviour) parameter, and models that relate the input parameter to the output parameter. The accuracy of the results of simulation is highly dependent on having an accurate definition for the system. It is common to assume the system model in Monte Carlo simulation to be non-random. However, modelling uncertainty can be incorporated in the analysis in the form of Bayas factors and additional variables, for example, coefficients of variation. All critical parameter should be included in the model. The definition of the input parameters should include their statistical or probabilistic characteristics, that is, knowledge of their moments and distribution types. The input parameters are generated and these values should then be substituted in the model to obtain output parameters. By repeating the procedure N times, N sets of output parameters are obtained.

The accuracy of the results is expected to increase by increasing the number of simulation cycles. The convergence of the simulation methods can be investigated by studying their limiting behaviour. Also, the efficiency, and thus the accuracy, of simulation methods can be increased by using variance reduction techniques. These techniques are known as Importance Sampling Method, Stratified Sampling Method, Latin Hypercube Sampling Method, Adaptive Sampling Method, Conditional Expectation Method, Antithetic Variates Method, Generalized Conditional Expectation Method and Response Surface Method, which are facile in most literature references.

#### *2.1.4.3 Advantage and Disadvantage*

The Monte Carlo simulation (numerical technique) offers many advantages over other techniques.

(1).The distributions of variables do not have to be approximated in any way

(2).Correlation and other inter-dependencies can be modelled

(3).The level of mathematics require to perform a Monte Carlo simulation is quite basic

(4).The computer does all the work required in determining the outcome distribution

(5).Software is commercially available to automate the tasks involved in the simulation

(6).Greater levels of precision can be achieved by simply increasing the number of iterations that are calculated

(7).Complex mathematics can be included (e.g. power function, log, IF statements, etc.) with no extra difficulty

(8).Monte Carlo simulation is widely recognized as a valid technique so its results are more likely to be accepted

(9).The behaviour of the model can be investigated with great ease

(10). Changes to the model can be made very quickly and the results compared with previous models

The disadvantage of Monte Carlo Simulation method is that the high accuracy of output needs quite large computation quantity. Furthermore, the Monte Carlo Simulation is only worth exploiting when the number of trials or simulations is less than the number of integration points required in numerical integration.

### 2.1.5 Probabilistic Finite Element Method

#### *2.1.5.1 Introduction*

Although the theory of statistics and structural reliability has been used successfully in modelling the uncertain nature of structure, loads environments, and in computing the probability of failure, its application is usually limited to simple structures with linear constitutive behaviour. Because of the complexity in the geometry, external loads, and non-linear material behaviour, especially in airspace structures, a new computational method called as Probability Finite Elements Method (PFEM), which combines the finite element method with statistics and reliability methods, was introduced to salve the linear and non-linear structural mechanics problems and fracture mechanics problems.

Extensive research on the PFEM has been conducted by Mr. W. K. Liu and his colleagues at the Northwestern University of America [20, 21, 22]. The finite element method couple with the first and second order reliability methods (FORM and SORM) has been developed by Der Kiureghian and Ke (1985, 1988)[23,24] for linear structural problems and by Liu and Der Kiureghian (1991)[25] for geometrically non-linear problems. The most critical step in this method is the development of an efficient search algorithm for locating the point at which the response surface is be expanded in a first- or second- order Taylor series. This point is obtained by an iterative optimisation algorithm, which involves repeated computation of the limit state function and response derivatives. Unlike the method of direct differentiation, the PFEM based on the perturbation approximation in conjunction with the FORM has been developed for the reliability analysis of brittle fracture and fatigue.

#### *2.1.5.2 Comments and Conclusion*

A number of researches has resulted that the accuracy and efficiency of PFEM in quantifying the statistic moments of a random system are in good agreement with Monte Carlo Simulation (MCS). The computational efficiency of the PFEM far exceeds the computational efficiency of the MCS. Because the PFEM discuss in reference [26] essentially involves solution of a set of deterministic problems, it is easily integral into any FEM-based code.

The PFEM couple with the first-order reliability method calculates the reliability index via an optimisation procedure and provides a powerful tool for the sensitivity analysis. Performance of this methodology is demonstrated on a single edge-cracked beam with a concentrated load and a classic model fatigue crack growth problem.

In addition to the PFEM, the random boundary element method, which combined the mixed boundary integral equation with the first order reliability method, is also present for the curvilinear fatigue crack reliability analysis. Because of the high degree of complexity and non-linearity of the response, direct differentiation couple with the response–surface method is employed to determine the response gradient. The reliability index and the corresponding probability of failure are calculated for a fatigue-crack growth problem with randomness in the crack geometry, defect geometry, fatigue parameters, and external loads. The response sensitivity of the initial crack length at the design point is also determined to show its role in the fatigue failure. The results show that the initial crack length at the design point is a critical design parameter, because crack lengths below the threshold of an inspection limit are likely to exhibit a large amount of scatter, which makes it imperative that the life expectancy of a structure be treated from a stochastic viewpoint.

In conclusion, the PFEM is a powerful tool for the calculation of structural reliability and fatigue life, particularly in cases of high complex structures and non-linear limit states. Now, there has already been commercial computer software on the PFEM available.

### 2.1.6 Probabilistic Fracture Mechanics

#### *2.1.6.1 Introduction*

Fracture mechanics is an engineering discipline that quantifies the conditions under which a load-bearing body became fail due to the enlargement of a dominant crack contained in that body (Kanninen and Popelar, 1985). Such enlargement can occur over an extended period, due to cyclic loading and/or adverse environmental effects. A conventional deterministic fracture mechanics provides the time (or cycles) to failure for a given set of initial (or current) conditions, as part of this process is evaluation of the critical crack size. However, many of the inputs to a fracture mechanics analysis are often subject to considerable scatter or uncertainty, such as the initial crack, stress driving force solution, applied stress, and material properties.

Therefore, it is realistic to consider some of key inputs to be random variables and viewing the output as a statistical distribution of lifetime.

Probabilistic fracture mechanics (PFM) is fracture mechanics that considers some of the inputs to be random variable. A prime example of a random is initial crack size. This is seldom accurately known and usually has a strong influence on lifetime. All other inputs, such as stress, cycle, subcritical crack growth characteristics, and fracture toughness, can also be considered as random variables.

Another factor that is naturally incorporated into PFM analyses is the effects by a given inspection procedure as a function of its size and the probability of accurately sizing the defect and satisfactorily repairing it.

Numerical techniques are required for the PFM; so as to it becomes mandatory that computer codes would be involved in the calculation for all but unrealistically simple problems. As is the case for deterministic problems, computer programs are often custom written for a specific application. However, some PFM software is publicly available, such as BLESS, PACIFIC, PROBAN, VISA, PCFAD, DA/DN, NASCRAC and many others.

### *2.1.6.2 Theory of PFM*

Probabilistic Fracture Mechanics (PFM) is based on Deterministic Fracture Mechanics (DFM). Figure 2-4 shows the model and arithmetic.

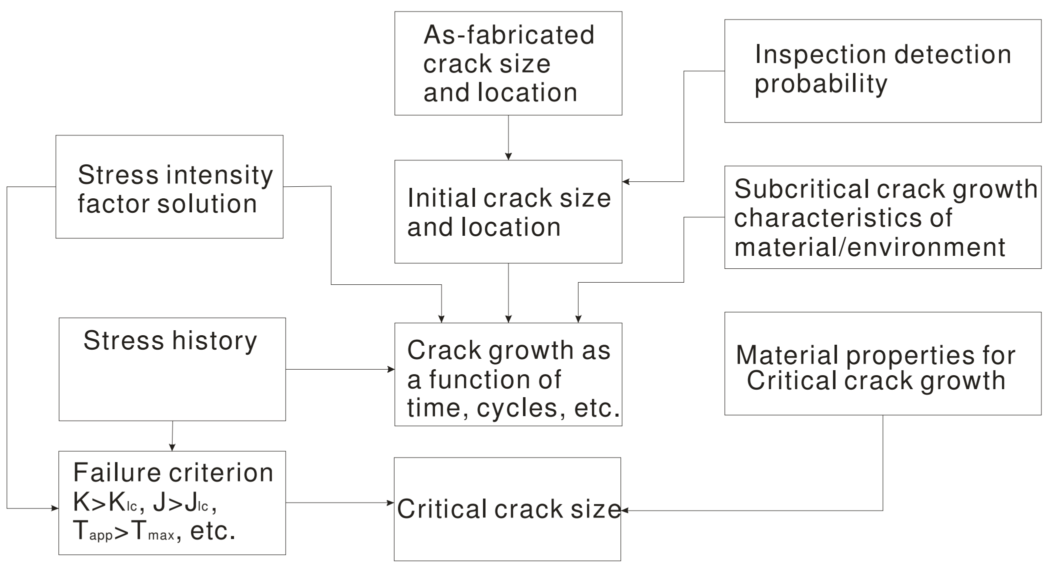


*Figure 2-4* DFM for prediction of crack growth and crack instability [27]

The behaviour of cracks is usually governed by their strain energy release rate. In linear elastic solids, this can be expressed in terms of the stress intensity factor, K. for non-linear elastic solids, which are often used to represent elastic-plastic metals; the value of the J-integral describes the strain energy release rate. The crack driving force depends on strain level, loading level and distribution, crack size, and body geometry. The K of a simple structure is usually expressed as

$$K = S\sqrt{(\pi a)} \quad \text{2- 13}$$

$$da/dN = c\Delta K^n \quad \text{2- 14}$$

For more complex geometries, the expression for K is similar, but contains factors related to crack and body geometry. The related details could be easily found in fracture mechanics textbook.

In the PFM, there are two aspects of variables that may be concerned. One of them is initial crack size (a) distribution as it does a strong influence on the calculated failure probability. Other is the detective effect of inspections, e.g. probability of detection (POD). For a given size, the probability of detection is POD (a). If $a^* > a_{repair}$ then some repair is performed. The inspection itself does not affect the reliability; only repairs or remedial actions performed as a result of the inspection have an effect.

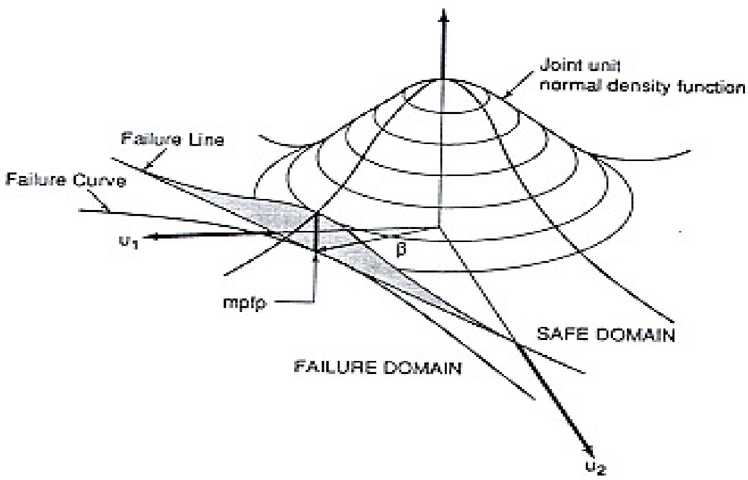


*Figure 2-5* Joint density function in unit variate space

Figure 2-5 shows the theory. The process of the PFM model construction does not include here, but it could be found in reference [28]. Once the distribution and the model were defined, the calculation for the failure probability appears formidable. Therefore, the numerical technique, such as Monte Carlo simulation, was involved in this procedure. Some illustrative examples are also list in reference [29].

#### *2.1.6.3 Data resource*

The PFM analyses generally require considerably more data than are necessary for the corresponding deterministic problem.

Data on distribution of material properties can be generated in the laboratory and/or gather from the literature. Uncertainties in distribution type and parameters of the distribution due to sparseness of data should be kept in mind, but have rarely been considered in PFM analyses.

Data on load histories or spectra can be gathered experimentally, such as directly from strain gages, can be based on engineering models that employ some well-characterized underlying forcing function, such as wind loads on past data.

Data on inspection reliability and uncertainty can be gathered in laboratory or from the literature.

Data on initial crack size distribution and location are important to any PFM analyses. Unfortunately, information on initial crack size distribution is sparse and expensive to gather. Estimates of initial crack size can be made on the basis of past experience and engineering judgment or by back calculation from failure data, which would be discussed in the following chapter.

#### *2.1.6.4 Conclusion*

The PFM is the fundamental and essential to structural lifetime prediction and reliability analysis. It normally was used in the components' crack growth and instability calculation and has been widely applied to structural reliability analysis program. The randomness of the initial crack sizes and flaw detection were very well considered in the model.

Future trends in PFM are expected to include increased usage for inclusion in decision tree and fault tree analyses of system reliability. This includes generation for various candidate inspection strategies that are used in cost and risk optimisation of inspections. A key input to such analysis, as for any PFM analysis, is the initial crack size distribution. Crack size information is sparse and usually expensive to generate. A credible technique for obtaining such estimates is highly desirable. The crucial progress will point to the calculation of flaw sizes distribution and frequencies. The use of prediction interval (rather than confidence interval on the mean) holds promise for future development of random variables that include both uncertainty due to sparse data and inherent randomness of the data.

### 2.1.7 Probabilistic Fatigue Analysis

#### *2.1.7.1 Reviewing*

As mentioned before, fatigue has been becoming the critical issue in aging aircraft structure since the end of 1970s. However, available information indicates that many fatigue failure result from poor details, which means uncertainty in the fatigue analysis process exit apparently. For example, the fatigue phenomenon in unpredictable, as evidenced by enormous statistical scatter in laboratory data, with cycle-to-fatigue data having coefficients of variation (COV) typically rang from 30~40% and sometimes as high as 150% [30]. Therefore, a probabilistic and statistical approach is particularly relevant.

Over the years, there have been numerous article and some books written on the topic of statistical analysis of fatigue data [31]. The standards published by the ASTM (American Society for Testing and Materials, 1987) provide a guideline for analysis of fatigue data. The lasted version is being reviewed and updated. In general, analyses of the fatigue process are complicated by following factors:

- The log-log transformation will not always linearize the data.
- The data tends to be heteroscedastic as the scatter band of life at a given stress broadens at lower stress levels.
- There will be some run out, or censored data.

These issues have been addressed by Schmee and Hahn (1979), Nelson (1984), and Hinkle (1991).

Since the 1980s, a number of researches have been achieved in the area of fatigue reliability analysis. Prof. Y. S. Feng (North-western Polytechnical University, 1982) set up a probabilistic model for aircraft structural fatigue analysis in his publication[32], and R. Melchers (1987, 1998)[33], Wirshing, P. H. (1990, 1995)[34,35], Ruggieri, C.(1997)[36], and Kordonsky, Kh (1997)[37] etc have made a lot of developments on fatigue reliability analysis. The procedure for practical application usually involved classification of structural details, cycle counting methods and the use of S-N curve. However, fracture mechanics approaches considering crack initiation and crack propagation have also been considered in the airspace structures. The details of the theories and formulations could be found in references [35]~[39]. A summary of the reliability methods involved in fatigue analysis is listed in table 2-2.

*Table 2-2* The reliability methods involved in probabilistic fatigue analysis[38]

| **Analysis Methods** |
|---|
| 1. Mean value first-order second moment (MVFOSM; Cornell, 1969) |
| 2. Hasofer-Lind generalized safety index (Hasofer and Lind, 1974) |
| 3. First order reliability method (FORM)<br>• Limit states represented by tangent hyperplanes at design points in transformed standard normal space (Madsen et al., 1986)<br>• Rackwitz-fiessler algorithm (1978) |
| 4. Second order reliability Method (SORM)<br>• Limit states represented by tangent hyperplanes at design points in transformed standard normal space (Madsen et al., 1986)<br>• Wu/FPI algorithm (WU and Wirshing, 1987) |
| 5. Advance Mean value (AMV) method (Wu et al., 1990) |

| Monte Carlo simulation |
| --- |
| 1. Direct Monte Carlo |
| 2. Importance sampling (Shinozuka, 1983) |
| 3. Domain-restricted sampling (Harbitz, 1986) |
| 4. Adaptive sampling (Bucher 1988) |
| 5. Directional sampling (Bjerager, 1990) |

#### *2.1.7.2 Comments*

The advantage of probabilistic fatigue analyses is to consider material properties, cycle stress and fatigue life as variables that very well describe the nature of their uncertainties. Therefore, reliability methods are particularly appropriate for quantifying structural performance for fatigue failure mode.

The lognormal format is particularly useful for the elementary case where fatigue strength by the form $NS^m$=A, in which the m and A are empirical constants, N is cycle to failure, and S is stress range.

The most areas for further work are in the areas of identification of critical locations for fatigue damage in aircraft structures and the corrosion fatigue mechanism.

### 2.1.8 Fuzzy reliability theories

#### *2.1.8.1 Theory Introduction*

The complexity in the engineering design arises from uncertainty. Uncertainty is usually consisted of randomness and vagueness. Randomness involves only uncertainties in the outcomes of an experiment; Vagueness, on the other hand, involves uncertainty in the meaning of the data. Examples of vagueness include experiments involving linguistic data, which for the purpose of information processing have to be modelled with greater care. Historically, probability theory has been the primary tool for representing uncertainty in mathematics. However, not all uncertainty is random. Some forms of uncertainty are non-random and hence not suited to treatment or modelled by probability theory. One prevalent way to convey information is our own means of communication: natural language. By its very nature, natural language is vague and imprecise. The underlying power of fuzzy set theory is that it uses linguistic variables, rather than quantitative variables, to represent imprecise concepts. Fuzzy set theory is a marvellous tool for modelling the kind of uncertainty associated with vagueness, with imprecision, and/or with a lack of information regarding a particular element of the problem at hand.

Fuzzy (set) logic has come a long way since it was first subjected to technical scrutiny in 1965, when Dr. Lotfi Zadeh (University of California) published his seminal work “Fuzzy sets” in the journal information and control. It was the early 1990s that fuzzy logic was introduced to the area of reliability engineering. The original work started from Dr. K. Y. Cai[39] (Study on Fuzzy Reliability, the dissertation of Dr. K. Y. Cai, 1990, Beijing University of Aeronautics and Astronautics). Afterward, other scientists and engineers, who pushed reliability theories forward a great step, have made many developments.

The main principle of fuzzy reliability is it tries to change two fundamental irrational assumptions in conventional reliability theory:

a) Binary fault assumption

b) Probability assumption

Dr. Cai thinks there is a process between “Normal state” and “Fault state” and the probability applied in describing “reliability” doesn’t satisfy presuppositions of probability theory. It is reasonable, he thinks, to describe the “reliability” with “possibility” concept instead of probability. The detailed theories and its applications of fuzzy reliability could be found in references [40, 41].

#### *2.1.8.2 Comments (Advantages and disadvantages)*

The relatively new Fuzzy Set concept is still developing. Much has yet to be explored in order to capitalize its applications, particularly in the area of probabilistic structural mechanics. Fuzzy reliability theory has shown its huge advantage in very well describing the vagueness nature of an event, both in random and in vagueness, since it was introduced into the field of reliability. It is undoubtedly leading the direction of reliability theory development. Although much progress has been made in fuzzy reliability modelling, the application of fuzzy reliability theory to structure reliability analysis still has a lot of big obstacles. For example, the issues concerned with determining of membership and its parameters. Owing to its fuzziness, assigning membership functions with few data may lead to erroneous result.

### 2.1.9 Reliability analysis of Structural Systems

#### *2.1.9.1 Introduction*

In reality, aircraft structure is composed of many members or elements and each component could involve various limit states such as bending action, shear, buckling, axial stress, deflection, etc. Furthermore, there may exist limit states for the structure as a whole rather than its elements (e.g. overall deflection, foundation settlement, residual stiffness) and the configuration of the structure itself may be of importance. It is likely, therefore, that the reliability assessment of structural systems will involve the need to consider multiple and perhaps correlated limit states.

Actually, structure system reliability analysis is quite complex in comparison to component risk assessment, although the same requirements are evident, such as an accurate statistical database, probabilistic description of random variable, and calculations of risks. The major difference is the system analysis requires the formulation and identification of the numerous potential collapse modes and their combination into a single assessment of system risk. In other structure configurations or with different materials, the failure of any single one of many significant members may lead to catastrophic consequences.

#### *2.1.9.2 Methodologies*

The fundamental system reliability problem is to extend the analyses of component reliability to an overall structure reliability assessment. One approach includes direct methods such as Monte Carlo, Simulation, Point estimation, and response surface generation. Direct methods that are relatively easy to apply to systems have been generally thought to be less accurate and offer little insight into the contributing variables that may affect system failure probability.

The second system formulation approach is failure mode analysis, which has received wide use in offshore platform, bridges, and other structural frameworks. Similar to fault tree analysis, the failure mode formulation lends itself to a physical representation and subsequent relationship between system and component failure probabilities. These allow opportunities for optimisation of both the member capacities and the structure topologies.

The third method is overall system descriptions. A system is an assemblage of components best characterized by its geometry, material response, and statistical correlation. The single failure mode of component can be solved by mentioned above methodologies, and the importance of the correlation (COV) between the different failure modes in the structural system appears. Prof. Fred Moses had ever proposed some models, such as Series model, parallel model, incremental Loading model etc. all those model has received extensive applications in large offshore platform structure (Nordal et al. 1987)[42] and bridge structures (Liu and Moses 1991).

#### *2.1.9.3 Comments*

System reliability is playing an important role in expressing the goals of structure safety. The formulation of system models must account for the potentially large number of failure modes, the statistical correlation between loading and between member strengths, performance of members after reaching their limit state condition, and geometry. Formulation of system models through failure tree searches is feasible even for large structures, such as aerospace or offshore structures. Further development is needed to make system models more accurate and consistent and also more accessible to designer for making risk-benefit tradeoffs.

## 2.2 *Tools in practice*

### 2.2.1 Damage Tolerance Assessment Software ---DARWIN

#### *2.2.1.1 Introduction*

Design Assessment of Reliability With Inspection (DARWIN) is a United States Federal Aviation Administration (FAA) certified probabilistic damage tolerant assessment software tool developed by the Southwest Research Institute of America (SwRI).

Based on the zone theory and the Monte Carlo method, DARWIN is specifically designed for fatigue life management of titanium aircraft rotor disks subject to the possibility of catastrophic failure due to the presence of hard alpha defects from the very beginning. As for any probabilistic assessment tool, the underlying assumptions that underpin the probabilistic models are one of the key aspects that govern the validity of any risk assessment. This is particularly the case for DARWIN, which may potentially be applied to future aircraft structure probabilistic assessment related issues [43].

#### *2.2.1.2 Zone-based probabilistic approach*

The difficulty in assessing the probability of failure of titanium rotor components due to fatigue crack initiation from hard alpha rests in the fact that such a defect occurs randomly at any locations within a rotor disk, with the size of these hard alpha defects being also random in nature. Whether a particular defect could cause a disk to fail would depend on its location, the stresses that it experiences, the fatigue crack growth rate at this location and the ability to detect, and remove, cracked rotor disks prior to failure.

The unique feature of DARWIN is the use of a so-called zone-based probabilistic approach to account for random defect location to compute the probability of failure of the entire rotor disk. This approach allows different areas on a disk to have their own defect occurrence rate, material properties, inspection parameters, loads and temperatures, so that the probability of failure of the entire disk made to be more realistically modelled. Further advantages of this approach are that it enables the critical areas to be identified, providing valuable information for design and inspection planning.

#### *2.2.1.3 Computational efficiency*

Any formal probabilistic analysis to compute the probability of failure by fatigue on a cycle-by-cycle basis for a number of different zones, as found in DARWIN, requires a great deal of computational time. The problem here is further complicated by the need to allow the merit of inspections and maintenance to be taken into account during both fatigue life prediction and probabilistic analysis. In addition, to repetitively perform the same procedures for all zones would mean that enormous computational evaluation time is required. For these reasons, the need for computationally fast and efficient probabilistic assessment procedures was recognized as a major objective during the development of DARWIN.

DARWIN uses the Monte Carlo Simulation method as the basis of the probabilistic methodology. However, this method is widely recognized as slow and computationally inefficient, particularly when evaluating low probabilities of failure, such as $10^{-6}$-$10^{-4}$, as required in the aerospace industry. The strategy adopted for improving the overall computational speed of DARWIN is to improve the computational efficiency during both probabilistic and fatigue crack growth analysis. Three major techniques and procedures are currently used in DARWIN to improve computational efficiency:

- Use of a pre-determined fatigue life distribution function;
- Use of the Life Approximation function;
- Use of the so-called importance sampling method.

#### *2.2.1.4 User interface*

The attractive feature of DARWIN is that a user friendly graphical user interface is available for handling and viewing input data, conducting fatigue crack growth analysis, setting up the multiple zone probabilistic analysis, and for viewing results. As part of the GUI, an excellent help file is also available, allowing easy troubleshooting and operation of the software to be made.

### 2.2.2 Risk Assessment Software ---PROF

#### *2.2.2.1 Introduction*

The USAF contracted the research institute of the University of Dayton to develop a computer program called PROF (PRobability Of Fracture) for aircraft structure risk assessment. The first version [44] was finished in 1991 to perform probabilistic risk analysis related to the primary failure mode of fatigue crack growth in a metallic aging aircraft structure, and it is used as a tool by the Air Logistics Centres in making decisions related to the timing of maintenance actions in the aging aircraft fleets.

Since the initial development of PROF, other damage scenarios associated with aging aircraft have been identified and investigated. The newly-identified risk analysis scenarios include:

a) Discrete source damage in the presence of MSD

b) WFD (both multi-site damage and multi-element damage)

c) The potential effect of corrosive thinning in a fleet.

Therefore, the American air force renewed the contract to develop the second version PROF program called WinPROF[45] in 1998. The primary goals of this version were to develop appropriate tools within the PROF analysis system to facilitate probabilistic risk analysis of these damage scenarios. Two additional capabilities were added to PROF to accomplish these goals: probabilistic risk assessment of discrete source damage (DSD) and a multi-run data management capability. A secondary, but necessary, goal was to modernize the windows user interface and improve the calculation algorithms of the original PROF code. All the FORTRAN codes in the first version were converted to $C^{++}$ (second window version). As well as the computation has been updated and the computing speed has increased dramatically. Problems that took up 30 minutes when PROF was first converted to PC now take second.

#### *2.2.2.2 The Procedure for computation*

The risk analysis model, PROF, addresses a single population of structure elements. The populations are defined in terms of all details that experience essentially equivalent stress history intensity factor coefficients. Such populations of potential crack sites are defined during the ASIP damage tolerance analysis. Each structural element in the population of details is assumed to contain a crack whose size at T spectrum hours is a random variable with a probability density.

There are three contexts for interpreting this distribution of cracks:

- An individual structure element

- A single airframe with many such ‘identical’ elements
- The fleet of airframes

In this program, the Probability of Fracture (POF) is calculated for a single flight and for any flight in the interval between the start of analysis and each inspection. POF is also calculated for any flight within each inspection interval. Maintenance costs are quantified in terms of the expected number of cracks that will be detected and repaired at each inspection and total expected costs of the planned maintenance scenario.

Risk assessment addresses both safety and durability. Safety is quantified in terms of the probability of a fracture resulting from the maximum load in a flight exceeding the critical load associated with the fracture toughness level. Durability is quantified in terms of the expected number and sizes of the cracks to be detected and repaired at each inspection and repair-if-necessary cycle and the expected costs of these repairs.

#### *2.2.2.3 Data required by PROF*

(1). Material or Geometry data: $K/\sigma$ versus a

(2). Distribution of fracture Toughness

PROF assumes fracture toughness values have a normal distribution and requested the mean and standard deviation of K for the particular material of the application

(3). Aircraft and aircraft usage data

Aircraft/usage data are specific to the past and expected usage of the fleet of aircraft being analysed. The initial structural design, manufacturing quality, and past usage determine the distribution of crack size that locations at the start of the analysis. The expected usage determines the projected growth of the crack and the operational stress peaks that may be encountered.

a. Aircraft population parameters

An individual execution of PROF is based on the analysis of a single distribution of crack sizes emanating from stress raisers in structure. The population modelled by this distribution can represent a single location in each airframe of the fleet. If there were multiple locations in each airframe that will experience essentially equivalent stress histories and have equivalent stress distribution factors, the crack size distribution would also apply to each of the stress raisers in the zone of equivalence. There are three fracture probabilities of interest to cover these populations:

- The POF at a single stress raiser
- The POF at any stress raiser in a single airframe of the fleet
- The POF for any stress raiser in any airframe of the fleet

b. Crack size versus flight time

Crack growth is a random phenomenon. If specimens containing cracks of "constant" size are subjected to a common stress history in the laboratory, a distribution of sizes will result.

PROF uses a deterministic correlation between spectrum flight hours and crack size as the basis for projecting the growth of the distribution of crack assumed to be in the population of structural details. This is accomplished by projecting percentiles of the crack size distribution base on the deterministic "a versus T" relationship for the expected stress sequence.

c. Maximum stress distribution

POF is calculated as the probability that an applied stress will exceed the residual strength of the cracked structure detail. For practical purpose, it can be assumed that the stress peak that will cause fracture is the largest peak to be encountered in a flight. Since available data might not extend to the largest stresses that might be encountered, a consistent basis for extrapolation was required. In PROF, the distribution of this maximum stress peak in a flight is modeled in terms of a Gumbel distribution of extreme values. The model, format, and example are described in

"Alan P. Berens, Peter W. Hovey, ' Risk Assessment for Aging Aircraft Fleets' WL-TR-91-3066, Vol. 1"

d. Initial crack size distribution

The calculation of PROF is independent of the method of modeling the initial crack size distribution; PROF requires only that the initial crack size distribution file contain a valid cumulative distribution function.

(4). Inspection/repair Data

a. Maintenance Times

The maintenance times are the number of flight hours at which the inspection and repair (if necessary) cycle is performed in the calculations of PROF.

b. Inspection Capability: Probability of Detection function (POD)

c. Repair crack size distribution

The equivalent repair crack size distribution is analogous to the equivalent initial quality distribution in concept.

d. Maintenance Cost

- Expected maintenance costs are not computed in PROF
- PROF provides an output from which expected maintenance costs can be calculated

#### *2.2.2.4 Output Data*

PROF output comprises three types of information: a screen plot, a tabular summary file, and data files. The details of the output information are as follows

(1) A summary of the input data either in form of file names and file description or the parameter values

(2) Single flight POF values for single details, single airframes and total fleet at ten time intervals between each inspection;

(3) Percent of inspection sites at which cracks are expected to be detected at each inspection;

(4) POF values for each usage interval for single details, single airframe, and total fleet;

(5) POF value for the total analysis interval (0 to t) for single details, single airframe, and total fleet;

(6) Crack size data at each inspection or repair - the crack size distribution before the inspection and after the cracks are repaired, the cumulative proportion of detected cracks, and the cumulative distribution of the size of the detected cracks.

#### *2.2.2.5 Sensitivity analysis*

Eight of the nine PROF input items can significantly affect the output of PROF. The sensitivity analysis of input variables can be used to evaluate items that cause the greatest change to the output. As some of the input to PROF reflects fleet maintenance actions, the results of the sensitivity analysis and trade off studies can be used to evaluate the results of specific actions in the fleet's maintenance. Appropriate fleet management action can then be taken to minimize these effects.

### 2.2.3 The Numerical Evaluation of Reliability Function---NERF

NERF[46], a computer software package for the Numerical Evaluation of Reliability Functions was developed by the Defence Science and Technology Organization in 1984, to enable a range of analysis options to be chosen for evaluating the risk of failure due to wear out processed such as fatigue or corrosion.

NERF evaluates time histories of reliability functions representing the aggregated fatigue behaviour of a population of structures or components, which possibly contain a crack at the location being studied. Given data describing the mean rate of fatigue crack growth and resultant strength degradation together with a statistical description among other functions, the risk rate, failure probability and survival possibility are gained directly. An important and useful facility of the program is that it allows for the effects of various inspection strategies and can be used as a development tool in selecting the best strategy for a particular population of structures.

NERF has been implemented using a friendly graphical user interface that can be used to specify and edit all the input data and options used by the analysis model. The probability of load exceedance, the median crack growth and the median strength functions are defined by sequences of ordered pairs to which NERF fits cubic splines. These functions can be imported from spreadsheets and can be edited within the user interface.

Further description on the functions, formulations, operations, required input data, and output data detailed in reference.

### 2.2.4 Computer programs available for FORM/SORM

Numerous computer programs have been developed by researchers to implement the FORM/SORM algorithms. Three of the commercially available programs in America are described here.

#### *2.2.4.1 NESSUS*

NESSUS [47] (Numerical Evaluation of Stochastic Structures Under Stress), developed at the Southwest Research Institute (1991) in San Antonio, Texas under sponsorship by NASA, combines probabilistic analysis with a general-purpose finite element/boundary element code. Structural analysis is performed using either the displacement method, or the mixed iterative formulation or boundary element method, and iterative perturbation is used for sensitivity analysis. Solution capabilities include transient, non-linear analysis for a large deformation/ displacement conditions, and fatigue/fracture problems. The program also includes techniques such as fast convolution, and curvature-based adaptive importance sampling. System reliability and risk assessment capabilities in the program use either fault tree analysis combined with adaptive importance sampling, or a structural reanalysis procedure to account for progressive damage. The program is available on Vax mainframes and SUN workstations.

#### *2.2.4.2 PROBAN*

PROBAN [48] (PROBability ANalysis) was developed at Det Norske Veritas, and designed to be a general probability analysis tool. Particularly efficient methods are available for computing small probabilities, which often arise in structural reliability problems. It can be applied in many different areas, including marine and offshore structures, mechanical and airspace structures, civil structures, and many other applications. PROBAN is capable of estimating the probability of failure, using the FORM/SORM for a single event, unions, intersections, and unions of intersections. It has a library of standard probability distributions. The approximate FORM/SORM results can be updated through importance sampling. The probability of general events can be computed by Monte Carlo simulation and directional sampling. Probability distribution computation can be performed by Monte Carlo simulation or Latin hypercube sampling. Sensitivity analysis by simulation is also available.

#### *2.2.4.3 CALREL*

CALREL [49] (CAL-RELiability) was developed at the University of California by Liu (1989). It incorporates four general techniques for computing the probability of failure

a) FORM

b) SORM

c) Directional simulation with exact or approximate surfaces, and

d) Monte Carlo simulation

CALREL has a large library of probability distributions for independent as well as dependent random variables. Additional distributions can be included through a user-defined subroutine. CALREL was written in FORTRAN-77 and operates on IBM-PC or compatible personal computers, as well as on computers with the Unix operation system.

### 2.2.5 Probabilistic Fracture Mechanics computer Software

Commercial computer softwares are now available on both deterministic fracture mechanics and probabilistic fracture mechanics. A summary of those follows as below.

#### *2.2.5.1 Computer software available for deterministic fracture mechanics*

*Table 2-3* Softwares for deterministic analyses of crack growth and instability

| Name | Reference | Available |
|---|---|---|
| NASA/FLAGRO | Forman et al. (1988) | Cosmic Code Centre, (Atlanta, GA) |
| NASCRAC | Harris et al. (1987) | Failure Analysis Associates (Mento Part, CA) |
| Pc-CRACK | | Structural Integrity Associates (San Jose, CA) |
| P/FATIGUE | | PDA engineering |
| Crack Growth Analysis | | Computational Mechanics Publ. |
| PCFAD | | The Babcock & Wilcox Company |
| DA/DN | Quinones et al (1988) | Electric Power Research Institute |

### 2.2.5.2 Computer software available for probabilistic fracture mechanics

*Table 2-4* Softwares for probabilistic fracture mechanics

| Name | Application | Ref. | Available from |
|---|---|---|---|
| PC-PRAISE | Stress corrosion and fatigue crack growth in Commercial power reactors | Harris et al. (1981,1992) | Livermore, CA |
| SAFER | Creep-fatigue crack growth in steam turbine rotors; includes thermal and stress analysis | Ammirato (1988) | Electric Power Research Institute, CA |
| BLESS | Creep-fatigue crack initiation and growth in boiler components; includes thermal and stress analysis | Grounloh et al. (1992) | Electric Power Research Institute, CA |
| PACIFIC | Fatigue crack growth | Dedhia and Harris, (1988 | Failure Analysis Associates (Menlo Park, CA |
| PROBAN | Comprehensive Structural reliability code; includes some fracture mechanics; can assist in development of inspection strategies | | DNV Industrial services Houston, TX |
| R/ring-life | Crack initiation and growth due to stress corrosion cracking in electrical generator relaining ring | Ricardella et al. (1991) | Electric Power Research Institute, CA |
| NESSUS | Comprehensive structural reliability code; includes some fracture mechanics | Millwater et al. (1992) | Southwestern Research Institute, NASA (San Antonio, TX) |
| PRISM | | | Bombardier, CA |
| COMPASS | | | Martec |
| PROMISS | | | Martec |
| PFAFAT | | | Cal Tech/JPL |
| TRACWFD | | | Battelle for the FAA |

#### 2.2.5.3 *Developed Reliability Analysis Software*

*Table 2-5* Overview of existing numerical tools

| Tools | Capability | Source |
|---|---|---|
| CALREL | • Reliability analysis for components and systems using FORM/SORM<br>• Sensitivity analysis, Monte Carlo Simulation and directional simulation<br>• Statistical Library<br>• Can be operated as a shell program in conjunction with other codes | University of California, USA |
| ISPUD | • Reliability analysis using Monte Carlo Simulation and adaptive IS<br>• Explicit limit state functions<br>• Response surface method (implicit limit state functions)<br>• Time dependent reliability analysis using the extreme value approach | University of Innsbruck, AU |
| COSSAN | • Stochastic Finite Element analysis (SFEM) Description of model uncertainties as stochastic fields rather than random variables<br>• Advanced Monte Carlo Simulation<br>• Response surface method<br>• Reliability based optimization | University of Innsbruck, AU |
| PROBAN | • FORM/SORM method<br>• Importance sampling and directional simulation<br>• Sensitivity analysis<br>• Stochastic material and model description<br>• Time dependent reliability analysis (first passage problems) | DNV software, Norway |
| STRUREL | • Reliability analysis for components and systems using FROM/SORM<br>• Package for Statistical analysis<br>• Structural and numerical analysis<br>• Time variant reliability analysis (out crossing rate) | RCP GmgH, German |
| VAP | • Windows based and user friendly environment<br>• Reliability analysis using FORM and Crude MCS | IBK, ETH Zurich Switzerland |

## 2.3 *Relative research perspective*

Numerous methodologies have been built up for structural elements and system reliability assessment since the 1980s and some new models for structural system reliability analysis have been proposing as well. Monte Carlo is a good simulation method, but the accuracy highly depends on the quantity of computation and usually it is time consuming. Fuzzy reliability theories are the direction of the development of reliability engineering, which very well describes the nature of both vagueness and randomness, but the present problems are lack of appropriate membership and its parameters. The further investigation would depend on test and field data collection.

Many applications of the existing methodologies can be found widely. However, although the basic principle of those applications is similar in whatever aircrafts or civil structures, the applied loads variables or spectrum and the concerned output objectives are quite different. Aircraft structures emphasize more on the risk (probability) of structural fracture and the determination of the relevant inspection intervals and cost. The probabilistic finite element method couple with FORM is strongly recommended for using in aircraft reliability analyses due to the highly complex structure and multiple composed random loads.

It must be kept in mind that probabilistic theories are not implemented for determining the exact variation of the true population but it is to allow reliable approximation of the population in a practical and prevailing fashion when data are scare. The ability of the analyst to ensure the population and important variables are statistically and effectively estimated by utilising the available approximation techniques effectively to avoid over-conservative assumptions is an important element in reliability analysis. These are some of the important factors that increase the readiness and feasibility of risk and reliability analysis for aircraft structures, component and fleet management.

A lot of computer softwares with different theories for different purposes now are available by commercial purchase. The Second version “WinPROF” developed by American Air force is a powerful tool for aging aircraft structural risk assessment and the fleet management associated with making decisions related to the timing of maintenance actions. Therefore, it is strongly recommended to be adapted for using in aging fleets management.

An efficient data base is very important to be built up for collecting the failure data, such as crack length versus flight hour, in the RAAF. The ways of gathering data varies from routing baseline inspection, teardown inspection, to laboratory test. However, the FRACAS (Failure Report And Corrective Action System) is the best way to the aircraft failure information collection, which has been successfully applied in the USA and China.

# CHAPTER 3 INITIAL CRACK SIZE EQUIVALENCE

## 3.1 *Introduction*

During the maintenance of aging aircraft structure, many damages are found to be pit as a result of corrosion. The influence of such corrosion on the fatigue performance of aircraft structures is of considerable importance in evaluation of the structural integrity of aging aircraft. As introduced in section 2.1.6, the initial crack size is critical as the input data for many failure probability softwares, such as BLESS, PACIFIC, PROBAN.

Many aspects of this problem have been studied extensively over the past decades. Models for pitting development and for the initiation of fatigue cracks under corrosion fatigue conditions have been proposed and have the validity for the range of materials and corrosion conditions under which they were derived. The fracture mechanic based approach was first suggested in 1961 by Paris, Gomez and Anderson, who related the crack extension per cycle (da/dN) to the maximum stress intensity factor $K_{max}$, which led to the well known Paris formula [50, 51].

In this chapter, the equivalent initial crack size approach is presented, which provides the fracture mechanics based solution for initial damage evaluation and for modelling fatigue life. FORTRAN program, based on the Paris law, is designed as a route in which corrosion damage could be quantitatively derived. The derived damage data is assessed conservative to predict the service life of corroded structures. In this approach, experiment data from Ref [52] is used for the sake of analysis, which described the fatigue lives under constant amplitude loading on pitted samples. To compare the experimental lives with predicted ones, the computer package AFGROW was also employed.

## 3.2 *Modelling*

At pit growth stage, the damage is expressed as the initial crack size $a_0$. In the literature, it is assumed to be the size of the nucleating particle, or it depends on the capability of NDE, or it can be derived from the dimension of pit perpendicular to the primary loading direction [53,54,55]. Considering the complicate loading effects on the crack size, the pit size parameters a and c are not able to be used as the direct input of $a_0$, while it is possible to calculate the equivalent initial crack size by applying extrapolation of the Paris law.

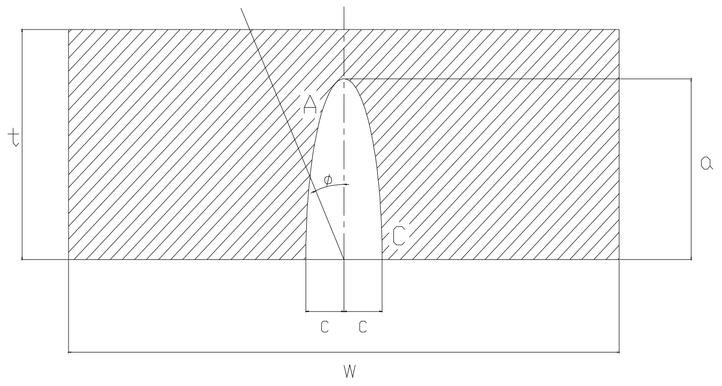


*Figure 3- 1* Sketch of semi-ellipsoidal pit

As shown in Fig3-1, a and c is able to be expressed as below applying Paris law:

$$\frac{da}{dN} = C_A(\Delta K_A)^m \qquad 3\text{- }1$$

$$\frac{dc}{dN} = C_C(\Delta K_C)^m \qquad 3\text{- }2$$

Where $\Delta K_A$, $\Delta K_C$ stands for the equivalent stress intensity factor range at extreme point A and C. da/dN is the crack growth per cycle (a is the crack length and N is the number of loading cycles), $C_A$, $C_C$and m are material constants.

Clearly, ΔK plays a role of "driving force" for fatigue crack advance in the Paris Law. It is a complicated process to work out this parameter as it exists in 3D dimension in fact. Newman and Raju had developed a useful formula widely referred in practice, by finite element analysis [56]:

The equation has the following format:

$$\Delta K = (\sigma_t + H\sigma_b)\sqrt{\frac{\pi a}{Q}}F(\frac{a}{c},\frac{a}{t},\frac{c}{w},\psi) \qquad 3\text{-}3$$

where $\sigma_t$ and $\sigma_b$ are the remote membrane and pure bending stresses at surface, respectively, H is a geometry parameter, which depends on the crack depth ratio a/t, aspect ratio a/c, Q indicates the shape factor for the circular hole, the boundary correction factor $F(\frac{a}{c},\frac{a}{t},\frac{c}{w},\psi)$ is a function of crack depth, aspect ratio, plate thickness, and the parametric angle ψ on the crack edge, where t and w stand for the material

The function $F(\frac{a}{c},\frac{a}{t},\frac{c}{w},\psi)$ can be rewritten as:

$$F = [M_1 + M_2(\frac{a}{t})^2 + M_3(\frac{a}{t})^4]gf_\psi f_w \quad 3\text{-}4$$

Where, when $\Phi=\frac{a}{c}\leq 1.0$

$$M_1 = 1.13 - 0.09\frac{a}{c} \quad 3\text{-}5$$

$$M_2 = -0.54 + \frac{0.89}{0.2+\frac{a}{c}} \quad 3\text{-}6$$

$$M_3 = 0.5 - \frac{1}{0.65+\frac{a}{c}} + 14(1-\frac{a}{c})^{24} \quad 3\text{-}7$$

$$g = 1 + [0.1 + 0.35(\frac{a}{t})^2](1-\sin\psi)^2 \quad 3\text{-}8$$

$$f_\psi = [(\frac{a}{c})^2\cos^2\psi + \sin^2\psi]^{\frac{1}{4}} \quad 3\text{-}9$$

$$f_w = [\sec(\frac{\pi c}{2w}\sqrt{\frac{a}{t}})]^{\frac{1}{2}} \quad 3\text{-}10$$

When $\Phi=\frac{a}{c}>1$, $f_w$ has the same format, while

$$M_1 = \sqrt{\frac{c}{a}}(1 + 0.04\frac{c}{a}) \quad 3\text{-}11$$

$$M_2 = 0.2(\frac{c}{a})^4 \quad 3\text{-}12$$

$$M_3 = -0.11(\frac{c}{a})^4 \quad 3\text{-}13$$

$$g = 1 + [0.1 + 0.35\frac{c}{a}(\frac{a}{t})^2](1 - \sin\psi)^2 \quad 3\text{-}14$$

$$f_\psi = [(\frac{c}{a})^2 \sin^2\psi + \cos^2\psi]^{\frac{1}{4}} \quad 3\text{-}15$$

Parameter Q has the following formula:

$$Q = 1 + 1.464(\frac{a}{c})^{1.65} \quad 3\text{-}16$$

The function H has the form:

$$H = H_1 + (H_2 - H_1)\sin^{\gamma}\psi \quad 3\text{-}17$$

$$\gamma = 0.2 + \frac{a}{c} + 0.6\frac{a}{t} \quad 3\text{-}18$$

$$H_1 = 1 - 0.34\frac{a}{t} - 0.11\frac{a}{t}(\frac{a}{c}) \quad 3\text{-}19$$

$$H_2 = 1 + G_1\frac{a}{t} + G_2(\frac{a}{t})^2 \quad 3\text{-}20$$

In the equation for $\mathrm{H}_2$, we have:

$$G_1 = -1.22 - 0.12\frac{a}{c} \quad 3\text{-}21$$

$$G_2 = 0.55 - 1.05(\frac{a}{c})^{0.75} + 0.47(\frac{a}{c})^{1.5} \quad 3\text{-}22$$

Accordingly, the case of a semi-elliptical surface crack stress intensity factor range under uniform remote tension gives approximately from the above equations:

$$\Delta K = \Delta\sigma\sqrt{\frac{\pi a}{Q}}[1.04 + 0.2(\frac{a}{t})^2 - 0.1(\frac{a}{t})^4][\sec(\frac{\pi a}{2w}\sqrt{\frac{a}{t}})]^{\frac{1}{2}} \quad 3\text{-}23$$

By integration, formula 3-1 can be transferred to:

$$\int_0^{N_f} dN = \int_{a_0}^{a_f}\frac{da}{C(\Delta K)^m} \quad 3\text{-}24$$

Where $N_f$ is fatigue life, $a_f$ is extreme fatigue size, which is known to the specific material.

The equivalent crack size is then could be obtained through the following none-linear equation:

$$F(a_0) = N_f - \int_{a_0}^{a_f} \frac{da}{C(\Delta K)^m} = 0 \qquad \text{3-25}$$

## 3.3 *None-linear Method*

### 3.3.1 Introduction

Monte Carlo and iterative method are selected to solve the complicated none-linear equation. By Monte Carlo integration, the estimated fatigue life would be:

$$N_f^{'} \approx \frac{a_f - a_0}{n} \sum_{i=0}^{n-1} \frac{1}{C(\Delta K)^m} \qquad \text{3-26}$$

Where n⇒ ∞, c and m are material constant, $\Delta K$ can be gained from formula 3-23, $a_i = a_0 + (a_f - a_0)\xi_i$ , and $\xi_i$ (i=0,1,2…n-1) is uniformly distributed random ranging in [0,1].

Then, based on the experiment data from [52], given the pit sizes and relative fatigue lives, the equivalent initial crack sizes are gained using iterative method.

Let$F_0 = F(a)$, $F_1 = F(a + r_i)$, keep running till $|F_1|$<$|F_0|$, hence a+r⇒ $a_0$ and $F_1 \Rightarrow F_0$; While $|F_1| \geq |F_0|$, let $r_i$=0.5*$r_i$ and repeat.

Where $r_i$ is uniformly distributed random between [-b, b], b>0 may be selected in program according to the initial pit size. The flow charts of Monte Carlo and iterative method are given as Fig 3-2 and Fig3-3.

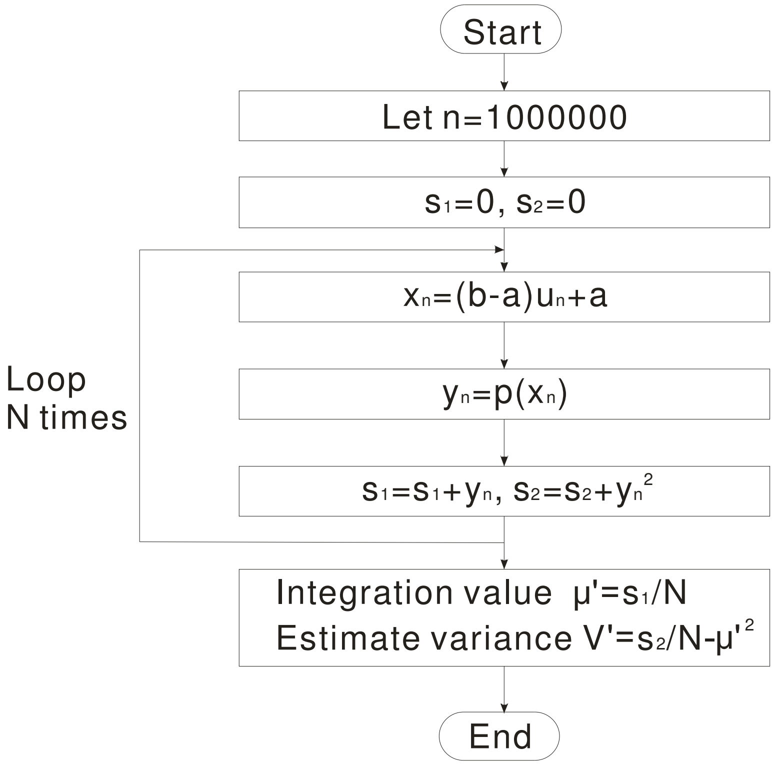


*Figure 3- 2* Flow chart of Monte Carlo simulation

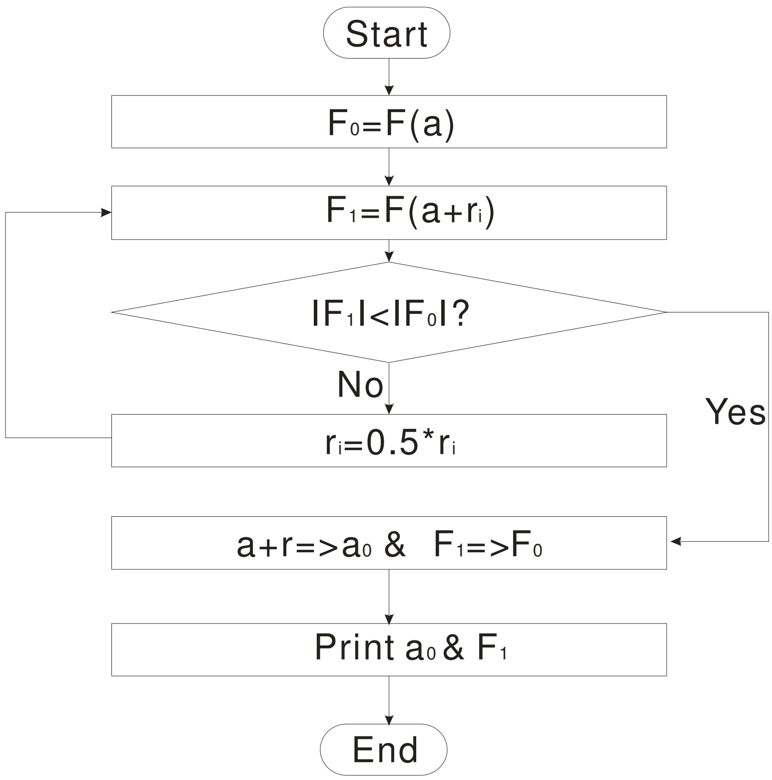


*Figure 3- 3* Flow chart of non-linear equation solving

### 3.3.2 Programming and calculation

Fortran 77/90 code is used in the research for calculation and simulation purpose [57]. Preferred compatible compiling platform is Fortran PowerStation4.0, where all the attached programs are compiled and executed. Full text of the code is attached in appendix. All programs are verified by the example as listed in the program.

Program **ITR** is worked out as listed in the appendix to calculate the integrate function in formula 3-25, incorporating the uniformly distributed random parameter and Monte Carlo simulation routine. It is a bit time consuming however robust comparing to traditional analytical or numerical methods. Only with simple modification/improvement, it is able to be used for simulations in other fields that require solutions for impractical or discrete problems.

In the program, Function f(a) is designed according to $f = \frac{1}{C(\Delta K)^m}$ in formula 3-26, which made the calculation much easier. Initial data are set as below:

m=3.67

c=0.0000000608

$\pi$=3.1415926

$\Delta\sigma$ =256.4 MPa

t=3500µm

w=26000 µm

While $c_0$ is set separately based on the experiment data from [52], accordingly, 113.4, 124.5, 152.9, and 178.5.

In the main body of the program, two large cycles are defined for simulation and iteration. Specific error (u=0.5) is also set initially as the judgement of root accuracy. Multi roots are largely possible as such the character of the non-linear equation, depending on the accuracy of error. Engineering empirical judgement is also necessary to rectify the multi roots. The other data is initialled as:

$$a_f = 6210\ \mu m$$

$$n = 1000$$

$$m = 10000$$

While $a_0$ and relative fatigue life $f_n$ are set separately each calculation, accordingly, 87.2, 115.8, 138.6, 159.4 and 105827, 92245, 80042, 71731.

## 3.4 *AFGROW Comparison*

AFGROW, developed by the Air Force Research Laboratory's Structural Mechanics Branch, is selected for equivalent initial crack size and crack growth life prediction comparison. AFGROW allows users to analyse crack initiation, fatigue crack growth, fracture, and assess the life of metallic structures. Being one of the fastest, most efficient and user-friendly crack life prediction tools available today, AFGROW is mainly used for aerospace structures experiencing fatigue crack [58].

The computer package is selected to compare experimental data with calculated ones using the non-linear equation model, as described in section3.3. The much easier window based input of material parameters, specified model, and load spectrum are presented as following sections.

### 3.4.1 Material input

Typical material is available from the software data base; for a material not included in, it is acceptable to define the material manually. Based on Walker's formula, the required parameter is listed:

| | |
|---|---|
| Crack growth coefficient c: | 1.008e-008 |
| Walker's exponent n: | 3.67 |
| Coefficient of Thermal Expansion: | 1.25e-005 |
| Young's Modulus: | 69580 MPa |

Yield Strength: 342.02 MPa

Poisson's Ratio: 0.33

Plane Stress Fracture Toughness: 100 MPa

Delta K threshold value: 2

### 3.4.2 Model description

Centre Semi-elliptic surface crack model is chosen for comparison, relative parameters are given:

Dimension (Fig 3-1): w=26mm

t=3.5mm

Initial size: (changing each calculation accordingly)

crack length, c=0.0978mm

crack depth, a=0.0753mm

### 3.4.3 Load spectrum

Constant amplitude loading is selected for the model, Fig 3-4, while specific actual spectrum is acceptable however not available. Stress Multiplication Factor of 9.5 is applied, which allows the normalized spectra to be used.

Stress ratio: R=0.02

Selected block size: 100

Duration of the block: 10

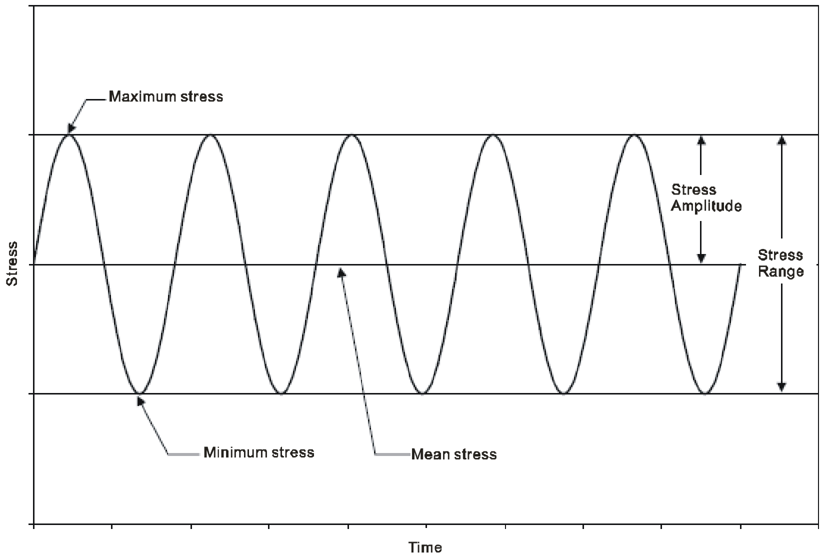


*Figure 3- 4* Loading spectrum

### 3.4.4 Output

Applying the designed model, different fatigue life could come out according to the input initial crack size. Following outputs are listed as an example:

AFGROW 4.11.14.0 6/25/2008 13:32

Crack Growth Model and Spectrum Information

Title: Example Problem

Initial crack depth (a) : 0.0753

Initial surface crack length (c): 0.0978

Thickness(t) : 3.500

Width(w) : 26.000

Young's Modulus =69580

Poisson's Ratio =0.33

Coeff. of Thermal Expan. =1.25e-005

No crack growth retardation is being considered

Determine Stress State automatically (2 = Plane stress, 6 = Plane strain)

The WALKER crack growth relation is being used

For Reff < 0.0, Kmax is used in place of Delta K

Material: User defined data

Number of segments: 1

| C | N | KCUT |
|---|---|---|
| 1.008e-008 | 3.67 | 0.5 |

Threshold: 2

Lower 'R' value boundary: -0.3

Upper 'R' value boundary: 0.8

Plane strain fracture toughness: 35

Plane stress fracture toughness: 100

Yield stress: 342.02

Failure is based on the current load in the applied spectrum

Vroman integration at 5% crack length

**Spectrum Information

Constant amplitude loading

Spectrum multiplication factor: 9.88

The spectrum will be repeated up to 999999 times

total Cycles: 100

Critical crack size in 'C' direction=3.58624, Stress State=6 (Based on Kmax criteria)

Critical crack size in 'C' direction=12.6017, Stress State=6 (Based on Net Section Yield criteria)

Transition will be based on Kmax or 95% thickness penetration Criteria

| | Crack size | Beta | R(k) | R(final) | Delta-K | Da/DN |
|---|---|---|---|---|---|---|
| C | 0.0978 | 0.644 | 0.0200000000 | 0.02000000 | 3.454e+000 | 9.894e-007 |
| A | 0.0753 | 0.741 | 0.0200000000 | 0.02000000 | 3.491e+000 | 1.028e-006 |

A/t ratio = 0.021514 A/C ratio = 0.76994

Max stress = 9.880000 R = 0.02

0 Cycles Constant amp.: 1 Pass: 1

| | Crack size | Beta | R(k) | R(final) | Delta-K | Da/DN |
|---|---|---|---|---|---|---|
| C | 0.10786 | 0.644 | 0.0200000000 | 0.02000000 | 3.628e+000 | 1.184e-006 |
| A | 0.083044 | 0.741 | 0.0200000000 | 0.02000000 | 3.666e+000 | 1.231e-006 |

A/t ratio = 0.023727 A/C ratio = 0.76994

Max stress = 9.880000 R = 0.02

9300 Cycles Constant amp.: 94 Pass: 94

------ (omit as repeated)

*********Fracture based on ' Kmax' Criteria (current maximum stress)

| | Crack size | Beta | R(k) | R(final) | Delta-K | Da/DN |
|---|---|---|---|---|---|---|
| C | 4.3808 | 1.083 | 0.0200000000 | 0.0200000 | 3.888e+001 | 7.146e-003 |

Max stress = 9.880000 R = 0.02

**108546 Cycles** Constant amp.: 1086 Pass: 1086

Stress State in 'C' direction (PSC): 6

Fracture has occurred - run time : 0 hour(s) 0 minute(s) 2 second(s)

3.152 hours have passed.

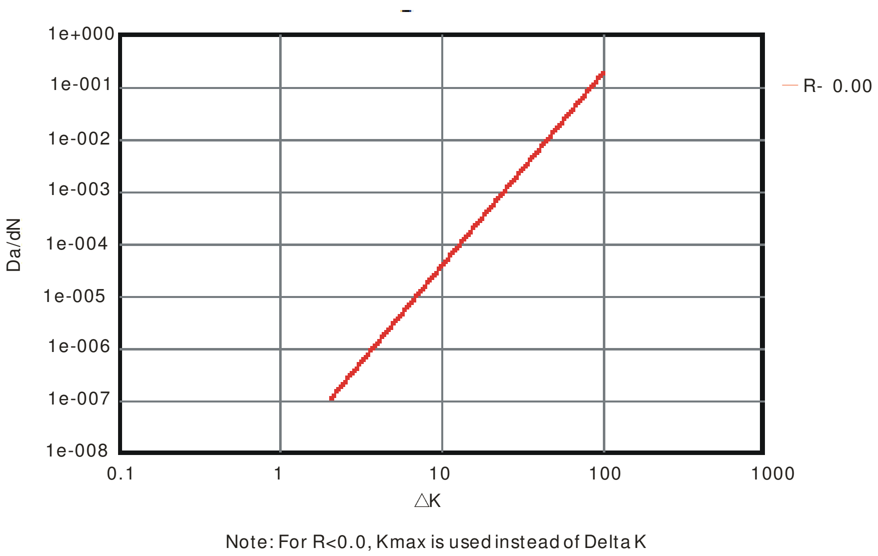


*Figure 3- 5* Crack growth rate

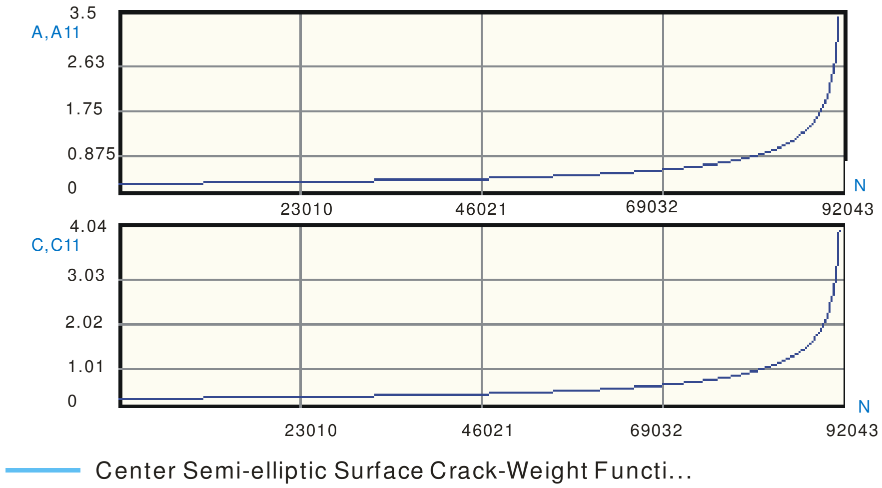


*Figure 3- 6* Crack length Increase

## 3.5 *Results and discussion*

As stated previously, the following data are used as input for the program **ITR** and AFGROW.

*Table 3- 1* Experiment data

| Corrosion time(days) | Temperature(T) | Experiment life(cycles) | Pit width (c-mm) | Pit depth (a-mm) | Aspect ratio Φ |
|---|---|---|---|---|---|
| 20 | 40 | 105827 | 113.4 | 87.2 | 0.77 |
| 20 | 60 | 92245 | 124.5 | 115.8 | 0.93 |
| 30 | 40 | 80042 | 152.9 | 138.6 | 0.91 |
| 30 | 60 | 71731 | 178.5 | 159.4 | 0.89 |

Running the program **ITR** for each set data in table 3-1, four relative equivalent initial pit sizes are gained, as shown in table 3-2. Applying program AFGROW, the expected time in cycles are gained accordingly to each input initial size, as shown in table 3-2. The relationship between pit depth and its equivalence is then concluded through correction factor λ.

*Table 3- 2* Execute result comparison

| Pit depth (a-mm) | Equivalence crack size (a0-mm) | Simulation loops | Calculation error(u) | Correction factor (λ) | Experiment life(cycles) | Expected life with a0 (cycles) | Expected life with pit size a (cycles) |
|---|---|---|---|---|---|---|---|
| 87.2 | 75.3 | 4090 | 0.5 | 0.864 | 105827 | 108546 | 98200 |
| 115.8 | 88.1 | 6325 | 0.5 | 0.761 | 92245 | 94127 | 72066 |
| 138.6 | 115.7 | 8645 | 1.0 | 0.835 | 80042 | 80850 | 62048 |
| 159.4 | 123.6 | 9608 | 1.5 | 0.775 | 71731 | 73949 | 56481 |

Discussion:

a) It is observed that the equivalent crack size calculated from program ITR is approximately 20% less than the measured pit size; however it is more accurate than direct use of pit size as the input of AFGROW, considering the experiment result.

b) Direct use of measured pit size for life prediction would apparently be conserve. It is more reasonable to incorporate the correction factor λ [0.77, 0.86] before put it into the designed life prediction model.

c) Recall the result data in table 3-2, there are only 4 sets of data are analysized, more data are highly desired for a better or more accurate correction coefficient *λ.*

d) It is possible that the non-linear equation has more than one root as the solution within the specific error. Careful selection of the roots according to its physics meaning is necessary.

e) The accuracy of the equivalent initial crack size obtained will thus depend on the Paris law used, where a few other parameters involved; The utility of the equivalent initial crack size lies in the requirement that the conditions in service will remain the same as outlined in the Paris law.

f) Multiple crack initiation at pits actually occurred in almost all tests in fact, which would result in irregular crack growth as either crack shielding or crack coalescence. Such effects are the exploring field for further research, however not included in this study.

# CHAPTER 4 LIFE PREDICTION AND RELIABILITY ASSESSMENT

## 4.1 *Introduction*

As described in section1.2 and 1.3, corrosion and fatigue are two major factors that contribute to the aging of aircraft, and they actually act synergistically in various ways. The corrosion and fatigue failure process of aircraft structure are directly concerned with many combined factors, such as load, material characteristics, corrosive environment and so on. Damage mechanism of those concurrent factors is not quite clear so far. Despite the mechanisms of these damages, here the thesis will only focus on the phenomenon and try to combine both corrosion and fatigue damages into the model.

Through decades of continuous research on corrosion and fatigue, a couple of effective models were developed very well to predict the corrosion fatigue life. Harlow and Wei [59] proposed firstly that corrosion fatigue life of aircraft structure should be composed of three stages: crack nucleation life, surface crack growth life and through crack growth life. A probabilistic model was also established. Vasudevan [60] thought that the fatigue life should be the sum of four stages. And a seven stage probabilistic model was proposed by Pan Shi and Mahadevan [61, 62].

This thesis intends to build a more elaborate model to assess aging structure reliability and to predict its life considering both randomness and fuzziness of the parameter that are existent in fact, applying constant amplitude spectrum from real time monitoring. For illustration purpose, example based on the selected data is presented.

### 4.2 *Model Construction*

The corrosion and corrosion fatigue damage process are assumed to begin with the nucleation of localized pit and subsequently corrosion fatigue grows and fail at last. Four stages corrosion fatigue life is adopted in the modal:

$$t_f = t_1 + t_2 + t_3 + t_4 \qquad 4\text{-}1$$

Where, $t_1$ is the time of pit nucleation; $t_2$ is the time of pit growth; $t_3$ is the time of short crack growth to long crack; $t_4$ is the time of long crack growth till failure.

Comparing to the traditional seven stages model, the following three stages are treated as fuzzy process in the model: transition from pit to short crack nucleation; transition from short crack to long crack, and failure criterion. Different membership functions are applied for each stage to identify the most appropriate pattern.

Corrosion fatigue lives under different required reliabilities are able to be carried out from the model. The failure probability of the aging structure at life t (corrosion fatigue induced) can be expressed as:

$$P_f(t) = P(t_f - t \leq 0) \qquad 4\text{-}2$$

### 4.2.1 Pit nucleation stage

After the failure of the corrosion prevention coating on the skin, the structure material is bare, directly exposed to corrosive medium and easily corroded. This stage includes the failure process of coating, subsequently the electrochemical corrosion process of aluminium alloy and the nucleation of a corrosion pit. The time of this stage depends on factors such as load, corrosion environment, and material properties of aluminium alloy, manufacture technology and so on. In addition, the damage mechanism is very complicated and not well understood yet. Only after the prevention coating has failed, is the corrosion environment able to corrode the material matrix. With the development of coating technology, the life of pit nucleation is lasting.

Based on empirical data, the time for pit nucleation $t_1$ is assumed as a Weibull random variable. In accordance with Refs.[63, 64], the operational life of the prevention coating is about 3-5 years and the mean value is 1500days.

### 4.2.2 Pit growth stage

Pit is formed from localized galvanic corrosion near the exposed constituent particles.[65] After its nucleation, it begins to grow, involving electrochemical processes affected by clusters of particles. The constituent particle in an aluminium alloy can be considered as cathodic or anodic relative to the matrix, where cathodic particles are considered essential for pit growth. As the pits grow, the exposed particles will interact and accelerate the growth subsequently. The following model considers the effect of the clustered particles on the pit and the fuzziness of transition criterion.

Applying Faraday's law,

$$\frac{dV}{dt} = \frac{d\left(\frac{2\pi ac^2}{3}\right)}{dt} = \frac{MI_{P0}}{nF\rho}\exp\left(-\frac{\Delta H}{RT}\right) \qquad 4\text{-}3$$

By integration of Eq. 4-3, the time for pit growth $t_2$ is obtained:

$$t_2 = \frac{2\pi nF\rho}{3MI_{P0}\emptyset^2}\exp\left(\frac{\Delta H}{RT}\right)\ (a_{ci}^3 - a_0^3) \qquad 4\text{-}4$$

Where M is the molecular weight of the material, n is the valence, F is the Faraday Constant, $\rho$ is the density, $\Delta H$ is the activation energy, R is the universal gas constant, T is the absolute temperature, $I_{P0}$ is the pitting current coefficient, which is dependent on the clustered particles. $a_{ci}$ is the critical pit size leading to short crack initiation; $a_0$ is the initial crack size. Φ is the aspect ratio described as following.

The shape of the pit is quite complex and is assumed to be semi-ellipsoidal here for illustration sake (refer to Fig. 3-1). The aspect ratio is incorporated and defined as:

$$\Phi = \frac{\mathrm{a}}{\mathrm{c}} \qquad 4\text{-}5$$

Where a and c denote half length of the major and minor axes. The pit can continue to grow deeply into material matrix in corrosion environment. Its aspect ratio $\Phi > 1$, and it is a random parameter. According to Ref. [66], the aspect ratio is in the range of [1, 4.72] and the mean value is 1.5.

The transition from pit to short crack growth occurs when the equivalent stress intensity factor range ΔK for the pit increases to the threshold driving force, $\Delta K_{th}$. For illustration purpose, it is assumed that the pit is the semi-elliptical surface crack in a semi-infinite body. According to ref.[64], The stress intensity factor range ΔK of the deepest point in the pit is then expressed as:

$$\Delta K = \frac{1.1K_t\Delta\sigma\sqrt{\pi a}}{(1+1.464\emptyset^{-1.65})^{1/2}} \qquad 4\text{-}6$$

The critical pit size can be found to be:

$$a_{ci} = \frac{(1+1.464\emptyset^{-1.65})}{\pi}\left[\frac{\Delta K_{th}}{1.1K_t\Delta\sigma_{max}}\right]^2 \qquad 4\text{-}7$$

Where $K_t$ is the stress concentration factor, $\Delta\sigma_{max}$ is the maximal stress range in the load spectrum.

### 4.2.3 Short crack growth stage

The short crack growth stage involves chemical and micro structural factors and their interactions. Random variables dependent on experimental data are used to build the probabilistic relationship. The Walker's formula is adopted to compute the crack growth rate, with consideration of the stress ratio.

$$\frac{da}{dN} = C_{sc}\,[(1-R)^{m-1}\Delta K]^{n_{sc}} \qquad 4\text{-}8$$

Where R is the stress ratio and is gained through test, m is Walker's exponent, $C_{sc}$ is the short crack growth coefficient of aluminium in corrosion environment; $n_{sc}$ is the short crack growth exponent and is assumed to be constant. Taking into account N= ft and combining Eq. 4-6 and Eq.4-8, the time for short crack growth is found to be:

$$t_3 = \frac{2(1+1.464\emptyset^{-1.65})^{n_{sc}/2}}{fC_{sc}(2-n_{sc})}\cdot\frac{(\sqrt{a_{th}})^{2-n_{sc}}-(\sqrt{a_{ci}})^{2-n_{sc}}}{[1.1(1-R)^{m-1}K_t\Delta\sigma\sqrt{\pi}]^{n_{sc}}} \qquad 4\text{-}9$$

Where $a_{th}$ is the critical crack size for the transition from the short crack to the long crack, f is the frequency of the load spectrum block. $\Delta\sigma$ is obtained through investigating operation status of the material/structure, where the load cycles are applied until transition from short crack to long crack growth occurs.

### 4.2.4 Long crack growth stage

As for long crack, according to ref.[64], the stress intensity factor range is given by

$$\Delta K = K_t \Delta\sigma\sqrt{\pi a} \tag{4-10}$$

According to Walker's formula, similar to the short crack growth, the long crack growth rate is expressed as:

$$\frac{da}{dN} = C_{lc}\,[(1-R)^{m-1}\Delta K]^{n_{lc}} \tag{4-11}$$

The time for long crack growth is then could be derived:

$$t_4 = \frac{2[(\sqrt{a_f})^{2-n_{lc}} - (\sqrt{a_{th}})^{2-n_{lc}}]}{fC_{lc}(2-n_{lc})[(1-R)^{m-1}K_t\Delta\sigma\sqrt{\pi}]^{n_{lc}}} \tag{4-12}$$

Where $a_f$ is the critical crack length.

### 4.2.5 Calculation Model

Considering all the discussed stages as above, the corrosion fatigue fuzzy reliability life $t_f$ can be expressed as a function of the following combined parameters:

$$t_f = t_f\,(I_{P0}, a_0, C_{sc}, C_{lc}, \Phi, t_1, \Delta K_{th}, a_{th}, a_f) \tag{4-13}$$

Where $I_{P0}$, $C_{sc}$ , $C_{lc}$ , $\Phi$ , $a_0$ and $t_1$ are random variables , $\Delta K_{th}$, $a_{th}$ and $a_f$ are fuzzy random variables. Combining those equations above (4-1, 4-4, 4-9, 4-12), $t_f$ can be found to be:

$$t_f = t_1 + \frac{2\pi nF\rho}{3MI_{P0}\emptyset^2}\exp\left(\frac{\Delta H}{RT}\right)\,(a_{ci}^3 - a_0^3) +$$

$$\frac{2(1+1.464\emptyset^{-1.65})^{n_{sc}/2}}{fC_{sc}(2-n_{sc})}\cdot\frac{(\sqrt{a_{th}})^{2-n_{sc}} - (\sqrt{a_{ci}})^{2-n_{sc}}}{[1.1(1-R)^{m-1}K_t\Delta\sigma\sqrt{\pi}]^{n_{sc}}} + \frac{2[(\sqrt{a_f})^{2-n_{lc}} - (\sqrt{a_{th}})^{2-n_{lc}}]}{fC_{lc}(2-n_{lc})[(1-R)^{m-1}K_t\Delta\sigma\sqrt{\pi}]^{n_{lc}}} \tag{4-14}$$

Applying the material constant, random variables and fuzzy variables function, it is able to compute the failure probability at the specified time t and the expected operation life under reliability requested using Monet Carlo simulation and scaling transformation method.

## 4.3 *Parameters estimation*

### 4.3.1 Fuzziness of the parameters

Other than the ordinary set theory, fuzziness theory expresses the parameters not just by either 1 or 0, but any value varying from zero to one, indicating the degree of the membership. Different shapes of membership function describe the relative degree trends.

With respect to failure criterion, the corrosion and corrosion fatigue failure are caused by accumulative damages that degrade the material performance. The process from “good condition” to “failure condition” is a gradual process from “quantitative change” to “qualitative change”. Ref. [61, 67] presents a critical size of crack as the failure criterion for the aircraft structure, which was 6 mm. It implied that a crack of a =6.00 mm would not cause failure, whereas a crack of a= 6.01 mm would cause failure. However, there is no substantive difference between 6.00 mm and 6.01 mm. The process from the safe status to the failure status is actually gradual and then the judgement criterion is fuzzy.

According to Ref.[63], the critical size $a_{th}$ of the transition of the short crack to the long crack is 1.0 mm in engineering field. It is an experiential data actually and similar to failure crack size $a_f$, the fuzziness is existing obviously.

As shown in Eq. 4-7, the critical crack size of the transition criterion from pit to short crack is determined by the random variable $\Delta K_{th}$, which is actually a material parameter and obtained through test. While the test conditions vary from time to time; the specimen is also different from the actual structure; and artificial assumptions existing during the test, the threshold driving force $\Delta K_{th}$ is therefore a typical fuzzy random variable. In this thesis, the fuzziness of the transition criterion from pit to short crack is described through $\Delta K_{th}$.

### 4.3.2 Membership Function

Membership function represents the degree of belief that one has on the fuzzy element. Owing to its subjective nature, membership function may take various shapes and forms. However, the membership values all relate to the fuzzy element, which may be described as specific formula. For simplicity and comparison, the following membership functions are selected for the fuzzy parameters ( $\Delta K_{th}$, $a_{th}$ and $a_f$ ) in this thesis: the semi-trapezoidal membership function; the normal membership function, which are commonly found in the literature.

The semi-trapezoidal membership function is adopted for $a_f$, which is described in Eq. 4-15 and Fig. 4-1.

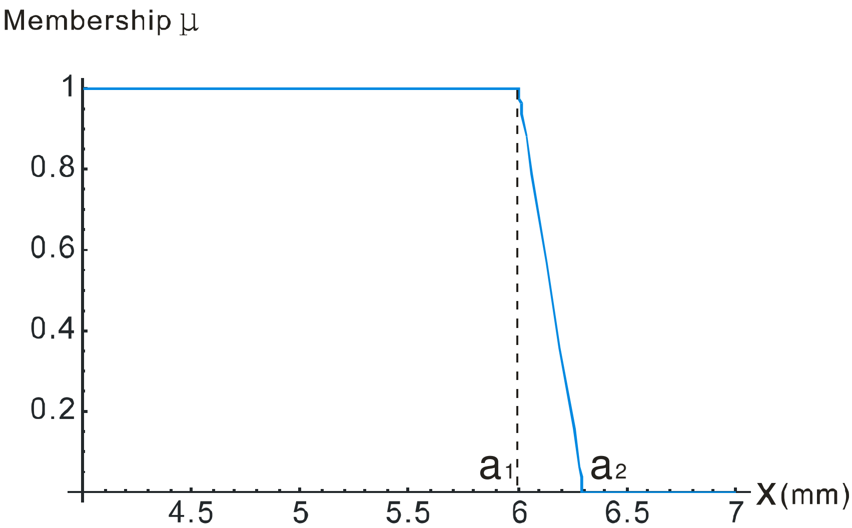


*Figure 4- 1* Semi-trapezoidal membership function

$$\mu_{af}(x) = \begin{cases} 1 & x \le a_1 \\ \frac{a_2 - x}{a_2 - a_1} & a_1 < x \le a_2 \\ 0 & x > a_2 \end{cases} \qquad \text{4-15}$$

Where $a_1$=6, indicating the minimum failure crack size, and $a_2$ is chosen in the calculation, which stands for the maximum failure crack size.

The semi-trapezoidal membership function is adopted for $a_{th}$, as shown in Eq. 4-16 and Fig. 4-2,

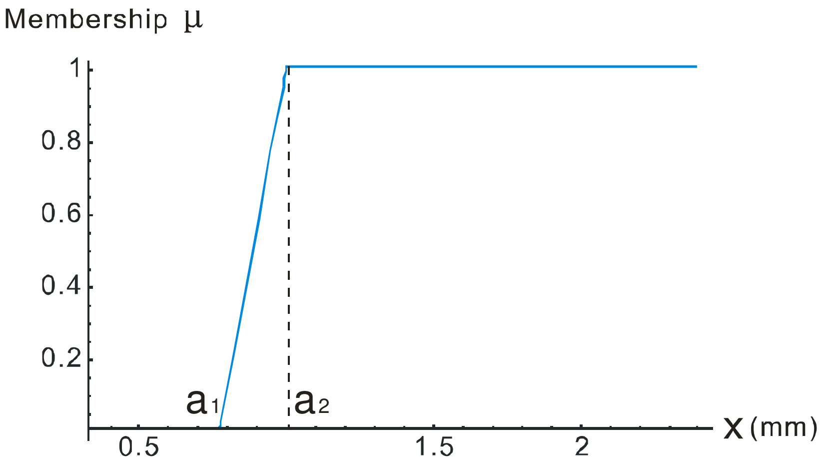


*Figure 4- 2* Semi-trapezoidal membership function

$$\mu_{ath}(x) = \begin{cases} 0 & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 < x \leq a_2 \\ 1 & x > a_2 \end{cases} \tag{4-16}$$

Where $a_2$ =1. 0, indicating the maximum critical crack size for the transition from the short crack to the long crack, and $a_1$ is chosen in the calculation, which stands for the minimum critical crack size.

The normal membership function is also selected for $\Delta K_{th}$ as shown in Eq. 4-17 and Fig.4-3.

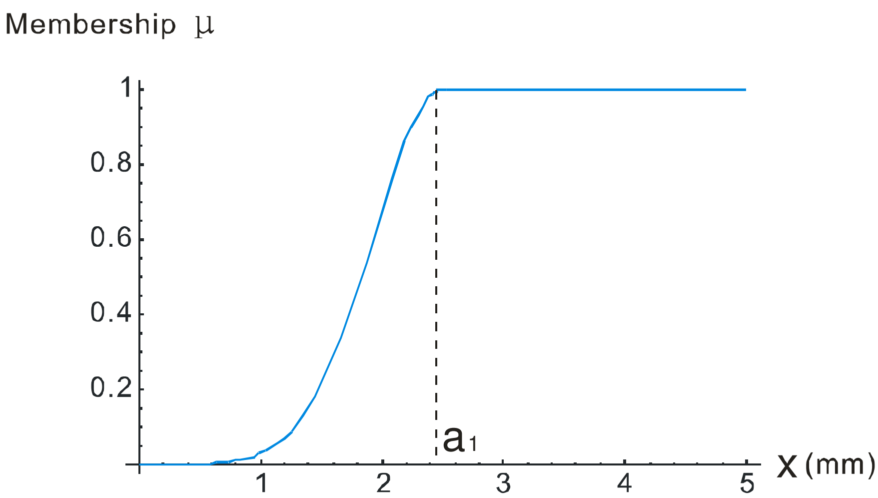


*Figure 4- 3* Normal membership function

$$\mu_{\Delta Kth}(x) = \begin{cases} \exp\left[-\frac{(x-a_1)^2}{a_2^2}\right] & x < a_1 \\ 1 & x \geq a_1 \end{cases} \qquad 4\text{-}17$$

Where $a_2$ =1. 0, indicating the shape of the curve, $a_1$ is chosen in the calculation, at which point the membership is 1.

### 4.3.3 Scaling based transformation

The current model is characterized by both the probabilistic and the fuzzy variables. For the feasibility of Monte Carlo simulation, the fuzzy variables are then necessary to be transferred to the equivalent random variables, relying on the fundamental concept of scaling based transformation.

Smith et al. [68] adopted a Bayesian approach in their work to reduce the conservatism of the possibility theory, scaling the membership function with respect to the area under the membership function does the transformation. The scaling factor is obtained to satisfy the axiom that the area under the PDF should be unity. It also intuitively satisfies the consistency principle that the possibility of an event should be greater than or equal to its probability. The corresponding probability density function of the fuzzy variables described in section 4.3.2 is obtained simply by scaling the membership function with k:

$$p_{af}(x) = k_1 \frac{a_2 - x}{a_2 - a_1} \qquad a_1 < x \leq a_2 \qquad 4\text{-}18$$

Where $k_1 = \frac{2}{a_2 - a_1}$

$$p_{ath}(x) = k_2 \frac{x - a_1}{a_2 - a_1} \qquad a_1 < x \leq a_2 \qquad 4\text{-}19$$

Where $K_2 = \frac{2}{a_2 - a_1}$

$$p_{\Delta Kth}(x) = k_3 \exp\left[-\frac{(x-a_1)^2}{a_2^2}\right] \qquad x < a_1 \qquad 4\text{-}20$$

Where $k_3 = \frac{2}{a2 * \mathrm{Erf}[a1/a2]\sqrt{\pi}}$=1.41(a2=0.8, a1=2.5) ; 1.05 (a2=1, a1=2.5)

### 4.3.4 Constant

The constants used in formula 4-14 are list as below:

| | |
|---|---|
| Density ρ | $2.7\times10^{3}$kg/m$^{3}$ |
| Molecular weight M | 27 |
| Valence n | 3 |
| The activation energy. $\Delta H$ | 50 kJ / mol |
| Faraday's constant F | 96514 C/ mol |
| The universal constant R | 8. 314 J / mol/K |
| The crack growth exponent $n_{sc} = n_{lc}$ | 3.14 |
| The impact exponent of stress ratio | 0. 66 |
| Temperature T | 293K |
| Applied stress $\Delta\sigma_{\max}$ | 90 MP |
| The load spectrum frequency f | 10 |
| The stress concentration factor $K_t$ | 3 |

### 4.3.5 Random variables

Weibull and Lognormal probability density function (PDF) is chosen for those random variables $I_{P0}$, $C_{sc}$ , $C_{lc}$ , Φ , $a_0$ and $t_1$, because of its much applicability.

For those Weibull distributed parameters $I_{P0}$, Φ, $a_0$ and $t_1$, the randomness is depicted by the different shape parameter α, the minimum value parameter γ, and the scale parameter ß. The three-parameter Weibull PDF is given by

$$f(x) = \frac{\alpha}{\beta}(\frac{x-\gamma}{\beta})^{\alpha-1}\exp[-(\frac{x-\gamma}{\beta})^{\alpha}] \quad 4\text{-}21$$

Accordingly, direct sample parameter is defined by:

$$\xi = F^{-1}(Z) = \beta[1 - \ln(1-Z)]^{1/\alpha} + \gamma \quad 4\text{-}22$$

Where, Z is the uniformly distributed random parameter coming from various random generators. Several uniformly distributed random generators are listed and incorporated in program.

The Weibull parameters of $I_{P0}$, Φ, $a_0$ and $t_1$ are given in the following table 4-1 according to Ref [69,70,71]. The variables are assumed to be statistically independent.

*Table 4- 1* Weibull distributed parameters

| Random variable | α | β | $\gamma$ | μ(mean value) |
|---|---|---|---|---|
| $\alpha_O$ (m) | 1.497 | $2.214\times10^{-6}$ | $0.272\times10^{-6}$ | $2.276\times10^{-6}$ |
| $Ip_0(c\ s^{-1})$ | 1.00 | k×6.5×10-5 | 6.5×10-5 | (k+1)×6.5×10-5 |
| T1 | 0.5 | 120 | 1000 | 1500 |
| $\Phi_K$;k = 4 | 2.255 | 0.510 | 0.00 | 0.452 |

Similarly, direct simulation of the fuzzy members as described in section4.3.3 could use the following deduced formula:

$$X1 = p_{af}^{-1}(x) = a2 - \frac{z}{k1}(a2 - a1) \quad a_1 < x1 \le a_2 \quad 4\text{-}23$$

$$X2 = p_{ath}^{-1}(x) = a1 + \frac{z}{k2}(a2 - a1) \quad a_1 < x2 \le a_2 \quad 4\text{-}24$$

$$X3 = p_{\Delta kth}^{-1}(x) = a1 + a2\sqrt{\ln(k3/z)} \quad x3 < a_1 \quad 4\text{-}25$$

Where z is the uniformly distributed random number in [0, 1].

A random variable *X* is said to have the lognormal distribution, with parameters μ and *d*, if ln(*X*) has the normal distribution with mean μ and standard deviation *d*. equivalently,

$$X = \exp(Y) \qquad 4\text{-}26$$

Where *Y* is normally distributed with mean μ and standard deviation *d*.

The Lognormal parameters of Fatigue coefficient for crack growth $C_{sc}$ and $C_{lc}$ are given in the following table 4-2 according to Ref. [72, 73]. The variables are assumed to be statistically independent.

*Table 4- 2* Lognormal distributed parameters

| Random variable | μ | d |
|---|---|---|
| Fatigue coefficient for short crack growth $C_{sc}$ (m/cycle) | $1.092\times10^{-9}$ | 0.5 |
| Fatigue coefficient for long crack growth $C_{lc}$ (m/cycle) | 1.86×10-11 | 0.2 |

### 4.3.6 Calculation and Programming

Upon fully defined variables and constants in formula 4-14, calculation was conducted through FORTRAN program. According to formula 4-2, the failure probability of the aging structure at life t can be carried out using Monte Carlo simulation, which is the failure rate of the N times simulation. Meanwhile, corrosion fatigue lives under different required reliabilities are able to be carried out using linear interpolation.

Program RELIABILITY is worked out to assess the reliability upon the specific time using Monte Carlo simulation. Effects of fuzzy and random variables are also analysized. It is described in the program that function Y(phi), function random_normal(), and function random_weibull(wa, wb, wg) are designed to generate process variable, lognormal distributed random variable, and weibull distributed random variable. In the main body of the program, constants are initially set, the four pit growth states are then defined separately incorporating the fuzzy or random variables according to the relative formula. $100X10000=10^6$ times calculation is executed each time of the reliability assessment to ensure the accurate.

Since Weibull and Lognormal distributions are the assumed CDFs for the random variables in this research, the random number generator has to be conducted respectively. A key ingredient of this method is a good source of randomness without any doubt. However, truly random entropy sources can only be found by measuring inherently unpredictable physical processes, which is impractical and unnecessary. In the program, several algebraic methods of generating random sequences are presented and compared to ensure it is uniformly distributed.

The flow chart of the program RELIABILITY is given as Fig.4-4

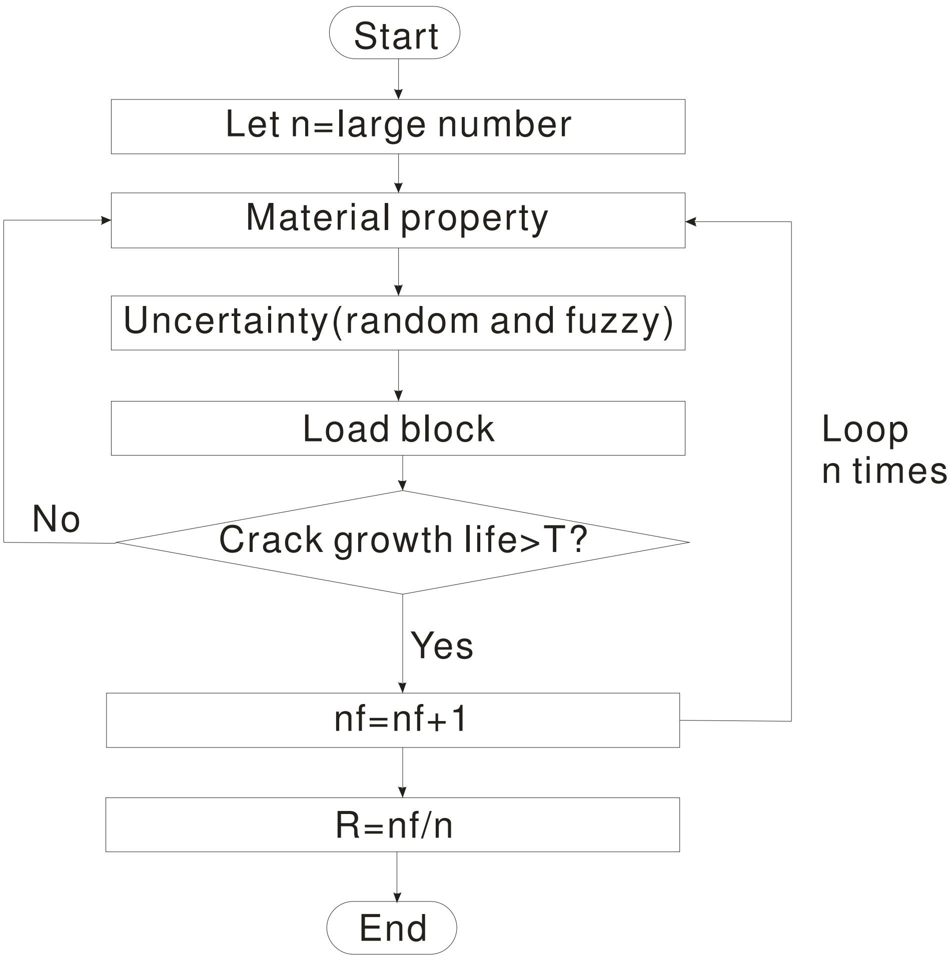


*Figure 4- 4* Flow chart of program reliability

## 4.4 *Assumptions*

In performing the analysis it was necessary to make several assumptions, either to simplify the problem or because of the lack of available data. These assumptions are detailed below:

a) Material data listed in this study does not come from the direct test of the aircraft structure either because it is proprietary or a sufficient sample size is not available. As a result, the parameter variation is determined using engineering judgment.

b) Only constant amplitude loading is considered in the model, which would be too idealist in engineering practice. The retardation of load sequences and overload effect are then not considered in the model. In order to develop a general model to consider the variable amplitude loading cases, more mechanisms should be incorporated into the model.

c) Due to little source of data, it is seldom possible to derive the exact probability distribution or fuzzy membership theoretically. Lognormal, Weibull distribution and trapezoidal, normal membership function are selected in the model, rather than conducting a particular one.

d) For the aging aircraft, most of the crack growth time is spent when the cracks are small. So, cracks are assumed to be single and therefore no interact. Although important, multi-site damage and its interaction are not the interesting of this paper.

e) Quasi-random number is assumed as uniformly distributed random number because of the difficulty of physically obtaining real random number. However, verification test is conducted to avoid the repeat cycle.

## 4.5 *Results and discussion*

### 4.5.1 Random variables effects

As described in section4.3.5, $I_{P0}$, $C_{sc}$ , $C_{lc}$ , Φ ,$a_o$ and $t_1$ are assumed as random variables. The reliability influence of each parameter is compared in table 4-3 and Fig4-5, where Reliability-hybrid is the reliability considering all the uncertainties, Reliability-phi, Reliability-$I_{po}$, Reliability-$a_o$, Reliability-$C_{sc}$, Reliability-$C_{lc}$, and Reliability-$t_1$ stand for the reliability without considering the randomness of the respective parameter, while mean values are used for the random parameters respectively.

*Table 4- 3* Random variables effects comparison

| Reliability-hybrid | Reliability-$C_{sc}$ | Reliability-$C_{lc}$ | Reliability-$t_1$ | Reliability-phi | Reliability-$I_{po}$ | Reliability-$a_o$ | life(days) |
|---|---|---|---|---|---|---|---|
| 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9996 | 0.9998 | 18000 |
| 0.9998 | 0.9994 | 0.9994 | 0.9996 | 0.9982 | 0.9990 | 0.9992 | 19000 |
| 0.9992 | 0.9990 | 0.9991 | 0.9992 | 0.9963 | 0.9968 | 0.9990 | 20000 |
| 0.9986 | 0.9987 | 0.9984 | 0.9985 | 0.9932 | 0.9960 | 0.9950 | 21000 |
| 0.9975 | 0.9970 | 0.9977 | 0.9970 | 0.9877 | 0.9929 | 0.9935 | 22000 |
| 0.9961 | 0.9963 | 0.9960 | 0.9960 | 0.9800 | 0.9880 | 0.9924 | 23000 |
| 0.9926 | 0.9923 | 0.9923 | 0.9923 | 0.9683 | 0.9805 | 0.9885 | 24000 |
| 0.9900 | 0.9902 | 0.9902 | 0.9985 | 0.9547 | 0.9720 | 0.9890 | 25000 |
| 0.9420 | 0.9424 | 0.9420 | 0.9417 | 0.8250 | 0.8830 | 0.9438 | 30000 |
| 0.7040 | 0.7038 | 0.7038 | 0.7044 | 0.4400 | 0.5520 | 0.7080 | 40000 |
| 0.4280 | 0.4280 | 0.4276 | 0.4275 | 0.1680 | 0.2540 | 0.4210 | 50000 |
| 0.2120 | 0.2122 | 0.2117 | 0.2118 | 0.0520 | 0.1060 | 0.2140 | 60000 |
| 0.1050 | 0.1045 | 0.1045 | 0.1053 | 0.0210 | 0.0430 | 0.1080 | 70000 |
| 0.0460 | 0.0462 | 0.0463 | 0.0464 | 0.0067 | 0.0021 | 0.0560 | 80000 |

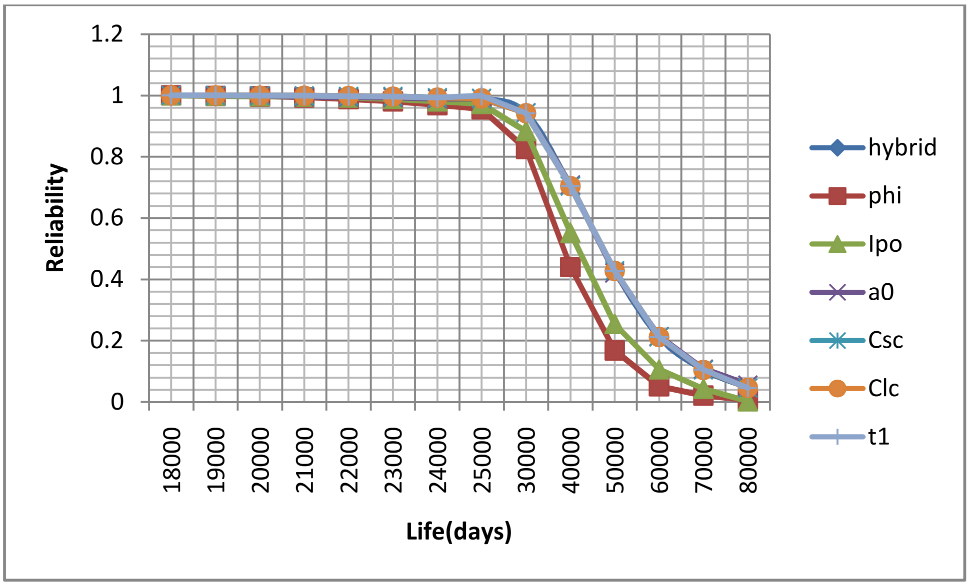


*Figure 4- 5* Reliability effects comparison

Discussion:

a) It is clear that all the curves fit quite well to each other, especially the reliability without considering $C_{sc}$, $C_{lc}$, $a_o$, $t_1$ and the so called hybrid reliability. Recall the formula 4-14, $C_{sc}$ , $C_{lc}$ lies in the short and long crack growth stage, which contribute the minority of the total life. And its selected lognormal distribution parameters are not quite broadly dispersed. For pit initial size $a_{o,}$ its effect actually covered by the critical pit size $a_{ci}$, which is quantitatively larger. As for $t_1$, it depends on the technology of coating and regular service of the aircraft. Comparing to the total operation life, its fluctuation according to its distribution would not significantly affect the reliability.

b) However, as far as $I_{P0}$ and Φ are treated as constant, the respective reliability would slightly decrease comparing to the hybrid reliability. Recall the formula 4-14,Φ exists in the pit growth and short crack growth stage, which contributes the majority of the total life. Moreover, as the crack shape parameter, its mechanism is even not clear so far, hence, the assumptions are inevitable.

c) It is observed that a sharp decrease happened at the time about 25000days in all the curves. Recall the formula 4-14, this is about the combined time of stage $t_1$ and $t_2$, which indicates that the aging aircraft structure would thought to be dangerous when short crack growth occurs.

d) Pitting current coefficient $I_{P0}$ is also an important parameter in pit growth stage, which would negatively affect the total life. As the parameter dependent on the clustered particles, its Weibull distribution is much wider than others which contribute to the decrease of reliability.

e) It is more than necessary to calculate the higher reliability result (>0.9999) for the aging aircraft structure, which is time consuming. Due to randomness of the parameters, reasonable error might exist in the result however would not change the curve trends.

f) The uncertainty of those random parameters results in the changing of reliability. Therefore, more experiment data are highly sought for more accurate parameter distribution, especially those of Φ and $I_{P0.}$

### 4.5.2 Sensitivity of membership function

Fuzzy variable is also incorporated into the hybrid model, by transferring the membership function into pdf. Where applicable, three typical membership functions are used to fit the fuzziness of $a_{th}$, $a_f$ and $\Delta k_{th}$. In order to determine the effect of each fuzzy variable, the reliability at specific time without considering the fuzziness is also investigated, namely, Reliability-$a_{th}$, Reliability-$a_f$, Reliability-$\Delta k_{th}$ , where mean values of the fuzzy parameters are assigned respectively.

The corrosion fatigue lives under different reliabilities are shown in Table 4-4 and Figure 4-6.

*Table 4- 4* Fuzzy member effects comparison

| Reliability-hybrid | Reliability-$a_{th}$ | Reliability-$a_f$ | Reliability-$\Delta k_{th}$ | Expected life(days) |
|---|---|---|---|---|
| 0.9999 | 0.9999 | - | 0.9996 | 18000 |
| 0.9998 | 0.9992 | 0.9999 | 0.9920 | 19000 |
| 0.9992 | 0.9989 | 0.9996 | 0.9300 | 20000 |
| 0.9986 | 0.9975 | 0.9995 | 0.7500 | 21000 |
| 0.9975 | 0.9970 | 0.9983 | 0.4380 | 22000 |
| 0.9961 | 0.9935 | 0.9951 | 0.1720 | 23000 |
| 0.9926 | 0.9910 | 0.9924 | - | 24000 |
| 0.9900 | 0.9840 | 0.9906 | - | 25000 |
| 0.9870 | 0.9810 | 0.9860 | - | 25500 |
| 0.9842 | 0.9800 | 0.9874 | - | 26000 |
| 0.9770 | 0.9710 | 0.9784 | - | 27000 |
| 0.9680 | 0.9630 | 0.9680 | - | 28000 |
| 0.9620 | 0.9610 | 0.9518 | - | 29000 |
| 0.9420 | 0.9400 | 0.9440 | - | 30000 |
| 0.7040 | 0.7020 | 0.7050 | - | 40000 |
| 0.4280 | 0.4210 | 0.4224 | - | 50000 |
| 0.2120 | 0.2110 | 0.2145 | - | 60000 |
| 0.1050 | 0.1000 | 0.1046 | - | 70000 |
| 0.0460 | 0.0400 | 0.0524 | - | 80000 |
| 0.0300 | 0.0200 | 0.0266 | - | 90000 |

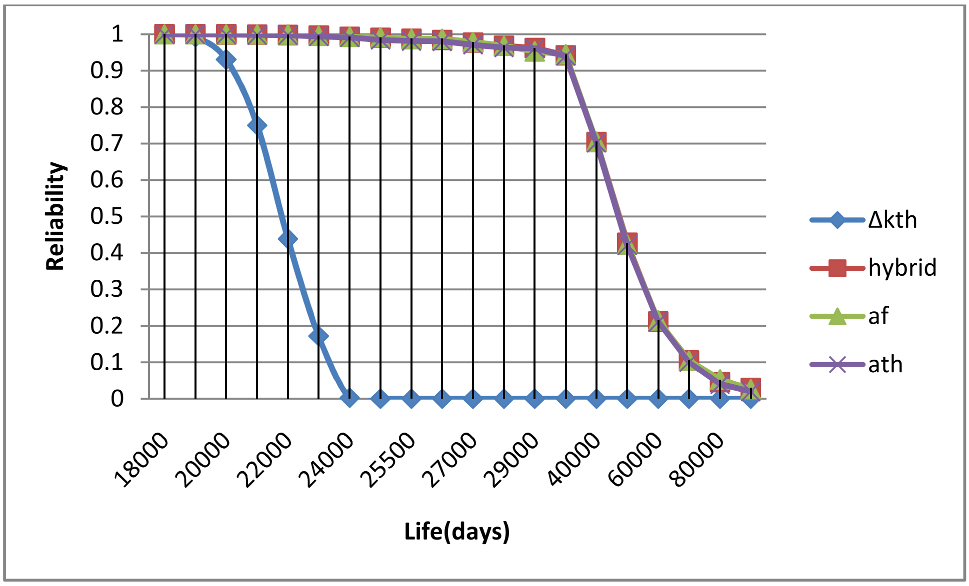


*Figure 4- 6* Fuzzy member comparison

Discussion:

a) Despite the reasonable calculation error, it can be seen from Figure 6-2 and table6-4, that $a_{th}$, $a_f$ has some but slight influence on the reliability. After careful study on the result, it can also be seen that the influences from these two parameters are actually different, while one increase the reliability, the other decrease the reliability.

b) It is surprising that $\Delta k_{th}$ has such a significant impact on the structure reliability that not only largely reduced the service life, but also severely shortened the high reliability duration. Based on this point, further research on $\Delta k_{th}$ is strongly recommended, especially on its actual value during the whole service life.

c) The fuzziness of $\Delta k_{th}$ should be emphasized that variation of this magnitude can severely alter the service life and the subsequent management decision. Further research is strongly recommended on this variable, especially on its membership and pdf transition.

d) The result is based on the data widely cited in the literature, however not demonstrated yet. As different conditions and requirements may be associated with the data, it is acceptable for illustration purpose but not suitable in the actual life prediction and reliability assessment. For example, when the hybrid reliability is 0.999, the structure reliability life is approximately 20000 days respectively, which will not well agree with the actual service life of the aircraft structure and would only happened under unearthly condition.

### 4.5.3 Life prediction and reliability assessment

The influences of parameter uncertainty on the structure reliability are discussed as above. Such influences on the service life can then be expected from the relationship as described in Table 4-3 and Table 4-4, using linear interpolation. For example, while requiring the predicted life under hybrid reliability of 0.9990, it could be approximately calculated by:

$$21000-(21000-20000)X(0.9990-0.9986)/(0.9992-0.9986)=20333(days)$$

Then, it is more than needed to analysis the influence on the service life under specific reliability.

However, for further discussion, Table4-5, 4-6 list the simulated life of each stage as described in section 4.2 and its mean value evaluated for life prediction.

*Table 4- 5* Mean value of each stage

| Mean t1 | Mean t2 | Mean t3 | Mean t4 | $t_f$ |
|---|---|---|---|---|
| 1493.44 | 37582.60 | 1648.26 | 146.70 | 40871 |

*Table 4- 6* Calculate process simulation

| T1 | T2 | T3 | T4 | Random number |
|---|---|---|---|---|
| 1307.16 | 48145.38 | 1206.29 | 146.70 | 2.45E-01 |
| 1350.77 | 51949.42 | 1098.17 | 146.70 | 1.98E-01 |
| 1534.37 | 3230.79 | 1843.27 | 146.69 | 6.51E-01 |
| 1178.67 | 41.76 | 3300.76 | 146.69 | 9.92E-01 |
| 1139.26 | 427.45 | 2831.29 | 146.69 | 9.07E-01 |
| 3074.38 | 31243.85 | 1446.26 | 146.70 | 3.89E-01 |
| 1214.16 | 153.32 | 3048.51 | 146.69 | 9.03E-01 |
| 1272.67 | 150.97 | 4191.50 | 146.69 | 9.73E-01 |
| 1690.11 | 229.26 | 3167.54 | 146.69 | 9.43E-01 |
| 3922.98 | 1065.59 | 2598.31 | 146.69 | 8.18E-01 |
| 1253.02 | 1800.01 | 1956.16 | 146.69 | 6.89E-01 |
| 1130.96 | 950.30 | 2109.08 | 146.69 | 7.01E-01 |
| 1145.91 | 6786.87 | 1400.98 | 146.70 | 4.44E-01 |
| 1623.41 | 48499.15 | 1259.60 | 146.70 | 2.66E-01 |
| 1317.22 | 2188.09 | 1336.27 | 146.70 | 5.36E-01 |
| 1656.23 | 60145.61 | 861.99 | 146.70 | 1.13E-01 |
| 1242.26 | 116188.50 | 955.35 | 146.70 | 1.52E-01 |
| 1440.47 | 6786.13 | 1629.20 | 146.70 | 5.40E-01 |
| 1909.12 | 42772.56 | 1209.17 | 146.70 | 2.89E-01 |
| 1121.00 | 18264.24 | 918.14 | 146.70 | 1.78E-01 |
| 1492.24 | 2412.15 | 1262.02 | 146.70 | 4.51E-01 |
| 1336.88 | 1230.31 | 2965.49 | 146.69 | 8.35E-01 |
| 1450.52 | 8109.40 | 1858.35 | 146.70 | 5.88E-01 |
| 2857.54 | 22327.91 | 773.95 | 146.70 | 1.43E-01 |
| 1310.27 | 17813.77 | 964.50 | 146.70 | 2.49E-01 |

Discussion

a) The fluctuations within the life of each stage and its causes have been discussed in the previous section. However, the detailed data provides the view of randomness which comes from the uniformly distributed random number. In the program, there are 100 samples randomly selected for calculation the mean value, trying to minimize the inevitable calculation error of the mean value (5%).

b) In the need of the adaptive simulation to converge to the reliability estimate, 100,000 random numbers are needed in the Monte Carlo method to converge to the high level of accuracy. The random number is paramount in conducting life prediction as it is the only source of uncertainty in the simulation.

c) Considering all the fuzzy factors, the structure reliability curve is getting steep suddenly at a specific time, which just covers the edge of the pit growth stage. It can be found from table 4-5 that the random number even has larger influence on this stage than the other stages.

d) Most of the aging aircraft has safely past the pit nucleation stage (T1), and is activate in the pit growth stage (T2) as it contributes the majority of the expected life(about 90%) and reliability(sharp curve occurs at the end of this stage). Short crack(T3) and long crack Stage(T4) are thought to be dangerous in practice and should be avoided through proper structure integrate program.

e) As can be expected, initial inspection is needed around T1 stage. Therefore, it is also essential to collect enough data of this stage for statistics purpose and to analysis the accurate distribution. Or on the other hand, mechanical model considering the pit nuclear mechanism and environment affection is needed for better understanding of the stage.

# CHAPTER 5 CONCLUSION

The following conclusions can be drawn from the current study:

a) Aging problems in the field of aircraft structure are initially addressed in the study, along with the most concerned issues, such as corrosion, fatigue, non-destructive evaluation and structural integrity program.

Because of budget constraints and affordability considerations, life extension of the current aging aircraft becomes necessary rather than costly replacement or updating. The extended use of those aging aircraft results in the more concern of the associated technical issues and operational needs.

Corrosion as one of the major causes of aging is recognized as a form of material damage caused by chemical and/or electro-chemical process from exposure to the corrosive environment. Despite the different types of corrosion, material and environment are key factors of corrosion mechanism. Corrosion prevention methods, techniques and tools could then be developed with respect to material treatment/replacement, environment protection, and even design refining.

Fatigue is another most important issue of aging aircraft, which is inevitable with the aging of the aircraft structure. Manufacturing flaw or defect of the structure component and dynamic loading are thought to be the key sources of fatigue. Crack growth law based fatigue analysis is developed to identify the initial crack size and to find out the factors that affect the crack growth rate and finally to extend the fatigue growth time.

Driven by detecting, measuring and screening the fatigue cracking, stress corrosion cracking, and corrosion conditions, many NDE techniques were developed, such as optical surface measurement, eddy current probe and Comparative Vacuum Monitoring (CVM) technology. Depending upon the measurement accuracy required and the specific geometry of the identified local problem areas different or combined NDE methods can be employed for proper quantification.

Reliable structural integrity of the aging aircraft assessment, considering all the issues stated as above, is the key to safe operation, mission effectiveness and proper maintenance. For aging aircraft, continuing airworthiness has traditionally been ensured by inspection programs and proper repairs. Fortunately, Australia had regulated airworthiness standards originating in the USA and Europe, which specified the complex issues arising.

Some recommendations on the aging problem of the aircraft structure follow:

- Design the reliability from the concept stage, which means applying the cutting-edging technology from the beginning of the design, aiming at the specific problem arisen. For example, reasonable application of compound material could avoid the corrosion associated with metal material; Sufficient dynamic analysis considering the possible change of design, change of aircraft use, and any other causes of loading change would reduce the risk of fatigue damage.

- As the corrosion and fatigue are acting simultaneously, more robust fatigue growth model considering the corrosion effects is highly desirable for more accurate life prediction and then more proper maintenance schedule.

- It is worth emphasizing the development and validation of analysis tools to predict the onset of WFD, such approach to WFD should be supplemented with advanced analysis methods and more extensive use of the results of detailed tear-down examinations of full-scale fatigue test or retired aircraft.

- Develop an integrated NDE capability based on life-cycle management program for the specific aging aircraft, which can recognize the interdisciplinary nature of NDE and the aging aircraft problem. Also, hybrid inspection technologies that use multiple techniques simultaneously are strongly recommended to increase the probability of flaw detection, including hidden corrosion and fatigue cracks associated with aging aircraft.

b) In an effort of well understanding the cutting-edge technology in life prediction and reliability evaluation, a wide range of methods and tools in the field of aging structure are carefully studied.

As the basic structural reliability method, the strength-load interference method is easy to use but hard to derive the analytical formulation, especially in non-linear limit state. Also, the assumption that the strength and stress are statistically independent may not be valid for some problems.

The FORM/SORM succeed in the structural reliability analysis and in the prediction of with arbitrary distribution variables and non-linear limit state function. However, they are not as accurate, and sometimes, the results from the calculation are conservative, although the SORM had made a progress in accuracy compared to the FORM.

Monte Carlo is a good simulation method, but the accuracy depends on the quantity of computation and usually it is time consuming. The efficiency, and thus the accuracy, of simulation methods can be increased by using variance reduction techniques.

The Probabilistic fatigue Method and probabilistic fracture mechanics consider both of the crack nucleation and subsequent propagation with random variables to calculate the probability of fracture of structures, which have been applied extensively. The randomness of the initial crack sizes and flaw detection were very well considered in the model.

With the increasing complexity of structure, a new computational method called as Probability Finite Elements Method (PFEM), which combines the finite element method with statistics and reliability methods, was introduced to solve the linear and non-linear structural mechanics problems, fracture mechanics problems, and fatigue problems. Although the PFEM couple with the first-order reliability method can calculate the reliability index via an optimisation procedure and provide a powerful tool for the sensitivity analysis, the computational results normally are approximate.

Fuzzy reliability theory has shown its huge advantage in very well describing the vagueness nature of an event, both in randomness and in vagueness, since it was introduced into the field of reliability. It is undoubtedly leading the direction of reliability theory development, but the present problems are lack of membership and its parameters. The further investigation would mainly focus on test and field data collection.

Structure system reliability analysis is quite complex in comparison to component risk assessment. The major difference is the system analysis requires the formulation and identification of the numerous potential collapse modes and their combination into a single assessment of system risk. Further development is needed to make system models more accurate and consistent and also more accessible to designer for making risk-benefit tradeoffs.

A couple of computer softwares designed for different purposes are introduced, such as fatigue life management software of titanium aircraft rotor disks (DARWIN), aircraft structure risk assessment tool (PROF), computer software package for the Numerical Evaluation of Reliability Functions (NERF), Computer programs available for FORM/SORM (NESSUS, PROBAN, CALREL), and many others. Among those, PROF and NERF are strongly recommended for using in aging fleets management.

Successful operation of any methods/theory or tools would depend on a sufficient and efficient database. It is quite urgent to build up such a database to provide accurate parameter distribution, variable relationship, and material property through teardown inspection and laboratory test.

The hybrid method incorporating the principle of two or more methods would be the trend of reliability assessment and life prediction of the aging aircraft structure. Such the combination of Monte Carlo simulation and fuzzy reliability method, the combination of Monte Carlo simulation and SORM are highly recommended.

c) The difficulty in analytically predicting the initiation and growth of small cracks arises, in part, from the potential that several different mechanisms (e.g., corrosion, fretting, fatigue, residual stress) could influence crack initiation. However, the equivalent initial crack size approach provides the fracture mechanics based solution.

Based on the widely cited Paris law, the equivalent initial crack size approach is presented, in which corrosion damage could be quantitatively derived. FORTRAN program is then designed to solve the non-linear equation derived from the Paris law. Given the measured pit size and tested fatigue life, the equivalent initial crack size is gained.

Monte Carlo method and iterative method are introduced to solve the complicated none-linear equation. In the process of simulation and iteration cycle, an error of 0.5 is carefully designed to balance the two conflicting calculation requirements: accuracy and convergence.

For verification purpose, fatigue life prediction software package AFGROW is introduced and applied with the data from reference. It is observed that direct use of measured pit size for life prediction and reliability evaluation would apparently be conserve. Concluded from the result gained, it is more reasonable to incorporate the correction factor λ [0.77, 0.86] for the measured initial pit size.

The accuracy of the equivalent initial crack size obtained will thus depend on the Paris law used, where a few other parameters involved; The utility of the equivalent initial crack size lies in the requirement that the conditions in service will remain the same as outlined in the Paris law.

The recommendation follows as below:

- More fatigue experiment with different material and loading spectrum should be conducted for calculation and for verification of the correction factor λ, which could be used to develop an EIF database, correlated with full-scale structural test.

d) Corrosion fatigue life of aircraft structure is composed of four stages: pit nucleation stage, pit growth stage, short crack growth stage, and long crack growth stage. While the transition from the short crack to the long crack, transition from the pit to short crack and failure criterion are treated as fuzzy process.

With consideration of the four stages corrosion fatigue life, incorporating the data obtained through maintenance practice and the data in the literature, a hybrid model considering both the randomness and fuzziness of parameters is presented, along with the FORTRAN code developed.

Those fuzzy factors in corrosion fatigue life are described by membership function. Through scaling based transformation method, the membership functions are then transferred to equivalent pdf, which enables the parameter simulation.

Lognormal distributed variables are expressed by mean value and standard deviation; for those Weibull distributed parameters, its randomness is depicted by the shape parameter, minimum value parameter, and the scale parameter. The inverse function of the relative pdf is then conducted for direct sampling using uniformly distributed random generator.

The reliability influence of each parameter is finally analysized through the established model. It is found that random parameters $\Phi$ and $IP_0$ result in the bigger changing of reliability comparing to the other random parameters: $C_{sc}$, $C_{lc}$, $a_o$, $t_1$, although there is only little difference.

In view of the analysis results in section 4-6, the aging structure reliability is significantly affected by the fuzzy parameter $\Delta K_{th}$, which is based on the material test. While the other two fuzzy parameters $a_{th}$ and $a_f$ only have slight influence on reliability.

As can be concluded from section 4-7, stage $t_1$ and $t_2$ contribute the majority of the expected life (about 90%), which indicates that most aging aircraft are active in the pit growth stage, that initial inspection are desired in pit nucleation stage and that while short or long crack occurs the structure is thought to be dangerous in terms of reliability.

Different material and loading spectrum will have different reliability and life. The current study is based on the data from literature; more accurate analysis can be conducted while actual fatigue test or inspection data are available. Hence, there is no guarantee that satisfactory prediction results could gained from the current model while it is used in other situations.

Finally, some recommendations on reliability assessment and life prediction follow:

- The need for further research and development in load interaction models is essential. The assumption of the large variation in loading spectrum would result in either over-inspecting the aircraft or flying unsafe aircraft. Rainflow cycle counting is proposed for better understanding of applied load spectrum.

- Future research on the pit nucleation stage should derive a mathematical model which can be implemented in practical applications and incorporate not only macrostructural effects, but also environmental effects.

- Multiple-site crack propagation analysis methodology considering interaction of the damages is also in the practical needs for the structural reliability.

- Further research on the parameter $\Delta K_{th}$ is needed to analysis the accuracy of its membership function, as well as evaluation of its sensitivity to aggressive environments, such as humidity, saltwater, fuel, or hydraulic fluids.

# REFERENCES


1. Tiffany, Charles F. —Aging of US Air Force Aircraft. " 1997 USAF Structural Integrity Program Conference, 11 December 1997, 57
2. ATSB: How old is too old, February 2007
3. Australian Transport Safety Bureau report. URL: http://www.atsb.gov.au/
4. Cope, D.A., 1998,"Corrosion Damage Assessment Framework, in Final Report on Corrosion/Fatigue Affects on Structural Integrity Contract, D500-13008-1", The Boeing Company, National Institute of Aviation Research, Wichita, KS.
5. Anderson, T.L., 1991, Fracture Mechanics: Fundamentals and Applications, 2nd Edition, CRC Press.
6. Kaplan, P.M., and Timothy, A.W., "Fatigue-Life Assessment", EAD Aerospace Inc.
7. Schutz, Walter. —A History of Fatigue. "Engineering Fracture Mechanics, 54 (1996): 263.
8. NDT Resource Centre, URL: http://www.ndt-ed.org/
9. CVM technology: http://www.smsystems.com.au/
10. Structural Integrity Associates Inc, URL: http://www.structint.com/pc-crack/lefm.html.
11. Grandt A.F., 2003, Fundamentals of Structural Integrity: Damage Tolerant Design and Non-destructive Evaluation, Wiley-Inter science.
12. Aircraft Airworthiness: http://www.casa.gov.au/rules
13. DEF STAN 00-970 (UK), Issued Oct 1987, Volume 1: Chapter 201: Fatigue Damage Tolerance.
14. Military Standard, Aircraft structural Integrity Program, Airplane requirement, MIL-STD-1530A (USAF), December 1975
15. C. (RAJ) Sundararajan: Probability Structural Mechanics Handbook---Theory and Industrial Applications, Chapman &hall, 1995, Page 16
16. R. E. Melchers: Structural Reliability Analysis and Prediction, Wiley, 1998, page 97~130
17. Rosentbleuth and Esteva (1972): Reliability Bases for Some Mexican Codes, ACI Publication SP-31:141
18. R. E. Melchers: Structural Reliability Analysis and Prediction, Wiley, 1998, P95~131

19. C. (RAJ) Sundararajan: Probability Structural Mechanics Handbook---Theory and Industrial Applications, Chapman &hall, 1999, P30~47
20. W. K. Liu, Probability Finite Elements for non-linear structural dynamics, Computer method in Applied Mechanics Engineering, 56:61-81, 1986b.
21. W. K. Liu and A. Mani, Finite Element methods in probabilistic mechanics, Probabilistic Engineering Mechanics, 2(4):201-213, 1988d.
22. Lua, Y. K and W. K. Liu, Curvilinear fatigue crack reliability analysis by stochastic boundary element method, International Journal for Numerical Methods in Engineering 36:3841-3858, 1992d.
23. Der Kiureghian, A., And B.-J. Ke (1985): Finite-element based reliability analysis of frame structures. Proceeding of the 4th international conference on structural safety and reliability, pp 395-404.
24. Der Kiureghian and P. L. Liu (1988): Optimisation algorithms for structural reliability, Computational Probabilistic Mechanic, New York: American society of mechanic Engineers, pp 185-196
25. Liu P. L. and Der Kiureghian (1991): Finite Element of geometrically non-linear uncertain structures, Journal of Engineering Mechanic Division of American Society of Civil Engineers 117(8): 1806-1825
26. W. K. Liu, T. Belytschko, and Y. J. Lua: Probabilistic Finite Element Method, The report of NASA, No.NAG3-822, 1995.
27. C. (RAJ) Sundarajan, Probabilistic Structure Mechanics Handbook, Chapman & Hall, 1995, Page 106
28. C. (RAJ) Sundarajan, Probabilistic Structure Mechanics Handbook, Chapman & Hall, 1995, Page 121~131
29. Ruggieri, C. (1997), An engineering Methodology for integrity Assessments Using probability fracture Mechanics, Proceeding of the ESREL'97 international conference on safety and Reliability, 17~20 June, Lisbon Portugal, 1997, page 1477~1484
30. John W. Lincoln, Aging Aircraft-USFA Experience and Actions, 15th Plateman Memorial Lecture
31. He Ren, An Application to Structural Fatigue of Fuzzy Reliability theory, Report Number: CR 9911, The Sir Lawrence Wackett Centre
32. Y. S. Feng, New Design Approaches of Modern Aircraft, (Chinese), NUP press, 1982

33. R. E. Melchers: Structural Reliability Analysis and Prediction, Ellis Horwood, 1987and Wiley, 1998, Page 247~250
34. Wirshing,P.H., etc.: Fatigue reliability and maintainability of marine structures, Marine Structures, Page 265~284, 1990
35. Wirshing,P.H, Probabilistic Structural Mechanics Handbook, Chapman & Hall, 1995, Page 146~162
36. Ruggieri, C. (1997), An engineering Methodology for integrity Assessments Using probability fracture Mechanics, Proceeding of the ESREL'97 international conference on safety and Reliability, 17~20 June, Lisbon Portugal, 1997
37. Kordonsky, Kh etc, Fatigue Crack Monitoring in Parallel time scales, Proceeding of the ESREL'97 international conference on safety and Reliability, 17~20 June, Lisbon Portugal, 1997, page 1485~1491.
38. C. (RAJ) Sundarajan, Probabilistic Structural Mechanics Handbook, Chapman & Hall, 1995, page 157.
39. K. Y. Cai, Study on Fuzzy Reliability, Ph.D dissertation of Beijing University of Aeronautics and Astronautics, 1990.
40. He Ren, A Brief Report on Fuzzy Reliability Theory and Its Applications, Report Number Cr9821, The Sir Lawrence Wackett Centre for Aerospace Design Technology, RMIT, 1998.
41. He Ren, The applications of fuzzy reliability theories on structural fatigue, Report Number Cr9911, The Sir Lawrence Wackett Centre for Aerospace Design Technology, RMIT, 1999.
42. Nordal, H. (1987), A structural system reliability case study of an eight-leg steel jacket offshore product platform, Proceeding of the Marine Structural reliability Symposium, Arlington, Virgina.
43. Y.C. Tong J. Hou and R.A. Antoniou Probabilistic Damage Tolerance Assessment: The Relative Merits of DARWIN, NERF AND PROF DSTO-TR-1789
44. Alan P. Berens, Risk analysis for Aging aircraft fleets, Volum 1- analysis, OH45469-0120, 1991.
45. Peter W. Hovey, Update of the PROF computer Program for Aging Aircraft Risk Analysis, OH 45469-0120, Air force Research Laboratory, Volume 1, 1998
46. Graham, A. D. ; Mallinson, G. D. A Computer Program for the Numerical Evaluation of Reliability Functions - User Manual. FEB 1984

47. Rajagopal, K.R. etc: Verification of NESSUS code on space propulsion components, Proceeding of the 5th International conference on structure safety and reliability, New York, 1989, Pages 2299~2306
48. Veritas Sesam Systems, PROBAN, Houston, Texas, 1991
49. Liu P. L, and H. Z. Lin: CALREL, Berkely, University of California, 1989
50. Pao PS, Gill SJ, Feng CR. On fatigue crack initiation from corrosion pits in 7075 T7351. Scripta Mater 2000;43(5):391–6.
51. Schijve J, Skorupa M, Skorupa A, Machniewicz T, Gruszczyn´ ski P. Fatigue crack growth in the aluminium alloy D16 under constant and variable amplitude loading. Int J Fatigue 2004;26(1):1–15.
52. REN Ke-liang LV Guo-zhi ZHANG You LY12-CZ corrosion test based correlation between corrosion pit with equivalent initial surface crack. STRUCTURE & ENVIRONMENT ENGINEERING Vol.33, No.2 1006-3919(2006)02-0050-08;
53. Willenborg, J., R. M. Engle, and H. A. Wood. —A Crack Growth Retardation Model Using an Effective Stress Concept." AFFDL-TM-71-1-FBR (1971).
54. Yang JN, Manning SD, Rudd JL, Bader RM. Investigation of mechanistically based equivalent initial flaw size approach. In: 18th Symposium of ICAF Melbourne Australia. ICAF 95.EMAS; 1995.
55. Fawaz, S. A. —Equivalent Initial Flaw Size Testing and Analysis," AFRL-VA-TR-2000-3024, Air Force Research Laboratory, Air Vehicles Directorate, Air Force Materiel Command, Wright-Patterson Air Force Base, OH 45433, 29.
56. Newman JC, Wu XR, Venneri SL, Li GC. Small crack effects in high strength aluminium alloys. NASA RP 1309, Hampton,VA, USA 1994.
57. William H. Press Saul A. Teukolsky William T. Vetterling Brian P. Flannery Numerical Recipes in Fortran 77-The Art of Scientific Computing Second Edition
58. Harter, James A., —AFGROW Users Guide and Technical Manual," AFRL-VA-WP-TR-1999-3016, February 1999.
59. Harlow D G , Wei R P. Probability approach for prediction of corrosion and corrosion fatigue life[J ] . AIAA Journal, 1994, 32 (10) : 2073 -2079.
60. Vasudevan A K , Sadananda K. Environmental effects on fatigue crack initiation and growth[ A ] . In : RTO meeting proceedings 18 [ C ] . France : NATO Research and Technology Organization , 1999. 144 -156.

61. Shi P , Mahadevan S. Probabilistic estimation of pitting corrosion fatigue life[ R] . AIAA-2000-1644 ,2000.
62. Shi P , Mahadevan S. Aircraft structure reliability under corrosion fatigue[ R] . AIAA-2001-1377 ,2001.
63. Liu W T , Li Y H , Chen Q Z. Accelerated corrosion environmental spectrums for testing surface coatings of areas of flight aircraft structures [J ] . Journal of Beijing University of Aeronautics and Astronautics, 2002, 28 (1) : 109 -112.
64. Chen Y L, LU G Z, Duan C M. A probability model for the corrosion damage of aircraft structure in service environment [J], Aeronautical and Astronautic Science. 2002, 23(3): 249-251 (in Chinese);
65. Wu X R. Handbook mechanical properties of aircraft structural metals (Vol. 3 Corrosion fatigue) [M ] . Beijing: China Aviation Industry Press, 1996. 227 -255.
66. Wu X R, Handbook of mechanical properties of aircraft structural (Vol.3 Corrosion fatigue) [M] . China Aviation Industry Press, 1996, 227 -255.
67. Rao S S , Sawyer J P. Fuzzy finite element approach for the analysis of imprecisely defined systems[J ]. AIAA Journal, 1995, 33(12): 2364-2370.
68. Smith SA, Krishnamurthy T, Mason BH. Optimized vertex method and hybrid reliability. 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, 2002; 1465.
69. Dawicke, D. S., A. F. Grandt Jr., and J. C. Newman Jr., ―Three-Dimensional Crack Closure Behavior," Engineering Fracture Mechanics, 1990 v 36 n 1.
70. de Koning, A. U., H. J. ten Hoeve, F. P. Grooteman, and C. J. Lof," Advances in the Modelling of Cracks and Their Behaviour in Space Structures." Proceedings of the European Conference on Spacecraft Structures, Materials, and Mechanical Testing, Braunschweig, Germany, 4-6 Nov. 1998.
71. Fawaz, S. A., J. Schijve, and A. U. de Koning. Fatigue Crack Growth in Riveted Lap Splice Joints, Proc. of the 19th Symposium of the International Committee on Aeronautical Fatigue, 16-20 June 1997, Edinburgh, Scot. Scotland, UK: EMAS/SoMat Systems International Ltd, 1997.
72. Schutz, Walter. ―A History of Fatigue." Engineering Fracture Mechanics, 54 (1996).
73. Newman Jr. JC, Raju IS. An empirical stress intensity factor equation for the surface crack. Engineering Fracture Mechanics 1981.

# APPENDIX FORTRAN PROGRAM CODES

## A1. NONE- LINEAR EQUATION

```
 ! USING MC ITERATION
 ! ERRORS ARE SET TO THE SPECIFIC INPUT
 ! MULTI ROOTS ARE POSSIBLE, SELECT ACCORDING TO ITS MEANING
 ! Function ran1(idum) IS SELECTED FOR RANDOM GENERATOR
program ITR
double precision exp
integer i,j, idum,n,m
real af, a0,fn,u,f0, Z,a1
parameter(af =6210., u=0.5)
parameter(n = 1000, m=10000)
!data initialize
fn=105827
!random seed
idum=200
exp = 0
a1=87.2
a0=a1
 ! Do n iterations
! CALL RANDOM_SEED()
 do 20 j=1,m
        do 10 i = 1, n
                x = (RAN1(idum) * (af - a0)) + a0
                exp = (exp * (i-1) + f(x)) / i
             10 continue
    f0 = exp * (af - a0)
     if(abs(f0-fn).GE.u) then
!          CALL RANDOM_NUMBER(Z)
```

```
            a0=(RAN1(idum)+0.5)*a1
      else
            a1=a0
            write(*,*) j,a1,f0
      end if
20 continue
! Print result
!write(*,*) j,a1
!write(*,*) exp * (af - a1)
end
 ! The integration function
function f(a)
real a,ki,deltas,q,pi,pm,t,w,ex
double precision c
pm=3.67
c=0.0000000608
pi=3.1415926
deltas=256.4
c0=113.4
!c0=124.5
t=3500
w=26000
q=1+1.464*(a/c0)**1.65
ki=deltas*(pi*a*0.001/q)**0.5*(1.04+0.2*(a/t)**2-
0.1*(a/t)**4)*(1+(tan(pi*a/2/w*(a/t)**0.5))**2)**0.25
ex=c*ki**pm
f=1/ex
return
end
```

## A2. MULTI INTEGRATION

```
C     GAOSI MULTI-INTEGRATE
C     TESTED WITH EXAMPLES
C     THE FUNCTION NEEDS TO BE INTEGRATABLE
      PROGRAM GAOSI INTEGRATION
      INTEGER K(3), S1(3), H(3)
        REAL A(3), B(3), C(3), X(3), D(4), R(4), U1(3), W1(3)
        DATA S1/2,2,2/
        DATA U1/-0.77459667, 0., 0.77459667/
        DATA W1/.55555555, 0.88888889, .55555555/
        CALL SUB0303(3,4,3,S1,U1,W1,K,H,A,B,C,X,D,R,Z)
        WRITE(*,10) Z
10    FORMAT(1X,2HZ=, F10.6)
        STOP
     END PROGRAM
C     THE INTEGRATION DIMENSION BE NOTED AS 3 IN THIS EXAPMLE
C     THE INITIAL DATA NEEDS TO BE RESET AS DIMENSION CHANGED
C     THE FUNCTION NEEDS TO BE REWRITE AS IT CHANGED

      SUBROUTINE SUB0303(N,N1,NP,S,U,W,K,H,A,B,C,X,D,R,Z)
      INTEGER K(N), S(N), H(N)
```

```
      REAL A(N), b(N), C(N),X(N),d(N1),R(N1),U(NP),W(NP)
      I1=1
      D(N1)=1.
      R(N1)=1.
40    DO 15 I=I1,N
      A(I)=F1(I,X)
      B(I)=F2(I,X)
      D(I)=(B(I)-A(I))/S(I)
      C(I)=0.5*D(I)+A(I)
      X(I)=0.5*D(I)*u(1)+C(I)
      R(I)=0.
      H(I)=1
      K(I)=1
15    CONTINUE
      I=N
33    Z1=F3(I,X)
      KI=K(I)
      R(I)=R(I+1)*D(I+1)*Z1*W(KI)+R(I)
      IF(K(I).LT.NP) GOTO 20
      IF(H(I).LT.S(I))GOTO 25
      I=I-1
```

```
      IF(I.EQ.0)GOTO 30
C   IF(I.EQ.0)GOTO 25
      GOTO 33
25    H(I)=H(I)+1
      C(I)=(H(I)-0.5)*D(I)+A(I)
      K(I)=1
      GOTO 35
20    K(I)=K(I)+1
35    KI=K(I)
      X(I)=0.5*D(I)*U(KI)+C(I)
      IF (I.EQ.N) GOTO 33
      I1=I+1
      GOTO 40
30    Z=.5**N*R(1)*D(1)
      RETURN
      END


 FUNCTION F1(J,X)
      REAL  X(3)
      GOTO(1,2,3), J
1     F1=-1
```

```
      GOTO 4
2     F1=-SQRT(1.-X(1)**2)
      GOTO 4
3     F1=-SQRT(1.-X(1)**2-X(2)**2)
4     RETURN
      END

 FUNCTION F3(J,X)
      REAL  X(3)
      GOTO (9,9,11),J
9     F3=1.
      GOTO 7
11    F3=1./(X(1)**2+X(2)**2+(X(3)+0.5)**2)
7     RETURN
      END

      FUNCTION F2(J,X)
      REAL  X(3)
      F2=-F1(J,X)
      RETURN
      END
```

## A3. MONTE CARLO INTEGRATION

```
! exp = expectation of f(x). n = no. of iterations
! i = no. of iterations done. t = time at start
! x, y = . x1, x2, y1, y2 = boundary points
! different random methods are tested
PROGRAM MC
double precision exp
integer i, idum
real x, y
parameter(y1 = 1, y2 =2, x1=1, x2=20)
parameter(n = 1000000)
! Seed the PRNG
!call random_seed
idum=380
exp = 0
! Do n iterations
  do 10 i = 1, n
         x = (RAN1(idum) * (x2 - x1)) + x1
     y = (RAN1(idum) * (y2 - y1)) + y1
     exp = (exp * (i-1) + f(x, y)) / i
         !WRITE(*,*)RAN1(IDUM)
  10 continue
! Print result
```

```
write(*,*) exp

write(*,*) exp * (y2 - y1)*(x2 - x1)

end    program

! The function

! several functions tested

function f(x, y)

real x, y

!f = exp(-(x-0.99*x)**2)*999.9996*exp(-0.2303*x)*20*(x-0.2)**19

 f=x**2+y**2

!f=log(x+y)

!f=sin(x)+cos(y)

!f=x+y**3+log(x)

return

end

   Function ran1(idum)

                 dimension r(97)

                parameter(m1=259200,ia1=7141,ic1=54773,rm1=3.85802e-6)

                 parameter(m2=134456,ia2=8121,ic2=28411,rm2=7.43738e-6)

                 parameter(m3=243000,ia3=4561,ic3=51349)

                 data iff/0/

                 if (idum.lt.0.or.iff.eq.0) then

                   iff=1

                          ix1=mod(ic1-idum,m1)
```

```
                ix1=mod(ia1*ix1+ic1,m1)

                ix2=mod(ix1,m2)

                ix1=mod(ia1*ix1+ic1,m1)

                ix3=mod(ix1,m3)

                do j=1,97

                  ix1=mod(ia1*ix1+ic1,m1)

                      ix2=mod(ia2*ix2+ic2,m2)

                      r(j)=(float(ix1)+float(ix2)*rm2)*rm1

        enddo

        idum=1

            end if

            ix1=mod(ia1*ix1+ic1,m1)

            ix2=mod(ia2*ix2+ic2,m2)

            ix3=mod(ia3*ix3+ic3,m3)

            j=1+(97*ix3)/m3

            if (j.gt.97.or.j.lt.1) pause

            ran1=r(j)

            r(j)=(float(ix1)+float(ix2)*rm2)*rm1

            return

            End

!       GENERATE RANDOM2 NUMBER
```

```
!         SUBROUTNE  RAND2

!         X(N+1)=C*X(N)*(MOD(M))

     SUBROUTINE RAND2(IX,IC,IM,YFL)

      IMPLICIT DOUBLE PRECISION(A-H,O-Z)

     IF(YFL.NE.0.) GO TO 1

     AX=FLOAT(IX)

     AM=FLOAT(IM)

     AC=FLOAT(IC)

     YFL=AX/AM

1     YFL=AC*YFL

     LII=YFL

     YFL=YFL-FLOAT(LII)

!   YFL=YFL-FLOAT(IFIX(YFL))

     RETURN

     END

!         GENERATE RANDOM NUMBER 3

!         SUBROUTNE  RAND3

!         X(0)=1~67108  125  , 2796203

     SUBROUTINE RAND3(IX,YFL)

      IMPLICIT DOUBLE PRECISION(A-H,O-Z)

!    WRITE(*,*) '****** ENTER SEED *******'
```

```
      IF(IX.EQ.0) IX=67107
      IX=125*IX
      IX=IX-IX/2796203*2796203
      YFL=FLOAT(IX)
      YFL=YFL/2796203.0
!    WRITE(*,*)'YFL=',YFL
      RETURN
      END
!          GENERATE NOMAL RANDOM NUMBER 4
!          SUBROUTNE  RAND4
!         Y1=(-2*LN(R1)**(1/2)*COS(2.*3.14*R2)
!         Y2=(-2*LN(R1)**(1/2)*SIN(2.*3.14*R2)
!        TRANSFORM (Y1,Y2) TO (W1,W2)~N(U,CGM)
!         V1=U+CGM*Y1
!         V2=U+CGM*Y2
      SUBROUTINE RAND4(XX,IC,IM,YFL,S,AM,V1,V2)
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       CALL DRAND(XX,YFL)
      V1=DSQRT(-2.*DLOG(YFL))
       CALL DRAND(XX,YFL)
      T=6.2831853*YFL
      V2=V1*DSIN(T)*S+AM
```

```
      V1=V1*DCOS(T)*S+AM
      RETURN
      END
       SUBROUTINE DRAND(XX,YFL)
       DOUBLE PRECISION A,P,XX,B15,B16,XHI,XALO,LEFTL,FHI,XK,YFL
!      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
1       REAL LEFTL
!      IMPLICIT DOUBLE  LEFTL
       DATA A/16807.D0/,B15/32768.D0/,B16/65536.D0/,P/2147483647.D0/
       XHI=XX/B16
       XHI=XHI-DMOD(XHI,1.D0)
       XALO=(XX-XHI*B16)*A
       LEFTL=XALO/B16
       LEFTL=LEFTL-DMOD(LEFTL,1.D0)
       FHI=XHI*A+LEFTL
       XK=FHI/B15
       XK=XK-DMOD(XK,1.D0)
       XX=(((XALO-LEFTL*B16)-P)+(FHI-XK*B15)*B16)+XK
       IF(XX.LT.0.D0) XX=XX+P
       YFL=XX*4.656612875D-10
       RETURN
       END
```

## A4. RELIABILITY ASSESSMENT

### PROGRAM RELIBILITY

```
real    :: x1,x2,x3,x4, deltaH=50000, R=8.314, T=293, phi, dkth, a0, oip

real    :: ath, csc, f=10., m=0.66, ratio=0.3, si=114.5

real    :: af, clc, ath1=0.77E-3, ath2=1.0E-3

real    :: z, af1=6.0E-3, af2=6.3E-3, sx1,sx2,sx3,sx4

real    ::       k3=1.05,dkth1=1, dkth2=2.5

!real   ::       k3=1.41,dkth1=0.8, dkth2=2.5

real    ::  tf=80000., x, q=0.

integer ::  n=100, k=10000, j, i

!k3=1.05 for comparasion

    open(2,file='=reliability-out.dat',status='new')

call random_seed()

!do 20 j=1,k

!sx1=0.

!sx2=0.

!sx3=0.

!sx4=0.

do 10 i=1,n

  ! pit nucleation stage weibull random t1
```

```
x1=random_weibull(0.55,120.0,1000.0)

!write(*,*) x1

!pit growth stage t2 initiation

oip=random_weibull(1.0, 26E-5, 6.5E-5)

! oip=58.6E-5

phi=random_weibull(2.255,0.51,0.3)

!phi=0.977

call random_number(z)

dkth=dkth1+dkth2*sqrt(log(k3/z))

!dkth=3.6

!a0=0.0000015

a0=random_weibull(1.497, 2.214E-6, 0.272E-6)

!write(*,*) oip, phi, dkth, a0

x2 = exp(deltaH/(R*T))*(0.57*Y(phi)**3*(dkth/si)**6-23400*a0**3)/(oip*phi**2)

!write(*,*)dkth,  0.57*Y(phi)**3*(dkth/si)**6, Y(phi),     23400*a0**3,  (oip*phi**2)

!short crack growth stage t3 initiation

ath=ath1+0.5*z*(ath2-ath1)**2

!ath=0.001

!csc=ABS(random_normal())*1.092E-9

csc=9.17E-11
```

```
  !write(*,*) dkth, ath, csc

  x3=0.0068*(7.5*Y(phi)*(dkth/si)**(-1.14)-Y(phi)**1.57*ath**(-0.57))/

  (f*csc*((1-    ratio)**(m-1)*si)**3.14)

  !long crack growth stage t4 initiation

  !call random_number(z)

  af=af2-0.5*z*(af2-af1)**2

  ! clc=abs(random_normal())*6.11E-11

  !af=0.006

  clc=6.11E-11

  !write(*,*) af, clc

  x4=0.0092*(ath**(-0.57)-af**(-0.57))/(f*clc*((1-ratio)**(m-1)*si)**3.14)

  write(2,*)x1, x2, x3, x4, z

  sx1=sx1+x1

  sx2=sx2+x2

  sx3=sx3+x3

  sx4=sx4+x4

  !x=x1+x2+x3+x4

10 continue

x=sx1/n+sx2/n+sx3/n+sx4/n

!if (x.LT.tf) then
```

```
  !q=q+1
  !write(*,*) sx1/n,sx2/n,sx3/n,sx4/n
  !stop
!end if
!20 continue
write(2,*) x, sx1/n,sx2/n,sx3/n,sx4/n
end    program
! The function Y(phi)
! constant to a/c
function Y(yphi)
real y,yphi
y=1+1.464*yphi**(-1.65)
return
end
! Lognormal distribution
! If X has a lognormal distribution, then log(X) is normally distributed.
! Here the logarithm is the natural logarithm, that is to base e, sometimes
! denoted as ln.  To generate random variates from this distribution, generate
! a random deviate from the normal distribution with mean and variance equal
! to the mean and variance of the logarithms of X, then take its exponential.
```

```
! Relationship between the mean & variance of log(X) and the mean & variance

! of X, when X has a lognormal distribution.

! Let m = mean of log(X), and s^2 = variance of log(X)

! Then

! mean of X     = exp(m + 0.5s^2)

! variance of X = (mean(X))^2.[exp(s^2) - 1]

FUNCTION random_normal() RESULT(fn_val)

!  The algorithm uses the ratio of uniforms method of A.J. Kinderman

!  and J.F. Monahan augmented with quadratic bounding curves.

REAL :: fn_val

!  Local variables

REAL     :: s = 0.449871, t = -0.386595, a = 0.19600, b = 0.25472,    &

            r1 = 0.27597, r2 = 0.27846, u, v, x, y, q, half

!  Generate P = (u,v) uniform in rectangle enclosing acceptance region

DO

  CALL RANDOM_NUMBER(u)

  CALL RANDOM_NUMBER(v)

  half =0.5*u

  v = 1.7156 * (v - half)

!     Evaluate the quadratic form

  x = u - s

  y = ABS(v) - t

  q = x**2 + y*(a*y - b*x)
```

```
!     Accept P if inside inner ellipse

  IF (q < r1) EXIT

!     Reject P if outside outer ellipse

  IF (q > r2) CYCLE

!     Reject P if outside acceptance region

  IF (v**2 < -4.0*LOG(u)*u**2) EXIT

END DO

!     Return ratio of P's coordinates as the normal deviate

fn_val = v/u

RETURN

END FUNCTION random_normal

! three parameters weibull distribution

FUNCTION random_weibull(wa, wb, wg) RESULT(fn_val)

!  The algorithm uses direct sampling

!  wa stands for the shape parameter, wg stands for the minimum value, wb stands
for the scale

!  every excute, resluting one random by this function

REAL :: fn_val, wa, wg, wb, z

!  Generate the weibull random according to the formula

CALL RANDOM_NUMBER(z)

fn_val = wb*(1-log(1-z))**(1/wa)+wg

RETURN

END FUNCTION random_weibullbull
```