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Equation Chapter 1 Section 1

The improvement of a simple theoretical model for the prediction of the sound insulation of double leaf walls

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This paper presents a revised theory for predicting the sound insulation of double leaf cavity walls that removes an approximation which is usually made when deriving the sound insulation of a double leaf cavity wall above the critical frequencies of the wall leaves due to the airborne transmission across the wall cavity. This revised theory is also used as a correction below the critical frequencies of the wall leaves instead of a correction due to Sewell. It is found necessary to include the "stud" borne transmission of the window frames when modelling wide air gap double glazed windows. A minimum value of stud transmission is introduced for use with resilient connections like steel studs. Empirical equations are derived for predicting the effective sound absorption coefficient of wall cavities without sound absorbing material. The theory is compared with experimental results for double glazed windows and gypsum plasterboard cavity walls with and without sound absorbing material in their cavities. The overall mean, standard deviation, maximum and minimum of the differences between experiment and theory are -0.6 dB, 3.1 dB, 10.9 dB at 1250 Hz and – 14.9 dB at 160 Hz respectively.

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I. INTRODUCTION

There are still no really satisfactory theoretical models for predicting the sound insulation of walls. This means that acoustical consultants have to use measured values of sound insulation when designing buildings. Measurements of sound insulation are expensive and time consuming to make. This paper improves the accuracy of a simple theoretical model of sound insulation.

The author (Davy, 1990, 1991, 2009a) derives a theoretical model for the sound transmission of double leaf cavity walls due to the air borne transmission across the wall cavity above the critical frequencies of the cavity wall leaves. In this derivation, the integral over angles of incidence from 0 ° to 90 ° is approximated by extending the range of integration to $-\infty$ to ∞ following the approach of Cremer (1942) for single leaf walls. This extension of range approximation is used to make the integral easier to evaluate analytically. However, it is possible to evaluate the integral analytically without extending the range of integration. Unfortunately, although the principles of integration involved are straight forward, the algebra becomes very messy. In this paper, the integral is evaluated analytically with the assistance of the Maple 11 software package.

The model, developed for use above the critical frequencies of the wall leaves, is also used as a correction below the critical frequencies by assuming that the non-existent critical angle is 90 °. This correction replaces Sewell's (1970) correction in the region just below the critical frequencies. This new correction has the advantage that it does not become ill-defined at the critical frequencies of the cavity wall leaves.

When the theory is compared with measurements on double glazed windows it is found that it is necessary to include "stud" borne transmission due to the window frames in order to be able to correctly predict the sound transmission for the wider air gaps. To model steel studs

correctly, the author's theory (Davy, 1993, 2009a) is extended to include a minimum stud transmission and a value for this quantity is derived by comparison with experimental data.

When there is no sound absorbing material in the wall cavity, this paper derives two empirical linear regression equations for the "effective" sound absorption coefficient of the cavity. One of these equations is for double glazing and the other is for gypsum plaster board cavity walls. Because of approximations made in the author's theory, the "effective" sound absorption coefficient of the cavity is expected to be larger than the actual physical sound absorption coefficient of the cavity.

The theory is compared with experimental results for narrow and wide air gap double glazing and gypsum plaster board cavity walls, both with and without sound absorbing material in the wall cavity. The gypsum plaster board comparison walls include the cases of no structural connection between the wall leaves and steel and wooden studs.

II. ABOVE THE CRITICAL FREQUENCIES

Above the lowest of the critical frequencies of the two wall leaves the method given by eqns. (1) to (18) of Davy (2009c) is followed. Because these equations are applied to both wall leaves, the subscript i = 1 or 2 is applied to the variables τ (transmission coefficient), Z (bending wave impedance), m (mass per unit area), η (total damping loss factor), ω_c (angular critical frequency), f_c (critical frequency), η_{int} (internal damping loss factor), η_{rad} (radiation damping loss factor), a (ratio of mass impedance to twice the characteristic impedance of air), and θ_c (coincidence angle). To avoid confusion with a symbol used later on, the variable r (ratio of frequency to critical frequency) is replaced with the variable ξ and the subscript i is also applied to it. Eqn. (2) of Davy (2009c) can also be written as [Cremer (1942) eqn. (8.19)]

$$\tau = 2\int_0^1 \tau(\theta)\sin\theta d(\sin\theta) \tag{1}$$

Eqn (18) of Davy (2009c) can be written as

$$\tau_i(\theta) = \frac{1}{s_i^2} \frac{1}{q_i^2 + (x - p_i)^2}, \tag{2}$$

where

$$p_i = 1 - \frac{1}{\xi_i}, \tag{3}$$

$$s_i = \frac{2a_i \xi_i}{\sigma(\theta_{ci})}, \tag{4}$$

$$q_i = \frac{1 + \dfrac{a_i \eta_i}{\sigma(\theta_{ci})}}{s_i} \tag{5}$$

and $\sigma(\theta_{ci})$ is the single sided forced radiation efficiency at the coincidence angle of the *i*th wall leaf. Following the approach of Davy (1990, 1991, 2009c), above the lower of the critical frequencies of each leaf, the sound transmission coefficient $\tau(\theta)$ of a double leaf cavity wall is approximated as

$$\tau(\theta) = \frac{\tau_1(\theta)\tau_2(\theta)}{\alpha^2}, \tag{6}$$

where α is the sound absorption coefficient of the wall cavity. The reason for adoption of this equation is explained at the end of the next section.

Putting eqn. (6) into eqn. (2) of Davy (2009c) gives the diffuse field sound transmission coefficient as

$$\tau = \frac{I}{s_1^2 s_2^2 \alpha^2} \tag{7}$$

where α is the sound absorption coefficient of the wall cavity of the double leaf wall and

$$I = \int_0^1 \frac{dx}{\left[q_1^2 + (x - p_1)^2\right]\left[q_2^2 + (x - p_2)^2\right]}. \tag{8}$$

The integral in eqn. (8) can be evaluated using the methods outlined in sections 2.101 to 2.103 of Gradshteyn and Ryzhik (1965). However the algebra is rather complicated. Thus the integral was evaluated with the assistance of the Maple 11 software package.

$$I = \frac{A + B + C}{D}, \tag{9}$$

where

$$A = q_1 q_2 \left(p_2 - p_1 \right) \ln \left\{ \frac{\left[q_1^2 + \left(p_1 - 1 \right)^2 \right] \left(q_2^2 + p_2^2 \right)}{\left[q_2^2 + \left(p_2 - 1 \right)^2 \right] \left(q_1^2 + p_1^2 \right)} \right\}, \tag{10}$$

$$B = q_1 \left[\left(p_1 - p_2 \right)^2 + q_1^2 - q_2^2 \right] \left[\arctan\left(\frac{p_2}{q_2} \right) - \arctan\left(\frac{p_2 - 1}{q_2} \right) \right], \tag{11}$$

$$C = q_2 \left[\left(p_2 - p_1 \right)^2 + q_2^2 - q_1^2 \right] \left[\arctan\left(\frac{p_1}{q_1} \right) - \arctan\left(\frac{p_1 - 1}{q_1} \right) \right], \tag{12}$$

$$D = q_1 q_2 \left[\left(p_2 - p_1 \right)^2 + \left(q_2 + q_1 \right)^2 \right] \left[\left(p_2 - p_1 \right)^2 + \left(q_2 - q_1 \right)^2 \right]. \tag{13}$$

If the two wall leafs have the same properties, $q_1 = q_2 = q$ and $p_1 = p_2 = p$. This makes eqn. (9) indeterminate. In this situation, eqn. (8) becomes

$$I = \int_0^1 \frac{dx}{\left[q^2 + \left(x - p \right)^2 \right]^2}. \tag{14}$$

Evaluating eqn. (14) using the Maple 11 software package gives

$$I = \frac{q^2 - p(p-1)}{2q^2 \left(q^2 + p^2 \right) \left[q^2 + \left(p - 1 \right)^2 \right]} + \frac{\arctan\left(\frac{p}{q} \right) - \arctan\left(\frac{p-1}{q} \right)}{2q^3}, \tag{15}$$

where use has been made of the fact that $q > 0$ and $p \geq 0$.

The integrand in eqn. (8) has local maxima when $x = p_i$. If $q_i << 1$, which is usually the case, and if $|p_1 - p_2| >> q_i$, the integrand is half its local maximum value when $|x - p_i| = q_i$. Since q_i is usually very much less than 1, and p_i is between zero and one if the frequency is greater than or equal to f_{ci}, the values of x where the integrand is significantly different from zero usually lie well inside the integral limits from 0 to 1. Because of this Davy (1990, 1991, 2009a) approximated the integral in eqn. (8) by extending the limits of integration from minus infinity to plus infinity.

$$I = \int_{-\infty}^{\infty} \frac{dx}{\left[q_1^2 + (x - p_2)^2\right]\left[q_1^2 + (x - p_2)^2\right]}. \tag{16}$$

This integral can be evaluated using the calculus of residues. This evaluation was carried out using the Maple 11 software package and gave

$$I = \frac{\pi(q_1 + q_2)}{q_1 q_2\left[(q_1 + q_2)^2 + (p_1 - p_2)^2\right]}. \tag{17}$$

If

$$\frac{a_i \eta_i}{\sigma(\theta_{ci})} \gg 1, \tag{18}$$

which is usually the case, then

$$q_i = \frac{\eta_i}{2\xi_i}. \tag{19}$$

Substituting eqns. (17) and (19) into eqn. (7) and evaluating gives

$$\tau = \frac{\pi\sigma^2(\theta_{c1})\sigma^2(\theta_{c2})(\eta_1\xi_2 + \eta_2\xi_1)}{2a_1^2 a_2^2 \eta_1\eta_2\alpha^2\left[(\eta_1\xi_2 + \eta_2\xi_1)^2 + 4(\xi_1 - \xi_2)^2\right]}. \tag{20}$$

If an infinite wall leaf is assumed, the forced radiation efficiency is given by

$$\sigma(\theta_{ci}) = \frac{1}{\sqrt{1-\frac{1}{\xi_i}}} = \frac{1}{\sqrt{1-\frac{\omega_{ci}}{\omega}}} = \frac{1}{\sqrt{1-\frac{f_{ci}}{f}}} = \frac{1}{\cos\theta_{ci}}. \tag{21}$$

Because eqn. (21) gives an infinite result at the critical frequency, a common approximation is to assume that the forced radiation efficiency is unity above the critical frequency. If this assumption is made, eqn. (20) becomes

$$\tau = \frac{\pi(\eta_1\xi_2 + \eta_2\xi_1)}{2a_1^2a_2^2\eta_1\eta_2\alpha^2\left[(\eta_1\xi_2 + \eta_2\xi_1)^2 + 4(\xi_1 - \xi_2)^2\right]}. \tag{22}$$

Eqn. (22) should agree with eqns. (30) to (34) in Davy (1990), eqns. (9) to (13) in Davy (1991) and eqns. (40) to (44) of Davy (2009a). It does not do so for two reasons. The first reason is errors in the older equations. Eqns. (33) in Davy (1990) and (12) in Davy (1991) should read $v = 4(\xi_1 - \xi_2)$ rather than $v = 4(\eta_1 - \eta_2)$. Also the factor α^2 is missing from the denominator of eqn. (34) of Davy (1990). The second reason is that the approximations made are slightly different. However if the critical frequencies of the two wall leaves are the same, then $\xi_1 = \xi_2 = \xi$ and eqn. (22) and eqns. (34) of Davy (1990) (after correction), (13) of Davy (1991) (after correction) and eqn. (44) of Davy (2009a) all reduce to

$$\tau = \frac{\pi}{2a_1^2a_2^2\eta_1\eta_2\xi\alpha^2(\eta_1 + \eta_2)}. \tag{23}$$

It should be noted that the ξ_i in Davy (1990), Davy (1991) and Davy (2009a) is the square root of the ξ_i used in this paper. If the masses per unity area and the damping loss factors of both wall leaves are the same, then $a_1 = a_2 = a$ and $\eta_1 = \eta_2 = \eta$ and all four equations reduce further to

$$\tau = \frac{\pi}{4a^4\eta^3\xi\alpha^2}. \tag{24}$$

At the critical frequency of the ith wall leaf, $\xi_i = 1$ and $p_i = 0$. This means that one of the maxima of the integrand in eqn. (8) occurs at zero which is the lower limit of integration in eqn.

(8). Thus the extension of the limits of integration which occurs in eqn. (16) means that this maximum contributes about double to the integral compared to what it did before the extension of the limits of integration. Thus in this paper, eqns. (5), (7) and (9) to (13) or (15) will be used to calculate the sound transmission coefficient instead of the further approximations given by eqns. (17), (19), (20), (22), (23) or (24).

Because eqn. (21) gives an infinite value for the forced radiation efficiency at the critical frequency, this paper uses Davy's (2009a) theory to calculate the forced radiation efficiency. This theory is an updated version of the theory in Davy (2004). First the cosine of the coincidence angle is calculated. Since eqns. (5), (7) and (9) to (13) or (15) are going to be used as a correction term below the critical frequency, the cosine of the coincidence angle is set to zero for frequencies below the critical frequency. The actual equations used are eqns. (33) to (39) of Davy (2009c).

To obtain better agreement with experimental results, $\sigma(\theta_{ci})$ is set equal to one if the frequency is greater than or equal to the lower of the two critical frequencies. If the frequency is between 0.9 times and 1 times the lower of the two critical frequencies, $\sigma(\theta_{ci})$ is linearly interpolated in the frequency domain between the value of $\sigma(\theta_{ci})$ at 0.9 times the lower of the two critical frequencies and the value one at the lower of the two critical frequencies.

An alternative approximation is used by Davy (1990, 1991, 2009a). It produces a different result from the approximation used in this paper if the critical frequencies of the two wall leaves are different. This other approximation follows Cremer (1942) and uses eqn. (1) rather than eqn. (2) of Davy (2009c). The $\sin\theta$ term in eqn. (1) is approximated by the average of $\sin\theta_{c1}$ and $\sin\theta_{c2}$. This is the approximation that causes the difference if the critical frequencies of the wall leaves are different. It is not needed if eqn. (2) of Davy (2009c) is used but works

satisfactorily if the critical frequencies are fairly similar. The second $\sin^4\theta$ term in eqn. (12) of Davy (2009c) is approximated as

$$\sin^4\theta \approx \sin^4\theta_{ci}\left(1+4\frac{\sin\theta-\sin\theta_{ci}}{\sin\theta_{ci}}\right). \tag{25}$$

Then the limits of integration are extended from minus infinity to plus infinity as in eqn. (16). Finally radiation efficiency is approximated to be unity.

III. BELOW THE CRITICAL FREQUENCIES

Below the lowest of the critical frequencies of the two wall leaves the method given by eqns. (18) to (35) of Davy (2009a) is followed. An important difference is that a correction factor (Sewell, 1970) for when the critical frequency is approached is omitted from eqn. (24) of Davy (2009a). That is, eqn. (24) of Davy (2009a) becomes

$$a_i = \frac{\omega m_i}{2\rho_0 c} \tag{26}$$

where $\rho_0 c$ is the characteristic impedance of air. Another difference is that the square root of the panel area $\sqrt{A}$ in eqn. (34) of Davy (2009a) is replaced by the length $2a$ of the side of an equivalent square panel defined by eqn. (34) of Davy (2009c). Note that this a is different from the a used earlier in this paper. To avoid confusion with symbols used earlier in this paper p is replaced with P and q is replaced with Q.

Davy (2009a) uses a variable limiting angle of integration (Sewell, 1970) and shows that the use of this angle for single leaf walls is equivalent to the use of the forced radiation efficiency of the finite size of the wall averaged over all angles of incidence. Thus the use of the variable limiting angle of integration is equivalent to the spatial windowing technique of Villot et al. (2001). For double leaf walls, eqn. (34) of Davy (2009a), which is used in this paper, limits the

maximum value of the limiting angle of integration to 61° in order to make the theory agree better with experimental results. Similarly, Schoenwald et al. (2008) found that the spatial windowing technique was not sufficient on its own for double leaf walls and combined it with Kang's (2000) Gaussian distribution of directional incident sound energy.

To compensate for the removal of Sewell's (1970) correction term in eqn. (26), the sound transmission coefficient, above the normal incidence mass-air-mass resonance frequency and below the upper of the two critical frequencies, is calculated as the sum of eqn. (33) of Davy (2009a) and eqn. (7) where eqns. (9) or (15) have been used to evaluate eqn. (7). At and above the upper of the two critical frequencies, only eqn. (7) is used. Eqn. (33) of Davy (2009a) can be included immediately above the upper of the two critical frequencies, but it causes problems if used a long way above both critical frequencies.

Q is half the sum of the ratio of the surface density of one wall leaf to that of the other with the inverse of this ratio. Thus if the wall leaves are the same $Q = 1$. Px is the product of the ratio of the mass impedance of one wall leaf to twice the specific acoustic impedance of air with the same ratio for the other wall leaf and with the sound absorption coefficient of the cavity and with the square of the cosine of the angle of incidence. Because Q is usually very much less than Px, eqn. (28) of Davy (2009a) can be approximated as follows,

$$\tau(\theta)=\frac{1}{a_1^2\cos^2(\theta)a_2^2\cos^2(\theta)\alpha^2}\,. \tag{27}$$

The first term in the denominator of eqn. (12) of Davy (2009c) is usually very much smaller than the second term. Ignoring this first term, assuming that ξ_i is very small and using the value of the radiation efficiency for an infinite panel which is given by eqn. (21) changes eqn. (12) of Davy (2009c) to

$$\tau_i(\theta) = \frac{1}{a_i^2 \cos^2(\theta)}. \tag{28}$$

Thus eqn. (27) can be written as eqn. (6). This is the reason for the adoption of eqn. (6).

IV. STUD BORNE TRANSMISSION

This paper is primarily concerned with air borne transmission across the cavity. However, to the author's surprise, it was found that it was necessary to include "stud borne" transmission between the edges of the glass panes due to the window frames in order to correctly predict the sound insulation of double glazing with larger air gaps. Thus this section gives the formulae for stud borne transmission across the cavity (Davy, 1993, 2009a). The actual equations used are eqns. (47) to (50) of Davy (2009a) where the symbols D (ratio of total to non-resonant radiation) and g (sum of the product of the mass per unit area of each wall leaf with the square root of the angular critical frequency of the other wall leaf) of Davy (2009a) have been replaced with the symbols H and G to avoid confusion with the use of the symbols D and g earlier in this paper. Eqn. (46) of Davy (2009a) is split into the following two equations.

$$\tau = \frac{32\rho_0^2 c^3 HJ}{G^2 b\omega^2}, \tag{29}$$

where the stud transmission ratio is

$$J = \frac{2}{1 + \left(1 - \frac{4\omega^{3/2} m_1 m_2 c C_M}{G}\right)^2}. \tag{30}$$

The studs have a mechanical compliance of C_M where $C_M = 0$ gives the rigid stud case. The mechanical compliance of a stud has dimensions of length per (force per length of the stud). The stud transmission J is the ratio of the stud borne sound transmission coefficient for the wall to the stud borne sound transmission coefficient for the same wall with rigid studs ($C_M = 0$). The stud transmission J is restricted to be greater than or equal to the set minimum stud transmission

K. C_M and K are selected to give the best agreement with experiment. This is different from Davy (2009a) which gives the formula for resilient studs but recommends setting the stud compliance to zero and using a set constant stud transmission J for resilient studs instead of the stud transmission J given by eqn. (30). In Davy (2009a), J is selected to give the best agreement with experiment.

Research by Poblet-Puig et al. (2006, 2009), Guigou-Carter et al. (1998) and Guigou-Carter and Villot (2006) has shown that a steel stud can be modelled as a translational spring with a translational stiffness which varies with frequency in the range from 10^5 to 10^8 Pa. The constant value of mechanical compliance used in the next section corresponds to a translational stiffness of 6 x 10^5 Pa which lies towards the bottom end of the above range. The value of the minimum stud transmission used in the next section is -23 dB. This also lies in the 0 to -40 dB stud transmission range determined by Poblet-Puig et al. (2006, 2009) for a standard steel stud. It would be possible to use the frequency dependent translational stiffness values determined by Poblet-Puig et al. (2006, 2009) with the theory developed in this paper.

σ_i, which appears in eqns (48) and (49) of Davy (2009a), is the single sided radiation efficiency of a free reverberant bending wave vibration field of the ith wall leaf. Note that, below the critical frequency of the ith wall leaf, this is different from the single sided radiation efficiency of a forced bending wave on the ith wall leaf which is given by eqns. (33) to (39) of Davy (2009c). The corrected versions of Maidanik's formulae for the single sided radiation efficiency of a free reverberant bending wave vibration field given by Vér and Holmer (1971) are used in this paper. However the maximum value of the radiation efficiency is limited to the value one. Previous research (Cremer, 1942, Davy, 1993, 2009a) has shown that this assumption works well for predicting the sound transmission of third octave bands of noise above the critical

frequency. It is also consistent with the restrictions placed on the result of eqns. (33) to (39) of Davy (2009c) above the critical frequency in section II.

The stud borne transmission is not included in the combined transmission below the mass-air-mass resonance frequency. In this frequency range, the wall leaves are already effectively coupled by the air cavity.

V. COMPARISON WITH EXPERIMENT

Quantitative measures of the differences between experiment and theory for each of the 7 cases shown in the following figures are given in Table I. This table shows the mean, the standard deviation, the maximum and the minimum in dB. Also shown are the frequencies in Hz at which the maximum and minimum differences occur. The overall column shows the average of the 7 means, the root mean square of the 7 standard deviations, the maximum of the 7 maxima and the minimum of the 7 minima. In figs. 2 and 3, only the combined theoretical results are used for the calculations whose results are shown in Table 1.

The first comparison is for the case where there is no vibration connection between the two leaves of the wall (except possibly at the edges) and hence only air borne cavity wall transmission is involved. In this first comparison there is sound absorbing material in the wall cavity. The value one is used for the cavity sound absorption coefficient for cavity walls with sound absorbing material in the cavity in this paper, since the experimental results show little dependence of sound insulation on the type or the thickness of the sound absorbing material in the cavity, providing that the sound absorbing material is not too thin or lightweight.

Fig. 1 compares theory with experimental results for five 40 mm double steel stud 16 mm gypsum plaster board cavity walls with cavity absorption. There is a 10 mm gap between the separate studs. The double studs are spaced on 610 mm centres. The experimental results in fig.

1 were measured by the National Research Council of Canada (NRCC). The last of the walls measured (TL-92-975) had no studs. The gypsum plaster board is assumed to have a density of 770 kg/m^3, a Young's modulus of 1.85 x 10^9 Pa, a Poisson's ratio of 0.3 and an internal damping loss factor of 0.03. The agreement between theory and experimental is good. Fig. 1 should be compared with fig. 3 of Davy (2009a) where the same experimental data is presented. Slight differences in the two theories can be seen in the frequency region around the peak below the critical frequency.

To determine the appropriate value of the "sound absorption coefficient" to be used to predict the sound insulation of cavity walls without sound absorbing material in the cavity, the NRCC measurements on double glazed windows (Quirt, 1981 and 1982) were analysed. It soon became apparent that it was necessary to include "stud borne" transmission between the edges of the glass panes due to the window frames in order to correctly predict the sound insulation of double glazing with larger air gaps. This is the reason for the inclusion of section IV in this paper. The window size is 2.02 by 1.8 m. The frame has two vertical dividers. Thus each layer of glass consists of three separate panes of glass. This gives a stud spacing of b = 2.02 / 3 = 0.67 m. The theoretical calculations assume that the density is 2500 kg/m^3, the Young's modulus is 6.5×10^{10} Pa and Poisson's ratio is 0.22. The "stud" compliance C_M of the window frames is assumed to be 0 Pa^{-1}.

The "sound absorption coefficient" of the cavity and the internal damping loss factor of the glass panes were assumed to be constant as functions of frequency. They were adjusted to provide the best fit between theory and experiment for each of the double glazing combinations that were tested. The "sound absorption coefficient" α of the cavity increased with increasing

cavity width d. A linear regression of the "sound absorption coefficient" as a function of cavity width produced the following equation.

$$\alpha = 0.027 + 2.4d \tag{31}$$

Note that the value of the "sound absorption coefficient" is limited by eqn. (35) of Davy 2009a. Because the second term on the right hand side of eqn. (27) of Davy (2009a) has been ignored, the effective "sound absorption coefficient" α of the cavity is expected to be considerably larger than the actual physical sound absorption coefficient of the cavity. Eqn. (31) is less accurate at larger cavity widths because, as will be seen, the "stud borne" transmission across the window air cavity is dominant in the theoretical values for larger cavity widths.

The average value of the internal damping loss factor of the glass panes was 0.064. Although this internal damping loss factor seems high for glass, it should be noted that Cremer (1942) assumed a damping loss factor of 0.1 for glass. This internal damping loss factor actually includes the loss of vibrational energy at the edges of the glass panes.

The narrowest window air cavity gap measured by NRCC was 3 mm. Fig. 2 shows a comparison of the air borne, stud borne and combined theoretical results with the experimental result for a double glazed unit consisting of a 3 mm glass pane, a 3 mm air gap and another 3 mm glass pane. Fig. 2 shows that the theoretical sound insulation is controlled by the air borne transmission across the cavity. Remember that the theoretical stud borne transmission is not used below the mass-air-mass resonance which occurs at about 550 Hz in this situation. The theory under estimates the experimental sound reduction index below 160 Hz and from 2500 to 4000 Hz. The agreement is good in the other frequency ranges. The under estimation in the 2500 to 4000 Hz range can be removed by increasing the apparent "sound absorption coefficient" in this frequency range.

The next comparison shows the case where the two "wall leaves" are different and where the window air cavity gap is large. Fig. 3 compares the air borne, stud borne and combined theoretical results with the experimental result for a double glazed window consisting of 4 mm and 6 mm glass panes separated by an air cavity of 100 mm width. Fig. 3 shows that the theoretical sound insulation is controlled by the stud borne transmission across the cavity above 160 Hz and by the airborne transmission across the cavity below 160 Hz. The combined theoretical results agree reasonably well with the experimental results, although the theory fails to predict the local maxima at 160 and 200 Hz.

To estimate the stud mechanical compliance C_M and the minimum stud transmission K for use in the case of steel studs, test data was taken for five walls from Halliwell et al. (1998). All five walls consisted of two layers of 16 mm gypsum plaster board mounted on each side of 90 mm steel studs at 406 mm centres. All five walls had 90 mm of porous sound absorbing material in the wall cavity. The first wall had sprayed cellulose fibre, while the second and third walls had glass fibre and the last two walls had mineral fibre. The best fit to the experimental data is obtained by setting the stud mechanical compliance C_M equal to 1.6×10^{-6} Pa^{-1} and the minimum stud transmission K equal to 0.005[1].

The gypsum plaster board is assumed to have a density of 770 kg/m^3 and a Poisson's ratio of 0.3. To ensure that the two layers of 16 mm gypsum plaster board have the same critical frequency as a single layer of 16 mm gypsum plaster board, the Young's modulus is set equal to one quarter of the 1.85×10^9 Pa used for a single 16 mm layer of gypsum plaster board. To obtain good agreement above the critical frequency, the internal damping loss factor is set to 0.02. The comparison of theory with experiment is shown in fig. 4. The theory overestimates at 100 Hz and in the 1000 to 2000 Hz range. The agreement is good at other frequencies.

The next comparison is for gypsum plaster board cavity walls without cavity absorption. The last three experimental results in fig. 5 (TL-92-265 through TL-92-267) are for the same construction as the results in fig. 1, except that there is no sound absorbing material in the cavity. In the first three experimental results (TL-92-262 through TL-92-264), the double 40 mm steel studs with a 10 mm gap are replaced with 90 mm steel studs on 813 mm centres. These 90 mm steel stud results have been included, because surprisingly they are as good as or better than the double 40 mm steel stud results. The theoretical curve includes stud borne transmission, but the theoretical stud borne transmission only has a small effect on the total theoretical sound insulation in the region of 100 Hz. Another experimental result for the case with no studs, which is not included here, produced lower results. Presumably this is because the studs help inhibit the oblique propagation of sound in the cavity in this case without sound absorption in the cavity. The experimental measurements in fig. 5 were also measured by the National Research Council of Canada (NRCC).

The original intention was to use eqn. (31) to predict the cavity absorption coefficient. However, comparison with the experimental results shown in this figure, and the results without cavity absorption from Halliwell et al. (1998) and from NAHB (1971), produced the following eqn.

$$\alpha = 0.043 + 0.73d \ . \tag{32}$$

The mean value of the damping loss factor was 0.044. This value of the damping loss factor and the value of sound absorption coefficient given by eqn. (32) were used for the theoretical calculations in figs. 5 and 7. The theory in fig. 5 under estimates the experimental results in the range from 500 Hz to 1600 Hz. Use of eqn. (31) instead of eqn. (32) does a better job of predicting the peak in this range at the expense of over estimating in the range from 160 to 630

Hz. Fig. 5 should be compared to Fig. 4 of Davy (2009a) where the same experimental data is plotted. The theory of Davy (2009a) is closer to the experimental results in the frequency range from 500 to 1600 Hz. The theory of this paper is closer to the experimental results in the 250 to 400 Hz range and at 2000 Hz.

Fig. 6 compares theory and experiment for the sound insulation of three 90 mm wooden stud 16 mm gypsum plaster board cavity walls with cavity absorption. The stud spacing is 406 mm. The first two experimental results are a new and an old measurement by the National Research Council of Canada (Halliwell et al., 1998 and Northwood, 1968). The third experimental measurement is by Owens/Corning Fibreglas (DuPree, 1981). The second and third experimental results also appear as an average in NAHB (1971). In order to obtain reasonable agreement above the critical frequency, the damping loss factor used to obtain the theoretical result had to be increased to 0.1.

The theory does not do a good job of predicting the experiment results below 250 Hz. Between 315 and 1250 Hz it is in reasonable agreement with the most recent measurement, but over predicts the two older measurements. It over predicts between 1600 and 2500 Hz. It is in very rough agreement with the experimental results above 2500 Hz, but only because of the adoption of a theoretical damping loss factor of 0.1.

Note that the predicted mass-air-mass resonance frequency of about 80 Hz is significantly less than the measured mass-air-mass resonance frequencies of 125 or 160 Hz. This may be due to a structural resonance which is not included in the theory described in this paper. Bradley and Birta (2001) have shown that the sound insulation of wood stud exterior walls can be significantly degraded by a structural resonance if the two wall leaves are rigidly coupled by the wooden studs. They explain this structural resonance in terms of the analysis conducted by Lin

and Garrelick (1977). The effects of this resonance can be reduced by structurally isolating the two wall leaves with resilient mounts, thin steel studs, staggered studs or double studs. The frequency of the resonance is about double the calculated mass-air-mass resonance and it reduces in frequency as the rigid stud spacing is increased and as the depth of the rigid studs is increased.

Bradley and Birta (2000) report the results of laboratory sound insulation measurements on typical Canadian building facades. These measurements showed the structural resonance at 125 Hz. However field measurements (Bradley et al. (2001), Bradley (2002)) with actual aircraft noise showed little effect due to this structural resonance.

Fig. 7 compares theory and experiment for 13 mm gypsum plaster board on each side of 90 mm thick wooden studs with no sound absorbing material in the wall cavity. The studs are spaced at 400 mm centres. The experimental results are the average results of two separate experimental measurements (NAHB, 1971). Although the theory over estimates the experimental results below 1000 Hz, it still does a reasonable job of predicting the general trend of the experimental data. The predicted results would compare better with the experimental ones if the value of the cavity absorption coefficient was modified. However, in order to have a prediction method, the cavity absorption given by the linear regression eqn. (32) has been used.

V. CONCLUSIONS

A simple theory for predicting the sound insulation of double leaf cavity walls has been revised. An approximation to the range of integration over angle of incidence for the theory above the critical frequency is removed. This enables the theory for above the critical frequency to be used as a correction to theory for below the critical frequency instead of Sewell's (1970) correction and thus gives continuous sound insulation values. The "stud" borne transmission via the window frames is included when modelling the sound insulation of double glazed windows.

This “stud” borne transmission via the window frames is particularly important for windows with wide air gaps. Linear regression equations for the “effective” sound absorption coefficient of a wall cavity without sound absorbing material in the wall cavity as a function of cavity width are derived for both double glazed widows and gypsum plaster board cavity walls. This “effective” sound absorption increases with increasing wall cavity width.

Comparison with experiment shows that the theory does a reasonably good job of predicting the general trend of the experimental values. The overall mean, standard deviation, maximum and minimum of the differences between experiment and theory are -0.6 dB, 3.1 dB, 10.9 dB at 1250 Hz and – 14.9 dB at 160 Hz respectively. The theory struggles most when attempting to predict the sound insulation of double leaf gypsum plasterboard walls without sound absorbing material in the wall cavity. This indicates that the assumption that the cavity sound absorption coefficient is constant with frequency, except at low frequencies, probably needs to be revised.

ENDNOTE

1. It should be noted that the value of the stud mechanical compliance C_M is similar to the value of 1×10^{-6} Pa^{-1} used by Davy (1990). The minimum stud transmission K is not needed in Davy (1990) because the theoretical airborne transmission across the cavity is greater in that paper because the limiting angle θ_l is not limited to a maximum value of 61 °.

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TABLE I. This table shows the mean, the standard deviation, the maximum and the minimum in dB of the differences between experiment and theory for each of the 7 cases presented in the accompanying figures. Also shown are the frequencies in Hz at which the maximum and minimum differences occur. The overall column shows the average of the 7 means, the root mean square of the 7 standard deviations, the maximum of the 7 maxima and the minimum of the 7 minima. In figs. 2 and 3, only the combined theoretical results are used for the calculations whose results are shown in this table.

	Fig. 1	Fig. 2	Fig. 3	Fig. 4	Fig. 5	Fig. 6	Fig. 7	Overall
Mean (dB)	-0.6	1.1	1.2	-2.1	0.9	-3.3	-1.5	-0.6
Standard deviation (dB)	2.4	2.7	2.7	3.1	3.9	4.3	2.3	3.1
Maximum (dB)	3.0	7.6	8.4	4.2	10.9	8.5	2.1	10.9
Minimum (dB)	-9.3	-2.0	-2.6	-12.5	-5.4	-14.9	-5.6	-14.9
Frequency of maximum (Hz)	200	80	160	50	1250	80	4000	1250
Frequency of minimum (Hz)	2000	400	80	2000	315	160	160	160

FIGURE CAPTIONS

Fig. 1. Comparison of theory and experiment (measured by National Research Council of Canada) for the sound insulation of five double steel stud 16 mm gypsum plaster board cavity walls with cavity absorption. The cavity width is 90 mm.

Fig. 2. Comparison of the air borne, stud borne and combined theoretical results with the experimental result (Quirt, 1981 and 1982) for the sound insulation of a sealed double glazed unit consisting of two 3 mm glass panes separated by an air cavity of 3 mm width.

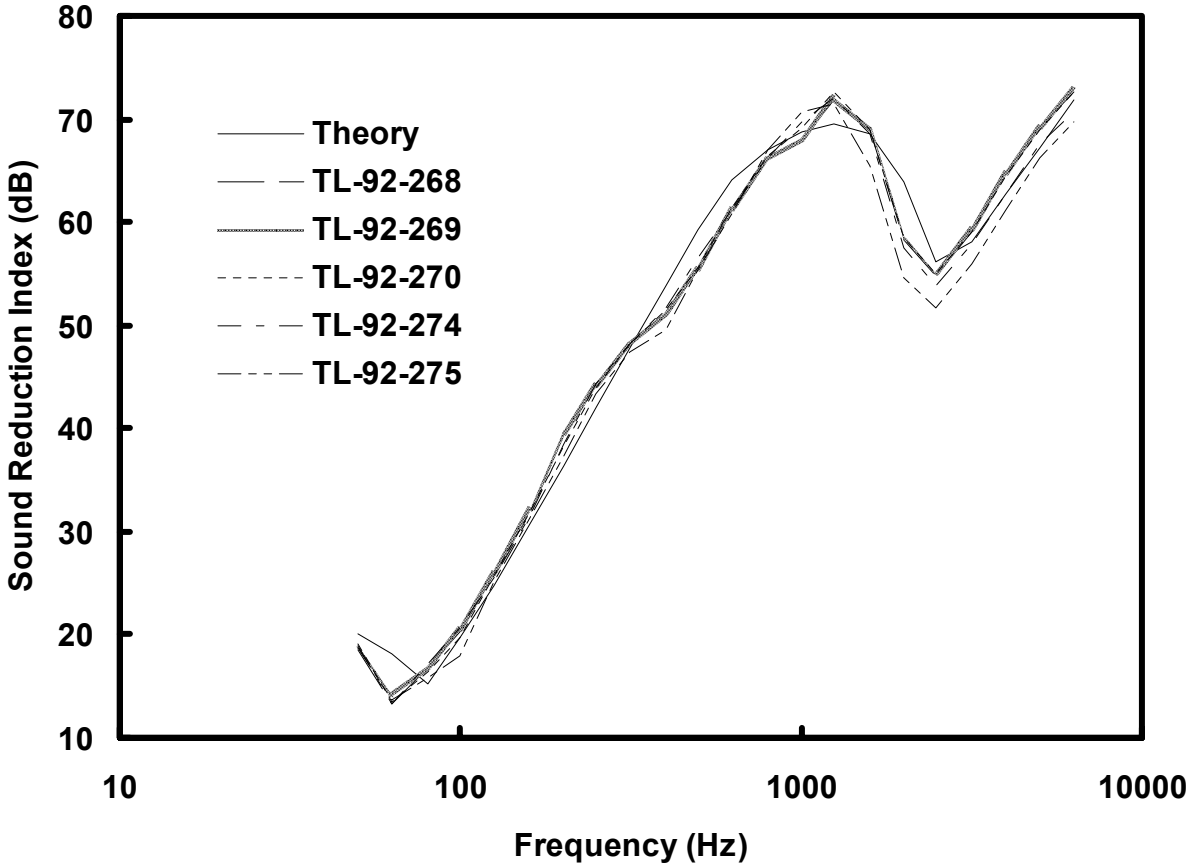
Fig. 3. Comparison of the air borne, stud borne and combined theoretical results with the experimental result (Quirt, 1981 and 1982) for the sound insulation of a double glazed window consisting of 4 mm and 6 mm glass panes separated by an air cavity of 100 mm width.

Fig. 4. Comparison of theory and experiment (Halliwell et al., 1998) for the sound insulation of five 90 mm steel stud cavity walls with cavity absorption. Each side of the steel studs has two layers of 16 mm gypsum plaster board attached.

Fig. 5. Comparison of theory and experiment (measured by National Research Council of Canada) for the sound insulation of three steel stud and three double steel stud 16 mm gypsum plaster board cavity walls without cavity absorption. The cavity width is 90 mm.

Fig. 6. Comparison of theory and experiment (Halliwell et al., 1998, Northwood, 1968 and DuPree, 1981) for the sound insulation of three 90 mm wooden stud 16 mm gypsum plaster board cavity walls with cavity absorption.

Fig. 7. Comparison of theory and experiment (NAHB, 1971) for the sound insulation of a 13 mm gypsum plaster board 90 mm cavity wall without cavity absorption.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50
40
30
20
10
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Theory
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TL-92-269
TL-92-270
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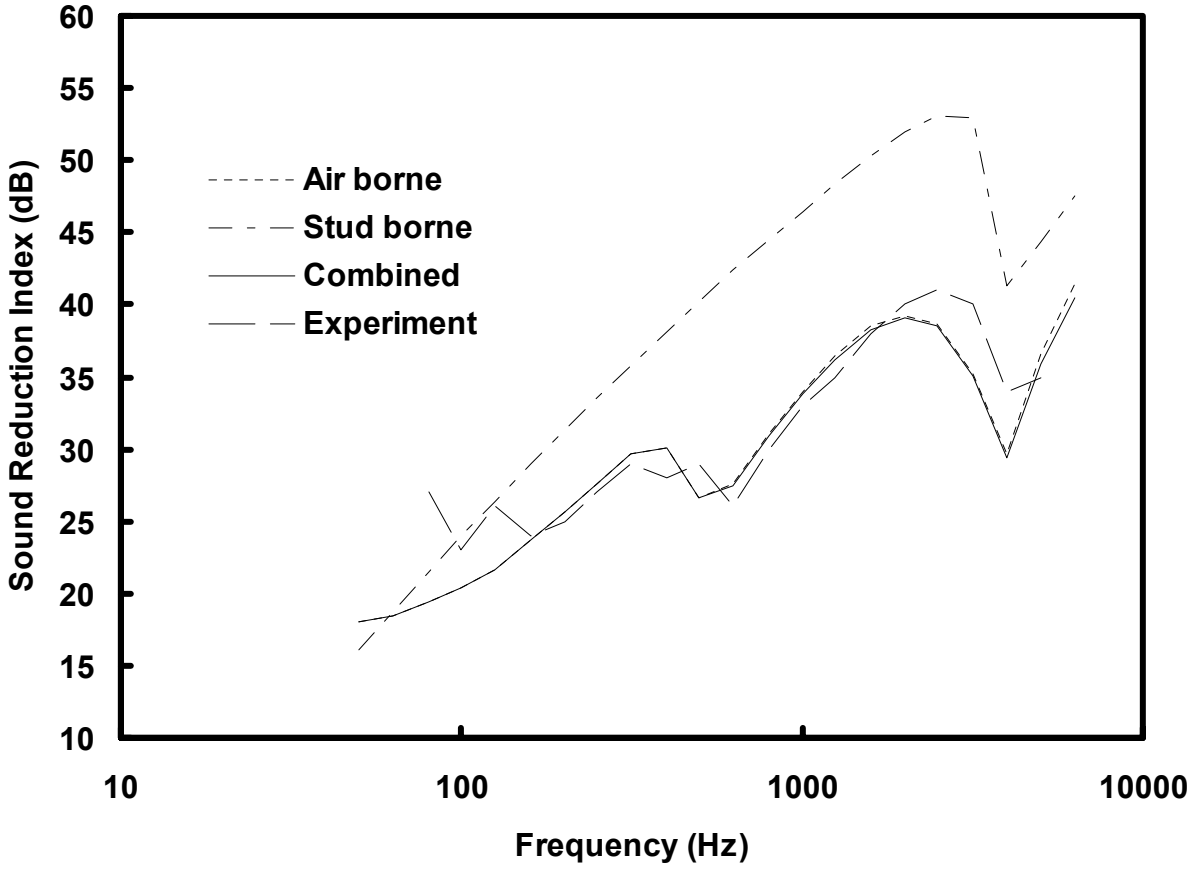
TL-92-275
10
100
1000
10000
Frequency (Hz)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Sound Reduction Index (dB)
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Stud borne
Combined
Experiment
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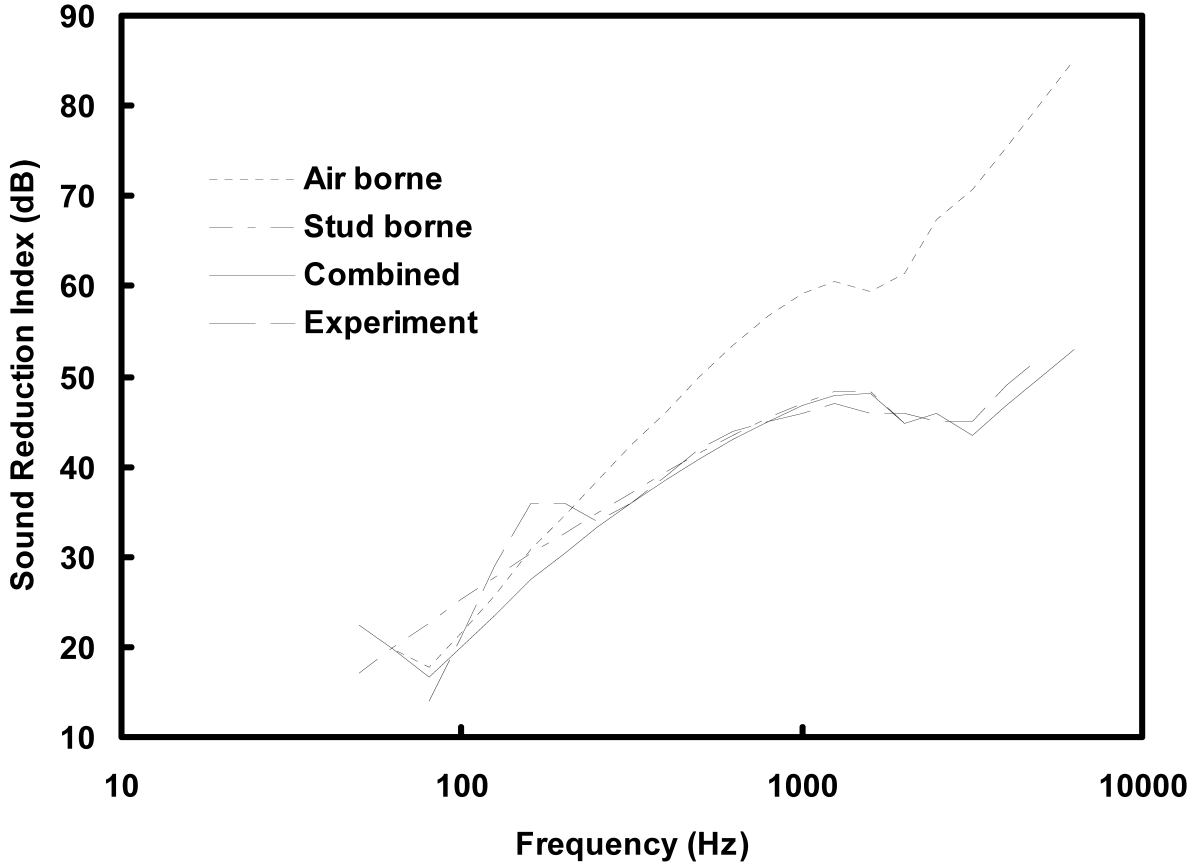
Frequency (Hz)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Stud borne
Combined
Experiment
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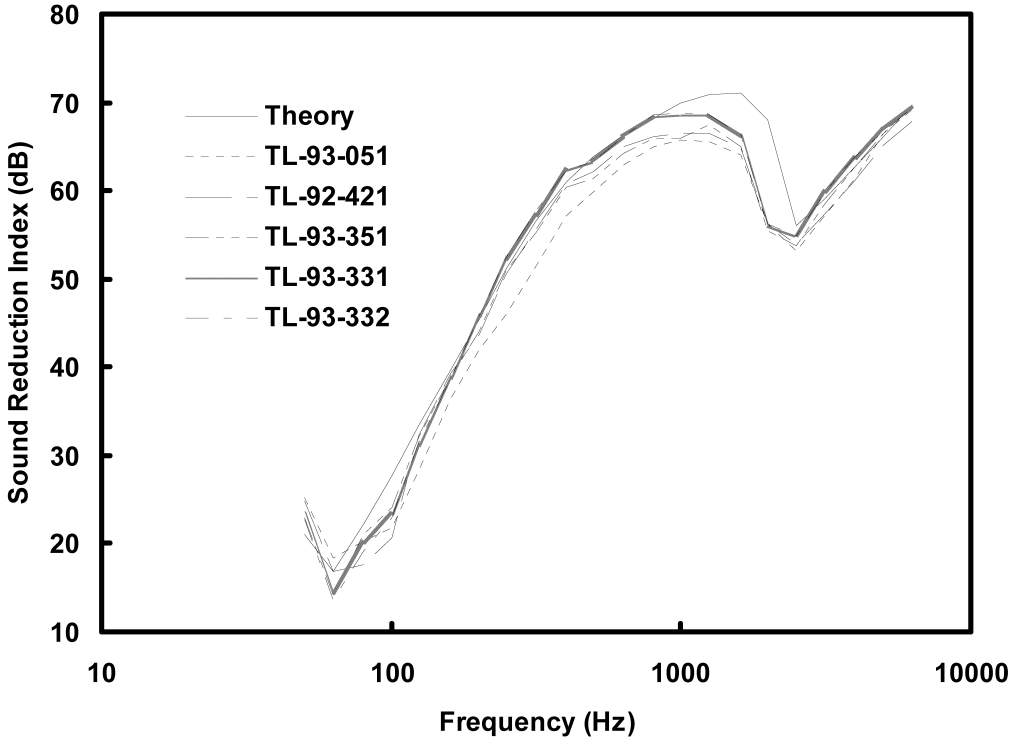
Frequency (Hz)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Theory
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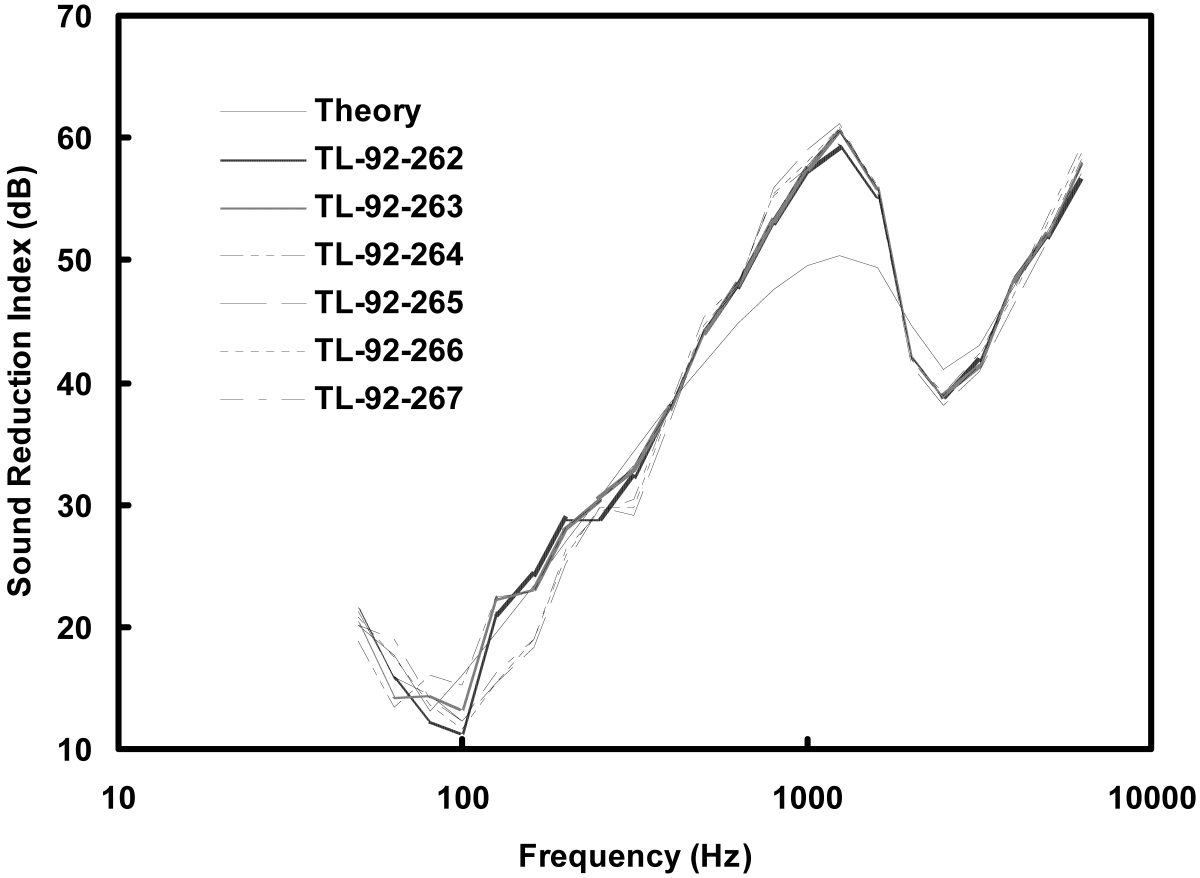
Frequency (Hz)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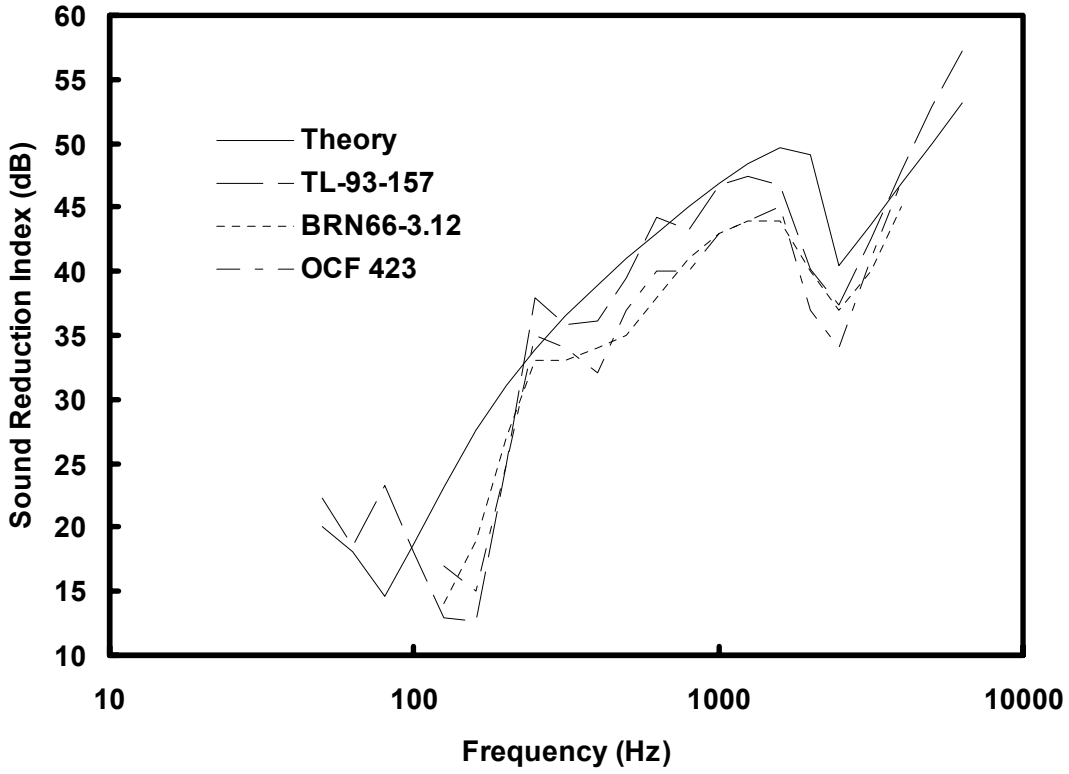
Frequency (Hz)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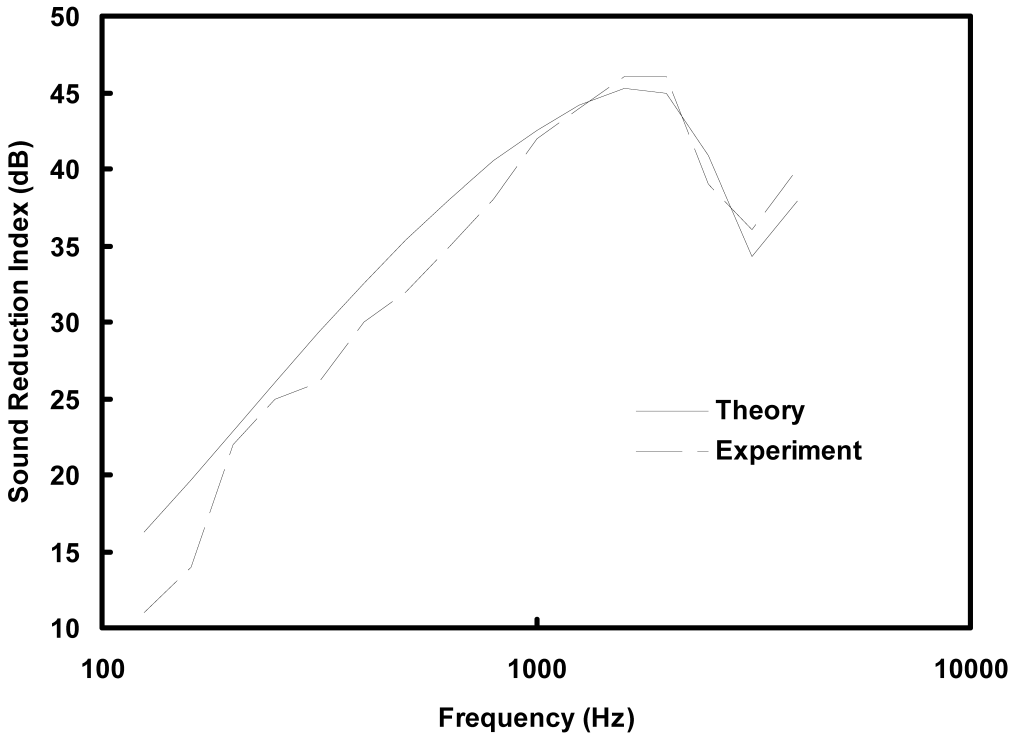
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