Improved Knight Method Based on Narrowed Search Space for Instantaneous GPS Attitude Determination

YONG LI

University of New South Wales, Sydney, Australia

KEFEI ZHANG and RON GRENFELL

RMIT University, Melbourne, Australia

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ABSTRACT: The Knight algorithm can instantaneously resolve integer ambiguities and improve significantly the search speed by skipping over most less-likely integer ambiguity candidates. However, its reliability relies on the volume of the integer ambiguity search space. Utilizing coarse attitude knowledge can reduce the size of the search space. This paper proposes a new method to narrow the search space by taking into account a geometric constraint of visible satellites. The constraint is formulated as a recursive procedure and is thus very suitable to be incorporated into the search loop of the Knight method. Experimental results demonstrate that the proposed method can improve the search efficiency and reliability of the Knight method.

INTRODUCTION

Efficient algorithms to fix integer ambiguities are crucial to the success of the Global Positioning System (GPS) applications where carrier phase measurements are fundamental observations, *i.e.* attitude determination and real-time kinematical positioning (RTK) applications. Two approaches have been developed to resolve the integer ambiguity problem for GPS based attitude determination. Those are either instantaneous (search-based) or dynamic (motion-based) techniques [1-6]. Motion-based methods need to collect data for a period and wait for obvious changes to occur either from the visible GPS constellation configuration or from the host platform rotation during data collection [4-6].

Instantaneous methods usually use a search procedure to find the most likely solution by using measurements of only one epoch. This makes them very suitable to real-time applications although they usually suffer from the possibility of getting wrong solution due to the measurement noise. Two techniques have evolved. First, the search is carried out in a real space that consists of all possible grid points of selected search parameters, such as elevation and azimuth angles of a baseline [7-8]. Second, the search is restricted in the integer space that consists of all possible combinations of candidates of integer ambiguities [1-3].

The efficiency and success of a search method highly relies on two factors: the mechanism employed to skip over the less-likely combinations or the strategies adopted to reduce the volume of search space. Several methods have been proposed to skip over the less-likely combinations, such as that using orthogonalized difference matrix [2] or that using QR factorization [3]. The method using QR factorization can also be found in the RTK application where it is used to isolate the least-squares residuals from the least-square solution space without the necessity of performing the least-squares solution itself [9]. Unlike methods above, Knight introduces a new algorithm that can instantaneously resolve the integer ambiguities effectively through skipping over the less-likely combinations [1]. It formulates the overall weighted fit residual as a recursive form. The residual is calculated at each recursive step of a

Kalman filter so that it can interrupt the calculation of the current integer ambiguity combination and “jump” to the next combination once having found the current residual exceeding the current minimum level of residuals. This innovation allows the Knight method to reduce the computational load dramatically.

Another factor affecting the efficiency of a search method is the volume of search space. The search space can be determined by stochastic properties of the observations in RTK applications. To improve the search efficiency, the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method uses a so-called Z-transformation to rescale the search space [10]. The fast ambiguity search filter method utilizes the information of the satellite geometry to reduce the computational load of the search procedure [11]. However, for GPS attitude determination, the search space can be completely determined by the known baseline length. Meanwhile, the coarse attitude knowledge and the geometry of satellites in view can greatly reduce the volume of search space, *e.g.* a method using the satellite geometry in the integer ambiguity search procedure for GPS attitude determination [2]; and a method incorporating the satellite geometry information into the QR factorization based search procedure [3].

In fact, a GPS attitude determination system usually uses a common local oscillator as the time reference to down convert received GPS radio frequency (RF) signals into the intermediate frequency (IF) signals [4]. IF signals are further correlated to demodulate GPS data and generate observations such as pseudorange, carrier phase and signal-to-noise ratio (SNR) etc. One benefit of using a common time reference is that the clock error becomes a common distribution to all carrier phase measurements and thus it can be cancelled by forming single difference. This makes the single differenced carrier phase (SDCP) measurements sufficient to derive the attitude solution. The solution derived from SDCP measurements are more accurate than that derived from double differenced measurements because SDCP measurements are less noisy. Regarding SDCP measurements as the principal observation for GPS attitude determination, this paper presents a new method to integrate the geometric constraint into the search loop of the Knight method.

This paper will first review the integer ambiguity search principle and briefly summarize the Knight algorithm. Then, a new method is presented through which the knowledge of satellite constellation is used to narrow the search space. Finally, a number of experiments have been carried out to test the performance of the new method. Results have demonstrated that the new method is of better efficiency and higher reliability.

THE INTEGER AMBIGUITY SEARCH PRINCIPLE

For the GPS based attitude determination system with common time reference, the principal observation is the single differenced carrier phase measurements which are the carrier phase difference between the GPS signals received by two antennas separated by a short baseline. This kind of measurements also reflects the projection of the baseline vector onto the line-of-sight (LOS) vectors to GPS satellites. Suppose there are n GPS satellites in view, the single differenced carrier phase measurements on a baseline can be expressed as follows,

$$\theta_i = \mathbf{s}_i^T \mathbf{x} - k_i \lambda + \beta + v_i \,, \quad i=1,2,\ldots,n \tag{1}$$

where θ_i is the single differenced carrier phase measurement on the baseline, $\mathbf{x}$ is the baseline vector, $\mathbf{s}_i$ is the unit LOS vector to the i^{th} satellite, k_i is the integer ambiguity associated with the i^{th} satellite, β is the line bias, λ is the wavelength of the carrier signal (*i.e.* 19.03 cm for L_1

signal), and v_i is the measurement error. Note that the line bias is assumed to have been obtained by an additional procedure [13], and so it is not considered and will be neglected in the following derivation in this investigation. Technical treatment of the line bias can be found in [14].

As a universal accepted procedure, the GPS attitude determination can be divided into two steps: first resolving the integer ambiguities and then determining the attitude matrix **A**. Once the integer ambiguities are fixed, they no longer need to be resolved in the latter processing unless cycle slips occur. This paper will focus on the integer ambiguity resolution.

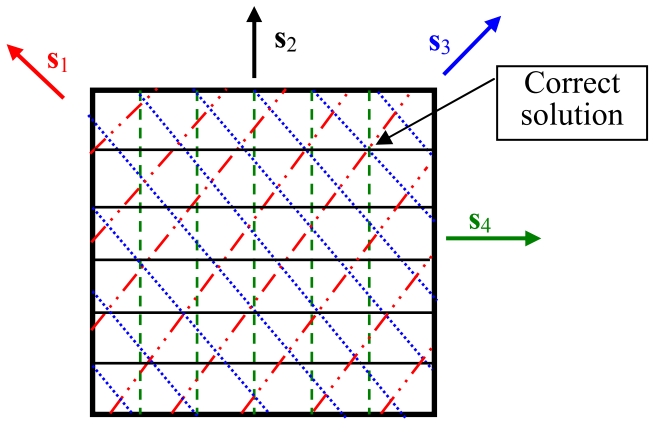

Figure 1. Illustration of search principle

Figure 1 illustrates the principle of routine search methods, where $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$ and $\mathbf{s}_4$ are LOS vectors to four GPS satellites in view. For simplification, the four LOS vectors are assumed to co-locate in a common plane with the baseline vector. Referring to Figure 1, there are four sets of parallel lines in the square box. Each of them represents carrier signal in integer cycles from the corresponding GPS satellite, for example the set in dot-dash lines represents the carrier signal from #1 satellite, the thin-solid lines from #2 satellite, the dot lines from #3 satellite, the dash lines from #4 satellite. The lines associated with the same satellite are separated by one wavelength of the carrier signal. Ideally, the correct solution lies at the conjoint point of four lines that come from different satellites separately. These intersecting lines can be called the potential solution lines. However, this conjoint point would never exist in practice because the lines are subject to measurement errors and do not intersect at a unique point. The correct solution must be the point that minimizes the deviation. The mathematics of the search method is given below.

According to [10], the least-squares principle of ambiguity resolution can be generally summarized as the selection of **x** and **k** to minimize the following cost function W,

$$W(\mathbf{x},\mathbf{k}) = \left\| \mathbf{y} - \mathbf{Bx} - \mathbf{Dk} \right\|^2, \qquad \mathbf{x} \in R^q, \quad \mathbf{k} \in Z^n \tag{2}$$

where **y** is the observation vector of order m, **x** and **k** are the unknown parameter vectors of order q and n respectively. The given design matrices **B** and **D** are of order m by q and m by n, respectively. $\left\| \cdot \right\|^2 = (\cdot)^T \mathbf{Q}_y^{-1}(\cdot)$, with $\mathbf{Q}_y$ the variance matrix of **y,** R^q the q-dimensional space of reals, and Z^n the n-dimensional space of integers.

As is proved in [12], the single differenced carrier phase measurements θ_i (i=1,2,…,n) are mathematically uncorrelated and one can make an assumption that the measurement errors v_i (i=1,2,…,n) show a random behavior resulting in a normal distribution with zero mean and variance σ_i^2, thus the probability density of θ_i can be written as follows,

$$p(\theta_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp[-\frac{(\theta_i - \mathbf{s}_i^T \mathbf{x} + k_i \lambda)^2}{2\sigma_i^2}] \tag{3}$$

where $p(\theta_i)$ is the probability density of θ_i, the exp (x) is the exponential function of x.

By taking into the assumption above, and applying equation (1) to equation (2), the problem can be equivalent to minimize the cost function W below,

$$W(\mathbf{x},\mathbf{k}) = \sum_{i=1}^{n} \frac{(\theta_i - \mathbf{s}_i^T \mathbf{x} + k_i \lambda)^2}{\sigma_i^2} \tag{4}$$

where $\mathbf{k} = [k_1, k_2, \cdots, k_n]^T$.

It is easy to prove that the solution of equation (4) maximizes the logarithm likelihood function below:

$$G(\boldsymbol{\theta}) = \ln p(\boldsymbol{\theta}) = -\sum_{i=1}^{n} \left[\frac{1}{2} \ln(2\pi\sigma_i^2) + \frac{(\theta_i - \mathbf{s}_i^T \mathbf{x} + k_i \lambda)^2}{2\sigma_i^2} \right] \tag{5}$$

where $p(\boldsymbol{\theta}) = \prod_{i=1}^{n} p(\theta_i)$ is the joint probability density of $\boldsymbol{\theta} = [\theta_1 \ \cdots \ \theta_n]^T$, ln(x) is the natural logarithm of x, G(θ) is the likelihood function and it would be maximized when W in equation (4) is minimized. In other words, the solution of equation (4) is the most-likely one in all possible candidates. Note that W also expresses the deviation of the solution lines from their ideal positions in Figure 1.

Given a set of integers k_i (i=1,2,…,n), the solution of **x** can be obtained by the least-squares method or a Kalman filter. According to equation (4), the overall residuals are not isolated from the solution. It implies that a complete set of integer trials must be given before the solution can be obtained and then the residuals can be calculated by equation (4). This implies that the search procedure has to test every trial of all integer combinations, or in other words, the search procedure has to run over the whole search space if no additional mechanism is adopted to isolate W from the solution.

THE KNIGHT METHOD

The Knight method isolates the residuals from the solution and thus can skip over less-likely integer combinations. The search mechanism of the Knight method is briefly summarized herein and more details of its mathematics can be found in [1]. Given a set of integers k_i (i=1,2,…,n), the baseline vector can be estimated by the following Kalman filter which can be implemented in the following recursive procedure,

$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{c}_i z_i \tag{6a}$$

$$z_i = \theta_i + k_i\lambda - \mathbf{s}_i^T \mathbf{x}_{i-1} \tag{6b}$$

$$\mathbf{c}_i = \mathbf{P}_{i-1}\mathbf{s}_i \,/(\mathbf{s}_i^T \mathbf{P}_{i-1}\mathbf{s}_i + \sigma_i^2) \tag{6c}$$

$$\mathbf{P}_i = (\mathbf{I} - \mathbf{c}_i \mathbf{s}_i^T)\mathbf{P}_{i-1} \tag{6d}$$

where z_i is a scalar, and $\mathbf{c}_i$ is a 3-dimension vector, $\mathbf{P}_i$ is a 3 by 3 matrix, $\mathbf{I}$ is a 3 by 3 identity matrix, and i=1,2, …, n.

Given initial values of $\mathbf{x}_0$ and $\mathbf{P}_0$, one can obtain the final estimate of $\mathbf{x}$ denoted as $\mathbf{x}_n$. The weighted residual of the solution shown in equations (6a)-(6d) can be calculated by the following iterative procedure,

$$W_i = W_{i-1} + \frac{(\theta_i - \mathbf{s}_i^T \mathbf{x}_n + k_i \lambda)^2}{\sigma_i^2} \tag{7}$$

Obviously, the final result W_n is equal to W in equation (4) if the initial value $W_0 = 0$. Referring to equation (7), the residual of the i^{th} step iteration, W_i has to be calculated after obtaining $\mathbf{x}_n$, and unfortunately, $\mathbf{x}_n$ can only be obtained after having a complete set of integer ambiguities over all satellites in view. In other words, calculation of W_i must employ the integers of k_j (where $j > i$). It means no combinations can be skipped over. The Knight method resolves this problem by reforming equation (7) to the following equivalent form,

$$W_i = W_{i-1} + (\theta_i - \mathbf{s}_i^T \mathbf{x}_{i-1} + k_i \lambda)^2 / (\mathbf{s}_i^T \mathbf{P}_{i-1} \mathbf{s}_i + \sigma_i^2) \tag{8}$$

Obviously, one can calculate W_i once $\mathbf{x}_{i-1}$ has been obtained. However, calculation of W_i in equation (7) has to wait for completing the calculation of $\mathbf{x}_n$, *i.e.* through equations from (6a) to (6d). More importantly, because the second term of the right hand side in equation (8) is always non-negative, W_i keeps increasing along with the increase of the index number i. This property makes it possible that the Knight method can skip over less-likely integer combinations and greatly reduces the computational load. Keep the fact in mind that the search method aims to find an integer combination with the minimum residual. Equation (8) can be implemented by iterations and at each iterative step, the current residual is compared with the stored threshold and once the current residual exceeds the stored threshold, the search procedure will jump out of the loop computing current combination and move to another new combination. Meanwhile, the threshold is updated once the residual is found to be smaller than the current threshold.

DETERMINATION OF THE SEARCH SPACE

The search space can be determined by stochastic estimates, i.e. the float solution and its error covariance. The LAMBDA method uses a so called Z-transformation to rescale the search space by decorrelation the covariance, and improves computational efficiency greatly [10]. However, the float solution is obtained by using measurements over a few minutes.

In contrast to the methods based on multiple-epoch measurements, instantaneous methods use measurements of only one epoch. With the known lengths of baselines in a GPS based attitude determination system, the search space for the single differenced integer ambiguities can be determined from equation (1) as follows,

$$k_i = (\mathbf{s}_i^T \mathbf{x} - \theta_i + v_i)/\lambda \tag{9}$$

The first term of the right hand side in equation (9) is equivalent to the dot product of vectors $\mathbf{s}_i$ and $\mathbf{x}$. The dot product requires that the two vectors are expressed in the same coordinate system, *i.e.* either the reference system or the body frame system for attitude determination applications.

As the LOS vector's projection in the reference system and the baseline's projection in the body frame system are known, there are three types of parameters to evaluate the integers in equation (9). The first is the angle of the two vectors, the second is the elevation and azimuth angles of the baseline vector, and the third is three attitude angles. Correspondingly, the integer k_i can be re-written as any of the following three functions.

For the first type,

$$k_i(\alpha_i) = (b\cos\alpha_i - \theta_i + v_i)/\lambda \tag{10}$$

where α_i is the angle between the LOS vector and the baseline, and b is the length of baseline. The fact that the LOS vector is a unit vector has been used in the derivation of equation (10).

For the second type,

$$k_i(el, az) = \frac{1}{\lambda}\left(\begin{bmatrix} s_{ix} & s_{iy} & s_{iz} \end{bmatrix}\begin{bmatrix} b\cos(el)\sin(az) \\ b\cos(el)\cos(az) \\ b\sin(el) \end{bmatrix} - \theta_i + v_i\right) \tag{11}$$

where el and az are the elevation and azimuth angles respectively, and (s_{ix}, s_{iy}, s_{iz}) are the three components of $\mathbf{s}_i$ in the reference coordinate system.

For the third type,

$$k_i(\gamma, p, y) = [\mathbf{s}_i^T \mathbf{A}^T(\gamma, p, y)\mathbf{b} - \theta_i + v_i]/\lambda \tag{12}$$

where (γ,p,y) are angles of roll, pitch and yaw respectively, $\mathbf{A}$(γ,p,y) is the attitude matrix parameterised as three attitude angles, and $\mathbf{b}$ is the baseline vector in the body frame system.

One can use any of the three types above to determine the boundaries of the search space. For example, according to equation (10), the variation range of the angle α_i is from 0 to π in radians. The range exceeding π means that the satellite is beneath the horizon of the baseline platform and its signal can not be received by the antennas. Noting the fact that the fractional carrier phase measurement as well as the phase noise is less than one cycle, the amount of possible integer combinations can be obtained from equation (10) as follows,

$$N \leq (2b/\lambda + 1)^n \tag{13}$$

where N is the number of all possible combinations of integer candidates.

For the case of a one-meter long baseline and six GPS satellites, the number of combinations is of the order of 11^6. There are two ways to reduce the search volume. The first is to shorten the baseline length. The second is to reduce the number of satellites involved as much as possible. On the other hand, a large number of satellites are desirable to improve the reliability of the search method. Thus, it is necessary to consider carefully the trade-off between the calculation cost and the reliability.

GEOMETRIC CONSTRAINT TO THE SEARCH SPACE

The integer ambiguities are usually treated as parameters independent of each other when determining the search space. For example, if considering the baseline length only, the angle between the LOS vector and the baseline vector will vary from 0 to π. Once the value of this angle is fixed, *i.e.* $\hat{\alpha}_i$ for the i^{th} satellite, the corresponding integer ambiguity can be completely determined as shown in the following section.

Suppose the integer ambiguity k_i corresponding to the i^{th} satellite lies in the set of $\Im_i = [k_i^{from}, k_i^{to}]$. The boundaries of the set, both k_i^{from} and k_i^{to} can be determined from equation (11) as follows:

$$k_i^{from} = \mathrm{int}[(-b - \theta_i - \varepsilon_i)/\lambda], \quad \text{when } \alpha_i = \pi \tag{14a}$$

$$k_i^{to} = \mathrm{int}[(b - \theta_i + \varepsilon_i)/\lambda], \quad \text{when } \alpha_i = 0 \tag{14b}$$

where $\mathrm{int}(x)$ is a function to truncate the integral part of the real number of x, and ε_i is the factor to reflect the noise level of carrier phase measurements, *i.e.* $\varepsilon_i = 3\sigma_i$ representing three times the standard deviation of the noise.

The whole search space can be constructed as a combination of all $\Im_i$ (i=1,2,..,n) by the logical-OR operation as follows,

$$\Im = \Im_1 \cup \Im_2 \cdots \cup \Im_n \tag{15}$$

where $\Im$ is the entire search space, and $\Im_i$ is the i^{th} child space.

By referring to equations (9) and (10), the child search space $\Im_i$ can be conceptually illustrated as Figure 2 wherein the i^{th} child space is a semi-spherical surface with the radius of b. One can image that the search space is generated as the rotation of the semi-circle (starting at the point of k_j^{from} (when $\alpha_i = \pi$) and ending at the point of k_j^{to} (when $\alpha_i = 0$) with the elevation angle of el_i. When the semi-circle rotates along the axis of **b** with the angle of el_i varying from 0 to π, the semi-circle scans a surface which is just the semi-spherical surface – the search space. The size of this full child search space can be evaluated by the area of the semi-spherical surface as follows,

$$sz_i = \int_0^{\pi}\int_0^{\pi}(b^2 \sin\alpha_i)d(el_i)d\alpha_i = 2\pi b^2 \tag{16}$$

where sz_i is the size of the full i^{th} child search space.

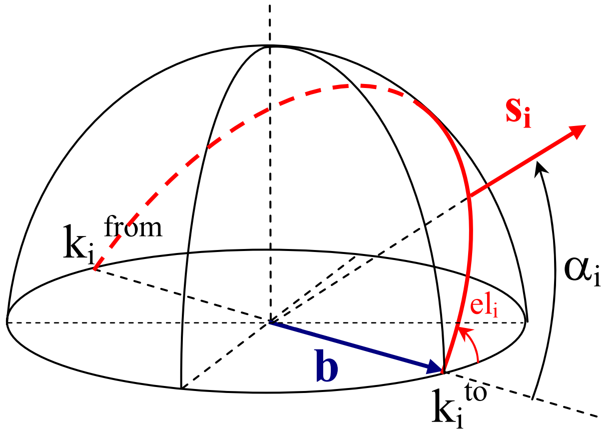

Figure 2. Illustration of the full child search space

The search space shown in equation (15) is based on the assumption that the child search spaces are independent to each other. In fact, this is not true. The reason can be explained as follows. Considering the case when the i^{th} integer ambiguity is known as $\hat{k}_i$, and by using equation (10), the angle between vectors **b** and $\mathbf{s}_i$ can be calculated as follows,

$$\hat{\alpha}_i = \angle(\mathbf{s}_i, \mathbf{b}) = \cos^{-1}[(\theta_i + \hat{k}_i\lambda)/b] \tag{17}$$

where $\hat{\alpha}_i \in [0, \pi]$ is the estimate of the angle between vectors $\mathbf{b}$ and $\mathbf{s}_i$ when the i^{th} integer ambiguity is equal to $\hat{k}_i$.

The measurement noise is neglected in the equation above. Furthermore we take the angle between two LOS vectors into account as follows,

$$\alpha_{ij}^{s} = \cos^{-1}(\mathbf{s}_i \cdot \mathbf{s}_j) \tag{18}$$

where α_{ij}^{s} is the angle between vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ and $\alpha_{ij}^{s} \in [0, \pi]$. Then the integer ambiguity of the j^{th} satellite is constrained by the knowledge of $\hat{\alpha}_i$ and α_{ij}^{s} as follows,

$$k_j^i = (b \cos \alpha_j^i - \theta_j + v_j) / \lambda \tag{19}$$

where k_j^i is the j^{th} integer with the constraint by the i^{th} satellite, and α_j^i is the j^{th} angle with the constraint by the i^{th} satellite and it satisfies the condition,

$$\max\{0, (\hat{\alpha}_i - \alpha_{ij}^{s})\} \le \alpha_j^i \le \min\{\pi, (\hat{\alpha}_i + \alpha_{ij}^{s})\}, \tag{20}$$

where max{x_1, x_2} returns the bigger one between x_1 and x_2, and min{ x_1, x_2} returns the smaller one between x_1 and x_2. Please note that equation (20) uses the fact that a satellite in view is always above the horizontal plane of the antennas. Thus, the left boundary of α_j^i should not exceed 0 and the left boundary of α_j^i should not exceed π.

Therefore the j^{th} constrained child space can be written as $\Im_j^i = [k_j^{i(from)}, k_j^{i(to)}]$ with the boundaries as follows,

$$k_j^{i(from)} = \begin{cases} \mathrm{int}([b\cos(\hat{\alpha}_i + \alpha_{ij}^s) - \theta_j + \mathrm{sign}(\cos(\hat{\alpha}_i + \alpha_{ij}^s)) \cdot \varepsilon_j]/\lambda & \text{when } (\hat{\alpha}_i + \alpha_{ij}^s) \le \pi \\ \mathrm{int}((-b - \theta_j - \varepsilon_j)/\lambda & \text{when } (\hat{\alpha}_i + \alpha_{ij}^s) > \pi \end{cases} \tag{21a}$$

and

$$k_j^{i(to)} = \begin{cases} \mathrm{int}([b\cos(\hat{\alpha}_i - \alpha_{ij}^s) - \theta_j + \mathrm{sign}(\cos(\hat{\alpha}_i - \alpha_{ij}^s)) \cdot \varepsilon_j]/\lambda), & \textit{when } \hat{\alpha}_i \ge \alpha_{ij}^s \\ \mathrm{int}((b - \theta_j + \varepsilon_j)/\lambda), & \textit{when } \hat{\alpha}_i < \alpha_{ij}^s \end{cases} \tag{21b}$$

where ε_j is the factor to reflect the noise level of carrier phase measurements, and $\mathrm{sign}(x)$ is the sign function which is equal to +1 (if $x \ge 0$) or –1 (if $x < 0$) .

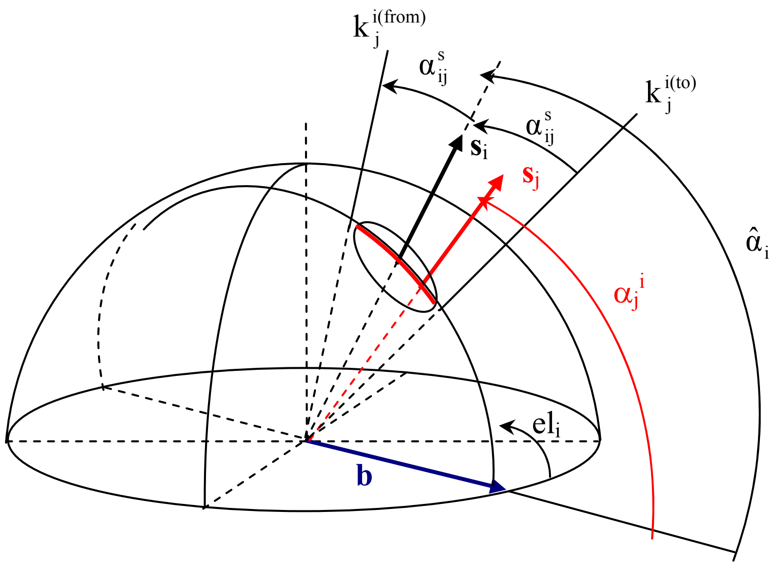

Figure 3. Illustration of constrained child search space

Figure 3 conceptually illustrates the j^{th} constrained child space, which can be imaginarily generated as the rotation of the arc (starting at the point of k_j^{from} (when $\alpha_j^i = \hat{\alpha}_i + \alpha_{ij}^s$) and ending at the point of k_j^{to} (when $\alpha_j^i = \hat{\alpha}_i - \alpha_{ij}^s$) with the elevation angle of el_i. When the arc rotates along the axis of **b** with the angle of el_i varying from 0 to π, the arc scans a surface which is a tyre-like zone on the semi-spherical surface with radius of b. The size of this constrained child search space can be evaluated by the area of the surface of the zone as follows,

$$sz_j^i = \int_{\hat{\alpha}_i - \alpha_{ij}^s}^{\hat{\alpha}_i + \alpha_{ij}^s} \int_0^{\pi} (b^2 \sin \alpha_j^i) d(el_i) d\alpha_j^i = 2\pi b^2 \sin \hat{\alpha}_i \sin \alpha_{ij}^s \tag{22}$$

where sz_j^i is the size of the j^{th} child search space constrained by the i^{th} satellite.

Note that equation (22) supposes that $(\hat{\alpha}_i - \alpha_{ij}^s) \geq 0$ and $(\hat{\alpha}_i + \alpha_{ij}^s) \leq \pi$. One can easily derive the corresponding sz_j^i by using actual boundaries if the case in equation (22) is not true. It is obvious that the size of the constrained search space is smaller than that of its original by comparing equations (22) and (16). For example, suppose $\hat{\alpha}_i = \alpha_{ij}^s = \pi/4$, according to equations (22) and (16), the size of constrained space is the half of its original space.

A given integer in the i^{th} child space constrains the search space of the j^{th} satellite. Similarly, a given integer in the s^{th} child space constrains the search space of the j^{th} satellite too. The final search space of the j^{th} satellite is the insertion of two constrained child spaces, constrained by i^{th} and s^{th} satellites respectively.

In general, if there are n satellites used in the search procedure, the narrowed search space can be determined by the following iterative procedure,

(1) Boundaries of the first child search space $\Im_1$ are determined by equations (14a) and (14b).

(2) Given an element in the space of $\Im_1$, it will constrain to the succeeding child search spaces. The constrained spaces can be denoted as $\Im_j^1$(j=2,...,n) and their boundaries can be determined by equations (21a) and (21b).

(3) Similarly, given an element in arbitrary child space of $\Im_i$, it will constrain its succeeding child spaces denoted as $\Im_j^i$ (j=i+1,…,n), and their boundaries can be determined by equations (21a) and (21b) as well. The process will proceed until i=n-1.

(4) The process above states the constraint made by one satellite. However, one can obtain the j[th] child space by accumulating the constraints made by (j-1) satellites. That is, for the j[th] satellite, there are (j-1) constrained child spaces that are denoted as $\Im_j^1, \Im_j^2, \cdots, \Im_j^{j-1}$ (corresponding to (j-1) satellites) respectively. The final resultant j[th] child space should be the intersection of these sets of $\Im_j^1, \Im_j^2, \cdots, \Im_j^{j-1}$ by the logical AND operation as follows,

$$\hat{\Im}_j = \Im_j^1 \cap \Im_j^2 \cap \cdots \cap \Im_j^{j-1}, \text{ with } \hat{\Im}_1 = \Im_1 \tag{23}$$

where $\hat{\Im}_j$ is the resultant j[th] child space.

Equation (23) can be rewritten in the following iterative form,

$$\hat{\Im}_j^{(i)} = \hat{\Im}_j^{(i-1)} \cap \Im_j^i, \text{ with } j\geq 2, \text{ and } i=1,\ldots,j-1, \text{ and } \hat{\Im}_j^{(0)} = \Im_j \tag{24}$$

Obviously

$$\hat{\mathfrak{I}}_j = \hat{\mathfrak{I}}_j^{(j-1)} \tag{25}$$

(6) The whole constrained search space of all n satellites is derived by the logical OR operation on all constrained child spaces as follows,

$$\hat{\mathfrak{I}} = \hat{\mathfrak{I}}_1 \cup \hat{\mathfrak{I}}_2 \cdots \cup \hat{\mathfrak{I}}_n \tag{26}$$

where $\hat{\mathfrak{I}}$ is the whole constrained search space.

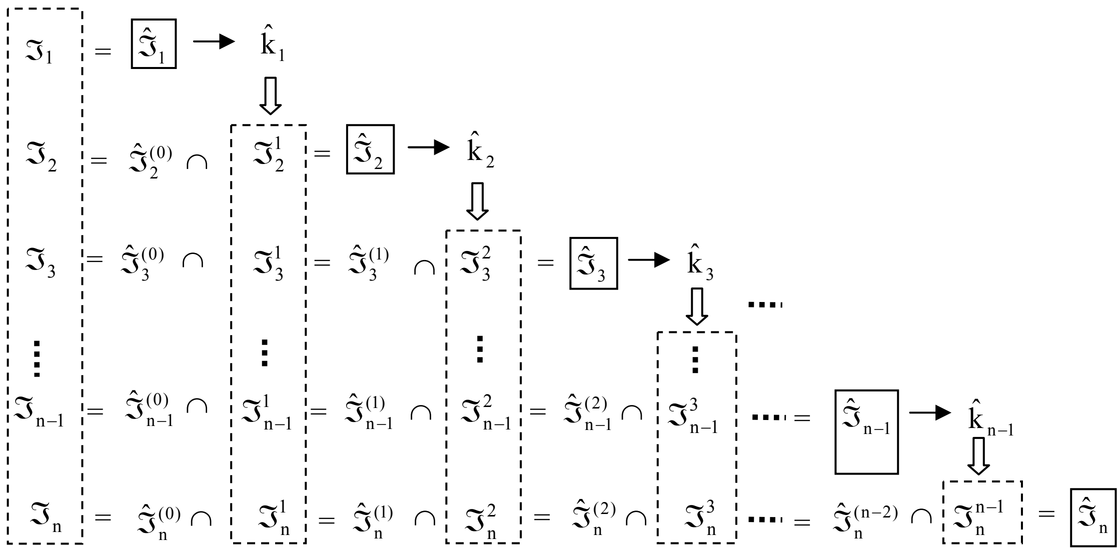

Figure 4. The schematic implementation of the search procedure in the narrowed space

The implementation of the search loop is illustrated in Figure 4. The parts surrounded by the rectangles of dashed lines are the child spaces constrained by specified integers (except for the first). Equations (21a) and (21b) can be used to calculate the boundaries of these child spaces. The child spaces in the first dash-line box are the normal space determined by equations (14a) and (14b). The child spaces surrounded by the solid-line rectangles are accumulated to form the final constrained child spaces. The boundaries of these spaces can be calculated by equations (24) and (25).

EXPERIMENTS

A number of experiments have been conducted to validate the new method proposed above. In the experiments, the raw single differenced carrier phase measurements and LOS vectors were received from a TANS Vector GPS receiver which is a solid-state attitude-determination and position location system with a four-antenna array [13].

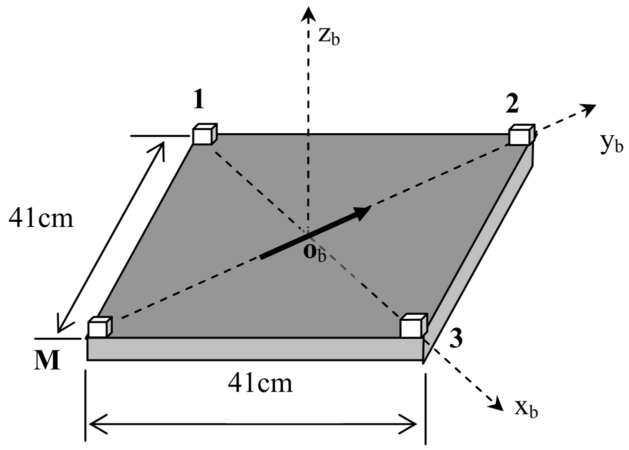

Figure 5. TANS Vector's square configuration of four antennas

Figure 5 shows a three-baseline configuration which four antennas are arranged in a 41 cm by 41cm square platform. These four antennas are denoted as "M", "1", "2" and "3" respectively. "M" represents the master antenna. The vector pointing from the master antenna to the j^{th} slave antenna is the j^{th} baseline vector and it is denoted as $\mathbf{b}_j$ (j=1,2,3). By referring to Figure 5, the baselines can be expressed in the antenna coordinate system or referred to the body-frame system as, $\mathbf{b}_1 = [-d \quad d \quad 0]^T$, $\mathbf{b}_2 = [0 \quad 2d \quad 0]^T$, $\mathbf{b}_3 = [d \quad d \quad 0]^T$, where d = 29 cm.

The receiver was placed at the open sky to consistently record data for about one hour. Then the data was processed by a notebook computer. The software is written using the Microsoft Visual *C*++ 6.0 development platform. The search procedure can be implemented by using a nested “for” loop procedure as presented in [1]. The primary disadvantage of such an implementation is that the nested “for” loop lacks flexibility to tolerate the amount of visible satellites varying from time to time. The more satellites, the more layers of nested loop are needed and thus the more codes are necessary. This investigation uses the recursive technique of C++ to implement the search loop. Because the recursion technique permits a function definition to call itself [15], the recursive loop can resolve all defects of the nested loop. Without losing efficiency, it can process any number of satellites using compact length-fixed codes. Two kinds of search loops are used in the software and both can achieve completely the same result.

All available satellites in tracking are used in the data processing. Because each RF port of TANS Vector has six channels, measurements from up to six satellites are processed in the search loop at an epoch. There were always at least four satellites being tracked during the experiment and about 87 percent of epochs had six satellites. Only epochs with six satellites are used in the evaluation of the two algorithms. There are 1,400 such epochs in total. The search procedure was forced to run at every epoch in order to evaluate the efficiency of the search procedure.

The number of iterative steps contained in a search loop is used to evaluate the computational load. Obviously, the more iterative steps, the more computational load. The computational result shows the average total iterative steps contained in the search loop for six satellites and three baselines (as shown in Figure 5) are 149,561. The normal Knight method reduces the iterative steps to 4,961, only 3.3 percent of the total amount, however the improved Knight method can further reduce the iterative steps to 663, 0.4 percent of the total amount. The comparison of computational load is depicted in Figure 6. The result shows that both the normal and the improved Knight methods can reduce the computational loads dramatically and the improved method can reduce more computational load than the normal method. The number of iterative steps of the improved method is only about 13.4 percent (663/4961) of that of the normal method.

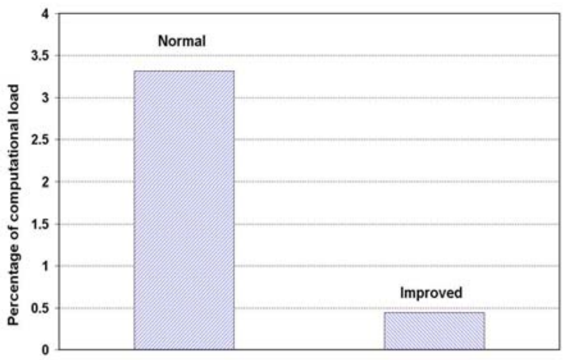

Figure 6. Comparison of the amount of iterative steps (Total steps = 149,561)

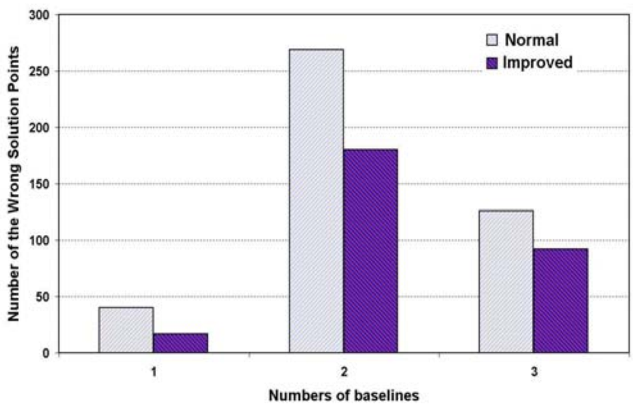

Figure 7. Comparisons of wrong solution points of the normal and improved Knight methods (Total Points = 1,400)

The term 'wrong solution rate' is used to represent the percentage of epochs when the algorithm gives a wrong solution. The lower the wrong solution rate, the higher the reliability. The result is listed in Table 1 where the reliability of two algorithms — the normal and improved Knight algorithms, is compared. The wrong solution rates and points on three baselines are listed respectively. It is evident that the improved algorithm can significantly reduce the wrong solution rate, *i.e.* reducing 23 wrong solution points on $\mathbf{b}_1$, 89 points on $\mathbf{b}_2$, and 34 points on $\mathbf{b}_3$ respectively (as mentioned above the total points are 1,400). Therefore, the improved Knight algorithm can greatly improve the reliability of the search procedure. The result is also depicted graphically in Figure 7 where the wrong solution points on three baselines are presented.

Algorithms	$\mathbf{b}_1$		$\mathbf{b}_2$		$\mathbf{b}_3$	
	percent	points	percent	points	percent	points
Normal	2.9	40	19.2	269	9.0	126
Improved	1.2	17	12.8	180	6.6	92

Table 1. Comparisons of wrong solution rates of the normal and improved Knight methods over three baselines $\mathbf{b_1}$, $\mathbf{b_2}$ and $\mathbf{b_3}$ (Total Points = 1,400)

Knowledge about the system, *e.g.* the lengths of baselines and the coarse attitude of the antenna array etc. may also help to narrow the search space. The coarse attitude knowledge may come from other independent sensors, *e.g.* an inertial measurement unit (IMU). How the search procedure benefits from coarse attitude knowledge is investigated as well in the experiment. A coarse attitude of the antenna platform is known within a particular uncertainty: 30 deg for azimuth and 15 deg for both pitch and roll. This coarse attitude knowledge is used to reduce the volume of search space. Table 2 shows the performance of the normal and improved methods in terms of the iterative steps using the coarse knowledge of the system and Table 3 compares the reliability of the normal and the improved methods. The average number of total iterative steps is greatly reduced from 149,561 to 3,686. Correspondingly, the average number of iterative steps for the normal Knight method is reduced from 4,961 to 533, and the average number of iterative steps for the improved Knight method is reduced from 663 to 249. The result is listed in Table 2 for clear comparison. Error rates of two methods in this case are compared in Table 3. Both can achieve almost the same reliability in this case, *i.e.* all can achieve more than 99 percent correct solution rates. The reason is that the search space is narrowed small enough by the coarse attitude and very few spaces are left for the constraints of the satellite geometric information. It is apparent that coarse attitude knowledge can efficiently reduce the iterative steps of search loops and so reduce computational load greatly.

Iterative steps	Total	Normal	Improved
Without attitude information	149,561	4,961	663
With attitude information	3,686	533	249

Table 2. Comparison of iterative steps, with/without coarse attitude information

(Uncertainties of the coarse attitude information: 30 deg for azimuth and 15 deg for both pitch and roll)

Algorithms	$\mathbf{b}_1$		$\mathbf{b}_2$		$\mathbf{b}_3$	
	percent	points	percent	points	percent	points
Normal	0.14	2	0.79	11	0.21	3
Improved	0.14	2	0.64	9	0.21	3

Table 3. Comparisons of wrong solution rates over three baselines $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$

(Total Points = 1,400, with coarse attitude information: uncertainty of 30 deg for azimuth and uncertainty of 15 deg for both pitch and roll)

CONCLUSIONS

A new method for ambiguity search has been presented. The proposed method can improve the integer ambiguity search efficiency and reliability by narrowing the search space. Due to its recursive nature, the procedure is very suitable to be integrated into the search loop of the Knight method. Thus it can benefit from both narrowing the search space and skipping over less-likely combinations. Experiments demonstrate that the improved Knight method significantly outperforms the normal Knight method, especially when any coarse attitude information is not available.

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