

On the Spectral Efficiency for Selection Combiner Diversity (SCD) over Slow Fading

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Abstract—In this paper we derive closed-form expressions for the single-user capacity of selection combining diversity (SCD) system, taking into account the effect of imperfect channel estimation at the receiver. The channel considered is a slowly varying spatially independent flat Rayleigh fading channel. The complex channel estimate and the actual channel are modelled as jointly Gaussian random variables with a correlation that depends on the estimation quality. Two adaptive transmission schemes are analyzed: 1) optimal power and rate adaptation; and 2) constant power with optimal rate adaptation. Our numerical results show the effect of Gaussian channel estimation error on the achievable spectral efficiency.

I. INTRODUCTION

It's well known that information bearing signals transmitted over wireless channels experience multipath fading that introduces both random phase shift and amplitude variation [1], resulting in a serious degradation in communication and increased bit error rate (BER). Diversity can help effectively in recovering the signal by providing the receiver with multiple faded replica of information bearing signal [1], [2], [3]. In particular, selection combining diversity (SCD) has been the most commonly implemented scheme in wireless communication systems owing to its simplicity.

Most system designs assume that perfect channel estimation is available at the receiver. However, in practical systems, the branch signal-to-noise ratio (SNR) estimates are usually combined with noise which makes it difficult to estimate them perfectly. In practice, a diversity branch SNR estimate can be obtained either from a pilot signal or data signals (by applying a clairvoyant estimator) [4]. For example, if a pilot signal is inserted to estimate the channel, a Gaussian error may arise in due the large frequency separation or time dispersion. Previous work on the analysis of imperfect channel estimation with no diversity can be found in [5] and [6]. In [7], a new closed-form expression for the probability density function (PDF) of the SCD combiner output with imperfect channel estimation was derived, based on the derivation of [4]. The author focused on deriving the average error probability, where it was shown that the degradation due to imperfect channel estimation induces error floors at relatively high SNR values.

Shannon's benchmark paper [8] established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a communication channel. The Shannon capacity of fading channels under different

assumptions about the knowledge of the channel information at the transmitter and the receiver was presented in [9] and [10], respectively. In [11], the capacity of a single-user flat fading channels with perfect channel information at the transmitter and the receiver was derived for various adaptation policies; namely, 1) optimal rate and power adaptation (*opra*), 2) optimal rate adaptation and constant power (*ora*), and 3) channel inversion with fixed rate (*cifr*), which is beyond the scope of our work. The first scheme requires channel information at the transmitter and receiver, whereas the second scheme is more practical since the transmission power remains constant. The last scheme is a suboptimal transmission adaptation scheme, in which the channel side information is used to maintain a constant received power by inverting the channel fading [11]. In [12], the general theory developed in [11] was applied to derive closed-form expressions for the capacity of Rayleigh fading channels under different adaptive transmission and diversity combining techniques. Recently, there has been some work dealing with the channel capacity of different fading channels employing different adaptive schemes such as [13],[14], and the references therein. Up to the knowledge of the authors, the capacity of SCD receivers with estimation errors has not been derived.

In this paper, we extend the results in [12] to obtain closed-form expressions for the single-user capacity of SCD system, in the presence of Gaussian channel estimation errors. The contributions of this paper are deriving closed-expressions for two adaptive transmission schemes including their asymptotic approximations and upper bounds and these schemes are:

- (1) optimal simultaneous power and rate adaptation (*opra*).
- (2) optimal rate adaptation with constant transmit power (*ora*).

The paper is organized as follows. In Section II, the system model used in this paper is discussed. In Section III, we derive closed-form expressions for the channel capacity under two adaptation schemes; *opra* and *ora* including their asymptotic approximations and upper bounds in sub-sections III-A and in III-B, respectively. Results are presented and discussed in Section IV. The main outcomes of the paper are summarized in Section V.

II. SYSTEM MODEL

Consider an L -branch diversity receiver in slow fading channels. Assuming perfect timing and no inter-symbol in-

terference (ISI), the received signal on the l th branch due to the transmission of a symbol s can be expressed as

$$r_l = g_l s + n_l, \quad l = 1 \dots L, \quad (1)$$

where g_l is a zero-mean complex Gaussian distributed channel gain, n_l is the complex additive white Gaussian noise (AWGN) sample with a variance of $N_0/2$, and s is the data symbol taken from a normalized unit-energy signal set with an average power P_s . An SCD receiver tracks the amplitude of the channel estimate \hat{g}_l from the L diversity branches, and selects the branch yielding the largest fading amplitude. Thus, if the SCD is employed with equal noise mean power at all branches, the decision criteria reduces to

$$m = \arg \max_{l=1 \dots L} \{|\hat{g}_l|\}, \quad (2)$$

where \hat{g}_m is the magnitude of the selected diversity branch gain at the output of the combiner. The channel estimate \hat{g} and the channel gain g can be accurately approximated as jointly complex Gaussian [4]. We further assume the actual channel gains of the L diversity branches are i.i.d. as well as the channel estimates. The actual channel gain g is related to the channel estimate \hat{g} [4] as follows

$$g_l = \rho \hat{g}_l + z_l, \quad (3)$$

where ρ is a complex number representing the normalized correlation between g and \hat{g} and z_l is a complex Gaussian random variable independent of \hat{g} with zero-mean and a variance of σ_z^2 . The PDF of the SCD receiver with imperfect channel estimation is given by [7]

$$p_\gamma(\gamma) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \frac{k+1}{\gamma_t (k+1 - k\rho^2)} \exp\left(\frac{-\gamma(k+1)}{\gamma_t (k+1 - k\rho^2)}\right), \quad (4)$$

where $\gamma_t = \frac{P_s}{N_0}$ is the average SNR per receive branch. In the following, the PDF in (4) is used to derive the channel capacity with SCD and channel estimation errors.

III. ADAPTIVE CAPACITY POLICIES

We recall the main results from [12] for channel capacities for the following transmission policies.

A. Power and Rate Adaptation

Given an average transmit power constraint, the channel capacity C_{opra} in (bits/seconds) of a fading channel [11], [12] is given by

$$C_{opra} = \frac{B}{\ln 2} \int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma) d\gamma, \quad (5)$$

where B (in hertz) is the channel bandwidth and γ_0 is the optimum cutoff SNR satisfying the following condition

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p_\gamma(\gamma) d\gamma = 1. \quad (6)$$

To achieve the capacity in (5), the channel fading level must be tracked at both transmitter and receiver. The transmitter has to adapt its power and rate accordingly by allocating high power levels and transmission rates for good channel conditions (large γ). Since the transmission is suspended when $\gamma < \gamma_0$, this policy suffers from outage, whose probability P_{out} is defined as the probability of no transmission and is given by

$$P_{out} = 1 - \int_{\gamma_0}^{\infty} p_\gamma(\gamma) d\gamma. \quad (7)$$

However, C_{opra} in (5) can be expressed in terms of the CDF of γ by applying integration by-parts resulting in

$$\frac{C_{opra} \ln(2)}{B} = - \int_{\gamma_0}^{\infty} \frac{1}{\gamma} F(\gamma) d\gamma. \quad (8)$$

Substituting (4) in (6) yields the equality

$$\sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} (k+1) \left[\exp\left(\frac{\gamma_0(1+k)}{\gamma_t [k+1 - k\rho^2]}\right) \left(\frac{[k+1 - k\rho^2]\gamma_t}{(1+k)\gamma_0} \right) - E_1\left(\frac{(k+1)\gamma_0}{\gamma_t [k+1 - k\rho^2]}\right) \right] = \gamma_t [k+1 - k\rho^2]. \quad (9)$$

The second term of (9) can be evaluated by making use of Exponential integral function of first order [16] defined as

$$E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t} dt. \quad (10)$$

Upon substitution of (10) into (9), it is found that the optimal cutoff SNR, γ_0 has to satisfy the following equality

$$\sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} (k+1) \times \left[\exp\left(\frac{\gamma_0(1+k)}{\gamma_t [k+1 - k\rho^2]}\right) \left(\frac{[k+1 - k\rho^2]\gamma_t}{(1+k)\gamma_0} \right) - E_1\left(\frac{(k+1)\gamma_0}{\gamma_t [k+1 - k\rho^2]}\right) \right] = \gamma_t [k+1 - k\rho^2]. \quad (11)$$

To obtain the optimal cutoff SNR, γ_0 in (11), we follow the following procedure. Let $x = \frac{\gamma_0}{\gamma_t}$ and define the function $f_{sc}(x)$ as

$$f_{sc}(x) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left[\exp\left(\frac{-x(1+k)}{\gamma_t [k+1 - k\rho^2]}\right) \left(\frac{[k+1 - k\rho^2]}{(1+k)x} \right) - E_1\left(\frac{(k+1)x}{\gamma_t [k+1 - k\rho^2]}\right) \right] - \frac{\gamma_t [k+1 - k\rho^2]}{k+1}. \quad (12)$$

Making change of variable where $\mu = (k+1)/(\gamma_t [k+1 - k\rho^2])$ and applying the first order derivative to (12) with respect to x , it yields

$$f'_{sc}(x) = - \sum_{k=1}^{L-1} \binom{L}{k+1} \exp\left(\frac{-\mu x}{\mu^2 x^2}\right). \quad (13)$$

Hence, $f'_{sc}(x) < 0, \forall x > 0$, meaning that $f'_{sc}(x)$ is a strictly decreasing function of x . Also, observing that

$$1) \quad \lim_{x \rightarrow 0^+} f_{sc}(x) = \infty \quad (14)$$

$$2) \quad \lim_{x \rightarrow \infty^+} f_{sc}(x) = -\frac{\gamma_t [k+1 - k\rho^2]}{k+1}, \quad (15)$$

Noting that, $f_{sc}(x)$ is a continuous function of x , which leads to a unique positive γ_o such that $f_{sc}(x) = 0$. Therefore, it is concluded that for each $\gamma_t > 0$ there is a unique γ_o satisfying (12). Numerical results using MATLAB show that $\gamma_o \in [0, 1]$ as γ_t increases, and $\gamma_o \rightarrow 1$ as $\gamma_t \rightarrow \infty$.

Now, substituting (4) into (5) yields the channel capacity with the *opra* scheme as follows

$$\frac{C_{opra}}{B} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \frac{k+1}{\gamma_t [k+1 - k\rho^2]} \underbrace{\int_{\gamma_o}^{\infty} \ln\left(\frac{\gamma}{\gamma_o}\right) \exp\left(\frac{-\gamma(k+1)}{\gamma_t [k+1 - k\rho^2]}\right) d\gamma}_{I_1} \quad (16)$$

where the integral I_1 in the above expression can be computed using the fact from [12], which states the following

$$\int_0^1 \ln x \exp(-\mu x) dx = E_1(\mu)/\mu. \quad (17)$$

Inserting (17) into (16) implies that the capacity C_{opra} per unit bandwidth (in bits/seconds/hertz) can be expressed as

$$\frac{C_{opra}}{B} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} E_1\left(\frac{(1+k)\gamma_o}{\gamma_t [k+1 - k\rho^2]}\right) \exp\left(\frac{-\gamma(k+1)}{\gamma_t [k+1 - k\rho^2]}\right) \quad (18)$$

1) Asymptotic Approximation: We can obtain asymptotic approximation C_{opra} using the series representation of Exponential integral of first order function [16] expressed as

$$E_1(x) = -E - \ln(x) - \sum_{i=1}^{+\infty} \frac{(-x)^i}{i \cdot i!}, \quad (19)$$

where $E = 0.5772156659$ is the Euler-Mascheroni constant. Then, the asymptotic approximation C_{opra}^{∞} per unit bandwidth (in bits/seconds/hertz) can be shown to be

$$\begin{aligned} \frac{C_{opra}}{B} &= \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left(-E \right. \\ &\quad \left. - \ln\left(\frac{(1+k)\gamma_o}{\gamma_t [k+1 - k\rho^2]}\right) + \left(\frac{\gamma(k+1)}{\gamma_t [k+1 - k\rho^2]}\right) \right) \\ &\quad \times \exp\left(\frac{-\gamma(k+1)}{\gamma_t [k+1 - k\rho^2]}\right). \end{aligned} \quad (20)$$

2) Upper Bound: The capacity expression of C_{opra} can be upper bounded by applying Jensen's inequality to (5) as follows

$$\frac{C_{opra}^{UB}}{B} = \ln\left(\mathbb{E}[\gamma]\right), \quad (21)$$

where $\mathbb{E}[\cdot]$ is the expectation operator. The expression in (24) can be evaluated by averaging it over the PDF in (4) and making help of [16] resulting in

$$\int_0^{\infty} x^n e^{-\mu x} dx = n! \mu^{-n-1}, \quad (22)$$

for $\text{Re}[\mu] > 0$. The resulting expression can be further simplified to obtain the upper bound for C_{opra} as follows

$$\frac{C_{opra}^{UB}}{B} = \ln\left(\sum_{k=0}^{L-1} \frac{(-1)^k}{\gamma_o} \binom{L}{k+1} \frac{\gamma_t [k+1 - k\rho^2]}{\gamma(k+1)}\right). \quad (23)$$

3) Opra Upper Bound: We upper-bound the capacity C_{opra} by applying Jensen's inequality to (5) as follows:

$$\frac{C_{opra}^{UpperBound}}{B} = \ln\left(\mathbb{E}\{\gamma\}\right). \quad (24)$$

We evaluate the expression in (24) by averaging γ over (4) with help of [16]:

$$\int_0^{\infty} x^n e^{-\mu x} dx = n! \mu^{-n-1}. \quad (25)$$

for $\text{Re}[\mu] > 0$. We simplify the resulting expression to obtain the upper bound as follows:

$$\frac{C_{opra}^{UpperBound}}{B} = \ln\left(\sum_{k=0}^{L-1} \frac{(-1)^k}{\gamma_o} \binom{L}{k+1} \frac{\gamma_t [k+1 - k\rho^2]}{\gamma(k+1)}\right). \quad (26)$$

B. Constant Transmit Power

By adapting the transmission rate to the channel fading condition with a constant power, the channel capacity C_{ora} [8], [9] is given by

$$C_{ora} = \frac{B}{\ln 2} = \int_0^{\infty} \ln(1+\gamma) p_{\gamma}(\gamma) d\gamma. \quad (27)$$

Substituting (4) into (27) results in

$$\frac{C_{ora}}{B} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left[1 - \frac{k\rho^2}{k+1}\right]^{-1} \underbrace{\int_0^{\infty} \ln(1+\gamma) \exp\left(\frac{-\gamma(k+1)}{\gamma_t [k+1 - k\rho^2]}\right) d\gamma}_{I_2} \quad (28)$$

The integral I_2 can be computed conveniently by using the change of variable $x = 1 + \gamma$ and applying (17), resulting

in a closed-form expression for the capacity C_{ora} per unit bandwidth (in bits/seconds/hertz) given by

$$\frac{C_{ora}}{B} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \exp\left(\frac{(1+k)}{\gamma_t [k+1 - k\rho^2]}\right) \times E_1\left(\frac{(1+k)}{\gamma_t [k+1 - k\rho^2]}\right). \quad (29)$$

1) *Asymptotic Approximation*: Following the same procedure in Section III-A, the asymptotic approximation C_{ora}^∞ per unit bandwidth (in bits/seconds/hertz) can be computed as

$$\frac{C_{ora}^\infty}{B} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \exp\left(\frac{(1+k)}{\gamma_t [k+1 - k\rho^2]}\right) \times \left(-E - \ln\left(\frac{(1+k)\gamma_0}{\gamma_t [k+1 - k\rho^2]}\right) + \left(\frac{\gamma(k+1)}{\gamma_t [k+1 - k\rho^2]}\right) \right). \quad (30)$$

2) *Upper Bound*: The capacity C_{ora} can be upper bounded by applying Jensen's inequality to (5) as follows

$$C_{ora}^{UB} = \ln\left(1 + \mathbb{E}[\gamma]\right), \quad (31)$$

and the upper bound can be written as

$$\frac{C_{ora}^{UB}}{B} = \ln\left(1 + \sum_{k=0}^{L-1} \frac{(-1)^k}{\gamma_t} \binom{L}{k+1} \frac{\gamma_t [k+1 - k\rho^2]}{\gamma(k+1)}\right). \quad (32)$$

IV. NUMERICAL RESULTS

In this section we provide some numerical results that illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of average receiver SNR (γ_t) in dB for different adaptation policies with SCD over slow Rayleigh fading with weight estimation errors. All curves provided are obtained using the closed-form expressions, (18), (20), (26), (29), (30), (32).

Figure 1 shows a comparison of the capacity per unit bandwidth for optimal power and rate adaptation (*opra*) and optimal rate adaptation and constant transmit power (*ora*). The results show the capacity of *opra* outperforms the capacity of *ora* for any average SNR (γ_t) per branch (dB). However, both *opra* and *ora* achieve the same result if the power adaptation is not considered at the transmitter for the *opra* policy. In addition, The results showed that for the same bandwidth, the capacity increases with the increase of the diversity order L and the increase of the average γ_t per branch for both *opra* and *ora*.

Figure 2 compares *opra* for different values of correlation between the channel and its estimate; namely, $\rho = 0.3$, $\rho = 0.5$, $\rho = 0.7$, $\rho = 0.9$ and $\rho = 1$. It can be noticed that the highest *opra* that can be achieved when $\rho = 1$. Furthermore, *opra* decreases when the value of ρ decreases where in this case the weight error increases. It can be observed from Figure

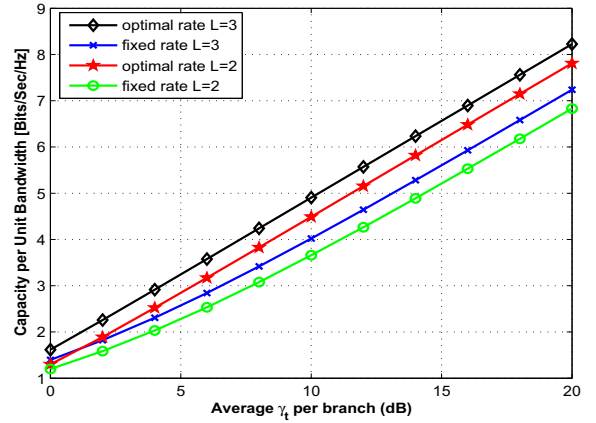


Fig. 1. Capacity per unit bandwidth for a Rayleigh fading with SCD diversity ($L=3$) for different adaptation schemes.

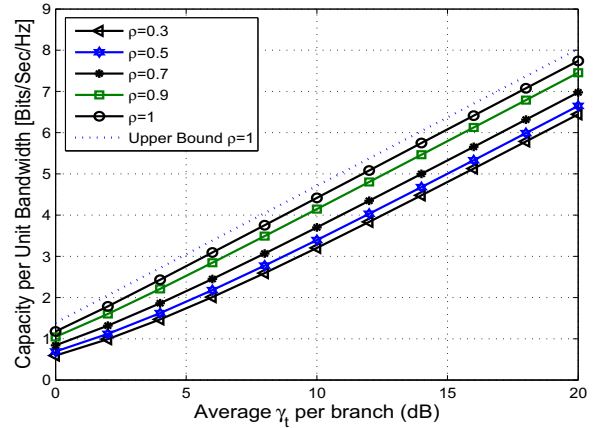


Fig. 2. Capacity per unit bandwidth for a Rayleigh fading with SCD diversity ($L=3$) and various values of ρ under power and rate adaptation.

2 that there is almost a 5 dB difference in *opra* between $\rho = 1$ and $\rho = 0.3$.

The upper bound shows a tight approximation of the exact average capacity. Figure. 3 shows the plot of *ora* as well as its asymptotic approximation as a function of the average received SNR γ_t for $L = 3$. The same figure shows that the *ora* policy is less sensitive to the estimation error than the *opra*.

V. CONCLUSION

The channel capacity for unit bandwidth for three different adaptation policies including their approximations and upper bounds over a slow Rayleigh fading channel for SCD with estimation error is discussed. Closed-form expressions for two adaptation policies are derived for L -selection combiner. Our numerical results showed that for the same bandwidth, the capacity increases with increase of the diversity order L and increase of of the average γ_t per branch. Also, simulation

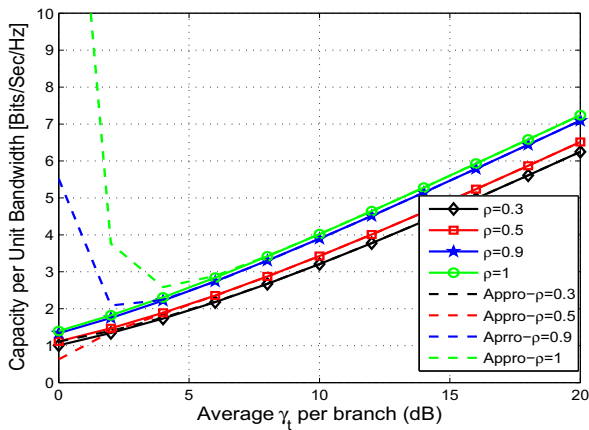


Fig. 3. Capacity per unit bandwidth for a Rayleigh fading with SCD diversity ($L=3$) and various values of ρ under rate adaptation and constant power.

showed that *opra* outperforms *ora*, however, *ora* is less sensitive to the estimation error than *opra*.

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