Abstract

This work is concerned with the use of wavelet transform based denoising techniques for improved direction-of-arrival (DOA) estimation and source localization. DOA estimation and source localization are problems of great interest in applications of array signal processing to RADAR, SONAR, seismic explorations, etc. Signals from several sources distributed in space are received by an array of sensors. Information provided by the sampled spatial structure of the wave-field is utilised to estimate the directions of arrival of the signals or the locations of the sources. Specific examples include plane wave DOA estimation by MUSIC, bearing estimation of underwater acoustic sources by the Subspace Intersection Method (SIM), and underwater acoustic source localization by Matched Field Processing (MFP). These methods are known to produce reliable estimates at relatively high SNR (say, SNR ≥ 0 dB). But values of SNR lower than this threshold are frequently encountered in practice, and the performance of all the DOA estimation / localization techniques degrades rapidly when the SNR dips below the threshold. The subthreshold performance of these techniques may be improved by performing an SNR enhancement prior to DOA estimation / localization.

The Wiener filter is the minimum mean square error (MMSE) estimator of signals in stationary noise if the signal and noise are jointly Gaussian, and the best linear estimator otherwise. The Wiener filter may be used to achieve SNR enhancement provided that the joint second order statistics of the signal and noise are known. But information about the signal statistics is often not available, and in such circumstances Wiener filtering cannot be used. It is in this context that the technique of wavelet denoising comes to the rescue. Wavelet denoising is a nonlinear filtering technique proposed by Donoho for the estimation of signals in white Gaussian noise. When the noisy signal is wavelet transformed, the signal energy is concentrated in a few coefficients in the approximation space whereas the energy of white noise is distributed uniformly over all coefficients in the approximation and detail spaces. By optimally thresholding the wavelet coefficients and then computing the inverse
wavelet transform, much of the noise energy can be eliminated without causing significant distortion of the signal. The process of wavelet denoising described above applies to single-sensor signals. The current work involves signals from multiple sensors.

The problem of wavelet denoising of signals received at an array of sensors has been addressed by Rao and Jones. Signals received by an array of sensors have significant spatial correlation whereas noise is normally uncorrelated across the array. The spatial correlation of the signal can be effectively used for denoising the signal received by the array. Based on the structure of Wiener filter for multisensor signals, Rao and Jones have proposed a wavelet array denoising (WAD) method wherein the noisy signal received at the array is spatially decorrelated using KLT, and each of the decorrelated components is then denoised separately. The denoised components are then recorrelated to obtain the denoised multisensor signal. Thus, not only is the intersensor signal correlation preserved, this correlation is effectively utilised to improve the denoising performance.

The method described above assumes that the spatial correlation statistics of the signal is known. In the case of absence of knowledge of spatial signal statistics, Rao and Jones have proposed the use of DFT in the spatial domain as a substitute for KLT. This substitution is motivated by the asymptotic decorrelation property of the DFT.

In this thesis, we propose the use of wavelet array denoising with DFT to enhance the SNR of noisy sensor data at the output of a uniform linear array (ULA), and then perform DOA estimation / source localization using the denoised data. In practice, the configuration of the array often deviates from the nominal ULA geometry. We have taken into account the effect of array perturbation on the spatial correlation statistics of the signal. The performance of the DFT based WAD is analysed by numerically investigating the SNR gain under different conditions. Three problems, viz, plane wave DOA estimation by MUSIC, bearing estimation of underwater acoustic sources in shallow ocean by SIM, and range-depth estimation of underwater acoustic sources by MFP are considered. It is shown that, the use of WAD data leads to a lower mean square (DOA estimation) error and a higher resolution as compared to the use of undenoised array data.

The denoising method described above requires effective compaction of the signal energy into the approximation space of the wavelet domain. In order to achieve this compaction and prevent the leakage of the signal energy into the detail spaces, it is necessary to ensure that the signal is sufficiently oversampled. This need for high sampling rate may limit the utility of the method. Moreover, in the case of random signals, it is not possible to ensure a sufficiently high sampling rate, and the consequent leakage of the signal energy into the detail spaces leads to a degradation of performance. A method of avoiding this
problem by performing wavelet array denoising in the spatial domain instead of the temporal domain is proposed in the thesis. The spatial wavelet denoising technique is particularly effective in denoising random narrowband signals. The spatial power spectral density of the signal received at a ULA from \( n \) narrowband sources has \( n \) distinct peaks riding over a plateau of height \( \sigma^2 \), assuming that the noise is *white* both spatially and temporally. One can use Spatial Wavelet Packet Transform (SWPT) to filter the peaks from the background noise, thereby achieving denoising. The SWPT divides the spatial frequency range into uniform subbands. The filtering is done by a block thresholding method wherein all coefficients in a subband are set to zero if the total energy in that subband is less than or equal to \( \beta \sigma^2 \), where \( \beta \) is a tolerance parameter greater than unity. The coefficients in a subband are retained if the energy in the subband exceeds the threshold \( \beta \sigma^2 \). This kind of block thresholding achieves spatial denoising by eliminating noise from directions that do not contain a source. Simulation results are presented to compare the performance of the SWPT denoising technique with that of the temporal WAD technique for DOA estimation of narrowband random signals. It is shown that the SWPT denoising technique achieves a higher SNR gain at lower sampling rate, and consequently a lower MSE and a higher resolution for DOA estimation using MUSIC.