Abstract

Knapsack problems are among the most intensively studied NP-hard combinatorial optimization problems. The applications of these problems span a wide canvas from industrial applications and financial management to electronic commerce and personal health-care. The common flavor in most of these problems is resource allocation. The allocation of a specific amount of a single resource among competitive alternatives is often modeled as a knapsack problem or its variants. In nonlinear knapsack problems (also called nonlinear resource allocation problems), there are multiple units of each item and the cost of inclusion varies nonlinearly with the number of units. This thesis is an investigation on a nonlinear resource allocation problem with non-convex and piecewise linear cost structure, called as the *piecewise linear knapsack problem* (PLKP). Such a cost structure is motivated and inspired by major applications in electronic commerce and electronic business, in particular, electronic procurement.

The thesis first deals with modeling and solving of the PLKP in detail, followed by modeling and solving of the bid evaluation problem in three representative procurement settings. The thesis is divided into three parts. The problem PLKP is introduced in Part 1. In this part, the PLKP is modeled as a mixed integer linear programming problem. The standard textbook models for modeling piecewise linear cost structure are presented followed by two new proposals: *precedence constrained knapsack model* (PCKM) and *multiple choice knapsack model* (MCKM). The proposed models PCKM and MCKM are explored for exploitable substructures to develop algorithms. Designing of algorithms for the PLKP is dealt with in detail in Part 2. The algorithms include *heuristics* based on greedy techniques, knapsack structure, linear programming relaxation, and Lagrangian relaxation; *exact algorithms* based on dynamic programming, branch-and-bound technique, and Benders' decomposition; and a *2-approximation algorithm* and a *fully polynomial time approximation scheme*. In Part 3, three different representative procurement scenarios, in which the PLKP is a subproblem, are considered. Modeling and solving of the bid evaluation problem in the three procurement scenarios are discussed using the algorithms and approaches discussed in Part 2.