Universal three-body parameter in ultracold ^{4}He*

S. Knoop,[1] J. S. Borbely,[1] W. Vassen,[1] and S. J. J. M. F. Kokkelmans[2]
[1]*LaserLaB, VU University, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands*
[2]*Eindhoven University of Technology, P. O. Box 513, 5600 MB Eindhoven, The Netherlands*
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We have analyzed our recently measured three-body loss rate coefficient for a Bose-Einstein condensate of spin-polarized metastable triplet ^{4}He atoms in terms of Efimov physics. The large value of the scattering length for these atoms, which provides access to the Efimov regime, arises from a nearby potential resonance. We find the loss coefficient to be consistent with the three-body parameter (3BP) found in alkali-metal experiments, where Feshbach resonances are used to tune the interaction. This provides evidence for a universal 3BP outside the group of alkali-metal elements. In addition, we give examples of other atomic systems without Feshbach resonances but with a large scattering length that would be interesting to analyze once precise measurements of three-body loss are available.

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I. INTRODUCTION

When the short-range interaction between particles gives rise to (near-)resonant scattering, few-body properties are expected to become universal, i.e., irrespective of the precise nature of the interaction and therefore applicable to nucleons, atoms, or molecules [1]. Within universal few-body physics a hallmark prediction is the Efimov effect, in which three particles that interact via a resonant short-range attractive interaction exhibit an infinite series of three-body bound states, even in the regime where the two-body interaction does not support a bound state [2]. The first experimental evidence of Efimov trimers came from an ultracold trapped gas of atoms [3] by tuning the strength of the interaction via a Feshbach resonance [4]. In the context of ultracold atoms, the universal regime is realized when the s-wave scattering length a, characterizing the two-body interaction in the zero-energy limit, is much larger than the characteristic range of the interaction potential. Signatures of Efimov states are imprinted on trap loss caused by three-body recombination, which typically determines the lifetime of an ultracold trapped atomic gas or Bose-Einstein condensate. So far, observations of Efimov features have been made in ultracold quantum gases of bosons: ^{7}Li [5–7], ^{39}K [8], ^{85}Rb [9], Cs [3,10,11], a three-spin component mixture of fermionic ^{6}Li [12–14], and the Bose-Bose mixture ^{41}K + ^{87}Rb [15].

In addition to the scattering length, a three-body parameter (3BP) is needed to fully describe the spectrum of Efimov trimers. The 3BP accounts for all the short-range information that is not contained in the scattering length, including a true three-body interaction. It can be parametrized as the location of the first Efimov resonance, a_-, on the $a < 0$ side of a Feshbach resonance. Initially, the 3BP was thought to be very sensitive to details of the short-range interaction and therefore different for each (atomic) system [16]. However, experiments around different Feshbach resonances and with different alkali-metal atoms found the ratio $|a_-|/r_{\rm vdW}$ in a narrow range between 8 and 10 [5,9,11], where $r_{\rm vdW} = \frac{1}{2}(mC_6/\hbar^2)^{1/4}$ is the range of the tail of the two-body potential (also called the van der Waals length), with m the atomic mass and C_6 the long-range coefficient. There is a vivid theoretical debate on the physical origin of this universal 3BP [17–22]. Most work points towards a three-body repulsive barrier that prevents the three atoms from probing the short-range interaction. An important question is how general the universal 3BP is. Experimental data outside the group of alkali-metal atoms could shed light on this issue.

In this paper we investigate the possibility of extracting the 3BP from our recently measured three-body loss rate coefficient in a Bose-Einstein condensate (BEC) of metastable triplet helium-4 (denoted as ^{4}He*) [23]. We will show that its value is consistent with those measured in alkali-metal systems, providing further experimental evidence of a universal 3BP. We will also discuss other atomic systems that can be analyzed in a similar fashion. The common feature is that in the absence of a Feshbach resonance, these atomic systems already have a scattering length that is much larger than the range of the potential. The mechanism for this is an almost resonant interaction potential, i.e., a bound state is almost degenerate with the collision threshold. This potential resonance is a simple single-channel effect. In contrast, a Feshbach resonance is a multichannel effect, where the width of the resonance introduces another length scale [4], which may give rise to nonuniversal physics. Therefore, potential resonances are more directly related to the universal description connected to a large scattering length than Feshbach resonances.

II. THREE-BODY LOSS IN ALKALI METALS

To relate our work to the alkali-metal experiments, we first summarize how the 3BP is extracted from three-body loss measurements around a Feshbach resonance [1,3]. In the limit of $|a| \gg r_{\rm vdW}$ the three-body loss rate coefficient L_3 for identical bosons is given by

$$L_3 = 3C_\pm(a)\frac{\hbar a^4}{m}, \tag{1}$$

where $C_\pm(a)$ are dimensionless prefactors that depend on a. Here we assume that three atoms are lost from the trap in the event of three-body recombination. The scattering length a is tuned by a magnetic field from $a > 0$ to $a < 0$ through resonance. The prefactors are given by

$$C_+(a) = 67.1e^{-2\eta_+}\{\cos^2[s_0 \ln(a/a_+)] + \sinh^2 \eta_+\} + 16.8(1 - e^{-4\eta_+}) \tag{2}$$

and

$$C_-(a) = \frac{4590 \sinh(2\eta_-)}{\sin^2[s_0 \ln(a/a_-)] + \sinh^2 \eta_-}, \tag{3}$$

respectively. On top of a strong a^4 scaling, L_3 shows, as a function of a, a series of resonances for $a < 0$ and minima for $a > 0$, and the locations of these Efimov features are determined by a_+ and a_-. The parameters $\eta_\pm$ are related to the decay of the trimers into atom-dimer pairs and provide a width to the Efimov features. Experimentally $a_\pm$ and $\eta_\pm$ are obtained by fitting Eqs. (2) and (3) to the measured L_3 spectrum as a function of a. For identical bosons $s_0 = 1.00624$, such that $C_\pm(a) = C_\pm(22.7a)$, and therefore a_+ and a_- are defined only within a factor 22.7^n, n being an integer. Universal theory requires a single 3BP and therefore the Efimov features for $a > 0$ and $a < 0$ are related, namely, via the relation $a_+/|a_-| = 0.96(3)$ [1]. A nonuniversal 3BP would manifest itself as random scatter of $|a_-|$ values in a range between 1 and 22.7 for different systems. However, the ratio $|a_-|/r_{\rm vdW}$ was found in a narrow range between 8 and 10 for experiments with different alkali-metal atoms [5,9,11,18], indicating a universal 3BP [24].

III. ANALYSIS OF THREE-BODY LOSS IN ^{4}He*

Recently we have measured the three-body loss rate coefficient in a ^{4}He* BEC, prepared in the high-field-seeking $m = -1$ Zeeman substate, and obtained the value $L_3 = 6.5(0.4)_{\rm stat}(0.6)_{\rm sys} \times 10^{-27}$ cm^6 s^{-1} [23]. For spin-polarized He* Penning ionization is strongly suppressed [25] and three-body loss dominates the lifetime of a ^{4}He* BEC. Scattering of spin-polarized He* is given by the $^5\Sigma_g^+$ potential, for which high-accuracy *ab initio* electronic structure calculations are available [26]. For ^{4}He* + ^{4}He* this potential supports 15 vibrational states. The highest excited vibrational state is weakly bound, which gives rise to a nearby potential resonance. Its binding energy is $h \times 91.35(6)$ MHz, measured by two-photon spectroscopy [27], from which a quintet scattering length of $141.96(9)a_0$ ($a_0 = 0.05292$ nm) was deduced, consistent with the *ab initio* theoretical value of $144(4)a_0$ [26]. It is indeed much larger than the range of the potential, as $r_{\rm vdW} = 35a_0$ [28], such that $a/r_{\rm vdW} = 4.1$. The binding energy of this weakly bound two-body state corresponds to 4.4 mK, which is much larger than the trap depth of about 10 μK and therefore both the formed dimer and the free atom leave the trap after three-body recombination. There are no broad Feshbach resonances in ^{4}He* because of the absence of nuclear spin [29].

We now consider Eq. (2) to find the set of a_+ and η_+ values that explains our observed value of L_3. Following the current convention, we present the 3BP in the form $|a_-|/r_{\rm vdW}$ by using the universal relation $a_+/|a_-| = 0.96$. In the alkali-metal experiments typically $\eta_+ \approx \eta_-$ and therefore in the following we will only use η. In Fig. 1 we show two sets of solutions of Eq. (2) that match our measured L_3 value, namely, $|a_-|/r_{\rm vdW} = 2.3$ (dashed lines) and 7.7 (solid lines), for different values of η. In both cases our data point is located far outside an Efimov minimum, giving rise to a weak dependence of η on L_3. That is the reason why our L_3 value, obtained for a single scattering length, provides information about a_-.

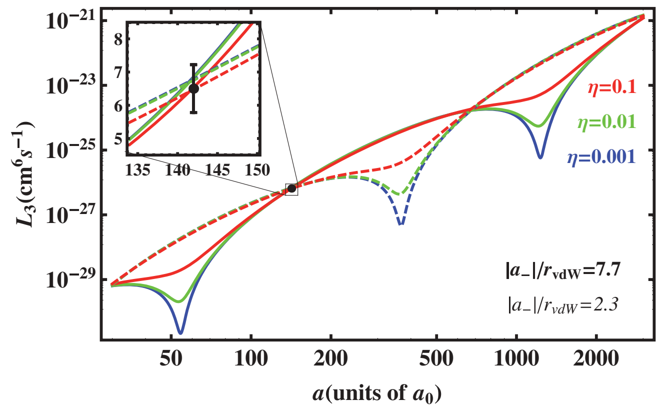

FIG. 1. (Color online) Universal three-body loss curves [Eq. (2)] for ^{4}He* with $|a_-|/r_{\rm vdW} = 2.3$ (dashed lines) and $|a_-|/r_{\rm vdW} = 7.7$ (solid lines), for different values of η, that match our measured L_3 value (see inset).

In Fig. 2 we show the set of solutions to Eq. (2) in $(|a_-|/r_{\rm vdW},\eta)$ parameter space for our value of L_3, represented by the black solid line, with the gray shaded area reflecting the experimental uncertainty in our measured L_3 value. Within the range of 1 to 22.7 for $|a_-|/r_{\rm vdW}$, we indeed find two narrow regions of $|a_-|/r_{\rm vdW}$ around 2 and 8, provided that η is not too large. For $\eta = 0.1$ we find $|a_-|/r_{\rm vdW} = 7.7(7)$ and 2.3(2). If η becomes larger than 0.5 the Efimov minima are washed out and their location becomes undefined, giving rise to a broad range of possible $|a_-|/r_{\rm vdW}$ values. For comparison, the 3BPs obtained from the different alkali-metal experiments are depicted by the colored symbols, with their numerical values

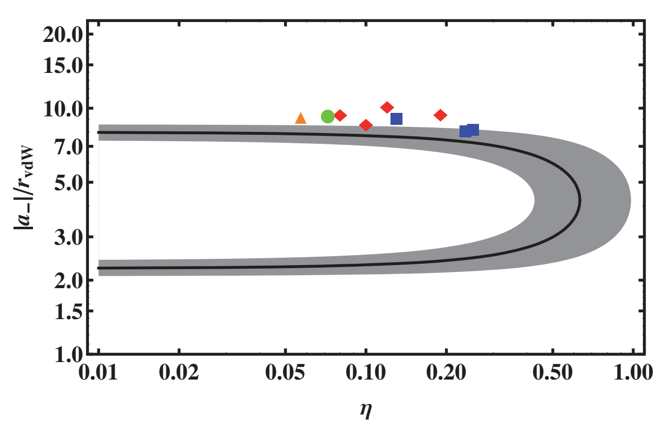

FIG. 2. (Color online) Graphic representation of the set of $|a_-|/r_{\rm vdW}$ and η values for which Eq. (2) matches our observed value of L_3, given by the black solid line, where the gray band corresponds to possible values based on our L_3 error bar. Also indicated are the $|a_-|/r_{\rm vdW}$ values for the alkali-metal experiments: Cs, 8.6(2),10.2(6),9.5(8),9.5(3) [11] (red diamonds), ^{7}Li 8.1(3) [5], 9.2(3) [6], 8.3(4) [7] (blue squares), ^{6}Li 9.3 [30] (green circle), ^{85}Rb 9.23(7) [9] (orange triangle), showing at the same time the observed η parameters.

given in the caption. We expect the value of η for ^{4}He* to be similar to those found in the alkali-metal systems, since Penning ionization will play no important role in the decay mechanism of the Efimov trimers. Figure 2 shows that our value is consistent with the 3BPs found in the alkali-metal system, considering the scatter shown in the available data and our uncertainty in L_3.

In our analysis we rely on two assumptions. The first assumption is that $a/r_{\mathrm{vdW}} = 4.1$ is sufficiently large for application of Eq. (2). Here we notice that the three-body loss data around a Feshbach resonance fit well for $|a|$ larger than a few r_{vdW}. Effects beyond universal theory [31–33] may be present, but are small enough not to alter our conclusion. The second assumption is that three atoms are lost for each three-body recombination event. For $a > 0$ additional resonances on top of the a^4 scaling have been observed in three-body loss spectra [6,8,34]. Those features are explained by secondary atom-dimer collisions that are resonantly enhanced near $a = a_*$, where a_* is the atom-dimer Efimov resonance position [1], which effectively leads to an enhancement of the number of atoms lost in a three-body recombination event. The precise underlying mechanism, and therefore what to extract from these additional resonances, is still under debate [35–37]. Here we note that if we take $|a_-|/r_{\mathrm{vdW}} = 8$, then $a_* = 300a_0$, which is far away from the actual value $142a_0$, such that secondary atom-dimer collisions are expected not to play a role for ^{4}He*.

IV. OTHER SYSTEMS

There are more atomic systems with a nearby potential resonance, for which a similar analysis as that performed for ^{4}He* can be done once a precise measurement of L_3 becomes available. Alkali-metal atoms prepared in a spin-stretched state (i.e., electron and nuclear spin maximally aligned) scatter only in the triplet potential. Therefore alkali metals with a large triplet scattering length provide the opportunity to extract the 3BP obtained from three-body loss in the presence of a potential resonance. Two candidates are ^{85}Rb [$a_T = -388(3)a_0$ [38], $r_{\mathrm{vdW}} = 82a_0$] and Cs [$a_T = 2440(24)a_0$ [39], $r_{\mathrm{vdW}} = 101a_0$]. An experimental challenge is to distinguish three-body loss from two-body loss processes, such as spin relaxation and hyperfine-changing collisions, especially in the case of Cs [40].

Another group of atoms that do not possess Feshbach resonances are the alkaline-earth-metal elements and Yb. In the electronic ground state the atoms have zero electron spin and therefore there is only a single two-body potential, which is of singlet character. Furthermore, the bosonic isotopes have zero nuclear spin and two-body loss processes are completely absent. An interesting example is Ca, for which potential resonances show up for all the bosonic isotopes [41]. In the following we will discuss two isotopes of Sr and Yb, for which a is accurately known, $a \gg r_{\mathrm{vdW}}$, and the first three-body loss measurements in BECs have already been reported.

For ^{86}Sr [$a = 798(12)a_0$ [42], $r_{\mathrm{vdW}} = 75a_0$], Stellmer *et al.* [43] report an upper limit of $L_3 = 6(3) \times 10^{-24}$ cm^6 s^{-1}, which is one order of magnitude larger than maximally allowed by Eq. (2). The authors indicate that secondary collisions, possibly enhanced by a resonance in the atom-dimer cross section, may explain this discrepancy. We note that if one tentatively assumes that the scattering length is indeed near the atom-dimer resonance, i.e., $a_* \approx 800a_0$, then $a_- \approx -750a_0$ and thus $|a_-|/r_{\mathrm{vdW}} \approx 10$. This is a hint that three-body loss in ^{86}Sr is consistent with the universal 3BP.

For ^{168}Yb [$a = 252(3)a_0$ [44], $r_{\mathrm{vdW}} = 78a_0$], Sugawa *et al.* [45] report an upper limit of $L_3 = 8.6(1.5) \times 10^{-28}$ cm^6 s^{-1}. If we perform a similar analysis as for ^{4}He* we find again two solutions of $|a_-|/r_{\mathrm{vdW}}$. Taking the upper limit, one of the two solutions lies in a narrow range between 8 and 9. Here a smaller L_3 leads to a larger $|a_-|/r_{\mathrm{vdW}}$, and a value between 10 and 11 is reached when the reported L_3 value is reduced by a factor of 2. This is a strong indication that three-body loss in ^{168}Yb is also consistent with the universal 3BP.

V. CONCLUSIONS

We find our measured L_3 coefficient in spin-polarized ^{4}He* to be consistent with the 3BP that was recently found in comparing measurements using alkali-metal atoms. We give further examples of atomic systems without a Feshbach resonance but in the presence of a nearby potential resonance for which the 3BP can be extracted from an accurately measured L_3, such as alkali-metal atoms in spin-stretched states and alkaline-earth-metal atoms. We find that the three-body loss measured in ^{168}Yb strongly indicates consistency with the universal 3BP.

We provide experimental evidence for a universal 3BP, outside the alkali-metal group and in the absence of a Feshbach resonance. A universal 3BP means that short-range three-body physics is not relevant for the Efimov spectrum. This implies that not only three-body observables in the universal regime are fully determined by two-body physics, but four-body [46–48] and N-body ($N > 4$) [49,50] observables as well.

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