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Branching ratio measurements of B_s decays

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We have just entered an era of precision measurements for B_s-decay observables. A characteristic feature of the B_s-meson system is B_s^0-$\bar{B}_s^0$ mixing, which exhibits a sizable decay width difference. The latter feature leads to a subtle complication for the extraction of branching ratios of B_s decays from untagged data samples, leading to systematic biases as large as $\mathcal{O}(10\%)$ that depend on the dynamics of the considered decay. We point out that this effect can only be corrected for using information from a time-dependent analysis and suggest the use of the effective B_s decay lifetime, which can already be extracted from the untagged data sample, for this purpose. We also address several experimental issues that can play a role in the extraction of effective lifetimes at a hadron collider, and advocate the use of the B_s branching ratios, as presented in this note, for consistent comparisons of theoretical calculations and experimental measurements in particle listings.

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I. INTRODUCTION

Weak decays of B_s mesons encode valuable information for the exploration of the standard model. The simplest observables are branching ratios, which give the probability of the considered decay to occur. Measurements of B_s branching ratios at hadron colliders, such as Fermilab's Tevatron and CERN's Large Hadron Collider (LHC), would require knowledge of the B_s production cross section, which presently makes absolute branching ratio measurements impossible. Hence, experimental control channels and the ratio of the $f_s/f_{u,d}$ fragmentation functions, describing the probability that a b quark hadronizes as a $\bar{B}_q$ meson [1], are required for the conversion of the observed number of decays into the branching ratio. At e^+e^- B factories operated at the $\Upsilon(5S)$ resonance, the total number of produced B_s mesons is measured separately and subsequently also allows for the extraction of the B_s branching ratio from the data [2].

A key feature of the B_s mesons is B_s^0-$\bar{B}_s^0$ mixing, which leads to quantum-mechanical, time-dependent oscillations between the B_s^0 and $\bar{B}_s^0$ states. In contrast to the B_d system, the B_s mesons exhibit a sizable difference between the decay widths of the light and heavy mass eigenstates, $\Gamma_{\rm L}^{(s)}$ and $\Gamma_{\rm H}^{(s)}$, respectively [3]. Currently the most precise measurement is extracted from the $B_s^0 \to J/\psi\phi$ channel by the LHCb Collaboration [4]:

$$y_s \equiv \frac{\Delta\Gamma_s}{2\Gamma_s} \equiv \frac{\Gamma_{\rm L}^{(s)} - \Gamma_{\rm H}^{(s)}}{2\Gamma_s} = 0.088 \pm 0.014; \qquad (1)$$

$\tau_{B_s}^{-1} \equiv \Gamma_s \equiv [\Gamma_{\rm L}^{(s)} + \Gamma_{\rm H}^{(s)}]/2 = (0.6580 \pm 0.0085)\ \text{ps}^{-1}$ is the inverse of the B_s mean lifetime τ_{B_s}.

In view of the sizable decay width difference, Eq. (1), special care has to be taken when dealing with the concept of a branching ratio. We shall clarify this issue and give an expression, allowing us to convert the experimentally measured B_s branching ratio into the corresponding "theoretical" branching ratio. The latter is not affected by B_s^0-$\bar{B}_s^0$ mixing and encodes the information for the comparison with branching ratios of B_d^0 decays, where the relative decay width difference at the 10^{-3} level [3] can be neglected, or branching ratios of B_u^+ modes.

The difference between these two branching ratio concepts involves y_s and is specific for the considered B_s decay, thereby involving nonperturbative parameters. However, measuring the effective lifetime of the considered B_s decay, the effect can be included in a clean way.

In experimental analyses, this subtle effect has so far been neglected or only been partially addressed; examples are the branching ratio measurements of the $B_s \to K^+K^-$ [5], $B_s \to J/\psi f_0(980)$ [6], $B_s \to J/\psi K_{\rm S}$ [7], $B_s \to D_s^+ D_s^-$ [8] and $B_s^0 \to D_s^- \pi^+$ [9] decays by the LHCb, CDF, D0 and Belle Collaborations.

II. EXPERIMENT VERSUS THEORY

What complicates the concept of a B_s branching ratio is the fact that the untagged decay rate is the sum of two exponentials [10]:

$$\begin{aligned}\langle\Gamma(B_s(t) \to f)\rangle &\equiv \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)\\ &= R_{\rm H}^f e^{-\Gamma_{\rm H}^{(s)} t} + R_{\rm L}^f e^{-\Gamma_{\rm L}^{(s)} t}, \qquad (2)\end{aligned}$$

corresponding to two mass eigenstates with different lifetimes. Using Eq. (1), we write

$$\begin{aligned}\langle\Gamma(B_s(t) \to f)\rangle &= (R_{\rm H}^f + R_{\rm L}^f) e^{-\Gamma_s t}\\ &\times \left[\cosh\left(\frac{y_s t}{\tau_{B_s}}\right) + \mathcal{A}_{\Delta\Gamma}^f \sinh\left(\frac{y_s t}{\tau_{B_s}}\right)\right], \qquad (3)\end{aligned}$$

where

$$\mathcal{A}_{\Delta\Gamma}^{f} \equiv \frac{R_{\mathrm{H}}^{f} - R_{\mathrm{L}}^{f}}{R_{\mathrm{H}}^{f} + R_{\mathrm{L}}^{f}} \tag{4}$$

is a final-state dependent observable.

In experiment, it is common practice to extract a branching ratio from the total event yield, ignoring information on the particles' lifetime. The "experimental" branching ratio can thus be defined as follows [10]:

$$\begin{aligned} \mathrm{BR}(B_s \to f)_{\mathrm{exp}} &\equiv \frac{1}{2}\int_0^\infty \langle \Gamma(B_s(t) \to f)\rangle dt = \frac{1}{2}\left[\frac{R_{\mathrm{H}}^f}{\Gamma_{\mathrm{H}}^{(s)}} + \frac{R_{\mathrm{L}}^f}{\Gamma_{\mathrm{L}}^{(s)}}\right] \\ &= \frac{\tau_{B_s}}{2}(R_{\mathrm{H}}^f + R_{\mathrm{L}}^f)\left[\frac{1 + \mathcal{A}_{\Delta\Gamma}^f y_s}{1 - y_s^2}\right]. \end{aligned} \tag{5}$$

Note that this quantity is the average of the branching ratios for the heavy and light mass eigenstates.

On the other hand, what is generally calculated theoretically are CP-averaged decay rates in the flavor-eigenstate basis, i.e.

$$\langle \Gamma(B_s(t) \to f)\rangle|_{t=0} = \Gamma(B_s^0 \to f) + \Gamma(\bar{B}_s^0 \to f). \tag{6}$$

This leads to the following definition of the theoretical branching ratio:

$$\begin{aligned} \mathrm{BR}(B_s \to f)_{\mathrm{theo}} &\equiv \frac{\tau_{B_s}}{2}\langle \Gamma(B_s^0(t) \to f)\rangle|_{t=0} \\ &= \frac{\tau_{B_s}}{2}(R_{\mathrm{H}}^f + R_{\mathrm{L}}^f). \end{aligned} \tag{7}$$

By considering $t = 0$, the effect of B_s^0-$\bar{B}_s^0$ mixing is "switched off". The advantage of this B_s branching ratio definition, which has been used, for instance, in Refs. [11,12], is that it allows a straightforward comparison with branching ratios of B_d^0 or B_u^+ mesons by means of the $SU(3)$ flavor symmetry of strong interactions.

The experimentally measurable branching ratio, Eq. (5), can be converted into the theoretical branching ratio defined by Eq. (7) through

$$\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}} = \left[\frac{1 - y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^f y_s}\right]\mathrm{BR}(B_s \to f)_{\mathrm{exp}}. \tag{8}$$

In the case of $y_s = 0$, the theoretical and experimental branching ratio definitions are equal.

Inspection of Eq. (8) reveals that y_s and $\mathcal{A}_{\Delta\Gamma}^f$ are required for the translation of the experimental branching ratios into their theoretical counterparts. Ideally, the latter quantities should eventually be used in particle compilations, in our opinion.

The decay width parameter y_s is universal and has already been measured, as summarized in Eq. (1). In Fig. 1, we illustrate Eq. (8) for a variety of values of $\mathcal{A}_{\Delta\Gamma}^f$ and observe that differences between $\mathrm{BR}(B_s \to f)_{\mathrm{theo}}$ and $\mathrm{BR}(B_s \to f)_{\mathrm{exp}}$ as large as $\mathcal{O}(10\%)$ may arise.

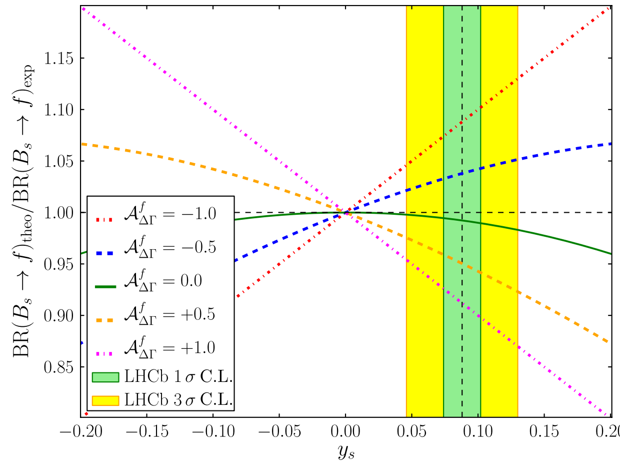

FIG. 1 (color online). Illustration of Eq. (8) for various values of $\mathcal{A}_{\Delta\Gamma}^f$. We also show the current LHCb measurement of y_s [4].

The simplest situation corresponds to flavor-specific decays such as $B_s^0 \to D_s^- \pi^+$, where $\mathcal{A}_{\Delta\Gamma}^{\mathrm{FS}} = 0$ and the correction factor is simply given by $1 - y_s^2$.

However, if both the B_s^0 and the $\bar{B}_s^0$ mesons can decay into the final state f, the observable $\mathcal{A}_{\Delta\Gamma}^f$ is more involved and depends, in general, on nonperturbative hadronic parameters, CP-violating weak decay phases and the B_s^0-$\bar{B}_s^0$ mixing phase ϕ_s. Assuming the standard model structure for the decay amplitudes and using the $SU(3)$ flavor symmetry to determine the hadronic parameters from relations to B_d decays, theoretical analyses of $\mathcal{A}_{\Delta\Gamma}^f$ were performed for the final states $J/\psi\phi$ [12], K^+K^- [13], $J/\psi f_0(980)$ [14], $J/\psi K_{\mathrm{S}}$ [15] and $D_s^+D_s^-$ [16].

III. USING LIFETIME INFORMATION

The simplest possibility for implementing Eq. (8) is to use theoretical information about the $\mathcal{A}_{\Delta\Gamma}^f$ observables. However, this input can be avoided once time information of the untagged B_s decay data sample becomes available. Then, the effective lifetime of the $B_s \to f$ decay can be determined, which is theoretically defined as the time expectation value of the untagged rate [17]:

$$\begin{aligned} \tau_f &\equiv \frac{\int_0^\infty t\langle \Gamma(B_s(t) \to f)\rangle dt}{\int_0^\infty \langle \Gamma(B_s(t) \to f)\rangle dt} \\ &= \frac{\tau_{B_s}}{1 - y_s^2}\left[\frac{1 + 2\mathcal{A}_{\Delta\Gamma}^f y_s + y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^f y_s}\right]. \end{aligned} \tag{9}$$

The advantage of τ_f is that it allows an efficient extraction of the product of $\mathcal{A}_{\Delta\Gamma}^f$ and y_s. Using the effective lifetime, Eq. (8) can be expressed as

$$\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}} = \left[2 - (1 - y_s^2)\frac{\tau_f}{\tau_{B_s}}\right]\mathrm{BR}(B_s \to f)_{\mathrm{exp}}. \tag{10}$$

TABLE I. Factors for converting $\mathrm{BR}(B_s \to f)_{\mathrm{exp}}$ [see Eq. (5)] into $\mathrm{BR}(B_s \to f)_{\mathrm{theo}}$ [see Eq. (7)] by means of Eq. (8) with theoretical estimates for $\mathcal{A}_{\Delta\Gamma}^{f}$. Whenever effective lifetime information is available, the corrections are also calculated using Eq. (10).

$B_s \to f$	$\mathrm{BR}(B_s \to f)_{\mathrm{exp}}$ (measured)	$\mathcal{A}_{\Delta\Gamma}^{f}$(SM)	$\mathrm{BR}(B_s \to f)_{\mathrm{theo}}/\mathrm{BR}(B_s \to f)_{\mathrm{exp}}$ From Eq. (8)	From Eq. (10)
$J/\psi f_0(980)$	$(1.29^{+0.40}_{-0.28}) \times 10^{-4}$ [18]	0.9984 ± 0.0021 [14]	0.912 ± 0.014	0.890 ± 0.082 [6]
$J/\psi K_S$	$(3.5 \pm 0.8) \times 10^{-5}$ [7]	0.84 ± 0.17 [15]	0.924 ± 0.018	N/A
$D_s^- \pi^+$	$(3.01 \pm 0.34) \times 10^{-3}$ [9]	0 (exact)	0.992 ± 0.003	N/A
K^+K^-	$(3.5 \pm 0.7) \times 10^{-5}$ [18]	-0.972 ± 0.012 [13]	1.085 ± 0.014	1.042 ± 0.033 [19]
$D_s^+ D_s^-$	$(1.04^{+0.29}_{-0.26}) \times 10^{-2}$ [18]	-0.995 ± 0.013 [16]	1.088 ± 0.014	N/A

Note that on the right-hand side of this equation, only measurable quantities appear and that the decay width difference y_s enters at second order. The measurement of effective lifetimes is hence not only an interesting topic for obtaining constraints on the B_s^0-$\bar{B}_s^0$ mixing parameters [17], but an integral part of the determination of the theoretical B_s branching ratios from the data.

In Table I, we list the correction factors for converting the experimentally measured branching ratios into the theoretical branching ratios for various decays. Here, we have used theoretical information for $\mathcal{A}_{\Delta\Gamma}^{f}$ and Eq. (8), or—if available—the effective decay lifetimes and Eq. (10).

The rare decay $B_s^0 \to \mu^+\mu^-$, which is very sensitive to new physics [20], is also affected by $\Delta\Gamma_s$. In Ref. [21], we give a detailed discussion of this key B_s decay, showing that the helicities of the muons need not be measured to deal with this problem, and that $\Delta\Gamma_s$ actually offers a new window for new physics in $B_s^0 \to \mu^+\mu^-$.

IV. $B_s \to VV$ DECAYS

Another application is given by B_s transitions into two vector mesons, such as $B_s \to J/\psi\phi$ [22], $B_s \to K^{*0}\bar{K}^{*0}$ [23] and $B_s \to D_s^{*+}D_s^{*-}$ [8]. Here, an angular analysis of the decay products of the vector mesons has to be performed to disentangle the CP-even and CP-odd final states, which affects the branching fraction determination in a subtle way, as recognized in Refs. [23,24]. Using linear polarization states 0, $\|$ with CP eigenvalue $\eta_k = +1$ and $\perp$ with CP eigenvalue $\eta_k = -1$ [25], the generalization of Eq. (8) is given by

$$\mathrm{BR}_{\mathrm{theo}}^{VV} = (1 - y_s^2)\left[\sum_{k=0,\|,\perp} \frac{f_{VV,k}^{\mathrm{exp}}}{1 + y_s \mathcal{A}_{\Delta\Gamma}^{VV,k}}\right]\mathrm{BR}_{\mathrm{exp}}^{VV}, \quad (11)$$

where $f_{VV,k}^{\mathrm{exp}} = \mathrm{BR}_{\mathrm{exp}}^{VV,k}/\mathrm{BR}_{\mathrm{exp}}^{VV}$ and $\mathrm{BR}_{\mathrm{exp}}^{VV} \equiv \sum_k \mathrm{BR}_{\mathrm{exp}}^{VV,k}$ so that $\sum_k f_{VV,k}^{\mathrm{exp}} = 1$. As discussed in Ref. [17], assuming the standard model structure for the decay amplitudes, we can write

$$\mathcal{A}_{\Delta\Gamma}^{VV,k} = -\eta_k \sqrt{1 - C_{VV,k}^2}\cos(\phi_s + \Delta\phi_{VV,k}), \quad (12)$$

where $C_{VV,k}$ describes direct CP violation, ϕ_s is the B_s^0-$\bar{B}_s^0$ mixing phase and $\Delta\phi_{VV,k}$ is a nonperturbative hadronic phase shift. The expressions given in Ref. [23] for the $B_s \to K^{*0}\bar{K}^{*0}$ decay take the leading order effect of y_s into account and assume $\phi_s = 0$ and negligible hadronic corrections.

The generalization of Eq. (10) is given by

$$\mathrm{BR}_{\mathrm{theo}}^{VV} = \mathrm{BR}_{\mathrm{exp}}^{VV} \sum_{k=0,\|,\perp}\left[2 - (1 - y_s^2)\frac{\tau_k^{VV}}{\tau_{B_s}}\right] f_{VV,k}^{\mathrm{exp}} \quad (13)$$

and does not require knowledge of the $\mathcal{A}_{\Delta\Gamma}^{VV,k}$ observables.

V. EXPERIMENTAL ASPECTS

Additional subtleties arise in the experimental determination of B_s branching ratios and effective lifetimes, in particular, at a hadron collider environment where many final-state particles are produced in the fragmentation.

Separating B_s signal decays from the background typically involves selection criteria which use the flight distance of the B_s meson or the impact parameter of its decay products, leading to a decay-time dependent efficiency. By rejecting short-living B_s meson candidates, the relative amounts of $B_{s,\mathrm{L}}$ and $B_{s,\mathrm{H}}$ mesons in the remaining data sample are altered, resulting in a biased result for the branching ratio determination. The extrapolation of the event yield to full acceptance is usually obtained from simulation, but this requires *a priori* assumptions of the values for y_s and $\mathcal{A}_{\Delta\Gamma}^{f}$. For example, the dependence of the branching fraction correction on the value $\mathcal{A}_{\Delta\Gamma}^{f}$ can be several percent if only decay times greater than 0.5 ps are considered. This systematic uncertainty is avoided by tuning the simulation using the measured value of the effective lifetime.

Furthermore, the presence of remaining background events with a different observed decay time distribution as the signal implies that it is experimentally unpractical to determine the time expectation value τ_f of the untagged rate as given in Eq. (9). Instead, the effective lifetime is commonly extracted by fitting a single exponential function to the untagged rate [6,19,26], which in general is described by two exponentials [see Eq. (2)]. In the Appendix, we demonstrate that such a fitting procedure leads to an unbiased determination of the effective lifetime in the case of a log likelihood fit and to a small bias for a χ^2 minimization procedure.

VI. CONCLUSIONS

The established width difference of the B_s mesons complicates the extraction of branching ratio information from the experimental data, leading to biases at the 10% level which depend on the specific final state. On the one hand, these effects can be included through theoretical considerations and phenomenological analyses. On the other hand, it is also possible to take them into account through the measurement of the effective $B_s \to f$ decay lifetimes, which is the preferred avenue. So far, these effects have not, or have only partially, been included, and we advocate to use the converted branching ratios for comparisons with theoretical calculations in particle listings.

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APPENDIX: EFFECTIVE LIFETIME FITS

An *effective lifetime* for a B_s decay channel is obtained in practice by fitting a single exponential function to its untagged rate. As an untagged rate is in general described by two exponentials, corresponding to two mass eigenstates with different lifetimes, the single exponential fit is an approximation.

In order to find analytic expressions for the fitted effective lifetime $\tau_{\rm eff}$, we let the untagged rate be the *true* probability distribution function, and the single exponent function the *fitted* probability distribution function, such that

$$f_{\rm true}(t) \equiv \frac{A(t)\langle\Gamma(t)\rangle}{\int_0^\infty A(t)\langle\Gamma(t)\rangle dt}, \tag{A1}$$

$$f_{\rm fit}(t;\tau_{\rm eff}) \equiv \frac{A(t)e^{-t/\tau_{\rm eff}}}{\int_0^\infty A(t)e^{-t/\tau_{\rm eff}}dt}, \tag{A2}$$

where $A(t)$ is an acceptance efficiency function. The likelihood or χ^2 function for the fit in question is then built using the above probability distribution functions, and maximized or minimized, respectively, in the limit of infinitesimally spaced bins. Specifically, for n events, we minimize the functions:

$$-\log L(\tau_{\rm eff}) = -n\int_0^\infty f_{\rm true}(t)\log[f_{\rm fit}(t;\tau_{\rm eff})]dt, \tag{A3}$$

$$\chi^2(\tau_{\rm eff}) = n\int_0^\infty \frac{[f_{\rm true}(t) - f_{\rm fit}(t;\tau_{\rm eff})]^2}{f_{\rm fit}(t;\tau_{\rm eff})}dt, \tag{A4}$$

for a maximum likelihood and a least-squares fit, respectively. In a modified least-squares fit, where data is used to estimate the error, the denominator in the χ^2 integrand should be replaced by $f_{\rm true}(t)$. For the maximum likelihood fit, taking the infinitesimal bin limit is equivalent to an unbinned fit.

The effective lifetime $\tau_{\rm eff}$ resulting from these fits is then given implicitly by the formula

$$\frac{\int_0^\infty te^{-t/\tau_{\rm eff}}A(t)dt}{\int_0^\infty e^{-t/\tau_{\rm eff}}A(t)dt} = \frac{\int_0^\infty tg(t;\tau_{\rm eff})A(t)dt}{\int_0^\infty g(t;\tau_{\rm eff})A(t)dt}, \tag{A5}$$

where

$$g(t;\tau_{\rm eff}) \equiv \begin{cases} \langle\Gamma(t)\rangle & \text{: maximum likelihood} \\ \langle\Gamma(t)\rangle^2 e^{t/\tau_{\rm eff}} & \text{: least squares} \\ \langle\Gamma(t)\rangle^{-1} e^{-2t/\tau_{\rm eff}} & \text{: modified least squares.} \end{cases}$$

The effective lifetime definition given in Eq. (9) is reproduced for the untagged rate given in Eq. (3) if we assume a trivial acceptance function, $A(t) = 1$, and apply a maximum-likelihood fit. For nonzero values of y_s, the least-squares fits give different analytic expressions for the effective lifetime. Fortunately, for the current experimental range of y_s, the differences are of the order 0.1%.

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