The influence of previously seen objects' sizes in distance judgments

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An object's retinal image size is determined by a combination of its physical size and its distance, so judgments of an object's size and distance from its retinal image size are coupled. Since one does not have direct access to information about the object's physical size, people may make assumptions about how large it is likely to be. Here we investigated whether the sizes of similar, previously encountered objects influence the assumptions about the physical size of an object and therefore the interpretation of its retinal image size in terms of its distance. Subjects moved their unseen index finger to the positions of binocular simulations of red cubes. For identical target cubes at the same position, they indicated a nearer position of the cube when the preceding cube was small than when it was big. This is in agreement with a tendency to expect the cube to be the same size as that on the previous trial. However, if the expectation were simply adjusted slightly on each trial, the cube would be judged to be nearer when preceded by two consecutive smaller cubes than when preceded by only one smaller cube. It was not, so there must be a more direct influence of the size in the previous trial on distance judgments.

Introduction

The visual system uses many cues to estimate distances. One of these cues is retinal image size. The relationship between object size, retinal image size, and distance is illustrated in Figure 1. From this figure it is easy to see why distance and size are perceptually coupled (Gillam, 1995; Van Damme & Brenner, 1997). An object of a certain size at a certain distance will produce a certain retinal image size, so if the object's size is known, its retinal image size can be used to estimate its distance (Sedgwick, 1986).

In order to use the retinal image size to judge distance, one must have some idea about the size of the object. If one is not familiar with the object, one will be uncertain about the object's size as well as its distance, so it is not immediately evident how retinal image size could contribute to one's judgments. Nevertheless, it has been shown that even when the size of the object is not known, its image size does contribute to judgments of its distance (Collet, Schwarz, & Sobel, 1991; Sousa, Brenner, & Smeets, 2011a, 2011b): People judge smaller objects to be further away than bigger ones. This suggests that people make assumptions about the size of the object. If the object size is assumed to be smaller than it really is, the same retinal image will be attributed to a nearer object (Figure 1). Similarly, if the object is assumed to be larger than it really is, the image will be attributed to an object that is further away. The converse is also true—that if the object is assumed to be nearer, it will be perceived to be smaller—but in this paper we will only consider how the assumed size influences the estimated distance.

Where do people's assumptions about the size of an object come from? People must use prior experience to build such assumptions. They could take their experience with a wide range of objects that vaguely resemble the present one into account, or only their experience with objects that are very similar to the present one. In

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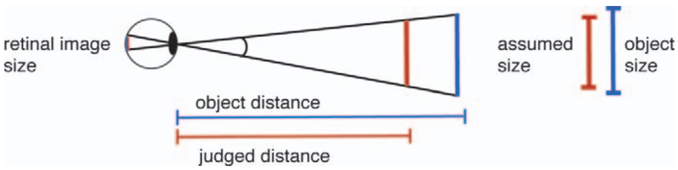

Figure 1. The relationship between retinal image size, (assumed) object size, and (judged) distance.

the latter case, if people are placed in an experiment in which similar objects are presented during successive trials, we expect them to consider the perceived sizes on previous trials to get a better estimate of the size than they could obtain by only using the directly available information. If previously experienced sizes contribute to the estimated size, and retinal image size contributes to judging the distance, previously encountered sizes will influence the judged distance.

How are the previously encountered sizes used? If the target object looks similar to a previously presented object, its size can be assumed to be similar to the *perceived* size of the previous one. If the perceived size of the previous object was partly based on assumptions about the size, and therefore recursively on the perceived size of objects that were presented before, the assumed size for the target will be based on a range of presented objects. Thus, the previously experienced sizes can constitute a likelihood distribution of possible sizes: a size prior that is given a weight that corresponds with its reliability (Mamassian & Landy, 2001) and of which the position of the peak shifts with experience (Adams, Graf, & Ernst, 2004).

The reliability of the size prior (i.e., the width of the range of likely sizes, which is a measure of how confident one can be about the object's size on the basis of previous experience) will influence the reliability of retinal image size as a cue for distance. Shifting the prior (i.e., shifting one's notion of the likely sizes) will influence the assumed size and therefore the distance judged from retinal image size. If one assumes that the prior is based on *recent* experience, one can make two predictions: that the variability of recently perceived sizes will influence the reliability of the size prior and consequently the weight given to image size when judging distance, and that recently experienced sizes will influence the position of the peak of the prior and consequently the interpretation of image size in terms of judged distance.

We recently showed that retinal image size is indeed given less weight for judging distances when the variability in recently experienced sizes is large: Subjects halved the weight given to size as a cue for distance when objects of a wide range of sizes were presented rather than only objects of a narrow range of sizes (Sousa et al., 2011b). In the present study we tested the second prediction: Do the most recently experienced sizes influence the judged distance by shifting the assumed object size? In other words, do people assume that an object that they see is smaller if they have just seen a similar object that was small? To find out we examined whether distance judgments are influenced by the object's simulated size on the previous trial (and on the trial before that).

We asked subjects to indicate the distance of virtual cubes by pointing at them in total darkness. We compared distance judgments for cubes at the same positions when preceded by a smaller or bigger cube. If the assumed object size (the prior) is shifted to a smaller value after seeing a small cube, the indicated distance should be nearer after seeing a small cube. Moreover, if the size prior is constantly updated on the basis of recent experience, having a small cube on two consecutive preceding trials should shift the assumed object size to a smaller value than having a small cube on only one preceding trial. Therefore, the indicated distance for a certain cube when preceded by two small cubes should be nearer than the indicated distance for the same cube when preceded by one small cube.

Methods

Subjects

Eight subjects took part in the experiment. None of them knew the purpose of the study and all of them had normal binocular vision.

Apparatus

We used the same set-up as in our previous studies (Sousa et al., 2011a, 2011b), with mirrors that reflect

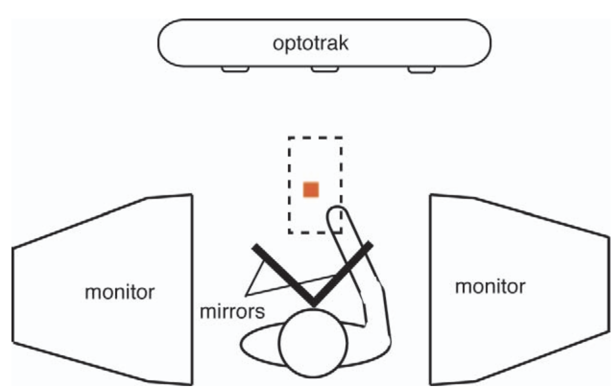

Figure 2. Schematic representation of a top view of the setup. The mirrors reflect the monitors' images so that virtual stimuli are presented in the area indicated by the dashed rectangle.

the images from two CRT monitors (1096 × 686 pixels, 47.3 × 30 cm) to the two eyes to produce simulations of three-dimensional (3-D) objects (see Figure 2). New images were created for each eye with the frequency of the refresh rate of the monitors (160 Hz). The 3-D positions of the subject's head and right index finger were recorded at 250 Hz using Infra-red Emitting Diodes (IREDs) and an Optotrak 3020 system (Northern Digital, Inc., Waterloo, Ontario, Canada).

One IRED was attached to the nail of the subject's right index finger and three others to a mouthpiece with a dental imprint. The positions of the subject's eyes relative to the mouthpiece were determined in advance. The measured position and orientation of the mouthpiece was used to adapt the images to the eyes' changing positions. This was necessary because subjects were allowed to move their head freely during the experiments (although they could not move very far since they had to look into the mirrors). The calibration procedure is described in detail elsewhere (Sousa, Brenner, & Smeets, 2010).

Stimuli

On each trial, the stimulus was a single cube (size 1, 1.5, or 2 cm) presented in total darkness. The cube's simulated surfaces had Lambertian reflectance with half the simulated illumination being ambient and the other half being from a distant light source above and 30° to the left of the subject. The cube was presented at pseudorandom positions in a volume of space of 8 × 8 × 20 cm (width × height × depth) with the depth direction being slanted 30° downwards so that the subjects pointed at a comfortable height. The ocular convergence that was required to fixate the cube, the motion parallax when the subject moved his or her head, and the relative disparity between the edges within the cubes were all consistent with the simulated distance. Positioning the cubes in the abovementioned volume of space meant that the range of possible heights and lateral positions in the visual field was larger for nearby objects, but more distant objects were not systematically higher in the visual field or further to one side.

Conditions

There were two conditions (see Figure 3). One condition in which trials with 1.5-cm cubes were alternated with trials with a 1- or 2-cm cube (*single condition*) and one condition in which each trial with a 1.5-cm cube was preceded by a pair of trials with 1- or 2-cm cubes (*double condition*). In the single condition there were 100 possible positions, 50 for the 1.5-cm cube and 50 for the 1- and 2-cm cubes. The 1.5-cm cube

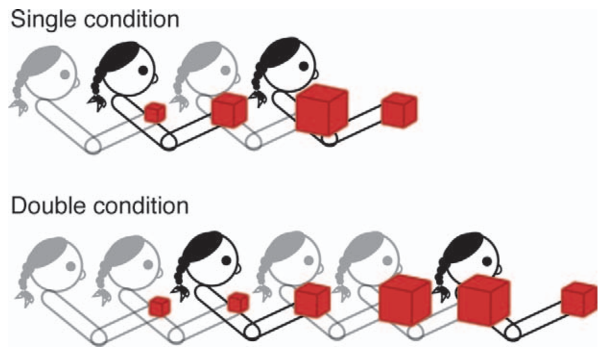

Figure 3. Schematic representation of consecutive trials for each of the two conditions. In the *single condition*, a 1 or 2 cm cube (of which the position is here indicated by a gray subject) preceded each 1.5-cm cube (indicated by a black subject). In the *double condition*, a pair of 1 or 2 cm cubes preceded each 1.5-cm cube.

was presented twice at each position, once preceded by a 1-cm cube and once preceded by a 2-cm cube. The preceding 1-cm cubes were also presented at the same positions as the preceding 2-cm cubes (which was a different position than that of the 1.5-cm cube). Altogether, the *single condition* had 200 trials that were presented in one session. The *double condition* was very similar to the single condition except that two 1- or 2-cm cubes were presented before each 1.5-cm cube. The same positions were used for the first of the two as for the one directly preceding the 1.5-cm cube (so each cube appeared twice at each of these positions), but the first positions were presented in a different random order so the two consecutive cubes of the same size were not also at the same distance. The *double condition* was presented in two sessions. In each session there were 26 positions for each 1.5-cm cube, so there were 156 trials per session and in total 52 positions for the 1.5-cm cube. In both conditions, the order of the positions of the 1.5-cm cubes and the order of the cube sizes between the trials with 1.5-cm cubes was random. Note that although we only used three simulated cube sizes, they were at many simulated distances, so our subjects could not identify the stimulus set (Keefe & Watt, 2009).

Procedure

Subjects started each trial with their hand near their body. They were instructed to move their unseen index finger to the center of the cube that appeared. The pointing movement was considered to have ended if the hand had moved less than 1 mm in 300 ms and was within 30 cm of the center of the volume of possible cube positions. At that moment the finger position was saved (as was that of the eyes) and the cube

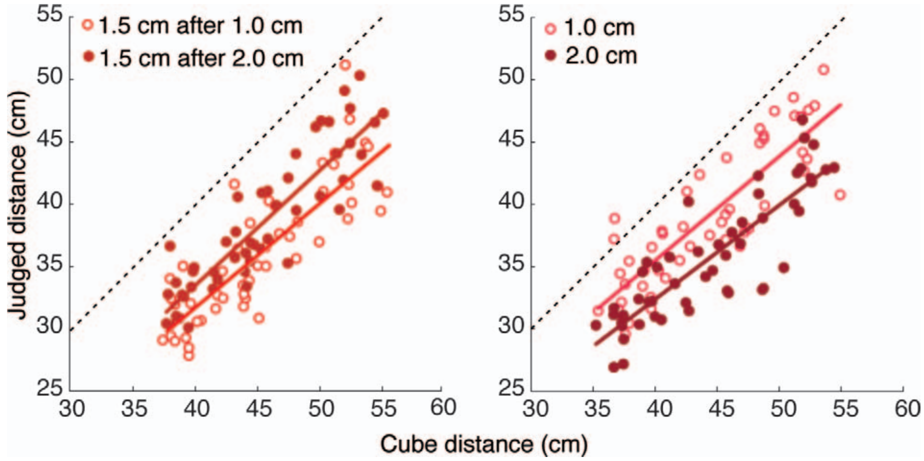

Figure 4. Judged distance as a function of simulated cube distance for one subject in the *single condition*. Each dot is a trial. Left panel: influence of the previous cube's size. Right panel: influence of the current cube's size. This subject is represented by the red symbols in Figures 5 and 6.

disappeared. The next cube only appeared after the subject had brought the hand back near the body.

Analysis

We defined the judged distance on each trial as the distance between the pointed location and the point halfway between the eyes. We calculated the difference in judged distance between cubes that were presented at the same positions. As mentioned before, the 1.5-cm cubes were presented twice at 50 different positions, once preceded by a 1-cm cube and once preceded by a 2-cm cube. For the *single* condition we subtracted the judged distance for the 1.5-cm cube preceded by a 1-cm cube from the judged distance for the matched 1.5-cm cube preceded by a 2-cm cube. For each subject we then averaged these values and determined the associated standard error. This gave us an estimate of the influence of the cube size on the previous trial. We also subtracted the judged distances for the 2-cm cubes from those for the 1-cm cubes at the same positions, which gave us an estimate of the influence of the current size.

We used these estimates from the *single condition* to make a prediction for the same measure in the *double condition*. In this prediction it was assumed that after every trial the size prior is updated in the direction of the size in that trial (Equation 3 of the Appendix). The rate at which size is updated was estimated from the average influences of the current and previous object size on the pointing distance in the *single condition* (Equation 9 of the Appendix). If consecutive trials of the same size are presented, and the assumed size is updated in the manner described above, the difference in pointing for the 1.5-cm cubes preceded by a pair of either 1-cm cubes or 2-cm cubes should be bigger than the difference in pointing for the 1.5-cm cubes preceded by only one 1-cm cube or one 2-cm cube. A quantitative prediction for the effect of having two preceding small or large cubes is given in Equation 15 of the Appendix. We checked whether the influence of the preceding cubes' sizes on subjects' average judged distances in the *single* condition was significantly different from that in the *double condition* with a paired t test. A similar test was used to examine whether judged distance in the *double condition* was significantly different from the prediction.

For each subject in the *double condition*, we subtracted the judged distance for the 1-cm cube preceded by another 1-cm cube (i.e., the second cube of each pair of 1-cm cubes) from the judged distance for the 2-cm cube preceded by another 2-cm cube (second cube of each pair of 2-cm cubes) that was presented at the same position. We then averaged these values and determined the associated standard error. The difference between judged distances for the 1- and 2-cm cubes preceded by a 1.5-cm cube (current size) should be larger than the difference between the judged distances for the 1- and 2-cm cubes preceded by another 1- or 2-cm cube (repeating current size), because when the second cube of the same size is presented, the assumed size will already have shifted slightly towards that size. The predicted difference in pointing distance is given in Equation 12 of the Appendix. We checked whether the effect of repeating the current size on judged distance was significantly different from the prediction with a paired t test.

Results

We found a clear effect of the size of the previous cube on judged distance. For example, Figure 4 shows

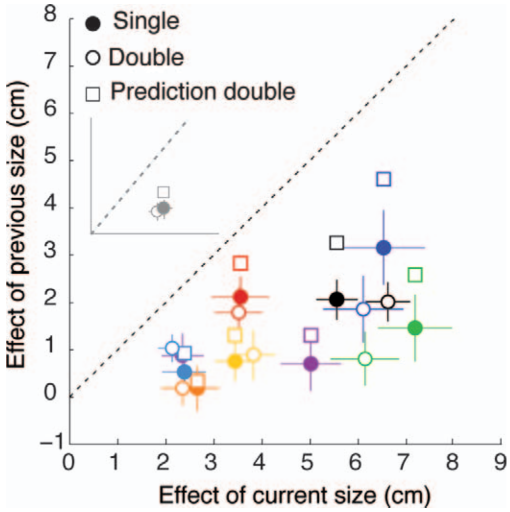

Figure 5. The difference between the judged distances of 1.5-cm cubes preceded by a single (filled dots) or pair of (open dots) 1 and 2 cm cubes, as a function of the difference between the judged distances of 1 and 2 cm cubes preceded by a 1.5-cm cube. Each dot is the data for one subject (average with standard errors). Different symbols with the same color represent the same subject. The open squares represent the predictions for the double condition (open dots) based on Equation 15 of the Appendix and the data of the single condition (filled dots). The inset shows the average results and prediction.

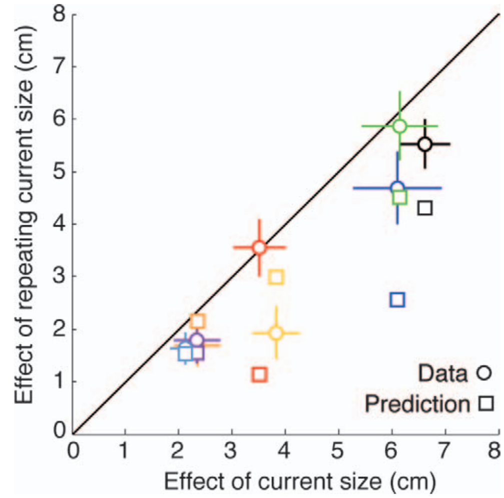

Figure 6. The difference between the judged distances of 1 and 2 cm cubes preceded by another cube of the same size (repeating current size) as a function of the difference between the judged distances of 1 and 2 cm cubes preceded by a 1.5-cm cube (current size). Each dot shows a subject's average values with standard errors. Color coding as in Figure 5. The squares represent predictions based on Equation 12 of the Appendix.

how one subject judged the 1.5-cm cube to be further away when preceded by a 2-cm cube than when preceded by a 1-cm cube. This pattern was present for all subjects (all vertical values are larger than zero in Figure 5). In the single condition, the effect of the size in the previous trial is smaller than the effect of the size in the current trial (smaller difference between the lines in the left plot than in the right plot of Figure 4; dots below diagonal in Figure 5).

In Figure 5, the filled dots are the data for the *single condition* and the open dots are the data for the *double condition*. The positive values for the effect of the previous size show that the size on the previous trial influences distance judgments, which is consistent with shifting a size prior on the basis of recent experience. Equation 15 of the Appendix was used to predict how much larger the effect would be if there were two preceding 1- or 2-cm cubes, rather than only one, when the 1.5-cm cube was presented (open squares). The measured values do not fit the prediction. The influence of the previous trial for the 1.5-cm cubes should have been bigger in the *double condition* than in the *single condition*. That is not the case. The vertical positions of the open and filled dots do not differ significantly ($p = 0.37$). The open dots' vertical positions are significantly different ($p = 0.02$) from those of the open squares, showing that the reasoning behind Equation 15 does not hold. No difference was expected between the *current size* effects in the two conditions (horizontal positions of the open and filled dots), because these are always the large or small cubes that are preceded by a 1.5-cm cube, and indeed no systematic difference is found ($p = 0.32$). The similarity of the effect of current size for the two conditions demonstrates that the differences between subjects were consistent across sessions.

Figure 6 shows that the difference between the judged distances for the 1-cm and 2-cm cubes preceded by a 1.5-cm cube (current size) is bigger than the difference between the judged distances for the 1- and 2-cm cubes preceded by another cube of the same size (repeating current size): The dots are below the line. This means that the subjects pointed nearer for the 2-cm cube when preceded by a 1.5-cm cube than when preceded by another 2-cm cube and pointed further for the 1-cm cubes when preceded by a 1.5-cm cube than when preceded by another 1-cm cube. This finding is consistent with the smaller preceding cube size, making subjects assume that the cube was smaller and therefore nearer (Figure 1). In three cases, the prediction fell outside the 95% confidence interval for the data. Although in most cases the effect of repeating the current size wasn't reduced as predicted, there is no systematic difference between the data and the prediction ($p = 0.13$).

Discussion

The object's size in the previous trial influences distance judgments. Subjects pointed nearer for the 1.5-cm cube when it was preceded by a smaller object (1-cm cube) than when it was preceded by a bigger object (2-cm cube). This is consistent with a size prior being shifted to a smaller value after the presentation of a smaller object and to a bigger value after the presentation of a bigger object. However, with such a mechanism the size prior should be shifted to an even smaller value after the presentation of two consecutive smaller objects, but when the 1.5-cm cube was preceded by two large objects, it was not judged as being further than when it was preceded by only one large object (open dots are not systematically above filled dots in Figure 5). This implies that if the effect is due to changes in the assumed size prior, the size prior does not depend on the size in previous trials as proposed in Equation 3. Rather, there seems to be a particularly strong effect of the immediately preceding trial. One possibility is that the effect is not due to a changing size prior, but that there is a direct effect of the difference between subsequent retinal image sizes. Judging the retinal image size to be larger rather than the physical object size to be smaller will have the same effect on the judged distance, but if the effect on judged retinal image size is only based on the contrast between the current and previously seen retinal image size, the effect will not accumulate across trials as shifts in a modifiable size prior would do.

We knew that the variability in presented object sizes influences the extent to which people rely on a size prior to help make distance judgments (Sousa et al., 2011a). Here we show that the simulated object size in the previous trial influences distance judgments. We considered that this arises from shifting a prior: changing the assumed object size. Priors can be shifted with experience (Adams et al., 2004), and different priors shift at different rates (Flanagan, Bittner, & Johansson, 2008). However, a size prior that is shifted by a proportion of the new size on every trial cannot account for our results. Instead, there may be a contrast effect between retinal images sizes that makes an object be perceived to be bigger if the previous retinal image size was smaller and vice-versa. Of course, a size prior may also contribute to the influence of preceding objects' sizes on judged distance, because assumptions about the size of the object are required for retinal image size to be used as a distance cue, and our results certainly do show that this cue is used. However this mechanism alone cannot account for our data. Thus, probably there is both a (fast) contrast effect on the perceived object size and a more stable prior for object size.

Keywords: depth perception, size, binocular vision, distance, pointing

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Appendix

We assume that the pointing distance (d_p) is related linearly to a weighted average of various cues to distance, d_i, one of which (d_s) is based on retinal image size (others could include vergence and accommodation):

$$d_p = a(w_s d_s + w_v d_v + w_a d_a + \cdots) + b \tag{1}$$

The distance indicated by the retinal image size cue on trial t depends on the object's distance on that trial (d_t) and on the relationship between the object's size (s_t) and its assumed size ($\hat{s}_t$) on that trial:

$$d_s = \frac{\hat{s}_t}{s_t} d_t \tag{2}$$

Our hypothesis in the present study is that the sizes encountered on previous trials influence the assumed size. A straightforward method to model this is by updating the assumed size on the basis of the encountered size by a fraction u on each trial:

$$\hat{s}_t = (1-u)\hat{s}_{t-1} + u s_t \tag{3}$$

On the basis of these equations, and a few approximations, we can make and test two predictions for our experiment. The first is about the effect of presenting two rather than one cube of a different size (**X**; which is either 1 or 2 cm) before the standard cube (**Y**; 1.5 cm). Since we will test the prediction on the basis of average values, our first approximation is to assume that when the first cube of a different size is presented (at $t = 1$), the former assumed size ($\hat{s}_0$) is **Y**, because this is both the size at that moment and the average across preceding trials each step back in time. Thus, the assumed size when the first cube of a different size is presented is:

$$\hat{s}_1 = (1-u)\mathbf{Y} + u\mathbf{X} \tag{4}$$

Similarly, when returning to the standard cube after that, the assumed size will be:

$$\hat{s}_2 = (1-u)\hat{s}_1 + u\mathbf{Y} = (1-u+u^2)\mathbf{Y} + (u-u^2)\mathbf{X} \tag{5}$$

So, if we look at the difference between the expected pointing distance for a 1 and a 2-cm cube ($\Delta_{1\mathrm{C}}$; effect of one current target of a different size), assuming that all other cues are identical in both cases and that all the distances are the same (as they are in our experiment), we can combine Equations 1 and 2 to get:

$$\begin{aligned}\Delta_{1\mathrm{C}} &= d_{p;X=1} - d_{p;X=2} = a w_s (d_{s;X=1} - d_{s;X=2}) \\ &= a w_s d_1 \left(\frac{\hat{s}_{1;X=1}}{s_{1;X=1}} - \frac{\hat{s}_{1;X=2}}{s_{1;X=2}}\right)\end{aligned} \tag{6}$$

which can be combined with Equation 4, filling in the actual sizes, to give:

$$\begin{aligned}\Delta_{1\mathrm{C}} &= a w_s d_1 \left(\frac{(1-u)1.5+u}{1} - \frac{(1-u)1.5+2u}{2}\right) \\ &= \frac{3}{4} a w_s d_1 (1-u)\end{aligned} \tag{7}$$

Similarly, for pointing at 1.5-cm cubes after being shown a 1 or a 2-cm cube ($\Delta_{1\mathrm{P}}$; effect of one previous target of a different size), we get:

$$\Delta_{1\mathrm{P}} = a w_s d_2 \left(\frac{\hat{s}_{2;X=1}}{s_{2;Y}} - \frac{\hat{s}_{2;X=2}}{s_{2;Y}}\right) = -\frac{2}{3} a w_s d_2 (u-u^2) \tag{8}$$

If we average the values of $\Delta_{1\mathrm{C}}$ and $\Delta_{1\mathrm{P}}$ across trials, the only parameter that differs across trials, the distance, can be removed from the equations, because $\bar{d}_1 = \bar{d}_2$. Subsequently, combining Equations 7 and 8 yields:

$$u = -\frac{9}{8}\frac{\Delta_{1\mathrm{P}}}{\Delta_{1\mathrm{C}}} \tag{9}$$

The value of u can thus be estimated from the measured values of $\Delta_{1\mathrm{P}}$ and $\Delta_{1\mathrm{C}}$ and used to make predictions for the condition in which there were two successive large or small cubes. For the second such cube, the assumed size will be:

$$\hat{s}_2 = (1-u)\hat{s}_1 + u\mathbf{X} = (1-u)^2\mathbf{Y} + (2u-u^2)\mathbf{X} \tag{10}$$

Note that the value of $\hat{s}_2$ is different than in Equation 5 because of the different cube size. Filling in this value for the first stage of Equation 8, we now get (for $\Delta_{2\mathrm{C}}$; the effect of two consecutive targets of a different size on pointing distance):

$$\Delta_{2\mathrm{C}} = a w_s d_2 \left(\frac{\hat{s}_{2;X=1}}{s_{2;X=1}} - \frac{\hat{s}_{2;X=2}}{s_{2;X=2}}\right) = \frac{3}{4} a w_s d_2 (1-u)^2 \tag{11}$$

For the average values (i.e., $\bar{d}_1 = \bar{d}_2$), we can combine Equations 11, 7, and 9 to give:

$$\Delta_{2C} = \Delta_{1C}(1-u) = \Delta_{1C} + \frac{9}{8}\Delta_{1P} \tag{12}$$

Finally, when pointing at 1.5-cm cubes after being shown either two 1 cm or two 2-cm cubes (Δ_{2P}; the effect of two previous targets of a different size), the assumed size will be:

$$\begin{aligned}\hat{s}_3 &= (1-u)\hat{s}_2 + u\mathbf{Y} \\ &= (1-2u+3u^2-u^3)\mathbf{Y} + (2u-3u^2+u^3)\mathbf{X}\end{aligned} \tag{13}$$

So, for the effect on pointing distance we get:

$$\begin{aligned}\Delta_{2P} &= aw_s d_3\left(\frac{\hat{s}_{3;X=1}}{s_{3;Y}} - \frac{\hat{s}_{3;X=2}}{s_{3;Y}}\right) \\ &= -\frac{2}{3}aw_s d_3(2u-3u^2+u^3)\end{aligned} \tag{14}$$

For the average pointing values (with $\bar{d}_1 = \bar{d}_3$), we can combine Equations 7, 8, 9, and 14 to give:

$$\Delta_{2P} = \Delta_{1P}\left(2 + \frac{9}{8}\frac{\Delta_{1P}}{\Delta_{1C}}\right) \tag{15}$$

Equations 15 and 12 are used for the predictions in Figures 5 and 6, respectively.