TI 2012-126/II

Tinbergen Institute Discussion Paper

Games with a Local Permission Structure: Separation of Authority and Value Generation

René van den Brink

Chris Dietz

Faculty of Economics and Business Administration, VU University Amsterdam, and Tinbergen Institute.

Games with a Local Permission Structure: separation of authority and value generation

René van den Brink* Chris Dietz†

November 23, 2012

*Department of Econometrics and Tinbergen Institute, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. E-mail: jrbrink@feweb.vu.nl.
†Department of Econometrics and Tinbergen Institute, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. E-mail: cdietz@feweb.vu.nl. This author is financially supprted by NWO grant

Abstract

It is known that peer group games are a special class of games with a permission structure. However, peer group games are also a special class of (weighted) digraph games. To be specific, they are digraph games in which the digraph is the transitive closure of a rooted tree. In this paper we first argue that some known results on solutions for peer group games hold more general for digraph games.

Second, we generalize both digraph games as well as games with a permission structure into a model called *games with a local permission structure*, where every player needs permission from its predecessors only in order to generate worth, but does not need its predecessors in order to give permission to its own successors. We introduce and axiomatize a Shapley value type solution for these games, generalizing the conjunctive permission value for games with a permission structure and the β-measure for weighted digraphs.

Keywords: Cooperative TU-game, peer group game, digraph game, game with a permission structure, local permission structure.

JEL code: C71

1 Introduction

Recently, the use of cooperative game theory to model economic situations is growing. In several of these situations there is an underlying hierarchical ordering of the players. Examples are e.g. the airport games of Littlechild and Owen (1973) where aircraft landings can be ordered by the cost of the landig strip they need, the auction situations of Graham and Marshall (1987) and Graham, Marshall and Richard (1990) where players can be ordered according to their valuation of a good, the sequencing games of Curiel, Potters, Rajendra Prasad, Tijs and Veltman (1993, 1994) where jobs are ordered in an initial queue, the queueing games of Maniquet (2003) where jobs are not in an initial queue but can be ordered according to their waiting cost, the water distribution problems of Ambec and Sprumont (2002) or polluted river problems of Ni and Wang (2007) where countries are ordered by their location on a river flowing from upstream to downstream.

All of these examples imply an underlying hierarchical structure where some agents are not able to cooperate fully without the presence of other agents. Cooperative game theory has been applied to study such structures. This has resulted in several types of games. One class of games in the study of hierarchical structures is formed by the so-called games with a permission structure in Gilles, Owen and van den Brink (1992), van den Brink and Gilles (1996), Gilles and Owen (1994) and van den Brink (1997). In these games with a permission structure it is assumed that the players in a cooperative game with transferable utility, shortly TU-game, are part of a hierarchical (permission) structure (represented by a directed graph or digraph) such that players need permission from other players in order to cooperate. A distinction is made between the conjunctive and disjunctive approach. In the conjunctive approach as developed in Gilles, Owen and van den Brink (1992) and van den Brink and Gilles (1996), a player needs permission from all its predecessors (if it has any) in the digraph in order to cooperate. Consequently, a player can only be in a feasible coalition if all its superiors (being all players from whom he can be reached by a directed path) are in the coalition. On the other hand, in the disjunctive approach as developed in Gilles and Owen (1994) and van den Brink (1997) for acyclic, quasi-strongly connected permission structures, a player needs permission from at least one of its predecessors (if it has any).

These games have been applied to describe several economic organizations, such as hierarchically structured firms in e.g. van den Brink (2008) and van den Brink and Ruys (2008). But also the above mentioned auction games of Graham and Marshall (1987) and Graham, Marshall and Richard (1990), the so-called DR-polluted river game of Ni and Wang (2007) and the dual of the airport game of Littlechild and Owen (1973) are special classes of these games with a permission structure. In fact, these last three applications are a special class of peer group games as introduced in Brânzei, Fragnelli and Tijs (2002), being a special class of games with a permission structure where the game is additive and the permission structure is a rooted tree. Since the permission structure in a peer group game is a rooted tree, in all these applications the conjunctive and disjunctive approaches coincide.

Besides being a special class of games with a permission structure, peer group games are also a special class of (weighted) digraph games that are introduced in van den Brink and Borm (2002). For a given (weighted) digraph, with weights assigned to the nodes, the associated digraph game assigns to every coalition of nodes the sum of the weights of all nodes in the coalition whose predecessors also belong to the coalition. To be specific, a digraph game is a peer group game if the digraph is the transitive closure of a rooted tree. We show that results stated in the literature for peer group games can be extended straightforward to weighted digraph games.

In this paper we develop a new, more local, approach to permission structures. In order for a player to generate value, it is no longer needed that all its superiors are present, but it is sufficient if its direct predecessors belong to the coalition. So, every player needs permission from its direct predecessors in order to cooperate, but it can give permission to its own successors without permission from its predecessors. In this way, this model allows a certain degree of separation between authority and value generation. Since a player can now give permission to its successors, without approval of its own predecessors, the value generating part of a coalition might not contain this player, although it is needed for the value generating part to be active.

For hierarchically structured firms we can say that in a 'standard' permission structure a worker at the bottom can only be 'activated' by obtaining permission from all of its superiors. This is in line with Williamson (1967) where it are only these bottom level workers who are able to generate worth, while the higher level managers only organize and manage the production process, but do not actively take part in it. However, rarely does permission really consist of a whole chain of predecessors being involved in every step of the production process. The local permission structures do not require this full line of approval.

Similar to the conjunctive (and disjunctive) approach, we associate with every game with a permission structure a new restricted game, called the *locally restricted game*, where the worth of any coalition equals the worth of its value generating set, that is the subset of players in the coalition whose predecessors all belong to the coalition. Whereas peer group games are a special case of games with a permission structure as well as digraph games, we show that the games with a local permission structure that we develop in this paper generalize the classes of games with a permission structure as well as digraph games in the sense that the conjunctive restricted game of a game with permission structure equals its locally restricted game if the digraph is transitive, and a weighted digraph game equals the locally restricted game of the additive game defined by the weights of the nodes on the digraph as permission structure.

After studying several properties of games with a local permission structure and the locally restricted game, we introduce the local permission value as the solution that is obtained by applying the Shapley value to the locally restricted game, and compare it with the 'standard' conjunctive permission value by providing an axiomatization with axioms similar to those that characterize the conjunctive permission value.

The paper is organized as follows. Section 2 contains preliminaries. Section 3 explores the relation between digraph games and peer group games. In Section 4 we introduce the new local restriction and study several properties. In Section 5 we introduce and axiomatize the local permission value. Finally, Section 6 contains concluding remarks.

2 Preliminaries

2.1 Cooperative TU-games

A situation in which a finite set of players $N \subset \mathbb{N}$ can generate certain payoffs by cooperation can be described by a *cooperative game with transferable utility* (or simply a TU-game), being a pair (N, v) where $v\colon 2^N \to \mathbb{R}$ is a *characteristic function* on N satisfying $v(\emptyset) = 0$. For any coalition $S \subseteq N$, $v(S) \in \mathbb{R}$ is the *worth* of coalition S, i.e. the members of coalition S can obtain a total payoff of $v(S)$ by agreeing to cooperate. Since we take the player set to be fixed, we denote a TU-game (N, v) just by its characteristic function v. We denote the collection of all TU-games on player set N by $\mathcal{G}^N$.

A *payoff vector* for game $v \in \mathcal{G}^N$ is an $|N|$-dimensional vector $x \in \mathbb{R}^N$ assigning a payoff $x_i \in \mathbb{R}$ to any player $i \in N$. A (single-valued) *solution* for TU-games is a function that assigns a payoff vector to every TU-game. One of the most famous solutions for TU-games is the *Shapley value* (Shapley (1953)) given by

$$Sh_i(v) = \frac{1}{|N|!} \sum_{\pi \in \Pi(N)} m_i^{\pi}(v),$$

where $\Pi(N)$ is the set of all permutations of N and for every permutation $\pi\colon N \to N$, the corresponding *marginal vector* $m^{\pi}(v)$ is given by $m_i^{\pi}(v) = v(\{j \in N \mid \pi(j) \leq \pi(i)\}) - v(\{j \in N \mid \pi(j) < \pi(i)\})$ for all $i \in N$.

The *core* (Gillies (1953)) of $v \in \mathcal{G}^N$ is the set of all efficient payoff vectors that are stable in the sense that no coalition can do better by separating, and is given by

$$core(v) = \left\{ x \in \mathbb{R}^N \;\middle|\; \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subset N \right\}.$$

As known, the Core of a game is nonempty if and only if the game is balanced, see e.g. Bondareva (1962) or Shapley (1967).

Game $v \in \mathcal{G}^N$ is *superaditive* if $v(E \cup F) \geq v(E) + v(F)$ for all $E, F \subseteq N$ such that $E \cap F = \emptyset$. Game $v \in \mathcal{G}^N$ is *convex* if $v(E \cup F) + v(E \cap F) \geq v(E) + v(F)$ for all $E, F \subseteq N$. For each $T \subseteq N$, $T \neq \emptyset$, the *unanimity* game u_T is given by $u_T(S) = 1$ if $T \subseteq S$, and $u_T(S) = 0$ otherwise. It is well-known that the unanimity games form a basis for $\mathcal{G}^N$. For every $v \in \mathcal{G}^N$ it holds that $v = \sum_{\substack{T \subseteq N \\ T \neq \emptyset}} \Delta_v(T) u_T$, where $\Delta_v(T) = \sum_{S \subseteq T} (-1)^{|T|-|S|} v(S)$ are the *Harsanyi dividends*, see Harsanyi (1959).

Finally, a game is *additive* or *inessential* if $v(S) = \sum_{i \in S} v(\{i\})$ for all $S \subseteq N$. Equivalently, we can say that a game is additive if only singletons have a nonzero dividend.

2.2 Games with a permission structure

A game with a permission structure describes a situation where some players in a TU-game need permission from other players before they are allowed to cooperate within a coalition. Formally, a permission structure can be described by a directed graph on N. A directed graph or *digraph* is a pair (N, D) where $N = \{1, ..., n\}$ is a finite set of nodes (representing the players) and $D \subseteq N \times N$ is a binary relation on N. We assume the digraph to be irreflexive, i.e., $(i, i) \notin D$ for all $i \in N$. Since we take the player set to be fixed, in the sequel we simply refer to D for a digraph (N, D), and we denote the collection of all irreflexive digraphs on N by $\mathcal{D}^N$. For $i \in N$ the nodes in $S_D(i) := \{j \in N \mid (i, j) \in D\}$ are called the *successors* of i, and the nodes in $P_D(i) := \{j \in N \mid (j, i) \in D\}$ are called the *predecessors* of i in D. For given $D \in \mathcal{D}^N$, a (directed) *path* from i to j in N is a sequence of distinct nodes $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in S_D(h_k)$ for $k = 1, \ldots, t-1$, and $h_t = j$. The *transitive closure* of $D \in \mathcal{D}^N$ is the digraph $tr(D)$ given by $(i, j) \in tr(D)$ if and only if there is a directed path from i to j. By $\widehat{S}_D(i) = S_{tr(D)}(i)$ we denote the set of successors of i in the transitive closure of D, and refer to these players as the *subordinates* of i in D. We refer to the players in $\widehat{P}_D(i) = \{j \in N \mid i \in \widehat{S}_D(j)\}$ as the *superiors* of i in D. A digraph $D \in \mathcal{D}^N$ is transitive if $D = tr(D)$. For a set of players $E \subseteq N$ we denote by $S_D(E) = \bigcup_{i \in E} S_D(i)$, respectively, $P_D(E) = \bigcup_{i \in E} P_D(i)$, the sets of successors, respectively predecessors of players in coalition E. Also, for $E \subseteq N$, we denote $\widehat{S}_D(E) = \cup_{i \in E} \widehat{S}_D(i)$.

A directed path $(i_1, \ldots, i_t)$, $t \geq 2$, in D is a *cycle* in D if $(i_t, i_1) \in D$. We call digraph D *acyclic* if it does not contain any cycle. Note that acyclicity of digraph D implies that D is irreflexive and has at least one node that does not have a predecessor. A digraph $D \in \mathcal{D}^N$ is a rooted tree if and only if (i) there is an $i_0 \in N$ such that $P_D(i_0) = \emptyset$ and $\widehat{S}_D(i_0) = N \setminus \{i\}$, and (ii) $|P_D(i)| = 1$ for all $i \in N \setminus \{i_0\}$. Note that this implies that D is acyclic. We denote the collection of all rooted trees on N by $\mathcal{D}^N_{tree}$.

A triple (N, v, D) with $N \subset \mathbb{N}$ a finite set of players, $v \in \mathcal{G}^N$ a TU-game and

$D \in \mathcal{D}^N$ a digraph on N is called a *game with a permission structure*. Again, since we take the player set N to be fixed, we denote a game with a permission structure just as a pair (v, D). In the *conjunctive approach* as introduced in Gilles, Owen and van den Brink (1992) and van den Brink and Gilles (1996) it is assumed that a player needs permission from all its predecessors in order to cooperate with other players. Therefore a coalition is feasible if and only if for any player in the coalition all its predecessors are also in the coalition. So, for permission structure D the set of *conjunctive feasible coalitions* is given by

$$\Phi_D^c = \{E \subseteq N \,|P_D(i) \subseteq E \text{ for all } i \in E\}.$$

For any $E \subseteq N$, let $\sigma_D^c(E) = \bigcup\{F \in \Phi_D^c \mid F \subseteq E\} = E \setminus \widehat{S}_D(N \setminus E)$ be the largest conjunctive feasible subset[1] of E in the collection Φ_D^c. Then, the induced *conjunctive restricted* game of the pair (v, D) is the game $r_{v,D}^c \colon 2^N \to \mathbb{R}$, given by

$$r_{v,D}^c(E) = v(\sigma_D^c(E)) \text{ for all } E \subseteq N, \tag{2.1}$$

i.e., the restricted game $r_{v,D}^c$ assigns to each coalition $E \subseteq N$ the worth of its largest conjunctive feasible subset. Then the *conjunctive permission value* φ^c is the solution that assigns to every game with a permission structure the Shapley value of the restricted game, thus

$$\varphi^c(v, D) = Sh(r_{v,D}^c) \text{ for all } (v, D) \in \mathcal{G}^N \times \mathcal{D}^N.$$

As shown by Algaba, Bilbao, van den Brink and Jiménez-Losada (2004a), when D is acyclic then the collection Φ_D^c is an antimatroid.

2.3 Peer group games

Brânzei, Fragnelli and Tijs (2002) define a peer group situation as a triple (N, a, T) where $N \subset \mathbb{N}$ is a set of players, $T \in \mathcal{D}_{tree}^N$ is a rooted tree, and $a \in \mathbb{R}_+^N$ is a vector of nonnegative weights assigned to the players. Again, since we take the player set N to be fixed, we denote a peer group situation just as a pair (a, T). To each peer group situation (a, T), they assign the *peer group game* $v_{a,T}^P$ given by $v_{a,T}^P(E) = \sum_{\substack{i \in E \\ \widehat{P}_T(i) \subseteq E}} a_i$, $E \subseteq N$. So, a coalition $E \subseteq N$ might be considered *feasible* if $\widehat{P}_T(i) \subseteq E$ for all $i \in E$ (i.e. when it belongs to the set of conjunctive feasible coalitions), and the worth of an arbitrary coalition $E \subseteq N$ is the sum of the weights of the players in its largest feasible subset. In terms of unanimity games a peer group game can be written as $v_{a,T}^P = \sum_{i \in N} a_i u_{\widehat{P}_T(i)}$.

[1]Every coalition having a unique largest feasible subset follows from the fact that Φ_D^c is union closed.

In Brânzei, Fragnelli and Tijs (2002) it is already mentioned that every peer group situation (a, T) can be seen as a game with a permission structure (w^a, T) where the permission structure T is a rooted tree and the game w^a is the additive game given by $w^a(E) = \sum_{i \in E} a_i$ for all $E \subseteq N$.

Since the conjunctive restricted game is the same for a game with permission structure (v, D) and that game v on the transitive closure $tr(D)$ of the permission structure, i.e. $r^c_{v,D} = r^c_{v,tr(D)}$ for all $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$, for peer group games it holds that $v^P_{a,T} = r^c_{w^a,T} = r^c_{w^a,tr(T)}$.

2.4 Digraph games

Another model of games with a digraph on the set of players are the *(weighted) digraph games* introduced in van den Brink and Borm (2002). A reflexive *weighted directed graph*, shortly refered to as *weighted digraph*, is a triple (N, δ, D) where $N \subset \mathbb{N}$ is a set of nodes, $D \in \mathcal{D}^N$ is an irreflexive digraph, and $\delta \in \mathbb{R}^N_+$ is a vector of nonnegative weights assigned to the nodes. The *(weighted) digraph game* corresponding to (N, δ, D) is the game $(N, \overline{v}_{\delta,D})$ where the players represent the nodes and the characteristic function is given by $\overline{v}_{\delta,D}(E) = \sum_{\substack{i \in E \\ P_D(i) \subseteq E}} \delta_i$, $E \subseteq N$. So, the worth of an arbitrary coalition $E \subseteq N$ is the sum of the weights of the players in that coalition for whom all predecessors belong to the coalition. In terms of unanimity games, a digraph game can be written as $\overline{v}_{\delta,D} = \sum_{i \in N} \delta_i u_{P_D(i) \cup \{i\}}$. Again, since we take the player set N to be fixed, we denote a weighted digraph and weighted digraph game on N as (δ, D), respectively, $\overline{v}_{\delta,D}$.

In van den Brink and Borm (2002) a relational power measure assigning values to every node in a weighted digraph is obtained by applying the Shapley value to the associated weighted digraph game. This power measure is refered to as the β-measure and is given by[2]

$$\beta_i(D) = Sh_i(\overline{v}_{\delta,D}) = \sum_{j \in S_D(i) \cup \{i\}} \frac{\delta_j}{(|P_D(j)| + 1)}.$$

3 Peer group games are digraph games

It is already mentioned in Brânzei, Fragnelli and Tijs (2002) that peer group situations are a special case of games with a permission structure. However, peer group situations

[2]In van den Brink and Gilles (2000) a similar game and measure are defined, but a node does not 'share' in the power over itself, i.e. they consider the game $v'_{\delta,D}(E) = \sum_{\substack{i \in N \\ P_D(i) \subseteq E}} \delta_i$, $E \subseteq N$, having Shapley value $\beta'_i(D) = Sh_i(v'_{\delta,D}) = \sum_{j \in S_D(i)} \frac{\delta_j}{|P_D(j)|}$. A disadvantage of this measure is that a node can do better in the associated ranking after 'being defeated' by more other nodes.

are also a special class of weighted digraph games in the sense that for every peer group situation, the associated peer group game coincides with the digraph game on the transitive closure of the peer group tree.

Proposition 3.1 *For every peer group situation (a,T) it holds that $v^P_{a,T} = \overline{v}_{a,tr(T)}$.*

Proof
Let (a,T) be a peer group situation. By definition of the transitive closure it holds that $P_{tr(T)}(i) = \widehat{P}_T(i)$. Therefore, taking weighted digraph (δ, T) with $\delta_i = a_i$ it holds that $v^P_{a,T}(E) = \sum_{\substack{i\in E \\ \widehat{P}_T(i)\subseteq E}} a_i = \sum_{\substack{i\in E \\ P_{tr(T)}(i)\subseteq E}} \delta_i = \overline{v}_{\delta,tr(D)}(E)$, for all $E \subseteq N$.

$\square$

On the other hand, not every weighted digraph game is a peer group game.

Example 3.2 *Consider the weighted digraph (δ, D) on $N = \{1,2,3\}$ given by $\delta = (1,1,1)^\top$ and $D = \{(1,2),(2,3)\}$. The corresponding weighted digraph game is $\overline{v}_{\delta,D} = u_{\{1\}} + u_{\{1,2\}} + u_{\{2,3\}}$. There is no peer group situation (a,T) such that $v^P_{a,T} = \overline{v}_{\delta,D}$ since $\overline{v}_{\delta,D}(\{1\}) = 1$ implies that 1 is the root of T. But then $v^P_{a,T}(\{2,3\})$ must be equal to zero, while $\overline{v}_{\delta,D}(\{2,3\}) = 1$.*

We have the following corollary.

Corollary 3.3 *For every peer group situation (a,T) it holds that $v^P_{a,T} = r^c_{w^a,T} = r^c_{w^a,tr(T)} = \overline{v}_{a,tr(T)}$.*

So, in the literature we encounter two classes of games with a hierarchical structure on the player set which generalize peer group games. Whereas the results on solutions for peer group games as given in Brânzei, Fragnelli and Tijs (2002, Proposition 1) do not hold for general games with a permission structure, they do hold for digraph games.[3]

Proposition 3.4 *Let (δ, D) be a weighted digraph and let $v = \overline{v}_{\delta,D}$ be the associated digraph game. Then*

(i) *The bargaining set of v coincides with the core of v;*

(ii) *The kernel of v coincides with the pre-kernel of v and consists of a unique point being the nucleolus of v;*

(iii) *The nucleolus of v occupies a central position in its core and is the unique point satisfying $Nu(v) = \{x \in C(v) \mid s_{ij}(x) = s_{ji}(x) \forall i,j\}$, where $s_{ij}(x) = \max\{v(S) - x(S) \mid i \in S \subseteq N \setminus \{j\}\}$;*

[3] We refer to the mentioned literature for definitions of the solutions.

(iv) *The core of v coincides with the Weber set of v (being the convex hull of all marginal vectors of v);*

(v) *The Shapley value $Sh(v)$ is the center of gravity of the extreme points of the core and is given by $Sh_i(v) = \sum_{j \in S_D(i) \cup \{i\}} \frac{\delta_j}{(|P_D(j)|+1)}$, $i \in N$;*

(vi) *The τ-value $\tau(v)$ is given by $\tau_i(v) = \begin{cases} \alpha\delta_i + (1-\alpha)M_i(v) & \text{if } P_D(i) = \emptyset \\ (1-\alpha)M_i(v) & \text{if } P_D(i) \neq \emptyset, \end{cases}$ where $M_i(v) = \sum_{j \in S_D(i) \cup \{i\}} \delta_j$ and $\alpha \in [0,1]$ is obtained such that $\sum_{i \in N} \tau_i(v) = v(N)$.*

(vii) *The core of v coincides with the selectope of v;*

(viii) *There exist population monotonic allocation schemes.*

Similar as in Brânzei, Fragnelli and Tijs (2002), the parts (i), (ii) and (iii) follow directly from convexity of $\overline{v}_{\delta,D}$, Maschler, Peleg and Shapley (1972) and Maschler (1992). (vii) follows from the game being totally positive (meaning that all dividends are non-negative), see Derks, Haller and Peters (2000) and Vasil'ev and van der Laan (2002), as already mentioned by van den Brink and Borm (2002). This then also implies (iv) since the Weber set always is a subset of the selectope and contains the core. Computation of the τ-value in (vi) is straightforward from its definition. Finally, (v) is shown in van den Brink and Borm (2002).

Part (vii) of Proposition 3.4 states that for digraph games the core (being equal to the convex hull of all marginal vectors if the game is convex) and selectope (being equal to the convex hull of all selectope vectors) coincide. Theorem 4.4 of van den Brink and Borm (2002) shows that the set of marginal vectors equals the set of selectope vectors if and only if the digraph D has no *anti-directed semi-circuit* being a sequence $(i_1, i_2, \ldots, i_t)$, $t \geq 2$ of t even distinct nodes such that (i) $i_k \in N$ for all $k \in \{1, \ldots, t\}$, (ii) $i_k \in S_D(i_{k-1}) \cap S_D(i_{k+1})$ if $k \neq t$ is even, and (iii) $i_t \in S_D(i_{t-1}) \cap S_D(i_1)$. Although a rooted tree does not have an anti-directed semi-circuit, the transitive closure of a rooted tree such that there is a path from the top node to another node that contains at least four nodes, has an anti-directed semi-circuit, and therefore the set of selectope vectors and marginal vectors of a peer group game can be different.

Example 3.5 *Consider the peer group situation (a, T) on $N = \{1, 2, 3, 4\}$ given by $a = (1, 1, 1, 1)$ and $T = \{(1, 2), (2, 3), (3, 4)\}$. The corresponding peer group game is $v^P_{a,T} = u_{\{1\}} + u_{\{1,2\}} + u_{\{1,2,3\}} + u_{\{1,2,3,4\}}$. (Note that this is also the digraph game corresponding to $\delta = a$ and $tr(T) = T \cup \{(1, 3), (1, 4), (2, 4)\}$.) One of the selectope vectors is $(2, 2, 0, 0)$ (assigning the dividend of $\{1\}$ and one of the other dividends to player 1, and two of the dividends to player 2). However, there is no permutation of the players such that this is the* *corresponding marginal vector of* $v = v^P_{a,T}$ *since* $m^\pi_1(v) = 1$ *if* $\pi(1) < \pi(2)$, *and* $m^\pi_2(v) = 0$ *if* $\pi(2) < \pi(1)$.

Brânzei, Fragnelli and Tijs (2002) conclude by mentioning some applications of peer group games. Since their above mentioned results on solutions also hold for more general digraph games, these results also can be applied to more situations, such as the ranking of teams in sports competitions, defining social choice correspondences or voting rules in social choice theory, and measuring relational power in social networks.

4 Locally restricted games

Comparing games with a permission structure with weighted digraph games, there are two essential differences, one considering the games and one considering the effect of the digraph on the restrictions in cooperation. First, games with a permission structure allow any game, but weighted digraphs only consider additive games. On the other hand, to have permission to cooperate, a player in a game with (conjunctive) permission structure needs permission from all its superiors, but in a weighted digraph game it needs permission only from its (direct) predecessors. Obviously, the digraph game associated to a transitive digraph equals the conjunctive restricted game of the corresponding additive game on that digraph as permission structure.

Proposition 4.1 *For every weighted digraph* (δ, D) *it holds that* $\overline{v}_{\delta,tr(D)} = r^c_{w^\delta,D}$, *where* $w^\delta(E) = \sum_{i\in E} \delta_i$ *for all* $E \subseteq N$. *In particular, if* D *is transitive then* $\overline{v}_{\delta,D} = r^c_{w^\delta,D}$.

Proof

Let (δ, D) be a weighted digraph. By definition of the transitive closure it holds that $P_{tr(D)}(i) = \widehat{P}_D(i)$. Therefore, $\overline{v}_{\delta,tr(D)}(E) = \sum_{\substack{i\in E\\ P_{tr(D)}(i)\subseteq E}} \delta_i = \sum_{\substack{i\in E\\ \widehat{P}_D(i)\subseteq E}} w^\delta(\{i\}) = w^\delta(\{i \in E \mid \widehat{P}_D(i) \subseteq E\}) = w^\delta(\sigma^c_D(E)) = r^c_{w^\delta,D}(E)$ for all $E \subseteq N$.

□

Next, we generalize the (weighted) digraph games as well as games with a permission structure in the sense that we consider pairs (v, D) where $v \in \mathcal{G}^N$ can be any game, $D \in \mathcal{D}^N$ can be any digraph, but every player needs permission only from its direct predecessors in order to cooperate. So, a player needs permission from its predecessors in order to cooperate with other players, but it can give permission to its own successors without permission from its predecessors. In this paper we only follow the conjunctive approach (remarks on a disjunctive approach are made in the final section).

For any $E \subseteq N$, let $\sigma^l_D(E) = \{i \in E \mid P_D(i) \subseteq E\}$ be the subset of players in E for whom all predecessors also belong to E. We refer to this as the *value generating set* of coalition E in D. Then, we define the *locally restricted* game $r^l_{v,D}$ associated to the pair $(v,D) \in \mathcal{G}^N \times \mathcal{D}^N$ by

$$r^l_{v,D}(E) = v(\sigma^l_D(E)) \text{ for all } E \subseteq N. \tag{4.2}$$

An important difference with the conjunctive feasible coalitions is the fact that $\sigma^l_D(\sigma^l_D(E))$ need not be equal to $\sigma^l_D(E)$, so the value generating set of a coalition need not be equal to its own value generating set. Note that in the conjunctive approach, for every digraph D it holds that $\sigma^c_D(\sigma^c_D(E)) = \sigma^c_D(E)$ for all $E \subseteq N$.[4]

Example 4.2 *Consider the digraph D on $N = \{1,2,3\}$ given by $D = \{(1,2),(2,3)\}$. Then $\sigma^l_D(\{2,3\}) = \{3\}$ but $\sigma^l_D(\{3\}) = \emptyset$.*

Because of this, the cooperation structure cannot be described just by a set of feasible coalitions. In the example above, the coalition $\{3\}$ can be considered not feasible, but there is a coalition, to be specific coalition $\{2,3\}$, such that $\{3\}$ is exactly the coalition that generates value. Therefore, we call $\{3\}$ a value generating set in D.

Proposition 4.3 *Let $(v,D) \in \mathcal{G}^N \times \mathcal{D}^N$ and $E \subseteq N$ be given.*

(i) *For all F such that $\sigma^c_D(E) \subseteq F \subseteq E$ it holds that $r^c_{v,D}(F) = v(\sigma^c_D(E))$*

(ii) *For all $F \subseteq E \setminus \sigma^c_D(E)$ it holds that $r^c_{v,D}(F) = 0$*

(iii) *For all F such that $\sigma^l_D(E) \cup P_D(\sigma^l_D(E)) \subseteq F \subseteq E$ it holds that $r^l_{v,D}(F) = v(\sigma^l_D(E))$*

(iv) *For all $F \subseteq E \setminus \sigma^l_D(E)$ it holds that $r^l_{v,D}(F) = 0$.*

Proof

Let $(v,D) \in \mathcal{G}^N \times \mathcal{D}^N$ and $E \subseteq N$ be given. (i) follows straightforward since $\sigma^c_D(F) = \sigma^c_D(E)$ for all F such that $\sigma^c_D(E) \subseteq F \subseteq E$. (ii) follows since $\widehat{P}_D(i) \not\subseteq E$ for all $i \in E \setminus \sigma^c_D(E)$, and thus $\sigma^c_D(F) = \emptyset$ for all $F \subseteq E \setminus \sigma^c_D(E)$. (iii) and (iv) follow in a similar way, but using $\sigma^l_{v,D}$ instead of $\sigma^c_{v,D}$, and $P_D(i)$ instead of $\widehat{P}_D(i)$. □

Part (i) implies that for conjunctive restricted games, if a coalition of players E is able to generate its own worth, then it does not need permission from players outside E to do so; value generation and permission imply one another. For this reason, this approach can be described in terms of sets of feasible coalitions Φ^c_D. This is not the case for locally restricted games as reflected in part (iii). This part states that the value generating set

[4] In most models of restricted cooperation, such as the communication graph games of Myerson (1977), and its generalization to games on union stable systems in Algaba, Bilbao, Borm and López (2000, 2001), it is the case that the worth of a feasible coalition in the restricted game equals the worth of this coalition in the original game.

of a coalition E can generate its worth together with its predecessors (which are in E), without these predecessors actually generating any value within this coalition themselves: these coalitions still earn the worth of $\sigma^l_D(E)$. Note that a coalition containing the value generating set of E, but not all its predecessors might generate a different worth. Although it is true that $\sigma^l_D(F) = \sigma^l_D(E)$ for all F such that $\sigma^l_D(E) \cup P_D(\sigma^l_D(E)) \subseteq F \subseteq E$, this does not necessarily hold for all F such that $\sigma^l_D(E) \subseteq F \subseteq E$. This is an important difference with the 'standard' conjunctive approach to games with a permission structure.

Example 4.4 *Consider the digraph D on $N = \{1, 2, 3\}$ given in Example 4.2, and the game $v = u_{\{3\}}$. For coalition $E = \{2, 3\}$, we have that $\sigma^l_D(E) \cup P_D(\sigma^l_D(E)) = \{3\} \cup \{2\} = \{2, 3\} = E$. However, taking $F = \{3\}$ we have $\sigma^l_D(E) = F \subset E$, but $r^l_{v,D}(F) = v(\sigma^l_D(F)) = v(\emptyset) = 0$ while $v(\sigma^l_D(E)) = v(\{3\}) = 1$. So, indeed the predecessor of the value generating set of $E = \{2, 3\}$ is necessary to generate its worth.*

Next, we introduce some notions to describe the value generation and permission in games with a local permission structure. For any $E \subseteq N$ we define $\overline{\alpha}^l_D(E) = \sigma^l_D(E) \cup P_D(\sigma^l_D(E))$ as the *active set* of E. These are the players that are necessary and sufficient to make the value generating set $\sigma^l_D(E)$ of E active.

Now, we call a set E *locally feasible* in D if $\overline{\alpha}^l_D(E) = E$. We denote the set of all locally feasible sets in D by Ψ_D. So,

$$\Psi_D = \{E \subseteq N \mid \overline{\alpha}_D(E) = E\}.$$

Let the *authorizing set* of E be given by $\alpha^l_D(E) = E \cup P_D(E)$, being the set of players in E together with all their predecessors. This is the set of players that is necessary and sufficient to make the players in E active. It is clear that for any coalition E, $\alpha^l_D(E)$ is locally feasible.

Example 4.5 *Consider the permission structure D of Example 4.2 and coalition $\{2, 3\}$. We already saw that its value generating set is $\{3\}$. Its active set is $\overline{\alpha}^l_D(\{2, 3\}) = \{2, 3\}$ since permission of 2 is necessary and sufficient to make its value generating set $\{3\}$ active. Its authorizing set is $\alpha^l_D(\{2, 3\}) = \{1, 2, 3\}$ since player 1 is necessary to make player 2 active who is not value generating in $\{2, 3\}$ but is still necessary to give permission to player 3. In this case, $\Phi^c_D = \{\{1\}, \{1, 2\}, \{1, 2, 3\}\}$ and $\Psi_D = \Phi^c_D \cup \{\{2, 3\}\}$.*

Again, the active sets and authorizing sets show the separation between value generation and permission which coincide in the standard conjunctive approach.[5]

[5] For the conjunctive approach, Gilles, Owen and van den Brink (1992) introduce the authorizing set of E as $\alpha^c_D(E) = E \cup \widehat{P}_D(E)$ which is the smallest conjunctive feasible set containing E.

Proposition 4.6 *Consider permission structure $D \in \mathcal{D}^N$. Then*

(i) $\emptyset \in \Psi_D$,

(ii) $N \in \Psi_D$,

(iii) *If* $E, F \in \Psi_D$ *then* $E \cup F \in \Psi_D$.

Proof

(i), (ii) By definition of the local permission structure, $\overline{\alpha}^l_D(\emptyset) = \emptyset$ and $\overline{\alpha}^l_D(N) = N$, and therefore $\{\emptyset, N\} \subseteq \Psi_D$.

(iii) If $E \in \Psi_D$ then $E = \overline{\alpha}^l_D(E)$, and thus for any player $i \in E$ it holds that either $i \in \sigma^l_D(E)$ or $i \in P_D(j)$ for some $j \in \sigma^l_D(E)$. The same holds for F. Therefore, for any player $i \in E \cup F$ it holds that either $i \in \sigma^l_D(E \cup F)$ or $i \in P_D(j)$ for some $j \in \sigma^l_D(E \cup F)$, and thus $E \cup F \in \Psi_D$.

□

Part (iii) shows that Ψ_D is union closed. The basis elements of Ψ_D are the sets $\{P_D(i) \cup \{i\} \mid i \in N\}$. The other elements of Ψ_D can be written as the union of two or more basis elements. However, unlike the conjunctive sovereign parts of two conjunctive feasible coalitions, for two coalitions $E, F \in \Psi_D$ it does not necessarily hold that $E \cap F \in \Psi_D$.

Example 4.7 *Consider the permission structure D of Example 4.2. Both coalitions $E = \{1, 2\}$ and $F = \{2, 3\}$ belong to Ψ_D. However, $E \cap F = \{2\}$ does not belong to Ψ_D since player 2 needs permission from 1 to generate its own worth, or cooperate with player 3 to generate the worth of 3.*

Next, we state some properties of value generating sets and authorizing sets, similar to properties that hold for the sets of conjunctive feasible coalitions in Gilles, Owen and van den Brink (1992, Proposition 3.5). The proof is similar to their result, but using $S_D(i)$ and $P_D(i)$ instead of $\widehat{S}_D(i)$ and $\widehat{P}_D(i)$ respectively, and is therefore omitted.

Proposition 4.8 *Consider $D \in \mathcal{D}^N$ and $E, F \subseteq N$. Then*

(i) $\sigma^l_D(E) \cup \sigma^l_D(F) \subseteq \sigma^l_D(E \cup F)$;

(ii) $\sigma^l_D(E) \cap \sigma^l_D(F) = \sigma^l_D(E \cap F)$;

(iii) $\alpha^l_D(E) \cup \alpha^l_D(F) = \sigma^l_D(E \cup F)$;

(iv) $\alpha^l_D(E \cap F) \subseteq \alpha^l_D(E) \cap \sigma^l_D(F)$.

The following result is similar to another result in Gilles, Owen and van den Brink (1992, Theorem 4.2), but while their proof uses the fact that $\sigma^c_D(E) = E$ for every $E \in \Phi^c_D$, for local restrictions we have seen that $\sigma^l_D(E)$ need not be equal to E for $E \in \Psi_D$.

Theorem 4.9 *For $E \subseteq N, E \neq \emptyset$ and $D \in \mathcal{D}^N$, it holds that $r^l_{u_E,D} = u_{\alpha^l_D(E)}$.*

PROOF
By definition of the locally restricted game, we have $r^l_{u_E,D}(T) = u_E(\sigma^l_D(T)) = 1$ if $E \subseteq \sigma^l_D(T)$, and $r^l_{u_E,D}(T) = 0$ otherwise. Since for $E \subseteq T$ we have $[E \subseteq \sigma^l_D(T)] \Leftrightarrow [E \cup P_D(E) \subseteq T] \Leftrightarrow [\alpha^l_D(E) \subseteq T]$, this implies that $r^l_{u_E,D} = u_{\alpha^l_D(E)}$.

□

Since Ψ_D is the set of coalitions E for which there is an $F \subseteq N$ with $\alpha^l_D(F) = E$, as a corollary (similar as Corollary 4.3 in Gilles, Owen and van den Brink (1992)), any locally restricted game can be described as a linear combination of the unanimity games of its locally feasible sets.

Corollary 4.10 *Let $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$. Then $r^l_{v,D} = \sum_{E \in \Psi_D} [\sum_{\substack{F \subseteq N \\ \alpha^l_D(F)=E}} \Delta_v(F)] u_E$*

Similar inheritance properties as for the conjunctive restriction hold for the locally restricted game.

Proposition 4.11 *Let $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$.*

(i) *If v is monotone then $r^l_{v,D}$ is monotone. Moreover, if v is also balanced, then $r^l_{v,D}$ is balanced as well.*

(ii) *If v is superadditive, then $r^l_{v,D}$ is superadditive.*

(iii) *If v is convex, then $r^l_{v,D}$ is convex.*

The proof is similar to the proof of Theorem 4.6 in Gilles, Owen and van den Brink (1992) and is therefore omitted. Part (iv) of their result states that the existence of a player i_0 such that $\widehat{S}(i_0) = N \setminus \{i_0\}$ is sufficient for the restriction of a monotone game to be superadditive and balanced. That is not true for locally restricted games.

Example 4.12 *Consider the permission structure D of Example 4.2 and game $v \in \mathcal{G}^N$ given by $v(E) = 1$ for all $E \subseteq N$, $E \neq \emptyset$. Then $\widehat{S}(1) = N \setminus \{1\}$ and v is monotone, but $r^l_{v,D}(\{2\}) = r^l_{v,D}(\{3\}) = 0$ and $r^l_{v,D}(E) = 1$ otherwise, and thus $r^l_{v,D}$ is not superadditive (since $r^l_{v,D}(\{1\}) + r^l_{v,D}(\{2,3\}) = 2 > 1 = r^l_{v,D}(\{1,2,3\})$) nor balanced (since $r^l_{v,D}(\{1\}) = r^l_{v,D}(\{2,3\}) = r^l_{v,D}(\{1,2,3\}) = 1$.*

Next we argue that the locally restricted approach to games with a permission structure generalizes the conjunctive approach as well as digraph games.[6]

The conjunctive restricted game of a game with a permission structure equals the locally restricted game of that game on the transitive closure of the permission structure. A weighted digraph game equals the locally restricted game of the additive game determined by the weights and the digraph as permission structure.

Proposition 4.13 (i) *For every $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$, it holds that $r^c_{v,D} = r^l_{v,tr(D)}$. In particular, if $D \in \mathcal{D}^N$ is transitive then $r^c_{v,D} = r^l_{v,D}$.*

(ii) *For every weighted digraph (δ, D) it holds that $\overline{v}_{\delta,D} = r^l_{w^\delta,D}$.*

PROOF

(i) For every $D \in \mathcal{D}^N$ it holds that $\widehat{P}_D(i) = P_{tr(D)}(i)$ for all $i \in N$. So, $\sigma^c_D(E) = \{i \in E \mid \widehat{P}_D(i) \subseteq E\} = \{i \in E \mid P_{tr(D)}(i) \subseteq E\} = \sigma^l_{tr(D)}(E)$, and thus $r^c_{v,D}(E) = v(\sigma^c_D(E)) = v(\sigma^l_{tr(D)}(E)) = r^l_{v,tr(D)}(E)$ for all $E \subseteq N$.

(ii) This follows straightforward since $\overline{v}_{\delta,D}(E) = \sum_{\substack{i \in E \\ P_D(i) \subseteq E}} \delta_i = \sum_{\substack{i \in E \\ P_D(i) \subseteq E}} w^\delta(i) = w^\delta(\{i \in E \mid P_D(i) \subseteq E\}) = w^\delta(\sigma^l_D(E)) = r^l_{w^\delta,D}(E)$ for all $E \subseteq N$.

$\square$

We end this section with an example illustrating some of the relations between the classes of games described above.

Example 4.14 *Consider the digraph $D = \{(1,3),(2,3),(3,5),(4,5)\}$ on $N = \{1,2,3,4,5\}$. If $v = u_{\{5\}}$ then v is additive and, clearly $r^l_{v,D} = u_{\{3,4,5\}} = \overline{v}_{\delta,D}$ with $\delta = (0,0,0,0,1)$. The corresponding local permission value is $\varphi^l(v,D) = (0,0,\frac{1}{3},\frac{1}{3},\frac{1}{3})$. Note that the conjunctive restriction $r^c_{v,D} = u_{\{1,2,3,4,5\}} = r^l_{v,tr(D)}$, and $\varphi^c(v,D) = \varphi^l(v,tr(D)) = (\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5})$.*
If $v = u_{\{1,5\}}$ then $r^l_{v,D} = u_{\{1,3,4,5\}}$. This cannot be the weighted digraph game of any $\delta \in \mathbb{R}^N_+$ on D since $v(E) = 0$ for all $E \subseteq N$ such that $E = P_D(i) \cup \{i\}$ for some $i \in N$, implying that all weights must be zero, but then $\overline{v}_{\delta,D}$ must be the zero game assigning worth zero to every coalition.
The game $r^l_{v,D} = u_{\{1,3,4,5\}}$ also cannot be the conjunctive restriction of some game $v \in \mathcal{G}^N$ on D since then $r^l_{v,D}(E)$ must be equal to $r^l_{v,D}(\sigma^c_D(E))$ for all $E \subseteq N$, but $u_{\{1,3,4,5\}}(\{1,3,4,5\}) = 1$ while $u_{\{1,3,4,5\}}(\sigma^c_D(\{1,3,4,5\}) = u_{\{1,3,4,5\}}(\{1,4\}) = 0$.

[6] We want to remark that the generalization of games with a permission structure to games with a local permission structure is different from the generalization to games on antimatroids (see Algaba, Bilbao, van den Brink and Jiménez-Losada (2003, 2004a, 2004b)) which considers cooperative games with restricted coalition formation where the set of feasible coalitions is an antimatroid. The set of locally feasible coalitions is not an antimatroid.

5 The local permission value

The *local (conjunctive) permission value* φ^l is the solution that assigns to every game with a permission structure the Shapley value of the locally restricted game, i.e.

$$\varphi^l(v, D) = Sh(r^l_{v,D}).$$

We provide an axiomatization of the local permission value using axioms similar to those used by van den Brink and Gilles (1996) for the conjunctive permission value. Player $i \in N$ is *inessential* in game with permission structure (v, D) if i and all its subordinates are *null* players in v, i.e., if $v(E) = v(E \setminus \{j\})$ for all $E \subseteq N$ and $j \in \widehat{S}_D(i) \cup \{i\}$. Player $i \in N$ is called *necessary* in game v if $v(E) = 0$ for all $E \subseteq N \setminus \{i\}$. A characteristic function $v \in \mathcal{G}^N$ is *monotone* if $v(E) \leq v(F)$ for all $E \subseteq F \subseteq N$. The class of all monotone characteristic functions on N is denoted by $\mathcal{G}^N_M$.

Efficiency For every $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$, it holds that $\sum_{i \in N} f_i(v, D) = v(N)$.

Additivity For every $v, w \in \mathcal{G}^N$ and $D \in \mathcal{D}^N$, it holds that $f(v + w, D) = f(v, D) + f(w, D)$, where $(v + w) \in \mathcal{G}^N$ is given by $(v + w)(E) = v(E) + w(E)$ for all $E \subseteq N$.

Inessential player property For every $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$, if $i \in N$ is an inessential player in (v, D) then $f_i(v, D) = 0$.

Necessary player property For every $(v, D) \in \mathcal{G}^N_M \times \mathcal{D}^N$, if $i \in N$ is a necessary player in v then $f_i(v, D) \geq f_j(v, D)$ for all $j \in N$.

These four axioms are satisfied by the conjunctive permission value φ^c as well as the local permission value φ^l. The next axiom is not satisfied by the local permission value.

Structural monotonicity For every $(v, D) \in \mathcal{G}^N_M \times \mathcal{D}^N$, if $i \in N$ and $j \in S_D(i)$ then $f_i(v, D) \geq f_j(v, D)$.

Theorem 5.1 *[van den Brink and Gilles (1996)] A solution is equal to the conjunctive permission value φ^c if and only if it satisfies efficiency, additivity, the inessential player property, the necessary player property and structural monotonicity.*

The local permission value does not satisfy structural monotonicity, as can be seen from the following example.

Example 5.2 *Consider the game with permission structures (v, D) on $N = \{1, 2, 3\}$ given by $D = \{(1, 2), (2, 3)\}$ and $v = u_{\{3\}}$. Then $\varphi^l(v, D) = (0, \frac{1}{2}, \frac{1}{2})$, and thus player 2 earns more than player 1, although 2 is a successor of 1 and the game is monotone.*

The local permission value satisfies a weaker version requiring the payoff of a player to be at least equal to the payoff of any of its successors in a monotone game if at least one of its successors is a necessary player.

Local structural monotonicity For every $(v, D) \in \mathcal{G}_M^N \times \mathcal{D}^N$, if $i \in N$ and $j \in S_D(i)$ are such that there exists at least one $h \in S_D(i)$ who is a necessary player in v, then $f_i(v, D) \geq f_j(v, D)$.

As mentioned above, the local permission value does satisfy the inessential player property. It even satisfies a stronger version of the inessential player property, requiring the payoff of a null player to be zero as soon as all its successors, but not necessarily all its subordinates, are null players in the game. We say that player $i \in N$ is *locally inessential* in game with permission structure (v, D) if i and all its successors are *null* players in v, i.e., if $v(E) = v(E \setminus \{j\})$ for all $E \subseteq N$ and $j \in S_D(i) \cup \{i\}$.

Local inessential player property For every $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$, if $i \in N$ is a locally inessential player in (v, D) then $f_i(v, D) = 0$.

It turns out that strengthening the inessential player property in this way and weakening structural monotonicity as done above characterizes the local permission value.

Theorem 5.3 *A solution is equal to the local permission value φ^l if and only if it satisfies efficiency, additivity, the necessary player property, the local inessential player property, and local structural monotonicity.*

Proof

It is straightforward to verify that the local permission value satisfies the five axioms.

The proof of uniqueness follows similar steps as the uniqueness proof for the conjunctive permission value in van den Brink and Gilles (1996). Suppose that solution f satisfies the five axioms. Let v_0 be the *null game* given by $v_0(E) = 0$ for all $E \subseteq N$. The local inessential player property then implies that $f_i(v_0, D) = 0$ for all $i \in N$.

Now, consider a scaled unanimity game (w_T, D) with $w_T = c_T u_T$ for some $c_T > 0$ and $\emptyset \neq T \subseteq N$. We distinguish the following three cases with respect to $i \in N$:

1. If $i \in T$ then the necessary player property implies that there exists a $c^* \in \mathbb{R}$ such that $f_i(w_T, D) = c^*$ for all $i \in T$, and $f_i(w_T, D) \leq c^*$ for all $i \in N \setminus T$.

2. If $i \in N \setminus T$ and $T \cap S_D(i) \neq \emptyset$ then local structural monotonicity implies that $f_i(w_T, D) \geq f_j(w_T, D)$ for every $j \in T \cap S_D(i)$, and thus with case 1 that $f_i(w_T, D) = c^*$.

3. If $i \in N \setminus T$ and $T \cap S_D(i) = \emptyset$ then the local inessential player property implies that $f_i(w_T, D) = 0$.

From 1 and 2 follows that $f_i(w_T, D) = c^*$ for $i \in T \cup P_D(T)$. Efficiency and 3 then imply that $\sum_{i \in N} f_i(w_T, D) = |T \cup P_D(T)|c^* = c_T$, implying that $c^* = \frac{c_T}{|T \cup P_D(T)|}$, and thus $f(w_T, D)$, is uniquely determined.
Next, consider (w_T, D) with $w_T = c_T u_T$ for some $c_T < 0$ (and thus we cannot apply the necessary player property and local structural monotonicity since w_T is not monotone). Since $-w_T = -c_T u_T$ with $-c_T > 0$, and $v_0 = w_T + (-w_T)$, it follows from additivity of f that $f(w_T, D) = f(v_0, D) - f(-w_T, D) = -f(-w_T, D)$ is uniquely determined because $-w_T$ is monotone.
Finally, since every characteristic function $v \in \mathcal{G}^N$ can be written as a linear combination of unanimity games $v = \sum_{T \subseteq N} \Delta_v(T) u_T$ (with $\Delta_v(T)$ the *Harsanyi dividend* of coalition T, see Harsanyi (1959)), additivity uniquely determines $f(v, D) = \sum_{T \subseteq N} f(\Delta_v(T) u_T, D)$ for any $(v, D) \in \mathcal{G}^N \times \mathcal{D}^N$. □

Instead of using local structural monotonicity, we can strengthen the necessary player property by saying that a player earns at least as much as any other player if this player is necessary or has at least one necessary successor in a monotone game.

Strong necessary player property For every $(v, D) \in \mathcal{G}_M^N \times \mathcal{D}^N$, if at least one of the players in $S_D(i) \cup \{i\}$ is a necessary player in v then $f_i(v, D) \geq f_j(v, D)$ for all $j \in N$.

Theorem 5.4 *A solution is equal to the local permission value φ^l if and only if it satisfies efficiency, additivity, the strong necessary player property and the local inessential player property.*

The proof is similar to that of Theorem 5.3, except that in case 2 the strong necessary player property is used instead of local structural monotonicity.

Another interesting difference between the conjunctive and local permission value is the following. Suppose that a player is going to veto one of its successors in the sense that in the original game the successor is not active without that player. So, for players $i \in N$ and $j \in S_D(i)$ we consider the game $v^{ij}(E) = v(E \setminus \{j\})$ if $E \subseteq N \setminus \{i\}$, and $v^{ij} = v(E)$ if $i \in E$.[7]

[7]This game is similar to one of the 'collusion' games in Haller, the proxy agreement game v_*^{ij}, where $v_*^{ij}(E) = v(E \setminus \{j\})$ if $E \subseteq N \setminus \{i\}$, and $v_*^{ij} = v(E \cup \{j\})$ if $i \in E$. So, the difference is that in v_*^{ij} player j is active if player i is present even when j itslef is not.

Veto monotonicity For every $(v, D) \in \mathcal{G}_M^N \times \mathcal{D}^N$ and $i, j \in N$ such that $j \in S_D(i)$, it holds that $f_i(v^{ij}, D) \geq f_i(v, D)$.

Proposition 5.5 *The conjunctive permission value φ^c satisfies veto monotonicity.*

As the following example shows the local permission value does not satisfy veto monotonicity.

Example 5.6 *Consider the game with permission structure (v, D) on $N = \{1, 2, 3\}$ given by $D = \{(1, 2), (2, 3)\}$ and $v = u_{\{3\}}$. Then $v^{23} = u_{\{2,3\}}$, and $\varphi^l(v, D) = (0, \frac{1}{2}, \frac{1}{2})$ while $\varphi^l(v^{23}, D) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. So, in (v, D) player 2 earns more than in (v^{23}, D).*

The reason why the local permission value does not satisfy veto monotonicity is clear from the example above. A player who is necessary but also has a necessary successor will share the payoff resulting from its own necessity with its predecessor. So, in case at least one of the successors of a player is necessary it is better for that player not to be neccessary since then it will still have its share in the payoff because it needs to give permission to a necessary successor, but because the player itself is not necessary it does not have to share with its own predecessors.

Applied to hierarchically structured firms this would imply that a manager is better off by having important (necessary) successors but not being necessary in the production process itself. In this way the manager will keep more influence because, when it would do a necessary task itself, it would have to ask permission from its predecessor to do this task. It is better for the manager to delegate this task to a successor, creating a situation that the execution of this necessary task is 'invisible' for its own predecessors. Another example are criminals who sometimes 'give' their property to others in order that for the tax office or prosecutor it looks as if they have no property. Of course, this works only if the authority relations within the criminal network are strong enough so that the top criminal can be sure that it has access to 'its' property when he or she wants.

6 Concluding remarks

In this paper we discussed a conjunctive approach to games with a local permission structure. As an alternative for the conjunctive approach, Gilles and Owen (1994) and van den Brink (1997) consider the disjunctive approach to games with an acyclic permission structure where it is assumed that a player needs permission of at least one of its predecessors (if it has any) in order to cooperate with other players. Therefore a coalition is feasible if and only if for every player in the coalition at least one of its predecessors (it it has any) is also in the coalition. So, for permission structure D the set of *disjunctive feasible coalitions* is given by $\Phi_D^d = \{E \subseteq N \,|\, P_D(i) \cap E \neq \emptyset$ for all $i \in E$ with $P_D(i) \neq \emptyset\}$.

For any $E \subseteq N$, let $\sigma_D^d(E) = \bigcup\{F \in \Phi_D^d \mid F \subseteq E\}$ be the largest disjunctive feasible subset of E in the collection Φ_D^d.[8] The induced *disjunctive restricted* game of the pair (v, D) in the disjunctive approach is the game $r_{v,D}^d \colon 2^N \to \mathbb{R}$, given by $r_{v,D}^d(E) = v(\sigma_D^d(E))$ for all $E \subseteq N$, i.e., the restricted game $r_{v,D}^d$ assigns to each coalition $E \subseteq N$ the worth of its largest disjunctive feasible subset. Then the *disjunctive permission value* φ^d is the solution that assigns to every game with a permission structure the Shapley value of the restricted game, thus $\varphi^d(v, D) = Sh(r_{v,D}^d)$.

Applying a disjunctive approach to games with a local (acyclic) permission structure, we can assume that every coalition can earn the worth that can be generated by those players in the coalition who have at least one predecessor in the coalition. A special case would be a new class of digraph games, where the weight of a player is earned by any coalition containing this player and any of its direct predecessors. Applying solutions such as the Shapley value then yield new power measures for digraphs. These *disjunctive digraph games* have a similarity with apex power games where, besides every coalition containing the apex player and at least one of its direct predecessors, additionally also the coalition of all direct predecessors earns the weight of a player, see van den Brink (2002) who defines power measures for digraphs using apex games. Whereas it seems that a disjunctive approach to games with a local permission structure can be done straightforward, it is less obvious how this generalizes the (standard) disjunctive games with a permission structure.

References

Algaba E, Bilbao JM, Borm P, López JJ (2000), "The position value for union stable systems", *Mathematical Methods of Operations Research*, 52, 221-236.

Algaba E, Bilbao JM, Borm P, López JJ (2001), "The Myerson value for union stable structures", *Mathematical Methods of Operations Research*, 54, 359-371.

Algaba, E., J.M. Bilbao, R. van den Brink, and A. Jiménez-Losada (2003), "Axiomatizations of the Shapley value for cooperative games on antimatroids,", *Mathematical Methods of Operations Research*, 57, 49-65.

Algaba, E., J.M. Bilbao, R. van den Brink, and A. Jiménez-Losada (2004a), "Cooperative games on antimatroids," *Discrete Mathematics*, 282, 1-15.

[8] Also, if D is acyclic then Φ_D^d is union closed.

Algaba, E., J.M. Bilbao, R. van den Brink, and A. Jiménez-Losada (2004b) "An axiomatization of the Banzhaf value for cooperative games on antimatroids", *Mathematical Methods of Operations Research*, 59, 147-166.

Ambec, S., and Y. Sprumont (2002), "Sharing a river", *Journal of Economic Theory* 107, 453-462.

Bondareva, O. (1963), "Some applications of linear programming methods to the theory of cooperative games", Problemy Kibernetiki, 10, 119-139 (in Russian).

Brânzei, R., V. Fragnelli, and S. Tijs (2002), Tree-connected peer group situations and peer group games, *Mathematical Methods of Operations Research*, 55, 93-106.

Brink, R. van den (1997), "An axiomatization of the disjunctive permission value for games with a permission structure", *International Journal of Game Theory*, 26, 27-43.

Brink, R.van den (2002), "The Apex power measure for directed networks", *Social Choice and Welfare*, 19, 845-867.

Brink, R. van den (2008), "Vertical wage differences in hierarchically structured firms", *Social Choice and Welfare*, 30, 225-243.

Brink, R. van den, and P. Borm (2002), "Digraph competitions and cooperative games", *Theory and Decision*, 53, 327-342.

Brink R. van den, and R.P. Gilles (1996), "Axiomatizations of the conjunctive permission value for games with permission structures", *Games and Economic Behavior*, 12, 113-126.

Brink, R. van den, and R.P.Gilles (2000), "Measuring domination in directed networks", *Social Networks*, 22, 141-175.

Brink, R. van den, and P.H.M. Ruys (2008), "Technology driven organizational structure of the firm", *Annals of Finance*, 4, 481-503.

Curiel, I., J. Potters, V. Rajendra Prasad, S. Tijs and B. Veltman (1993), "Cooperation in one machine scheduling", ZOR: Methods and Models of Operations Research, 38, 113-129.

Curiel, I., J. Potters, V. Rajendra Prasad, S. Tijs and B. Veltman (1994), "Sequencing and cooperation", Operations Research, 54, 323-334.

Derks, J., H. Haller, and H. Peters (2000), "The selectope for cooperative TU-games", *International Journal of Game Theory*, 29, 23-38.

Gilles, R. P., and G. Owen (1994), Cooperative games and disjunctive permission structures, Department of Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Gilles R.P., G. Owen, and R. van den Brink (1992), "Games with permission structures: the conjunctive approach", *International Journal of Game Theory*, 20, 277-293.

Gillies DB (1953) Some theorems on n-person games. Ph. D. Thesis, Princeton University Press, Princeton, New Jersey.

Graham, D., R. Marshall (1987), "Collusive behavior at single-object second-price and english auctions", *Journal of Political Economy*, 95, 1217-1239.

Graham, D.A., R.C. Marshall, and J-F. Richard (1990), "Differential payments within a bidder coalition and the Shapley value", *American Economic Review*, 80, 493-510.

Harsanyi, J.C. (1959), A bargaining model for cooperative n-person games, in: *Contributions to the Theory of Games IV* (A.W. Tucker and R.D. Luce, eds.), Princeton University Press, Princeton, NJ, pp. 325-355.

Littlechild, S.C, and G. Owen (1973), "A simple expression for the Shapley value in a special case", *Management Science*, 20, 370-372.

Maniquet, F. (2003), "A characterization of the Shapley value in queueing problems", *Journal of Economic Theory* 109, 90-103.

Maschler, M. (1992), The bargaining set, kernel, and nucleolus. In: Aumann RJ, Hart S(eds.) Handbook of Game Theory, with Economic Applications, volume 2, North Holland, pp. 591667.

Maschler, M., B. Peleg, and L.S. Shapley (1972), "The kernel and bargaining set for convex games", *International Journal of Game Theory* 1, 73-93.

Myerson, R. B. (1977), "Graphs and cooperation in games", *Mathematics of Operations Research*, 2, 225-229.

Ni, D., and Y. Wang (2007), "Sharing a polluted river", *Games and Economic Behavior* 60, 176-186.

Shapley, L. (1953), A value for n-person games, in: *Contributions to the Theory of Games II* (H.W. Kuhn and A.W. Tucker, eds.), Princeton University Press, Princeton, NJ, pp. 307-317.

Shapley, L. (1967), "On balanced sets and cores", *Naval Research Logistics Quarterly*, 14, 453-460.

Vasil'ev, V.A. and G. van der Laan , (2002), "The Harsanyi set for cooperative TU-games", *Siberian Advances in Mathematics*, 12, 97-125.

Williamson, O.E. (1967), "Hierarchical control and optimum firm size", *Journal of Political Economy*, 75, 123-138.