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*Hugo E. Silva*

*Erik T. Verhoef\**

*Faculty of Economics and Business Administration, VU University Amsterdam.*

*\* Tinbergen Institute.*

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# Optimal pricing of flights and passengers at congested airports: the efficiency of atomistic charges\*

Hugo E. Silva\*, Erik T. Verhoef<sup>1</sup>

*Department of Spatial Economics, VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands*

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## Abstract

This paper investigates optimal airport pricing when airlines provide imperfect substitutes products, and make decisions on capacity, scheduling and pricing. We show that the first-best toll per flight may be higher than the simple market-shares formulae that were recently derived for Cournot models, and approaches the atomistic toll (which ignores the airlines' internalization of self-imposed congestion) as products become closer substitutes. This increases the relevance of congestion pricing and does not require leadership behavior. We also find that an airport requires two pricing instruments to achieve the first-best outcome: per-passenger subsidies to counteract airlines' market power, and per-flight tolls to correct congestion externalities. We numerically analyze second-best policies of having only one tax instrument, as well as the performance of atomistic pricing, and find that the latter may offer a more attractive alternative than what is suggested by simpler Cournot models.

*Keywords:* Airport pricing, Congestion internalization, Atomistic tolls

*JEL codes:* H21, H23, L13, L93, R48

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## 1. Introduction

Delays at airports have been consistently increasing over the last years, becoming a major problem worldwide (see, for example, Rupp (2009) and Santos and Robin (2010)). Besides capacity enlargements, the price mechanism has been widely discussed and proposed to manage congestion. This approach consists of an airport authority setting user charges, with the possibility to charge passengers, airlines or both.<sup>2</sup> Many papers have studied optimal airport pricing: for example, Brueckner (2002) first showed that in oligopoly, airlines competing in a Cournot fashion internalize congestion imposed on themselves; therefore, the optimal charge should account only for the fraction of congestion that is imposed on competitors. We will refer to this as the “Cournot toll”, as opposed to the “atomistic toll” that considers marginal congestion costs imposed on all other flights and passengers, regardless of the operator. One important implication of a Cournot toll is that a dominant airline should pay a lower charge per flight than small airlines (Brueckner, 2005). Pels and Verhoef (2004) extend the analysis by explicitly considering market power distortions. They show that a welfare maximizing airport has to correct two effects: airlines' market power by subsidizing them, and congestion externalities with the Cournot toll. Further extensions found that this congestion pricing rule remains optimal.<sup>3</sup>

However, optimal congestion toll has been subject of continuous debate over the last decade, and the desirability of the Cournot toll has been questioned, also from the empirical side. In contrast to the road case, where users behave atomistically, the relevant question in aviation markets is what share of congestion airlines actually do internalize when making scheduling decisions of flights. If they internalize a high proportion of congestion costs, charges that optimally account for this will be relatively low and thus should have a small impact on flight patterns and social welfare. This internalization hypothesis, based on Brueckner's analysis, is supported by empirical evidence by Mayer and Sinai (2003) with U.S. data, and by Santos and Robin (2010) with European data. Nevertheless, Daniel (1995), who first identified the potential for internalization of congestion, argues—with a simulation model—that atomistic behavior may in fact be more pertinent from an empirical point of view; i.e., that airlines do not take into account self-imposed congestion when making scheduling decisions. As

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\*Corresponding author

*Email addresses:* h.silvamontalva@vu.nl (Hugo E. Silva), e.t.verhoef@vu.nl (Erik T. Verhoef)

<sup>1</sup>Affiliated to the Tinbergen Institute, Roetersstraat 31, 1018 WB Amsterdam, The Netherlands.

<sup>2</sup>Another approach to manage airport congestion is to use slot sales and slot trading; see Brueckner (2009), Basso and Zhang (2010) and Verhoef (2010).

<sup>3</sup>Zhang and Zhang (2006) explicitly incorporate airport's costs and capacity decisions, and Basso (2008) includes airline differentiation, origin and destination airports and schedule delay costs. Basso and Zhang (2007) analyze rivalry between two congestible facilities, which again could be airports.

a consequence, the optimal toll should be the so-called atomistic toll that ignores any internalization, and that is equal to total marginal congestion costs. The atomistic behavior of airlines is further supported by empirical evidence by Daniel and Harback (2008) and by Rupp (2009), who contributes to the internalization debate by showing that “airlines are not internalizing passenger delay cost, and are hence behaving more like atomistic competitors” (p. 26). Moreover, from a different perspective, Morrison and Winston (2007) argue in favor of levying atomistic tolls at congested airports, because they find a small net benefit loss when an airport charges the atomistic toll instead of the Cournot toll.

This divergence between views on the extent to which atomistic versus Cournot tolls are more desirable has been addressed before. For example, Brueckner and Van Dender (2008) present a theoretical analysis showing that when one airline acts as a Stackelberg leader and interacts with a large number of fringe carriers, the leader behaves atomistically as long as the products are perfect substitutes. In this case, the optimal congestion toll would be the atomistic charge, because congestion is not being internalized by any carrier. Although theoretically valid, the Stackelberg setting with carriers behaving in a perfectly competitive fashion seems to be restrictive. Czerny and Zhang (2011) present a different argument and state that, when travelers with different values of time are considered, and in absence of price discrimination by carriers, it might be useful to increase the airport charge towards the atomistic toll. This is to protect passengers with a relatively high value of time from congestion caused by passengers with a relatively low value of time. Obviously, their analysis cannot explain differences in findings between models that consider homogeneous values of time.

The purpose of this paper is to show that the optimal congestion charge per flight can be higher than the Cournot toll and can be significantly close to the atomistic toll, under rather plausible assumptions. We find that airlines’ internalization of congestion can be close to atomistic behavior, even when airlines fully recognize the impact of their flight scheduling on airport congestion, and in absence of leadership behavior (in contrast to Brueckner and Van Dender (2008)), and without heterogeneity in values of time (in contrast to Czerny and Zhang (2011)). We show, in a duopoly setting where outputs are imperfect substitutes, that airlines can internalize less than the self-imposed congestion, because they take into account the fact that an extra flight imposes congestion on its competitor’s passengers, affecting positively its own demand and profit. This new scheduling incentive for airlines, yields an optimal congestion toll that lies between the marginal congestion cost imposed on the competitors’ passengers (the Cournot toll) and the atomistic toll. We also find that the size of the deviation from the self-imposed internalization result of Cournot competition depends only on the degree of product differentiation. The entire range of tolls can be optimal: when the differentiation is strong, the optimal congestion charge is close to the Cournot toll. Conversely, if the differentiation is weak, airlines should be charged a toll very close to the atomistic one, even when one of them is a dominant airline with a high market share. This result may help explaining why the empirical and simulation studies provide such a wide range of estimations regarding internalization, and this finding provides new arguments in favor of applying congestion pricing at airports.

The result holds in a setting where airlines compete simultaneously taking rival’s decisions (aircraft size, frequency and fares) as given; but also when airlines are engaged in a competition where fares are chosen in the last stage, while frequency and aircraft size competition occurs in previous stages, as long as they are unable to observe rival’s play. On the other hand, when they are engaged in this sequential competition, but they observe the competitor’s play, the outcome is the same as if they take rival’s traffic (quantity) as given, therefore the optimal toll is the Cournot toll. We also reproduce the result of Brueckner and Van Dender (2008) for a Stackelberg leader with a Cournot follower, and extend it to the case of a Stackelberg leader with a Bertrand follower, finding optimal tolls that lie between the Cournot toll and the atomistic toll for both players. In this last setting, the optimal toll again approaches the atomistic toll as the differentiation is weaker.

Various policy implications follow from the analysis. If the firms behave taking the rival’s fare as given (instead of traffic), the optimal toll can be anywhere in between the marginal congestion cost imposed on the competitors’ passengers, and the atomistic toll. Perloff et al. (2007), in an empirical study on airlines conduct in a duopoly market, estimate that this behavior, in fact, better describes the industry.<sup>4</sup> If this is the case, congestion pricing may bring more significant welfare gains and congestion reductions. We present some numerical examples to assess the relative efficiency of second-best policies, and, for example, find that only using a congestion per-flight charge and levying atomistic tolls yield substantial benefits when airlines do not behave in a Cournot fashion, and when the degree of product differentiation is not too strong.

The paper is organized as follows. First, in Section 2, we introduce the model that includes aircraft size, fare and frequency decisions in an oligopolistic airline market, and that formally takes into account market power exertion and (potential) congestion internalization. In Section 3 we derive analytical solutions for the airports’ problem, specifically first-best tolls and optimal capacity investment. We show that a welfare maximizing airport can only reach the first-best outcome by using two tax instruments, namely per-flight and per-passenger tolls. Moreover, congestion and market power effects are separate: the market power exertion can only be corrected by means of a per-passenger subsidy, while the optimal congestion charge should only be charged with the per-flight toll. Section 4 presents numerical exercises to quantify the analytical results, to assess the efficiency of second-best policies, and to study the performance of levying atomistic tolls. Finally, Section 5 concludes.

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<sup>4</sup>This is found using the dataset of Brander and Zhang (1990) and Oum et al. (1993), that has been used to support Cournot behavior.

## 2. Airlines' duopoly model

For the analysis, we consider a vertical setting on a single market, i.e., a single origin-destination pair. In the first stage an airport chooses capacity, toll per flight and toll per passenger charged to the carriers that use the facility. In the second stage, a duopoly of airlines compete with aircraft size, frequency and fare as decision variables. Following Zhang and Zhang (2006), we model only one airport for analytical simplicity, but the conclusions remain the same if the other airport is included, as long as the airports share the objective function (e.g. they perform joint welfare maximization).<sup>5</sup>

For the airlines' market, we consider the differentiated duopoly proposed by Dixit (1979), assuming that demands arise from the following quadratic utility function:

$$U(q_i, q_j) = A \cdot (q_i + q_j) - (B \cdot q_i^2 + 2 \cdot E \cdot q_i \cdot q_j + B \cdot q_j^2)/2, \quad (1)$$

where  $q_i$  is the amount of good  $i$  (hereafter, when subscript  $j$  appears in the same expression with  $i$ , it refers to the rival airline),  $A$ ,  $B$  and  $E$  are positive parameters with  $B > E > 0$  so that goods are imperfect substitutes.<sup>6</sup> This utility function gives rise to a linear demand structure, with inverse and direct demands:

$$\theta_i = A - B \cdot q_i - E \cdot q_j, \quad (2)$$

$$q_i = a - b \cdot \theta_i + e \cdot \theta_j, \quad (3)$$

where  $\theta_i$  is the full price of good  $i$  and parameters  $a$ ,  $b$  and  $e$  satisfy  $a = A/(B + E)$ ,  $b = B/(B^2 - E^2)$  and  $e = E/(B^2 - E^2)$ . The full price of traveling with airline  $i$  is assumed to be:

$$\theta_i = p_i + D + g_i, \quad (4)$$

where  $p_i$  is the fare and the passengers' cost of congestion is represented by  $D$ : the congestion delay experienced at airports.  $D$  depends on airport capacity ( $K$ ) and on the total number of take offs and landings at the congested airport ( $F = f_i + f_j$ ). Finally,  $g_i$  is the schedule delay cost faced by a passenger that travels with airline  $i$ , which depends only on the flight frequency of the airline ( $f_i$ ). The fact that schedule delay does not depend on rival's frequency, as congestion does, reflects our assumption that in the differentiated duopoly, frequency is perceived as an airline-specific attribute.

We make the plausible assumptions that  $D$  is differentiable in  $F$ , that  $g_i$  is differentiable in  $f_i$  and that:

$$\begin{aligned} \frac{\partial D}{\partial f_i} > 0, \quad \frac{\partial^2 D}{\partial f_i^2} = \frac{\partial^2 D}{\partial f_i \partial f_j} \geq 0, \quad \frac{\partial D}{\partial K} < 0, \\ \frac{\partial^2 D}{\partial K \partial f_i} < 0, \quad \frac{\partial g_i}{\partial f_i} < 0, \quad \frac{\partial^2 g_i}{\partial f_i^2} > 0, \quad \forall i. \end{aligned} \quad (5)$$

Congestion thus increases with the number of flights and the effect is stronger when congestion is more severe; congestion decreases with airport's capacity; schedule delay cost decreases with airline-specific frequency, and that effect is smaller when frequency is higher.<sup>7</sup>

Following Brueckner (2004), we model airlines' cost ( $C_i$ ) as a function of aircraft size and frequency in the following way:

$$C_i = f_i \cdot (\gamma_i^f + \gamma_i^s \cdot s_i), \quad (6)$$

where  $\gamma_i^f$  and  $\gamma_i^s$  are positive cost parameters and  $s_i$  is the number of seats per flight. The underlying assumption is that cost per flight is a linear function of the number of seats, a relation that has been also found in a cost-engineering study for airlines by Swan and Adler (2006).<sup>8</sup> Congestion costs for airlines are not considered in the analysis because we focus on

<sup>5</sup>If airports are not regulated by the same authority or if airports independently maximize profits both airports have to be formally modelled. For a discussion on the implications of two local welfare maximizing airports see Pels and Verhoef (2004) and for a discussion on independent profit maximization see Basso (2008).

<sup>6</sup>The imperfect substitution allows to have airlines in equilibrium with different generalized price and traffic. The fact that an airline can have demand despite having a higher generalized price than the competitor has been also modelled with a brand-loyalty variable that gives the additional gain from travelling with a specific airline relative to travel with the other airline (Brueckner and Flores-Fillol, 2007; Flores-Fillol, 2010). In our model it is also possible to model a preference for a specific carrier by letting the demand parameters ( $B$  and  $E$ ) vary across carriers.

<sup>7</sup>This set of assumptions is common in the literature: the linear delay function used by Pels and Verhoef (2004) and the convex function used by Zhang and Zhang (2006) satisfy the assumptions regarding  $D$ . The schedule delay function that is inversely proportional to the airline frequency satisfies the conditions for  $g_i$  (see Brueckner (2004) and Basso (2008) for a discussion).

<sup>8</sup>In this work, a cost function per trip is calibrated using distance and aircraft size as explanatory variables, then holding distance fixed the function is linear in number of seats.

passengers' congestion, but including them would not change the results in any essential way because it only changes  $C_i$  and does not modify the demand structure.<sup>9</sup>

With the cost function defined, we can now write the profit of airline  $i$  as:

$$\pi_i = q_i \cdot p_i - f_i \cdot (\gamma_i^f + \gamma_i^s \cdot s_i) - f_i \cdot \tau_i^f - q_i \cdot \tau_i^q, \quad (7)$$

where  $\tau_i^f$  is the per-flight toll charged by the airport and  $\tau_i^q$  the toll charged per passenger.

One of the goals of this paper is to assess the impact of different kinds of strategic interaction. In modelling the airlines' competition, we study a closed-loop game where airlines, first, simultaneously decide frequency and aircraft size, and fares in a second stage. We thereafter look at an open-loop game, where airlines do not observe the play of the others or, equivalently, aircraft size, frequency and fares are decided simultaneously. Finally, we study two Stackelberg settings, where the leader chooses all the relevant variables prior to a follower who takes the rival's traffic volume as given, or the rival's fare as given.

### 2.1. Closed-Loop

In this game setting, there are two stages: airlines first simultaneously choose aircraft size and frequency and, in the second stage, they compete on fares (note that an airline can transport at most  $f_i \cdot s_i$  passengers, hence in the first stage they also make a capacity decision). We solve this game using backward induction, finding that airlines' unique equilibrium is to choose frequency, fares and aircraft size according to the following conditions (the complete derivations are presented in Appendix A).

$$p_i = \gamma_i^s + \tau_i^q + q_i \cdot B, \quad (8)$$

$$\gamma_i^f + \tau_i^f = -q_i \cdot \left( \frac{\partial D}{\partial f_i} + \frac{\partial g_i}{\partial f_i} \right), \quad (9)$$

$$q_i = s_i \cdot f_i. \quad (10)$$

Equation (8) states that the fare charged by an airline has three terms: (i) the marginal cost per capacity unit ( $\gamma_i^s$ ); (ii) the airport charge per passenger ( $\tau_i^q$ ) and (iii) a conventional monopolistic markup reflecting carrier's market power, which is related to the sensitivity of demand and own demand ( $q_i \cdot B$ ). Equation (9) states that airline's marginal cost per flight equals marginal benefits for own passengers (marginal congestion savings plus marginal schedule delay benefits); therefore, airlines internalize own-passenger congestion. The last equation shows that airlines do not have idle capacity.

These rules basically describe that airlines internalize congestion on their own passengers and charge a markup which equals  $q_i \cdot \partial \theta_i / \partial q_i$ , a result analogous to the rules obtained previously in Cournot competition (e.g. Pels and Verhoef, 2004; Zhang and Zhang, 2006; Basso, 2008).<sup>10</sup>

Moreover, solving a game with aircraft size ( $s_i$ ), frequency ( $f_i$ ) and traffic ( $q_i$ ) as decision variables, we also obtain equations (8)–(10) as first-order conditions. Therefore, this closed-loop game is equivalent to assuming that airlines act simultaneously taking competitor's quantity as given.<sup>11</sup> From now on, we refer to Cournot internalization to the result obtained in this game setting, i.e. internalization of congestion imposed on own-passengers. This closed-loop setting is also equivalent with a three stage game where airlines make the decision of aircraft size in the first stage, because, once the fare and frequency are chosen, demands are already set. Hence, idle seat capacity is not optimal.

It is worth noting some aspects about airlines' behavior that arises from this model. Airlines do not charge passengers for congestion because they set frequency and fare separately; for any given demand, they internalize own-passengers congestion by setting frequency according to (9) and adjusting aircraft size to accommodate the passengers. In a fixed-proportions model, an additional passenger necessarily increases delays and the only way to internalize this is by charging self-imposed marginal congestion costs to passengers. But, when aircraft size is a strategic variable, this is no longer desirable, because they can accommodate a new passenger by increasing aircraft size by  $1/f_i$  at a cost of  $\gamma_i^s$  (which they do charge to passengers, see (8)) without raising delays. This also explains why the per-flight toll ( $\tau_i^f$ ) is absent in the airlines' fare: it affects frequency and aircraft size setting, while keeping the cost per passenger constant at  $\gamma_i^s + \tau_i^q$ .

<sup>9</sup>Congestion imposed on airlines works out in a way similar to congestion imposed on passengers. If congestion costs are included, airlines would not internalize congestion costs imposed on the competitors' flights and this should be corrected in first-best tolls, as found by Basso (2008), Brueckner (2009) and Verhoef (2010).

<sup>10</sup>The second term in the right-hand side of equation (9) is present in previous studies including schedule delay cost. It is in Brueckner (2004) for a monopoly and it is in Basso (2008), but does not appear in the pricing rule of airports because it is set optimally by a private airline from a social welfare perspective.

<sup>11</sup>The intuition of this result comes from Kreps and Scheinkman (1983) who show that under mild demand assumptions, Cournot outcomes are the unique equilibrium in a two-stage game where firms first choose capacity and then prices. In our game, the key differences are the imperfect substitutability and the fact that first stage is more than capacity investment since it affects generalized price through frequency. Proof is in Appendix A.

## 2.2. Open-Loop

The open-loop game can take the same two-stage dynamic game structure as the closed-loop, but airlines cannot observe rival's play. This setting is analytically equivalent to simultaneous decisions. Therefore, the problem faced by an airline is to maximize profit (equation 7) taking rival's frequency, aircraft size and fare as given. Because having idle seat capacity only decreases profit, it is straightforward that an airline will set the number of seat such that aircraft is filled ( $s_i = q_i/f_i$ ); this allows us to express profit in terms of fare and frequency. Using the same argument, this open-loop game is equivalent with a two-stage game where airlines first compete on aircraft size and then on frequency and fares.

Rewriting equation (7) and explicitly including the functions' arguments (without including rival's variables since they are taken as given) we get:

$$\pi_i(p_i, f_i) = q_i(p_i, f_i) \cdot (p_i - \gamma_i^s - \tau_i^q) - f_i \cdot (\gamma_i^f + \tau_i^f). \quad (11)$$

Then, first-order conditions with respect to fare and frequency yield:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Rightarrow p_i = \gamma_i^s + \tau_i^q + \frac{q_i}{b}, \quad (12)$$

$$\frac{\partial \pi_i}{\partial f_i} = 0 \Rightarrow \gamma_i^f + \tau_i^f = (p_i - \gamma_i^s - \tau_i^q) \cdot \frac{\partial q_i}{\partial f_i}. \quad (13)$$

Again, the fare charged by an airline includes the marginal cost per capacity unit ( $\gamma_i^s$ ), the airport charge per passenger, and a market-power markup. The difference with the closed-loop game is that the market power effect is now weaker ( $1/b < b/(b^2 - e^2) = B$ ). The equivalence between the closed-loop game and Cournot competition mentioned earlier helps explaining why the market power effect is stronger in the closed-loop game: when airlines take rival's price as given, the outcome is more competitive than when they take rival's quantity as given (see Singh and Vives (1984) for a discussion in a general context).

From the frequency first-order condition (13), we get that frequency is set optimally by equating marginal cost per flight to the revenue gain from an extra flight, as found by Brueckner (2004). For further interpretation, note that using equation (3) we can rewrite  $\partial q_i/\partial f_i$  as:

$$\frac{\partial q_i}{\partial f_i} = -b \cdot \frac{\partial \theta_i}{\partial f_i} + e \cdot \frac{\partial \theta_j}{\partial f_i} = -b \cdot \left( \frac{\partial D}{\partial f_i} + \frac{\partial g_i}{\partial f_i} \right) + e \cdot \frac{\partial D}{\partial f_i}. \quad (14)$$

This expression shows that the effect of an increase in the airline's number of flights, has two effects on its demand: it changes both schedule delay cost and congestion for own passengers, but it also increases the congestion experienced by competitor's passengers when its frequency is fixed (or taken as given). The second effect has a positive impact for the airline, since increasing competitor's congestion raises own demand due to the fact that airlines offer (imperfect) substitute outputs.

Using  $p_i - \gamma_i^s - \tau_i^q = q_i/b$  from equation (12) and equation (14), we can rewrite (13) as:

$$\gamma_i^f + \tau_i^f = -q_i \cdot \left( \frac{\partial D}{\partial f_i} \cdot \left( 1 - \frac{e}{b} \right) + \frac{\partial g_i}{\partial f_i} \right). \quad (15)$$

This equation defines how an airline sets frequency. It differs from equation (9) for the closed-loop game in the term multiplying marginal congestion costs. In this game, an airline internalizes congestion imposed on its own passengers but also takes into account the congestion imposed on its competitor, as explained above. This is represented by the degree of differentiation  $e/b$  that appears in equation (15).

This term causes a difference with the common internalization finding, and with the closed-loop game result, because now airlines are not taking the competitor's traffic as given. In Cournot competition, airlines believe that they are not able to influence the competitor's traffic by raising congestion, simply because they do not "see" the effect by assumption. On the other hand, when taking the competitor's fare together with frequency as given, traffic is the result of setting the generalized price through the two variables. Therefore, airlines realize that they can influence competitor's traffic, or increase own demand, by raising the rival's congestion.

The size of the deviation from the traditional result of internalization depends directly on the degree of differentiation  $e/b$ . This ratio ranges from 0 when products are completely independent to 1 when products are perfect substitutes. The fraction of runway congestion considered by an airline, when setting frequency, is given by the ratio of congestion terms from equation (15) and total marginal congestion costs:

$$\frac{q_i \cdot D' \cdot (1 - e/b)}{(q_i + q_j) \cdot D'} = \frac{q_i}{q_i + q_j} \cdot \left( 1 - \frac{e}{b} \right). \quad (16)$$

As  $e < b$ , carriers act as if they internalize less congestion than what is imposed on their own passengers, as in the closed-loop game and in Brueckner's (2002) Cournot model. Only when products are close to be independent, the effective internalization approaches the market share. When they are close to be homogeneous, airline behavior approaches atomistic behavior. For example, if the output is symmetric and the ratio of differentiation is 0.5, airlines internalize only 25 percent of congestion costs, instead of one half.

Which one of the settings presented so far is more appropriate depends on market-specific conditions. One argument in favor of the closed-loop setting is the contrast between the high number of fare adjustments performed by airlines and the comparatively slow process of changing frequency and aircraft size. On the other hand, the private nature of airlines and the fact that carriers serve several routes can be used to support the difficulty of observing rival's capacity decisions on a specific market, which leads to the open-loop game outcome.

There are few empirical studies that estimate airlines' strategy. Brander and Zhang (1990) and Oum et al. (1993) estimations—which have been used to support Cournot behavior—are well summarized by the latter's conclusion that “the overall results indicate that the duopolists' conduct may be described as somewhere between Bertrand and Cournot behavior, but much closer to Cournot, in the majority of the sample observations” (p. 189). This means that fares generally exceed marginal costs (Bertrand outcome) and by less than the “Cournot markup” (in our notation  $q_i \cdot \partial \theta_i / \partial q_i = q_i \cdot B$ ). The results of the closed-loop game are consistent with Cournot behavior while the open-loop fare (equation 12) is somewhere between marginal cost and Cournot fare, because airlines take rivals' fare as given in a context of imperfect substitution. In a recent work, Perloff et al. (2007) estimate airlines pricing strategies using the same database as the previous studies, but with several methods and allowing for the possibility that airlines provide differentiated services on a route. They show that the average Bertrand Lerner index is virtually the same as the average Lerner index estimated from the data with all the methods, while the Cournot and collusive indexes are much higher. This provides empirical evidence that supports the open-loop setting as representative.

### 2.3. Stackelberg leader and Cournot follower

In the Cournot-Stackelberg setting we suppose that airline  $i$  is a leader and airline  $j$  the follower that chooses traffic ( $q_j$ ), frequency ( $f_j$ ) and aircraft size ( $s_j$ ) taking the leader's strategic variables as given. As we explained in Section 2.1, this is equivalent to the closed-loop setting and therefore the follower's behavior is characterized by first-order conditions (8)-(10). The leader maximizes profit knowing the response of the follower to its own decisions. As in previous settings, it is not optimal to have idle capacity, so the profit function of the leader is:

$$\pi_i(q_i, f_i) = q_i \cdot [p_i(q_i, f_i, q_j, f_j) - \gamma_i^s - \tau_i^q] - f_i \cdot (\gamma_i^f + \tau_i^f), \quad (17)$$

where both  $q_j$  and  $f_j$  depend on the leader's choice variables.

First-order conditions with respect to traffic and frequency yield:

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow p_i = \gamma_i^s + \tau_i^q + q_i \cdot \left( B + E \cdot \frac{\partial q_j}{\partial q_i} + \frac{\partial D}{\partial f_i} \cdot \frac{\partial f_j}{\partial q_i} \right), \quad (18)$$

$$\frac{\partial \pi_i}{\partial f_i} = 0 \Rightarrow \gamma_i^f + \tau_i^f = -q_i \cdot \left[ \frac{\partial D}{\partial f_i} \cdot \left( 1 + \frac{\partial f_j}{\partial f_i} \right) + \frac{\partial g_i}{\partial f_i} + E \cdot \frac{\partial q_j}{\partial f_i} \right]. \quad (19)$$

Since the follower's responses are downward-sloping (the proof is in Appendix B), the leader sets a higher quantity than an airline that takes rival's traffic as given (as the follower does). As a consequence, the leader's market power effect is weaker (for a given frequency, higher traffic implies a lower fare).

Regarding frequency setting (first-order condition 19), as the leader anticipates the way the follower reacts, the incentives to reduce frequency are different from those in the Cournot case, because of two effects. First, the leader predicts that any frequency reduction is partially offset by an increase on number of flights by the follower ( $\partial f_j / \partial f_i < 0$ ). As can be seen in (19) the term involving marginal congestion is cut by this expression which situates leader's internalization in between the Cournot case of self-imposed congestion and the atomistic behavior, just as pointed out by Brueckner and Van Dender (2008). The second effect—not directly related to marginal congestion—is the last term multiplying  $q_i$  on the right hand-side of equation (19), which further reduces the internalization. The leader realizes that any frequency increase induces a reduction on follower's traffic, therefore the frequency reduction incentive is diminished. The overall effective internalization is in between congestion imposed on own passengers and atomistic behavior, but the magnitude of the deviation depends, among other things, on the degree of differentiation. In Section 4, we quantify the deviation and expand more on this matter.



#### 2.4. Stackelberg leader and Bertrand follower

Finally we solve a Stackelberg game with airline  $i$  as a leader and airline  $j$  as a follower that chooses fare ( $p_j$ ), frequency ( $f_j$ ) and aircraft size ( $s_j$ ). The only difference with the previous setting is that the follower takes the leader's fare as given instead of traffic. We refer to this setting as the Bertrand-Stackelberg game.

The problem for the follower is the same as in the open-loop game, i.e. maximize profit (equation 11) with respect to fare and frequency, yielding the same first-order conditions (equations 12 and 13). For the leader, the profit function now is:

$$\pi_i(p_i, f_i) = q_i(p_i, f_i, p_j, f_j) \cdot (p_i - \gamma_i^s - \tau_i^q) - f_i \cdot (\gamma_i^f + \tau_i^f). \quad (20)$$

First-order conditions with respect to fare and frequency yield:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Rightarrow p_i = \gamma_i^s + \tau_i^q + \frac{q_i}{b}, \quad (21)$$

$$\frac{\partial \pi_i}{\partial f_i} = 0 \Rightarrow \gamma_i^f + \tau_i^f = -q_i \cdot \left[ \frac{\partial D}{\partial f_i} \left(1 - \frac{e}{b}\right) \left(1 + \frac{\partial f_j}{\partial f_i}\right) + \frac{\partial g_i}{\partial f_i} - \frac{e}{b} \left( \frac{\partial p_j}{\partial f_i} + \frac{\partial g_j}{\partial f_j} \frac{\partial f_j}{\partial f_i} \right) \right] \cdot \frac{b}{\tilde{b}}, \quad (22)$$

where  $\tilde{b} \equiv b - e \cdot \partial p_j / \partial p_i - \partial q_i / \partial f_j \cdot \partial f_j / \partial p_i$ .

In this game, follower's reaction functions with respect to frequency are downward-sloping while the reaction functions with respect to fare are upward-sloping (see Appendix B). This feature makes that in both first-order conditions for the leader (equations 21 and 22), there are opposite effects that prevent from quantifying analytically the effects. In fare setting it is not possible to state whether the market power effect is higher or lower in comparison with the open-loop game.

Concerning frequency choice, we find the same effect that reduces internalization due to the follower at least partly offsetting reductions in the number of flights ( $\partial f_j / \partial f_i < 0$ ). But, in this case, the sign of the second effect (last term in brackets on the right-hand side of equation 22) cannot be determined a priori, so the internalization reduction with respect to the open-loop might well be counteracted. In Section 4 we study the magnitude of these effects with numerical examples, suggesting that the leader internalization can be less than in the open-loop game, but still in between Cournot and atomistic behavior.

As we discuss in Section 1, there are empirical studies that support the internalization hypothesis as well as studies that reject it. We believe that our model helps in explaining such a wide range of findings. When airlines take rival's fare, frequency and aircraft size as given (open-loop and Bertrand-Stackelberg setting) the degree of internalization is in between non-internalization and the self-imposed congestion. Moreover, it depends on demand-structure parameters (the ratio of differentiation  $e/b$ ) and there is empirical evidence supporting this behavior. Therefore, it is possible that in some market an airline behaves almost atomistically, even if it is dominant, while in others it internalizes a big share of congestion.

With the airlines market characterized, we now analyze airport pricing and capacity investments for the airlines model we propose.

### 3. Airport pricing and capacity investment

We consider the first-best case of a (unweighted) welfare maximizing airport with capacity and both per-flight and per-passenger toll as instruments. We solve the airport maximization problem analytically in this section, and numerically in Section 4. The derivation of first-order conditions presented in this section is in Appendix C.

Social welfare is defined as the sum of benefits for each agent: consumer surplus, airlines' profits and airport's profit. The first of this, with quantities and full-prices being  $q_i$ ,  $q_j$ ,  $\theta_i$  and  $\theta_j$ , is just  $U(q_i, q_j) - \theta_i \cdot q_i - \theta_j \cdot q_j$ . Using (1) and (2), straightforward calculations yield the following expression:

$$CS = \frac{B}{2} \cdot (q_1^2 + q_2^2) + E \cdot q_1 \cdot q_2. \quad (23)$$

We assume that airport's costs are separable and proportional to traffic, frequency and capacity, shaping airport profits in the following way:

$$\Pi = \sum_i \left( q_i \cdot (\tau_i^q - c_q) + f_i \cdot (\tau_i^f - c_f) \right) - K \cdot r, \quad (24)$$

where  $c_q$  is the (constant) operating cost per-passenger,  $c_f$  the (constant) operating cost per-flight,  $r$  the cost of capital and  $K$  the capacity of the airport.

Adding airlines profit (equation 7) we can express social welfare as a function of traffic, frequencies and fares:

$$SW = \left[ \frac{B}{2}(q_1^2 + q_2^2) + E \cdot q_1 \cdot q_2 \right] + \left[ \sum_i q_i \cdot (p_i - c_q) \right] - \left[ \sum_i f_i \cdot (\gamma_i^f + s_i \cdot \gamma_i^s + c_f) \right] - [K \cdot r], \quad (25)$$

where the first bracketed term is consumer surplus, the second bracketed term is airlines' and airport's per-passenger revenues minus costs (airport's revenues from tolls cancel out against airlines' costs from tolls), the third is per-flight revenues minus costs (again airport's tolls cancel out) and the last bracketed term is capacity costs. Both traffic volumes are a function of fares and frequencies, but we omit the arguments.

Straightforward calculations lead to the following conditions for optimal fares.

$$p_i - \gamma_i^s - c_q = 0, \quad \forall i. \quad (26)$$

This result states that optimal fare must equal airlines' marginal cost per capacity unit plus airport marginal operating cost per passenger. The welfare maximizing fare that should be charged to passengers does not include any congestion term because—as explained in the previous section—airlines take congestion into account only in their frequency setting. From first-order conditions for frequency we obtain the following:

$$-(q_i + q_j) \cdot \frac{\partial D}{\partial f_i} - q_i \cdot \frac{\partial g_i}{\partial f_i} = c_f + \gamma_i^f, \quad \forall i. \quad (27)$$

This means that the optimal frequency must be such that the marginal cost per flight (right-hand side) equals marginal net benefits of all passengers from congestion plus schedule delay savings (left-hand side). We define “total marginal congestion costs” as the congestion cost that an extra flight imposes on all passengers ( $\partial D / \partial f_i \cdot (q_i + q_j)$ ). If airlines do not internalize any congestion at all, they should be charged this amount plus the airport's marginal operating cost per flight ( $c_f$ ), the so-called atomistic toll.

Finally, the optimal investment rule for the airport is:

$$-(q_i + q_j) \cdot \frac{\partial D}{\partial K} = r. \quad (28)$$

This shows that airport capacity is increased until marginal cost equals marginal benefits from congestion reductions. Having established the first-order conditions for social optimal fares and frequencies, we can now derive the optimal toll per passenger ( $\tau_i^q, \tau_j^q$ ) and per flight ( $\tau_i^f, \tau_j^f$ ) by using airlines' fare and frequency first-order conditions of each game (e.g., for the open-loop game, we use equations (12), (15), (26) and (27)). Results now follow treated by game type.

- Closed-loop game

For the closed-loop game, equivalent to Cournot behavior, it can be expected that optimal tolls are consistent with the earlier airport pricing literature. Indeed, the per-passenger toll in (29) is marginal operating cost plus a subsidy equal to market power markup, and the per-flight toll in (30) equals marginal operating cost plus congestion imposed on the rival airline passengers.

$$\tau_i^q = c_q - q_i \cdot B, \quad (29)$$

$$\tau_i^f = c_f + q_j \cdot D'. \quad (30)$$

However, the subsidy is separate from the congestion toll, and the per-passenger first-best toll is negative when the market power effect is bigger than airport per-passenger marginal operating cost. Hereafter, we use the term “Cournot toll” for the charge that accounts only for the congestion costs imposed on the competitor's passengers (equation (30)).

- Open-loop game

For the open-loop game, we find:

$$\tau_i^q = c_q - q_i \cdot \frac{1}{b}, \quad (31)$$

$$\tau_i^f = c_f + \left( q_j + \frac{e}{b} \cdot q_i \right) \cdot D'. \quad (32)$$

The per-passenger toll in the open-loop game is marginal operating cost plus a subsidy equal to the market power markup. For the per-flight toll, we obtain that it is marginal operating cost per flight plus a congestion toll, which is however different

Brueckner's (2002) Cournot toll. A first-best airport charges the congestion costs imposed on rival's passengers ( $q_j \cdot D'$ ) plus an additional term. This new term is the own-passengers marginal congestion cost ( $q_i \cdot D'$ ) times the degree of differentiation ( $e/b$ ). When this ratio is zero, the goods are independent and the optimal per-flight toll is the congestion imposed on the rival's passengers. When it is one, goods are perfect substitutes, and the first-best toll is the so-called atomistic toll. Any other feasible value of  $e/b$  (between zero and one) yields a charge that is somewhere in between the atomistic toll and the Cournot toll. The toll reflects that airlines are internalizing less than self-imposed congestion. Hence, first-best tolls are closer to the atomistic toll than in Cournot competition; and, the weaker the differentiation is, the higher the first-best toll should be.

The intuition of this result, which to the best of our knowledge is new in the airport pricing literature, is that when goods are independent, there are two monopolies using the same facility in order to serve two independent markets. Therefore, a global welfare maximizing airport charges to each carrier the congestion imposed on the competitor, which was entirely ignored by the operators because of the independence. In the other extreme, where the ratio equals one, goods are perfect substitutes and—since airlines take the rival's fare as given—the Nash equilibrium is the perfectly competitive outcome (as in Bertrand competition with homogeneous goods). In this extreme case, an airline will expect that any reduction in its own flight volume will be offset by an equally big increase in the competitor's flight volume. As a consequence, the total number of flights, thus airport congestion, will remain unchanged, and internalization of own congestion makes no sense.

This new result on optimal congestion pricing suggests that welfare maximizing per-flight tolls might be higher than what is suggested by the Cournot model, and hence have a more substantial impact in airlines' scheduling decisions, and therefore might yield more sizable welfare gains. Even if the market is highly concentrated, the dominant airline has to be charged for a high proportion of total marginal congestion costs if the differentiation between the airlines is not too strong.

- Stackelberg leader and Cournot follower game

Since the first-order conditions for the follower are the same as in the closed-loop setting, the first-best tolling rules also coincide. The per-passenger toll is marginal operating cost plus the market power subsidy (see equation 29) and the per-flight toll is marginal operating cost plus congestion imposed on the rival (equation 30). For the leader ( $i$ ), the first-best tolls are:

$$\tau_i^q = c_q - q_i \cdot \bar{B}, \quad (33)$$

$$\tau_i^f = c_f + \left( q_j + q_i \cdot \left| \frac{\partial f_j}{\partial f_i} \right| \right) \cdot D' + q_i \cdot E \cdot \left| \frac{\partial q_j}{\partial f_i} \right|, \quad (34)$$

where  $\bar{B} = B + E \cdot \partial q_j / \partial q_i + D' \cdot \partial f_j / \partial q_i$  and derivatives in absolute value are negative (see Appendix B).

The interpretation of these tolls is the same as in previous settings. The first-best per-passenger toll is the marginal operating cost plus a subsidy equal to the market power markup, and the per-flight toll corrects for uninternalized congestion. Because the leader—when considering the effect of its own decisions on the followers—alters internalization, the optimal congestion toll charged is in between the Cournot and the atomistic toll. This optimal congestion toll reproduces the result of Brueckner and Van Dender (2008) in their Stackelberg behavior with a Cournot follower game.

- Stackelberg leader and Bertrand follower game

The optimal tolling rules for the follower are identical to the open-loop tolls (equations 31 and 32), although the actual tolls will differ because the equilibrium is different. On the other hand, the optimal tolls charged to the leader ( $i$ ) are:

$$\tau_i^q = c_q - q_i \cdot \frac{1}{\bar{b}}, \quad (35)$$

$$\tau_i^f = c_f + \left( q_j + q_i \frac{e}{\bar{b}} \right) \cdot D' + q_i \left[ \frac{\bar{b} - b}{\bar{b}} (D' + g'_i) + \frac{e - b}{\bar{b}} D' \frac{\partial f_j}{\partial f_i} - \frac{e}{\bar{b}} \left( \frac{\partial p_j}{\partial f_i} + g'_j \frac{\partial f_j}{\partial f_i} \right) \right], \quad (36)$$

where  $\bar{b} \equiv b - e \cdot \partial p_j / \partial p_i - \partial q_i / \partial f_j \cdot \partial f_j / \partial p_i$ .

The optimal toll per passenger is airport's marginal operating cost per passenger plus the subsidy that corrects market power. Since  $\bar{b}$  is the derivative of own traffic with respect to own fare (taking into account the effect on the follower) the interpretation is the same. The per-flight toll corrects the frequency setting so that the leader sets the welfare maximizing frequency. As the sign of  $\bar{b} - b$  cannot be determined a priori, the per-flight toll for the leader has to be studied numerically. In Section 4 we do this, finding that in the numerical examples the optimal toll is in between the Cournot and the atomistic toll, and can be above or below the open-loop toll depending on the degree of differentiation.

We have shown in this section that a welfare maximizing airport needs to use two taxes, namely per-passenger and per-flight tolls, to reach the first-best outcome. It corrects the market power effect with a per-passenger toll and the frequency

inefficiency with a per-flight charge. The former ( $\tau_i^q$ ) is below airport marginal operating cost per passenger because it counteracts the airline market power exertion by means of a subsidy. As a consequence, this toll is negative when airlines' markups exceed airport's marginal operating costs. Conversely, the first-best per-flight toll is always above airport marginal operating cost, because airlines partially internalize congestion. If only one tax can be applied, the airport is facing a second-best problem, and which instrument is better to apply depends on market specific conditions. If the airlines' market power effect is stronger than the congestion effect, then using only per-passenger tolls is more efficient than charging only per-flight, and vice versa. In the extreme case of monopoly operation, the per-flight toll is unnecessary, and the first-best is attained with per-passenger subsidies only.

The results also imply that the first-best outcome cannot be reached by only charging passengers, because also charges per movement are necessary. If the authority or the facility wants to charge airlines per flight and passengers per trip, a positive tax for the passengers is not consistent with welfare maximization if airlines have market power. We further expand on this in the numerical analysis below.

Two recent analyses have put a question mark on the desirability of the traditional Cournot congestion toll, i.e. total marginal congestion costs times the market share of each airline (in our model the closed-loop toll in equation 30). Morrison and Winston (2007) find a small difference between the net benefits of charging the Cournot toll versus the atomistic toll that ignores any internalization. Then, Brueckner and Van Dender (2008) shows that Stackelberg behavior with a Cournot follower yields optimal airport tolls that lie in between of both policies.

The results of this model give new insights into the debate concerning the desirability of the traditional Cournot congestion toll: first, the optimal toll might well be close to the atomistic toll without assuming leadership behavior and without abstracting from airlines' market power exertion. This is the open-loop case, simultaneous competition with aircraft size, fare and frequency as strategic variables, and the closed-loop setting if moves cannot be observed by the competitor. From equation (32), it is straightforward that the closeness of the optimal toll to the atomistic toll depends on market-specific characteristics (ratio of differentiation  $e/b$ ). Therefore, within the same setting, the entire range of tolls can be optimal. We also confirm the internalization result of Brueckner and Van Dender (2008) for a Stackelberg leader with a Cournot follower and extend it to the case of a Stackelberg leader with a Bertrand follower, finding even higher optimal tolls. These findings may lead to optimism on the relative efficiency of atomistic congestion pricing in aviation markets. As we cannot compare welfare and equilibrium values analytically, in Section 4 we solve numerically the equilibrium for an airport charging the atomistic toll and make the comparisons with the first-best to assess the relative efficiency.

#### 4. Numerical analysis

In this section we present a numerical analysis that allows making comparisons that are not possible analytically. We also analyze the performance of second-best policies. Despite the simplified structure of the model, we use parameters that are as much as possible calibrated so as to reflect realistic values. We use the following functional forms for the schedule delay cost ( $g_i$ ) and passengers' congestion cost ( $D$ ):

$$g_i = \gamma \cdot \frac{1}{f_i}, \quad (37)$$

$$D = \alpha \cdot \left( \frac{f_i + f_j}{K} \right)^\beta, \quad (38)$$

where the schedule delay cost in (37) is inversely proportional to the airline frequency and  $\gamma$  is a constant representing the monetary value of a unit of schedule delay time (as it would be with uniformly distributed desired departure times, and equally spaced flights), as is usual in the literature (e.g. Brueckner, 2004; Basso, 2008). The congestion delay at the airport in (38) is a function of the volume capacity ratio, with  $\alpha$  being proportional to the passengers' value of travel time,  $K$  the capacity, and  $\beta$  the power of the function.

Our reference scenario for calibration has symmetric airlines and assumes a marginal-operating-cost pricing airport, i.e. a toll per passenger of  $c_q$  and a toll per flight of  $c_f$ . We use the closed-loop and open-loop settings for calibration purpose, and leave the Stackelberg settings for analyses. The parameters and equilibrium outputs are:

Table 1: Parameter values.

|  | Demand parameters |      |      |     |          | Cost parameters |          |         |       |
|--|-------------------|------|------|-----|----------|-----------------|----------|---------|-------|
|  | A                 | B    | E    | $a$ | $\gamma$ | $c_0$           | Airlines | Airport | $c_q$ |
|  | 1250              | 1.04 | 0.63 | 750 | 30       | 4390.7          |          |         | 10    |
|  |                   | $b$  | $e$  | 1.5 | $\alpha$ | 337.7           |          |         | 1000  |
|  |                   |      |      | 0.9 | $\beta$  |                 |          | $r$     | 6000  |

Table 2: Equilibrium outputs, marginal operating cost pricing airport.

| Game setting | Total traffic | Fare | Total Frequency | Aircraft size | $K$ | Airlines Profit |
|--------------|---------------|------|-----------------|---------------|-----|-----------------|
| Open-loop    | 745           | 596  | 2.5             | 295           | 3.6 | 171,452         |
| Closed-loop  | 639           | 680  | 2.1             | 311           | 3.0 | 201,493         |

Table 3: Generalized price.

| Game setting | Gen. price | Fare share | Congestion share | Schedule delay share | Demand Elasticity |
|--------------|------------|------------|------------------|----------------------|-------------------|
| Open-loop    | 629        | 0.95       | 0.014            | 0.036                | -1.01             |
| Closed-loop  | 718        | 0.95       | 0.011            | 0.039                | -1.35             |

Airlines' operating cost parameters are from Swan and Adler (2006), using a reference distance of 1.500 km and an adjustment in the per-passenger cost as suggested by the authors. Values of time are from Morrison and Winston (1989), while congestion, delay and demand parameters are set such that equilibrium elasticity with respect to generalized price in both settings is close to -1.146, the mean value of 204 observations reported by Brons et al. (2002). Values are set to the same monetary units (U.S. dollars) throughout the calibration process.

#### 4.1. Internalization

We first solve the problem with a welfare maximizing airport and a duopoly of symmetric airlines, to assess the equilibrium share of congestion that is internalized by each player. In this case, each carrier has an equilibrium share of 50 percent of passengers. Figure 1 shows the percentage of congestion that is internalized by each agent for a feasible range of the ratio of differentiation,  $e/b$ . Recall that a value of 0 means independent goods, whereas 1 represents pure substitutes. According to the first-order conditions derived in Section 2, in a closed-loop setting each airline will internalize half of the total marginal congestion cost (see equation 9). For the open-loop setting, internalization is in between one half and zero, and decreases linearly in the ratio of differentiation; see (16). For the game with a Stackelberg leader and a Cournot follower, the leader will internalize less than half of total marginal congestion and the follower will act as in the closed-loop game, internalizing exactly half of total marginal congestion costs. Finally, Bertrand follower's internalization is the same as in the open-loop game, while the leader behavior cannot be quantified a priori.

The self-imposed internalization of Cournot behavior and the linear decrease of the internalization for airlines in an open-loop setting are shown in Figure 1 in black lines. To illustrate the possible implications of our results, we may look at parameters estimated by Perloff et al. (2007) for a linear demand structure (as in equation (3)) in a duopolistic competition. They find values for the ratio  $e/b$  between 0.4 and 0.7, which implies that airlines would internalize between 12 and 30 percent of congestion costs, and therefore should be charged for a remaining 70 to 88 percent of the total marginal congestion costs.

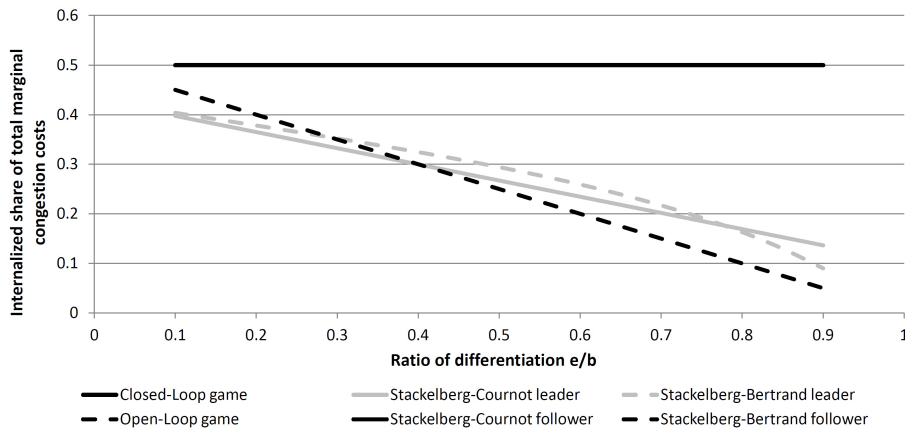


Figure 1: Internalization.

For both Stackelberg games, we find that the leader internalization is less than the self-imposed congestion, and decreases towards atomistic behavior as the ratio  $e/b$  increases. In the case of Cournot behavior (the two solid lines), the leader

always internalizes less than the follower, whose internalization is always the congestion imposed on own passengers. For Bertrand behavior in the Stackelberg setting (the two dashed lines), we find that the leader internalizes roughly the same congestion as the follower, both being less than the self-imposed and approaching to zero as the ratio of differentiation grows. Brueckner and Van Dender (2008) also analyze Stackelberg behavior with a Cournot follower, but suppressing market-power by assuming perfectly elastic demand, and assuming that the outputs of the carriers are perfect substitutes. We extend this to the case of price-sensitive demand and imperfect substitution, finding similar results regarding internalization of congestion: the leader internalizes less than self-imposed congestion, but do not fully reach atomistic behavior. This is represented in Figure 1 by the solid grey line.

From this analysis it follows that, for Bertrand behavior in the open-loop game as well as in Stackelberg competition, optimal congestion charges are close to the atomistic charge when the differentiation is not too strong (on the right end of Figure 1). For this reason, optimal per-flight congestion tolls might have a more significant impact on airlines' scheduling decisions and, therefore, in alleviating congestion, than what was suggested by earlier Cournot models.

## 4.2. Alternative policies

We next look at alternative policies: (i) the second-best case when an airport can use only one tax instrument, and (ii) the relative performance of atomistic pricing.

### 4.2.1. Using one tax instrument

As shown in Section 3, a welfare maximizing airport needs two pricing instruments to reach the first-best outcome. It corrects the market power effect with a per-passenger subsidy and the frequency inefficiency with a per-flight charge. We now look at what happens when it can only use one instrument. For this purpose, we define the relative efficiency  $\Omega^p$  as the welfare gain due to a policy ( $p$ ), relative to the first-best gain:

$$\Omega^p = \frac{SW^p - SW^{mc}}{SW^{fb} - SW^{mc}}, \quad (39)$$

where superscript  $fb$  refers to the first-best case, and  $mc$  to an airport charging marginal operating costs (as in the reference scenario for calibration).

Figure 2a shows the relative efficiency of both second-best policies for the base calibration. The results show that, in our calibration, the market power effect dominates the congestion effect, yielding a high relative efficiency for a per-passenger subsidy and a low one for the per-flight toll. The results also show a significant effect of game type on the performance of a policy. As we discuss in Section 2, the market power exertion and the amount of internalized congestion are always higher in a closed-loop setting than in an open-loop setting. This leads to a higher relative efficiency of the per-passenger subsidy, and a lower efficiency of the per-flight charge, in the closed-loop game than in the open-loop game. For the Stackelberg games, the relative efficiency lies in between the closed-loop and open-loop, but the ranking is sensitive to the parameters (as is the degree of internalization). The results furthermore show that, for the chosen parametrization, the social gains for the two instruments are nearly additive. That is, the gains from introducing the one instrument are almost insensitive to the other instrument being in place already. This underlines the lack of substitution between the policies.

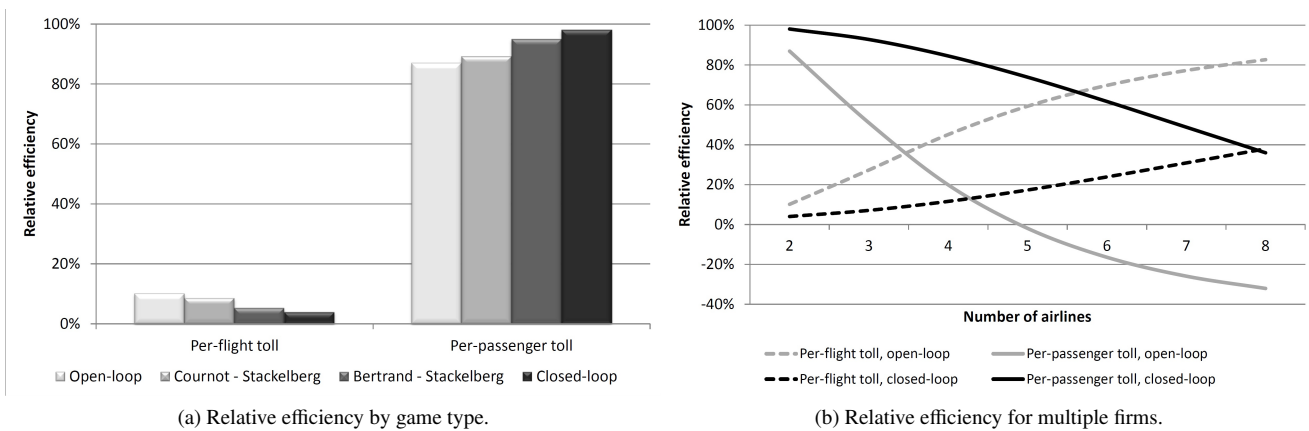


Figure 2: Second-best relative efficiency analysis.

The second analysis we perform has the purpose of assessing the relative efficiency of both second-best policies when the market power and the congestion effect have different comparative importance. We do this by increasing the number

of (symmetric) airlines that participate in the market, for a given demand structure, because it captures in one parameter the relative importance of both effects: increasing the number of firms makes the market power effect weaker because of the increased competition and, for the same reason, the congestion externality becomes more severe. As Figure 2b shows, the number of firms participating in the market affects the policies in a different manner: the relative efficiency of the per-flight toll increases with the number of airlines, while the opposite occurs for the per-passenger subsidy. The intuition is straightforward: when the number of airlines increases, each airline's market share of passengers decreases, which leads to less internalization as well as less ability for exerting market power. Both effects explain the performance improvement of the per-flight toll and the reduction of gains from counteracting the airlines' markup with a per-passenger subsidy. The negative relative efficiency of the per-passenger toll, in the open-loop game for 5 or more airlines, is because congestion inefficiencies are significantly more important than market power exertion in those cases, therefore, the positive per-flight toll ( $\tau_f = c_f$ ) of the reference scenario, which is removed in the per-passenger toll scenario, gives higher social welfare gains than the second-best per-passenger toll, with a zero per-flight toll.

Which second-best option is better clearly depends on the balance between the inefficiencies. The relative efficiency of per-passenger subsidies is higher than the per-flight toll efficiency for the closed-loop game up to 8 airlines; on the other hand, in the open-loop game, the per-flight toll outperforms the per-passenger subsidy already with 4 airlines, and exceeds 70 percent of the first-best with 6 airlines.

We illustrated these points by means of increasing the number of firms, but the increasing performance of the per-flight toll in the open-loop setting can also be found with an increase of the ratio of differentiation  $e/b$ . As the differentiation diminishes, the performance of the per-flight toll rises.<sup>12</sup>

This exercise provides some useful insights into the performance of second-best policies. When the market power effect is stronger than the congestion effect, it is better for social welfare to give a per-passenger subsidy instead of charging a per-flight toll, and vice versa. If negative tolls are not feasible, it is wiser to charge only a per-flight congestion toll that performs well if the internalization is low, because of small market shares or because differentiation is not too strong when airlines behave as in the open-loop setting. Finally, a positive per-passenger toll cannot be supported from an efficiency perspective, unless the per-flight toll is not feasible and the airlines' market power markup is small compared to the airport's marginal operating cost per passenger.

#### 4.2.2. Atomistic pricing

Finally, we assess the efficiency of levying atomistic congestion tolls to airlines. For this purpose, we look at the welfare gain due to atomistic tolls relative to the second-best case of only having per-flight tolls ( $SW^f$ ), and having a marginal-operating-cost pricing airport as a reference ( $SW^{mc}$ ). The aim of measuring the efficiency relative to this second-best policy, is to isolate the welfare gain that comes from the per-passenger subsidy.<sup>13</sup> The performance measure, for this case, is defined in the following way (using superscript *atom* for atomistic tolls):

$$\widetilde{\Omega}^{atom} = \frac{SW^{atom} - SW^{mc}}{SW^f - SW^{mc}}. \quad (40)$$

Figure 3a shows that  $\widetilde{\Omega}^{atom}$  has an intuitive relationship with the amount of congestion internalized by carriers. This policy achieves the lowest benefits when airlines behave as in the closed-loop game, where the first-best toll is half of total marginal congestion costs. Moreover, the performance of atomistic pricing in the closed-loop setting is only moderately sensitive to the differentiation ratio, as it varies between 77 and 79 percent.

When airlines are characterized by an open-loop game, the performance of atomistic pricing rapidly improves as the differentiation is weaker ( $e/b$  approaches 1). This is because the amount of congestion that is internalized diminishes (see Figure 1) and, therefore, the first-best toll approaches the atomistic toll as we showed analytically in Section 3. Figure 3a also shows that the welfare gains from levying atomistic charges, when the differentiation is not too strong, are almost the same as the gains from charging the optimal per-flight toll (in absence of per-passenger subsidies). Using again the ratios  $e/b$  obtained by Perloff et al. (2007), atomistic pricing would yield roughly between 85 and 95 percent of the maximum social benefit that can be obtained with per-flight tolls.

The benefits that atomistic pricing generates in Stackelberg games follow the internalization patterns; for the Bertrand-Stackelberg game,  $\widetilde{\Omega}^{atom}$  is almost the same as in the open-loop game, because carriers are internalizing approximately the same amount of congestion in both games. For the Cournot-Stackelberg game, we showed, both analytically as well as numerically, that the follower internalizes the same amount of congestion as in the closed-loop game, and that the leader's

<sup>12</sup>We assess this numerically, finding that for a ratio  $e/b$  of 0.9,  $\Omega^f$  reaches 67 percent for an open-loop duopoly.

<sup>13</sup>When the comparison is carried out with respect to the first-best, the welfare gains of charging atomistic tolls are also almost as high as the gains of using the (optimal) per-flight toll. On the other hand, the relative efficiency of atomistic tolls together with (second-best) per-passenger (negative) tolls varies between 99 and 100 percent. This is because market power effect is dominating.

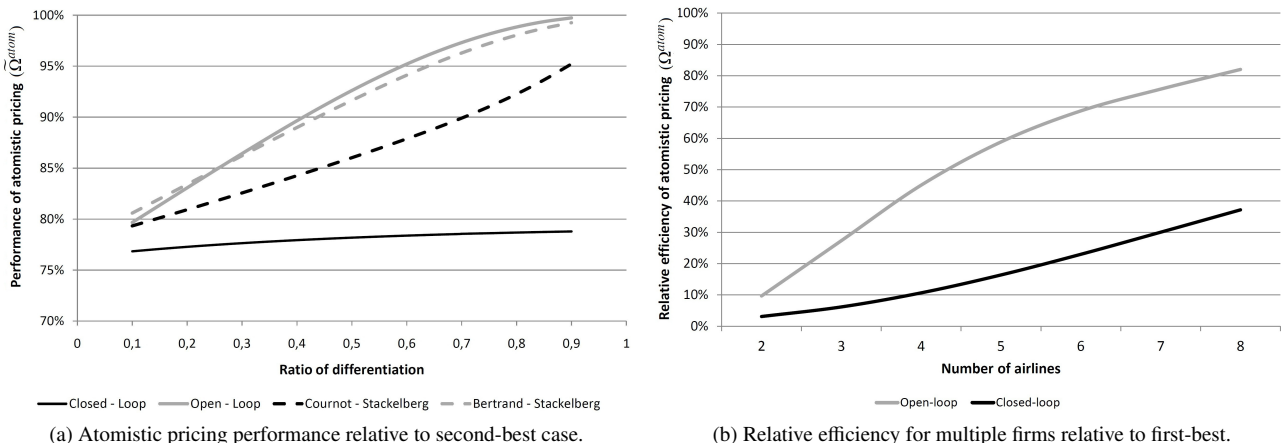


Figure 3: Efficiency of atomistic pricing.

internalization is similar to the one observed in the open-loop game. As a consequence, the performance of atomistic pricing, in the Cournot-Stackelberg setting, is lower than in the open-loop setting, and higher than in the closed-loop setting.

In Figure 3b, we show the relative efficiency of atomistic pricing compared to the first-best (i.e. using  $\Omega^{atom}$  from (39) as the performance measure), when the number of symmetric firms increases in the open-loop and closed-loop setting. The performance of atomistic pricing, in this case, is also very similar to the performance of the second-best policy of charging only per-flight tolls, as can be seen by comparing Figure 2b and Figure 3b. As the number of airlines increases, the performance of atomistic pricing is better, reaching high values when airlines compete as in the open-loop setting. This implies that atomistic congestion pricing can perform, in terms of social welfare, in a way comparable to the optimal per-flight toll.

These results also give new insights to the airport pricing literature: if congestion is a major issue and the industry is more adequately described as in the open-loop setting, atomistic pricing may offer a more attractive alternative. When per-passenger subsidies are given or, as a second-best policy, per-passenger tolls are set to zero, the airport financial deficit can then be reduced without significant welfare losses. If closed-loop behavior is more adequate, then naive atomistic pricing for a duopoly can be less attractive. For the Stackelberg games, where first- and second-best tolls differ among carriers, the uniform atomistic toll still produces a small welfare loss with respect to the maximum benefit that can be obtained with a per-flight charge.

## 5. Conclusion

The present analysis shows that the amount of congestion that airlines internalize when making flights scheduling decisions, may be smaller than the simple market shares formulae from Cournot models, and more so if firms are closer substitutes. As a consequence, the gains from congestion pricing may be higher, but the optimal charge depends strongly on the prevailing type of strategic interaction. We show analytically that, if the open-loop setting is relevant, optimal charges are higher than the Cournot toll, and may be close to the so-called atomistic toll, even for dominant airlines with high market shares. The size of the toll depends critically on the degree of differentiation between airlines' outputs: if the differentiation is not too strong, the optimal charge is sizable and might have a significant impact on airlines scheduling decisions and welfare. On the other hand, if the closed-loop setting is more relevant, the optimal congestion charge approaches zero as the market share of an airline is close to 100 percent. This result contributes to the internalization debate from a theoretical side, and provides new arguments in favor of applying congestion pricing at airports.

We also provide numerical examples to assess the performance of levying atomistic tolls and the relative efficiency of second-best policies. The analysis confirms that when the airlines' market power effect is larger than the congestion effect, it is wiser to give a per-passenger subsidy instead of a per-flight charge, and vice versa. Numerical examples also suggest that only using a per-flight charge and levying atomistic tolls can yield substantial benefits when the degree of differentiation is not too strong, and airlines do not behave in a Cournot fashion. This is, in our framework, when they behave as in the open-loop setting, or as in the Stackelberg-Bertrand setting. The good performance of atomistic pricing, although to a smaller extent, is also found numerically in the Stackelberg-Cournot setting.

From the analysis, a number of issues emerge for future research. As airline behavior determines the optimality of congestion charges, and in some settings the degree of differentiation also has a major influence on the size of optimal tolls, estimation of airline conduct parameters stands as an important topic for future research. Our approach abstracts from



studying entry barriers, which might yield different incentives and outcomes for existent firms, and this framework can be used for studying such potential incentives from a dual tax perspective. Another qualification of our model is that it relies on symmetric product differentiation and, although this does not critically affects our main conclusions, a more realistic demand structure should be considered especially for studying interactions between asymmetric airlines. For example, studying the interactions between the legacy and the emerging low-cost carriers, may require a more elaborate specification of the demand structure.

We see regulation of private airports and the role of commercial (concession) operations in airport pricing as another natural extension of the present analysis. Finally, we perform the analysis in a single market with one airport for analytical simplicity and to focus on the main insights, but network effects and airports having different objective functions are also seen as a logical extension for future research.

## Appendix A. Closed-Loop airlines' equilibrium

The problem for airlines in the second stage is:

$$\begin{aligned} \max_{p_i} \Phi_i &= q_i \cdot p_i - f_i \cdot (\gamma_i^f + \gamma_i^s \cdot s_i) - f_i \cdot \tau_i^f - q_i \cdot \tau_q \\ \text{s.t.} \quad q_i &\geq f_i \cdot s_i, \end{aligned} \quad (\text{A.1})$$

with  $q_i$  as a function of prices and (given) frequencies (equation 3). Let  $C_i$  be the costs faced by the airline which in this stage are irrelevant, then the associated lagrangian is:

$$L_i = q_i \cdot (p_i - \tau_i^q) - C_i + \lambda [f_i \cdot s_i - q_i], \quad (\text{A.2})$$

and the optimum must satisfy KKT conditions:

$$(p_i - \tau_i^q) \cdot \frac{\partial q_i}{\partial p_i} + q_i - \lambda \cdot \frac{\partial q_i}{\partial p_i} = 0, \quad (\text{A.3a})$$

$$f_i \cdot s_i - q_i \geq 0, \quad (\text{A.3b})$$

$$\lambda \geq 0, \quad (\text{A.3c})$$

$$\lambda \cdot [f_i \cdot s_i - q_i] = 0. \quad (\text{A.3d})$$

Thus, the solution can be with or without the constraint binding.

- Case (A): Non-binding constraint.

In this case  $\lambda = 0$ , therefore, from equation (A.3a), we have:

$$p_i = \tau_i^q - \frac{q_i}{\frac{\partial q_i}{\partial p_i}} = \tau_i^q + \frac{q_i}{b}. \quad (\text{A.4})$$

And we can obtain the reaction function for case A:

$$p_i^A(p_j) = \frac{a - b \cdot (D_i + g_i) + e \cdot (p_j + D_j + g_j) + b \cdot \tau_i^q}{2b}, \quad (\text{A.5})$$

that is valid if and only if the constraint is binding, i.e. if  $f_i \cdot s_i > q_i(p_i^A(p_j), p_j, f_i, f_j)$ . As this condition depends on  $p_j$ , we can write the condition for  $p_j$ . As a result, we get that case (A) is valid, if and only if,  $p_j$  satisfies:

$$p_j < \frac{2 \cdot f_i \cdot s_i - [a - b \cdot (D_i + g_i) + e \cdot (D_j + g_j) - b \cdot \tau_i^q]}{e} \equiv \bar{p}_j, \quad (\text{A.6})$$

- Case (B): Binding constraint.

Now  $\lambda \geq 0$ , and from equation (A.3a), we obtain:

$$p_i = \tau_i^q - \frac{q_i}{\frac{\partial q_i}{\partial p_i}} + \lambda = \tau_i^q + \frac{q_i}{b} + \lambda, \quad (\text{A.7})$$

$$f_i \cdot s_i = q_i. \quad (\text{A.8})$$

The last equation determines the reaction function  $p_i^B(p_j)$ . Solving,  $p_i^B(p_j)$  is given by:

$$p_i^B(p_j) = \frac{a - b \cdot (D_i + g_i) + e \cdot (p_j + D_j + g_j) - f_i \cdot s_i}{b}. \quad (\text{A.9})$$

Now, we prove that there is a unique pure-strategy equilibrium in this stage. First, noting that if  $p_j \geq \bar{p}_j$  the case A is not feasible, we can write the fare reaction function for firm  $i$  as:

$$R_i(p_j) = \begin{cases} p_i^A & : p_j < \bar{p}_j \\ p_i^B & : p_j \geq \bar{p}_j \end{cases}.$$

The first step is show that the reaction function is continuous in  $p_j$ . For this, is enough to see that  $p_i^B(\bar{p}_j) = \frac{f_i \cdot s_i}{b} + \tau_i^q$  and, since in this case  $f_i \cdot s_i = q_i$ , we have that  $p_i^B(\bar{p}_j) = p_i^A(\bar{p}_j)$ .

Secondly, the slope of  $p_i^A$  and  $p_i^B$  are  $e/(2b)$  and  $e/b$  respectively, meaning that reaction functions are upward-sloping and the slope is less than unity, ensuring the uniqueness of the equilibrium.

Now, we prove that at the equilibrium of the full game, the constraints are binding. Suppose  $(p_i^*, p_j^*, f_i^*, f_j^*, s_i^*, s_j^*)$  is the equilibrium and at least one firm ( $i$ ) does not have all the seats filled. Then  $p_i^*$  is given by  $p_i^A$  which is independent of  $s_i$ . Hence, in this equilibrium, it is not an optimal choice for firm  $i$  because it can improve its profit by reducing the number of seats. Finally, we solve the full game for two firms with binding constraints. The intersection of both reaction functions yield the following equilibrium fares.

$$p_i^* = A - (D_i + g_i) - B \cdot f_i \cdot s_i - E \cdot f_j \cdot s_j, \quad (\text{A.10})$$

$$p_j^* = A - (D_j + g_j) - B \cdot f_j \cdot s_j - E \cdot f_i \cdot s_i. \quad (\text{A.11})$$

Airline's fare is a function of frequency and traffic ( $q_i = f_i \cdot s_i$  because the constraint is binding), and matches the result of clearing fare from inverse demand function  $\theta_i = A - B \cdot q_i - E \cdot q_j$ . Therefore, as a result of the second stage, airlines will set fares as if they take quantities as given. On the first stage airlines solve the following maximization problem.

$$\max_{f_i, s_i} \Pi_i = f_i \cdot s_i \cdot p_i^* - f_i \cdot (\gamma_i^f + \tau_i^f) - f_i \cdot s_i \cdot (\gamma_i^s + \tau_i^s). \quad (\text{A.12})$$

First-order condition with respect to aircraft size yields:

$$\frac{\partial \Pi_i}{\partial s_i} = 0 \Rightarrow f_i \cdot \left( p_i^* + s_i \cdot \frac{\partial p_i^*}{\partial s_i} - \gamma_i^s - \tau_i^s \right) = f_i (p_i^* - f_i \cdot s_i \cdot B - \gamma_i^s - \tau_i^s) = 0. \quad (\text{A.13})$$

And with respect to frequency,

$$\frac{\partial \Pi_i}{\partial f_i} = 0 \Rightarrow s_i \cdot \left( p_i^* + f_i \cdot \frac{\partial p_i^*}{\partial f_i} - \gamma_i^f - \tau_i^f \right) - (\gamma_i^f + \tau_i^f) = 0. \quad (\text{A.14})$$

Using equation (A.13) and replacing the derivative, we obtain:

$$-f_i \cdot s_i \cdot \left( \frac{\partial D_i}{\partial f_i} + \frac{\partial g_i}{\partial f_i} \right) - (\gamma_i^f + \tau_i^f) = 0. \quad (\text{A.15})$$

Both equations, (A.13 and A.15), are equivalent to the ones presented as the result of the closed-loop game in Section 2.

## Appendix B. Reaction functions

- Cournot Stackelberg

First-order conditions for the follower  $j$  can be written as:

$$A - 2 \cdot B \cdot q_j - E \cdot q_i - D - g_j - \gamma_j^s - \tau_j^s = 0, \quad (\text{B.1})$$

$$-q_j \cdot \left( \frac{\partial D}{\partial f_j} + \frac{\partial g_j}{\partial f_j} \right) - \gamma_j^f - \tau_j^f = 0. \quad (\text{B.2})$$

To derive how the follower outputs vary when the leader changes quantity and frequency, we differentiate the system, write the result in matrix notation, and apply Cramer's rule. After some straightforward calculations, we get:

$$\frac{\partial q_j}{\partial q_i} = \frac{-E \cdot q_j \cdot (D'' + g_j'')}{R} \leq 0, \quad (\text{B.3})$$

$$\frac{\partial f_j}{\partial q_i} = \frac{E \cdot (D' + g_j')}{R} \leq 0, \quad (\text{B.4})$$

$$\frac{\partial q_j}{\partial f_i} = \frac{-q_j \cdot (D' \cdot g_j'' - D'' \cdot g_j')}{R} \leq 0, \quad (\text{B.5})$$

$$\frac{\partial f_j}{\partial f_i} = \frac{-2 \cdot B \cdot q_j \cdot D'' + D' \cdot (D' + g_j')}{R} \leq 0, \quad (\text{B.6})$$

where  $R = 2 \cdot B \cdot q_j \cdot (D'' + g_j'') - (D' + g_j')^2$  is, by definition, the determinant of the Hessian matrix of airlines profit. Since we assume existence of a maximum,  $R > 0$ . Because of assumptions (5), and the fact that an equilibrium with positive traffic implies  $D' + g_j' < 0$ , it is clear that all the reaction functions in this case are non-positive.

- Bertrand Stackelberg

We proceed in the same way as above to calculate the reaction functions. First-order conditions for the follower  $j$  are:

$$p_j - \gamma_j^s - \tau_j^q - \frac{q_j}{b} = 0, \quad (\text{B.7})$$

$$\gamma_j^f + \tau_j^f - (p_j - \gamma_j^s - \tau_j^q) \cdot \frac{\partial q_j}{\partial f_j} = 0. \quad (\text{B.8})$$

And denoting  $\widehat{p}_j = p_j - \gamma_j^s - \tau_j^q$ , we obtain:

$$\frac{\partial p_j}{\partial p_i} = \frac{-\frac{e}{b} \cdot \frac{\partial^2 q_j}{\partial f_i^2} \cdot \widehat{p}_j}{R} \geq 0, \quad (\text{B.9})$$

$$\frac{\partial p_j}{\partial f_i} = \frac{-\frac{1}{b} \cdot \frac{\partial q_j}{\partial f_i} \cdot \frac{\partial^2 q_j}{\partial f_i^2} \cdot \widehat{p}_j + \frac{1}{b} \cdot \frac{\partial q_j}{\partial f_j} \cdot \frac{\partial^2 q_j}{\partial f_i \partial f_j} \cdot \widehat{p}_j}{R} \leq 0, \quad (\text{B.10})$$

$$\frac{\partial f_j}{\partial p_i} = \frac{\frac{e}{b} \cdot \frac{\partial q_j}{\partial f_i}}{R} \geq 0, \quad (\text{B.11})$$

$$\frac{\partial f_j}{\partial f_i} = \frac{2 \cdot \frac{\partial^2 q_j}{\partial f_i \partial f_j} \cdot \widehat{p}_j + \frac{1}{b} \cdot \frac{\partial q_j}{\partial f_j} \cdot \frac{\partial q_j}{\partial f_i}}{R} \leq 0, \quad (\text{B.12})$$

where  $R = 2 \cdot \frac{\partial^2 q_j}{\partial f_j^2} \cdot \widehat{p}_j - \frac{1}{b} \cdot \left(\frac{\partial q_j}{\partial f_j}\right)^2 > 0$ , as we assume existence of a maximum. Because of assumptions (5),  $\partial q_j / \partial f_i < 0$ ,  $\partial^2 q_j / \partial f_j^2 < 0$  and  $\partial^2 q_j / \partial f_i \partial f_j < 0$ . An equilibrium with positive traffic implies  $\partial q_j / \partial f_j > 0$ , therefore we can show that reaction functions have the sign that is presented above.

### Appendix C. Welfare maximizing airport first-order conditions

Since airlines choose aircraft size such that  $q_i = s_i \cdot f_i$ , we use this relation before maximizing welfare. Then, first-order condition with respect to fare is:

$$\begin{aligned} \frac{\partial SW}{\partial p_i} &= Bq_i \frac{\partial q_i}{\partial p_i} + Bq_j \frac{\partial q_j}{\partial p_i} + Eq_i \frac{\partial q_i}{\partial p_i} + Eq_j \frac{\partial q_j}{\partial p_i} + (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_i} + q_i + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_i} \\ &= q_i \left( B \frac{\partial q_i}{\partial p_i} + E \frac{\partial q_j}{\partial p_i} + 1 \right) + q_j \left( B \frac{\partial q_j}{\partial p_i} + E \frac{\partial q_i}{\partial p_i} \right) + (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_i} + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_i}. \end{aligned} \quad (\text{C.1})$$

And noting that

$$B \frac{\partial q_i}{\partial p_i} + E \frac{\partial q_j}{\partial p_i} = B \cdot -b + E \cdot e = \frac{-b^2}{b^2 - e^2} + \frac{e^2}{b^2 - e^2} = -1, \quad (C.2)$$

$$B \frac{\partial q_j}{\partial p_i} + E \frac{\partial q_i}{\partial p_i} = B \cdot e + E \cdot -b = \frac{b \cdot e}{b^2 - e^2} + \frac{-b \cdot e}{b^2 - e^2} = 0, \quad (C.3)$$

we obtain, using the analogous calculations for  $\partial S W / \partial p_j$ , first-order conditions for fares:

$$\frac{\partial S W}{\partial p_i} = (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_i} + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_i} = 0, \quad (C.4)$$

$$\frac{\partial S W}{\partial p_j} = (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_j} + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_j} = 0. \quad (C.5)$$

Both conditions can only be satisfied if fares fulfill:

$$(p_i - \gamma_i^s - c_q) = 0, \quad \forall i. \quad (C.6)$$

For the frequency first-order conditions, let  $\bar{p}_i = p_i - \gamma_i^s - c_q$  and  $\bar{p}_j = p_j - \gamma_j^s - c_q$ . Then, the derivative of SW with respect to  $f_i$  is:

$$\frac{\partial S W}{\partial f_i} = B q_i \frac{\partial q_i}{\partial f_i} + B q_j \frac{\partial q_j}{\partial f_i} + E q_i \frac{\partial q_j}{\partial f_i} + E q_j \frac{\partial q_i}{\partial f_i} + \bar{p}_i \cdot \frac{\partial q_i}{\partial f_i} + \bar{p}_j \cdot \frac{\partial q_j}{\partial f_i} - (\gamma_i^f + c_f). \quad (C.7)$$

Using  $\bar{p}_i = \bar{p}_j = 0$  from equation (26), we can write,

$$\frac{\partial S W}{\partial f_i} = q_i \left( B \frac{\partial q_i}{\partial f_i} + E \frac{\partial q_j}{\partial f_i} \right) + q_j \left( B \frac{\partial q_j}{\partial f_i} + E \frac{\partial q_i}{\partial f_i} \right) - (\gamma_i^f + c_f). \quad (C.8)$$

Note that,

$$\frac{\partial q_i}{\partial f_i} = -b \frac{\partial D}{\partial f_i} - b \frac{\partial g_i}{\partial f_i} + e \frac{\partial D}{\partial f_i}, \quad (C.9)$$

$$\frac{\partial q_j}{\partial f_i} = -b \frac{\partial D}{\partial f_i} + e \frac{\partial D}{\partial f_i} + e \frac{\partial g_i}{\partial f_i}. \quad (C.10)$$

And using these equations, first-order condition can be written as,

$$\begin{aligned} \frac{\partial S W}{\partial f_i} = & -(\gamma_i^f + c_f) + q_i \left( \frac{\partial D}{\partial f_i} (-bB + eE) + \frac{\partial D}{\partial f_i} (eB - bE) + \frac{\partial g_i}{\partial f_i} (-bB + eE) \right) \\ & + q_j \left( \frac{\partial D}{\partial f_i} (-bB + eE) + \frac{\partial D}{\partial f_i} (eB - bE) + \frac{\partial g_i}{\partial f_i} (eB - bE) \right). \end{aligned} \quad (C.11)$$

Finally, because  $-b \cdot B + e \cdot E = -1$  and  $e \cdot B - b \cdot E = 0$ , the first-order condition is:

$$-(q_i + q_j) \cdot \frac{\partial D}{\partial f_i} - q_i \cdot \frac{\partial g_i}{\partial f_i} = c_f + \gamma_i^f. \quad (C.12)$$

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