

Model independent analysis of gluonic pole matrix elements and universality of transverse-momentum-dependent fragmentation functionsL. P. Gamberg,^{1,*} A. Mukherjee,^{2,†} and P. J. Mulders^{3,‡}¹*Division of Science, Penn State University-Berks, Reading, Pennsylvania 19610, USA*²*Physics Department, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India*³*Department of Physics and Astronomy, VU University, NL-1081 HV Amsterdam, The Netherlands*

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Gluonic pole matrix elements explain the appearance of single spin asymmetries (SSA) in high-energy scattering processes. They involve a combination of operators which are odd under time reversal (T-odd). Such matrix elements appear in principle both for parton distribution functions and parton fragmentation functions. We show that for parton fragmentation functions, these gluonic pole matrix elements vanish as a consequence of the analytic structure of scattering amplitudes in quantum chromodynamics. This result is important in the study of the universality of transverse-momentum-dependent (TMD) fragmentation functions.

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Cross sections for high-energy scattering processes are given by a convolution of partonic cross sections with parton distribution functions (PDF) and parton fragmentation functions (PFF). These functions interpreted as momentum distributions and parton decay functions, respectively, are given as matrix elements of quark and gluon operators [1–4]. Among such observables, gluonic pole or Qiu-Sterman matrix elements involve a combination of operators which is odd under time reversal (T-odd). These matrix elements have been extensively studied [5–12]. They explain the appearance of single spin asymmetries (SSA) in high-energy scattering processes. These correlation functions show up in combination with calculable hard parts that may differ from the partonic cross sections through specific calculable factors and signs [9,13,14]. In this paper, we use general properties of scattering amplitudes in quantum chromodynamics (QCD) to study the support properties of these parton correlation functions. Specifically, assuming unitarity and analyticity properties to hold for forward parton-hadron scattering amplitudes, we uncover their singularity structure which then determines the corresponding properties of the PDFs and PFFs. Using this analysis for fragmentation functions, we show that single gluon and multigluon pole matrix elements vanish in the limit when the momenta of these gluons become zero. Since these multigluonic pole matrix elements appear in integrated and weighted transverse-momentum-dependent (TMD) fragmentation functions, as a consequence of them being zero, all leading T-odd effects in the matrix elements are all part of the final state interactions among the fragmentation remnant and final state hadron [4,15] rather than from T-odd partonic operator combinations. Thus, when transverse-momentum-dependent T-odd fragmentation functions appear in

observables like SSA, they are convoluted with the standard partonic cross sections. The vanishing of gluonic pole matrix elements for any number of gluons provides a general proof of the universality of these TMD fragmentation functions [16].

Spectral studies of the gluonic pole matrix elements for fragmentation, specifically for quark-quark-gluon matrix elements with just one gluon field [17,18] already indicated that they vanish. Our arguments presented here are more general and can be applied to multigluonic pole matrix elements. They depend only on the analytic structure of scattering amplitudes, yet they do not depend on the details of partonic or hadronic masses, and are insensitive to integrations over transverse momentum.

We begin our analysis by considering high-energy scattering processes where the structure of hadrons is accounted for using quark and gluon correlators, which are Fourier transforms of forward matrix elements of non-local quark and gluon operators between hadronic states. For instance, the quark-quark correlator

$$\Phi_{ij}^{[U]}(x, k_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \times \langle P | \bar{\psi}_j(0) \mathcal{U}_{[0;\xi]} \psi_i(\xi) | P \rangle_{\text{LF}}, \quad (1)$$

where $\mathcal{U}_{[\eta;\xi]} = \mathcal{P} \exp[-ig \int_C ds \cdot A^a(s) t^a]$ is the gauge link that ensures gauge invariance [19–22]. The nonlocality of the matrix elements needed to describe the distribution functions is restricted to the light-front (LF), and it is convenient to use the Sudakov decomposition

$$k = xP + \sigma n + k_T, \quad (2)$$

in terms of a generic lightlike four-vector n , satisfying $n^2 = 0$ and $P \cdot n = 1$. In a particular hard process, its role is played by other momenta that are hard with respect to the hadron under consideration, e.g. $n \approx P'/P \cdot P'$. We can then also work with light-cone coordinates. Including

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mass effects, one would have $n_- = n$ and $n_+ = P - \frac{1}{2}M^2 n$; with $k^\pm \equiv k \cdot n_\mp$. These are $k^+ = k \cdot n = x$ and $k^- = k \cdot P - \frac{1}{2}xM^2 = \sigma + \frac{1}{2}xM^2$. The transverse momentum is orthogonal to n and P . In a hard process, the dependence on k^- of a particular correlator is not important and it is integrated over, leaving us with the restricted LF nonlocality $\xi^+ = 0$. The expansion of these correlators (in Dirac space) contains the TMD distribution functions depending on momentum fraction x and transverse momentum k_T^2 . Upon integration over k_T , one obtains the collinear correlators

$$\Phi(x) = \int \frac{d(\xi \cdot P)}{2\pi} e^{ix\xi \cdot P} \langle P | \bar{\psi}(0) \mathcal{U}_{[0;\xi]}^n \psi(\xi) | P \rangle_{\text{LC}}, \quad (3)$$

where nonlocality is restricted to the light-cone (LC: $\xi \cdot n = \xi_T = 0$) and the gauge link is unique, being the straightline path along n . These collinear correlators are expanded in the ‘‘standard’’ parton distribution functions, depending solely on the momentum fraction x . Here, we will not discuss the scale dependence [23–26].

The quark-quark LF correlator that plays a role in the fragmentation of partons is

$$\Delta_{ij}^{[U]}(z, k_T) = \sum_X \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 | \mathcal{U}_{[0;\xi]} \psi_i(\xi) \times | P, X \rangle \times \langle P, X | \bar{\psi}_j(0) | 0 \rangle_{\text{LF}}, \quad (4)$$

with the quark momentum, $k = \frac{1}{z}P + k_T + \sigma n$, i.e. a Sudakov expansion with $x = 1/z > 1$. In this case, one often refers to the hadron transverse momentum $P_\perp = -zk_T$ (in a frame in which the parton does not have a transverse momentum ($k_\perp = 0$)). Diagrammatically, these correlators are represented in Fig. 1. In this paper, we start the investigation of multiparton correlators by looking at the case with one additional gluon, as given in Fig. 1(c).

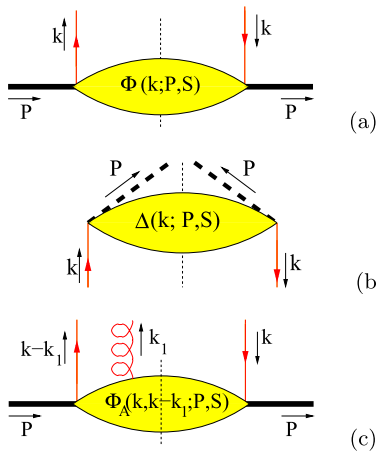


FIG. 1 (color online). The quark-quark correlators that establish the *nonperturbative* connection between partons and hadrons for distribution functions (a) or fragmentation functions (b) and a quark-quark-gluon (multiparton) correlator (c).

They appear in azimuthal asymmetries involving the k_T -weighted correlator,

$$\begin{aligned} \Phi_\partial^{\alpha[U]}(x) &= \int d^2 k_T k_T^\alpha \Phi^{[U]}(x, k_T) \\ &= \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]} \pi \Phi_G^\alpha(x, x), \end{aligned} \quad (5)$$

which is decomposed in pieces $\tilde{\Phi}_\partial$ and Φ_G , that contain T-even and T-odd operator combinations, respectively [22]. The T-odd parts come with calculable gluonic pole factors $C_G^{[U]}$ that depend on the gauge link. In this paper, we focus on the connection of this part to the zero momentum ($x_1 \rightarrow 0$) limit of a quark-quark-gluon correlator

$$\begin{aligned} \Phi_G^\alpha(x, x - x_1) &= \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix_1(\eta \cdot P)} e^{i(x-x_1)(\xi \cdot P)} \\ &\times \langle P | \bar{\psi}(0) \mathcal{U}_{[0;\eta]}^n g G^{n\alpha}(\eta) \mathcal{U}_{[\eta;\xi]}^n \psi(\xi) | P \rangle_{\text{LC}}, \end{aligned} \quad (6)$$

where $G^{n\alpha} = n_\mu G^{\mu\alpha}$ represents specific components of the color field strength tensor (α being transverse). The zero momentum limit of this correlator is the gluonic pole matrix element mentioned above. It is the support of $\Phi_G(x, x - x_1)$ that we are after, and specifically the model independent proof that it vanishes in the limit $x_1 \rightarrow 0$ for the case of the fragmentation correlators ($|x| > 1$).

The first step in all considerations is the observation that the k^- and k_1^- -integrations in the quark-quark and quark-quark-gluon correlators lead to LF correlators, for which time-ordering is irrelevant. Therefore, the matrix elements can be considered as forward matrix elements of time-ordered products of operators. These represent scattering amplitudes and their analytic structure enables one to make statements about the support of the associated parton correlation functions. This can be done for quark-quark [27] and multiparton correlators [28], and for TMD as well as collinear correlators [29]. In this language, the diagrams of Figs. 1 are just hadron-parton amplitudes, e.g. the quark-quark correlator Eq. (3) related to the forward antiquark-hadron scattering amplitude $\mathcal{A}(k^2; s, u)$ (see Fig. 2). Depending on the precise structure these are untruncated Green functions (time-ordered) or related to such Green functions via the LSZ formalism [30].

The second step is the study of the analytic structure of an amplitude and, in particular, the singularities arising from cuts in the forward amplitudes. These are cuts in the (untruncated) legs, in particular, the parton virtuality k^2 and the Mandelstam invariants. The virtualities are conventionally placed just below the real axis, or the invariants are replaced by $p^2 + i\epsilon$, $s + i\epsilon$. The integrations over k^- and k_1^- imply integrations over some of the invariants. At this point, one must make the standard assumption that it is possible to use analyticity for QCD-amplitudes. We illustrate this step first for the standard quark-quark correlators. In that case, one works with the four-point Green functions, shown in the

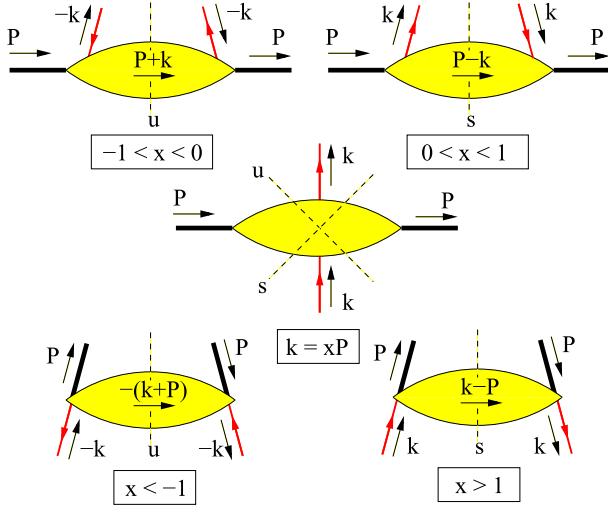


FIG. 2 (color online). Integrating parton correlators over k^- allows connecting them to a single antiparton—hadron scattering four-point function $\mathcal{A}(k^2; s, u)$ (middle). Depending on the value of x , the imaginary part of this amplitude represents the (anti)-parton distribution or fragmentation correlators.

middle of Fig. 2, where the cut amplitude depending on the particular value of x gives the quark distributions ($0 < x < 1$), or the quark fragmentation functions ($0 < z < 1$ or $x > 1$) while analytic continuation to negative x describes the antiquark distributions ($-1 < x < 0$) and antiquark fragmentation ($-1 < z < 0$ or $x < -1$).

The forward amplitude itself has singularities (cuts) for positive parton virtuality k^2 and in the Mandelstam variables, for which we choose $s = (P - k)^2$ and $u = (P + k)^2$. For the forward amplitude, the invariants are constrained to $s + u = 2k^2 + 2M^2$, where we will neglect the hadron masses (they do not play any essential role in our proof). These singularities then constitute the s -cut (for $s > 0$ and $u < 0$) and the u -cut (for $u > 0$ and $s < 0$). Using the expansion for k in Eq. (2), one has $k^2 = 2xk^- + k_T^2$ or $s = 2(x - 1)k^- + k_T^2$ and a similar expression for u . The transverse momenta, just as any of the parton or hadron masses, have little bearing on our results so we omit them in the expressions for k^- ,

$$k^- = \frac{s + i\epsilon}{2(x - 1)} = \frac{u + i\epsilon}{2(x + 1)} = \frac{k^2 + i\epsilon}{2x}, \quad (7)$$

and one sees that for distribution functions the k^- integration with respect to s , u and k^2 singularities follows the (dashed) contour in Fig. 3. The integration contours can be wrapped around the s and u -cuts for positive and negative x -values, respectively, if $|x| < 1$, cuts that (in k_1^-) smoothly vanish when $|x| \rightarrow 1$. Neither masses nor transverse momenta matter and the support properties are valid for collinear and TMD PDFs. We get

$$\Phi(x) = \theta(x)\theta(1 - x)\text{Disc}_{[s]}\mathcal{A} + \theta(-x)\theta(1 + x)\text{Disc}_{[u]}\mathcal{A}. \quad (8)$$

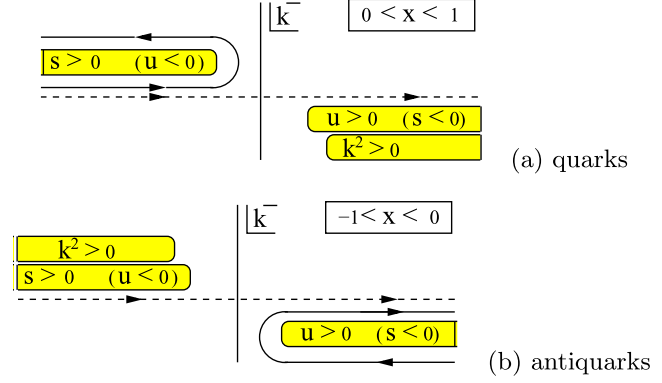


FIG. 3 (color online). The integration contours for the k^- integration with respect to the kinematic singularities in the (forward) antiparton—hadron scattering amplitude for the case of (nonvanishing) distribution functions for quarks (a) and antiquarks (b).

As discussed for instance in Ref. [31], the case for fragmentation is different since one in essence discusses the parton propagator for positive k^2 (sitting on the cut). For $|x| > 1$, one simply has

$$\Delta(x) = \theta(x - 1)\text{Disc}_{[s]}\mathcal{A} + \theta(-1 - x)\text{Disc}_{[u]}\mathcal{A}. \quad (9)$$

In wrapping the integration around the s - or u -cut, we have to assume convergence in the variable k^- (or k^2), or use subtracted relations.

It is important to mention here that the integration of Eq. (1) and $\mathcal{A}(k^2; s, u)$ over k_T leads to ultraviolet divergences [26,28,29]. However, this does not invalidate the assumption that the integral over k^- alone is sufficiently well behaved when k^+ and k_T are fixed. Integration over k_T as well as weighting with k_T is anyway intimately linked with QCD evolution of TMDs [32].

We can extend this analyticity analysis to the multiparton distribution and fragmentation functions in Eq. (6), by looking at the multiparton amplitude $\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k - k_1)^2)$ shown in Fig. 4, by studying the contours for the additional integrations. Defining the momentum of the additional parton as k_1 as shown in Fig. 1, one retains the definitions and relations for s and u . For given positive s (s -channel, $x > 0$) or positive u (u -channel, $x < 0$) one gets additional invariants $s_1 = (P \mp k \pm k_1)^2$ and $u_1 = (P \mp k_1)^2$ (cf. Figures 4(a) and 4(b), respectively). Note that $t_1 = k^2$ in both cases. Furthermore, one has parton virtualities. Depending on if one is dealing with the s - or u -cut discontinuities, one has slightly different constraints for $s_1 + u_1$, but for given values of s and u in the two cases of Fig. 4, one has cuts along $s_1 > 0$ ($u_1 < 0$) and $u_1 > 0$ ($s_1 < 0$) as well as for positive parton virtualities. The relevant singularities for k_1^- are found from

very general assumptions of analyticity for QCD amplitudes that this TMD correlator is universal and will be convoluted with the standard partonic cross sections. There is no proliferation of functions originating from the structure of the gauge links [14]. This universality thus applies to all TMD fragmentation functions, T-even or T-odd, and for quark as well as for gluon PFFs. Most well-known are the T-odd ones for quarks such as the Collins function H_1^\perp [4,15] and the polarization fragmentation function D_{1T}^\perp [4,34]. These functions are simply allowed T-odd parts in the fragmentation correlator $\Delta(z, k_T)$ being a decay function. The corresponding T-odd TMD distribution functions, the Boer-Mulders function h_1^\perp and the Sivers function f_{1T}^\perp originate from the difference $\Phi^{[u_+]}(x, p_T) - \Phi^{[u_-]}(x, p_T)$ of correlators with different gauge links and as a consequence will be convoluted with nonstandard gluonic pole cross sections [13,14,35]. Our result

also implies universality for the TMD fragmentation functions of gluons [36]; including, for instance, the T-even TMD fragmentation functions $H_1^{\perp(g)}$, which just as the corresponding distribution function $h_1^{\perp(g)}$ has a nontrivial gauge link dependence [35,37,38]. The T-even fragmentation function $H_1^{\perp(g)}$, however, is universal. In the case of these T-even functions, the nontrivial gauge link dependence only becomes visible in even k_T -moments involving contributions from T-even multigluonic pole matrix elements with an even number of gluons, all of which for fragmentation functions, however, will vanish.

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