



An ODE-based Wall Model for Turbulent Flow Simulations

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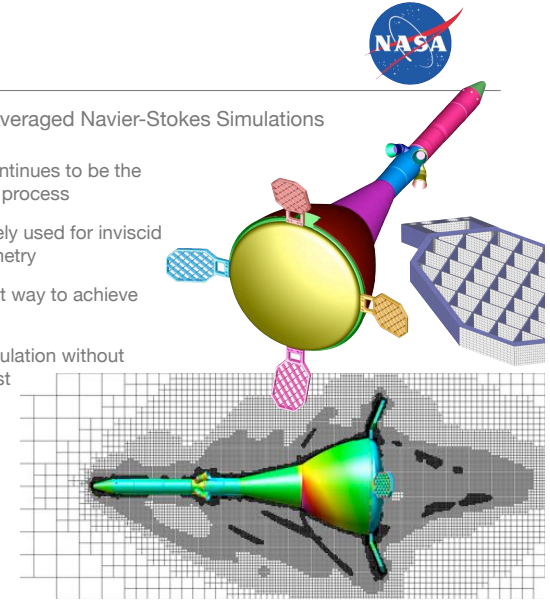
23 February, 2017

2

Motivation

Fully automated meshing for Reynolds-Averaged Navier-Stokes Simulations

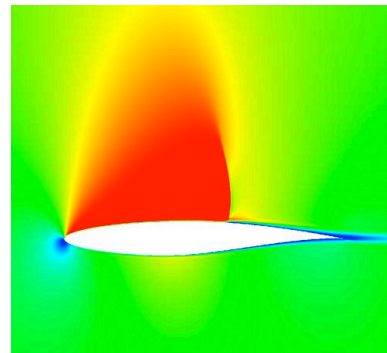
- Mesh generation for complex geometry continues to be the biggest bottleneck in the RANS simulation process
- Fully automated Cartesian methods routinely used for inviscid simulations about arbitrarily complex geometry
- These methods lack of an obvious & robust way to achieve near wall anisotropy
- **Goal:** Extend these methods for RANS simulation without sacrificing automation, at an affordable cost
- Note: Nothing here is limited to Cartesian methods, and much becomes simpler in a body-fitted setting.



3

Outline

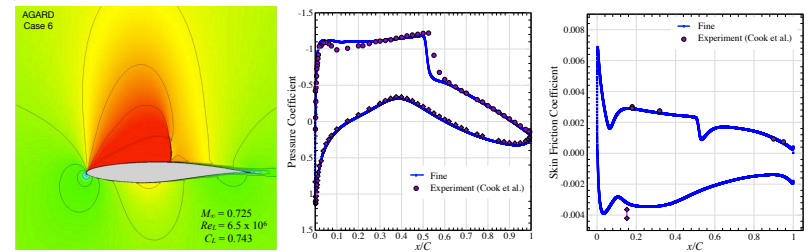
- Previous work & analytic wall functions
- ODE-based wall models
 - A New ODE wall model
- Numerical examples
- Conclusions



4

Previous Work

Analytic wall functions



- Pure Cartesian cut-cell approach
- Coupled applied analytic wall functions with cut-cell Cartesian meshes in 2012*
- Results comparable to body-fitted methods using wall functions

Conclusion: Cartesian RANS is viable, but wall functions alone are probably not sufficient to make the approach cost competitive

5

*AIAA 2012-1301, "Progress Towards a Cartesian Cut-Cell Method for Viscous Compressible Flow", Berger, Aftosmis & Allmaras



Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.
$$\frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right]$$

- The diffusion model assumes that velocity is small and ZPG

$$\frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = 0$$

6
6



Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.
$$\frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right]$$

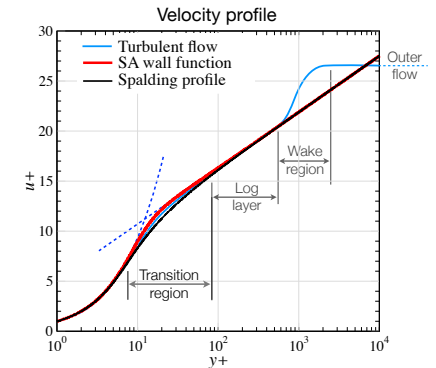
- The diffusion model assumes that velocity is small and ZPG

$$\frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = 0$$

- Wall functions solve the diffusion model closed with a mixing-length model for eddy viscosity: $\nu_t \sim$ distance to the wall

$$\mu_t = \rho \nu_t = \rho \kappa \nu y^+$$

- Excellent agreement with velocity data up through the log layer



7
7



Previous Work

Analytic wall functions

- Spalding model:
$$y^+(u^+) = u^+ + e^{-\kappa B} \left(e^{\kappa u^+} - 1 - \kappa u^+ - \frac{1}{2}(\kappa u^+)^2 - \frac{1}{6}(\kappa u^+)^3 \right)$$

- SA wall function (2012):
Derived, using a limiting form of SA turbulence model and integrating the diffusion model

$$u^+(y^+) = \bar{B} + c_1 \log((y^+ + a_1)^2 + b_1^2) - c_2 \log((y^+ + a_2)^2 + b_2^2) - c_3 \arctan 2(y^+ + a_1, b_1) - c_4 \arctan 2(y^+ + a_2, b_2)$$

- Prefer SA wall function, since it gives direct relationship for velocity as a function of distance
- Knowing u at a point F , iterate to find u_τ , so that $u^+(y_F^+) = u_F^+ = u_\tau u_F$

8
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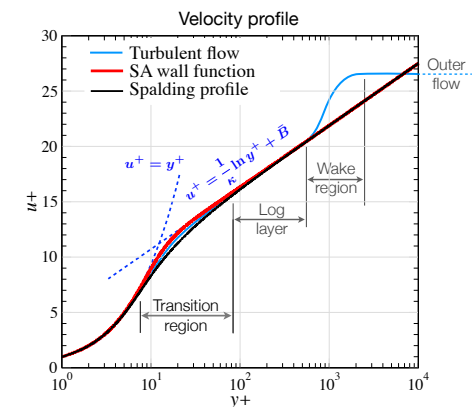
Previous Work

Analytic wall functions

- Spalding model: $y^+(u^+) = \dots$
- SA wall function (2012): $u^+(y^+) = \dots$

Both:

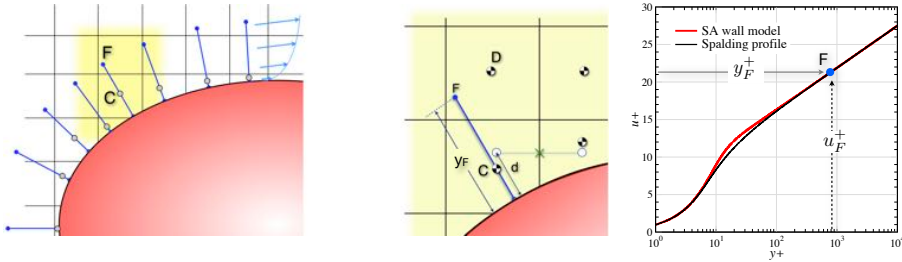
- Are good approximations and give accurate wall shear stress when anchored to the log-layer
- Are inappropriate beyond the log layer
 - Predict exponentially increasing velocity
 - Don't consider pressure gradients
 - Ignore the wake



9
9



Coupling and Forcing Point Construction



- Construct forcing points at uniform distance from wall
- Interpolate data to point F from cell centered solution on outer grid
- With velocity at distance at forcing point, use wall function to find u_τ and wall shear $\tau_{\text{wall}} = \rho u_\tau^2$
- Feed back to outer meshes via viscous fluxes in the cut cells

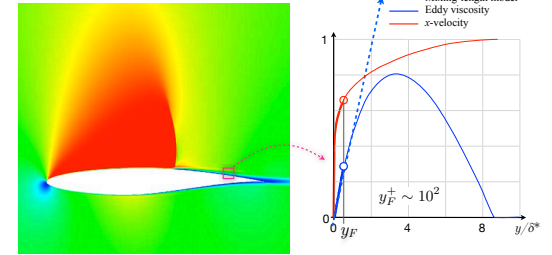
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Good wall functions gone bad

Thick boundary layer

- $y_F^+ \approx 10^2$
- Forcing point in log layer
- Mixing length model gives good estimate eddy viscosity
- Analytic wall function is appropriate



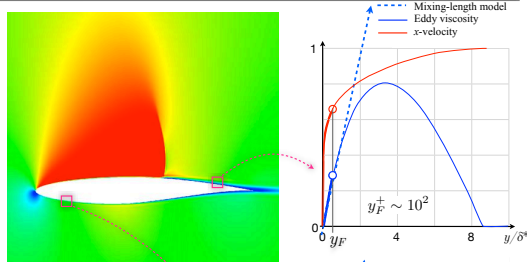
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Good wall functions gone bad

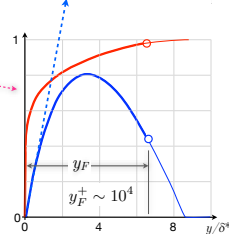
Thick boundary layer

- $y_F^+ \approx 10^2$
- Forcing point in log layer
- Mixing length model gives good estimate eddy viscosity
- Analytic wall function is appropriate



Thin boundary layer

- At the same distance from the wall, $y_F^+ \approx 10^4$
- Forcing point is now in the wake layer
- Eddy viscosity highly non-linear
- Mixing length model is a poor approximation, analytic wall function inappropriate



12
12



Outline

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 - A New ODE wall model
- Numerical examples
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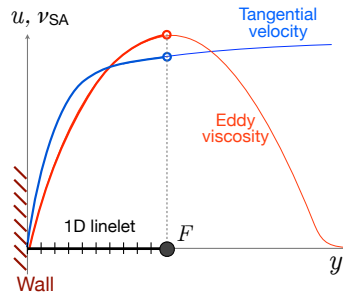
13
13



ODE-Based Wall Models

Proposed by several authors* in last decade

- Solve ODE on 1D “linelet” normal to surface
- Solve:
 - Diffusion eq. for streamwise momentum
 - Turbulence model in wall-normal direction
- Produces a system of 2-point, 2nd-order BVPs
- Coupling: *Just like an analytic wall function*



* see: Kalitzin et al., J. Comp. Phys., 204, 2005, Bond & Blottner, Intl. J. Num. Methods Fluids, 66, 2011, or Capizzano, AIAA J. 54(2), 2016



ODE-Based Wall Models

SA-BVP: Diffusion equation coupled with wall-normal SA turbulence model

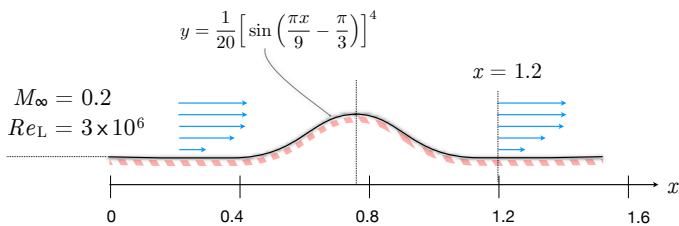
$$x\text{-momentum: } \frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = 0$$

$$\text{SA model on linelet: } \frac{\partial}{\partial y} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial y} \right) = \underbrace{-c_{b2} \left(\frac{\partial \tilde{\nu}}{\partial y} \right)^2}_{\text{wall-normal diffusion}} + \text{Production} - \text{Destruction}$$



ODE-Based Wall Models

Compare SA-BVP with SA wall function on turbulent bump in channel



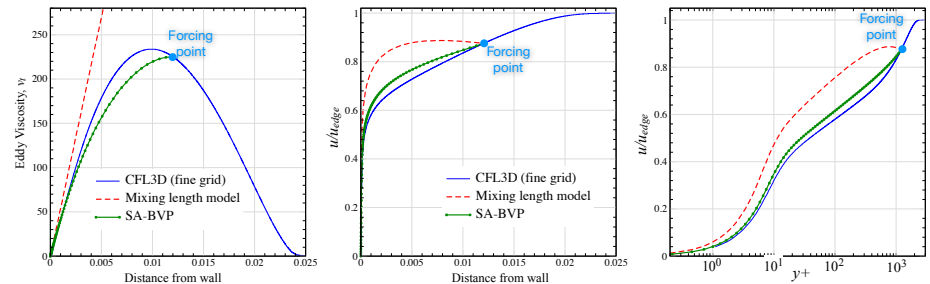
$$x\text{-momentum: } \frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = 0$$

$$\text{SA model on linelet: } \frac{\partial}{\partial y} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial y} \right) = -c_{b2} \left(\frac{\partial \tilde{\nu}}{\partial y} \right)^2 + \text{Production} - \text{Destruction}$$



ODE-Based Wall Models

Compare SA-BVP with SA wall function on turbulent bump in channel @ x = 1.2



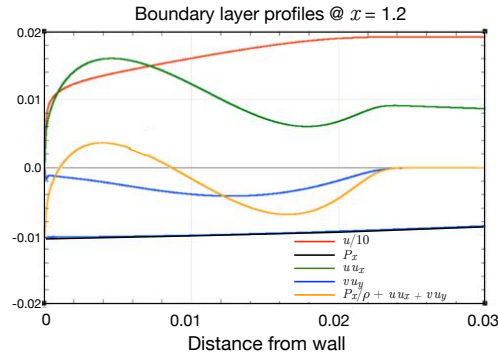
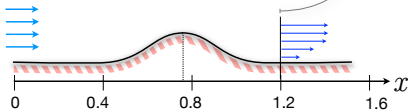
- Forcing point well out in wake layer, $y = 0.012$, $u = 0.85 u_{edge}$
- Mixing length eddy viscosity inappropriate, so diffusion model alone does poorly
- Improved eddy viscosity makes a significant difference



ODE-Based Wall Models

Streamwise momentum in the wake

$$M_\infty = 0.2$$
$$Re_L = 3 \times 10^6$$



- Thin layer streamwise momentum: $((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vv_y)$
- Examine relative magnitude of terms as we move away from the wall

18
18

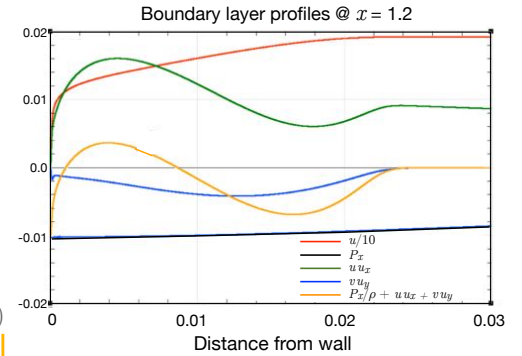


ODE-Based Wall Models

Streamwise momentum in the wake

- At the wall, we have $((\mu + \mu_t)u_y)_y = p_x$
- Outside the boundary layer we approach: $0 = p_x + \rho(uu_x + vv_y)$
- In between we have the full streamwise momentum eq.

$$((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vv_y)$$

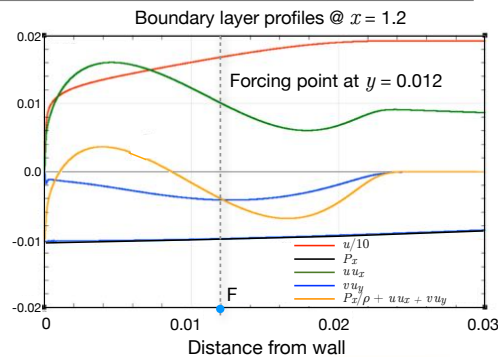
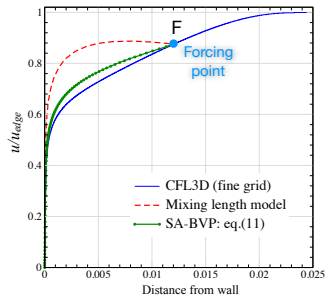


19
19



ODE-Based Wall Models

Streamwise momentum in the wake



- Forcing point @ $y = 0.012$ is in the wake. $((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vv_y)$

20
20



A New ODE-based Wall Model

The *bvp4* wall model

- The complete *bvp4* model becomes

$$\frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \psi(y) \rho \left[u_F \frac{\partial u}{\partial x} \Big|_F + v_F \frac{\partial u}{\partial y} \Big|_F \right]$$

$$\frac{\partial}{\partial y} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{v}}{\partial y} \right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$

- 1D ODE for streamwise momentum, & SA turbulence model
- Locally computable
- Includes pressure gradient and model for the streamwise momentum balance

21
21



ODE-Based Wall Models

Including the convective balance

- At the wall, velocity is zero, \therefore convective balance is zero

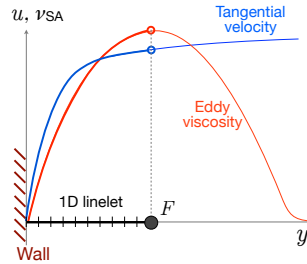
$$((\mu + \mu_t)u_y)_y = p_x$$

- But at the forcing point, it has the same magnitude as p_x

$$((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vu_y)|_F$$

- Computing wall-normal variation of convective balance introduces streamwise coupling, and means computing the wall-normal velocity & interpolating derivatives
– *prefer not to do this*

- Instead, we *choose* to model the wall normal variation of the convective balance
– keeps the stencil local
– well behaved on irregular meshes



22



ODE-Based Wall Models

Including the convective balance

- Introduce a cutoff function, $\psi(y)$, to shut down the convective balance approaching the wall

$$((\mu + \mu_t)u_y)_y = p_x + \psi(y)[\rho(uu_x + vu_y)]_F$$

- Require: $\psi(0) = 0$, and $\psi(y_F) = 1$

Desire: Scales like velocity, since through the log-layer the convective balance roughly follows velocity

23



ODE-Based Wall Models

Including the convective balance

- Introduce a cutoff function, $\psi(y)$, to shut down the convective balance approaching the wall

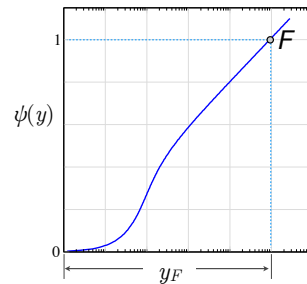
$$((\mu + \mu_t)u_y)_y = p_x + \psi(y)[\rho(uu_x + vu_y)]_F$$

- Require: $\psi(0) = 0$, and $\psi(y_F) = 1$

Desire: Scales like velocity, since through the log-layer the convective balance roughly follows velocity

- Choose: $\psi(y) = \frac{u_{SA}(y)}{u_{SAF}} = \frac{u_{SA}^+(y)}{u_{SAF}^+}$

- Blends momentum in exponentially in log region and linearly in laminar sublayer, and gives full convective balance at the forcing point, F

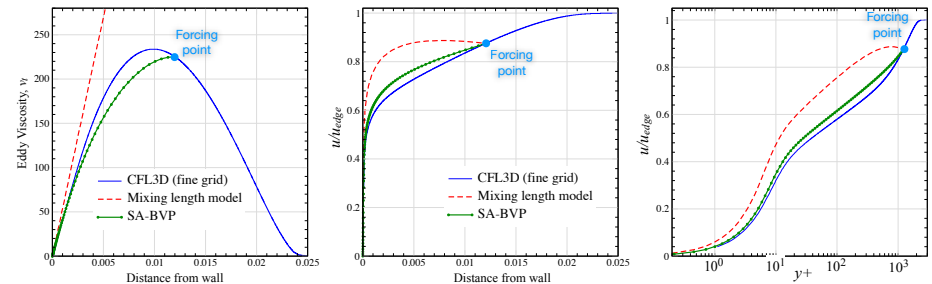


24



A New ODE-based Wall Model

bvp4 wall model: Include streamwise convective balance and pressure gradient



bvp4

$$x\text{-momentum: } \frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \psi(y)\rho \left[u_F \frac{\partial u}{\partial x} \Big|_F + v_F \frac{\partial u}{\partial y} \Big|_F \right]$$

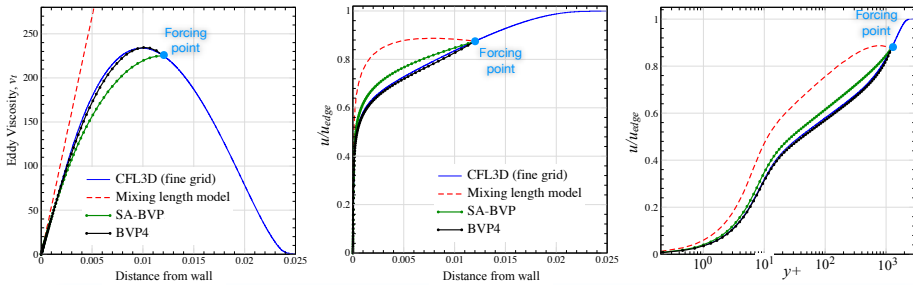
$$\text{SA model on linelet: } \frac{\partial}{\partial y} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{v}}{\partial y} \right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$

25



A New ODE-based Wall Model

bvp4 wall model: Include streamwise convective balance and pressure gradient



bvp4

$$x\text{-momentum: } \frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \psi(y)\rho \left[u_F \frac{\partial u}{\partial x} \Big|_F + v_F \frac{\partial u}{\partial y} \Big|_F \right]$$

$$\text{SA model on linelet: } \frac{\partial}{\partial y} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial y} \right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$

26
26



A New ODE-based Wall Model

ODE solver for *bvp4*

$$\frac{\partial}{\partial y} \left((\mu + \mu_t) \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \psi(y)\rho \left[u_F \frac{\partial u}{\partial x} \Big|_F + v_F \frac{\partial u}{\partial y} \Big|_F \right]$$

$$\frac{\partial}{\partial y} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial y} \right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$

- Reformulate 2nd order equations as system of four 1st order BVPs
- Solve with 6th order adaptive ODE solver from Shampine and Muir
- Use warm starts on each linelet after initial solve ~2x cost of analytic WF
- Other details of implementation and coupling in paper

27
27



Numerical Results

Verification and Validation using examples from the NASA Turbulence Modeling Resource

Computational Examples from TMR

1. Turbulent bump in channel
2. NACA 0012
3. NACA 4412 with trailing edge separation

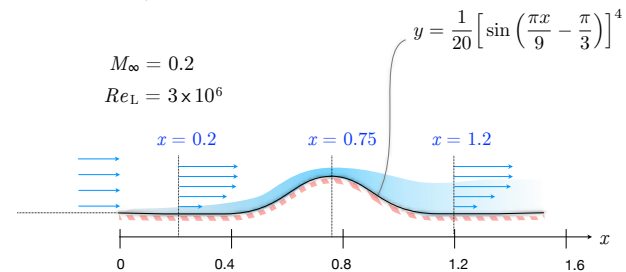
Mesh convergence studies in the paper – just include highlights here

28
28



Turbulent Bump In Channel

TMR: "VERIF/2DB: 2D Bump-in-channel Verification Case"



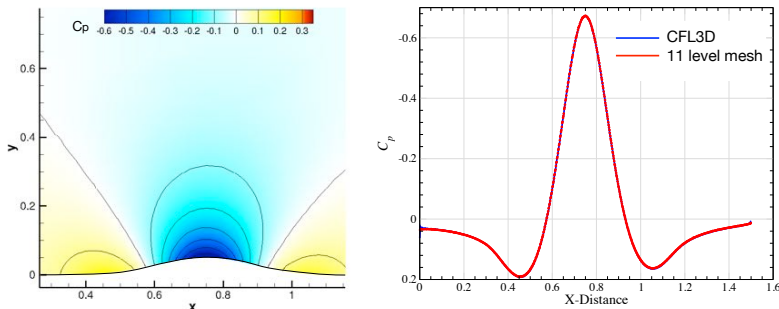
- Inlet & exit 25 units away, symmetry plane 5 units above
- Mesh-converged body-fitted results on 1409 x 641 mesh (~900 k points)
- Compare results with CFL3D reference solution with SA turbulence model on finest mesh

29
29



Turbulent Bump In Channel

Bump: Isobars and surface pressure comparison



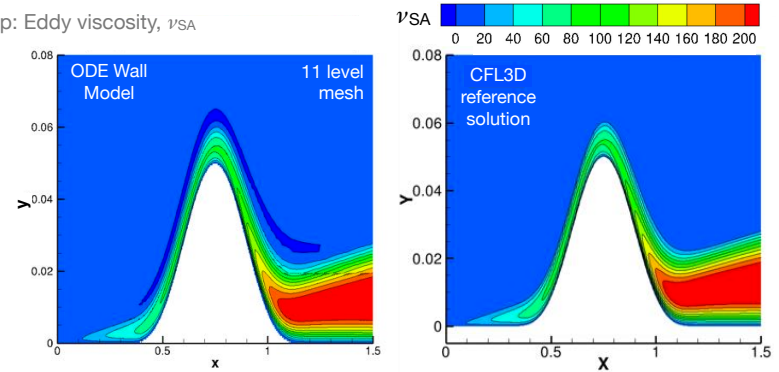
	# Cells or Points	Minimum Cell Dimensions
Coarse	10 levels	52 k $\Delta y = 0.0012$
Medium	11 levels	70 k $\Delta y = 0.0006$
Fine	12 levels	91 k $\Delta y = 0.0003$
Reference solution		900 k $\Delta y = 2.5e-7$

30



Turbulent Bump In Channel

Bump: Eddy viscosity, ν_{SA}



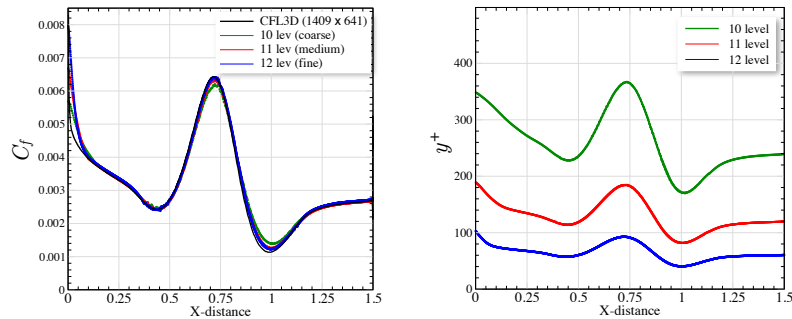
- Good agreement for evolution and peak eddy viscosity
- Slight negative values of ν_{SA} outside of boundary-layer due to 2nd-order advective terms, easily controlled by negative-SA turbulence model

31



Turbulent Bump In Channel

Bump: - Skin friction & y^+ distribution comparison with bvp4



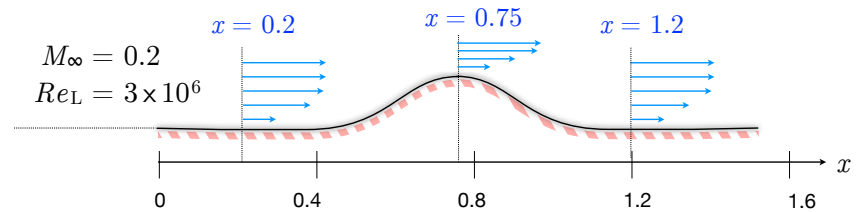
- Smooth C_f historically challenging for cut-cell meshes, but look good here
- Slight noise from HLLC flux when face-normal velocity passes through zero
- Good agreement progressing toward mesh convergence, results ordered by dissipation

32



Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles



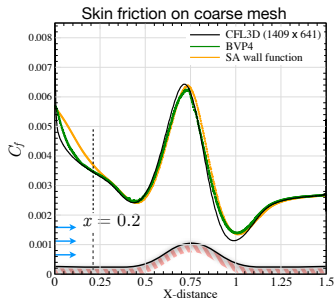
- Paper examine mesh convergence at all 3 stations
- Boundary layer thickens by approx. factor of 2 at each station
- Since resolution of Cartesian mesh is constant, resolution roughly doubles each time we move downstream

33



Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

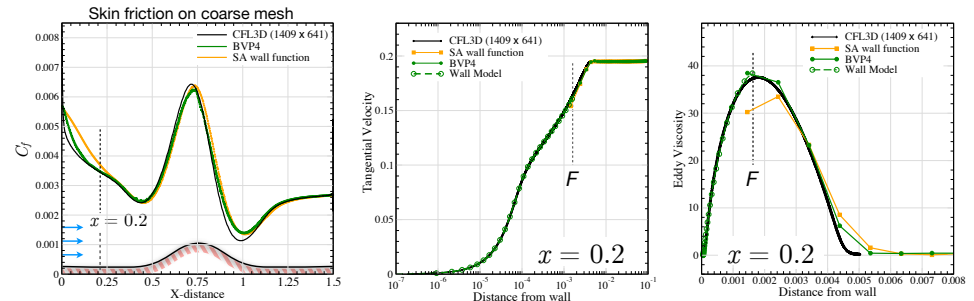


- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)



Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

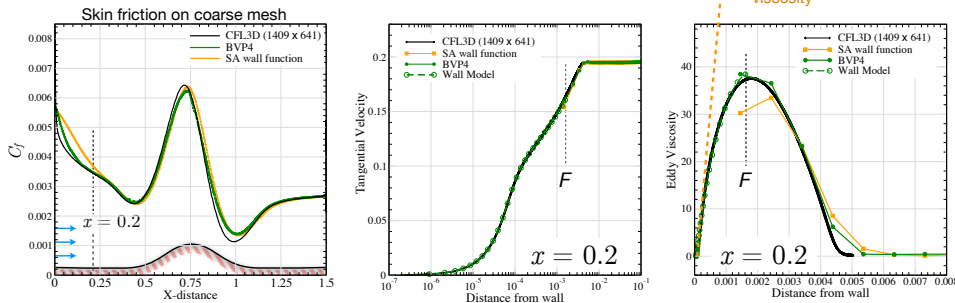


- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)
- Skin friction discrepancy comes from misprediction of eddy viscosity by analytic wall function since it assumes a mixing-length model



Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

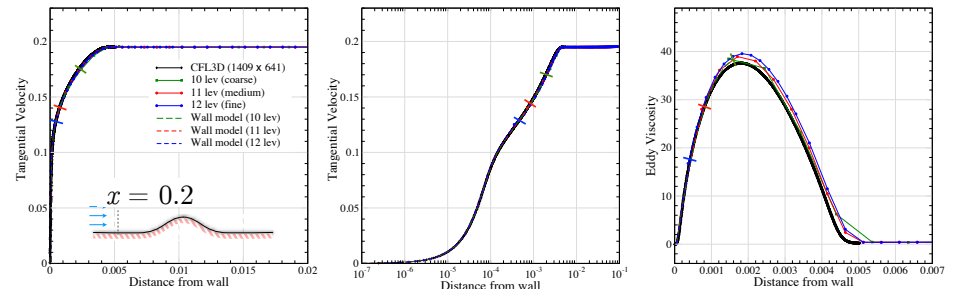


- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)
- Analytic wall function overpredicts eddy viscosity by about factor of 3, – is inconsistent with outer solution



Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

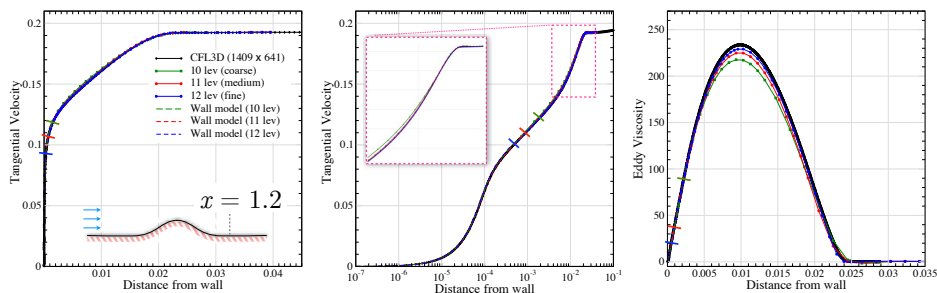


- 7 curves on each plot, (wall model & field solution) x (coarse, med, fine) + CFL3D
- Very good agreement for velocity, good agreement for eddy viscosity
- $x = 0.2$ is the most under resolved station, ~4-5 Cartesian cells in boundary layer



Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles



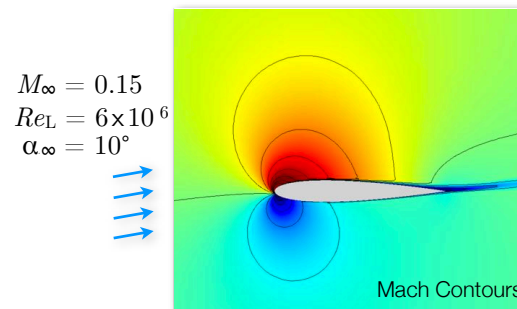
- B-L about 4x thicker by $x=1.2$
- Aft of bump, slight adverse pressure gradient, thick boundary layer
- Velocity profiles show very good agreement -- even on semi log scale
- Eddy viscosity peak being eroded slightly by dissipation on outer mesh

38



NACA 0012

Modified NACA 0012 geometry with sharp trailing edge



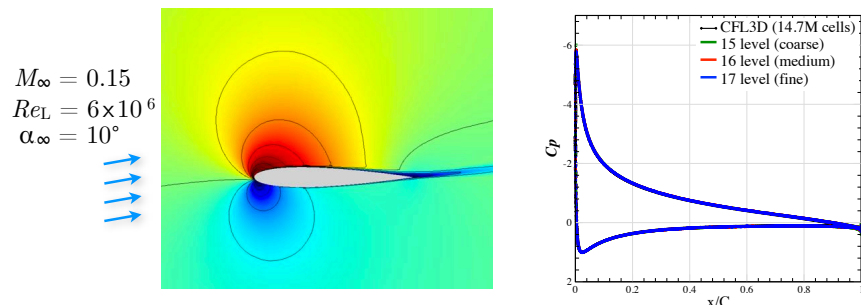
- Validation example "2DN00: 2D NACA 0012 Airfoil Validation Case" of TMR website
- Refinement studies on grids up to 14.7M points
- Compare with CFL3D, SA model with no circulation correction
- Use this case to dissect the role of various terms in the bvp4 model

39



NACA 0012

Modified NACA 0012 - Surface pressure mesh convergence



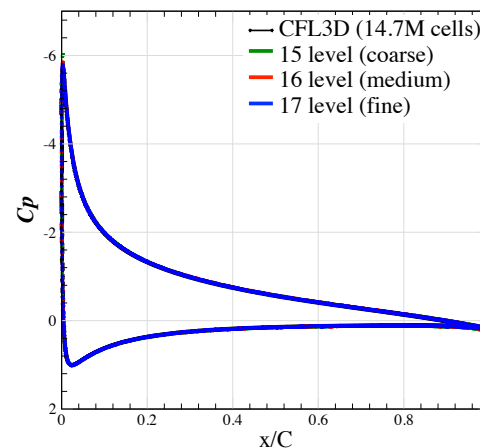
	# Cells or Points	Minimum Cell Dimensions
Coarse	15 levels 58 k	$\Delta x = \Delta y = 1.7e-3 C$
Medium	16 levels 80 k	$\Delta x = \Delta y = 8.4e-4 C$
Fine	17 levels 133 k	$\Delta x = \Delta y = 4.2e-4 C$
Reference solution		$1.e-7 C \times 1.25e-5 C$

40



NACA 0012

Modified NACA 0012 - Surface pressure mesh convergence

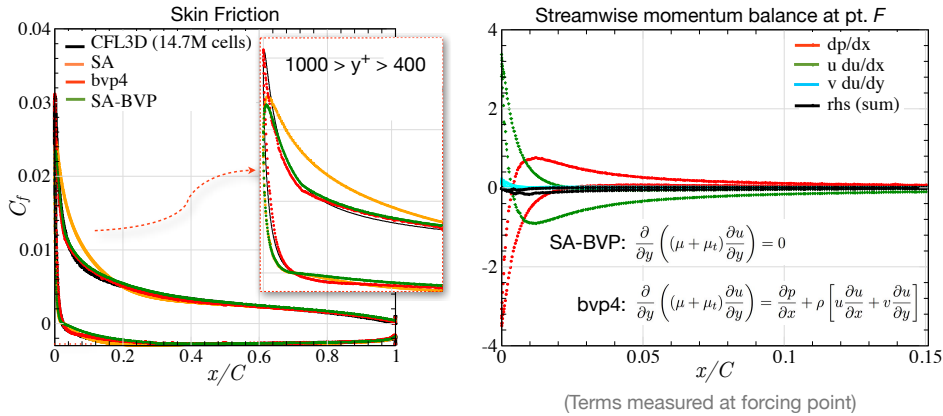


41



NACA 0012

Modified NACA 0012 - skin friction and streamwise momentum

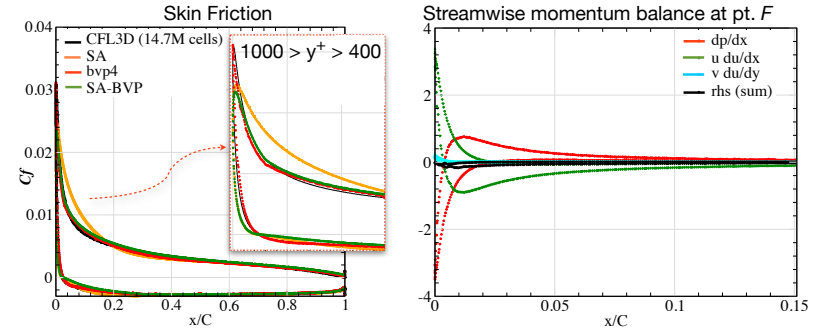


42
42



NACA 0012

Modified NACA 0012 - Compare bvp4 with analytic SA-BVP & SA wall function



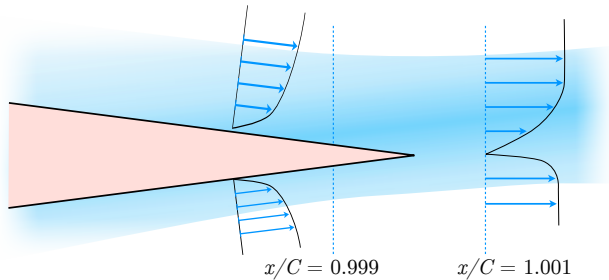
- SA-wall function lags both bvp4 model and SA-BVP skin friction, good around $y^+ = 300$
- SA-BVP does a good job, but misses peak and pressure-side near leading edge
- bvp4 sees p_x at the wall, but by the time you reach F , p_x is largely shutoff by the convective balance ($u u_x + v u_y$) and RHS nearly vanishes

43
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NACA 0012

Modified NACA 0012 - Wake surveys of velocity and eddy viscosity

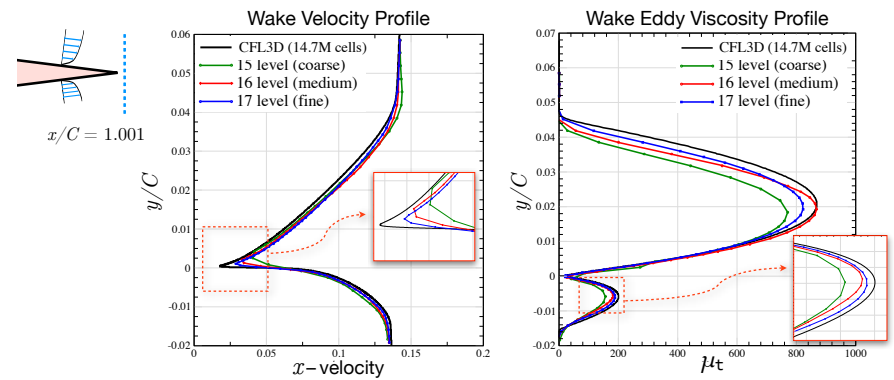


- Comparison data exist for profiles at $x/C = 0.999$ and 1.001
- Examine data at $x/C = 1.001$, (similar results at $x/C = 0.999$)

44
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NACA 0012



- Low dissipation inviscid flux helps resolution in cusp – good mesh convergence behavior
- Lower surface eddy viscosity shows good mesh convergence
- Upper surface eddy viscosity not yet mesh converged – literature shows slow convergence

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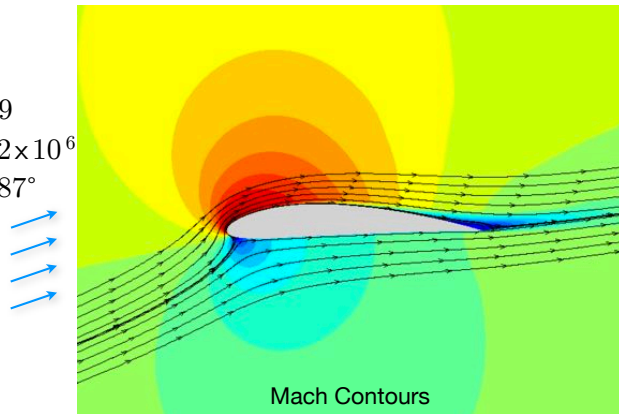
NACA 4412

Modified NACA 4412 with trailing edge separation bubble

$$M_\infty = 0.09$$

$$Re_L = 1.52 \times 10^6$$

$$\alpha_\infty = 13.87^\circ$$



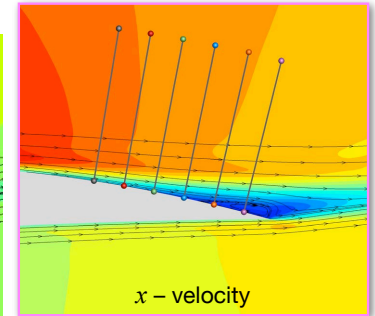
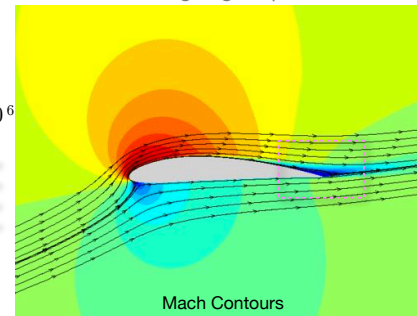
NACA 4412

Modified NACA 4412 with trailing edge separation bubble

$$M_\infty = 0.09$$

$$Re_L = 1.52 \times 10^6$$

$$\alpha_\infty = 13.87^\circ$$

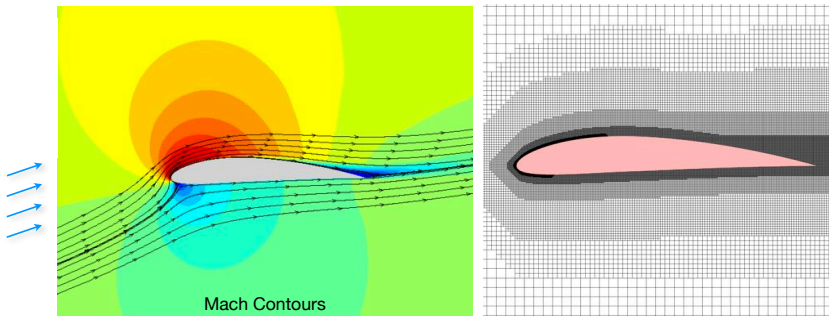


- Validation example in the “Extended cases” section of NASA TMR
- Smooth-body separation bubble near maximum lift conditions
- Experiment by Coles & Wadcock (1979) with hot-wire velocity profiles
- Reference data form CFL3D on 897 x 257 grid ($\approx 230k$ points)



NACA 4412

Modified NACA 4412 with trailing edge separation bubble

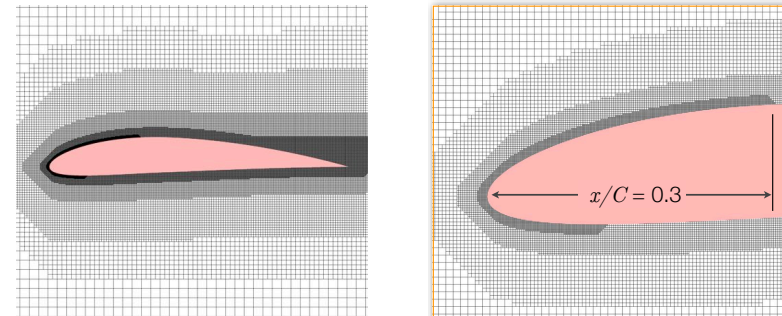


- Multilevel Cartesian mesh with $\sim 59k$ cells
- 1-level of mesh refinement near leading edge
- Leading edge, $\Delta x = 0.1\%C$, trailing edge $\Delta x = 0.2\%C$, ~ 1200 cut cells



NACA 4412

Modified NACA 4412 with trailing edge separation bubble



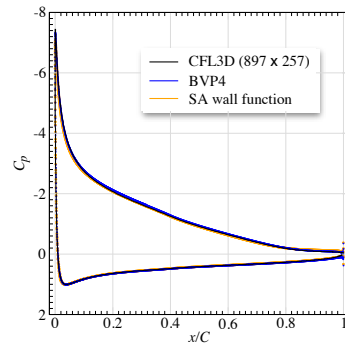
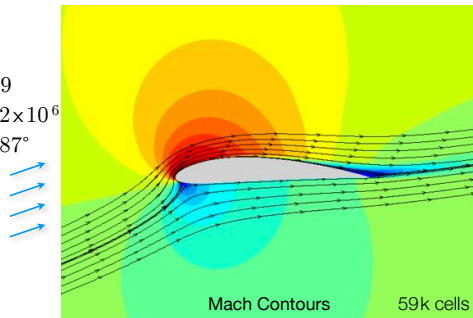
- Multilevel Cartesian mesh with $\sim 59k$ cells
- 1-level of mesh refinement near leading edge
- Leading edge, $\Delta x = 0.1\%C$, trailing edge $\Delta x = 0.2\%C$, ~ 1200 cut cells



NACA 4412

Modified NACA 4412 – Surface pressure comparison

$M_\infty = 0.09$
 $Re_L = 1.52 \times 10^6$
 $\alpha_\infty = 13.87^\circ$

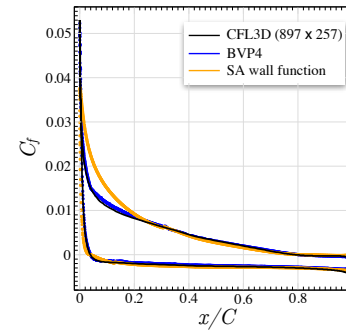


- Good comparison of surface pressure coefficient with both models
- SA-wall function & bvp4 nearly indistinguishable from CFL3D results



NACA 4412

Modified NACA 4412 – Skin friction comparison

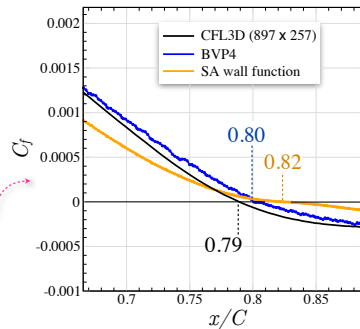
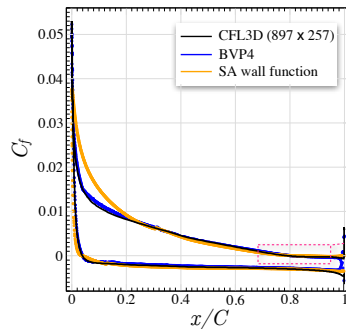


- Leading edge very under resolved, $y^+ \sim 500$
- bvp4 substantially outperforms wall function near leading edge with thin boundary-layer & steep pressure gradient



NACA 4412

Modified NACA 4412 – Skin friction comparison

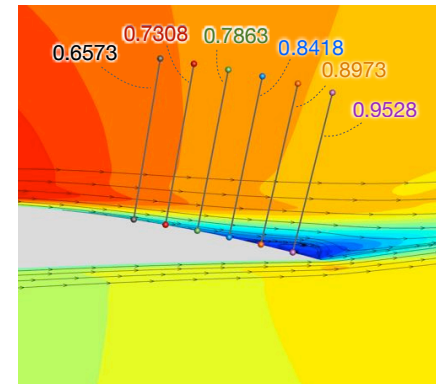


- bvp4 substantially outperforms wall function near leading edge with thin boundary-layer & steep pressure gradient
- bvp4 predicts separation location within 1% of mesh resolved CFL3D result
- Noise in bvp4 due to interpolation of $(u u_x \& v u_y)$ at forcing point



NACA 4412

Modified NACA 4412 – Velocity comparison near separation

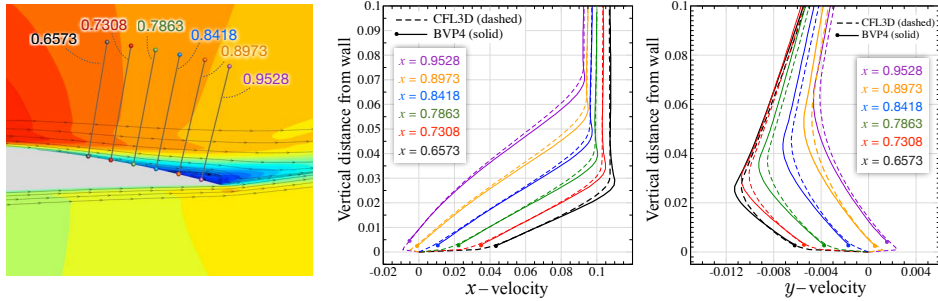


Locations of hot-wire surveys in experiment (Coles & Wadcock, 1979)



NACA 4412

Modified NACA 4412 – Velocity comparison near separation



- Good prediction of both x and y components of velocity through separation bubble
- Vertical velocity about an order of magnitude smaller than horizontal
- Slight “viscous overshoot” due to coarseness of Cartesian mesh, $\Delta x = \Delta y = 0.2\%C$

54
54



Summary

- Presented V&V studies for a new ODE-based wall model for RANS equations. Demonstrated on several well-studied flows including one with smooth body separation
- *bvp4* model:
 - Solves a coupled set of ODEs posed as two-point boundary value problems for the streamwise velocity and the turbulent viscosity
 - Includes both the streamwise pressure gradient and the momentum balance and is valid at significantly larger values of y^+ than analytic wall functions
 - All the ODE solves are completely local and are driven by data at a single field point
 - Cost is about 2-3x the computational cost of analytic wf's on same mesh
- Permits wall spacing on the outer mesh that is 4 to 8 times coarser than is possible with analytic wall functions
- Can be applied on both body-fitted & non-body fitted meshes
- Shows promise for non-body-fitted RANS

55
55



Acknowledgements

Many many thanks!

- NASA Turbulence Modeling Resource <https://turbmodels.larc.nasa.gov>
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- Steven Allmaras (MIT)
- Mike Olsen (NASA ARC)

56
56

Questions?



57