Quantification of SLS Dynamic Model Validation Metrics using Uncertainty Propagation from Requirements

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SLS Dynamic Model & Requirement

- The Space Launch System, NASA's new large launch vehicle for long range space exploration, is presently in the final design and construction phases, with the first launch scheduled for 2019.
- The Structural Dynamic model is critical for generation of natural frequencies for guidance, navigation, and control (GNC), as well as mode shapes used in calculation of interface loads.
- Single modal test of the unfueled SLS will be performed while bolted down to the Mobile Launch Pad just before the first launch. (Integrated Vehicle Modal Test, IMT).
- The validation of the SLS flight model will be achieved using a probabilistic optimization technique defined as the "Best Model Estimate (BME)" method developed by Stewart, 2017*.
- Question will remain whether model is adequate to meet loads and GNC requirements.
  - Our interpretation of the GNC requirement is that the true frequency can be no more than 3% less than the post-test prediction (flight model), which comes out of BME process.

IMT

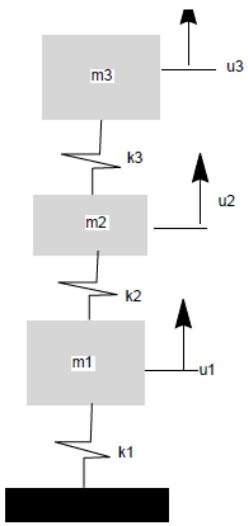
**Stewart, E., Hathcock, M., "Using Dispersed Modes During Model Correlation", AIAA SciTech Forum, 2017*

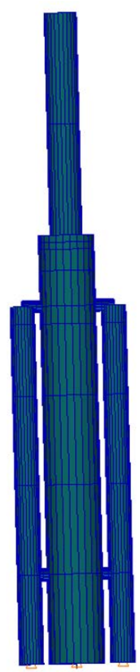
SLS Dynamic Model Uncertainty Quantification & Configurations

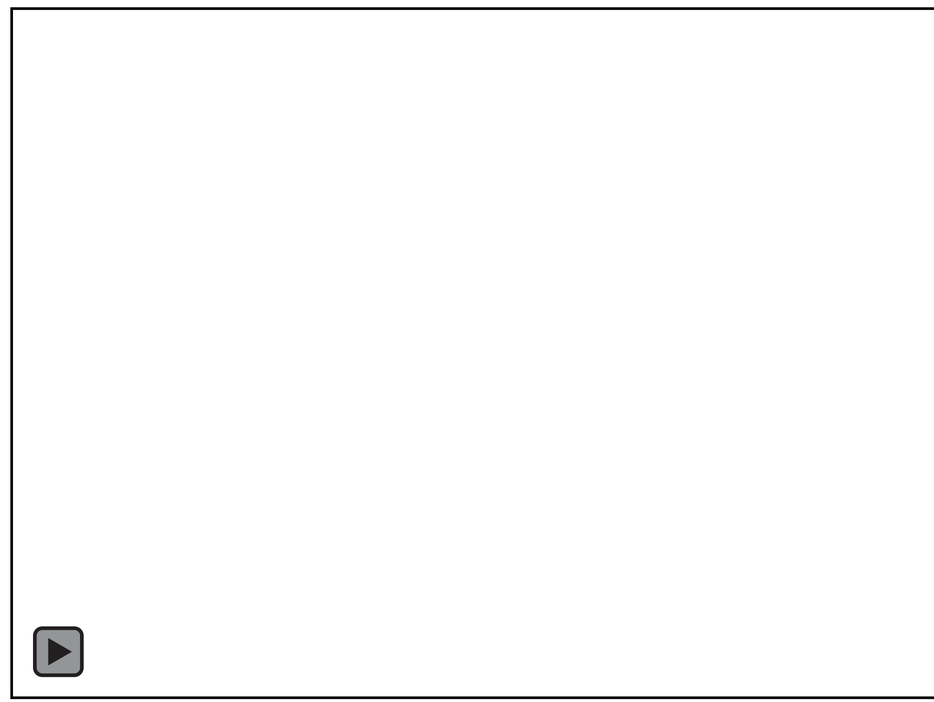
- Our goal is generate an Uncertainty Propagation and Quantification (UP and UQ) technique to develop a quantitative set of validation metrics that is based on the flight requirements.
- Considerable research on UQ and UP and validation in the literature, but very little on propagating the uncertainties from specific requirements; most validation metrics are "rules-of-thumb."
- Natural Frequencies differ between SLS test configuration and flight configuration
  - SLS structure mounted on Mobile Launcher, no liquid fuel in tanks.
    - Frequency of Ground-Modal Test Configuration test Fgt - scalar since very little uncertainty in measured natural frequency.
    - Frequency of Ground-Modal Test Configuration model {Fgm} - vector incorporating modeling uncertainty.
  - SLS fully fueled, free-free boundary conditions.
    - Frequency of Flight Configuration "test/truth" Fft - our goal, scalar value of actual natural frequency of mode of concern.
    - Frequency of Flight Configuration model {Ffm} – vector incorporating model uncertainty.
- Central question is how to extrapolate uncertainties from ground to flight configurations.

Procedure Development - Flight to Ground Model Ratio Extrapolated to Test (true) Values

- A number of possible methodologies were examined, and applied to increasingly more complicated geometries.
- First step in process is to identify and quantify uncertain model parameters in both configurations.
- Techniques were examined using 3-DOF spring/mass model, then simplified NASTRAN model of SLS, using LS-Opt to run Monte Carlo, where response variables are fundamental flight configuration frequency {Ffm} and candidate ground configuration frequencies {Fgm}.







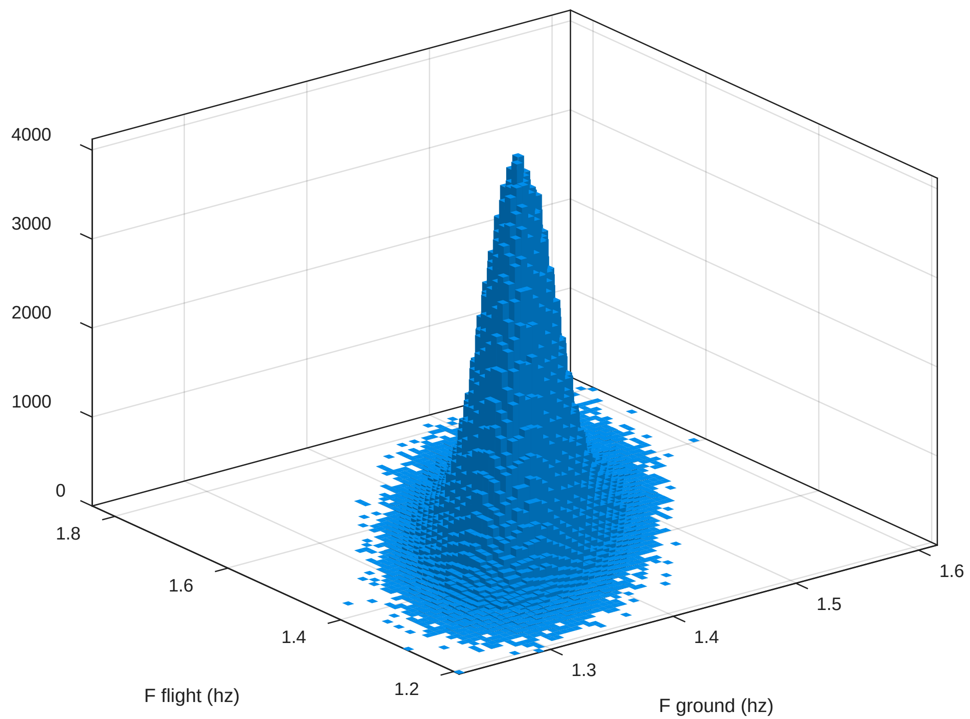
Extrapolation of Ground-Test Frequency to Pseudo-Test Flight Configuration Frequency

- Correlation between Ffm and each candidate Fgm calculated, and pair with highest r chosen.

$$r = \frac{\sum z_x z_y}{N} = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum (X-\bar{X})^2 \sum (Y-\bar{Y})^2}}$$

- The two distributions are assumed to be normal and a multi-variate normal distribution created.

Multi-Variate Histogram of Simple SLS flight 1st Bending vs F3 ground models



Uncertainty Quantification

- We now see this problem may be cast as regression analysis.
- If our data can be modeled with a bivariate normal distribution of variables X and Y, this implies a linear regression curve for the mean of Y with constant variance. If X equals a specific value x, then the parameters of the conditional distribution for Y are

$$\mu_{Y|X} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X) \quad \text{and} \quad \sigma^2_{Y|X} = \sigma^2_Y (1 - \rho^2)$$

- So here, let random variable Fgm be equal to specific value Fgt, and we assume that

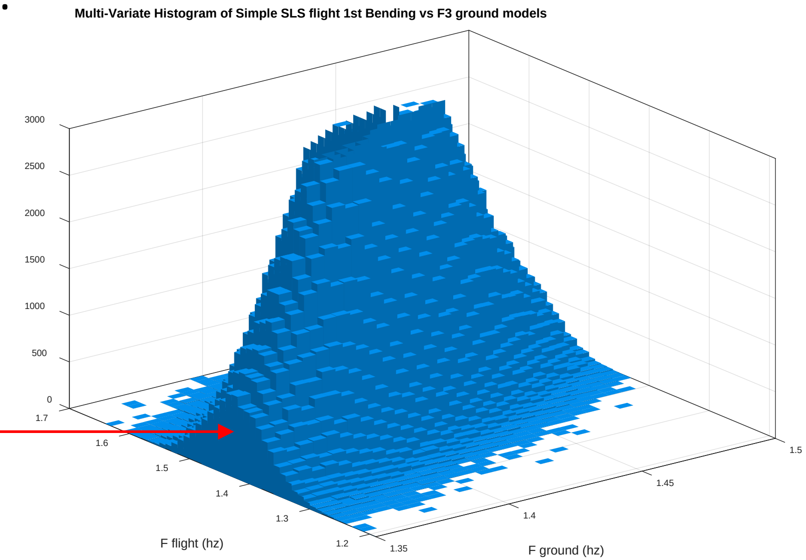
$$\mu_{F_{ft}} = \mu_{F_{fm}|F_{gm}=F_{gt}} \qquad \sigma^2_{F_{ft}} = \sigma^2_{F_{fm}|F_{gm}=F_{gt}}$$

which are in turn defined from the theorem as

$$\mu_{F_{fm}|F_{gm}=F_{gt}} = \mu_{F_{fm}} + \rho \frac{\sigma_{YF_{fm}}}{\sigma_{F_{gm}}}(F_{gt} - \mu_{F_{gm}}) \qquad \sigma^2_{F_{fm}|F_{gm}=F_{gt}} = \sigma^2_{F_{fm}}(1 - \rho^2)$$

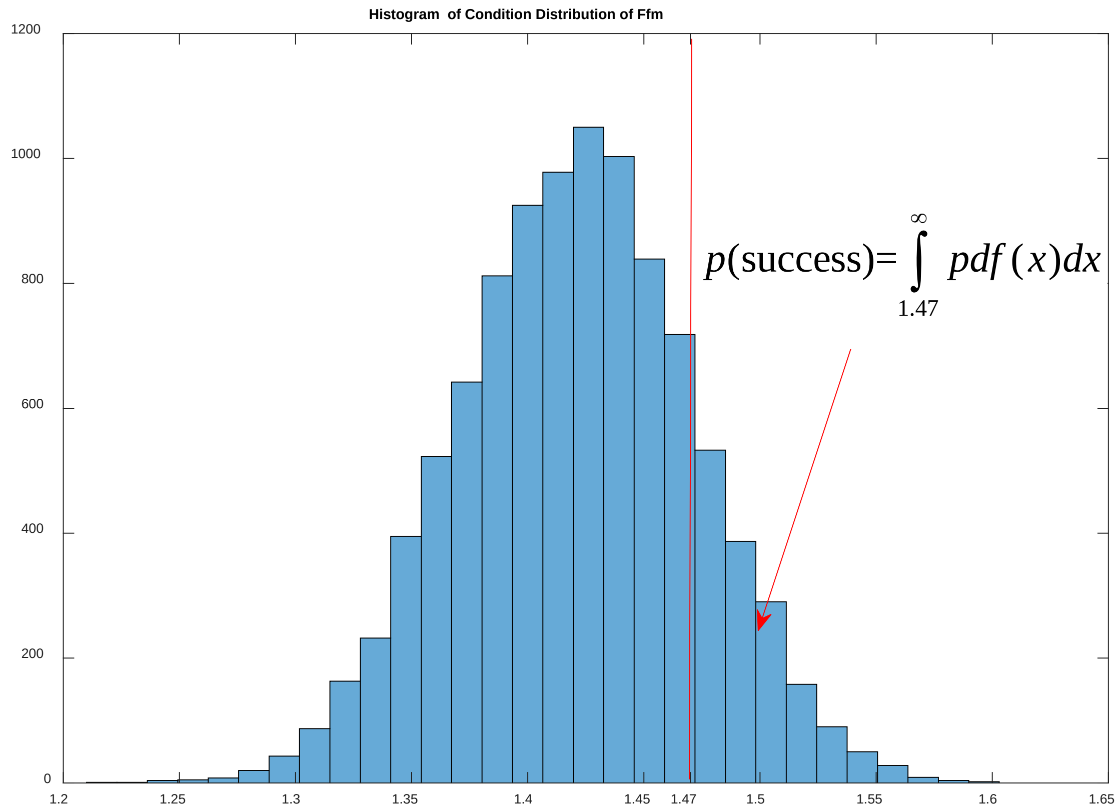
where Fgt is the measured ground test frequency.

- E.G., for a 500 sample case of the simple SLS model, if Fgt=1.35, the conditional distribution of Ffm is the the 2-D slice of the multi-variate distribution, which is itself a normal distribution, scaled up to an area of 1.0.



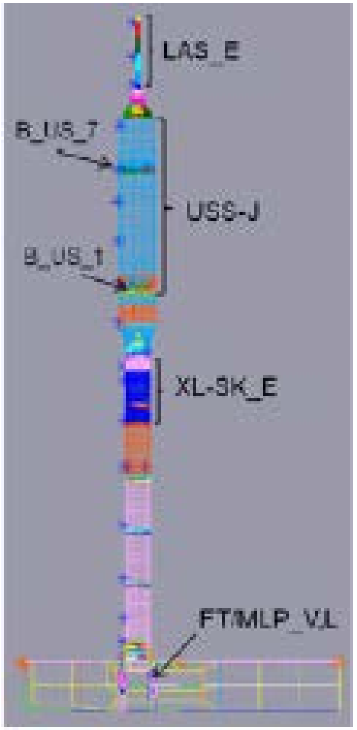
Calculation of Probability of Success from Conditional PDF

- Restating our requirement, “the true frequency (flight “pseudo-test”) can be no more than 3% less than the post-test prediction (flight model)”
- The “post-test prediction” of the flight model is simply the mean μ_{fm} =1.5123 hz.
- Therefore, we use conditional distribution of the flight model, and the probability of success i.e, the probability that the flight model frequency is greater than the true value of the flight natural frequency, is the integral of that PDF above 0.97*μ_{fm} =1.47 hz.
- For the Simple SLS case shown, p(success)=0.18 .



Application to Ares I-X as Test Case

- Ares I-X was a Constellation Program test vehicle flown in 2009.
- For Ares I-X, we have not only models and ground test data but also have flight measurement of "true" frequencies, so can use as validation case.
- Used same parameter set and distributions as "Finite Element Model Calibration for Ares I-X Flight Vehicle, L Horta

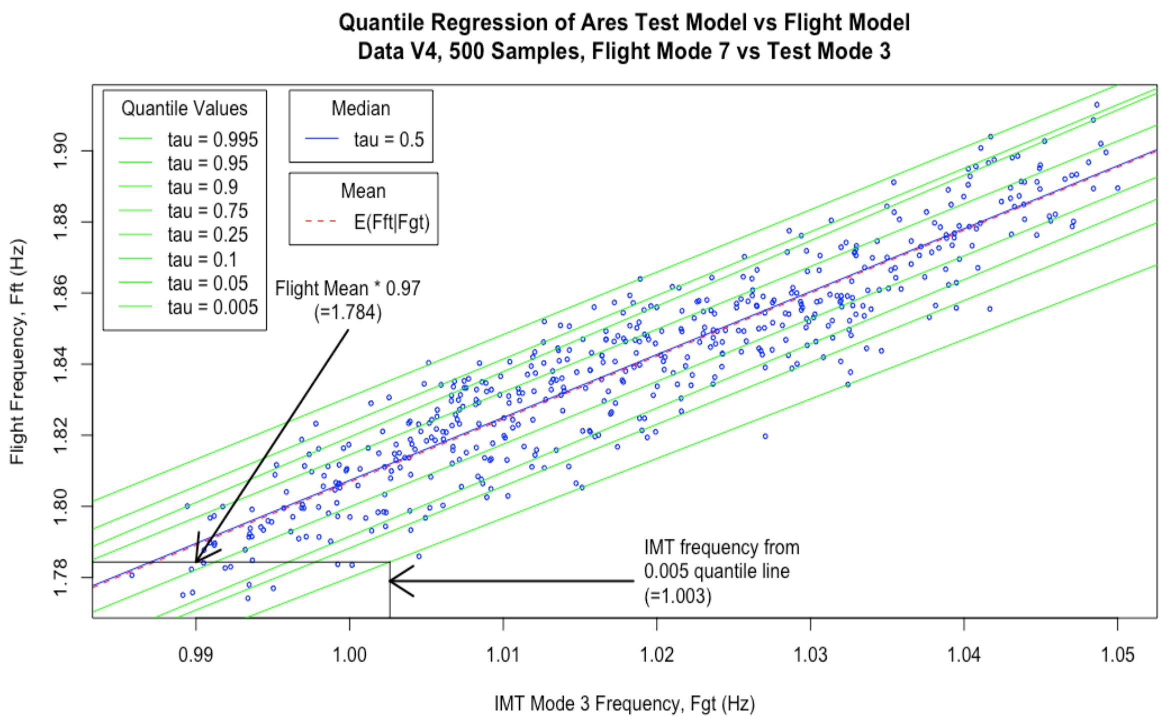


No.	Parameter ID	Lower bound	Nominal	Upper bound
1	USS_J (lbs/in)	4.84E+06	1.00E+07	2.63E+07
2	FTV/MLP_V (lbs/in)	2.43E+08	2.70E+08	2.97E+08
3	FTV/MLP_L (lbs/in)	2.43E+08	2.70E+08	2.97E+08
4	B_US_1 (lbs-s^2/in^4)	6.44E-04	7.32E-04	8.27E-04
5	B_US_7 (lbs-s^2/in^4)	5.63E-04	7.32E-04	8.20E-04
6	XL_Fwd_Skirt_E (lbs/in^2)	2.70E+07	3.00E+07	3.30E+07
7	LAS_E (lbs/in^2)	8.91E+06	9.90E+06	1.09E+07

- 500 sample Monte Carlo Analysis performed for flight configurations 1st bending mode at 93s into flight, which data shows is 1.95hz, and first several ground configuration modes.
- Flight Model 1st Bending Frequency μ_{fm}=1.8395 hz

Probability of Success using Quantile Linear Regression

- "Normal Conditional" methodology assumes normality of response variables.
- Numerical Distribution testing (and examination) indicated that for Ares I-X, they are not normal, though.
  - Given distributions of primitive rv's are uniform distributions to encompass overall stiffness uncertainty.
  - Bulk of sensitivity is due to a single PRV, so Central Limit Theorem doesn't apply.
- Alternate "Quantile Linear Regression" (QLR) methodology therefore applied on the most correlated ground-flight mode pair.
- For a correlated pair of rv's, QLR generates specific values for any given quantile for each value of the independent variable.
- For example, if Fgt=1.003, then the quantile crossing the lower limit of 1.7844 hz (97% of μ_{fm}) is .005.

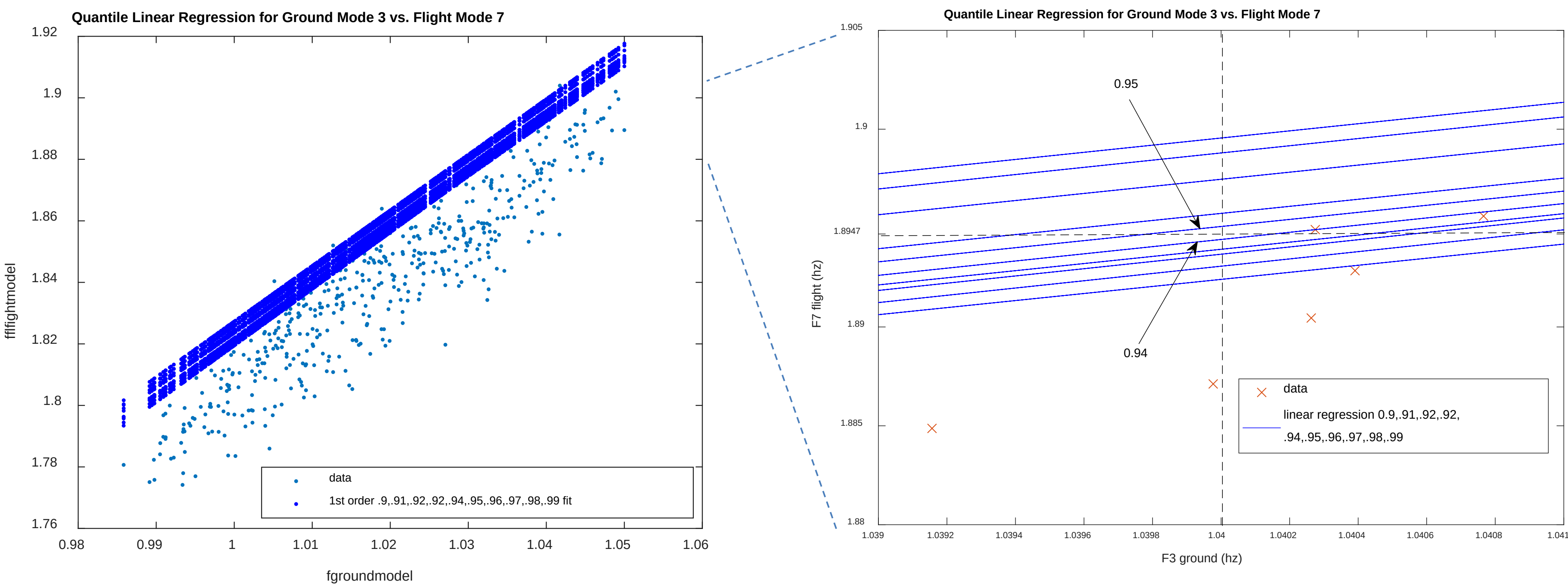


Example/Validation – Ares I-X

- To identify the exact probability crossing the lower limit, an iterative procedure is required.
  - If we just need to show that the probability is greater than some program-determined quantile (e.g., 95%), then no iteration is needed.
- For Ares I-X the Fgt=1.059 hz is higher than the distribution {Fgm} (overly-compliant model), so the QLR is not valid.
- We can go back to the Normal Conditional methodology, though, and generate the p(success)=1.0, as expected.
- It’s hard to say that this validates the method, since the model is so compliant that there’s no doubt it would pass the “1-sided 3%” requirement.

Example/Validation – Ares I-X

- However, hypothetically, if a “2-Sided 3% Requirement” existed (implying more of a test of the model’s accuracy), then the PDF integral between 1.7844 hz and 1.8947 hz would give us a better validation test.
  - The F_{gt} is off the chart, so the QLR still isn’t really applicable.
- For illustration, though, if the ground test was 1.04 hz, an iterative procedure would be used to identify the exact probability crossing the low and high limit.
- Since the low limit has a negligible probability by observation, we only need to obtain the probability for the high limit, which is =.945. This matches almost exactly the value obtained just using the Normal Conditional method, not surprisingly.



Conclusions/Future work

- Two methods identified that can give probabilistic assessment of the capability of the SLS dynamic model to meet program requirements:
  - Normal Conditional Method, assumes bivariate normal distribution of flight vs ground models, uses ground modal test as conditional value to obtain distribution of flight model.
  - Quantile Linear Regression, does not require normal assumption, uses data to identify specific quantiles in a linear regression between flight and ground models and uses ground modal test as conditional value to obtain probability.
- Apply to actual SLS model, requiring estimation of many uncertain parameters.
  - SLS Element (Substructure) Modeling Uncertainties
  - Integration of Reduced Craig-Bampton-Hurty Models Uncertainties
  - Integration of Bulk Data Models Uncertainties
  - Best Model Estimate Uncertainties
  - Flight Configuration Modelling Uncertainties
  - Integrated Vehicle Modal Test (IMT) Uncertainties