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Kant's conception of proper science

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Abstract Kant is well known for his restrictive conception of proper science. In the present paper I will try to explain why Kant adopted this conception. I will identify three core conditions which Kant thinks a proper science must satisfy: systematicity, objective grounding, and apodictic certainty. These conditions conform to conditions codified in the Classical Model of Science. Kant's infamous claim that any proper natural science must be mathematical should be understood on the basis of these conditions. In order to substantiate this reading, I will show that only in this way it can be explained why Kant thought (1) that mathematics has a particular foundational function with respect to the natural sciences and (2) as such secures their scientific status.

Keywords Kant · Proper science · Objective grounding · Mathematics

1 Introduction

The Preface to the *Metaphysical foundations of natural science* (1786) contains one of Kant's few systematic attempts at finding the notion of a proper science. Kant defines a proper science as a body of cognition that (i) is a system, (ii) constitutes a rational interconnection of grounds and consequences, and (iii) provides apodictically certain cognition. In addition, Kant states that any proper natural science must allow for the application of mathematics.[1] The Preface does not contain a detailed analysis of these conditions, nor does it explain why we should accept them. However, the implications

[1] Kant (1902, IV, pp. 467–471).

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of these conditions are rich. They enable Kant to argue that natural description (the classification of natural kinds), natural history (the historical study of changes within nature), chemistry and empirical psychology are improper sciences.

This does not mean that Kant did not take an active interest in the experimental sciences. Recent research has shown that Kant, throughout his life, provided significant philosophical analyses of these sciences.[2] This raises the question why Kant adopted his restrictive conception of proper science. In the present paper I will try to answer this question by describing the conceptual background of Kant's idea of proper science. I will analyze Kant's conditions for proper science one by one and indicate how they are related to each other. I will also argue that several of these conditions correspond to conditions of the Classical Model of Science as set out by de Jong and Betti (2008). In the first section I will discuss the condition of systematicity. In the second section I will discuss Kant's claim that any proper science must provide a rational ordering of grounds of consequences. This claim is sometimes interpreted as stating that any proper science must have a priori principles.[3] In my opinion, it can be better understood as stating that any proper science must satisfy a grounding-relation, i.e., provide explanative demonstrations. It is Kant's third condition, discussed in section three, that *implies* that proper sciences must have a priori principles. Finally, section four will provide an interpretation of the claim that any proper natural science must allow of mathematization.

2 Systematicity

The first condition that any proper science must satisfy is that of systematicity.[4] In the *Critique of pure reason* Kant explicates the concept of system as follows:

> If we survey the cognitions of our understanding in their entire range, then we find that what reason quite uniquely prescribes and seeks to bring about concerning it is the **systematic** in cognition, i.e., its interconnection based on one principle. This unity of reason always presupposes an idea, namely that of the form of a whole of cognition, which precedes the determinate cognition of the parts and contains the conditions for determining a priori the place of each part and its relation to the others. Accordingly, this idea postulates complete unity of the understanding's cognition, through which this cognition comes to be not merely a contingent aggregate but a system interconnected in accordance with necessary laws. (Kant 1787, *A*, p. 645/*B*, p. 673, original emphasis)

These remarks require explanation. First, note that the systematic unity of cognition is said to be brought about by the *faculty of reason*. This follows from Kant's conception of reason as a faculty that organizes cognition. In particular, reason logically

2 See, for example, the collection of essays in Watkins (2001).

3 Watkins (2007, p. 5), Pollok (2001, pp. 56–62).

4 This notion has received considerable attention. For recent discussion see Falkenburg (2000, pp. 376–385), Fulda and Stolzenberg (2001), Guyer (2005, pp. 11–73). My account is indebted to Falkenburg, from whose analysis of Kant's theory of science I have greatly benefited.

orders cognition and thus unifies it.[5] The term 'cognition' refers to both *concepts* and *judgments*. I will however restrict my discussion to concepts. Second, Kant claims that the unity of cognition effected by reason is based on an idea of the "form of a whole of cognition" which postulates a "complete unity" of cognition. Hence, a system of cognition constitutes a complete whole. Finally, Kant claims that the place of the parts (cognitions) within a system of cognition and the relation of these parts to each other is *determined* a priori in accordance with certain conditions. In the following, we will see that Kant takes a system of cognition to be constructed by following certain *logical rules* establishing necessary relations among cognitions. The concept 'conditions' refers, among others, to these rules. In short, a system is a complete whole composed of parts that are necessarily related to each other in accordance with rules. As such, it is distinguished from an aggregate.

If we turn our attention to Kant's discussion of systematicity in the *Jäsche Logik*, it becomes clear that systematicity must be understood as a *logical* requirement concerning the form of cognition.[6] Kant explicates this requirement in the Doctrine of Method of the *Logik*, which specifies the conditions of scientific cognition in general. Kant describes the requirement of systematicity, together with those of distinctness (*Deutlichkeit*) and thoroughness (*Gründlichkeit*), as *logical perfections*. These perfections provide ideals of scientific cognition.[7] With respect to the ideal of systematicity, Kant remarks that the combination of cognitions in a systematic whole depends on the "distinctness of concepts both in regard to what is contained *in* them and in respect of what is contained *under* them".[8] Here, Kant employs traditional logical terminology to elucidate the notion of systematicity. When Kant speaks of that which is contained in a concept, he refers to the totality of partial concepts comprising the intension (*Inhalt*) of this concept.[9] Thus, for example, the partial concepts 'animal', 'rational' and 'mortal' are *contained in* the concept 'man'. Conversely, when Kant speaks of that which is *contained under* a concept, he refers to the totality of concepts comprising its extension (*Umfang*). For example, the concepts 'gold', 'silver', 'copper' and so forth are contained under the concept 'metal', functioning as a characteristic (*Merkmal*) of these concepts.[10] Finally, a concept is distinct if we possess a clear representation of its characteristics, i.e., if we are conscious of the partial concepts *contained in* this concept. A concept is made distinct by analyzing it.[11] Kant's claim that the connection of cognitions into a systematic whole requires the distinctness of concepts in regard to what is *contained in* and *under* them can now be understood as follows: systematicity is brought about both by the *analysis* of the intension of concepts and by the *specification* of their extension.[12] As such, Kant's notion of systematicity, when applied

[5] Kant (1787, *A*, pp. 298–302/*B*, pp. 355–359). Cf. Falkenburg (2000, pp. 376–385).

[6] This is also emphasized by Longuenesse. Longuenesse (1998, pp. 149–153).

[7] Kant (1902, IX, pp. 139–140).

[8] Ibid.

[9] Kant (1902, IX, p. 95).

[10] Kant (1902, IX, p. 96).

[11] Kant (1902, IX, pp. 61–62).

[12] Cf. Longuenesse (1998, pp. 150–151).

to concepts, expresses the conditions posed on concepts in the Classical Model of Science as described by de Jong and Betti (2008), i.e., that a science *S* has a number of fundamental concepts and that all other concepts are composed of these fundamental concepts.[13]

Kant's conception of systematicity is exemplified by hierarchical systems of concepts (trees), proceeding from an elementary concept (*genus summum*) to more specific and complex concepts by adding *differentiae*.[14] In the appendix to the Transcendental Dialectic of the first *Critique* Kant endorses a similar conception. There Kant provides a detailed discussion of the logical principles by means of which we establish systematic unity among cognitions. These logical principles function as rules for the construction of systems and are described by Kant as principles of (i) homogeneity, (ii) specification, and (iii) continuity.[15] Rule (i) directs us to subsume any concept under a higher and more general concept. Rule (ii) directs us to divide or specify any given concept into more particular concepts (comprising subsets of the former). Finally, by means of rule (iii) we postulate that different levels of concepts within our classification are continuously related, guiding the attempt to specify continuous transitions from one level of concepts to another. By following these principles we order concepts in terms of their extension and intension and obtain a hierarchy of concepts with the "greatest unity alongside the greatest extension".[16]

How should we understand Kant's claim that a system of cognition should be complete? In the *Metaphysik Volckmann* it is argued that the completeness of any system requires (a) the specification of upper and lower limits (*terminus* a priori and *terminus* a posteriori), and (b) principles by means of which all the parts of a system can be related. Kant presents a closed genealogical tree ordered by the relation 'generated by' as an example of a system.[17] Similarly, in constructing a system of concepts we can specify a highest genus and a lowest species (*infima species*) and relate them in terms of their extension or intension. It is important to note, however, that Kant denies the existence of an *infima species*. In principle, the specification of any concept can proceed indefinitely. *Infima species* are specified by convention.[18] Kant also claims that the assumption of the existence of a highest genus is one of *reason*, for we cannot empirically identify a highest genus. Hence, although in constructing a system we conventionally specify upper and lower limits we cannot establish their objective reality. The *ordo cognoscendi* does not necessarily mirror the *ordo essendi*.

Let us return to the description of the concept 'system' given in the beginning of this section. There, a system was described as a whole consisting of parts. This description secures the generality of the notion of a system, for 'parts' can refer to concepts, judgments or material parts. In addition, Kant stated that a system is complete and that the parts of a system must be necessarily related to each other and to the whole in

13 de Jong and Betti (2008).

14 The theory of concepts adopted by Kant is analyzed in detail by de Jong: (1995, pp. 620–627).

15 Kant (1787, *A*, pp. 657–658/*B*, pp. 685–686).

16 Kant (1787, *A*, p. 643/*B*, p. 671).

17 Kant (1902, XXVIII, pp. 355–356).

18 Kant (1787, *A*, p. 655/*B*, p. 683; 1902, IX, p. 97).

accordance with certain conditions. On the basis of our discussion we can now claim that these conditions comprise (i) logical *rules* or principles by means of which we establish specific relations among cognitions, and (ii) the specification of upper and lower limits of a system. These conditions secure that a system is a complete and ordered whole.

3 Objective grounding

The requirement of systematicity provides a condition that any science must satisfy. In the Preface to the *Metaphysical foundations of natural science* (1786) Kant takes natural description, natural history, and chemistry to be systematic doctrines. However, he denies them the status of a *proper* science.[19] Hence, systematicity is not sufficient for distinguishing science from science proper. To make this distinction, Kant adds a second condition that any proper science must satisfy. According to Kant, any proper science must be systematically ordered and constitute an interconnection of *grounds* and *consequences*. This condition provides a basis for distinguishing mere science from rational science, where being a *rational* science must be understood as a necessary but not sufficient condition for being a *proper science*:

> Any whole of cognition that is systematic can, for this reason, already be called science, and if the connection of cognition is an interconnection of grounds and consequences, even *rational* science. (Kant 1902, IV, p. 468)

In other words, any rational science is a system of cognition containing a grounding-relation.[20] This condition is similar to the Proof Postulate of the Classical Model of Science, as described by de Jong and Betti (2008), which states that all non-fundamental propositions of a science *S* are ultimately grounded in fundamental propositions.[21] However, in the Model a neat distinction is made between the conceptual and the propositional ordering, and the Proof Postulate is related to the order of propositions or judgments. Kant does not neatly distinguish the order of concepts from that of judgments. Moreover, Kant takes the grounding-relation to obtain between both concepts and judgments. In the following, I will try to identify some core elements of Kant's conception of grounding by analyzing passages from both his pre-critical and critical writings.

19 Kant (1902, IV, pp. 467–468, 471).

20 As indicated in note 3, Pollok and Watkins interpret this condition as claiming that proper sciences must have a priori principles. Pollok further argues that Kant denies that natural description and natural history are proper sciences because they lack a priori principles. This is problematic because: (i) Kant does not criticize these doctrines in these terms, and (ii) Kant seems to allow that chemistry, based on *empirical* principles, provides a rational interconnection of grounds and consequences (IV, p. 468). More generally, I take this reading to conflate an *epistemic* condition that proper sciences must satisfy (Kant's third condition), with the condition of grounding, which I interpret as the condition that proper sciences must provide explanative demonstrations reflecting the *order of nature*. In terms of the Classical Model of Science, Kant's second condition relates to the *ordo essendi* and not to the *ordo cognoscendi*. In this context, we may also refer to Friedman (1992b), who in his discussion of a priori grounding of natural laws interprets grounding solely in terms of epistemic justification.

21 de Jong and Betti (2008).

Kant provides an extensive discussion of the concept ‘ground’ (*ratio*) in his *New elucidation* (1755).[22] Here, a ground is defined as that “which determines a subject in respect of any of its predicates”.[23] In addition, Kant defines ‘to determine’ as “to posit a predicate while excluding its opposite”.[24] Hence, a ground is a reason for predicating some concept *P* of a subject-concept *S*, while excluding *not-P*. Cognition of grounds is a condition for asserting the truth of judgments since it provides a reason for asserting a judgment ‘S is P’ while excluding the contradictory judgment ‘S is not P’. In the absence of such cognition there would be no knowledge of truths, since all judgments would be merely taken as possibly true.[25] This claim concerns the epistemic function of cognition of grounds but does not capture Kant’s grounding condition.

In the *New elucidation*, Kant interprets the concepts ‘ground’ and ‘consequence’ *ontologically*, i.e., as referring to existing objects. Hence, strictly speaking the relation of ground to consequence obtains between objects. This relation can be represented conceptually: a grounding-relation can be represented by relations holding between *concepts* and by relations holding between *judgments*.[26] Any structure of concepts or judgments can thus *express* an objective grounding-relation. For example, Kant takes a grounding-relation to be expressed in the judgment “a triangle has three sides”.[27] The concept ‘triangle’ provides us with a reason for predicating ‘three-sidedness’ of it because a triangle is defined as a three-sided figure. Kant provides an example of a grounding-relation expressed by *judgments* when he distinguishes between an ‘antecedently determining ground’ and a ‘consequentially determining ground’. The former is a ground of being or becoming, the reason *why*, while the latter is a ground of cognition, the reason *that*.[28] For example: the eclipses of the satellites of Jupiter are a ground for cognizing that light is propagated with a finite velocity, whereas (following Descartes) the elasticity of the globules of the atmosphere in which light is propagated is a ground of being for the finite velocity of light.[29] The eclipses of Jupiter’s satellites are a consequence of the finite velocity of light and allow us to demonstrate this fact.[30] These eclipses are not the cause of the finite velocity of light. Accordingly, they provide us with a ground of cognition, not a ground of being, for the truth that light has a finite velocity. By contrast, Descartes hypothesis that the propagation of light must be understood as a series of impacts of elastic globules identifies a cause, *a ground of*

22 Longuenesse has provided detailed accounts of the concept ‘ground’ in Kant’s pre-critical and critical writings. Longuenesse (1998, pp. 345–358; 2001). Different from Longuenesse I focus on the role of this notion in Kant’s views on scientific explanation.

23 Kant (1902, I, pp. 391–392).

24 Ibid.

25 Kant (1902, I, pp. 393–394).

26 Many commentators, in discussions of Kant’s views on the foundation of scientific cognition, focus exclusively on relations between *judgments*. Cf. Guyer (2005, pp. 11–55), Friedman (1992b). This is not incorrect but does not do justice to the fact that conceptual orderings can also satisfy grounding relations. This is the case, e.g., for systems of classification given in natural history, though these systems do not express relations obtaining between real grounds and real consequences.

27 Kant (1902, I, p. 392).

28 Kant (1902, I, pp. 391–392).

29 Kant (1902, I, pp. 392–393).

30 Ibid. Cf. Longuenesse (2001, p. 69).

being, for the finite velocity of light. The distinction between a ground of being and a ground of cognition can be related to the distinction between a *demonstratio propter quid* and a *demonstratio quia*.[31] Since Descartes' hypothesis identifies the ground of being of the finite velocity of light, his account of the velocity of light reflects the objective order of ground and consequence and allows us to give a *demonstratio propter quid* of this phenomenon. By contrast, cognition of the eclipses of the satellites of Jupiter merely provides *subjective justification* for the truth that light has a finite velocity. In Kant's terms, a ground of being is the source for the truth of judgments, i.e., a ground for some phenomenon (described by a judgment) to obtain, whereas a ground of cognition "does not bring the truth into being; it only displays it".[32]

In the *New elucidation*, Kant took grounding to be a relation that can be expressed by relations holding between concepts and judgments. This view is retained in the critical period. In the *Jäsche Logik*, Kant argued that a concept can be taken as a *ground of cognition* with respect to the set of representations comprising its extension.[33] For example, the concept 'metal' functions as a ground of cognition with respect to the concepts 'gold', 'silver', etc. Kant's idea is that a genus can function as a ground of cognition for its species: the relation of species to genus provides a ground for cognizing that gold is a metal.

I will try to specify how Kant understood the relation holding between ground and consequence. In the first *Critique*, Kant explicates the relation between ground and consequence in terms of *logical inference*:

> In every inference there is a proposition that serves as a ground, and another, namely the conclusion, that is drawn from the former, and finally the inference (consequence) according to which the truth of the conclusion is connected unfailingly with the truth of the first proposition. (Kant 1787, *A*, p. 303/*B*, p. 360)

Kant takes a logical inference to be a function of thought that relates true judgments and shows that the truth of the conclusion follows from the premisse(s). As types of inference Kant lists: 'immediate inference', i.e., subalternation, contraposition and the like, and 'mediate inference', i.e., syllogistic inference. If we employ modern terminology and strictly distinguish between logical inference and logical derivability or consequence (which Kant does not), we might say that Kant takes a logical inference to express a relation of logical derivability holding between true judgments and that the grounding-relation can be understood in terms of derivability among truths. This is problematic as the notion of grounding is stronger than that of derivability. Grounding p means providing an explanative demonstration of p.[34] This is not necessarily the case for a derivation of p. In addition, grounding is a relation obtaining between truths, whereas (from a modern point of view) derivability can obtain between falsities.

[31] de Jong and Betti (2008).

[32] Kant (1902, I, p. 394).

[33] Kant (1902, IX, p. 96).

[34] Cf. de Jong and Betti (2008).

These two difficulties can be resolved by employing Kant's distinction between a ground of cognition and a ground of being. In Kant's view, the logical derivation of a true judgment β from a true judgment α establishes that what is asserted by α is a *ground of cognition* for the truth of β. However, derivability does not show that α grounds β in the sense of providing an explanative demonstration for the truth of β. This type of grounding requires that α specifies the *ground of being* for what is asserted by β (as in the case of the Cartesian explanation of the finite velocity of light). It is the latter type of grounding relation that must obtain between scientific cognitions, since science must provide objective explanations representing the order of nature.

This becomes clear in Kant's lectures on metaphysics, the *Metaphysik Volckmann*. Here, a ground is defined as that which, if it is posited, something else is posited. Kant distinguishes between the relation holding between a logical ground and logical consequence, and that holding between a real ground and real consequence.[35] The first relation obtains within analytic judgments, e.g., in the hypothetical judgment "if a being is an animal, it is mortal".[36] In such cases, Kant claims that the relation between ground and consequence can be established by means of the principle of identity, i.e., is analytical.[37] The truth of this hypothetical can thus be proven logically. Such a proof can be interpreted as establishing a relation between a judgment (the consequent) and a *ground of cognition* for its truth (expressed in the antecedent), i.e., a ground for cognizing *that* animal beings are mortal. The ground of being of the mortality of animals is, however, not specified by this logical proof. Kant explicates this by stating that the concept of ground, as pertaining to *logic*, is "treated in so far it is a ground of cognition".[38] If we understand Kant's notion of logical inference as derivability even this is saying too much. For establishing a relation between a judgment and its ground of cognition via logical proof is tantamount to providing a ground for the *truth* of the latter, whereas the relation of derivability can hold between false judgments. However, as said Kant does not share our modern conception of derivability, for he takes logical inferences to be valid only if the premises are true.[39] For this reason, Kant thinks that logical inference allows us to show that what is asserted in the antecedent of a hypothetical judgment is a *ground of cognition* for the truth of what is asserted in the consequent.

Kant's distinction between real grounds and real consequences indicates that judgments that are not logically inferred can also satisfy a grounding-relation. Thus, α can ground β even if β is not derivable from α. This distinction prohibits us from explaining Kant's notion of grounding solely in terms of derivability.[40] According to Kant, the relation between a real ground and real consequence cannot be established analytically. A real ground is defined as that which "if it is posited, something else is

35 Kant (1902, XXVIII, pp. 401–402). For a thorough analysis of the notion of ground in the *Metaphysik Volckmann*, cf. Longuenesse (1998, pp. 354–356).

36 Kant (1902, XXVIII, p. 397).

37 Kant (1902, XXVIII, p. 402).

38 Kant (1902, XXVIII, p. 399).

39 Kant (1902, IX, p. 121).

40 For this reason, I cannot follow Falkenburg, who explicates Kant's notion of 'grounding' in terms of deducibility. Falkenburg (2000, pp. 368–370).

posited, but not according to the principle of identity".[41] Here, the relation is *synthetic*. This relation obtains, for example, in the hypothetical 'if I have been exposed to the cold, I will come down with the flu'. In this case, a grounding-relation obtains between antecedent and consequent, although the latter cannot be logically inferred from the former. In physics, according to Kant, we are concerned with the relation between real ground and real consequence.[42] Thus, judgments of physics that cannot be logically derived from one another can ground each other (express relations between real grounds and consequences). The same holds for mathematical theorems, which are synthetic and do not allow of logical proof. Kant emphasizes that the concept 'real ground' must not be interpreted as a ground of cognition, but as a *ground of being*.[43] This implies that within mathematics and physics we establish relations between judgments that *express* relations between real grounds and consequences and thus provide demonstrations *propter quid*.

The manner in which Kant takes mathematical judgments to be grounded cannot be explicated within this paper. Judgments of physics satisfy a grounding-relation because in a proof of physics they can be related in such a manner that they express a relation between *cause* and *effect*, which is an instance of a relation between ground of being and consequence. This is clear in the *Metaphysik Volckmann*, where two methods of proof for the truth of cognitions are distinguished: (i) an a posteriori method in which one proceeds from cognition of the consequence to cognition of its ground, e.g., observation of the world allows us to prove that God exists. In this case, we specify a *ground of cognition* for the truth that God exists. (ii) An a priori method, in which we proceed from cognition of the ground to cognition of its consequence. This is the true method of natural science which consists in specifying causes of effects.[44] A proof in which one proceeds, for example, from premises expressing relations between cause and effect to a conclusion expressing the effect would fit this method quite nicely.

In the *Metaphysical foundations*, natural description is denied the status of a proper science on the basis of Kant's grounding condition.[45] This doctrine does not provide "cognition through reason of the interconnection of natural things".[46] I take this to mean that natural description does not provide demonstrations *propter quid*. Natural description is defined as a "system of classification for natural things in accordance with their similarity".[47] Kant employs this notion to characterize classifications of natural kinds given in disciplines such as zoology or botany. According to Kant, cognitions making up such classificatory systems are not properly grounded. Take for example the taxonomy of organisms based on morphological criteria as given by Linnaeus in his *Systema naturae*. If we take this taxonomy to be correct, we are provided with a

41 Kant (1902, XXVIII, p. 403).

42 Ibid.

43 Kant (1902, XXVIII, p. 399).

44 Kant (1902, XXVIII, p. 355). The same conception of scientific demonstration, entitled 'dogmatic proof', is articulated in the *Danziger Physik*. Cf. Kant (1902, XXIX, pp. 103–104).

45 It must be noted that Kant's views on the scientific merit of natural description and natural history varied throughout his philosophical career. Cf.: Sloan (2006, pp. 627–648).

46 Kant (1902, IV, pp. 467–468).

47 Ibid.

ground for cognizing the truth that, say, a lion is a feline. However, it does not provide us with a ground of being, a reason why lions are feline. Linnaeus' taxonomy does not provide us with relationships holding between real grounds and consequences. Hence, this taxonomy does not allow us to explain why certain organisms have specific morphological characteristics. For this reason, Kant takes natural description to lack explanatory power.

The status of natural history is problematic. In his 1788 essay on teleological principles Kant construes natural history as a discipline investigating relations between present properties of natural objects and their historical causes.[48] Causal regularities relating present effects with earlier causes are derived from the observation of forces presently operative in nature and inferences by analogy, supporting the claim that these forces have been operative in the past and have produced similar effects as presently observed. Since causal relations constitute relations between objective grounds and consequences, natural history may be interpreted as providing objective explanations, e.g., of the origin of human races.[49] However, Kant emphasized that inferences by analogy merely provide empirical (non-apodictic) certain cognition[50] and stressed that natural history is a novel science in need of further development.[51] This may explain why natural history is classified as a *doctrine* rather than a *science* of nature.

4 Apodictic certainty

The third and final condition that any system of cognitions must satisfy in order to be a proper science is that its cognitions are apodictically certain, i.e., that we are conscious of their necessary truth:

> What can be called *proper science* is only that whose certainty is apodictic; cognition that can contain mere empirical certainty is only *knowledge* improperly so-called. (Kant 1902, IV, p. 468)

In the *Logik*, Kant defines knowledge (*Wissen*), opinion (*Meinung*) and belief (*Glaube*), as modes of holding-to-be-true (*Fürwahrhalten*). Holding something to be true is, in turn, defined as a judgment through which something is *subjectively* "represented as true".[52] In other words, opinion, belief and knowledge are terms that indicate different modes of *epistemic justification*. Kant's final condition of scientificity corresponds to what is called the 'Knowledge Postulate' in the Classical Model of Science, which relates to the *ordo cognoscendi* and states that any proposition of a science is known to be true.[53] In Kant's work the 'Knowledge Postulate' is intimately

48 Kant (1902, VIII, pp. 61–62).

49 Hence, I cannot subscribe to Sloan's thesis that Kant, from the 1780s onwards, gave theoretical preference to natural description over natural history. Sloan (2006, p. 629).

50 Kant (1902, IX, p. 133).

51 Kant (1902, VIII, p. 62).

52 Kant (1902, IX, pp. 65–66).

53 de Jong and Betti (2008). The fact that Kant's third condition, stating that the cognitions of a science must be apodictically certain, relates to the *ordo cognoscendi*, indicates that this condition should be

related to the ‘Necessity Postulate’ of the Classical Model of Science, which states that all propositions or judgments of a science are necessary, since he argues that we only have knowledge of a proposition or judgment if we assert its necessary truth.

Kant describes the three modes of epistemic justification as follows. We have an *opinion* if we judge without having sufficient subjective or objective grounds for the truth of this judgment. In this context, the concept ‘ground’ refers to a *ground of cognition*, a ground on the basis of which we take a judgment to be true. A ground is *subjectively* sufficient for taking a judgment to be true if it is sufficient for myself, and a ground is *objectively* sufficient for taking a judgment to be true if it is sufficient or valid for everyone.[54] We opine if in the act of judging we take the judgment to be problematic i.e., take the judgment to be merely possibly true. *Believing* is taking something to be true based on a ground of cognition that is objectively insufficient but subjectively sufficient, e.g., one can rationally believe that God exists since this belief “depends on subjective grounds (of moral disposition)”.[55] We believe something if in the act of judging we assert the truth of the judgment. *Knowing* is taking something to be true based on grounds that are both objectively and subjectively sufficient. I have knowledge if I have a judgment that is *apodictically certain*, i.e., if I take the judgment to be necessarily true.[56]

In the *Logik*, Kant further distinguishes between two types of knowledge: empirical knowledge, based on experience, and rational knowledge, based on reason. Rational knowledge is apodictically certain and can be divided into knowledge that is mathematically (intuitively) certain or philosophically (discursively) certain.[57] This distinction relates the epistemic status of mathematical and philosophical cognition to the methods of proof employed within mathematics and philosophy. Mathematical knowledge is intuitively certain because it is proven on the basis of a priori construction in pure intuition. In particular, mathematical theorems are mediately certain synthetic a priori propositions demonstrated from immediately certain (intuitive) synthetic a priori principles (axioms). Philosophical propositions are mediately certain propositions derived from (discursive) synthetic a priori principles. Both mathematical theorems and philosophical propositions are apodictically certain because they are proven on the basis of a priori principles. Empirical knowledge, justified merely empirically, is empirically certain or contingent. However, empirical knowledge is apodictically certain “insofar as we cognize an empirically certain proposition from principles a priori”.[58]

Footnote 53 continued
distinguished from Kant’s grounding condition discussed in Sect. 3, which relates to the *ordo essendi*. See note 23 for the relevance of this distinction.

54 Kant (1787, *A*, pp. 820–822/*B*, pp. 848–850).

55 Kant (1787, *A*, p. 829/*B*, p. 857).

56 Cf. Falkenburg (2001, pp. 364–365). Chignell (2007) argues that objective grounds for knowing propositions indicate that propositions have an objective *probability* of being true. This cannot be true if, as I will argue, objective grounds of cognition must typically be understood as a priori principles on the basis of which we take propositions to be *necessarily* true, i.e., have *knowledge* of these propositions.

57 Kant (1902, IX, pp. 70–71).

58 Kant (1902, IX, p. 71).

The foregoing shows that the epistemic justification that we have for judgments in a particular science is determined by the relation of these judgments to the *principles* (fundamental judgments) of this science. In particular, a judgment is apodictically certain if it can be proven by means of a priori principles. These principles are necessary and strictly universal truths, providing subjectively and objectively sufficient grounds of cognition for the truth of judgments somehow derivable from them. It follows that scientific judgments only provide us with *knowledge*, if they can be proven by means of a priori principles. In the *Metaphysical foundations*, Kant expresses this point by stating that the principles of a proper science must be a priori.

Kant's conception of 'proper science' can now be summarized as follows: in order to be a proper science, any body of cognition must be (i) systematically organized, (ii) express relations between objective grounds and consequences, (iii) have a priori principles on the basis of which the non-fundamental judgments of a science can be proven. These conditions comprise Kant's model of 'proper science'. However, the Preface to the *Metaphysical foundations* is infamous for a different claim. This is the claim that any proper natural science must allow for the application of mathematics, which Kant employs to deny that chemistry and psychology are sciences proper. In the final section I will deal with Kant's mathematization requirement. I will argue that the latter requirement follows from the requirement that any proper science must have a priori principles. Such a view is suggested by Kant's criticism of chemistry. For Kant argues that because the principles of chemistry do not allow of mathematization, we lack a priori cognition of the principles underlying chemical appearances.[59] It is because of the latter reason that chemistry is denied the status of a proper science. However, it is not clear why Kant thinks that the mathematization of a doctrine secures an a priori foundation of that doctrine. Kant's view becomes clearer if we take into account that he takes mathematization to be a necessary condition of the scientific status of doctrines of *nature*. Kant takes this view because he interprets mathematics as a science that provides us with a priori cognition of individual *corporeal* objects. In particular, mathematics provides a priori *grounds of cognition* that ground apodictic certain cognition of corporeal objects. As such, mathematics allows us to give an a priori (epistemic) foundation of natural sciences. Before developing this interpretation, I will first discuss a more instrumental interpretation of Kant's mathematization requirement.

5 Mathematics and a priori justification

The requirement that a proper natural science must allow of mathematization is often taken to be equivalent to the requirement that the concepts of such a science be quantifiable. Kant's claim that "in any special doctrine of nature there can be only as much *proper* science as there is *mathematics* therein",[60] is accordingly read as stating that only doctrines dealing with measurable magnitudes qualify as proper natural

59 Kant (1902, IV, pp. 470–471).

60 Ibid.

sciences.[61] Kant's mathematization requirement is thus simply taken to express the importance of measurability.

This reading certainly captures part of Kant's intentions in emphasizing the importance of mathematics within natural science. In the modern period mathematics was often thought of as providing a quantitative description of empirical objects.[62] Nevertheless, I do not think this reading can explain Kant's mathematization requirement. A difficulty confronting the above reading is that it conflates the notion of mathematization and that of measurability.[63] It is true that Kant thought that the mathematical representation of magnitudes enables the measurement of magnitudes. Nevertheless, one should carefully distinguish the notion of mathematization from that of measurability. In the *Critique of judgment*, Kant states that we measure natural objects by assigning numbers to particular objects and that measurement requires the selection of a unit of measurement. The selection of a unit is *arbitrary* or context dependent.[64] In a purely mathematical context we can, e.g., represent numbers and their relations in terms of relations between line segments. In measuring natural objects we empirically specify a particular kind of object as unit of measurement. Hence, mathematics does not by itself provide a measurement procedure. Consequently, we must distinguish between Kant's conception of mathematization and that of measurability.

If we focus on Kant's argument for the claim that proper natural sciences require mathematics, it becomes clear that considerations concerning measurability do not play any role. This argument, contained in Preface to the *Metaphysical foundations*, is based on the premise that proper natural sciences are based on "a priori cognition of natural things".[65] Kant continues his argument by stating that "to cognize something a priori means to cognize it from its mere possibility".[66] I take this to mean that cognition of the logical possibility of an object can be gained a priori by means of the analysis of its concept. However, according to Kant such a procedure does not enable us to cognize "the possibility of determinate natural things".[67] From this it is concluded that "in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the intuition corresponding to the concept be given a priori, that is, that the concept be constructed".[68] And this is a task for mathematics, since mathematical cognition is defined as cognition obtained through the construction of concepts.

61 This view has been endorsed by several commentators. Cf. Okruhlik (1986, p. 313), Nayak and Sotnak (1995, pp. 133–151). The latter authors assume that, according to Kant, the sole purpose of the application of mathematics within natural sciences is to allow for the *measurability* of the objects of these sciences. In the following I will argue, in contrast, that mathematics provides a priori principles securing the apodictic certainty of cognitions pertaining to the natural sciences.

62 Christian Wolff, for example, defines mathematics in his *Mathematisches Lexicon* as "a science that aims to measure everything that can be measured". Wolff (1965, pp. 863–864).

63 Nayak and Sotnak conflate these two conceptions. Cf. Nayak and Sotnak (1995, pp. 113, 142, 144).

64 Kant (1902, V, p. 251).

65 Kant (1902, IV, p. 470).

66 Ibid.

67 Ibid.

68 Ibid.

In order to understand Kant's position we must explain why only mathematical construction allows us to have a priori cognition of determinate natural things. I take Kant to hold that only mathematics provides us (i) with a priori cognition of natural objects by means of (ii) singular and immediate representations of these objects. This reading follows from the claim that a priori cognition of determinate natural things requires the *construction* of their concept. In the 'Discipline of Pure Reason' of the first *Critique*, Kant explains that mathematical reasoning is based on the construction of concepts, which is defined as follows: "to *construct* a concept means to exhibit a priori the intuition corresponding to it".[69] The term 'intuition' refers to a particular instance of a concept. In contrast to concepts, i.e., general representations representing their object mediately (via intuitions), Kant further interprets intuitions as *singular* representations that represent their object *immediately*.[70] Hence, mathematical cognition, based on the construction of concepts, concerns singular and immediate representations of objects (the term 'object' is explicated below). Moreover, since within mathematics the constructed intuition is exhibited a priori, which is to say that singular representations employed within mathematical demonstrations (e.g., an isosceles triangle) can represent all intuitions falling under the same concept (all isosceles triangles),[71] a characteristic of mathematical demonstration securing the universality of what is demonstrated, mathematics provides a priori cognition of objects.

With which objects is mathematics concerned? Kant accepts the traditional conception of (pure) mathematics as a science of magnitude. Geometry, for example, is construed as providing a priori cognition of space and spatial relations and thus concerns continuous magnitude. How, then, does mathematics provide a priori cognition of *natural* objects? In the first *Critique*, Kant argues that mathematical concepts relate to "data for experience" by means of the a priori construction of figures or images.[72] A figure or image is an intuition: it is a particular (sensible and concrete) instance of a concept. In the *Prolegomena*, Kant further explains that (geometrically) constructed images agree with empirical phenomena.[73] As an example, we can think of line segments as geometric images of the velocity (speed plus direction) of corporeal bodies. Kant thus construes mathematics as providing a priori cognition of mathematical constructs, *images* or *models*, that represent quantitative features of natural objects. That Kant entertains this position is not surprising, for his views on mathematics stem from a tradition that took mathematical cognition to be descriptive of the empirical world. For example, Christian Wolff, in his *Mathematisches Lexicon*, defines geometry as "a science of the space taken up by corporeal things in their length, breadth, and width".[74] Moreover, since all things occupy space, Wolff argues, geometry is applicable to all such objects and provides cognition of the latter. This position is similar to that of Kant, for Kant took geometry, insofar as it provides cognition of the structure

69 Kant (1787, *A*, p. 713/*B*, p. 741).

70 Kant (1902, IX, p. 91; 1787, *A*, p. 68/*B*, p. 93).

71 Kant (1787, *A*, pp. 713–714/*B*, pp. 741–742).

72 Kant (1787, *A*, p. 240/*B*, p. 299).

73 Kant (1902, IV, p. 287).

74 Wolff (1965, p. 665). On Wolff's views on mathematics in relation to Kant see Shabel (2003).

of space, to provide cognition of the formal features of perceptible spatiotemporal objects. The general conception motivating Kant's views on mathematics is thus that mathematics provides knowledge of the formal (spatiotemporal) features of corporeal objects.

In order to substantiate the present reading we can cite Kant's well-known claim that in mathematical problems the question is not about "existence as such at all, but about the properties of the objects in themselves".[75] Thus, Kant does not attribute existence to mathematical objects (pure intuitions). In a similar vein, Kant states:

> Through determination of the former [pure intuition] we can acquire a priori cognitions of objects (in mathematics), but only as far as their form is concerned as appearances; whether there can be things that must be intuited in this form is still left unsettled. Consequently all mathematical concepts are not by themselves cognitions, except insofar as one presupposes that there are things that can be presented to us only in accordance with the form of that pure sensible intuition. **Things in space** and **time**, however, are only given insofar as they are perceptions (representations accompanied with sensation), hence through empirical representation. (Kant 1787, *B*, p. 147)

This passage conveys Kant's view that the objective reality of mathematical concepts, i.e., the *possible existence* of objects falling under such concepts, requires their possible application to *empirical intuitions*, i.e., to perceivable empirical objects, a view that implies that mathematics yields a body of truths only insofar as it is applicable to empirical objects.[76] Hence, when Kant claims that by means of mathematical construction we cognize the form of objects (appearances), he is referring to empirical objects (phenomena). Note that Kant emphasizes that cognition of the objective reality of mathematical concepts requires a *philosophical* justification. The construction of mathematical concepts in pure intuition (space and time) guarantees their objective reality if we presuppose that "there are things that can be presented to us only in accordance with the form of that pure sensible intuition". This supposition requires the philosophical justification, given in the *Transcendental Aesthetic*, that space and time are pure forms of sensible intuition. Given the truth of this supposition mathematical construction establishes the objective reality of mathematical concepts. In particular, by means of mathematical construction we show the possible existence of empirical objects, the form of which is given by the construction. A different way of putting this is that mathematics provides a priori models that possibly represent (formal features of) existing and empirically given natural objects. With this in mind we can turn to the foundational role that Kant attributes to mathematics.

A nice illustration of the foundational role of mathematics can be found in § 38 of the *Prolegomena*.[77] In this paragraph Kant gives various examples that elucidate

[75] Kant (1787, *A*, p. 719/*B*, p. 747).

[76] On these and the following points, see: Thompson (1992, pp. 97–101), Parsons (1992, pp. 69–75), Friedman (1992, pp. 98–104).

[77] This paragraph had been subjected to a very detailed and subtle interpretation by Friedman, to which my interpretation is indebted. Cf. Friedman (1992, Chap. 4). I employ § 38 of the *Prolegomena* as providing an example that allows us to understand (i) the particular foundational role that Kant assigns to

the transcendental claim that the understanding prescribes a priori laws to nature. The *pièce de résistance* is an example taken from physical astronomy: "a physical law of reciprocal attraction, extending to all material nature, the rule of which is that the attractions decrease inversely with the square of distance from each part of attraction."[78] In other words, the main focus of § 38 is the law of gravitation. Note, however, that Kant focuses on the dependency on distance of gravitation, i.e., the fact that gravity is an inverse-square force ($1/r^2$). How is this law prescribed to nature by the understanding?

The first example of § 38 is mathematical. Kant refers to proposition 35 from Book III of the *Elementa* of Euclid, stating that if two straight lines intersect one another in a circle at point E, and intersect the circle at A, C and B, D, it holds that $AE \times EC = BE \times ED$. According to Kant, this law is dependent on the understanding because it can be demonstrated "only from the condition on which the understanding based the construction of this figure, namely, the equality of the radii".[79] Hence, the proof of the above law is based on the condition that all straight lines from the centre of the circle to its boundary are equal, a condition expressed in Euclid's definition of a circle. The second example is construed by Kant as a generalization of the above property of circles to conic sections. This proposition states that chords intersecting in a conic section intersect in such a way that the rectangles from their parts "stand to one another in equal proportions".[80] Thus, the products of the lengths of the segments of the chords of any conic section stand to one another in equal proportions. If we let chord AC intersect chord BD at E, and let chord $A'C'$ intersect chord $B'D'$ at E', then for all conic sections $(AE \times EC) : (BE \times ED) = (A'E' \times E'C') : (B'E' \times E'D')$.[81] It is this property of conic sections that Kant takes as a basis for inferring that gravity is an inverse-square force.

This choice of inference is understandable if we, following Friedman,[82] consider the Newtonian background of Kant's argument. In particular, we must take into account Newton's derivation of the inverse square law given in propositions 11–13 of Book I of the *Principia*. In proposition 11 Newton employs an instance of the property of conic sections described above to prove that: if a body P moving along an ellipse is subject to a force f centrally directed toward a focus S, then f is inversely proportional to SP^2.[83] In proposition 1 and 2 of Book I, Newton had shown that a force acting on a body with uniform linear motion is centrally directed towards a given point *if and only if* this motion describes equal areas in equal times with respect to that point (i.e., satisfies Kepler's law of areas).[84] Hence, Newton's proof of proposition 11 shows that if a body moving along an elliptical orbit describes equal areas in equal times with respect to

Footnote 77 continued
mathematics with respect to physics (which Friedman does not fully explicate), and (ii) Kant's claim that *only* mathematical natural sciences constitute *proper* natural sciences.

78 Kant (1902, IV, p. 321).

79 Ibid.

80 Ibid.

81 Cf. Friedman (1992, p. 191).

82 Friedman (1992, pp. 191–194).

83 Newton (1999, pp. 462–463).

84 Newton (1999, pp. 444–448).

the focus of the ellipse, it is subject to a central force that is inversely proportional to the square of the distance from that focus. In propositions 12 and 13 Newton proves that the same holds for hyperbolic and parabolic orbits. Corollary 1 of proposition 13 conversely shows that a moving body subject to a centripetal inverse-square force will orbit along a conic section. In short, Propositions 11–13 of Book I prove that orbital motion along a conic section satisfying the law of areas implies an inverse-square force (and vice versa). Kant's argument in § 38 suggests that he had this derivation in mind.

Propositions 11–13 provide us with mathematical demonstrations of particular equivalences. Kant suggests that these demonstrations allow us to infer that gravity is an inverse-square force. This is understandable if we recognize that the mathematical principles developed in Book I provide a basis for Newton's derivation of the law of gravitation in Book III of the *Principia*.[85] In particular, Kant seems to envision an argument along the following lines. Newton's proposition 11 of Book I proves that if a body moves in an ellipse and satisfies the laws of areas with respect to a focus, this motion is governed by an inverse-square centripetal force directed toward the focus. We know empirically, by means of Newton's "phenomena", that the satellites of primary bodies orbit in an ellipse and satisfy Kepler's law of areas with respect to their primary bodies situated at a focus. Hence, we can infer mathematically that these satellites are subject to an inverse-square force directed towards their primary bodies. Newton himself employs this type of reasoning in the first three propositions of Book III, insofar as he applies mathematically demonstrated relations, obtaining between centrally directed inverse-square forces maintaining a body in orbit and this motion satisfying Kepler's laws, to phenomena. These relations allow him to infer that the satellites of Jupiter and Saturn are subject to an inverse-square force directed toward the center of their primary planets (prop. 1),[86] that the planets are subject to an inverse-square force directed towards the sun (prop. 2), and that the moon is subject to an inverse-square force directed towards the earth (prop. 3). These propositions provide us with the first steps in Newton's argument for the law of gravitation and allow him to conclude that gravity is an inverse-square force.

The use of mathematics within natural science described above suggests that mathematics can be interpreted as providing models of the physical world, by means of which we cognize quantitative relations obtaining between individual objects. This is a three step procedure: (i) we mathematically establish that motion along a conic section satisfying (one of) Kepler's laws implies a centripetal inverse-square force; (ii) we empirically observe that heavenly bodies orbit in a conic section and satisfy (one of) Kepler's laws; (iii) we infer the existence of a centripetal inverse-square force. Kant took mathematics as providing models of physical objects and would have interpreted Newton's application of mathematical principles to phenomena accordingly. In this

[85] As Newton puts it himself in the Introduction to Book III, the task of Book III is to "exhibit the system of the world from these same principles." The locution 'same principles' refers to the mathematical principles of philosophy expounded in Books I and II. Newton (1999, p. 793).

[86] Newton does not, however, employ proposition 11 of Book I, but proposition 2 of Book I, allowing us to use the law of areas to infer the existence of a centripetal force, and Corollary 6 of proposition 4 of Book 1, allowing us to use the harmonic law to infer the existence of an inverse-square force. For an analysis of Newton's argument: Harper (2002, pp. 174–201).

case, a mathematically constructed conic section functions as a model of the motions of heavenly bodies, enabling the inference from (i) and (ii) to (iii) by subsumption. Note that this inference is successful because we have a fit between our mathematical model and observable phenomena. This need not be the case. In his discussion of the mathematical-mechanical mode of explanation in the *Metaphysical Foundations*, Kant states that this mode is based on concepts that can be mathematically represented (e.g., 'extension', 'corpuscle', 'space').[87] As an example, one may think of an explanation of density differences by means of the mathematical representation of varying amounts of empty space interspersed among material particles. According to Kant we can, then, construct mathematical models on the basis of concepts central to the mathematical-mechanical mode of explanation. Nevertheless, this mode of explanation is rejected since it employs empty concepts such as 'void space' or 'absolute impenetrability', i.e., concepts the object of which cannot be cognized as existing since their instances are not observable. Consequently, mathematical models based on empty concepts can not be taken to represent actual existing objects. This is probably a consequence of the fact that, insofar as these models are based on empty concepts, they are interpreted in a manner that cannot be *adequate* to existing objects (e.g., by taking mathematical points to represent absolutely impenetrable corpuscles). Thus, although the construction of a mathematical model secures its objective reality, i.e., its possible application to objects, these models can only be taken to represent particular existing objects if they are interpreted correctly.[88] A possible method to determine that a mathematical model represents existing objects is via the empirical confirmation of consequences inferred on the basis of this model (i.e., the empirical confirmation of (iii)).[89]

The upshot of the previous paragraph is that we take (correctly interpreted) mathematical models to represent existing objects on the basis of empirical confirmation of inferences made on the basis of these models. Hence, the use of mathematics within natural science is empirically conditioned. Nevertheless, mathematics allows the a priori justification of physical cognition. The derivation of the inverse-square law, discussed above in relation to § 38, allows us to see this. For the claim that gravity is an inverse-square force is based on mathematically demonstrated (a priori cognizable) relations. We apply these relations to observable phenomena in order to infer mathematically that, e.g., the planets are subject to an inverse-square force directed towards the sun. The latter claim is thus proven on the basis of a priori principles

87 Kant (1902, IV, pp. 524–525).

88 Abstracting from Kant's own terminology, we can say that a correct interpretation of mathematical models employed within physics is guaranteed metaphysically. For Kant takes it to be the task of metaphysics to provide a priori principles in accordance with which the concept of physics must be mathematically constructed. Cf. Kant (1902, IV, p. 473).

89 Because of the reasons sketched in this paragraph, I cannot follow commentators who interpret Kant's mathematization requirement as following from his view that the mathematical construction of the concepts of a science secures their *objective reality*. Cf. Falkenburg (2000, p. 289), Pollok (2001, pp. 86–87). Although constructability secures objective reality, it does not secure that mathematical models are *adequate* to particular natural objects. This adequacy is required, however, if mathematics is supposed to provide a priori justification of scientific cognitions. Hence, this particular interpretation cannot explain how mathematics fulfils a foundational function with respect to natural sciences.

and is accordingly *apodictically certain*. Note, moreover, that it provides cognition of a specific quantitative relationship obtaining between individual objects. Hence, mathematics allows us to obtain a priori grounded cognition of *determinate natural things*. Finally, we may note that mathematics provides a priori *grounds of cognition* of determinate natural things. Mathematical models do not specify (in Kant's terms) grounds of being of, e.g., the fact that planets are subject to an inverse-square force, for this ground is given by the physical force of gravitation. Hence, mathematics fulfills a strictly *epistemic* function with respect to physics. Such a reading is confirmed by the fact that Kant occasionally describes mathematics as an *organon* of the sciences, which is to say that mathematics is an instrument for bringing about certain cognition.[90] In short, mathematical demonstration of judgments in natural science secures *knowledge* but does not guarantee that judgments ground each other, i.e., express objective relations between grounds and consequences.

The interpretation developed above allows us to explain, in conclusion, why *only* mathematical sciences are proper sciences. According to Kant, it is mathematics alone that provides *a* priori insight of specific quantitative properties of individual physical objects. This is a consequence of the fact that mathematics provides a priori models (individual and concrete representations) of physical objects. Philosophy or metaphysics provide *discursive* a priori principles valid for natural objects. However, in contrast to mathematics, philosophical or metaphysical cognition is not based on the construction of concepts. Consequently, it does not provide a priori models of individual physical objects that can be applied to nature in order to obtain a priori cognition of specific quantitative relations obtaining between individual objects. This type of cognition can only be obtained by means of mathematics.

6 Conclusion

The upshot of the final section is that Kant argues for the necessity of applying mathematics within the study of nature since mathematics provides a priori cognition of corporeal objects. As such, mathematics can provide a priori principles for doctrines that aim to explain specific features of corporeal objects. The foundational function of mathematics is exemplified in the case of physics, where mathematics provides a priori principles for cognizing physical laws. Kant's claim that proper natural sciences require mathematics follows from the claim that any proper science must have a priori principles, which is meant to secure that scientific judgments are apodictically certain. This latter condition, in turn, builds on the condition that proper sciences must have principles or grounds, securing that they are explanatory. These conditions correspond to conditions (6) and (3) of the Classical Model of Science. Finally, the condition of systematicity secures that sciences possess a logical order and coherence, incorporating condition (2) of the Classical Model of Science. Kant's conception of proper science is thus a natural consequence of a classical ideal of science.

[90] Cf. Kant (1902, IX, p. 13). Here, Kant further argues that an *organon*, such as mathematics, *anticipates the matter* of the sciences. This claim, I take it, is nicely illustrated by the interpretation of mathematics as providing (a priori) models of physical objects, providing grounds of cognition for cognitions pertaining to physics.

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