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Friendship Networks Through Time: An Actor-Oriented Dynamic Statistical Network Model

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Abstract

We propose a class of actor-oriented statistical models for closed social networks in general, and friendship networks in particular. The models are random utility models developed within a rational choice framework. Based on social psychological and sociological theories about friendship, mathematical functions capturing expected utility of individual actors with respect to friendship are constructed. Expected utility also contains a random (unexplained) component. We assume that, given their restrictions and contact opportunities, individuals evaluate their utility functions and behave such that they maximize the expected amount of utility. The behavior under consideration is the expression of like and dislike (choice of friends). Theoretical mechanisms that are modelled are, e.g., the principle of diminishing returns, the tendency towards reciprocated choices, and the preference for friendship relations with similar others. Constraints imposed on individuals are, e.g., the structure of the existing network, and the distribution of personal characteristics over the respondents. The models are illustrated by means of a data-set collected among university freshmen at 7 points in time during 1994 and 1995.

Keywords: rational choice, friendship, Markov processes, random utility models, simulation, empirical test

1. Introduction

If, within a certain context, a group of individuals (either initially mutual strangers or not), has the opportunity to interact for a certain period of time, friendly relationships and friendships will undoubtedly be established and as a result, an affective relation network will arise. The structure of such a network can be described according to a variety of network measures, such as the degree of reciprocity, the degree of transitivity, the degree of segmentation, and clusterability. The structure of a friendship network is a result of simultaneously acting individuals who continuously have the opportunity to choose to initiate, establish, maintain, and dissolve relationships. These choice processes are not based on pure chance, but can, up to a certain point, be described by some principles that underlie individual behavior. The main objective of this study is to provide an explanation

regarding the choices people make with respect to the formation of affective relationships. In order to do so, we utilize rational choice theory.

In a nutshell, rational choice theory claims that, given constraints and opportunities, individual behavior can be modeled as if individuals base their actions on a cost-benefit analysis (Lindenberg, 1983; Coleman, 1990). Individuals are resourceful, restricted, expecting, evaluating, maximizing man, better known as RREEMM (Lindenberg, 1983). They are resourceful in the sense that they search for solutions when confronted with problems. They possess the capabilities to learn from past experiences and from experiences from others, in other words, they may learn from their social resources. Constraints make that they are confronted with scarce goods, and consequently they have to make choices. They have expectations regarding the outcomes of the choices they make, in other words, they attach subjective probabilities to the results of alternative behavior. They evaluate these choices according to their preference, and finally, they maximize expected utility when choosing how to behave or what to do. In other words, individuals behave rationally. On the other hand, individuals are bounded in their rationality, they have limited strategic foresight and limited other cognitive abilities, so that although they try to follow an optimal course of action, they do not necessarily succeed. Because of this, we explicitly add a random component to our rational choice based models to account for this uncertainty. The result is a random utility model in which the actor chooses among the possible actions with probabilities that are an increasing function of the uti-lity as calculated from the variables in the model. We emphasize that rational choice theory is not a theory that argues that individuals behave rationally, but it claims that human behavior can be modeled *as if* it is rational. In order to stress our individual-oriented approach, we will build our discussion regarding affective relationships around a rational arbitrary male actor called ego and some nameless rational arbitrary male alter(s).

In the remaining of this section we briefly discuss the process of friendship formation. In Section 2 we present some features of existing statistical network models, paying most attention to Markov models. The formal elements of our actor-oriented statistical model for social networks in time are also presented in Section 2. In Section 3 we introduce the main ingredients of the formal friendship model. Each substantive argument is followed by a formal representation in terms of expected utility. 7 measurements are collected among a group of university freshmen in the academic year 1994–1995. The data set consists of individual characteristics (sex, age, residence or hometown, education program, and smoking behavior) and sociometric data. Friendship is operationalized as a 6 category variable. We discuss results in Section 4.

1.1. Friendly Relationships and Friendship

Compared to friendship, friendly relationships are of a less intimate and demanding nature and are built upon a less voluntary foundation (Kurth, 1970). They serve two main non-exclusive purposes, a social and an instrumental one. The former refers to friendly relationships that make daily interaction with, in this case, fellow students run more smoothly (Fine, 1986). The latter refers to friendly relationships that, in times of need, are ‘handy’ or

supportive. Friendly relationships with fellow students make it easier for ego to ask them for help, for instance concerning their study (Argyle and Henderson, 1985).

Friendships are of a more intimate and personal nature. They are more demanding, more voluntary, but also more unique relationships than friendly relationships. Although intuitively everybody has some idea regarding the meaning of friendship, however, it is very hard to come up with a clear and plain formal definition of friendship. We all speak in terms of best friends, friends, acquaintances, etc., suggesting that, at least in general, people know what friendship is, and that they are able to make a distinction between various degrees of friendship. Nevertheless, friendship is an ambiguous, vague, and multidimensional concept. The meaning of friendship depends on someone's social class, age, stage in the life cycle, and gender. Furthermore, inhabitants of dissimilar societies may have different notions of what friendship entails (among others: Hess, 1972; Verbrugge, 1977; La Gaipa, 1977; Fisher, 1982; Winstead and Derlega, 1986; Dykstra, 1987; Hays, 1988; Allan, 1989).

A friendship is a voluntary relationship in the sense that ego is free to choose with whom he wants to be friends, or better, with whom he prefers to be friends. Ego is entirely free to shape relationships as he pleases and by their nature they are not supported by institutional structures, blood ties, or social arrangements (Allan, 1989; Blieszner and Adams, 1992). As such, ego's friendships have an achieved instead of an ascribed character. Ego needs friends for the fulfillment of his social needs; for confirmation of his behavior; for a dose of self-worth; for all sorts of (instrumental) help (if needed); for emotional support and stability; for social companionship. The range of requirements for a friend is rather broad. This suggests that ego needs several relationships of varying strengths to realize all his needs. Let us assume that ego has some sense regarding the potential rewardingness of alters. He does not randomly pick out the first person he runs into, in times of need. Subsequently the question is: whom does ego choose to satisfy his social needs?

Our starting point is a rather uncomplicated (formal) context: Interaction between ego and alters and among alters takes place under relatively unambiguous conditions that do not differ considerably from person to person. This can be answered for a group of university freshmen. At first,[1] the major concern of ego is to get to know others, to achieve a friendly level with (some of) them, and on the whole, to have a good time. During initial verbal and non-verbal interaction ego concentrates on visual characteristics, such as gender and race, but also on physical attractiveness, behavior, and visible indicators regarding membership of some sub-culture. Ego observes his own behavior and that of alters during the interaction, and evaluates this in terms of his own values, norms, interests, etc. Ego does not only pay attention to interaction in which he plays part himself, but also to interaction among alters. The more information ego has collected about all alters, the more reliable not only ego's estimation of the suitability of alters in terms of continuation of the relationship, but also his estimation of the willingness of alters to reciprocate his personal interest.

If ego's expectations are positive regarding certain persons, he will put more effort and spend more energy and time in the continuation of the contact, hoping that some friendly relationships evolve into a friendship. Once a relationship between ego and alter reaches a friendly level, common interests and values and agreement on third alters in the given context become a determinant of attraction. The degree of closeness in a relationship can be characterized by type of behavior (Levinger, 1980). The closer the relationship, the higher the probability that intimate activities are carried out within the relationship. Dykstra (1987)

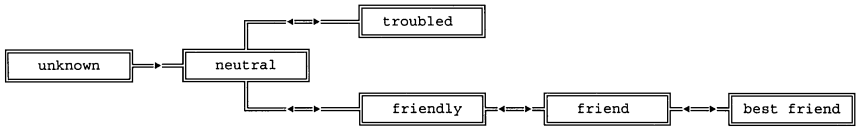

Figure 1. Possible transitions between categories of a relationship.

and Argyle and Henderson (1985) found similar results. When within a friendly relationship ego finds out that he and alter do not share interests, values, and opinions regarding third alters, he will probably not intensify the contact.

Relationships also dissolve. Just as friendship formation is not an event but a long term process, so is friendship dissolution. There are several causes for a friendship to dissolve. Ego may not get what he expected from the relationship. Ego may find out some negative characteristics of alter, that were at first not visible. Whether ego actually decides to break off the relationship is not depending only on relational dissatisfaction, but also on the availability of satisfactory alternatives (Thibaut and Kelly, 1959; Homans, 1961; Kelly and Thibaut, 1978), on investments already put in the relationship (Rusbult, 1980a, 1980b, 1991), and on structural embeddedness (Salzinger, 1982; Milardo, 1986), e.g., the presence of common friends. Furthermore, contact costs may play a role (e.g., associated with distance).

Putting this together, we distinguish five stages in the process of friendship formation. Initially ego and alter do not know each other. Once ego is aware of alter, he places alter in the category 'neutral' (along with most other alters). After this, his relationship with alter stabilizes or evolves (1) in the direction of a troubled relationship (which we will not elaborate at the moment) or (2) in the direction of a friendly relationship. Again, ego has the option to stabilize the relationship, strengthen the relationship into a friendship, or weaken the relationship into a neutral one. The closest relationship possible is called a best friend relationship. At each moment in time ego has the opportunity to evaluate his relationships in terms of costs and rewards and change them, but only to the adjacent categories (see figure 1).

In the next section we present rational choice based statistical models that explicitly take into account the process of friendship formation at the individual level, and that are a combination of Markov and random utility modeling.

2. Actor-Oriented Modeling

The network literature does not provide many formal models that express the dynamic aspects of the development of affective relation networks. Most network models are static in the sense that time does not play a role. There exist two important classes of dynamic stochastic network models, however. First, network models that are based on the loglinear approach (among others: Wasserman and Iacobucci, 1988; Van Duijn and Snijders, 1996). Second, continuous-time Markov chains (among others: Wasserman, 1980; Leenders, 1995, 1996a, 1996b). Each approach has its own limitations, however.

First, Markov analysis, as elaborated by Wasserman and Leenders, is restricted to dichotomous network data. The loglinear approach can deal with valued relations with ordered

categories (see for instance, Wasserman and Faust, 1994). Second, the loglinear approach produces an overflow of parameters. For instance, if g is the group size, the original p_1-model estimates $2g + 1$ parameters. Third, loglinear modeling allows only categorical attributes in the analysis. Continuous attributes, such as age and tenure have to be categorized. Markov analysis can incorporate categorical and continuous attributes, and also allows a larger number of attributes. Fourth, loglinear analysis models changes between the state of a network at $time = t$ and $t = t + 1$ as discrete steps. The problem is that the interpretation of the loglinear parameters depends on the length of the interval between the observations. When intervals are not of equal length (as in our research among students), the interpretation becomes difficult. Continuous-time Markov models, that are explicitly based on processes in continuous time, do not have these problems. The interpretation of the change rate parameters does not depend on the length of the intervals. Fifth, structural effects, such as transitivity or balance violate the assumption of dyad independence and therefore cannot be expressed in loglinear models. These effects also are not expressed satisfactorily in the Markov models for dichotomous network data published up to now.

So far it seems that for modeling networks in time, the Markov approach has, compared to loglinear analysis, some advantages. However, Markov analysis is still restricted by the assumption of independence between dyads. This assumption excludes most network theories in general, and friendship theories in particular, to be put to a test. For instance, if we want to estimate a parameter for the tendency to establish balanced relationships (which explicitly stresses the dependence of pairs of friends), this cannot be properly done by means of Markov analysis. Furthermore, although Markov analysis comes close, there is a limited connection between theory and data analysis. Preferably we need a model, that integrates the mathematical formulation of a social scientific theory with a statistical model that can be used for an empirical test of the applied theory. This would provide us with a model that is a direct expression of the theory.

A theoretically interesting but non-statistical approach was followed by Zeggelink (1993, 1994, 1995; Zeggelink et al., 1996). She proposed mathematical rational choice theory based models in order to explain which structural characteristics of friendship networks emerge in closed heterogeneous groups of initially mutual strangers, given a certain context in which the individuals will interact for a certain amount of time. The models were defined using an individual oriented modeling approach, but the individual behavior rules underlying the process of friendship formation cannot easily be tested at the individual level. Furthermore, it is not possible to estimate the relative weights of the issues involved in the process. Thus, the models proposed by Zeggelink offer us important insights in the evolution of friendship networks, but lack procedures in order to test relative weights of the behavior rules. A way to overcome these problems is to define rational choice theory based statistical models that explicitly take into account the process of friendship formation at the individual level, and that are a combination of Markov and random utility modeling. In the next section, we present a formal outline of such a statistical actor-oriented model (for details we refer to Snijders (1996) and Snijders and Van Duijn (1997)).

The actor-oriented statistical network models describe the development of a social network in time as a result of the actions of individual actors. Given the restrictions determined by the structure of the network, the distribution of actor attributes, and external factors, each individual actor maximizes his utility. The models combine random utility and Markov

theory, and are especially suited for rational choice theory based dynamic network models, e.g., for the development of influence networks with respect to decision making processes and for the evolution of affective relation networks. The formal components of the actor-oriented models are introduced and explicated here for the development of friendship networks among university freshmen, and, if necessary for clarification purposes, among employees of a work organization.

Starting point is a closed set of actors (1), denoted G, in a certain context, consisting of g actors, who potentially are involved in social relationships with each other (2). The relationships are directed, and may be valued and/or multidimensional. For instance, although friendship is often modeled as a mutual binary (on/off) relationship, it is clear that, first, friendship exists at several levels of intimacy, and second, two individuals sometimes have distinct perceptions regarding the strength of relationship between them. Furthermore, a friendship relation can almost always be characterized according to other typologies, for instance, as co-workers, as neighbors, as family, etc. Moreover, friendship between two actors can be described in terms of the activities that they are engaged in. The actors can be described in terms of l individual attributes (3), that remain constant, e.g., gender and age, or that (may) change in time, e.g., attitudes such as job satisfaction, behavior and behavior tendencies, such as smoking and watching certain television programs. Both the relation network and the individual attributes are time dependent, and are represented as a $g \times g$ matrix $F_{(t)}$ and a $g \times l$ matrix $Z_{(t)}$, respectively. Together they define the state of the model at time point t.

For each actor we define a so-called actor state (4) as a function of the state of the model. This state vector defines the most parsimonious collection of actor attributes that actors need to evaluate their situation with respect to the formation and maintenance of friendships. For each actor we define a utility function or preference function (5) that is based on the actor state given in (4). The utility function is what is actually maximized by the actors. It must be based on substantive arguments from the field of the subject under consideration, and constructed such that it represents the costs and rewards for an actor to be in a specific actor state at a certain moment in time. This is the core of model, and as such it determines its success or failure. The individual utility functions also will contain elements that are not represented in the model by measured variables; this is modeled as a random component. Also the substantive part of the utility function is not completely known to the researcher. Therefore, the utility function contains statistical parameters that have to be estimated from the data. These are the weight factors $\alpha_1, \ldots, \alpha_d$ as we will discuss in Section 3. The utility function may differ between actors. The model assumes that each actor has at least information regarding his own state (6). In the present version of the model, the only differences of utility functions between actors are those differences which can be expressed as functions of covariates (e.g., related to similarity effects). Extending the model with differences between actors in popularity (i.e., indegree) or activity (i.e., outdegree), not explained by covariates, will be a subject for future research.

For each actor we define a set of admissible actions (7). In this paper, the actions refer to relations with the others in the network, i.e., actors may initiate, maintain, or dissolve a relationship. These actions are partly constrained by the social structure, e.g., being a member of some informal group with specific norms and values may prohibit certain behavior.

As an indicator of the points in time at which the actions can take place, we define a time schedule (8). The time parameter of the models is continuous, so that the actors can act on arbitrary moments. The model implies stochastic waiting times specifying the moments of action. The waiting times are assumed to have the negative exponential distribution with parameter λ. This rate of change, i.e., the reciprocal of the expected length of these waiting times, is also a statistical parameter that has to be estimated from the data.

Finally, the actors choose their actions according to a heuristic (9) that determines which action in which situation is most likely to be chosen in view of optimizing expected utility. The word heuristic is used because the actors are assumed to have limited strategic foresight and limited other cognitive abilities, so that although they try to follow an optimal course of action, they do not necessarily succeed.

The resulting probability model is a Markov process in continuous time. The heuristic that actors use if they have the opportunity to act at a certain time point t, is modeled as a random utility model in which the actor chooses among the possible actions with probabilities that are an increasing function of the expected utility as calculated from the variables in the model. In the next paragraph we briefly discuss this type of models.

Discrete choice models are used in situations in which the dependent variable assumes values in a discrete set of choices. In this paper, the choice refers to a change in some relationship; in other applications it could be, e.g., the choice of a political party to vote for or the choice of a transport medium. In general, the choice of action for actor i at time point t, a_{it} (including 'doing nothing') from a set of allowed actions A is based on a number of independent variables. It is assumed that an actor is able to calculate the expected effects of each allowed action. Hence, each action is associated with a change in utility $\Delta U_{it}(a)$. Since the choice of action can also be based on utility arguments that are not explicitly modeled in the utility function, and because of measurement and/or specification errors, we assume that ego chooses the action that maximizes

$$\Delta U_{it}(a) + E_{it}(a), \tag{2.1}$$

in which $E_{it}(a)$ is a random disturbance that is assumed to be distributed with mean 0 and scale parameter σ. A convenient choice of this distribution provides us with the multinomial logit model (McFadden, 1973; Maddala, 1983; Agresti, 1989) where the probability of choosing action a is

$$p_{it}(a) = \frac{\exp(\Delta U_{it}(a)/\sigma)}{\sum_{a \in A} \exp(\Delta U_{it}(a)/\sigma)} \tag{2.2}$$

Expressed verbally, if the expected change in utility is approximately the same for all actions, ego's choice is almost entirely based on pure chance. If, however, compared to other actions, one action is associated with a relatively large increase of expected utility, the probability that ego chooses this specific action is also relatively large. Note that because of the random component $E_{it}(a)$, ego does not necessarily choose the action that is associated with the largest expected increase of utility.

The above model, when elaborated for a practical application, contains unknown parameters that have to be estimated from observed data by a statistical procedure. It is assumed

that the states and behaviors of the actors are observed at two or more points in time. Since the proposed stochastic models are too complex for the application of classical maximum likelihood estimation procedures and testing methods, we use the method of moments and Monte Carlo computer simulation to analyze and to approximate expected values of relevant statistics.

The *method of moments* is a statistical approach for estimating relevant statistics. Suppose a statistical model contains k parameters. In our model (see Section 3) the statistical parameter is $\theta = (\lambda, \alpha_1, \alpha_2, \ldots, \alpha_d)$ in which $k = d + 1$. For each of the k parameters a statistic is chosen such that it captures the variability in the set of possible data that can be accounted for by the parameter. The parameters are estimated by equating the observed and expected values of the k statistics. In order to use the method of moments, the researcher should have some idea regarding the distribution of the k statistics. For example, if we take a random sample from the normal distribution suitable statistics are the mean and the variance. The observed and expected values of these two statistics are equated and the two parameters are estimated. Since in the actor-oriented statistical models the equations cannot be solved numerically, we use the Robbins-Monro stochastic approximation method. Therefore, we define a k-dimensional random variable Z as the difference between the value of these k statistics for a stochastic network with the specified distribution according to the actor-oriented model and the observed value of the k statistics. The Robbins-Monro procedure provides us with a recursive method for finding the value of θ that solves

$$E_\theta Z = 0, \tag{2.3}$$

i.e., that equates the expected to the observed statistics. This procedure is based on simulating the actor-oriented model. Further explanation can be found in Snijders (1996) and Snijders and Van Duijn (1997). The software that is used is called SIENA, and can be downloaded from Snijders' home page (http://stat.gamma.rug.nl/snijders).

Methods for testing the significance of the separate parameters and the model as a whole have not been investigated intensively yet. Since the Robbins-Monro provides the researcher with estimates of θ and the covariance matrix of this estimate, a test can be applied with test statistic

$$\frac{\hat{\theta}_j}{S.E.(\hat{\theta}_j)}. \tag{2.4}$$

Snijders (1996) proposed to use an approximate standard normal distribution. This means that we consider values smaller than -2 or values larger than 2 to be statistically significant at the 5% level.

3. The Friendship Model

F_{ij} denotes whether there exists an affective relationship between ego i and alter j, or not. S_{ij} is the matrix in which cell (ij) represents the perceived similarity between ego and alter regarding some similarity attribute. The higher the value of cell (ij) in S_{ij}, the more similar

ego and alter. Finally, P_{ij} is the proximity (or opportunity for contact) matrix in which cell (ij) stands for the number of contact opportunities between ego and alter regarding some proximity dimension. The higher the value of cell (ij) in P_{ij}, the higher the amount of opportunities for ego to have contact with alter. All matrices may be asymmetric, but in practice this only holds for F_{ij}. The number of actors is denoted by g.[2]

Our goal is to construct a time dependent individual utility function of the form

$$EU^i = \sum_{d=1}^{D} \alpha_d EU^i_{(d)}. \tag{3.1}$$

EU^i stands for the amount of utility ego expects given the network structure and the actor attributes. Ego's amount of expected utility consists of D substantive components of which the relative importance is indicated by α_d. Ego uses (3.1) plus a random component in his evaluation regarding his course of action. At random time points ego is 'allowed' to take one action. He compares all possible states and chooses, with the highest probability, the one which he expects to provide him with the largest increase of utility. In the next section we introduce the substantive elements of the actor-oriented general friendship model.

So far, we did not and will not explicitly distinguish between rewards and costs of initiating and maintaining a relationship. Our theoretical considerations are expressed in terms of expected utility, in which the sign of the effect depends on the exact definition of the utility component. In general, a positive sign means that establishing a friendship is associated with an increase of expected utility, which implies that in that case the relative amount of costs is smaller than the relative amount of rewards. Consequently, aspects of friendship formation and dissolution that could be more appealing in terms of costs, rewards, and restrictions are nevertheless expressed in amounts of expected utility. For instance, 'the larger the number of contact opportunities for ego to meet alter, the lower the relative amounts of costs of ego to become friends with alter,' is translated into 'the larger the number of contact opportunities for ego to meet alter, the larger ego's amount of expected utility to become friends with alter'.

In the next sections we discuss the friendship components of the utility functions.

3.1. Having Relationships

The arising of friendships is an universal event. At all time, in all places, and in every context people meet other people, people interact, people like and dislike each other, and as a result of each individual's need for social contact, friendships develop. At this stage, we simply assume that ego wants to make friends.

For each relationship, the social need component of the utility function is defined as

$$EU^i_{(\text{frd})} = \sum_{j=1,\, j\neq i}^{g} F_{ij}. \tag{3.2}$$

Interpretation: Initiating or strengthening a relationship with alters increases ego's amount of expected utility. We refer to this, as the main effect of friendship.

3.2. *Principle of Diminishing Marginal Returns*

The social need effect does not take into account the fact that friendships also entail costs, e.g., they require time. We assume that the costs and rewards involved in making and maintaining relationships is a nonlinear function of the number of individual friendships ego already has. At a certain moment the marginal costs of starting a new relationship outweigh the marginal rewards. This is formalized by

$$EU^i_{(\text{mar})} = (F_{i+})^2 - F_{i+} \tag{3.3}$$

in which

$$F_{i+} = \sum_{j=1,\, j\neq i}^{g} F_{ij}. \tag{3.4}$$

Marginal returns of friendship are diminishing if this component has a negative weight in (3.1). The diminishing marginal returns effect has no real meaning if the social need effect is not part of the utility function too. In (3.5) they are combined and the subsequent weight factors are added. α_{frd} and α_{mar} are the social need parameter and the marginal return parameter, respectively. If $\alpha_{\text{mar}} = 0$, there is only a social need effect. This yields

$$\begin{aligned} EU^i_{(\text{frd+mar})} &= \alpha_{\text{frd}} F_{i+} + \alpha_{\text{mar}}\big((F_{i+})^2 - F_{i+}\big) \\ &= (\alpha_{\text{frd}} - \alpha_{\text{mar}}) F_{i+} + \alpha_{\text{mar}} (F_{i+})^2. \end{aligned} \tag{3.5}$$

Interpretation (for $\alpha_{\text{mar}} < 0$): Initiating or strengthening a relationship with alters will increase ego's amount of expected utility to a larger extent if ego has not many friends yet than if ego has already many friends.

3.3. *Popularity*

Popularity in the group can be seen as a form of social status. The more members of the group show their friendship to ego, the more popular ego is in the group, the more status ego has with respect to a relative ranking of social relationships. Indirect social status can be obtained by being friends with popular others. Thus, ego can produce status by having many friends, and by being friends with popular others. The indirect popularity component of the utility function is defined as

$$EU^i_{(\text{pop})} = \sum_{j=1,\, j\neq i}^{g} F_{ij} F_{+j}. \tag{3.6}$$

Interpretation: Initiating or strengthening a relationship with popular alters (i.e., who have a relatively high indegree) will increase ego's amount of expected utility to a larger extent than with unpopular alters (i.e., who have a relatively small indegree).

In (3.7) the main effect of friendship and the indirect popularity component are combined and the subsequent weight factors are added. α_{frd} and α_{pop} are the social need parameter

and the popularity parameter, respectively. If $\alpha_{\text{pop}} = 0$, there is only a social need effect. The result is

$$EU^i_{(\text{pop})} = \alpha_{\text{frd}} F_{i+} + \alpha_{\text{pop}} \sum_{j=1,\, j\neq i}^{g} F_{ij} F_{+j}. \tag{3.7}$$

This can be rewritten as

$$\begin{aligned} EU^i_{(\text{frd+pop})} &= \alpha_{\text{frd}} F_{i+} + \alpha_{\text{pop}} \sum_{j=1,\, j\neq i}^{g} F_{ij}(F_{+j} - \bar{F}_{+\cdot} + \bar{F}_{+\cdot}) \\ &= \alpha_{\text{frd}} F_{i+} + \alpha_{\text{pop}} \sum_{j=1,\, j\neq i}^{g} F_{ij}(F_{+j} - \bar{F}_{+\cdot}) + \alpha_{\text{pop}} F_{i+} \bar{F}_{+\cdot}, \\ &= (\alpha_{\text{frd}} + \alpha_{\text{pop}} \bar{F}_{+\cdot}) F_{i+} + \alpha_{\text{pop}} \sum_{j=1,\, j\neq i}^{g} F_{ij}(F_{+j} - \bar{F}_{+\cdot}) \end{aligned} \tag{3.8}$$

in which $\bar{F}_{+\cdot}$ is the average popularity of all alters. (3.8) shows that if $\alpha_{\text{pop}} = 0$, the main effect of friendship is $\alpha_{\text{frd}} F_{i+}$. If on the other hand α_{pop} is positive, the first part of (3.8) concerns ego's expected utility when he chooses to initiate an additional affective relationship with an alter of *average* popularity. The second part of (3.8) shows that ego expects greater gain of utility if he initiates an affective relationship with an alter who is more popular than average, and a smaller gain of utility if alter is less popular than average. If α_{pop} is negative, there exists a preference to initiate affective relationships with less than average popular alters.

3.4. The Opportunity Structure

In general, proximity is a necessary condition for friendship to arise (e.g., Newcomb, 1961). In the present study, this condition is satisfied. Students have chosen for the same study, and they will meet each other without doubt. Nevertheless, the number of contact opportunities is not the same for each pair of students, and the probability to get to know each other and, in some cases, become friends differs also. The exact definition of the opportunity structure depends on the actual, more or less formalized, context within which the group operates.

The proximity (or opportunity) matrices can be defined in numerous ways. We choose one based on absolute differences, such that the higher cell (ij) of P_{ij}, the more opportunities for contact. The proximity component of ego's utility function is defined as

$$EU^i_{(\text{opp})} = \sum_{j=1,\, j\neq i}^{g} F_{ij} P_{ij}. \tag{3.9}$$

Interpretation: Initiating or strengthening a relationship with alters to whom ego is proximate (i.e., has many opportunities for contact) will increase ego's amount of expected utility to a larger extent than initiating or strengthening a relationship with alters to whom ego is less proximate (i.e., has less opportunities for contact).

3.5. Similarity, Balance and Transitivity

In general, if individuals strive after some goal they will not randomly 'try things and hope for the best', but they possess some knowledge, skills, and experience regarding the options they have to accomplish these goals. Furthermore, they have some foresight in the outcomes and effectiveness of their behavior. The same holds for relationships. Ego has several social needs (see above) and ego 'knows' (or has learned) that friendly relationships and friendships are instruments to fulfil some of these needs. Ego also 'knows' that some relationships are potentially more rewarding than others. Similar persons have had similar experiences and have faced similar problems, and as a result ego expects them to have similar attitudes, values, and interests (among others: Lazarsfeld and Merton, 1954; Morton, 1959; Newcomb, 1961). In general, similarity may be a good predictor for the cost-reward ratio of relationships.

For each relationship between ego and alter, the similarity component of ego's utility function regarding individual characteristics is defined as

$$EU^i_{(\mathrm{sim})} = \sum_{j=1,\, j\neq i}^{g} F_{ij} S_{ij} \tag{3.10}$$

Interpretation: Initiating or strengthening a relationship with alters to whom ego is similar with respect to certain characteristics will increase ego's amount of expected utility to a larger extent than initiating or strengthening a relationship with alters to whom ego is less similar.

Balance (Heider, 1958) can be treated as a variation on the preference for similar alters, namely by focusing on attitudes regarding third alters. This is expressed by the following formula (where third alters are indicated by k),

$$EU^i_{(\mathrm{bal})} = -\sum_{j=1,\, j\neq i}^{g} \sum_{k=1,k\neq i,\, j}^{g} \left|F_{(ik)} - F_{(jk)}\right| F_{(ij)}. \tag{3.11}$$

Interpretation: Initiating or strengthening a relationship with alters with whom ego 'thinks alike' about others in the group will increase ego's amount of expected utility to a larger extent than initiating or strengthening a relationship with alters with whom ego does not 'think alike' about others in the group.

Balance in the sense as discussed above appeared to lead to problems in the statistical estimation process.[3] Therefore we use transitivity instead. This is represented in the amount of transitive triplets in which ego is involved,

$$EU^i_{(\mathrm{trs})} = \sum_{j=1}^{g} \sum_{k=1,k\neq i,\, j}^{g} F_{(ij)} F_{(ik)} F_{(jk)}. \tag{3.12}$$

Interpretation: Initiating or strengthening a relationship with alters with whom ego has more friends in common will increase ego's amount of expected utility to a larger extent than initiating or strengthening a relationship with alters with whom ego has less friends in common.

3.6. Mutuality Within a Relationship

Ego establishes relationships of varying strengths with all alters in the group. These processes are to a large degree a result of needs (main effect of friendship), preferences (effects of similarity and complementarity), opportunities (effect of proximity), and chance. Once ego is engaged with alter in a relationship, ego has, depending on the strength of the relationship, certain expectations regarding the content and quality of the relationship. In general, the closer the relationship the higher the expectations and amount of obligations regarding the relationship. The relationships ego labels as neutral involve hardly any expectations and obligations, except for the usual norms for interaction. Those with whom ego maintains friendly relationships also do not involve many obligations, but once ego categorizes alter as his friend or best friend his (and alter's) expectations rise. One of ego's expectations is equality regarding the rewards but also the costs that come with the relationship. As we discussed previously, this is one of the reasons why ego prefers to establish relationships with alters who possesses similar characteristics. Since ego 'knows' that, depending on the strength, each relationship involves certain obligations, differences between ego's judgement and ego's perception of alter's judgement of the strength the relationship suggest different expectations regarding the amount of obligations involved in the relationship, and consequently, regarding the level of equality within the relationship. As an indicator of (in)equality we use the comparison of ego's and alter's categorization of the relationship. We refer to this effect as reciprocity.

The reciprocity effect means that once ego likes alter he prefers alter to like him as much as he does alter.

$$EU^{i}_{(\mathrm{rec})} = \sum_{j=1,\, j\neq i}^{g} F_{ij} F_{ji} \tag{3.13}$$

Interpretation: The more ego and alter are similar in their perception of the strength of the relationship, the larger the amount of expected utility regarding this relationship.

3.7. Investments

Once the relationship between ego and alter has reached a state of mutuality, ego (and alter) have put efforts, energy, time, and maybe even money in the relationship. In other words, they have invested in their relationship (e.g., Rusbult, 1981). Ego looses the investments he put in the relationship with alter, if he chooses to dissolve the relationship. This effect cannot be expressed straightforwardly as a utility component, because it refers to the value of a change rather than of a situation. Suppose that ego and alter are involved in a mutual relationship. If ego dissolves his relationship with alter (in between time point $time = t$ and $time = t + \Delta$), he looses the investments he made in his relationship with alter. This is formalized as

$$EU^{i}_{(\mathrm{inv})} = \begin{cases} F_{ij(t+\Delta)} - \min\left\{F_{ij(t)}, F_{ji(t)}\right\} & \text{if this is negative} \\ 0 & \text{otherwise} \end{cases} \tag{3.14}$$

Interpretation: Dissolving or weakening a reciprocated relationship with alters will decrease the amount of expected utility, whereas dissolving or weakening a not by alter reciprocated relationship does not lead to a decrease of expected utility.

This model component, that cannot be represented as a term in the utility function, was represented in Snijders and Van Duijn (1997) by a so-called gratification function. It is a negative bonus that ego gets if he one-sidedly dissolves a reciprocated relationship. The minimum in (3.14) is needed because otherwise ego will also loose investments if he 'dissolves' an asymmetric relationship. Figure 2 shows the general idea behind investments.

State relationship: *time*=t	State relationship: *time*=$t+\Delta$ (0,0)	(0,1)
(1,0)	$\alpha_{inv}(F_{ij}(t+\Delta) - \text{MIN}\{F_{ij}(t),F_{ji}(t)\}) = 0$	two transitions, thus not defined
(1,1)	two transitions, thus not defined	$\alpha_{inv}(F_{ij}(t+\Delta) - \text{MIN}\{F_{ij}(t),F_{ji}(t)\}) = -\alpha_{inv}$

Figure 2. Interpretation of the investments parameter.

Figure 2 shows that if ego withdraws his friendship choice, he will loose α_{inv} in case the relationship was mutual, whereas he does not loose utility if the relationship was asymmetric.

The effect of investments in combination with the main effect α_{frd} and the reciprocity effect α_{rec} are summarized in figure 3.

Behavior of ego	State relationship from the point of view of alter: $F_{ji} = 0$	$F_{ji} = 1$
dissolving his relationship with alter	$-\alpha_{frd}$	$-\alpha_{frd} - \alpha_{rec} - \alpha_{inv}$
establishing a relationship with alter	α_{frd}	$\alpha_{frd} + \alpha_{rec}$

Figure 3. Investments and reciprocity combined.

Figure 3 shows directly that on top of a loss of $\alpha_{\text{frd}} + \alpha_{\text{rec}}$ ego gets a negative bonus of α_{inv} if he one-sidedly dissolves his mutual relationship with alter.

3.8. The Total Student Friendship Model

The total utility function (see (3.1)) for ego is defined as

$$EU^i = \alpha_{\text{frd}} EU^i_{(\text{frd})} + \alpha_{\text{mar}} EU^i_{(\text{mar})} + \alpha_{\text{pop}} EU^i_{(\text{pop})} + \alpha_{\text{opp}} EU^i_{(\text{opp})} + \alpha_{\text{sim}} EU^i_{(\text{sim})} + \alpha_{\text{trs}} EU^i_{(\text{trs})} + \alpha_{\text{rec}} EU^i_{(\text{rec})} + \alpha_{\text{inv}} EU^i_{(\text{inv})}. \quad (3.15)$$

Next to the main effect of friendship, the parameters α indicate the relative importance of diminishing marginal returns, indirect popularity, the opportunity structure, similarity, transitivity, reciprocity, and investments, respectively. Since (3.15) contains many terms that are specific effects of friendship (analogous to interactions of F_{ij} with other variables), the main effect parameter α_{frd} does not have a direct clear interpretation (for a more extensive elaboration of the interpretation of the parameters of the friendship model, we refer to Van de Bunt (1999)).

Summarizing, ego can maximize his expected utility by taking several actions, i.e., initiating, maintaining, strengthening or dissolving his affective relationships. In the beginning ego and alters do not know each other. The relationships are valued as zero. Once they learn to know each other, the relationship becomes a neutral one. Once a relationship has reached the neutral state, relationships can become 'troubled' or can grow into a friendly relationship, and ultimately end up in a 'real' friendship. In figure 1 all allowed immediate changes of behavior are given. These actions can take place in continuous time between the observation periods. Figure 1 shows that we do not allow for transitions that skip a phase in the formation of friendship (and vice versa). However, since we collected data at discrete time points, the empirical data may show that ego's evaluation of alter does skip a phase from one observation to another, but this is modeled as if ego made all the transitions needed to come from one type of relationship into another in the unobserved period between the observations. Finally, we assume perfect information, no transaction costs for initiating and maintaining relationships, and furthermore, that ego is immediately aware of changes in the strength of alters' relationships.

Troubled relationships do not receive special attention in this paper, and are treated as the same as neutral relationships.

4. Description of the Data and Results

We collected data among a group of university freshmen who, except for a few existing relationships (acquaintances from a former school), did not know each other at the first measurement ($time = t_0$). The data were collected at 7 time points. The first four time points are three weeks apart, whereas the last three time points are six weeks apart. The original group consisted of 49 students, but due to 'university drop-outs' and after deleting those who did not fill in the questionnaire at four to six times, we ended up with a group of 32 students for whom almost complete data are available.

In this section we present the results. The transition from not knowing each other to knowing each other (regardless of the status of the relationship) we assume to be a function of the opportunity structure, and not so much a result of more complex factors, such as popularity, transitivity, and investments already put in the relationship. Therefore, we use Leenders' (1995, 1996) Markov approach to model this transition. Furthermore, two affective relation transitions are discussed. We use the actor-oriented approach in order to model, first, the transition from at most a neutral to at least a friendly relationship, and second, the transition from at most a friendly relationship to at least a friendship. Both transitions are considered for six transitions in time (between $time = t_i$ and $time = t_{i+1}$ for $i = 1, 2, \ldots, 5$ and from $time = t_1$ to $time = t_6$).

The students were asked to rate their relationships on a six point scale (see Table 1).

Next to the sociometric data, we have access to a large set of individual characteristics, of which we used sex, age, residence, education program, and smoking behavior (see Table 2).[4] We decided to concentrate on sex and age because these have shown to be important variables with respect to the choice of friends. As opportunity component of the utility function, we use smoking behavior, residence, and education program. Smoking behavior because smoking is only allowed in special areas. As a consequence, the 'smokers' have

Table 1. Description of the response categories with regard to social relationships.

Label	Description of the response categories
Best friendship	Persons whom you would call your 'real' friends
Friendship	Persons with whom you have a good relationship, but whom you do not (yet) consider a 'real' friend
Friendly relationship	Persons with whom you regularly have pleasant contact during classes. The contact could grow into a friendship
Neutral relationship	Persons with whom you have not much in common. In case of an accidental meeting the contact is good. The chance of it growing into a friendship is not large
Unknown person	Persons whom you do not know
Troubled relationship	Persons with whom you can't get on very well, and with whom you definitely do not want to start a relationship. There is a certain risk of getting into a conflict

to separate themselves from the 'non-smokers' if they want to smoke (which they often do during coffee and lunch breaks). Thus, contact opportunities differ between actors because of their smoking behavior. We selected the attribute education program because, although they started to study at the same moment in time, three groups follow different courses. During the first months all programs overlap largely, but after a few months, the programs diverge. Especially the 2-year program is quite different from the other two programs. Therefore, this attribute also gives information on the individuals' contact opportunities. We chose for one's hometown or residence because those who do not live in the university town have more opportunities to meet than those who live elsewhere.

Table 2. Individual attributes.

Variable	Categories	Freq.
Sex	1 Male	8
	2 Female	24
Residence	1 University town	24
	2 Elsewhere	8
Education program	1 Regular 4-year program	16
	2 Special 3-year program	10
	3 Special 2-year program	6
Smoking	1 Yes	13
	2 No	19

4.1. Markov Models: The Meeting Process

The getting-to-know-model shows which variables play a significant role during the meeting process. The parameter estimates for the transition from a situation in which ego does not know alter to a situation in which he does know alter are given in Table 3. We discuss the

Table 3. University freshmen: Estimated parameters of the transition from a situation in which ego does not know alter to a situation in which ego at least knows alter (troubled, neutral, or friendly relationship, friendship or best friendship) at six transitions in time (based on a Markov analysis). Standard errors between parentheses. Parameters that are not shown are smaller than twice their standard errors and are fixed at zero, after which the model is estimated again.

	Getting to know each other					
	t_0 to t_1	t_1 to t_2	t_2 to t_3	t_3 to t_4	t_4 to t_5	t_5 to t_6
λ_0	−1.03 (0.07)	−1.00 (0.13)	−1.47 (0.19)	−1.77 (0.28)	−2.78 (0.41)	−1.66 (0.24)
υ_{sex}	0.26 (0.08)	n.s.	n.s.	n.s.	−0.88 (0.33)	n.s.
υ_{smk}	0.28 (0.08)	n.s.	n.s.	0.82 (0.31)	1.04 (0.28)	0.69 (0.23)
υ_{edu}	n.s.	0.31 (0.10)	0.60 (0.18)	1.16 (0.25)	0.88 (0.21)	0.35 (0.16)
μ_0	0.94 (0.09)	0.43 (0.14)	n.s.	n.s.	0.84 (0.30)	n.s.

interpretation of the parameters in more detail following the specification of the final model. In order to stimulate the meeting process, F_{ij} is dichotomized, such that $F_{ij} = 0$ if ego does not know alter and $F_{ij} = 1$ in case ego does know alter. Ego knows alter if he describes his relationship with alter as being a neutral, troubled, or friendly relationship, a friendship, or a best friendship.

The model contains several parameters; λ_0 measures the rate of change if the initial situation is a null relation (in both directions) and μ_0 (comparable with α_{rec} in the actor-oriented models) measures the importance of a reciprocated relationship. It shows that reciprocity plays an important role especially during the initial six weeks. Two further parameters describe the effect of the opportunity structure. This structure is a function of smoking behavior and education program with parameter estimates υ_{smk} (comparable to α_{smk} in the actor-oriented models) and υ_{edu} (comparable to α_{edu} in the actor-oriented models), respectively. For each individual characteristic we define a proximity matrix, so that we can estimate the effect of each characteristic separately. We expect the parameter estimates of the three variables to be positive. The results show that most effects are statistically significant and the signs as expected. So it seems that the chance that people get to know each other can be viewed as a function of the opportunities for contact, regardless of the time period. Whether ego lives in the university town is of no importance.

Both smoking behavior and education program are indicator variables for the amount of contact opportunities between two persons. However, during the initial weeks most students attend the same courses. Within the group with whom ego has the opportunity for contact, easily visible characteristics could play a role in the choice process. Therefore we added two covariates to the final models, namely sex and age. Analogous to the proximity component, we define two similarity matrices so that we can estimate the effect of each characteristic separately. $S_{ij} = 1$ if ego and alter are similar and $S_{ij} = 0$ if they are not. Table 3 shows the final model for each of the six transitions in time. During the transition from $time = t_0$ to $time = t_1$, sex absorbs the effect of education program (which was significant before we added sex to the model). Also regarding the transition from $time = t_4$ to $time = t_5$, sex has a significant effect, whereas age never plays a significant role in the getting to know process.

A positive effect of sex suggests that persons of the same sex get to know each other more often than persons of the opposite sex. We do not present the transition from $time = t_0$ to $time = t_6$. Since during this period almost all students have learned to know each other, it is not possible to estimate the effects, separately.

Interpretation of the parameters (for details we refer to Leenders (1996, 1996a, 1996b)):

Take a closer look at transition $time = t_0$ to $time = t_1$. The final model is

$$\lambda_{0ij} = e^{-1.03+0.94F_{ji}+0.26\times\text{sex}+0.28\times\text{smk}}. \tag{4.1}$$

This can be rewritten as

$$\lambda_{0ij} = e^{-1.03}e^{0.94F_{ji}}e^{0.26\times\text{sex}}e^{0.28\times\text{smk}}. \tag{4.2}$$

The intensity of a change from a non-choice to a choice depends on, first, whether alter j has already chosen ego (F_{ji}),[5] and second, the similarity between ego and alter with respect to gender and smoking behavior. e raised to the powers $0.26 \times S_{ij(\text{sex})}$ and $0.28 \times S_{ij(\text{smk})}$ are factors by which the intensities change when these variables change by one unit. Remember that a value of zero on the variable sex and smoking behavior mean that ego and alter are dissimilar. Consider two smoking women i and j, of whom actor j already chose actor i, then $\lambda_{0ij} = 0.45 \times 2.56 \times 1.30 \times 1.32 = 1.98$. If actor j did not smoke $time = t_0$, then $\lambda_{0ij} = 0.45 \times 2.56 \times 1.30 \times 1 = 1.50$. This shows that smoking behavior is important during the meeting process.

4.2. Actor-Oriented Models: From Neutral to Friendly to Friendship

In this section we present the results of the actor-oriented models for the transition from at most a neutral relationship to at least a friendly relationship and from at most a friendly relationship to a friendship or closer. We immediately show the final models. For a more extensive presentation of the results we refer to Van de Bunt (1999). We briefly describe the models, its parameters, and expectations.

The change rate parameter λ indicates the expected number of changes of relations with another person per time period per person. Since the transition step from a neutral relationship to a friendly relationship or back, is easier to take than the transition step from a friendly relationship to a friendship (or best friendship) or back, we expect λ to be smaller for the latter transition. Since the time periods in between the data collection are not constant over time, we calculate λ per week, simply by dividing the parameter by the length of the actual time period.

The final model represents all effects discussed in Section 3. It contains the friendship parameter α_{frd} that we referred to as the main effect of friendship, but which is difficult to interpret (for details see Snijders and Van Duijn, 1997). The diminishing marginal returns effect α_{mar} we expect to be negative. It represents our expectation that the more affective relationships ego already maintains the lower ego's expected utility of starting a new affective

relationship or intensifying an existing affective relationship. We expect the diminishing marginal returns effect to be smaller for the transition from neutral to friendly than for the transition from friendly to friend, because the relative costs of adding a friendly relationship to ego's existing group of friendly relationships is lower than the relative costs of adding a friend to ego's existing circle of friends.

The opportunity structure is a function of three variables, namely ego's residence, ego's education program, and ego's smoking behavior. We expect the three opportunity parameters (α_{res}, α_{edu}, and α_{smk}) to be positive. This would mean that the more opportunities for contact ego has in order to interact with alter, the higher ego's expected amount of utility regarding that specific relationship. Visible characteristics that we consider are sex and age, We expect the effects to be positive, implying that the more similar ego and alter are with respect to sex or age, the larger the amount of expected utility for ego to start or intensify a relationship with alter. Following among others Heider (1958) and Lazarsfeld and Merton (1954), we expect visible attributes to play a more important role in the transition from neutral to at least a friendly relationship than for the transition from at most a friendly relationship to at least a friendship.

Finally, structural parameters are added to the model. The following effects are taken into account: a popularity effect, a reciprocity effect, an investments effect, and a transitivity effect (instead of balance, see Section 3.5). The aspiration effect α_{pop} shows whether ego prefers to have affective relationships with popular alters (i.e., individuals with a high in-degree) and we expect it to positive. The reciprocity effect α_{rec} represents ego's preference for reciprocated relationships. We expect it to be positive. The investment effect α_{inv} expresses the relative costs of one-sidedly dissolving an existing relationship. We expect ego to take into account the investments put in the relationship, thus expecting a positive investment effect. Finally, we expect ego to strive after transitive relationships. If it occurs as expected, α_{trs} should be positive.

The final model contains parameters that are at least as large as once their standard error. We start with the main effect and we add new features one by one (in the same order as we discussed them in Section 3). If a new effect is significant it remains in the model. As a result of the addition of a new parameter, parameters that were significant until then may become less important. If they are less than once their standard error they are removed from the model. In the next step a new effect is added to the model, and the whole procedure starts all over again. The final model contains, regardless of their signs, all friendship parameters that are at least as large as approximately once their standard error. The model is based on 500 simulation runs. In order to make the random change parameters comparable over time, we present them as rate per week. Standard errors of the estimated parameters are presented between parentheses.

4.2.1. Results: From Neutral to Friendly. Table 4 shows the results regarding the transition from at most a neutral to at least a friendly relationship. In order to see the net effects of the opportunity structure and similarity, we present a model that does not include the structural effects.

Discussion of Table 4: Generally, the estimated rate of change (per week) decreases continuously. During all transitions the main effect is negative. This means that in the long

Table 4. University freshmen: The friendship model containing the marginal returns effect, the opportunity effects and the similarity effects. Estimated parameters of the transition from at most a neutral relationship (unknown, troubled, or neutral relationship) to at least a friendly relationship (friendly relationship, friendship or best friendship) at six transitions in time. Standard errors between parentheses.[6] Significant parameters (at least twice their standard error) are printed boldface.

	Formation of friendly relationships among students						
	t_0 to t_1	t_1 to t_2	t_2 to t_3	t_3 to t_4	t_4 to t_5	t_5 to t_6	t_1 to t_6
λ	1.40 (0.15)	0.92 (0.13)	0.95 (0.14)	0.79 (0.13)	0.49 (0.07)	0.55 (0.08)	0.30 (0.03)
frd	−0.46 (0.85)	**−0.48 (0.21)**	**−0.59 (0.18)**	**−0.81 (0.26)**	**−0.82 (0.18)**	**−0.89 (0.18)**	−0.36 (0.20)
mar	n.e.[a]	**0.22 (0.06)**	0.07 (0.04)	0.03 (0.02)	**0.09 (0.04)**	−0.01 (0.03)	**0.13 (0.04)**
Opportunity structure							
res	**0.62 (0.25)**	0.07 (0.39)	0.44 (0.37)	0.48 (0.42)	0.56 (0.39)	0.35 (0.29)	n.e.[a]
edu	0.07 (0.16)	0.42 (0.22)	0.06 (0.23)	**0.33 (0.23)**	**0.49 (0.21)**	**0.67 (0.21)**	**1.02 (0.18)**
smk	**0.52 (0.22)**	**0.56 (0.28)**	0.34 (0.28)	0.90 (0.51)	**0.95 (0.35)**	**0.86 (0.26)**	**0.81 (0.20)**
Similarity							
sex	**1.10 (0.25)**	0.03 (0.41)	0.44 (0.34)	0.05 (0.35)	0.68 (0.36)	0.07 (0.27)	**0.45 (0.27)**
age	**0.17 (0.05)**	−0.05 (0.06)	**0.17 (0.07)**	0.15 (0.08)	−0.06 (0.08)	0.03 (0.05)	−0.07 (0.05)

[a]The diminishing marginal returns effect and the overall effect of residence are too small and therefore α_{mar} and α_{res} cannot be estimated. In this respect n.e. stands for not estimated.

run ego maintains friendly relationships with less than half of the group. In Section 3.2 we explained the general notion of diminishing marginal returns. In the simulation study itself we used a slightly different version, but the idea remains the same. The diminishing marginal returns component of the utility function is represented as

$$\begin{aligned} EU^i &= \alpha_{\text{frd}} F_{i+} + \alpha_{\text{mar}}\big((F_{i+} - c)^2 - F_{i+}\big) \\ &= (\alpha_{\text{frd}} - \alpha_{\text{mar}}) F_{i+} + \alpha_{\text{mar}}(F_{i+} - c)^2 \\ &= (\alpha_{\text{frd}} - (2c+1)\alpha_{\text{mar}}) F_{i+} + \alpha_{\text{mar}}(F_{i+})^2 + c^2\alpha_{\text{mar}}. \end{aligned} \tag{4.3}$$

In (4.3) c is a constant, that we fixed to five. This constant c is necessary because otherwise the effect of $(\alpha_{\text{frd}} - \alpha_{\text{mar}})F_{i+}$ and $\alpha_{\text{mar}}(F_{i+})^2$ are rather strongly collinear which leads to problems in the estimation algorithm. It has hardly implications for the size of the parameters. With $c = 5$ (4.3) leads to

$$EU^i = (\alpha_{\text{frd}} - 9\alpha_{\text{mar}}) F_{i+} + \alpha_{\text{mar}}(F_{i+})^2 + 25\alpha_{\text{mar}}. \tag{4.4}$$

We expected α_{frd} and α_{mar} to be negative. What we found instead is that in all models α_{mar} appears to be positive. What does this mean? Literally, the shape tells us that ego prefers to have either a very small number of friendly relationships or lots of friendly relationships. This is not what we expected. It is related to an outdegree variance that is larger than expected on the basis of randomness. This would suggest that the diminishing marginal returns component of the utility function could be a wrong specification in the sense that

there is another, stronger, effect which is omitted in the present model: variation in social activity between individuals. It could also express that people have different notions or thresholds when referring to someone as a friend. This will be a subject of future research.

The parameters α_{edu}, α_{smk}, and α_{res} show the effect of the opportunity structure. It shows that the opportunity structure is rather important during the whole period: smoking behavior during the whole period, residence during the initial period, and education program mainly during the final periods. The latter observation is as expected, because during the initial periods all students have the same education program. If we take a look at the last column, we see that over the whole period one's residence is not important, but smoking behavior and education program are. Smokers are more likely to be engaged in friendly relationships with smokers, while non-smokers have a preference for non-smokers. The same holds for education program. Those who take part in the same education program are more often involved in friendly relationships with each other than with participants of another education program.

With respect to ego's preference for similar alters we see that sex and age are not very important, although almost all signs are as expected. At some transitions in time people more often have friendly relationships with persons of the same sex and age. Over the whole period, we find a significant effect for sex but not for age.

Table 4 shows that most effects are statistically significant at many transitions in time. For instance, the effect of smoking behavior is significant at all transitions in time except from $time = t_2$ to $time = t_3$ and from $time = t_3$ to $time = t_4$. This could be a result of pure chance, or better an error of the second kind. The other way around is also possible. Several effects are statistically significant at only one or two transitions in time. A test statistic that combines the results of all transitions in time is

$$\frac{\sum_{t=1}^{T} \hat{\alpha}_t}{\sqrt{\sum_{t=1}^{T} se_t^2}}, \tag{4.5}$$

in which t stands for the transition in time. This is a t-statistic that can be tested in the standard normal distribution assuming the number of degrees of freedom is large. The t-values for residence, education program, and smoking behavior are approximately 2.89, 3.68, and 5.10, respectively. We can conclude that the opportunity structure is a significant factor during the process of friendship formation. For sex and age the t-values are 2.78 and 2.56, so also similarity with respect to sex and age are significant factors during the process.

Table 5 shows the results of the final model, which also includes structural effects.

Discussion of Table 5: The structural effects show that especially the effects of reciprocity and transitivity are of great importance. During all transitions in time the effect of reciprocity is positive and at most times also statistically significant. This means that people generally strive after reciprocal relationships. It also implies a tendency to abandon asymmetric relationships. It is not clear from the results, however, whether people withdraw their choices from a not-reciprocated relationship, or whether people answer positively to a friendship choice. The positive and often statistically significant transitivity effects show that throughout the whole year people prefer transitive relationships. Also over the whole

Table 5. University freshmen: The final friendship model. Estimated parameters of the transition from at most a neutral relationship (unknown, troubled, or neutral relationship) to at least a friendly relationship (friendly relationship, friendship or best friendship) at six transitions in time. Standard errors between parentheses. Significant parameters (at least twice their standard error) are printed boldface. Parameters that are not shown are smaller than once their standard errors and are fixed at zero, after which the model is estimated again.

Formation of friendly relationships among students							
	t_0 to t_1	t_1 to t_2	t_2 to t_3	t_3 to t_4	t_4 to t_5	t_5 to t_6	t_1 to t_6
λ	1.50 (0.17)	1.03 (0.15)	1.27 (0.21)	0.95 (0.17)	0.52 (0.48)	0.66 (0.10)	0.36 (0.03)
frd	**−1.70 (0.59)**	**−1.82 (0.25)**	**−1.39 (0.21)**	**−2.31 (0.43)**	**−1.81 (0.35)**	**−1.79 (0.45)**	**−1.42 (0.32)**
mar	n.s.	**0.16 (0.07)**	0.09 (0.05)	n.s.	n.s.	n.s.	0.07 (0.06)
res	n.s.	n.s.	−0.30 (0.28)	n.s.	n.s.	n.s.	n.s.
edu	0.43 (0.34)	0.37 (0.22)	n.s.	0.39 (0.24)	**0.46 (0.21)**	**0.66 (0.17)**	**0.73 (0.16)**
smk	0.57 (0.31)	0.48 (0.36)	0.28 (0.24)	0.97 (0.53)	0.59 (0.32)	**0.75 (0.30)**	**0.76 (0.25)**
sex	0.78 (0.42)	n.s.	n.s.	n.s.	n.s.	n.s.	0.19 (0.28)
age	0.11 (0.08)	n.s.	**0.13 (0.05)**	n.s.	n.s.	n.s.	n.s.
pop	n.s.	n.s.	n.s.	n.s.	n.s.	1.44 (1.12)	n.s.
rec	**2.79 (0.79)**	**1.59 (0.40)**	0.88 (0.61)	n.s.	**1.07 (0.34)**	0.86 (0.86)	0.69 (0.56)
inv	n.s.	n.s.	**4.05 (1.31)**	**5.59 (1.48)**	n.s.	**3.42 (1.27)**	**3.12 (0.82)**
trs	**2.10 (1.04)**	**0.75 (0.18)**	n.s.	**0.80 (0.21)**	**0.48 (0.20)**	n.s.	**0.45 (0.16)**

period it is still a strong effect. The effect of investments put in the relationship can only be found once reciprocal relationships have been formed. It is significant during three transitions in time, and over the whole period. Popularity plays a minor role. Except for $time = t_5$ to $time = t_6$ we did not observe it. The effects of sex and age are somewhat reduced, but smoking behavior and education program remain significant.

On the whole we can say that the formation of friendly relationships is strongly influenced by the opportunity structure. Furthermore, people prefer reciprocated transitive relationships, and once they are engaged in a reciprocated relationship they are reluctant to dissolve one-sidedly the relationship. Finally, we find a weak preference for same-sex relationships.

4.2.2. Results: From Friendly to Friendship. Table 6 shows the results regarding the transition from at most a friendly relationship to at least a friendship. As expected there are not as many friendships as there are friendly relationships. This has an impact on the analyses. It is not possible to put all effects in one model and estimate them at the same time. Compared to the number of friendships we would have to deal with too many parameters. Therefore we do not make the distinction between a model that only includes the effects of the opportunity structure and similarity, and the final model that also includes structural effects. This also means that we cannot calculate the t-values. We immediately discuss the final model that is presented in Table 6.

Discussion of Table 6: In contrast with the transition from a neutral relationship to at least a friendly relationship, the estimated rate of change per time period with respect to the

Table 6. University freshmen: The final friendship model. Estimated parameters of the transition from at most a friendly relationship (unknown, troubled, neutral, or friendly relationship) to at least a friendship (friendship or best friendship) at six transitions in time. Standard errors between parentheses. Significant parameters (at least twice their standard error) are printed boldface. Parameters that are not shown are smaller than once their standard errors and are fixed at zero, after which the model is estimated again.

Formation of friendships among students							
	t_0 to t_1	t_1 to t_2	t_2 to t_3	t_3 to t_4	t_4 to t_5	t_5 to t_6	t_1 to t_6
λ	0.22 (0.05)	0.39 (0.07)	0.21 (0.06)	0.11 (0.04)	0.31 (0.07)	0.11 (0.04)	0.13 (0.02)
frd	n.s.	−0.79 (0.59)	0.22 (0.72)	**−2.78 (1.08)**	**−1.32 (0.43)**	−4.33 (2.66)	**−1.76 (0.70)**
mar	n.s.	n.s.	n.s.	n.s.	**0.19 (0.07)**	n.s.	n.s.
res	0.63 (0.56)	n.s.	n.s.	−2.78 (1.80)	n.s.	n.s.	n.s.
edu	n.s.	n.s.	n.s.	n.s.	0.56 (0.34)	1.71 (0.89)	**0.66 (0.26)**
smk	n.s.	n.s.	**1.53 (0.54)**	**3.80 (1.90)**	1.34 (0.75)	2.08 (1.50)	**1.42 (0.66)**
sex	**2.21 (0.99)**	**1.69 (0.53)**	n.s.	n.s.	n.s.	n.s.	**1.29 (0.65)**
age	0.30 (0.17)	**0.21 (0.09)**	**0.38 (0.16)**	n.s.	n.s.	n.s.	n.s.
rec	n.s.	n.s.	n.s.	n.s.	**2.21 (0.63)**	4.18 (2.52)	**4.00 (1.83)**

transition from a friendly relationship to a friendship does not really decrease throughout the year. This suggests that establishing or dissolving a friendship is not a monotonic function of the time the group is together, although changes occur on a somewhat higher rate during the initial period. The estimated rate of change is remarkably lower, however, than that of the transition from a neutral to at least a friendly relationship. This implies that the latter step or back is easier to take than the step from a friendly relationship to a friendship or back. The main effect of friendship is negative during all transitions. This is the same as we found in Tables 4 and 5. This hints at the fact that people are reluctant to call someone a friend.

The effect of the opportunity structure is in line with our expectations, although the parameter estimates are not always statistically significant. What is remarkable, however, is that the opportunity structure is of greater importance after approximately six weeks, whereas the effects of sex and age are more prominent in the initial period. For education program this is understandable, because during the initial periods all students are attending the same courses. The significance of sex and age during the initial weeks suggest that those who make friends very quickly rely to a large degree on visible characteristics, whereas those who establish friendships later depend more strongly on the opportunities for interaction. Over the whole period we see that, except for residence, the opportunity structure and preference for same sex friendships are essential in the process of friendship formation. There are hardly any structural effects, however. Reciprocity plays a role from $time = t_4$ and further and over the whole period. This suggests that in the beginning ego makes friendship choices although he is not (yet) chosen by alters. After a period of asymmetry, either alter decides to accept ego's friendship choice or ego withdraws his initial choice.

We did not find transitivity effects. This suggests that friends of freinds are not always friends. Perhaps it is sufficient that friends of ego only have friendly relationships. We also did not find an investments and a popularity effect. This means that during the year of research there dissolved hardly any friendships one-sidedly by either ego or alter. It

could be that friendships dissolved two-sidedly, but this cannot be shown by means of the investments effect.

We can conclude that the opportunity structure is by far the most important factor during the process of friendship formation. The only variable with respect to similarity that is significant over the whole period is sex. We did not find strong effects of age. However, the direction of almost all effects that we did find were as expected.

There are several possible explanations for the relative paucity of strong significant effects. First, we may have looked at the wrong variables. Second, we misspecified parts of the model. Third, the total number of respondents and/or number of friendships was small so that statistical power was low. Fourth, we have to do with a very peculiar set of students, but we have no reason to believe that this is the case. Fifth, our operationalization of friendship was not optimal. Perhaps it is better to use a social activity scale instead. Sixth, the effect of structure on the process of friendship formation is not as profound as is always thought. Since we did find structural effects explaining the transition from a neutral to a friendly relationship, it could well be that structural effects are less important during the transition from a friendly relationship to a friendship. This is in contradiction to what it is proclaimed in the literature. Concluding, it looks as if most of the structural mechanisms that are put forward in the literature with respect to the process of friendship formation are, at least in this specific group, more applicable to the transition from a neutral to at least a friendly relationship and vice versa, than to the transition from a friendly relationship to a friendship and vice versa. Factors that could have played a role in the process of friendship formation but have been left out in the analyses are the size of one's network outside this specific group of students, one's engagement in social activities, one's time budget, and attributes such as satisfaction with one's study, study results, interests, and personal characteristics.

Notes

1. We stress that it is our goal to provide a rough sketch. What is actually going on in the minds of people is not an issue here.
2. In the analyses we have to make the assumption that ego's perception of alter's relationships and attributes equals alter's perception.
3. We are working on these problems, and it seems that they are almost solved.
4. Age is measured on an interval scale and is not presented in Table 2. At the start of the research project, the youngest student is 18 years old while the oldest is 29.
5. In the original notation of Leenders (1995, 1996), F_{ji} is replaced by a_{ji}.
6. For readability of the tables we shorten our notation of the parameters of the actor-oriented models by leaving out α. We present only the substantive abbreviation of the parameter. In the text, however, we use the original notation. For example, in the tables α_{frd} is presented as frd, whereas in the text we use the usual α_{frd}.

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