Step tolling with price sensitive demand: Why more steps in the toll makes the consumer better off

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Abstract

   Most dynamic models of congestion pricing use fully time-variant tolls. However, in practice, tolls are uniform over the day or at most have a few steps. Such uniform and step tolls have received surprisingly little attention from the literature. Moreover, most models that do study them assume that demand is insensitive to price. This seems an empirically questionable assumption that, as this paper finds, strongly affects the implications of step tolling for the consumer. First-best tolling has no effect on the generalised price, and thus leaves the consumer equally well off as without tolling. Conversely, *under price-sensitive demand*, step tolling increases the price and lowers the number of users, making consumers worse off. The more steps the step toll has, the closer it approximates the first-best toll, thereby increasing the welfare gain and making consumers better off. This makes it important for real-world tolls to have as many steps as possible: this not only raises welfare, but also increases the political acceptability of the scheme by making consumers better off.

Key words: Congestion pricing, step tolls, bottleneck model, price sensitive demand, consumer surplus, political acceptability
JEL codes: D62, R41, R48

1. Introduction

Theoretical models of dynamic road congestion pricing generally use a fully time-variant toll. However, in practice, there are no such tolls. In practice, tolls are constant over the day or at most have a few steps in them. For example, the Oslo toll ring has a uniform toll that is constant over the day (Odeck and Bråthen, 1997). The London scheme has a uniform toll that is constant between 7:00 and 18:00.[1] In contrast, Singapore uses step tolls: the toll is at its lowest level in the early morning, and then increases in steps up to its highest level during the centre of the morning peak; thereafter, it decreases again in steps (see Fig. 1 in Section 2 for an example step toll). For the evening peak a similar pattern holds, but this paper will ignore the evening peak. The "Bugis-Marina Centre (Nicoll Highway)" in Singapore has 7 steps in the toll during the weekday morning peak.[2] The Stockholm pricing scheme has 5 steps in the

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[1] This follows www.tfl.gov.uk/tfl/roadusers/congestioncharge/whereandwhen/ as retrieved on 18 January 2010

[2] Rates for 3 May to 2010 to 1 August 2010, as retrieved on 11 May 2010 from
  www.onemotoring.com.sg/publish/onemotoring/en/on_the_roads/ERP_Rates.html

morning.[3] But step tolls *are also used* in the USA, for example on SR-91 and San Francisco-Oakland Bay Bridge in California and the SR-520 and SR 16 Tacoma Narrows bridges.[4] Such uniform and step tolls have received surprisingly little attention in the literature. Moreover, models of step tolls generally assume that demand is fixed and thus insensitive to price.[5] This seems an implausible assumption, as empirical research shows that transport demand does vary with the generalised travel price (or price for brevity). For a review of these price elasticities see, for example, Brons, Nijkamp, Pels and Rietveld (2002) and Graham and Glaiser (2004).

In the bottleneck model, first-best pricing changes the departure rate of drivers (i.e. it changes behaviour), thereby halving marginal social cost and generalised user cost (or user cost for brevity) for a given number of users. For social optimum, marginal social cost should equal demand. Due to the halving of marginal social cost, this occurs when the number of users in the first-best equilibrium is the same as in the no-toll equilibrium. Consequently, the price and consumer surplus remain the same as before the toll. A uniform toll is constant throughout the peak and causes no change in the departure rate. It can only limit congestion cost by reducing demand. The optimal uniform toll equals marginal external cost (i.e. marginal social cost minus user cost) (see also Arnott, de Palma and Lindsey, 1993). Uniform tolling raises the price and lowers the number of users and consumer surplus; this scheme is thus comparable to tolling in the textbook static-congestion model, where tolling is also harmful for the consumer and has a much lower gain than the first-best bottleneck toll.

Step tolling is in between uniform and first-best tolling: it somewhat changes the departure pattern, but also raises the price. This makes it important to control for price sensitivity of demand when considering step tolling. The more steps there are, the more marginal social cost is reduced, the lower the price is, and the higher consumer surplus is. As the number of steps goes to infinity, the step toll generally approaches the first-best toll, and the consumer becomes equally well off as without tolling. This conclusion has an important policy implication: it is essential to give a toll as many steps as possible, as this not only raises welfare, but also increases the acceptability of congestion pricing by making it less harmful for the consumer.

This paper investigates step tolling in three different models that use bottleneck congestion. The first model is the ADL model following Arnott, de Palma and Lindsey (1990, 1993); the second model is the Laih model of Laih (1994, 2004); and the third model is the Braking model of Lindsey, Van den Berg and Verhoef (2010). In the Laih model, an m-step toll lowers total cost by a fraction $\frac{1}{2} \cdot m/(1+m)$: so with a single step, the reduction is $\frac{1}{4}$ (or half

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[3] http://en.wikipedia.org/wiki/Stockholm_congestion_tax as retrieved on 30 May 2011.

[4] Respectively www.octa.net/91_schedules.aspx, bata.mtc.ca.gov/tolls/schedule.htm, and www.wsdot.wa.gov/Tolling/TollRates.htm as retrieved on 22 January 2012.

[5] Interesting exceptions are Arnott et al. (1993),Chu (1999) and Ge and Stewart (2010).Here, the latter two have fixed overall demand. But Chu (1999) has a logit distribution of users over driving alone, carpool and bus, making the number of (effective) vehicles dependent on the toll; and in Ge and Stewart (2010) only two of the three routes are tolled, making the number of toll-route users dependant on the tolls.

that of the first-best toll); with two steps, it is 1/3; and as the number of step goes to infinity, the toll approaches the first-best toll (Laih, 2004). In the ADL model, the gain is larger for a finite number of steps, while the toll also approaches the first-best toll as *m* goes to infinity. The Braking model takes into account that drivers have an incentive to wait passing the tolling point just before the toll is lowered, as this substantially lowers the toll they pay while only marginally increasing travel time and schedule delay. A consequence of this is that the bottleneck capacity will go unused for some time during the peak, and this inefficiency raises total costs. The inefficiency only becomes larger as the number of steps increases, and thus the Braking toll never approaches the first-best toll and always has a lower gain. The other two models are only stable if the government can prevent the braking (Lindsey et al., 2010). Note that this braking behaviour has been observed in practice in Singapore with the area licence scheme up to 1994 (Png, Olszewski and Menon, 1994) and with the electronic road pricing from 1994 (Chew, 2008), at the San Francisco-Oakland Bay Bridge (Lee and Frick, 2011), and in Stockholm (Fosgerau, 2011).

The next section starts with a general model of step tolling for any model of dynamic congestion. Then, Section 3 turns to the bottleneck model and discusses the equilibria without tolling, with first-best tolling, and with step tolling in the Laih, ADL, and braking model. Section 4 presents numerical examples to illustrate the effect of price sensitivity. Finally, Section 5 concludes.

2. General step toll model

This section derives the optimal level of the time invariant part of the toll for a congestion model and a form of the step part of the toll. The solution thus assumes that there is a formula for the time-variant part, and is based on the results of Arnott et al. (1993) on a uniform toll and a coarse toll (i.e. a single-step toll). The toll τ for arrival time t consists of the time-invariant part θ and a time-variant step part ρ^i (where the level of the step part depends on t):

$$\tau[t] = \rho_i + \theta. \tag{1}$$

The i indicates the ith toll level. As Fig. 1 shows, the central toll (ρ^1) is around the preferred arrival time (t^*) and is indicated by 1. The further out a step part is, the higher its indicator and the lower its level. I allow the levels of the ith early (before t^*) and the ith late toll (after t^*) to be different, but the number of early and late tolls is the same. I indicate an early step part by superscript $^+$ and a late step part by $^-$. The ith early part starts at   and ends at   ; the times for the ith late part are t_{i+1}^- and t_i^-. This is not a restrictive assumption, as this turns out to be optimal in all models. With step tolling, the peak starts at t_s' and ends at t_e'. These times are generally different than without tolling (t_s and t_e), as the numbers of users differ.
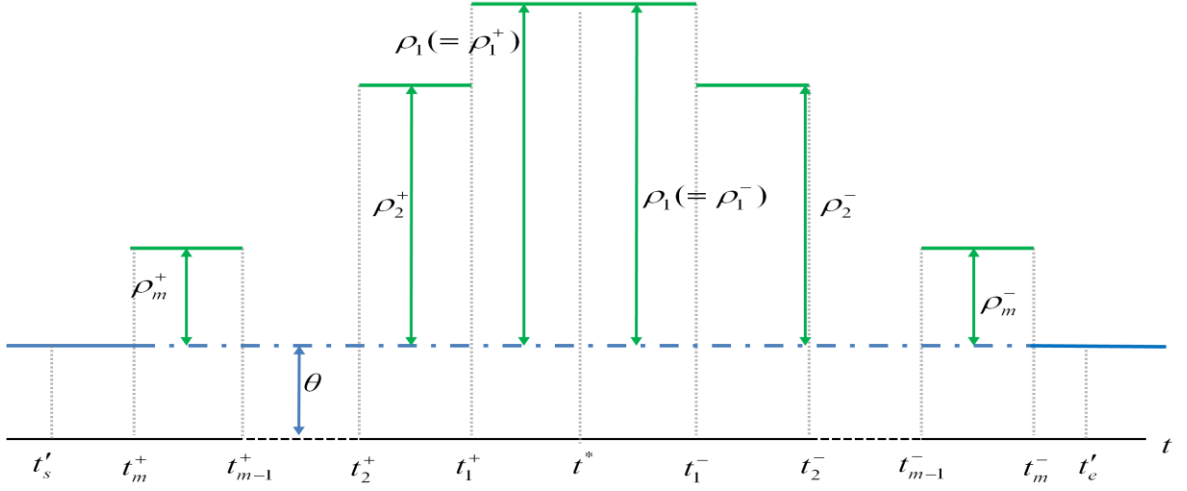
Figure 1: Notation in the multi-step toll models

Welfare equals the integral of (inverse) demand D minus Total Costs (TC), which equals average user cost ($E[c]$) multiplied by the number of users (N):

$$W = \int_0^N D[n]dn - TC = \int_0^N D[n]dn - E[c] \cdot N \qquad (2)$$

To find the optimal time-invariant part (θ) for a given pattern of the step toll, the following Lagrangian is maximised:

$$L = W + \lambda\left(D - E[c] - E[\rho] - \theta\right) = \int_0^N D[n]dn - E[c] \cdot N + \lambda\left(D - E[c] - E[\rho] - \theta\right), \qquad (3)$$

where $E[\cdot]$ indicates an average.[6] I use this operator as user cost and step toll vary over time. The first order conditions are:

$$\partial L / \partial N = 0 = D - E[c] - N \cdot \partial E[c] / \partial N + \lambda\left(\partial D / \partial N - \partial E[c] / \partial N - \partial E[\rho] / \partial N\right), \qquad (4a)$$

$$\partial L / \partial \theta = 0 = \lambda, \qquad (4b)$$

$$\partial L / \partial \lambda = 0 = D - E[c] - E[\rho] - \theta. \qquad (4c)$$

These equations imply:

$$\theta = MSC - E[c] - E[\rho] = E[MEC] - E[\rho] = N \cdot \partial E[c] / \partial N - E[\rho]. \qquad (5)$$

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[6] This solution assumes that the dynamic congestion model has a reduced form that only depends on the total number of users: so, just as in the bottleneck model, there is a formula for costs and step part of the toll at time t as a function of the total number of users. Further, the system is in user equilibrium, so that the price is constant throughout the peak. This allows me to use Lagrangian optimisation instead of optimal control theory which is difficult to use in the context of the bottleneck model (see Yang and Huang, 1997). Because of the assumption that the price is constant over time, one only needs the single constraint that states that the price for the average user ($E[c]+E[\rho]+\theta$) should equal inverse demand (D).

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The *MSC* is marginal social cost, which is the derivative of total cost to the number of users. The *MEC* is marginal external cost, which at *t* equals the difference between *MSC* and user cost *c*[*t*]. Just as Arnott et al. (1993) showed for a single-step toll, in general, the *θ* is set such that the price equals the average *MSC* (or alternatively the average toll equals the average marginal externality), and accordingly the average user internalise her external cost.

Since a uniform toll is a zero step toll, the above discussion implies that this toll should equal the average externality. Step tolling changes the equilibrium departure pattern and lowers the externality and price for a given number of users. But again the average toll, E[*τ*], equals the average marginal externality.

3. The bottleneck model

3.1. No-Toll (NT) equilibrium

I keep the discussion of the no-toll (NT) and first-best (FB) equilibria brief as these are extensively discussed in, for example, Arnott, de Palma and Lindsey (1990, 1993), as well as in textbooks such as Small and Verhoef (2007). The *N* identical users travel alone by car from the origin to the destination, which are connected by a road that is subject to bottleneck queuing congestion. Free-flow travel time is normalised to zero. Without a queue, an user thus departs from the origin, passes the bottleneck, and arrives at the destination all at the same moment. User cost for an arrival at *t* is the sum of travel delay (c^{TD}) and schedule delay costs (c^{SD}) from arriving at a different time than the common preferred arrival time (t^*):

$$c[t] = c^{TD} + c^{SD} = \alpha \cdot T[t] + \beta \cdot \text{Max}\left[0, t^* - t\right] + \gamma \cdot \text{Max}\left[0, t - t^*\right]. \tag{6}$$

The *α* is value of travel time or the unit cost of an hour of travel delay, *β* is the value of schedule delay early (i.e. the cost of arriving an hour before t^*), and *γ* is the value of schedule delay late.

The peak starts at t_s and ends at t_s. At these moments travel delay is zero while schedule delay cost is at its highest:

$$t_s = t^* - \frac{\gamma}{\beta + \gamma} \cdot \frac{N}{s}, \tag{7a}$$

$$t_e = t^* + \frac{\beta}{\beta + \gamma} \cdot \frac{N}{s}. \tag{7b}$$

In these equations, *s* is the capacity of the bottleneck. Equilibrium user cost is given in (8). Since the toll is zero, the generalised price P^{NT} equals c^{NT}. Total costs in (9) are $c^{NT} \cdot N$. Half of this total is travel delay cost while the other half is schedule delay cost. Marginal social costs are twice user cost, which makes marginal external cost (*MEC*) in (10) equal user cost. Here, superscript NT indicates the No-Toll equilibrium. The preference parameter *δ* is used to shorten the algebra and equals (*β·γ*)/(*β+γ*).

$$c^{NT} = p^{NT} = \delta \cdot N / s \tag{8}$$

$$TC^{NT} = c^{NT} \cdot N = \delta \cdot N^2 / s \tag{9}$$

$$MEC^{NT} = \delta \cdot N / s \tag{10}$$

For the interpretation of step tolling it is instructive to adapt the textbook demand and cost diagram for a static-flow-congestion model to the bottleneck model. Fig. 2 gives such a diagram for the no-toll equilibrium. I can use what is basically a static diagram, because reduced-form costs in the bottleneck model only depend on the total number of users. User equilibrium is found where demand equals user cost. But at this point the *MSC* is above the equilibrium price as part of the social cost is external to the consumer. And it are these marginal external costs that we would like the user to take into account by setting a congestion toll.
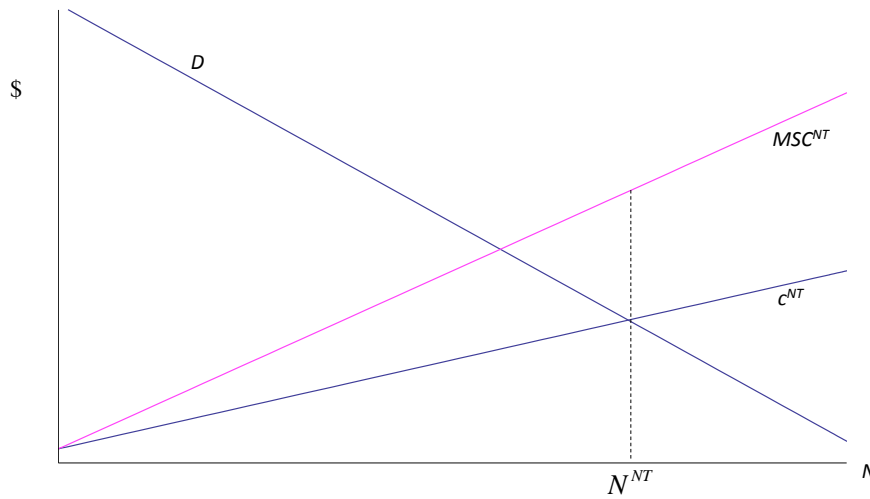

Figure 2: No-toll equilibrium

3.2. First-Best (FB) equilibrium

Travel delays are a pure deadweight loss: drivers could arrive at the same arrival times (hence having the same schedule delays) but with zero travel delays, if their rate of arrival at the bottleneck would equal capacity. Moreover, this would also leave t_s and t_e unchanged, and thus the equilibrium price remains equal to $-\beta \cdot t_s = \delta \cdot N / s$. The first-best toll, $\tau^{FB}[t]$, that achieves this equals, at each arrival time (t), the travel delay cost at t in the NT case:

$$\tau^{FB}[t] = \delta \cdot N / s - \beta \cdot \text{Max}\left[0, t^* - t\right] - \gamma \cdot \text{Max}\left[0, t - t^*\right]$$

This FB toll halves total cost, average Marginal External Cost (E[*MEC*]), and average user cost (E[*c*]) as the travel delays are converted into toll payments; while the price is unaffected:

$$E[c^{FB}] = \tfrac{1}{2}\,\delta \cdot N / s, \tag{11}$$

$$P^{FB} = \delta \cdot N / s, \tag{12}$$

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$$\text{E}[MEC^{FB}] = \tfrac{1}{2}\,\delta{\cdot}N/s, \tag{13}$$

$$TC^{FB} = c{\cdot}N = \tfrac{1}{2}\,\delta{\cdot}N^2/s. \tag{14}$$

Since the price is unaffected, total number of users and consumer surplus remain the same. Welfare increases since half of the total costs are converted into toll revenue. At each point in time, the toll equals the *MEC*, and thus is the externality fully internalised.

The equilibrium in Fig. 2 for the no-toll case was basically the same as in a static model. But Fig. 3 for first-best tolling is different as *MSC* and user cost (*c*) are halved. As the MSC is twice user cost, the *MSC* curve tilts down to the level of the NT user-cost curve, while user cost tilts down to half of its NT level. This also explains why FB tolling does not change the number of users: at $N^{NT}=N^{FB}$, the new *MSC* equals demand, which is what is needed for optimum. The price and the *MSC* are constant over time, but the toll and user cost are not; this is why, in Fig. 3, the cost and toll are labelled with the Expectance operator (E[·]).
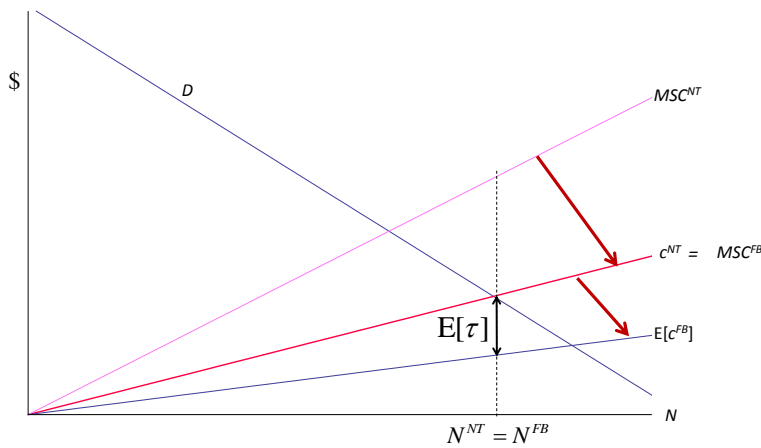

Figure 3: First-best equilibrium

3.3 Uniform toll

The uniform toll does not affect the departure rate. It can only limit congestion cost by reducing demand. Hence, the formulas for marginal social cost and user cost remain the same as in the NT case. In Fig. 4, this implies that there is no downward tilt of the curves. To ensure that marginal social cost equal demand, the number of users must be reduced from N^{NT} to N^{U} (superscript ^{U} indicates the Uniform toll). This is done by setting a time-invariant toll equal to *MEC=MSC−c*, just as in the textbook static model. Just as in the static model, uniform tolling lowers the number of users and consumer surplus.

As Fig. 5 shows, the uniform toll is generally higher than the average first-best toll because the marginal external cost is higher for a given number of users. The *MEC* in the uniform case is $\delta{\cdot}N^{U}/s$; in the first-best case it is on average $\tfrac{1}{2}{\cdot}\delta{\cdot}N^{FB}/s$. So the *MEC* is higher with a uniform toll as long as N^{FB} is not more than twice N^{U}. As Fig. 4 indicates, uniform tolling raises the price as long as demand is not perfectly elastic (i.e. a flat curve *D*) or perfectly inelastic (i.e. fixed). Hence, under these conditions, uniform tolling lowers consumer surplus, even though it does raise welfare.
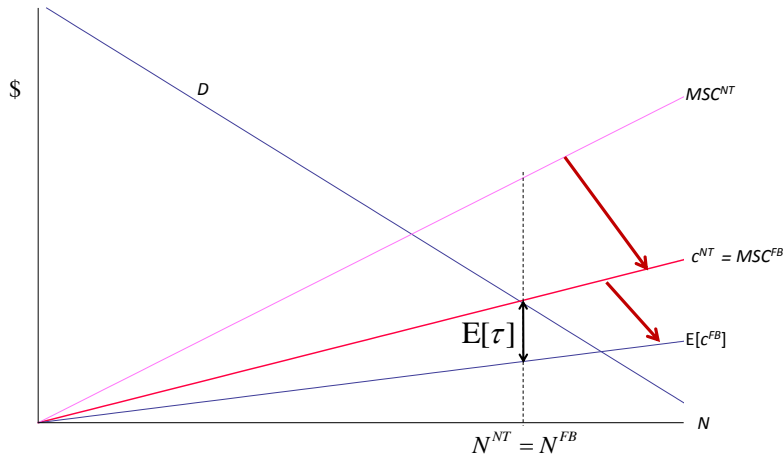
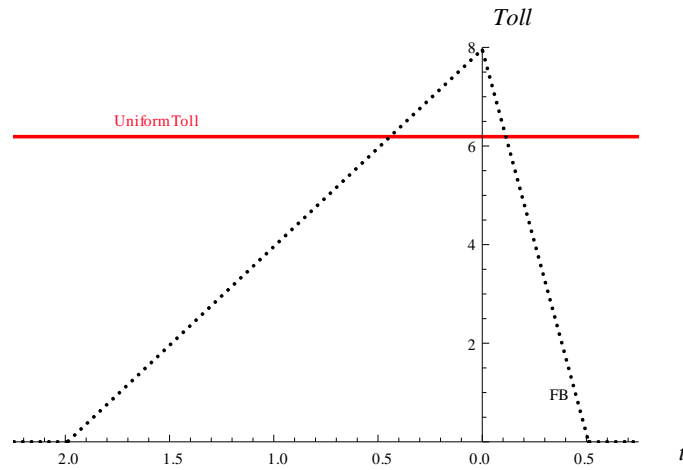*Figure 4: Equilibrium with the uniform toll*

Figure 5: Level of the uniform toll

3.4. Laih step toll

The Laih (1994; 2004) model is the simplest step-toll model for the bottleneck model to solve; the Braking and ADL model are more tedious. The models differ in how they achieve that prices before and after a toll decrease are equal; before t^*, when the toll only increases, the three models have the same set-up. In the Laih model, there are separate queues for drivers who pass the tolling point before and after a toll decrease at t_i^-, where the users who arrive after t_i^- start waiting in front of the tolling point before t_i^- In the ADL model, there are no separate queues; instead a mass departure at t_i^- equalises expected prices before and after t_i^-. In the Braking model, there is a single queue and no mass departure, instead users start waiting to pass the tolling point well before the toll is lowered to lessen the toll they pay.

The Laih model is easiest to solve because the step part of the toll does not change the arrival window and price. If the number of users were the same, at each t, the sum of user cost, $c[t]$, and step part of the toll, ρ^i, would equal the price without tolling. But as the optimal time-invariant part is positive, Laih tolling increases the price.

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Following Laih (1994, 2004), the start and end time of the peak follow the same formulas as those for the NT and FB equilibria in (7a,b):

$$t_s' = t^* - \frac{\gamma}{\beta + \gamma} \cdot \frac{N}{s}, \tag{15a}$$

$$t_e' = t^* + \frac{\beta}{\beta + \gamma} \cdot \frac{N}{s}. \tag{15b}$$

But total costs are lower for given N, as part of the travel time costs are converted into toll payments, which are not costs but transfers from the consumer to the government. Total costs and total toll revenue are

$$TC = \delta \frac{N^2}{s} \left(1 - \frac{1}{2} \frac{m}{1+m} \right), \tag{16}$$

$$TR = TR^{step} + TR^{fixed} = \frac{1}{2} \frac{m}{1+m} \delta \frac{N^2}{s} + \theta \cdot N. \tag{17}$$

Here, superscript ^{s} indicates the new step-tolling equilibrium. In optimum, the step parts of the toll are symmetric in the Laih model: i.e. the ith early and late toll are equal and $\rho_i^+ = \rho_i^-$. The step part of the toll follows

$$\rho_1 = \frac{m}{m+1} \cdot \delta \cdot \frac{N}{s}, $$

$$\rho_i = \rho_i^+ = \rho_i^- = \frac{m+1-i}{m} \cdot \rho_1, \quad i = 2,...,m. \tag{18}$$

The step part has a simple pattern: the second ρ_2 is a fraction $(m-1)/m$ of the central ρ_1 that is around t^*, the third toll a fraction $(m-2)/m$. This pattern is this same as with a fixed number of users, since this minimises total cost for given N. Conversely, the θ is set to optimise N so that demand equals the *MSC* for a given cost structure.

The average step part of the toll, $E[\rho]$, equals TR^{step}/N. Average marginal external cost is

$$E[MEC] = \delta \frac{N}{s} \left(1 - \frac{m}{2+2m} \right); \tag{19}$$

where the externality decreases with the number of steps, just as marginal social cost and user cost do. For given N, a single step toll reduces the average MEC by a fraction 1/4, with 2 step, this is 1/3; and as m goes to infinity, the MEC approaches the first-best MEC of $\frac{1}{2} \cdot \delta \cdot N/s$. Using that the average step toll should equal the average MEC, the optimal time-invariant part of the toll must be

$$\theta = E[MEC] - E[\rho] = \delta \frac{N}{s} \frac{m}{1+m}. \tag{20}$$

This θ approaches zero as the number of steps m approaches infinity, as then the step part of the toll approaches the first-best toll that at each t equals the *MEC*.

The price at the start of the peak at t_s' is the sum of scheduling cost, $\beta\left(t_s' - t^*\right) = \delta N / s$ and the time-invariant part of the toll θ; the time-variant toll and travel time are zero. For other used arrival times the price is the same, but travel time and toll are generally non-zero:

$$P = MSC = E[c] + E[\rho] + \theta = \delta \cdot \frac{N}{s} + \theta. \tag{21}$$

Fig. 6 compares the single-step Laih toll with the FB and uniform toll and is based on the numerical example of Section 4. Fig. 7 shows the equilibrium for a single-step Laih toll. Step tolling tilts down the cost curves, which means that the price increase is less and the average toll is lower than with uniform tolling. For a finite number of steps, the price with Laih tolling will be higher than with FB tolling.
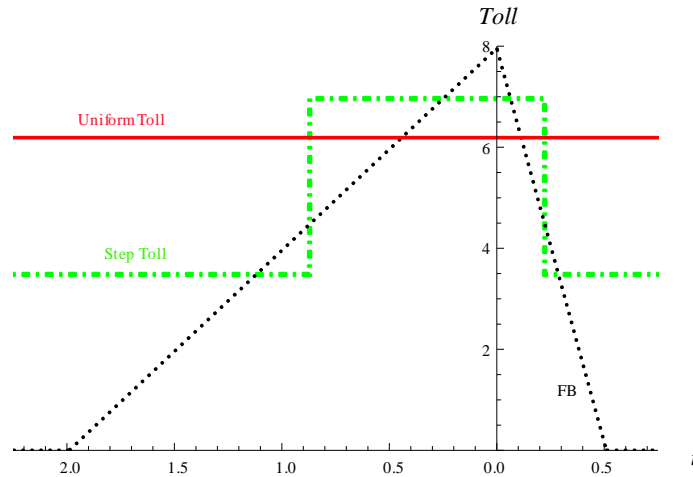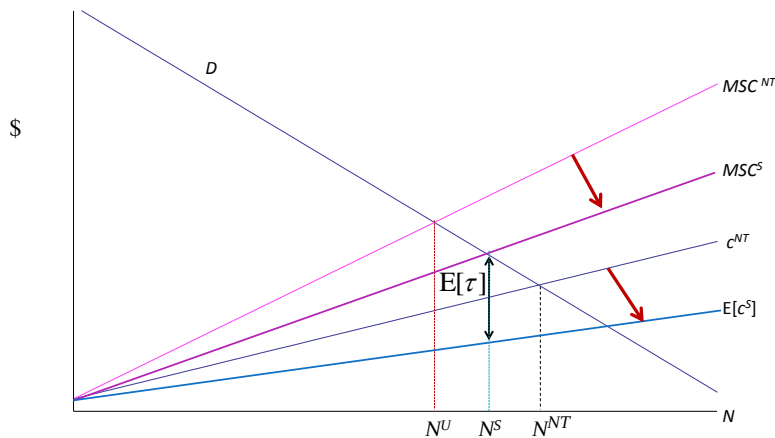

Figure 6: A single-step toll

Figure 7: Equilibrium with a single-step toll

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3.5. ADL step toll

With the ADL toll of Arnott et al. (1990; 1993) and fixed demand, step tolling lowers the price and shifts the peak to later (i.e. the start and end times are later). Each time the toll drops a level there is a mass departure of users. If $\alpha < \gamma$, then just after the last user of the *i*th mass arrives at t_i^-, there is the mass departure of the users who pay the *i−1*th toll.[7] Without the shift of the peak, the price for a user in a mass departure would be lower than for a user that travels during the rest of the peak. By having more drivers in the masses and fewer drivers outside, expected prices are made constant over time. And it is this that shifts the peak to later and lowers the equilibrium price.

Lindsey et al. (2010) find that generalising the ADL model to *m* steps is much harder than for the other models. Already for two or three steps the formulas are very complex. This analytical section focuses on single- and two-step tolls; the numerical model goes up to 10 steps. The ADL toll has, for a finite *m*, a larger welfare gain than the Laih toll, but also approaches the FB toll as *m* goes to infinity. Due to the shift of the peak and mass departures, the ADL toll is asymmetric with the *i*th late toll being higher than the *i*th early toll. Following Lindsey et al. (2010), the early step parts of the toll follow the same formula as in the Laih model (see (18)):

$$\rho_i^+ = \frac{m+1-i}{m} \cdot \rho_1, \quad i = 1,...,m, \quad i = 1,...,m. \tag{22}$$

Conversely, the late tolls are not simple fractions of the central toll, ρ_1:

$$\rho_i^- = (i-2)\rho_3^- - (i-3)\rho_2^-, \quad i = 2,...,m. \tag{23}$$

It is due to this more complex formula that there are no simple solutions for the ADL toll. To at least give some analytical insight, I give the analytics for one and two steps below.

3.5.1. Coarse (or single-step) ADL toll

For a single-step toll, the peak starts and ends at

$$t_s' = t^* - \frac{\gamma}{\beta + \gamma} \delta \frac{N}{s} \left(\frac{\gamma(\beta + 2\gamma) + \alpha(3\beta + 2\gamma)}{2(\alpha + \gamma)(\beta + \gamma)} \right),$$

$$t_e' = t^* + \frac{\gamma}{\beta + \gamma} \delta \frac{N}{s} \left(\frac{\alpha(\beta + 2\gamma) + \gamma(3\beta + 2\gamma)}{2(\alpha + \gamma)(\beta + \gamma)} \right).$$

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[7] Lindsey et al. (2010) and Daniel (2009) show that with $\alpha > \gamma$, there are normal departures after the *i*th mass that still pay the *i*th toll. This then ensures that there is no shift in the peak; and therefore the price and toll formulas are the same as in the Laih model, and hence the ADL model simplifies to the Laih model. I focus on $\alpha < \gamma$, as this seems more likely for car travel.

These equations only differ from the equations for the NT and Laih equilibria by the terms in brackets. For t_s' the term is smaller than 1 for relevant parameter values (i.e. $\gamma>\alpha>\beta>0$), while the term for t_e' is above 1. Hence, the peak starts and ends later for a given N.

Total costs and toll revenue are

$$TC = \delta \frac{N^2}{s}\left(1 - \frac{1}{4}\frac{\gamma(\alpha+2\beta+\gamma)}{(\alpha+\gamma)(\beta+\gamma)}\right), \tag{24}$$

$$TR = TR^{step} + TR^{fixed} = \delta \frac{N^2}{s}\left(1 - \frac{\delta}{2\beta} - \frac{\delta}{\alpha+\gamma}\right) + \theta \cdot N. \tag{25}$$

Marginal External Cost is on average

$$E[MEC] = \delta \frac{N}{s}\left(1 - \frac{1}{4}\frac{\gamma(\alpha+2\beta+\gamma)}{(\alpha+\gamma)(\beta+\gamma)}\right). \tag{26}$$

This equation shows that, for a given N, the ADL externality is lower than in the Laih model in (20), since $\frac{1}{4}\frac{\gamma(\alpha+2\beta+\gamma)}{(\alpha+\gamma)(\beta+\gamma)} > \frac{m}{2+2m} = \frac{1}{4}$ when $\gamma>\alpha>\beta>0$. Still, the time-invariant part, θ, follows the same formula as in the single-step Laih model, which implies that the average step part, $E[\rho]$ is lower (for given N):

$$\theta = E[MEC] - E[\rho] = \frac{1}{2}\delta\frac{N}{s}. \tag{27}$$

Due to the larger downward tilt of the cost curves, the consumer is better off with a single-step ADL toll than with a single-step Laih toll.

3.5.2. Two-step ADL toll

The formulas with two steps are much more difficult than with a single step, and the more steps there are, the more complex the formulas become. The early and late tolls are now asymmetric:

$$\rho_1 = \frac{2}{3}\delta\frac{N}{s}\left(1 + \frac{2\alpha\beta(\gamma-\alpha)}{3\alpha^3 + 3\gamma^3 + \alpha^2(8\beta+9\gamma) + \alpha\gamma(16\beta+9\gamma)}\right), \tag{28a}$$

$$\rho_2^+ = \frac{\rho_1}{2}, \tag{28b}$$

$$\rho_2^- = \rho_1\left(\frac{\alpha^3 + 5\alpha^2\gamma + 3\gamma^3 + \alpha\gamma(16\beta+7\beta)}{4(\alpha^3 + \gamma^3 + 3\alpha\gamma(2\beta+\gamma) + \alpha^2(2\beta+3\gamma))}\right). \tag{28c}$$

Hence, the early toll ρ_2^+ is half the central toll ρ_1, while the late toll ρ_2^- is somewhat higher. Total costs follow

$$TC = \left(1 - \frac{\gamma\left(2\alpha^3 + \gamma^2\left(3\beta + 2\gamma\right) + \alpha^2\left(11\beta + 6\gamma\right) + 2\alpha\left(8\beta^2 + 9\beta\gamma + 3\gamma^2\right)\right)}{2\left(\beta + \gamma\right)\left(3\alpha^3 + 3\gamma^3 + \alpha^2\left(8\beta + 9\gamma\right) + \alpha\gamma\left(16\beta + 9\gamma\right)\right)}\right)\delta\frac{N}{s}. \tag{29}$$

Unlike with a single step, the formula for the time-invariant part of the toll now differs from the one in the Laih model:

$$\theta = E[MEC] - E[\rho] = \frac{1}{1+2}\delta\frac{N}{s}\left(1 + \frac{4\alpha\beta\left(\alpha - \gamma\right)}{3\alpha^3 + 3\gamma^3 + \alpha^2\left(8\beta + 9\gamma\right) + \alpha\gamma\left(16\beta + 9\gamma\right)}\right). \tag{30}$$

In the Laih model, the term between brackets equals 1. In this ADL model, the term is below 1 when $\gamma > \alpha > \beta > 0$. Accordingly, the θ and average toll are lower for a given N. Observing the complexity of the ADL toll, I will keep the analytical discussion of the ADL model to two steps, and will now continue with the analytical framework for the braking toll.

3.6. Braking step toll

The ADL and Laih model overlook that drivers have an incentive to delay reaching the tolling point when the toll is about to drop if the waiting cost they incur is outweighed by the money they save. The Braking model of Lindsey et al. (2010) takes this incentive into account (see their paper for a more detailed discussion of the model). Users stop passing the tolling point a time Δt_i before the ith level decreases to the $i-1$th level at t_i^-. The first users to pay the $i-1$th level arrive just after t_i^-. The last users to pay the ith toll arrive at $t_i^b = t_i^- - \Delta t_i^-$. For the prices at these two arrival times to equal, the Δt_i must equal $\left(\rho_i - \rho_{i-1}\right)/\left(\alpha + \gamma\right)$. The total time the bottleneck is idle, Δt, is the sum of all the Δt_i. It depends only on the level of ρ_1 and preference parameters α and γ:

$$\Delta t = \rho_1/\left(\alpha + \gamma\right). \tag{31}$$

The step part of the toll follows the same formula as in the Laih model, but the levels are generally different, as, with price sensitive demand, the numbers of users differ:

$$\rho_1 = \frac{m}{m+1}\cdot\delta\cdot\frac{N}{s}, \quad \rho_i = \rho_i^+ = \rho_i^- = \frac{m+1-i}{m}\cdot\rho_1, \quad i = 2,...,m. \tag{32}$$

The idle time Δt is an inefficiency and pure deadweight loss that raises costs and makes step tolling more harmful for the user. The idle time does not disappear as m becomes larger.

Actually, it only increases with m, since ρ_1 increases with m, and following (31) Δt is an increasing function of ρ_1. This implies that the braking toll does not approach the first-best toll: even for an infinite m, its gain will be lower. The formulas for total cost and toll revenue are more complex with braking than in the Laih model, as they contain the fraction $\delta/(\alpha+\gamma)$:

$$TC = \delta \frac{N^2}{s}\left(1 - \frac{m}{1+m}\frac{1}{2}\left(1 - \frac{\delta}{\alpha+\gamma}\right)\right),$$
(33)

$$TR = TR^{step} + TR^{fixed} = \delta \frac{N^2}{s}\frac{m}{1+m}\frac{1}{2}\left(1 + \frac{\delta}{\alpha+\gamma}\right) + \theta \cdot N.$$
(34)

From (33) the average MEC can be derived, and it turns out to be again similar to in the Laih model but for the $\delta/(\alpha+\gamma)$ term:

$$E[MEC] = \delta \frac{N}{s}\left(1 - \frac{m}{1+m}\frac{1}{2}\left(1 - \frac{\delta}{\alpha+\gamma}\right)\right).$$
(35)

For given N, the average Braking MEC is higher than in the Laih model. This higher MEC is due to the extra travel costs caused by the time that the bottleneck is idle. Interestingly, as the $\delta/(\alpha+\gamma)$ term is in both the user cost as in the average step part of the toll ($E[\rho]=TR^{step}/N$), the fixed part of the toll, θ, simplifies to the same formula as in the Laih model:

$$\theta = E[MEC] - E[\rho] = \delta \frac{N}{s}\frac{1}{1+m}$$
(36)

For given N, the time-invariant toll is thus the same in the Laih and Braking model, while the step-part is higher with braking. The extra marginal external costs due to the braking are internalised for the average user via the step toll, which means that the time-invariant toll can follow the same formula as in the Laih model. Braking tolling is more harmful for the consumer than Laih tolling for two reasons: (1) the time the bottleneck is idle raises costs, and (2) the average toll is higher for the same number of users in the braking model than in the Laih model. This makes preventing braking even more important with price-sensitive demand than with fixed demand, where only the first effect occurs.

The model assumes that there is no direct cost to the user of braking. This seems unrealistic: standing still besides a road can be very dangerous, which means that there are costs from the increased risk of an accident. Further, this standing still is likely to be a traffic violation, meaning that there is also a risk of a fine. With such costs of braking, introducing more steps in the toll might solve the braking problem, the toll saving becomes ever smaller, while the extra cost from risk of accident and fine remain. Limiting braking was one of the reasons why Singapore introduced extra steps in 2003 (Chew, 2008). Moreover, it also seems important for the government to actively control for cars standing still and fine those that do,

as this behaviour is not only dangerous for the driver as well as for other drivers (i.e. it imposes and extra accident externality), but also reduced the gain from step tolling.

4. Numerical model

This section illustrates the effects of step tolling with price-sensitive demand using a numerical example. The section looks at tolls with one up to ten steps. I use the following preference parameters: the unit cost of an hour of travel delay (i.e. the value of time) is α=8, the value of schedule delay early is β=4, and the value of schedule delay late is γ=15.6. The bottleneck capacity is s=3600 cars an hour. The no-toll equilibrium has 9000 drivers, so the peak lasts 2.5 hours. The inverse demand follows a linear function, and the elasticity with respect to generalised price is −0.4 in the NT equilibrium.

Fig. 8 shows the patterns of the tolls with 5 steps. The ADL toll is the solid (blue) line, the (red) stripped curve is the Braking toll, the (green) dot-dashed curve the Laih toll, and the (black) dotted curve is the FB toll. The five-step toll is on average lower than a single-step toll: in Fig. 8, the 5-step tolls hug the first-best toll; in Fig. 6, the single-step toll is much higher than the FB toll. With price sensitive demand, the toll at the start and end of the peak equals the time-invariant term and is above the FB toll. The peak lasts much longer with the Braking toll than with the other step tolls due to the time the bottleneck is idle, even though it has the lowest number of users. The ADL and Laih peak last shorter than the FB peak, as these tolls result in a lower number of users. Before t^* the ADL and Laih toll are very similar in their levels, after t^* the ADL toll tends to be higher than the Laih toll.
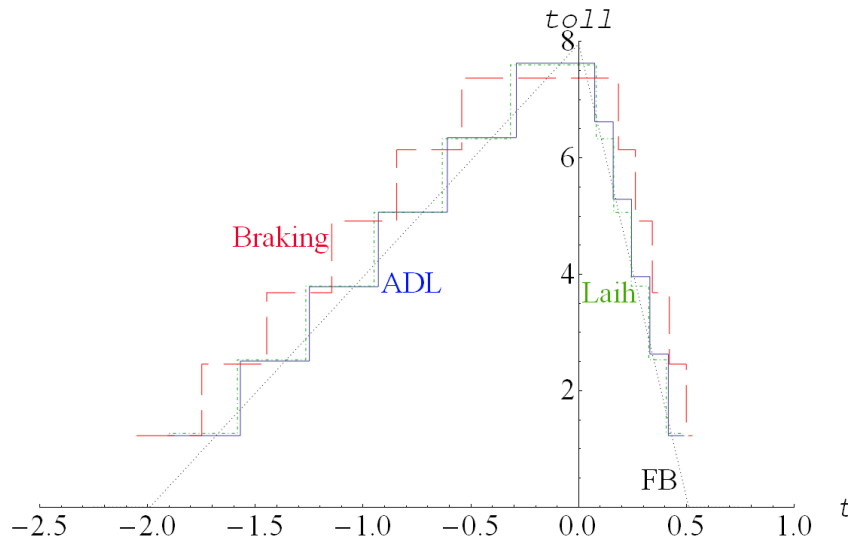
Figure 8: 5-steps toll for the ADL, Laih, and Braking model

Figs. 9-11 compare for the three regimes the (generalised) price, percentage change in consumer surplus, and relative efficiency (i.e. welfare gain from the NT case relative to the FB gain).

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The more steps there are, the better off the consumers: the price is lower, while consumer surplus and number of users are higher. A uniform toll or a step toll with few steps is a crude instrument to reduce the congestion problem; they (primarily) equate the private price with marginal social cost by lowering the number of users. The fully-time-variant FB toll equalises MSC and private price by halving the *MSC,* while keeping the number of users the same. The more steps a step-toll has, the closer its approximation of the FB toll, and the more it alters the departure pattern, shortens total travel delay, and lowers the MSC.

The price development over the number of steps differs strongly between fixed and price sensitive demand. With price sensitivity, the price decreases with m, since the time-invariant toll becomes lower. With fixed demand, there is no time-invariant toll (or more precisely it is undefined and therefore arbitrarily set to zero); and the price is independent of m in the Laih model, while it increases with m in the ADL and braking model (Lindsey et al., 2010).
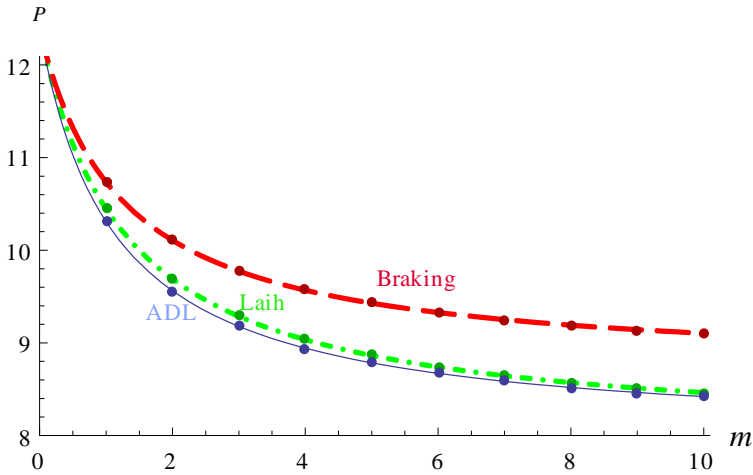
Figure 9: Generalised price as a function of the number of steps
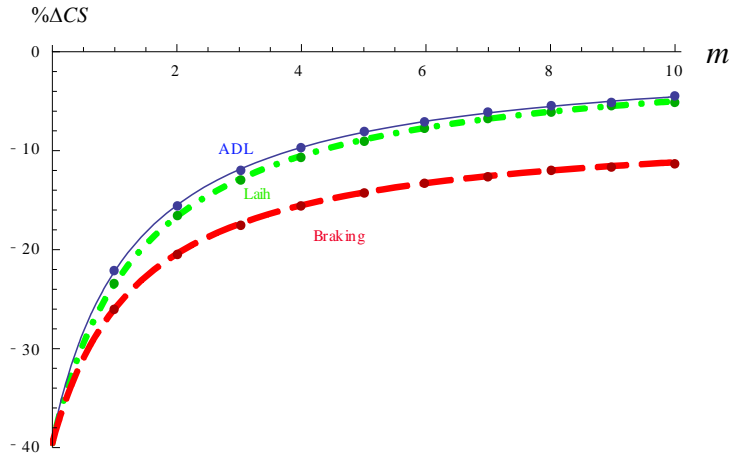
Figure 11: Percentage change in the consumer surplus as a function of the number of steps
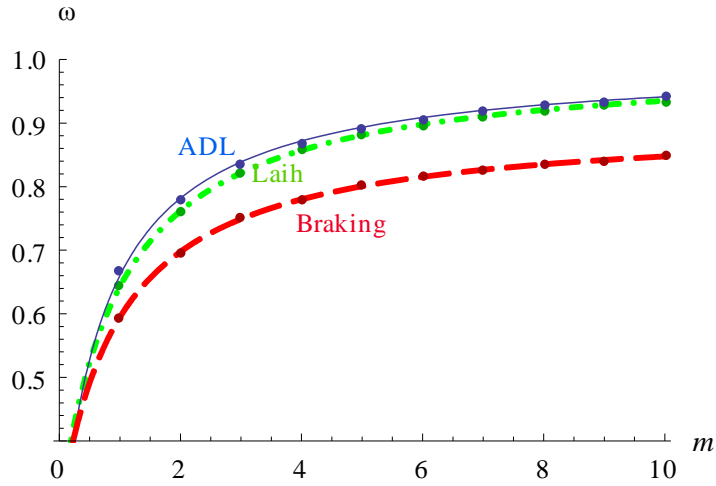
Figure 11: Relative efficiency as a function of the number of steps

5. Effect of the price sensitivity

Figure 12 compares the relative efficiencies of the three step tolls over different price elasticities: pane (a) does this for the ADL toll, pane (b) for the Laih toll, and pane (c) for the braking toll. Figure 13 compares the change in average consumer surplus (or alternatively the change in the average distance between demand function and price). The figures show that not only is welfare higher with more steps, the consumer is also better off: with a price elasticity of -0.4, a single-step toll decreases average consumer surplus by 1.17 (or a 22% lower total consumer surplus), while a 10-step toll by only 0.25 (or a 4.99% lower total surplus).

The effects of step tolling depend strongly on the price sensitivity. The gain of step tolling is higher and the consumer surplus loss is lower with more price sensitive demand: this is because it becomes easier for users to adapt their demand and toll revenue becomes larger relative to the consumer surplus loss. Note that this effect of price sensitivity on consumer surplus also occurs in a static model of congestion. Still, with more sensitive demand, step tolling lowers the number users more, and the percentage loss in consumer surplus is larger. The latter is because the initial surplus in the NT equilibrium decreases. In the limit, as the demand becomes perfectly elastic (i.e. a flat demand function) consumer surplus becomes zero in all regimes.
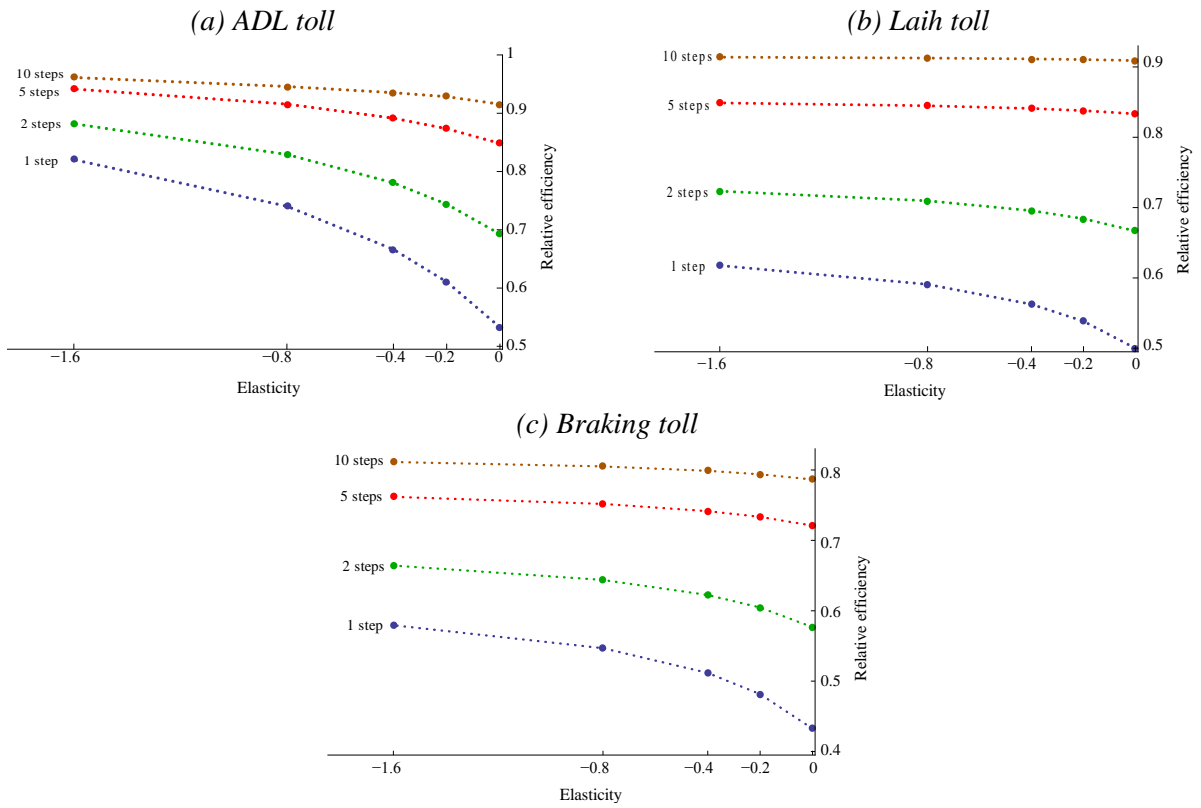
Figure 12: Relative efficiency and price sensitivity for (a) the ADL, (b) Laih, and (c) braking model

Figure 13: Change in average consumer surplus due to step tolling and price sensitivity for (a) the ADL, (b) the Laih, and (c) the braking model

5. Conclusion

Models of (step) tolling usually assume that demand is price insensitive. This assumption seems empirically questionable, and, as this paper found, has important implications for the effects of step tolling. In the bottleneck model, a first-best toll that is fully time variant leaves the price unchanged, and thus price sensitivity has no effect. Conversely, step tolling raises the price, and thus reduces consu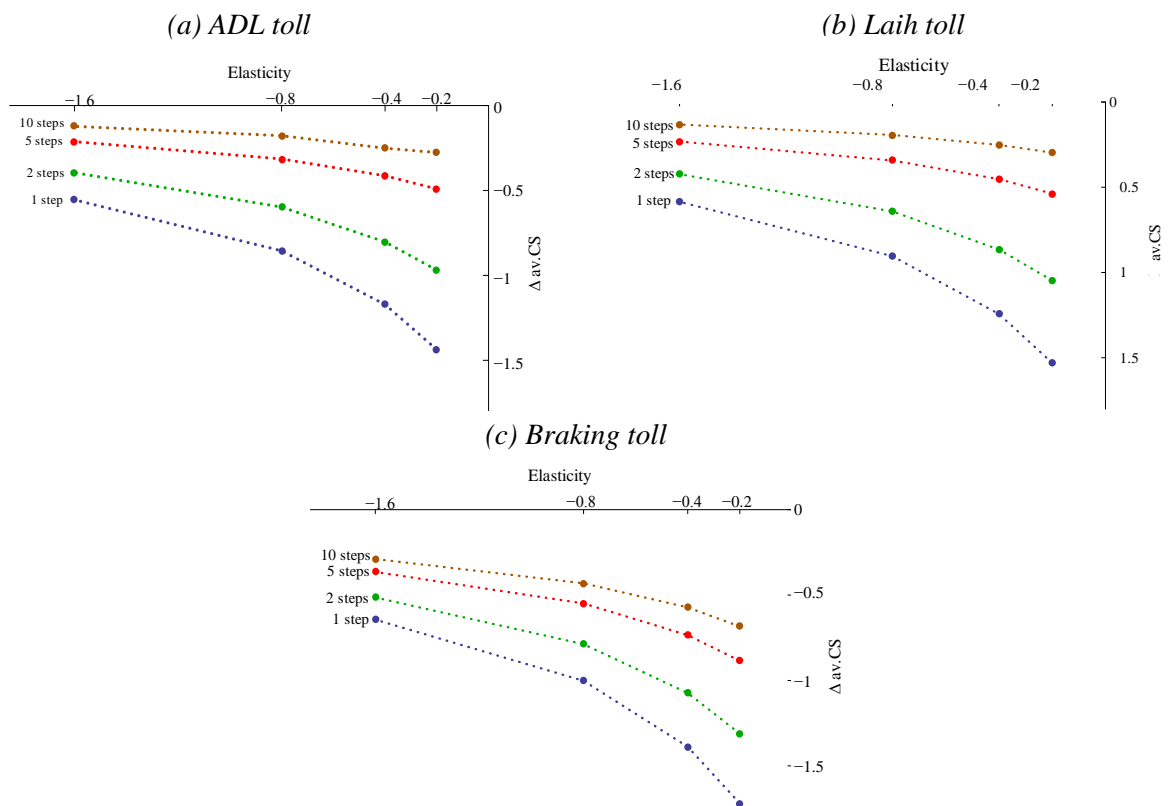mer surplus and the number of users. Accordingly, the welfare and consumer surplus changes due to step tolling depend strongly on the price sensitivity. The more steps there are in the toll, the closer it approximates the first-best toll, and the better off the consumer is. This makes it extra important for real-world tolling systems to have as many steps in the toll as possible: this not only raises the welfare gain of tolling, but also raises the political acceptability of tolling.

For future research it is interesting to study the effect of step tolling under price sensitive demand and heterogeneous preferences, this would allow studying of both aggregate welfare effects as well as distributional effects. Xiao, Qian and Zhang (2011) study the ADL single-step toll under fixed demand and the heterogeneity from Vickrey (1973) that varies the values of time, schedule delay early, and schedule delay late in fixed proportions. They find that step tolling decreases total users cost more with this heterogeneity than with homogeneity, because users with high values self select to the tolled period, while those with low values travel during the untolled period. It would also be interesting to include heterogeneity in the ratio of value of time and schedule delay early and in ratio of values of schedule delay early and late. Van den Berg and Verhoef (2011) study fully time-variant tolling under two dimensions of heterogeneity: (1) the heterogeneity from Vickrey (1973), and (2) in the ratio of value of time to value of schedule delay. They find that whether a certain user wins or loses depends on her values of time and schedule delay, the extent of both types of heterogeneity, and all the price sensitivities. The heterogeneity in the ratio of value of schedule delay early and late should also have important effects, as this ratio affects when a user arrives with and without step tolling (see Section 4 of Arnott, de Palma and Lindsey (1988, 1993) on the no-toll and first-best equilibria with this heterogeneity).

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