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EXPOSITA NOTE

On striking for a bargain between two completely informed agents

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Abstract This paper provides a thorough equilibrium analysis of a wage contract negotiation model where the union must choose between strike and holdout between offers and counter-offers. When the union and the firm have different discount factors, delay in reaching an agreement may Pareto dominate many immediate agreements. We derive the exact bounds of equilibrium payoffs and characterize the equilibrium strategy profiles that support these extreme equilibrium payoffs for all discount factors. In particular, our analysis clarifies open issues on the maximal wage in this model when the union has a higher discount factor than the firm.

Keywords Bargaining · Negotiation

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1 Introduction

In the contract negotiation model studied by Fernandez and Glazer (1991), Haller (1991), and Haller and Holden (1990), a union and a firm negotiate a new contract

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after an old one expired. In contrast to Rubinstein (1982), the union decides whether to strike or holdout after an offer is rejected in any period before they agree on a new contract. Fernandez and Glazer (1991) show that an *alternating-strike* strategy supports the *maximal wage*, the union's best equilibrium payoff. This strategy profile specifies that the union strikes (provided it is credible) only after its offer is rejected and both make acceptable offers. Bolt (1995) demonstrates that, however, this strategy profile fails to be an equilibrium when the union is more patient than the firm. Haller (1991), and Haller and Holden (1990) consider the case in which both players have the same discount factor and, therefore, Bolt's (1995) criticism does not apply. Bolt provides a *no-concession* strategy to the firm and shows that it can be sustained by equilibrium. Recently, one of Slantchev's (2003) claims implies that firm's *no-concession* strategy always supports the maximal wage when the union is more patient than the firm. This, however, contradicts with Bolt's (1993) finding that an *always-strike* strategy profile sometimes yields an even higher payoff to the union. Instead of invoking Shaked and Sutton's (1984) method to derive extreme equilibrium payoffs, Bolt (1993, 1995) and Slantchev (2003) simply verify whether a given strategy profile constitutes an equilibrium. Muthoo (1999) also notices this issue on extreme equilibria in a general model of this kind, called the negotiation model due to Busch and Wen (1995), and provides a set of bounds for equilibrium payoffs, that are not necessarily the tightest bounds. Busch and Wen (1995) also deal with the case of a common discount factor, so it is not subject to the current debate. This ongoing debate is still not settled, because it is still unclear what has been missing in those studies and what the union's best equilibrium payoff is in this celebrated model, particularly when the union is more patient than the firm.

In this paper, we explain why complications arise in analyzing the contract negotiation model when the union and firm have different discount factors. It has been noticed in other contexts that Pareto improvement is possible through intertemporal trade between agents with different discount factors, see e.g., Ramsey (1928), Bewley (1972), and more recently, Lehrer and Pauzner (1999). In the contract negotiation model, such Pareto improvement is also possible through a delayed agreement, which has not been formally recognized in the bargaining literature. Such Pareto improvement could be so dramatic that it may lead to payoff vectors above the bargaining frontier. This fact has been overlooked in most of the existing studies in this literature, and implies that we should also question the validity of the method used in these studies. More specifically, it is not innocent to assume that players always reach an agreement, or equivalently, players' continuation payoffs are always bounded by the bargaining frontier when applying Shaked and Sutton's (1984) technique.

For that reason, we formally incorporate the possibility of unacceptable offers in deriving extreme equilibrium payoffs in the contract negotiation model. In this paper, we confirm and validate Fernandez and Glazer's (1991) analysis if and only if the union is less patient than the firm. When the union is more patient than the firm, however, the assumption that the continuation payoffs are always bounded by the bargaining frontier is violated. Depending upon the discount factors, we completely characterize all extreme equilibria in this model. Roughly speaking, when the firm's discount factor is not far below the union's, delay happens in the union's best equilibrium. In this case, the firm adopts the no-concession strategy against the union's alternating-strike strategy. The rationale for the firm to choose its no-concession strategy is that the continuation payoff from delay is above the bargaining frontier due to their different discount factors. Consequently, there is no mutually acceptable agreement available. However, delay for more than one period would not be effective in supporting the union's best equilibrium. When the firm's discount factor is significantly below the union's, the alternating-strike strategy is no longer effective since the union can manoeuvre the firm in a worse situation with its always-strike strategy, even though now the firm makes acceptable offers.

Fudenberg and Tirole (1991) are the first to consider the possibility of making unacceptable offers in Rubinstein (1982). Since all continuation payoffs are bounded by the bargaining frontier in Rubinstein (1982), it is without loss of generality to examine acceptable offers only when applying Shaked and Sutton's (1984) technique. As discussed, this is no longer true in the contract negotiation model. As shown in our analysis, the same holds in the stochastic bargaining model of Merlo and Wilson (1995, 1998). As Merlo and Wilson (1995) explain on p. 372, however, their model excludes the contract negotiation model studied here. Since the contract negotiation model lies outside the scope of these references, our findings here are similar to theirs, but resulting from a different extension.

Finally, Houba and van Lomwel (2001) study the maximal wage strategies of Fernandez and Glazer (1991) under short-term contracts, common discount factors, and productivity growth where intertemporal trade cannot take place. Such growth makes the union less patient than the firm and, for that reason, they forego investigating equilibrium bounds. Our results hint that there is no reason to explicitly derive such bounds in their model.

2 The model

Consider the contract negotiation model of Fernandez and Glazer (1991). After contract $w_0 \in [0, 1]$ expires, a union and a firm, called players u and f, negotiate how to share firm's future gross profit, normalized to be 1 per period, over infinitely many periods. The union makes offers in all even periods (including period 0) and the firm makes offers in all odd periods. In contrast to Rubinstein (1982), the union decides whether to strike or holdout in any period after an offer is rejected. During strike, both the union and the firm receive 0. During holdout, the union receives w_0 and the firm receives $1 - w_0$. From any outcome path of the model, a player receives the sum of its discounted payoffs from all periods. Denote player i's discount factor as $\delta_i \in (0, 1)$ for $i = u$ and f. For all $(\delta_u, \delta_f) \in (0, 1)^2$ and $w_0 \in [0, 1]$, there is a stationary SPE (subgame perfect equilibrium) where the union receives w_0 and the firm receives $1 - w_0$ in every period. Obviously, this stationary SPE is simultaneously the union's worst equilibrium and the firm's best equilibrium. What is less clear is the union's best SPE (the firm's worst SPE), which is the key issue of the current debate.

An important issue that has not been properly addressed is the Pareto frontier of all possible continuation payoffs. In Rubinstein (1982), this Pareto frontier simply consists of all possible agreements, i.e., the *bargaining* frontier $\{(w, 1 - w) : w \in [0, 1]\}$, whether players have the same discount factor or not. Unlike Rubinstein (1982), some feasible outcomes may lead to payoffs strictly above the bargaining frontier when the union and the firm have different discount factors. For example, consider a feasible outcome path where the union holds out for T periods, followed by an agreement $w \in [0, 1]$. The sum of their payoffs

$$\left[\left(1-\delta_u^T\right) w_0+\delta_u^T w\right]+\left[\left(1-\delta_f^T\right) (1-w_0)+\delta_f^T (1-w)\right]=1+\left(\delta_u^T - \delta_f^T\right) (w - w_0),$$

which is equal to 1 if $\delta_u = \delta_f$. The resulting payoff vector, however, is above the bargaining frontier if either $\delta_u < \delta_f$ and $w < w_0$, or $\delta_u > \delta_f$ and $w > w_0$. Therefore, not all continuation payoffs are bounded from above by the bargaining frontier. This implies that the Pareto frontier consists of two parts. For example in case $\delta_u > \delta_f$, for all $w \leq w_0$ this frontier coincides with the bargaining frontier for all, while for all $w > w_0$ it must lie above the bargaining frontier because the sum of payoffs is above 1.

In the context of repeated games, Lehrer and Pauzner (1999) demonstrate how to construct Pareto optimal outcome paths from two payoff vectors when two players have different discount factors. A feasible outcome path in the contract negotiation model is not as flexible as in a repeated game. By default, an agreement ceases any future payoff variation. Accordingly, a Pareto efficient outcome path must consist of the union's holdout for $T \geq 0$ periods with probability $1 - p \in [0, 1]$ and for $T + 1$ periods with probability p, followed by either $w = 1$ when $\delta_u > \delta_f$, or $w = 0$ when $\delta_u < \delta_f$.[1] To have such a continuation in an equilibrium, two players have to eventually agree on an equilibrium contract. Since neither $w = 1$ nor $w = 0$ is an equilibrium contract in this model, all Pareto efficient outcome paths with a sum of payoffs above the bargaining frontier are unachievable in equilibrium. Instead, we need to consider continuations with w being the minimum equilibrium contract when $\delta_u < \delta_f$ and w being the maximum equilibrium contract in case of $\delta_u > \delta_f$. Since no equilibrium contract can be less than w_0, there will be no effective continuation payoff above the bargaining frontier in supporting the maximal wage when $\delta_u < \delta_f$. This validates previous analysis on this issue in this case. On other hand, when $\delta_u > \delta_f$, we must consider feasible continuation payoffs above the bargaining frontier in supporting the maximal wage, which is why the conventional analysis breaks down.

3 Necessary conditions and unacceptable offers

In order to derive the union's highest SPE payoff, it is necessary to incorporate unacceptable offers in applying Shaked and Sutton's (1984) technique. Let M_u be the supremum of the union's SPE payoffs in any even period where the union makes an offer, and m_f be the infimum of the firm's SPE payoffs in any odd period where the firm makes an offer. Note that both M_u and m_f generally depend on $(\delta_u, \delta_f) \in (0, 1)^2$ and $w_0 \in [0, 1]$. Since w_0 is the union's worst SPE payoff, we have

[1] Here, players are allowed to play correlated strategies for technical simplicity, so that the Pareto frontier can be described by a continuous function.

$$w_0 \leq M_u \leq 1 \quad \text{and} \quad w_0 \leq 1 - m_f \leq 1. \tag{1}$$

In what follows, we derive a set of necessary conditions for M_u and m_f by analyzing players' equilibrium strategies in the union's best possible situation.

In any odd period, the firm may make either an unacceptable offer or an *irresistible offer* that will certainly induce the union to accept. If the union holds out after rejecting the firm's offer, the union will receive at most $(1-\delta_u)w_0+\delta_u M_u$ and certainly accepts any offer that is higher. Thus, the firm could obtain, at least, $1-(1-\delta_u)w_0-\delta_u M_u$ from making the least irresistible offer. Alternatively, the firm could obtain at least

$$(1-\delta_f)(1-w_0)+\delta_f(1-M_u) = 1-(1-\delta_f)w_0-\delta_f M_u$$

from making any unacceptable offer. The firm will make either the least irresistible offer or an unacceptable offer; whichever yields a higher value to the firm. Note that holdout is always credible for the union to carry out after rejecting a firm's offer.

On the other hand, if the union strikes after rejecting the firm's offer, the union's continuation payoffs will not be more than $\delta_u M_u$. The firm will obtain at least $1-\delta_u M_u$ from making the least irresistible offer, or $\delta_f(1-M_u)$ from making an unacceptable offer. Due to $(\delta_u, \delta_f) \in (0, 1)^2$ and $M_u \leq 1$, we have $(\delta_u - \delta_f)M_u \leq 1-\delta_f$ and, therefore,

$$\delta_f(1-M_u) \leq 1-\delta_u M_u, \tag{2}$$

which implies that the firm will never make an unacceptable offer if the union threatens to strike after rejecting the firm's offer. Unlike holdout, strike is credible if and only if $\delta_u M_u \geq w_0$, i.e., the union's highest continuation payoff is not less than w_0 due to the stationary SPE.

Obviously, m_f cannot be less than the minimum (with respect to either holdout or strike) of the firm's highest continuation payoff from making either the least irresistible offer or an unacceptable offer. That is, for all $(\delta_u, \delta_f) \in (0, 1)^2$ and $w_0 \in [0, 1]$,

$$m_f \geq \min \begin{cases} \max \begin{cases} 1-(1-\delta_u)w_0-\delta_u M_u, & (3a) \\ 1-(1-\delta_f)w_0-\delta_f M_u, & (3b) \end{cases} \\ 1-\delta_u M_u \quad \text{subject to } \delta_u M_u \geq w_0, & (3c) \end{cases} \tag{3}$$

Note that if we did not consider the possibility of unacceptable offers, $(3b)$ would disappear and (3) would simply reduce to $(3c)$, which would yield flawed results in Sect. 4. From condition (3), we obtain

Proposition 1 *For all* $(\delta_u, \delta_f) \in (0, 1)^2$ *and* $w_0 \in [0, 1]$,

$$m_f \geq \begin{cases} 1-\delta_u M_u, & \text{if } (\delta_u-\delta_f)M_u \geq (1-\delta_f)w_0, \\ 1-(1-\delta_f)w_0-\delta_f M_u, & \text{if } (\delta_u-\delta_f)M_u < (1-\delta_f)w_0, \quad \delta_f < \delta_u, \\ 1-(1-\delta_u)w_0-\delta_u M_u, & \text{if } \delta_f \geq \delta_u. \end{cases} \tag{4}$$

The proofs of this and all other propositions are deferred to the appendix. The condition in the first case of (4) implies both $\delta_f < \delta_u$ and the credibility constraint for (3c). Consequently, the union would never strike after rejecting a firm's offer when $\delta_f \geq \delta_u$. The firm makes an unacceptable offer if and only if the continuation payoffs Pareto dominate an immediate agreement, which is the second case of (4), because for $\delta_f < \delta_u$ the sum of their continuation payoffs is

$$(1-\delta_u)w_0+\delta_u M_u+(1-\delta_f)(1-w_0)+\delta_f(1-M_u)=1+(\delta_u-\delta_f)(M_u-w_0)\geq 1.$$

In any even period, if the union holds out after its offer is rejected, the firm will receive no less than $(1-\delta_f)(1-w_0)+\delta_f m_f$ by rejecting the union's offer. Therefore, the union's SPE payoffs cannot be more than $1-(1-\delta_f)(1-w_0)-\delta m_f$ from making the least acceptable offer, or $(1-\delta_u)w_0+\delta_u(1-m_f)$ from making an unacceptable offer.

Likewise, the union may threaten to strike if the firm rejects its offer, which is credible if and only if $\delta_u(1-m_f)\geq w_0$. In this situation, the union's SPE payoffs cannot be more than $1-\delta_f m_f$ from making the least acceptable offer, or $\delta_u(1-m_f)$ from making an unacceptable offer. Given $(\delta_u, \delta_f)\in(0,1)^2$ and $m_f\geq 0$, we have

$$1-\delta_u \geq (\delta_f-\delta_u)m_f \quad \text{if and only if} \quad 1-\delta_f m_f \geq \delta_u(1-m_f), \tag{5}$$

which implies that if the union threatens to strike, the union will not make an unacceptable offer to the firm in an even period. To summarize, the union's SPE payoffs cannot be more than the maximum of the union's continuation payoffs from all three possible cases discussed above. That is, for all $(\delta_u, \delta_f)\in(0,1)^2$ and $w_0\in[0,1]$,

$$M_u \leq \max \begin{cases} 1-(1-\delta_f)(1-w_0)-\delta_f m_f, & (6a)\\ (1-\delta_u)(1-w_0)+\delta_u(1-m_f), & (6b)\\ 1-\delta_f m_f, \qquad \text{subject to } \delta_u(1-m_f)\geq w_0. & (6c)\end{cases} \tag{6}$$

Again, if we ignored the possibility of unacceptable offers, then (6b) would disappear from (6), which would also introduce flawed results. From (6), we obtain

Proposition 2 *For all* $(\delta_u, \delta_f)\in(0,1)^2$ *and* $w_0\in[0,1]$, *we have*

$$M_u \leq \begin{cases} 1-\delta_f m_f, & \text{if } \delta_u(1-m_f)\geq w_0,\\ 1-(1-\delta_f)(1-w_0)-\delta_f m_f, & \text{if } \delta_u(1-m_f)< w_0, \quad \delta_f\geq\delta_u,\\ 1-(1-\delta_u)(1-w_0)-\delta_u m_f, & \text{if } \delta_u(1-m_f)< w_0, \quad \delta_f<\delta_u.\end{cases} \tag{7}$$

4 The union's best SPE

We now derive the union's best SPE payoff from the necessary conditions obtained in the previous section. To simplify the exposition, we will consider the cases of $\delta_f \geq \delta_u$ and $\delta_f < \delta_u$, respectively.

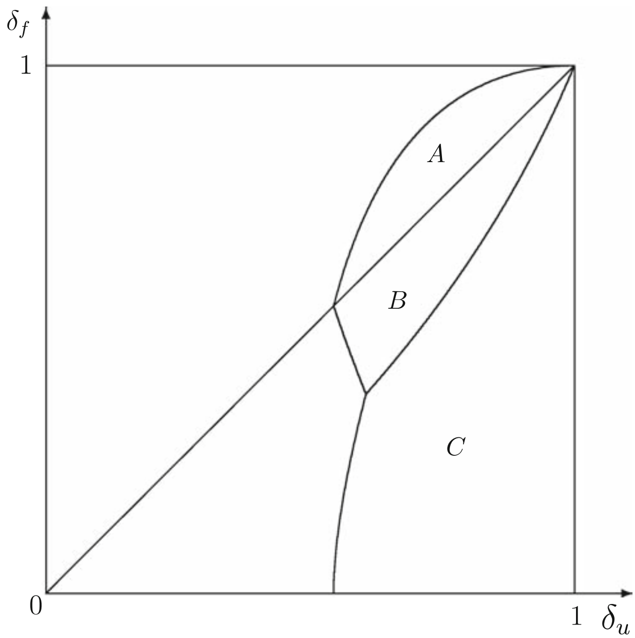

Fig. 1 The sets A, B and C for $w_0 = 0.3$

Proposition 3 *When $\delta_u \leq \delta_f$, (4) and (7) yield*

$$M_u = \begin{cases} w_0 + \frac{(1-\delta_f)(1-w_0)}{1-\delta_u\delta_f}, \\ w_0, \end{cases} \quad m_f = \begin{cases} \frac{(1-\delta_u)(1-w_0)}{1-\delta_u\delta_f}, & \text{if } \left(\delta_u, \delta_f\right) \in A, \\ w_0, & \text{if } \left(\delta_u, \delta_f\right) \notin A, \end{cases} \tag{8}$$

$$\text{where } A = \left\{(\delta_u, \delta_f) : \ \delta_u \leq \delta_f, \delta_u(\delta_u - w_0)\delta_f \leq (1 - w_0)\delta_u^2 + w_0\delta_u - w_0\right\}. \tag{9}$$

Proposition 3 validates Fernandez and Glazer's (1991) finding on the union's best SPE if and only if $\delta_u \leq \delta_f$. Their Lemma 4 provides the SPE strategy profile to support M_u and m_f for $(\delta_u, \delta_f) \in A$. Strike is credible if and only if $(\delta_u, \delta_f) \in A$, as illustrated in Fig. 1, which requires that the union be patient enough, at least $\delta_u > w_0$, and the firm's discount factor be bounded by

$$\delta_f \leq \frac{(1 - w_0)\delta_u^2 + w_0\delta_u - w_0}{\delta_u(\delta_u - w_0)}.$$

When $\delta_f < \delta_u$, the union best SPE becomes more complicated since it is possible to have continuation payoffs above the bargaining frontier.

Proposition 4 *When $\delta_f < \delta_u$, (4) and (7) yield*

$$M_u = \begin{cases} \frac{1+w_0\delta_f}{1+\delta_f}, \\ \frac{1-\delta_f}{1-\delta_u\delta_f}, \\ w_0, \end{cases} \quad m_f = \begin{cases} \frac{1-w_0}{1+\delta_f}, & \textit{if } \left(\delta_u, \delta_f\right) \in B, \\ \frac{1-\delta_u}{1-\delta_u\delta_f}, & \textit{if } \left(\delta_u, \delta_f\right) \in C, \\ 1 - w_0, & \textit{if } \left(\delta_u, \delta_f\right) \notin B \cup C, \end{cases} \tag{10}$$

$$\textit{where } B = \left\{ (\delta_u, \delta_f) : \delta_f < \delta_u, \ \delta_f \geq \frac{\delta_u - w_0}{1 - \delta_u w_0}, (\delta_u - w_0)\delta_f \geq w_0(1-\delta_u) \right\}, \tag{11}$$

$$C = \left\{ (\delta_u, \delta_f) : \delta_f \leq \frac{\delta_u - w_0}{1-\delta_u w_0}, (\delta_u - w_0)\delta_f \leq \frac{\delta_u^2 - w_0}{\delta_u} \right\}. \tag{12}$$

For $(\delta_u, \delta_f) \in B$, Proposition 4 implies that the union's alternating-strike strategy and the firm's no-concession strategy suggested by Bolt (1995), also implied by Slantchev (2003), constitute the union's best SPE. In this case, the union is patient enough to threaten to strike, but the firm is unable to gain from making the least irresistible offer. As illustrated in Fig. 1, for given δ_f, δ_u is too low for the union to benefit from the always-strike strategy. As a consequence, the firm capitalizes $1 - w_0$ every other period. For $(\delta_u, \delta_f) \in C$, however, Proposition 4 implies that the always-strike strategy suggested by Bolt (1993) implements the union's best SPE. In this second case, the union's best SPE features immediate agreement in every subgame since the corresponding continuation payoff is bounded from above by the bargaining frontier.

When $(\delta_u, \delta_f) \notin A \cup B \cup C$, the contract negotiation model has a unique SPE where the union receives w_0. The union's best SPE payoff is continuous, except on the left boundary of $A \cup B \cup C$ where strike just becomes credible and the union's best SPE payoff jumps to a level above w_0.

As a final result, there are many equilibria whose payoffs are above the bargaining frontier when $(\delta_u, \delta_f) \in B \cup C$. Consider the following pure strategy profile: The union holds out for $T > 0$ periods during which no party makes acceptable offers, followed by the union's best equilibrium. Such a strategy profile is a SPE that is not Pareto dominated by any other SPE, because of the discussion in Sect. 2. The correlation device in Sect. 2 may be assumed to connect these points. As a consequence, all SPE with immediate agreement, except the union's worst and best SPE, are not Pareto efficient and delay is Pareto improving.

To summarize, our paper settles a long-standing debate concerning on the contract negotiation model when players have different discount factors: efficient delay has been overlooked and its presence requires a careful and modified implementation of Shaked and Sutton's (1984) method. More importantly, our paper raises the awareness of a similar issue in the bargaining literature at large: In many applications of Shaked and Sutton's backward induction argument, it is often presumed that continuation payoffs are bounded from above by the bargaining frontier without verification of this presumption. Our paper emphasizes that we need to verify any presumption in such analysis for its integrity.

Appendix: Proofs

Proof of Proposition 1 In this proof, we will identify when each of the three cases in (3) applies to bound the value of m_f.

First, if $\delta_u \leq \delta_f$, then $m_f \geq (3a)$ because $(3a) \geq (3b)$ due to (1) and $(3c) > (3a)$ due to $(1-\delta_u)w_0 > 0$. This establishes the last case in (4).

Second, if $\delta_f < \delta_u$, then $(3b) > (3a)$ due to (1). Condition (33) implies that m_f is not less than the minimum of $(3b)$ and $(3c)$. Notice that

$$(3b) < (3c) \quad \text{if and only if } (\delta_u - \delta_f)M_u < (1-\delta_f)w_0.$$

Therefore, if $\delta_f < \delta_u$ and $(\delta_u - \delta_f)M_u < (1-\delta_f)w_0$, condition (3) implies that $m_f \geq (3b)$, which establishes the second case in (4).

Lastly, if $\delta_f < \delta_u$ and $(\delta_u - \delta_f)M_u \geq (1-\delta_f)w_0$, then $(3c) \leq (3b)$ and strike is credible because $\delta_u M_u - w_0 \geq \delta_f(M_u - w_0) \geq 0$ due to (1). Hence, condition (3) implies $m_f \geq (3c)$, which is the first case of (4). In the first case of (4), the credibility constraint is implied by the condition stated for this case. □

Proof of Proposition 2 First, when strike is not credible, i.e., $\delta_u(1-m_f) < w_0$, M_u is less than or equal to the maximum of $(6a)$ and $(6b)$. Then $\delta_u(1-m_f) \leq w_0$ implies that $(6a) \geq (6b)$ if and only if $\delta_f \geq \delta_u$, which establish the last two cases in (7).

Second, when strike is credible, i.e., $\delta_u(1-m_f) \geq w_0$, it is obvious that $(6c) \geq (6a)$ since $(1-\delta_f)(1-w_0) \geq 0$. It is also true that $(6c) \geq (6b)$ since

$$(6c) \geq (6b) \quad \text{if and only if } (1-\delta_u)(1-w_0) \geq (\delta_f - \delta_u)m_f,$$

which is trivial due to the fact of $m_f \leq 1 - w_0$ by (1) and $\delta_f < 1$. Therefore, $M_u \leq (6c)$ when $\delta_u(1-m_f) \geq w_0$, the first case of (7). □

Proof of Proposition 3 When $\delta_f \geq \delta_u$, there are two possible cases to consider from (4) and (7)

Case 1 Suppose that $\delta_u(1-m_f) \geq w_0$. (4) and (7) then imply that

$$m_f \geq (1-\delta_u)(1-w_0) + \delta_u(1-M_u) \quad \text{and} \quad M_u \leq 1 - \delta_f m_f,$$

which yield the upper bound of M_u and the lower bound of m_f as stated in the first part of (8). Substituting the lower bound of m_f into the credibility constraint $\delta_u(1-m_f) \geq w_0$, we have

$$\delta_u\left[1 - \frac{(1-\delta_u)(1-w_0)}{1-\delta_u\delta_f}\right] \geq w_0.$$

From the last inequality, we obtain

$$\delta_u(\delta_u - w_0)\delta_f \leq (1-w_0)\delta_u^2 + w_0\delta_u - w_0,$$

which is true if and only if $(\delta_u, \delta_f) \in A$ as defined by (9) when $\delta_u \leq \delta_f$.

Case 2 Suppose that $\delta_u(1-m_f) < w_0$. Inequalities (4) and (7) imply that

$$m_f \geq (1-\delta_u)(1-w_0) + \delta_u(1-M_u) \quad \text{and} \quad M_u \leq 1-(1-\delta_u)(1-w_0) - \delta_u m_f,$$

which yields $m_f \geq 1-w_0$ and $M_u \leq w_0$. Together with (1), we have $M_u = 1-m_f = w_0$, the second part of (8). In this case, strike is obviously not credible and there is a unique SPE. □

Proof of Proposition 4 When $\delta_f < \delta_u$, we need to consider the following four possible cases from (4) and (7)
Case 1 Suppose that $(\delta_u - \delta_f)M_u \geq (1-\delta_f)w_0$ and $\delta_u(1-m_f) < w_0$. Inequalities (4) and (7) then imply that

$$m_f \geq 1-\delta_u M_u \quad \text{and} \quad M_u \leq (1-\delta_u)w_0 + \delta_u(1-m_f),$$

which yield $M_u \leq \frac{1}{1+\delta_u} w_0 < w_0$. However, this contradicts with the fact that $M_u \geq w_0$ by (1). Therefore, Case 1 is not possible.
Case 2 Suppose that $(\delta_u - \delta_f)M_u < (1-\delta_f)w_0$ and $\delta_u(1-m_f) \geq w_0$. Inequalities (4) and (7) imply that

$$m_f \geq (1-\delta_f)(1-w_0) + \delta_f(1-M_u) \quad \text{and} \quad M_u \leq 1-\delta_f m_f,$$

which yield the upper bound of M_u and the lower bound of m_f as stated in the first part of (10). With these bounds, we have

$$\begin{aligned} (\delta_u - \delta_f)M_u < (1-\delta_f)w_0 \quad &\text{if and only if} \;\; (1-w_0\delta_u)\delta_f > \delta_u - w_0, \\ \delta_u(1-m_f) \geq w_0 \quad &\text{if and only if} \;\; (\delta_u - w_0)\delta_f \geq (1-\delta_u)w_0, \end{aligned}$$

which are true if and only if $(\delta_u, \delta_f) \in B$ defined by (11). This establishes the first part of (10).
Case 3 Suppose that $(\delta_u - \delta_f)M_u \geq (1-\delta_f)w_0$ and $\delta_u(1-m_f) \geq w_0$. Inequalities (4) and (7) then imply that

$$m_f \geq 1-\delta_u M_u \quad \text{and} \quad M_u \leq 1-\delta_f m_f,$$

which yield the upper bound of M_u and the lower bound of m_f as stated in the second part of (10). These bounds are the SPE payoffs in Bolt (1995) corresponding to the always-strike strategies. With these bounds, we have

$$\begin{aligned} (\delta_u - \delta_f)M_u \geq (1-\delta_f)w_0 \quad &\text{if and only if} \;\; (1-w_0\delta_u)\delta_f \leq \delta_u, \\ \delta_u(1-m_f) \geq w_0 \quad &\text{if and only if} \;\; \delta_f\delta_u(\delta_u - w_0) \geq \delta_u^2 - w_0, \end{aligned}$$

which are true if and only if $(\delta_u, \delta_f) \in C$ defined by (12). This establishes the second part of (10).

Case 4 Suppose that $(\delta_u - \delta_f)M_u \leq (1 - \delta_f)w_0$ and $\delta_u(1 - m_f) < w_0$. (4) and (7) imply that

$$m_f \geq (1 - \delta_f)(1 - w_0) + \delta_f(1 - M_u) \quad \text{and} \quad M_u \leq (1 - \delta_u)w_0 + \delta_u(1 - m_f),$$

which yield that $M_u \leq w_0$ and $m_f \geq 1 - w_0$. Obviously, the two conditions for this case hold. Together with (1), we have $M_u = 1 - m_f = w_0$, which establishes the last part of (10) where there is a unique SPE. □

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