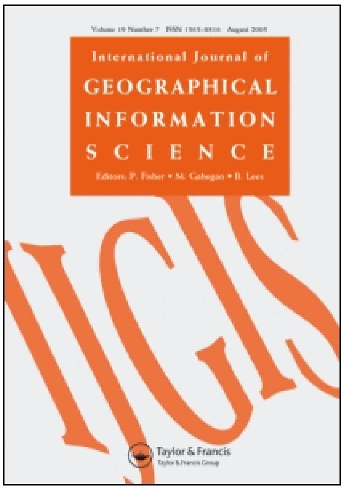

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Research Article

Using simulated annealing for resource allocation

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Abstract. Many resource allocation issues, such as land use- or irrigation planning, require input from extensive spatial databases and involve complex decision-making problems. Spatial decision support systems (SDSS) are designed to make these issues more transparent and to support the design and evaluation of resource allocation alternatives. Recent developments in this field focus on the design of allocation plans that utilise mathematical optimisation techniques. These techniques, often referred to as multi-criteria decision-making (MCDM) techniques, run into numerical problems when faced with the high dimensionality encountered in spatial applications. In this paper we demonstrate how simulated annealing, a heuristic algorithm, can be used to solve high-dimensional non-linear optimisation problems for multi-site land use allocation (MLUA) problems. The optimisation model both minimises development costs and maximises spatial compactness of the land use. Compactness is achieved by adding a non-linear neighbourhood objective to the objective function. The method is successfully applied to a case study in Galicia, Spain, using an SDSS for supporting the restoration of a former mining area with new land use.

1. Introduction

The decision framework of a spatial decision support system (SDSS) is a stepwise approach guiding the decision-maker through a decision-making process. An example is the framework for analysis (FFA), which consists of five steps (figure 1) (Findeisen and Quade 1985, Twillert 2000). Steps 1 and 2 comprise the definition of the problem, the (multiple-) objectives, evaluation criteria and constraints. Step 3 involves the determination of 'exogenous' influences. Step 4 is the 'computational step', where alternatives ('possible solutions') are designed and evaluated. Step 5 presents the preferred alternative.

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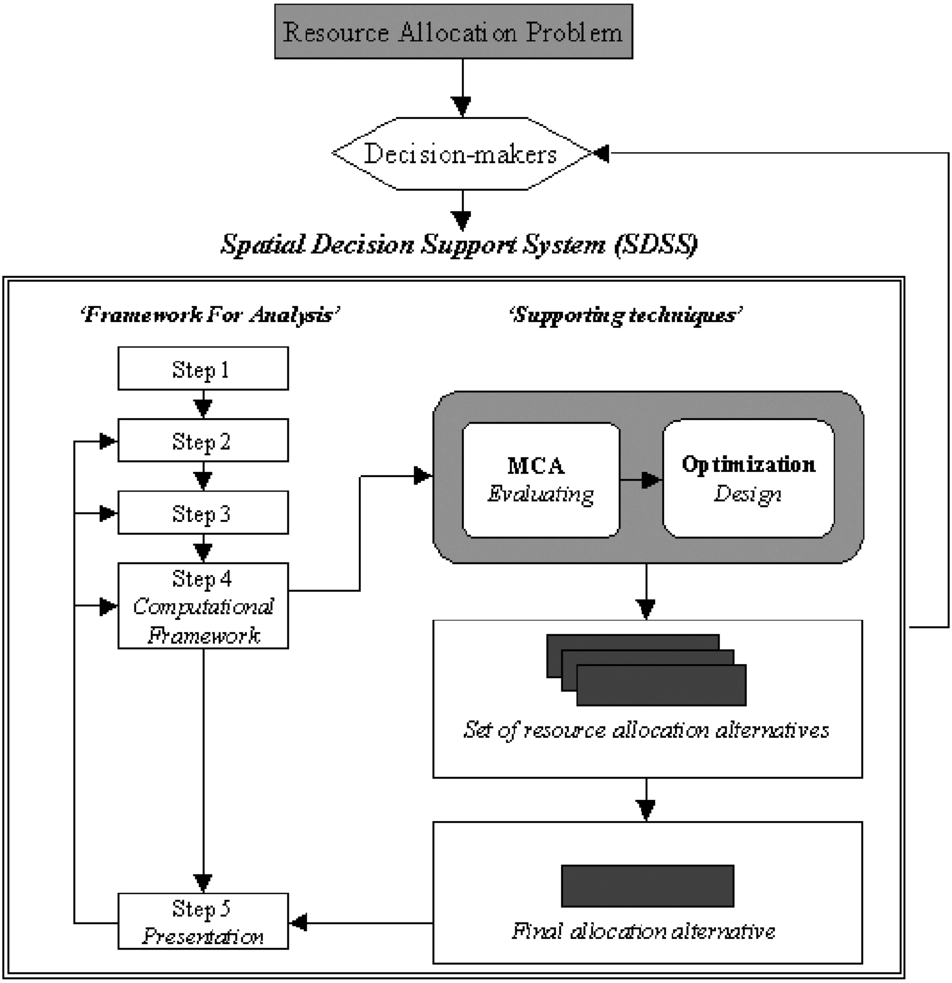

Figure 1. Structure of an SDSS for resource allocation with the framework for analysis (*left*) and optimisation techniques (*right*).

When the alternatives are defined beforehand, they are simply evaluated against each other. An evaluation thus involves the comparison of a limited number of alternatives, usually about three to five, and rarely more than ten. A well-known evaluation technique that has been used to evaluate (spatial) allocation alternatives is multi criteria analysis (MCA) (Voogd 1983 1995). Examples, which apply MCA in a geographical information system (GIS) environment are described by Carver (1991), Eastman *et al.* (1993), Pereira and Duckstein (1993) and Herwijnen (1999).

One major drawback of MCA is that it does not allow the comparison of a large number of alternatives. With only a few alternatives to be evaluated, it is almost certain that the best alternative chosen from the set is in fact a sub-optimal solution. This problem can be avoided by applying a *design* technique. Design techniques generate an optimal solution from a much larger or possibly infinite set of alternatives, where the set of alternatives to choose from is implicitly defined through the constraints defined in step 2 of the decision framework. In other words, the optimal solution is created or designed by the optimisation procedure itself. The optimal solution must be cleverly obtained by employing sophisticated search algorithms. Often, numerical optimisation techniques such as linear integer programming (LIP) are used for this purpose, and these are assembled under the term multi-objective mathematical programming (MMP) (Ridgley *et al.* 1995, Greenberg 2000). MCA and MMP together are referred to as multi criteria decision-making (MCDM) techniques (e.g. Zeleny 1973).

Some recent examples that use MMP in combination with GIS are presented by Ridgley *et al.* (1995), Williams and Revelle (1996, 1998), Snyder and Revelle (1996), Tkach and Simonovic (1997), Ridgley and Heil (1998), Cova (1999) and Cova and Church (2000). The application of MMP techniques in a spatial context is far from straightforward, though. One major difficulty is the large dimensionality of the problems, which may involve solving a resource allocation model for an area measuring more than 1000 by 1000 cells. Furthermore, some of the criteria involved introduce non-linearities, in particular those that require the solution to be spatially compact. For instance, in forestry research harvest schedules are to be optimised while dealing with strict adjacency constraints (e.g. Jones *et al.* 1991, Lockwood and Moore 1993, Murray and Church 1995). Although the power of LIP techniques is steadily increasing (Church *et al.* 1996, Cova 1999), as yet the use of MMP techniques for resource allocation problems is limited to areas with a much reduced spatial resolution or restricted grid size (up to 25 by 25 cells).

The main objective of this paper is to investigate whether simulated annealing, a heuristic algorithm that has successfully been applied to many other optimisation problems, is an attractive alternative to the analytically-driven MMP techniques for solving non-linear-high-dimensional resource allocation problems. We consider simulated annealing to be successful when it (1) can handle a large grids (at least 250 by 250 grid cells) (2) can handle non-linearities such as the implementation of a spatial compactness objective and (3) can be easily implemented in an SDSS.

2. Simulated annealing for resource allocation optimisation

2.1. *Basic optimisation model*

A combinatorial optimisation problem can be formulated as a minimisation or a maximisation problem (there is no real difference) and is specified by a set of problem instances. A problem instance of a combinatorial optimisation problem is defined as a pair (S, f), where the solution space S denotes the set of all possible solutions and f denotes the cost function. In the case of minimisation, the problem is to find an optimal solution $i_{opt} \in S$, which satisfies:

$$f(i_{opt}) \leqslant f(i) \qquad \text{for all } i \in S \tag{1}$$

As an example, consider a rectangular area to be allocated with land use. First, the area is divided to a grid with N rows and M columns. Let there be K different land use types, and let $P_k (k=1, \ldots, K)$ be the pre-determined proportion of land that must be allocated with land use k. We now introduce a binary variable x_{ijk} which equals 1 when land use k is assigned to cell (i,j) and equals 0 otherwise. Furthermore, development costs (C_{ijk}) are involved with each land use type k in cell (i,j). These costs vary with location because they may depend on specific physical attributes of the area, such as soil type, elevation and slope.

The goal is to minimise development costs, while satisfying the constraint of pre-set percentages of land use types. Accordingly, the problem may be written as follows:

Minimise

$$\sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{M} C_{ijk} x_{ijk} \tag{2}$$

Subject to

$$\sum_{k=1}^{K} x_{ijk} = 1$$

$$\forall\ i=1, \mathrm{K}, N, j=1, \mathrm{K}, M \quad x_{ijk} \in \{0, 1\} \tag{3}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{M} x_{ijk} = N.M.P_k \ \forall\ k=1,\mathrm{K},K \tag{4}$$

Equation (3) specifies that one and only one land use must be assigned to each cell. Equation (4) guarantees that the required percentage of land use is attained for each land use type. Because decision variable x_{ijk} must be either 0 or 1, the model is defined as an integer programme (IP).

2.2. *Principles of simulated annealing*

Before we apply simulated annealing (SA) to the basic optimisation model, we first briefly explain the principles behind the technique.

Kirkpatrick *et al.* (1983) introduced the concept of annealing in combinatorial optimisation. This concept is based on a strong analogy between combinatorial optimisation and the physical process of crystallisation. This process has inspired Metropolis *et al.* (1953) to propose a numerical optimisation procedure known as the Metropolis algorithm, which works as follows. Starting from an initial situation with 'energy level' $f(0)$, a small perturbation in the state of the system is brought about. This brings the system into a new state with energy level $f(1)$. If $f(1)$ is smaller than $f(0)$, then the state change is accepted. If $f(1)$ is greater than $f(0)$, then the change is accepted with a certain probability. A movement to a state with a higher energy level is sometimes allowed to be able to escape from local minima. The probability of acceptance is given by the Metropolis criterion (Aarts and Korst 1989, Rogowski and Engman 1996):

$$P(accept\ change) = \exp\left(\frac{f(0) - f(1)}{s_0}\right) \tag{5}$$

where s_0 is a control or freezing parameter. Next, the freezing parameter is slightly decreased and a new perturbation is made. The energy levels are again compared and it is decided whether the state change is accepted. This iterative procedure is repeated until a maximum number of iterations is reached or until change occurrences have become very rare.

The analogy with spatial optimisation assumes that physical states are replaced by *solutions* and *energy levels* by costs. Although it cannot be proven that simulated annealing guarantees an optimal solution, practice has shown that a sufficiently slow decrease of the freezing parameter yields in almost all cases the optimal solution (Lockwood and Moore 1993, Sundermann 1995).

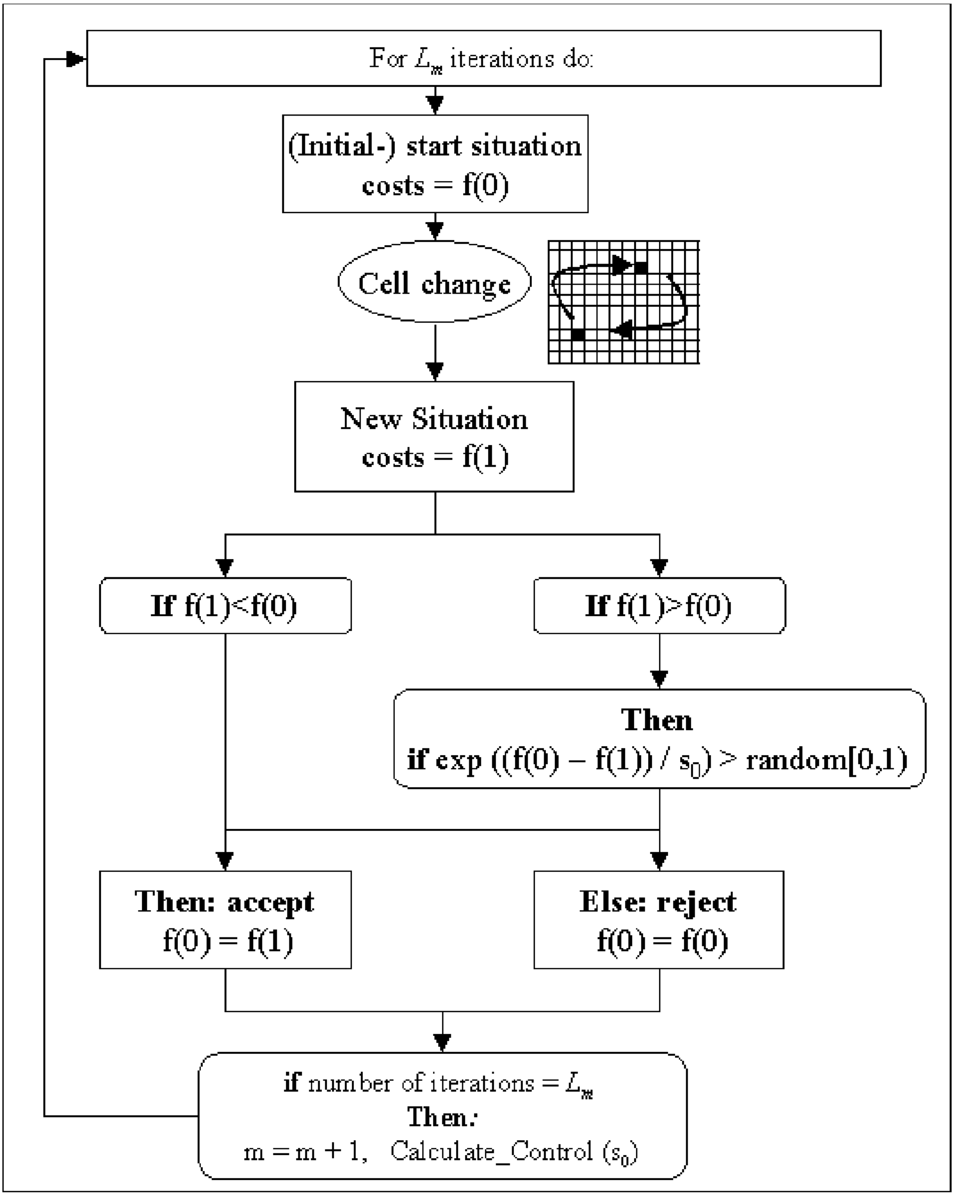

Figure 2. Flow diagram of the simulated annealing algorithm.

Figure 2 presents a flow diagram of the simulated annealing algorithm. A crucial element of the procedure is the gradual decrease of the freezing parameter s_i (Laarhoven 1987). Usually, this is done using a constant multiplication factor:

$$s_{i+1} = r \cdot s_i \tag{6}$$

where $0 < r < 1$. This effectively means that jumping to higher energy (read: costs) becomes less and less likely towards the end of the iteration procedure (Levine 1999).

The decrease of the freezing parameter using equation (6), is conducted only once every L iterations.

The 'cooling schedule' refers to the choice of the three parameters of the SA algorithm. These are the iteration length per temperature stage L, the initial value of the freezing parameter s_0 and the decrease factor r. As a rule of thumb, s_o should be chosen so that initially about 80% of the changes that increase the cost function are accepted (Laarhoven 1987). The correct parameter value can be obtained by running the algorithm shortly for a fixed s_0, calculating the corresponding acceptance rate, and adjusting s_0 until an acceptance rate of about 80% positive changes is achieved. Typical values for the decrease factor r are between 0.80 and 0.98 (Laarhoven 1987), although it is difficult to make generally valid statements. The total number of iterations L per temperature stage is chosen by keeping the temperature constant until the cost function has reached a constant value, or until it is oscillating around this constant value (Sundermann 1995).

Examples of studies that use SA for spatial optimisation can be found in the area of image enhancement (Sundermann 1995), in ecological research (Church *et al.* 1996) and in forestry research (Lockwood and Moore 1993, Boston and Bettinger 1998). SA has also been used to optimise spatial sampling (Van Groenigen and Stein 1998) and for generating realizations of random fields (Goovaerts 1997).

2.3. *Application of simulated annealing to the basic optimisation model*

We now use SA to solve the basic optimisation model. Consider three land use types *lu*1, *lu*2 and *lu*3 ($K=3$) and divide the area to a 10×10 grid ($N=M=10$). The required spatial coverage of the three land use types is taken as 57% for *lu1*, 29% for *lu2* and 14% for *lu*3 ($P_1=0.57$, $P_2=0.29$ and $P_3=0.14$). We also use fictitious development costs. These are given in figure 3.

The initial situation is a random distribution of the three land use types over the area, though satisfying the required percentage coverage per land use type. The associated development cost is denoted by $f(0)$. Following the flow diagram of figure 2, we now swap the land use of two randomly chosen cells. This yields a new situation, with new development costs $f(1)$. Whether the change from state 0 to

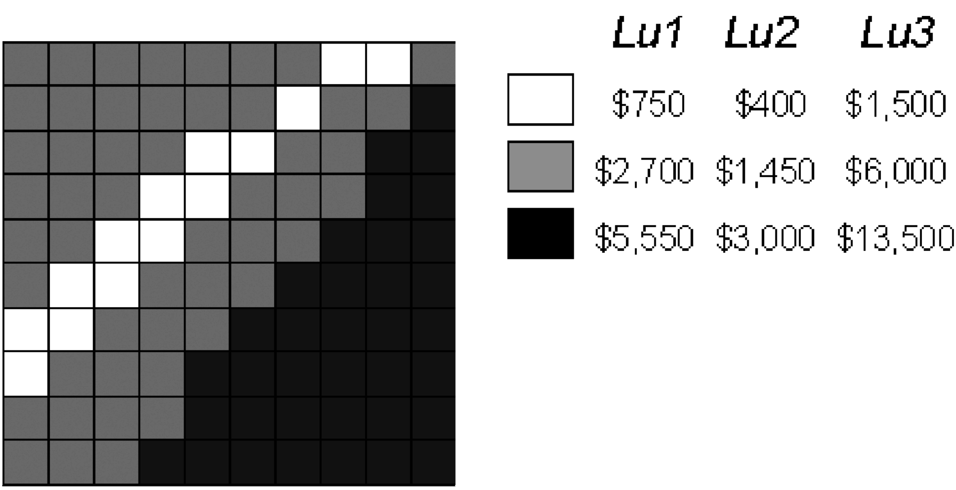

Figure 3. Map of development costs per land use type.

state 1 is accepted, depends on the difference in costs $f(1)–f(0)$. Once this is decided the swapping procedure is repeated, and it is decided whether the change is accepted. Next, a new swap is generated, and so on. Whenever the costs $f(i+1)$ are smaller than the costs $f(i)$, the cell change is accepted. When $f(i+1)$ greater than $f(i)$, costs are accepted with a certain probability following the Metropolis criterion expressed in Equation 5. This is achieved by comparing the value of the Metropolis criterion with a random number drawn from a uniform [0, 1] distribution (figure 2).

The starting values for the parameters s_0, r and L are given in table 1. Four different combinations are considered.

2.4. *Results for the basic model*

Figure 4 shows the cost function f against the total number of iterations $m.L$. The different runs A, B, C and D are the results of runs with the four different parameter sets given in table 1. Run A represents the start situation. For run B, the decrease factor r was much smaller than for run A. This obviously speeded up the search process. However, it should be noted that this increased the risk of ending up in a local minimum. Apparently, this did not occur here because runs A and B yield the same final costs. Run C shows the result of applying a relatively slow cooling schedule with $L=10\,000$. Although not displayed in figure 4, it achieves the same optimal result as runs A and B, but then only after 170 000 iterations. Graph

Table 1. Parameter setting used for the basic model.

	C_0	r	L
Run A	12 438	0.8	1000
Run B	12 438	0.2	10 000
Run C	12 438	0.8	10 000
Run D	12 438	0.2	1000

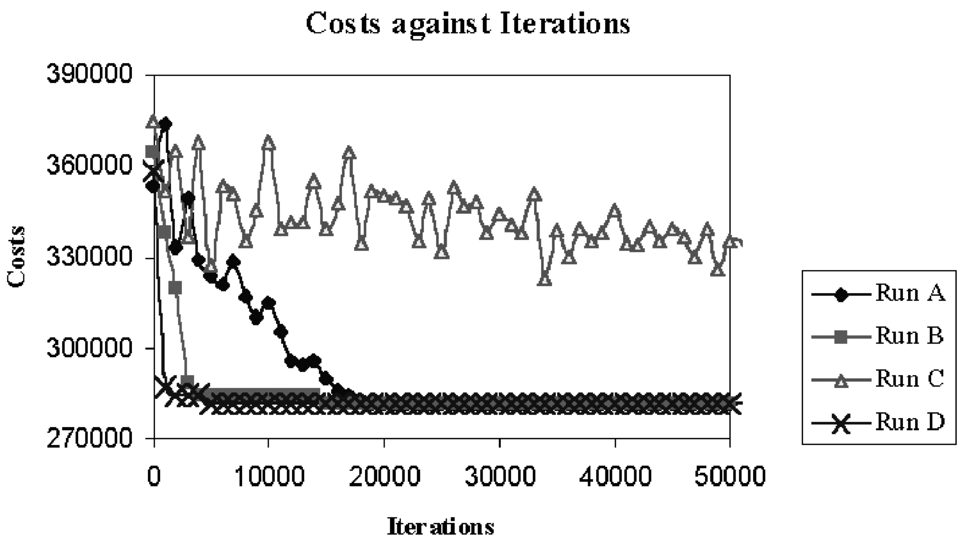

Figure 4. Total costs against number of iterations using simulated annealing for a 10 × 10 grid.

D shows a run with an extremely small decrease factor $r=0.2$. Again, the same optimum is found as for the parameter settings used in A, B and C. It is remarkable that with such an extreme fast cooling schedule the same optimum is obtained. Perhaps this may be explained by the straightforward optimisation criterion used here yielding a fairly smooth cost surface.

Figure 5 shows different stages in the iteration process using parameter set A (table 1). The map at the *far left* shows the initial—random—situation. The map to the *far right* shows the final situation, achieved after a total of 23.1000=23 000 iterations. At this stage, the objective function could no longer be improved and the iteration was terminated. The other three cases B, C and D yield a similar final land use allocation pattern. Although there are small differences between them, the associated costs are the same. Apparently, in this case there is no unique optimum solution and all four solutions are considered equally optimal.

2.5. *Adding spatial compactness criteria to the basic model*

In the previous section development costs was the only objective to be minimised. In practice, however, there will be additional spatial-pattern objectives that should be included in the optimisation procedure. For instance, a land resource allocation plan that creates noisy, patchy spatial elements is not attractive. Instead, large and compact areas of the same land use are preferred. In other words, we need to extend the cost function with a term that encourages compactness of land use. This may be achieved by rewarding cases where neighbouring cells have equal land use. Consider cell (i,j) with land use x_{ijk}, with the following neighbour aggregate variable:

$$b_{ijk}=x_{i-1jk}+x_{i+1jk}+x_{ij-1k}+x_{ij+1k}$$
$$k=1, \ldots, K,\ i=1, \ldots, N,\ j=1, \ldots, M \qquad (7)$$

Thus b_{ijk} is the number of cells neighbouring cell (i,j) that have land use k.

Here we have used a neighbourhood of four cells (top, down, left, right), but alternatively a larger neighbourhood may be defined. Note also that equation (7) must be modified for cells at the boundary of the area. The easiest way to do this is by assuming that:

$$x_{ijk}=0 \qquad \forall k=1, \ldots, K, i \in \{0, N+1\}, j \in \{0, M+1\} \qquad (8)$$

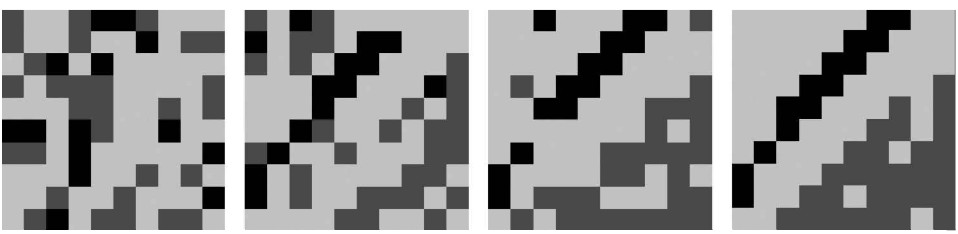

Figure 5. Different stages in the optimisation process. *Far left*: initial situation with a random assignment of land use; *left*: after 5 iteration stages; *right*: after 10 iteration stages; *far right*: final situation after 23 iteration stages. Land use *lu1* (57 %) is presented in light grey, *lu2* (29 %) in dark grey and *lu3* (14 %) in black.

If we define the objective function as:

$$\sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{M} -b_{ijk}x_{ijk} \tag{9}$$

then minimisation of equation (9) yields solutions that are spatially compact. Note that equation (9) is non-linear because the coefficients b_{ijk} depend on x_{ijk} through equation (7). The non-linearity of the objective function implies that analytical solutions to the optimisation problem are difficult to obtain.

The overall cost function of the basic model now contains two terms: development costs as in equation (2), and compactness costs as defined in equation (9). In order to allow for a preference for either of these two objectives, a weighting factor β is introduced. The objective function becomes:

Minimise:

$$\sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ijk}x_{ijk} - \beta \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ijk}x_{ijk} \tag{10}$$

Subject to equations (3, 4 and 7).

Minimisation of equation (10) is achieved using the SA procedure described before. This is done for various values of β, so as to compare how incorporating compactness criteria affects the optimal solution. Using the same initial state and parameter settings as given in table 1 under A, four different values for β were evaluated ($\beta=1$, $\beta=3$, $\beta=5$ and $\beta=10$). All four cases reach the optimum in about the same number of iterations (about 28 000). This is not surprising given that the same parameter settings were used.

Figure 6 depicts the optimal solutions for the four values of β. The smallest β value obviously yields a solution that most resembles the optimal solution obtained with only development costs involved (figure 6, *far right*). As β increases and compactness becomes more important, the optimal solution becomes more compact and thus allocates land use where development costs are relatively high but where the compactness objective is better met. The largest value of β yields an almost completely rectangular-shaped pattern.

3. Case study: restoration of an open mining area

Consider the open cast lignite mine of As Pontes in Galicia, in the North Western part of Spain covering a total area of about 25 km^2 (figure 7). The lignite is used to generate electricity in a power plant, which is situated just outside the mining area. The mining area can be divided into two main areas. The exploitation area, which consists of two pits of about 200–250 m deep, and the dump area where the waste

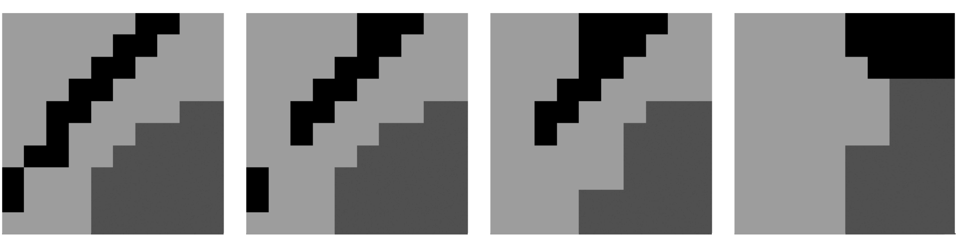

Figure 6. Final results of the optimisation using different values for β. *Far left*: $\beta=1$; *left*: $\beta=3$; *right*: $\beta=5$; *far right*: $\beta=10$.

Figure 7. Location of Galicia in North West Spain (*upper left*). The aerial photograph shows the dump-site with a relatively white reflection caused by the bare soil.

material is stored. Basically, all waste material (50 % of the total extracted material) is stored in the dump area (Aerts 1999). Within five years, the mine will be closed after which the area has to be restored as much as possible to the original state.

3.1. *Development and use of an SDSS for restoration of the As Pontes mining area*

Within the EU funded project 'Asterismos' (Aerts 1999), an SDSS using the framework for analysis (FFA) was developed for supporting the evaluation and generation of possible restoration plans. Workshops were organised in which mining experts and other stakeholders from the neighbouring municipality participated. These workshops provided the information for Steps 1 to 3 of the FFA, which included all parameter values, boundary conditions and objective functions as required by the optimisation model. Within Step 4 ('computational step') and Step 5 ('presentation'), possible restoration plans are generated. For this, the study area has been divided into a grid of 300 by 300 cells of 25 m × 25 m. The simulated annealing model, which fits into Step 4, was developed separately from the other Steps, using a combination of Delphi (Borland

2001) and ESRI/Mapobjects (ESRI 2001) software to import, manipulate and visualize the input data and to present the generated optimal land use map.

Recently enforced European legislation states that mining companies are obliged to produce a restoration plan for any mining area, and do so while the mine is still in operation. The main requirement for such a plan is that the land uses that were lost because of the mining exploitation will be restored as close as possible to the pre-mining situation. This means that if 1000 hectares of pine forest are cleared for mining activities, then 1000 hectares of pine forest have to be restored after the activities have been stopped. There are no rules as to where within the area these 1000 hectares of pine forest need to be re-allocated, and neither are there legal requirements that firm that all land uses have to be re-established in exactly the same sizes. This leaves room for designing the restoration plan such that the legal requirements are met at lowest restoration costs. Thus, the main problem that came out of Step 1 of the FFA is to design an optimal restoration plan that complies with European legislation.

Within Step 2, we narrowed the main problem down by considering the objectives, evaluation criteria and constraints that pertain to the As Pontes case. First, it was observed that due to the considerable lowering of the elevation in the mining pit, this part of the area would automatically transform into a lake through groundwater flow. It was clear to all parties involved that the costs associated with preventing this from happening would be far too high. Therefore, the restoration plan focuses only on the dumpsite. The mining experts expressed that the main interest for the mining company was simply to restore the area at the lowest costs, whilst satisfying the legislative requirements. Other parties involved in the workshops claimed that an important additional objective should be to create closed patches of land use, as large closed areas of forest and water represent a higher natural value than fragmented areas and because less fragmented areas have an increased potential for recreational activities. It was also stated that the restoration works could be carried out more economically with less fragmented areas of the same land use, although the expected reduction of costs were not quantified and thus were not included in the optimisation model.

The main constraints refer to the required division of land use types: the restored land use has to be of a similar type and size as in the pre-mining situation. Knowing the potential land use types for restoration (forest, shrub and water), constraints were set for the required surface cover of these potential land use types. Approximately 60 %, 22 % and 18 % of the area was originally covered by forest, shrub and water, respectively. The percentages were derived from classified Landsat TM images that were acquired before mining exploitation in 1984.

Deriving constraints in Step 2 also entails the definition of development costs (C). Although development costs encompasses a broad range of activities, we estimated an average cost per land use type k that primarily depends on the physical attributes 'elevation' and 'slope' (Aerts 1999). Another simplification was to ignore the spatial requirement for creating wildlife corridors. This requirement would involve an adjustment within the compactness objective. Both elevation and slope were derived from remote sensing data using SPOT stereo pairs at a resolution of $25\,\mathrm{m}\times25\,\mathrm{m}$ (figure 8). The cost function is (in \$ per m^2):

$$C_k = a_k \times \text{elevation} + b_k \times \text{slope} \tag{11}$$

where elevation is in meters and slope in degrees. The values of the parameters a_k and b_k

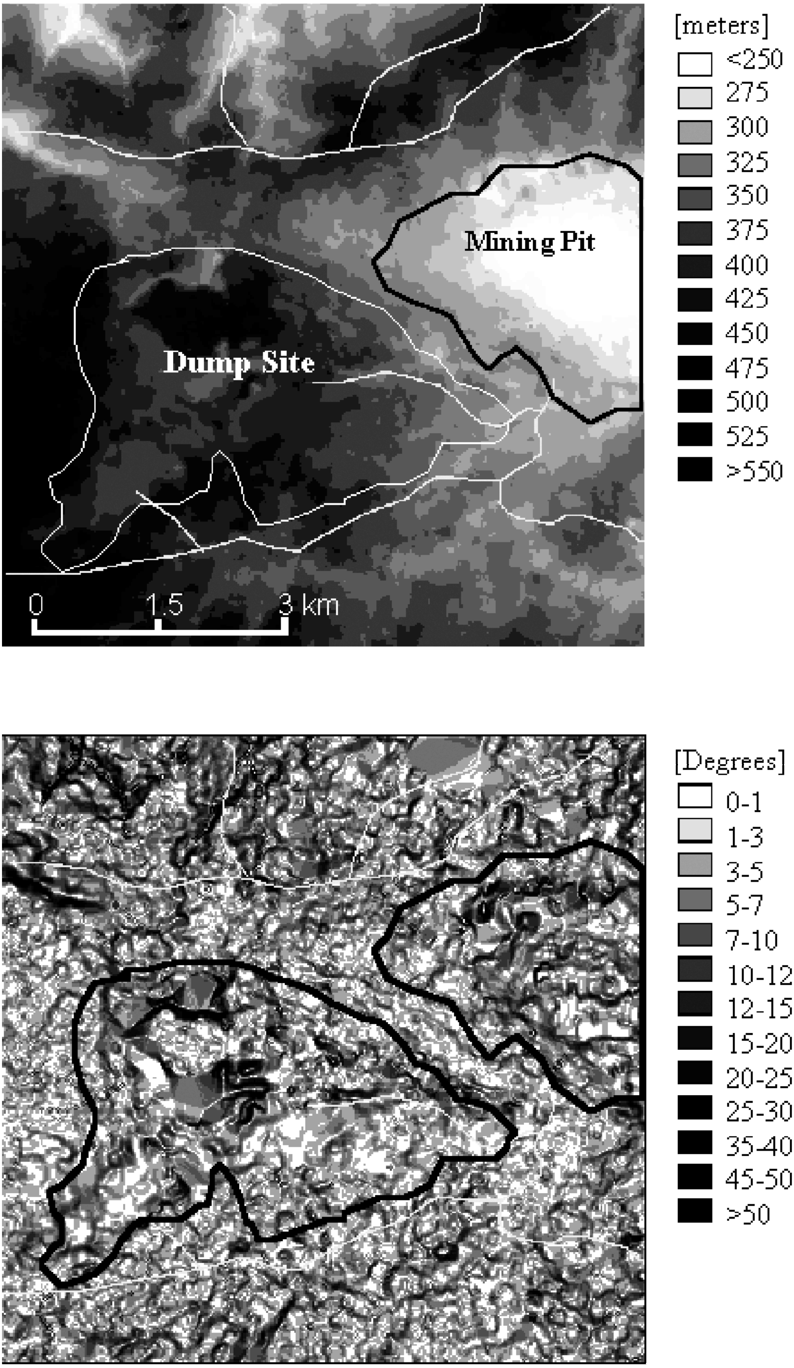

Figure 8. Digital elevation model (DEM) (*top*) and slope map (*down*) of the As Pontes mining area. The data has been derived from SPOT stereo pairs at a resolution of $25\,m \times 25\,m$.

depend on the land use type k, where a_k varies between 1 and 1.5 and b_k has been assigned to values between 1.5 and 3.0. The allocation of water only depends on elevation.

The last decision to be made within Step 2 of the FFA is to assign weights to the objective functions. A choice was made to initially weigh the compactness with $\beta=3$ (see §2), using parameter setting A (table 1). From here, β has been varied in an iterative fashion to examine its influence on the compactness of the land use. Using different values for the weight β and visual inspection of the corresponding solution. A value $\beta=3$ appeared to be an attractive compromise between 'costs' and 'compactness'.

There were no external influences of importance to the restoration problem. Therefore, Step 3 of the FFA was skipped.

3.2. *Case study results*

Figure 9 shows the initial situation at the beginning of the optimisation run and the final optimised allocation plan for the dumpsite. The optimised plan shows clearly that water is allocated towards the mining pit, where the elevation of the landscape drops. The figure shows furthermore separate spots of water, which indicate lower elevation 'pockets' within the dump area. This does not necessarily mean that these areas are indeed the most suitable areas for creating small wetlands, because the circumstances for creating wetlands depend largely on the available water within the neighbouring catchment. Additional catchment analysis is required to really determine whether these spots have a potential for being developed into a wetland area.

Figure 10 shows the development of the cost function. The graph shows a logarithmic decrease in costs, which indicates a slow cooling process. When comparing the graphs of figures 10 and 4, it appears, not surprisingly, that the total number of iterations is much larger for the large area of 300×300 cells. The optimised budget lies around $\$45\times10^6$, which is a reasonable estimate when compared to the figures of the mining company. They estimated an average of \$490 per cell of $25\,m^2$, which amounts to a total of $\$44.1\times10^6$.

The cost functions have been formulated such that cost for planting forest on steeper slopes is higher compared to the development of shrubs on similar steep slopes. Restoration experience in the area reveals that shrubs show an autonomous growth in the area and only need some maintenance once in three years, whereas maintenance and fertiliser cost for growing forest on steeper slopes is relatively expensive. Accordingly, forest tends to be allocated on the flatter slopes.

4. Discussion and conclusions

The main goal of this paper was to investigate whether simulated annealing (SA), is an attractive alternative for designing resource allocation alternatives. This has been evaluated on the basis of three main criteria: (1) capacity for handling large spatial datasets, (2) handling of non-linear functions, particularly the spatial compactness objective, and (3) implementation of SA within an SDSS.

The case study for allocating new land use in a former mining area showed clearly that SA is capable of solving large combinatorial optimisation problems, involving large amounts of spatial data. The problem was to restore the area with three land use types according to a fixed division over the area. We successfully solved the problem using SA on a grid of 300 cells $\times$ 300 cells.

The second research criterion was whether SA is suitable for optimising a—non-linear—spatial compactness objective. Sundermann (1995) indicates that it is possible to include 'a factor' in the cost function to derive uniform clusters of land use, but did not indicate how. We included a *neighbour* function in the objection of the SA

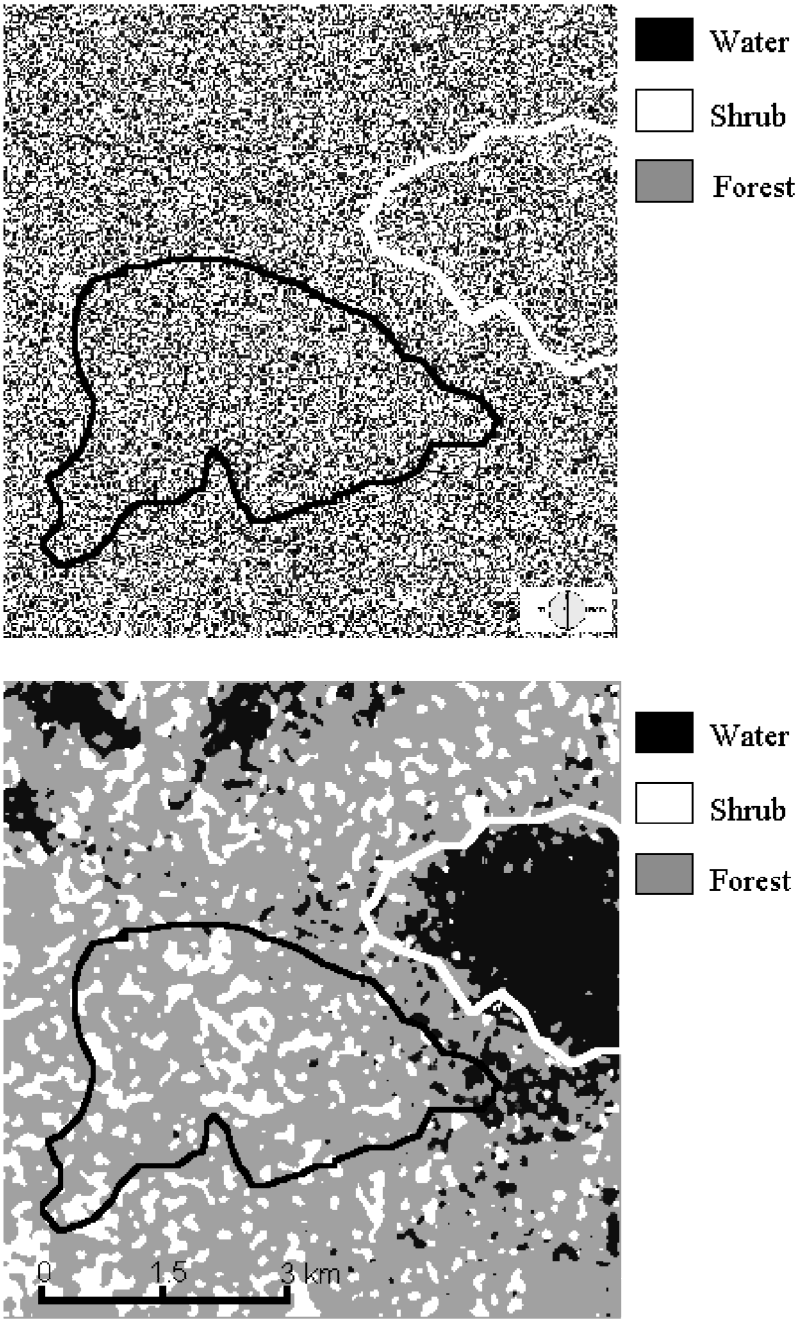

Figure 9. The dump site (black line) and the mining pit (white line). The picture depicts the initial random situation *(top)* and the optimised allocation plan (*down*) using three land use types within an area of 300 × 300 cells.

algorithm. The neighbour function is based on the principle that when a neighbour cell is allocated with land use of equal type, a 'cost bonus' is subtracted from the total development costs.

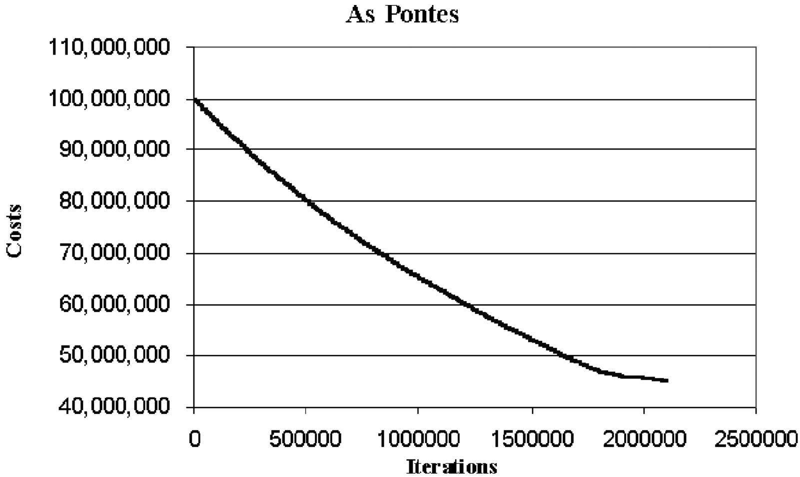

Figure 10. Costs in $ against number of iterations for the As Pontes case study.

The best restoration plan is the one that satisfies both objectives (minimise costs and maximise compactness). The preference of the decision-maker for either two of the objectives is expressed through a weighting factor β. For the mining example, we recommend the model variant with a β value of 3. This model produced a result with low development costs as well as closed patches of allocated land use. The optimal pattern resembled a known-optimal-pattern of land use—unlike other model variants, which produced land use patches with an 'un-natural' rectangular shape. However, implementing a variant with a different β value remains a question of decision-making preferences, and is thus subject to debate.

Although not explicitly worked out in this paper, the SA methods proved to be suitable for integration within an SDSS. We demonstrated this by using an SDSS developed for a mining company. The SDSS has been used to derive the input data (objectives, criteria and constraints), necessary to feed the SA based optimisation model. In this way it has been demonstrated that SA fits into the computational Step 4 of the SDSS, as models in this step are required to use the information from the previous SDSS steps. The SA software was developed as a 'loose coupling' approach but can be easily incorporated into the SDSS software, which is similarly developed with Delphi. Furthermore, the SA software is easy to handle by non-experts, as it only needs to be activated by an optimisation button and a slide bar for expressing the user's preference for the compactness level of the land use.

When comparing the SA application on an area with different grid sizes (10×10, 50×50, 250×250 and 300×300) it turned out that the optimisation time increases rapidly with the grid size. Although smaller grids are solved within seconds, larger maps were only optimised after a few hours on an average PC. The latter obviously decreases the practical use of SA within an SDSS using larger grids. However, hardware development is promising, such that the application for larger grids in an SDSS is only a matter of time. For now, grids up to 50×50 can be easily implemented in comparable SDSSs.

The complexity of higher-level-decision-making, as often found in resource allocation problems, is largely due to the different objectives of the stakeholders involved. Furthermore, complexity is often increased by the large amount of data, the uncertainty of the data, the complexity of models and the non-technical expertise of decision-makers. Therefore, an SDSS for these problems should be simple and transparent to be useful. We therefore simplified the case study at some points trying to demonstrate the use of optimisation techniques. It must however be clear that, although the optimisation results yield a good *indication* for restoring a mining area, a final realistic plan requires more detailed modelling. SA as presented in this paper, should therefore be regarded as a fast and simple technique useful in an early stage of the decision-making process.

There is a growing need for straightforward SDSSs to support decision-makers in solving resource allocation problems. In order to maintain the transparency of an SDSS, robust and simple techniques as simulated annealing are promising and may be well integrated in an SDSS. We therefore suggest that this approach is suitable for implementation within the computational step of an SDSS. It may furthermore be concluded that SA can be used to generate resource allocation alternatives and is capable for including spatial preferences.

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