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Double-spin $\cos\phi$ asymmetry in semi-inclusive electroproduction

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Abstract

We consider the double-spin $\cos\phi$ asymmetry for pion electroproduction in semi-inclusive deep inelastic scattering of longitudinally polarized leptons off longitudinally polarized protons. We estimate the size of the asymmetry in the approximation where all twist-3 interaction-dependent distribution and fragmentation functions are set to zero. In that approximation at HERMES kinematics a sizable negative $\cos\phi$ double-spin asymmetry for π^+ electroproduction is predicted.

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1. Introduction

Semi-inclusive deep inelastic scattering (SIDIS) of leptons off a nucleon is a suitable process to extract information on the quark–gluon structure—correlations between the spins of hadron or quark/gluon and the momentum of the quark/gluon with respect to that of the hadron. The parton intrinsic transverse momentum allows in the SIDIS cross section particular non-perturbative correlations, which can be probed in measurements of azimuthal asymmetries. The complete tree-level result for the SIDIS cross section in terms of distribution (DF) and fragmentation (FF) functions at leading and subleading order in $1/Q$ has been given in Ref. [1]. In particular, the combination with T-odd fragmentation functions leads to single-spin asymmetries. The HERMES Collaboration has recently reported on the measurement of such single target–spin asymmetries in the distribution of the azimuthal angle ϕ of produced pions relative to the lepton scattering plane, in semi-inclusive charged and neutral pion production on a longitudinally polarized hydrogen target [2,3].

Other consequences of non-zero intrinsic transverse momentum of partons are the spin-independent $\cos\phi$ and $\cos 2\phi$ asymmetries [4] and double-spin azimuthal asymmetries [1,5]. In this Letter we investigate the specific $\cos\phi$ azimuthal asymmetry at order $1/Q$ in semi-inclusive π^+ production with longitudinally polarized electrons on longitudinally polarized protons.

The kinematics of SIDIS is illustrated in Fig. 1: k_1 (k_2) is the 4-momentum of the incoming (outgoing) charged lepton, $Q^2 = -q^2$, where $q = k_1 - k_2$, is the 4-momentum of the virtual photon. The momentum P

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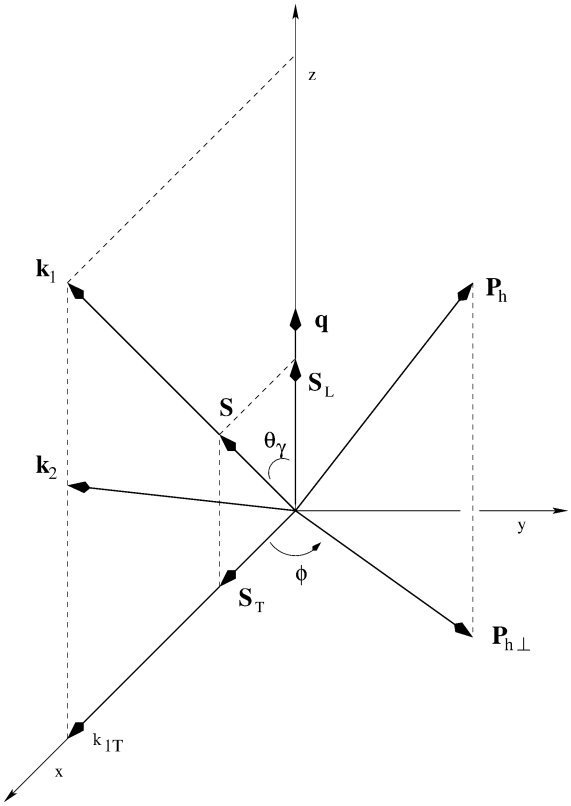

Fig. 1. The kinematics of SIDIS.

(P_h) is the momentum of the target (observed) hadron. The scaling variables are $x = Q^2/2(P \cdot q)$, $y = (P \cdot q)/(P \cdot k_1)$, and $z = (P \cdot P_h)/(P \cdot q)$. The momentum k_{1T} is the incoming lepton transverse momentum with respect to the virtual photon momentum direction, and ϕ is the azimuthal angle between $P_{h\perp}$ and k_{1T}. We will consider the case of a polarized beam, the helicity being denoted by λ_e. Note that for the specific case in which the target is polarized parallel (antiparallel) to the beam a transverse spin in the virtual photon frame arises which only can have azimuthal angle 0 (π). The value of this transverse spin component is [6]

$$|S_T| = |S| \sin\theta_\gamma, \tag{1}$$

where θ_γ is the virtual photon emission angle and S is target polarization parallel/antiparallel to the incoming lepton momentum.

The quantity $\sin\theta_\gamma$ is of order $1/Q$ and given by

$$\sin\theta_\gamma = \sqrt{\frac{4M^2x^2}{Q^2 + 4M^2x^2}\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right)}, \tag{2}$$

where M is the nucleon mass. We will distinguish the situations by referring to $(LL)_{\mathrm{lab}}$ which will produce both LL and LT polarization in the virtual photon frame.

2. The semi-inclusive cross section

The cross section for one-particle inclusive deep inelastic scattering is given by

$$\frac{d\sigma^{\ell+N\to\ell'+h+X}}{dx\,dy\,dz\,d^2P_{h\perp}} = \frac{\pi\alpha^2 y}{2Q^4 z} L_{\mu\nu} 2M\mathcal{W}^{\mu\nu}. \tag{3}$$

The quantity $L_{\mu\nu}$ is the well-known lepton tensor. The full expression for the symmetric and antisymmetric parts of the hadronic tensor $\mathcal{W}^{\mu\nu}$ at leading $1/Q$ order are given by Eqs. (77), (78) of Ref. [1]. In order to investigate the $\cos\phi$ azimuthal asymmetry we keep only the terms producing contributions in the cross section[1] that are ϕ-independent or proportional to $\cos\phi$

$$\begin{aligned}
&2M\mathcal{W}^{\mu\nu} \\
&\quad = 2z \int d^2k_T\, d^2p_T\, \delta^2\left(\boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z} - \boldsymbol{k}_T\right) \\
&\qquad \times \left\{ -g_\perp^{\mu\nu} f_1 D_1 + i\epsilon_\perp^{\mu\nu} g_{1s} D_1 + \frac{2\hat{t}^{\{\mu} k_\perp^{\nu\}}}{Q} f_1 \widetilde{D}^\perp \right. \\
&\qquad\quad + \frac{2\hat{t}^{\{\mu} p_\perp^{\nu\}}}{Q} x f^\perp D_1 + i\frac{2\hat{t}^{[\mu}\epsilon_\perp^{\nu]} k_{\perp\rho}}{Q} g_{1s} \widetilde{D}^\perp \\
&\qquad\quad \left. + i\frac{2\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]} p_{\perp\rho}}{Q}\left[x g_L^\perp D_1 + \frac{M_h}{M} h_{1s}^\perp \widetilde{E} \right] \right\},
\end{aligned} \tag{4}$$

where $\{\mu\,\nu\}$ indicates symmetrization of indices and $[\mu\nu]$ indicates antisymmetrization. In the above expression we have used the shorthand notation g_{1s}

$$\begin{aligned}
&g_{1s}(x, \boldsymbol{p}_T) \\
&\quad = \left[S_L g_{1L}\left(x, \boldsymbol{p}_T^2\right) + g_{1T}\left(x, \boldsymbol{p}_T^2\right) \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M} \right]
\end{aligned} \tag{5}$$

and similarly for $h_{1s}^\perp$.

[1] To avoid ambiguities, we will use the same notations as in Ref. [1].

The contraction of leptonic and hadronic tensors leads to the cross section with the following terms

$$\frac{d\sigma^{\ell+N\to\ell'+h+X}}{dx\,dy\,dz\,d^2P_{h\perp}} = \frac{\pi\alpha^2}{Q^2 y}\sum_q e_q^2\sigma^q, \tag{6}$$

where

$$\begin{aligned}
\sigma^q = \int d^2p_T\, d^2k_T\, z^2\delta^2\big(\boldsymbol{P}_{h\perp} - z(\boldsymbol{p}_T - \boldsymbol{k}_T)\big) \\
\times \Big\{ 2\big[1+(1-y)^2\big] f_1^q\big(x, \boldsymbol{p}_T^2\big) D_1^q\big(z, z^2\boldsymbol{k}_T^2\big) \\
- 8(2-y)\sqrt{1-y}\,\frac{1}{Q} \\
\times \Big[k_{Tx} f_1^q\big(x, \boldsymbol{p}_T^2\big)\widetilde{D}^{\perp q}\big(z, z^2\boldsymbol{k}_T^2\big) \\
+ p_{Tx} x f^{\perp q}\big(x, \boldsymbol{p}_T^2\big) D_1^q\big(z, z^2\boldsymbol{k}_T^2\big)\Big] \\
+ 2\lambda_e S_L y(2-y) g_1^q\big(x, \boldsymbol{p}_T^2\big) D_1^q\big(z, z^2\boldsymbol{k}_T^2\big) \\
- 8\lambda_e S_L y\sqrt{1-y}\,\frac{1}{Q} \\
\times \Big[k_{Tx} g_{1L}^q\big(x, \boldsymbol{p}_T^2\big)\widetilde{D}^{\perp q}\big(z, z^2\boldsymbol{k}_T^2\big) \\
+ p_{Tx}\Big(x g_L^{\perp q}\big(x, \boldsymbol{p}_T^2\big) D_1^q\big(z, z^2\boldsymbol{k}_T^2\big) \\
+ \frac{M_h}{M} h_{1L}^{\perp q}\big(x, \boldsymbol{p}_T^2\big)\widetilde{E}^q\big(z, z^2\boldsymbol{k}_T^2\big)\Big)\Big] \\
+ 2\lambda_e y(2-y)\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M} \\
\times g_{1T}^q\big(x, \boldsymbol{p}_T^2\big) D_1^q\big(z, z^2\boldsymbol{k}_T^2\big)\Big\}.
\end{aligned} \tag{7}$$

Here by k_{Tx} (p_{Tx}) we denote the x component of the final (initial) parton transverse momentum vector.

3. The correlation functions

In the asymmetries considered in this Letter a number of functions appear beyond the well-known leading twist DF's f_1^q, g_1^q and h_1^q and the FF D_1^q. Note that we do not consider polarization in the fragmentation process. These additional functions are

- The DF's g_{1T}^q and $h_{1L}^{\perp q}$, interpreted as distributions of longitudinally and transversely polarized quarks (of flavor q) in transversely and longitudinally polarized nucleons, respectively. The most interesting p_T-integrated functions in these case are the transverse moments

$$g_{1T}^{q(n)}(x) \equiv \int d^2p_T\left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n g_{1T}^q\big(x, \boldsymbol{p}_T^2\big). \tag{8}$$

- The DF's $f^{\perp q}$ and g_L^q appear at subleading $(1/Q)$ order in the above expression.
- The FF's $D^{\perp q}$ and E^q appear at subleading $(1/Q)$ order.

For all of these functions the relevant transverse moments can be expressed into the leading twist functions f_1, g_1 and h_1 and interaction-dependent functions, indicated with a tilde. The relations are of the same type as the Wandzura–Wilczek relation [7] for the subleading function g_T^q measured in inclusive leptoproduction with a transversely polarized target.

The relations needed in our case and details on them are found in Refs. [1,8]. For DF one needs

$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 dy\,\frac{g_1(y)}{y} - \frac{m}{M}\int_x^1 dy\frac{h_1(y)}{y^2} - \int_x^1 dy\,\frac{\tilde{g}_T(y)}{y}, \tag{9}$$

$$\frac{h_{1L}^{\perp(1)}(x)}{x^2} = -\int_x^1 dy\frac{h_1(y)}{y^2} + \frac{m}{M}\int_x^1 dy\,\frac{g_1(y)}{y^3} + \int_x^1 dy\,\frac{\tilde{h}_L(y)}{y^2}, \tag{10}$$

$$f^\perp(x) = \frac{f_1(x)}{x} + \tilde{f}^\perp, \tag{11}$$

$$g_L^\perp(x) = \frac{g_1(x)}{x} + \frac{m}{M}\frac{h_{1L}^\perp(x)}{x} + \tilde{g}_L^\perp, \tag{12}$$

while for DF we need

$$E(z) = \frac{m}{M_h} z D_1(z) + \widetilde{E}(z), \tag{13}$$

$$D^\perp(z) = z D_1(z) + \widetilde{D}^\perp(z). \tag{14}$$

The approximation that we will use below consists in setting all interaction dependent (tilde) functions to zero. There is in fact no justification for this, except the observation that the same approximation for g_T^q (the Wandzura–Wilczek approximation (WW)) seems to work well [9]. We want to add another point concerning potential contributions in the cross section that have already been neglected, namely those proportional to $\alpha_s(Q^2)$. Contributions proportional to $\alpha_s(Q^2)\cdots f_1\cdots$ will likely appear at the same point where the function $(M/Q)\cdots f^{\perp}$ appears [10], while contributions proportional to $\alpha_s(Q^2)\cdots g_1\cdots$ will likely appear at the same point where the function $(M/Q)\cdots g_L^{\perp}$ appears.

4. Weighted cross section

We will consider the differential cross section integrated over the transverse momentum of the produced hadron with different weights and denote them by [11, 12]

$$\langle W\rangle_{AB} = \int d^2 P_{h\perp} W \frac{d\sigma^{\ell+N\to\ell'+h+X}}{dx\,dy\,dz\,d^2P_{h\perp}}, \tag{15}$$

where $W = W(P_{h\perp},\phi,\phi_S)$. With the subscripts AB we denote the polarization of lepton and target hadron, respectively. We use U for unpolarized, L for longitudinally polarized and T for transversely polarized particles. From Eq. (7) we then obtain a number of asymmetries. For each of them we have indicated the results after setting all interaction dependent functions equal to zero, i.e., only keeping the twist-2 functions. The results are

$$\sigma^1_{UU} \equiv \langle 1\rangle_{UU} = \frac{[1+(1-y)^2]}{y} f_1(x)D_1(z), \tag{16}$$

$$\Delta\sigma^1_{LL} \equiv \langle 1\rangle_{LL} = \lambda_e S_L(2-y)g_1(x)D_1(z), \tag{17}$$

$$\begin{aligned}\sigma^2_{UU} &\equiv \big\langle |P_{h\perp}|\cos\phi\big\rangle_{UU}\\ &= -\frac{4}{Q}\frac{(2-y)\sqrt{1-y}}{y}\\ &\quad\times\big[M^2 x f^{\perp(1)}(x)zD_1(z)\\ &\qquad - M_h^2 f_1(x)z\widetilde{D}^{\perp(1)}(z)\big] \qquad (18)\\ &\overset{\text{WW}}{\Rightarrow} -\frac{4}{Q}\frac{(2-y)\sqrt{1-y}}{y}M^2 f_1^{(1)}(x)zD_1(z), \qquad (19)\end{aligned}$$

$$\begin{aligned}\Delta\sigma^2_{LL} &\equiv \big\langle |P_{h\perp}|\cos\phi\big\rangle_{LL}\\ &= 4\lambda_e\frac{S_L}{Q}\sqrt{1-y}\\ &\quad\times\big[M_h^2 g_1(x)z\widetilde{D}^{\perp(1)}(z)\\ &\qquad - M^2 x g_L^{\perp(1)}(x)zD_1(z)\\ &\qquad - M_h M h_{1L}^{\perp(1)}(x)z\widetilde{E}(z)\big] \qquad (20)\\ &\overset{\text{WW}}{\Rightarrow} -4\lambda_e\frac{S_L}{Q}\sqrt{1-y}\,M^2 g_1^{(1)}(x)zD_1(z), \qquad (21)\end{aligned}$$

$$\begin{aligned}d\sigma^3_{LT} &\equiv \big\langle |P_{h\perp}|\cos(\phi-\phi_S)\big\rangle_{LT}\\ &= \lambda_e|S_T|(2-y)Mg_{1T}^{(1)}(x)zD_1(z) \qquad (22)\\ &\overset{\text{WW}}{\Rightarrow} \lambda_e|S_T|(2-y)M\left[\int_x^1 dy\frac{g_1(y)}{y}\right]zD_1(z). \qquad (23)\end{aligned}$$

The particular $\cos\phi$ moment in the SIDIS cross section for which we will give an estimate is the following weighted integral of a cross section asymmetry,

$$\begin{aligned}&\big\langle |P_{h\perp}|\cos\phi\big\rangle_{(LL)_{\text{lab}}}\\ &= \frac{\int d^2P_{h\perp}|P_{h\perp}|\cos\phi(\sigma^{++}+\sigma^{--}-\sigma^{+-}-\sigma^{-+})}{\frac{1}{4}\int d^2P_{h\perp}(\sigma^{++}+\sigma^{--}+\sigma^{+-}+\sigma^{-+})}.\end{aligned} \tag{24}$$

Here $\sigma^{++},\sigma^{--}(\sigma^{+-},\sigma^{-+})$ denote the cross section with antiparallel (parallel) polarization of the beam and target, respectively.[2] They are given by $d\sigma^3_{LT}$ with $\phi_S = \pi(0)$ for σ^{++} and σ^{-+} (σ^{--} and σ^{+-}), respectively. The quantity M_h is the mass of the final hadron. Using the Eqs. (16)–(23) and assuming 100% beam and target polarization one obtains

$$\big\langle |P_{h\perp}|\cos\phi\big\rangle_{(LL)_{\text{lab}}} = 4\frac{\Delta\sigma^2_{LL} - d\sigma^3_{LT}}{\sigma^1_{UU}}. \tag{25}$$

For the experimentally measured cross sections that is determined without weighing with the transverse momentum of the produced hadron we use a further approximation,

$$A^{\cos\phi}_{(LL)_{\text{lab}}} \approx \frac{1}{\langle P_{h\perp}\rangle}\big\langle |P_{h\perp}|\cos\phi\big\rangle_{(LL)_{\text{lab}}}. \tag{26}$$

For the numerical estimate of $A^{\cos\phi}_{(LL)_{\text{lab}}}$ asymmetry we use the approximation, where only the twist-2 distribution and fragmentation functions are used, i.e., the interaction-dependent twist-3 parts are set

[2] This leads to positive $g_1(x)$.

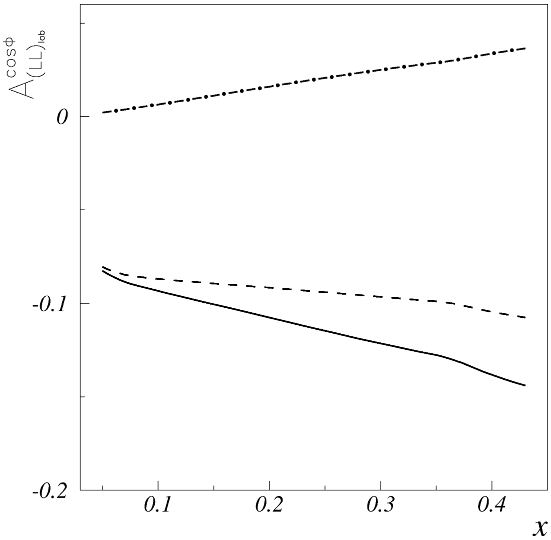

Fig. 2. $A^{\cos\phi}_{(LL)_{\rm lab}}$ for π^+ production as a function of Bjorken x. The dashed line corresponds to contribution of the $\Delta\sigma^2_{LL}$, dot-dashed one to $d\sigma^3_{LT}$ and the solid line is the difference of those two.

to zero. It is important to point out that in this approximation the $\cos\phi$ asymmetry reduces to a kinematical effect conditioned by intrinsic transverse momentum of partons similar to the $\cos\phi$ asymmetry in unpolarized SIDIS [4].

Assuming a Gaussian parameterization for the distribution of the initial parton's intrinsic transverse momentum, p_T, in the helicity distribution function $g_1(z, p_T^2)$ one can get

$$g_1^{(1)}(x) = \frac{\langle p_T^2\rangle}{2M^2} g_1(x). \tag{27}$$

It is worth to note that these approximations lead to similar results obtained in the simple quark–gluon model with non-zero intrinsic transverse momentum in polarized SIDIS [5]. To estimate the transverse asymmetry contribution $d\sigma^3_{LT}$ into the $A^{\cos\phi}_{(LL)_{\rm lab}}$, we proceed in the same way as in the Ref. [13].

In Fig. 2, the asymmetry $A^{\cos\phi}_{(LL)_{\rm lab}}(x)$ of Eq. (26) for π^+ production on a proton target is presented as a function of x-Bjorken. The curves are calculated by integrating over the HERMES kinematic ranges corresponding to $1\ \text{GeV}^2 \leqslant Q^2 \leqslant 15\ \text{GeV}^2$, $4.5\ \text{GeV} \leqslant E_\pi \leqslant 13.5\ \text{GeV}$, $0.2 \leqslant z \leqslant 0.7$, $0.2 \leqslant y \leqslant 0.8$, and taking $\langle P_{h\perp}\rangle = 0.365$ GeV as input. The latter value is obtained in this kinematic region assuming a Gaussian parameterization of the distribution and fragmentation functions with $\langle p_T^2\rangle = (0.44)^2\ \text{GeV}^2$ [14]. For the sake of simplicity, Q^2-independent parameterizations were chosen for the distribution, $g_1(x)$ [15], and fragmentation, $D_1(z)$ [16], functions.

From Fig. 2 one can see that the approximation where all twist-3 DF's and FF's are set to zero gives the large negative double-spin $\cos\phi$ asymmetry at HERMES energies. The 'kinematic' contribution to $A^{\cos\phi}_{(LL)_{\rm lab}}(x)$ coming from the transverse component of the target polarization is small (up to 25% at large x-Bjorken).

5. Conclusion

The $\cos\phi$ double-spin asymmetry of SIDIS of longitudinally polarized electrons off longitudinally polarized protons was investigated. We only kept the $(1/Q)$-order contribution to the spin asymmetry that arises from intrinsic transverse momentum effects related to twist-two DF and FF similar to the $\cos\phi$ asymmetry in unpolarized SIDIS. With that approximation, a sizable negative $\cos\phi$ asymmetry is found for HERMES kinematics. It is shown that the 'kinematical' contribution from target transverse component (S_T) is small. The approximation used to estimate the double-spin $\cos\phi$ asymmetry is not complete in $1/Q$ order: it contains only $1/Q$ 'kinematical' twist-3 contribution. It is similar to Cahn's approach [4] in unpolarized SIDIS, which describes well the experimental results from EMC [17] and E665 [18]. The complete behavior of azimuthal distributions needs the inclusion of higher-twist and pQCD contributions. Nevertheless, if one consider the kinematics with $P_{h\perp} < 1$ GeV and $z < 0.8$, the estimate shows the non-perturbative effects from the intrinsic transverse momentum of the partons in the nucleon. The double-spin $\cos\phi$ asymmetry is a good observable to investigate the importance of leading and subleading effects at moderate Q^2.

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