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Fermion masses and mixings in a S_4 based model

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Abstract

It has been recently claimed that the symmetry group S_4 yields to the Tri-bimaximal neutrino mixing in a "natural" way from the group theory point of view. Approving of this feature as an indication, we build a supersymmetric model of lepton and quark masses based on this family symmetry group. In the lepton sector, a correct mass hierarchy among the charged leptons is achieved together to a neutrino mass matrix which can be diagonalized by the Tri-bimaximal pattern. Our model results to be phenomenologically inequivalent with respect to other proposals based on different flavour groups but still predicting the Tri-bimaximal mixing. In the quark sector a realistic pattern for masses and mixing angles is obtained. The flavour structures of the mass matrices in both the sectors come from the spontaneously symmetry breaking of S_4, due to several scalar fields, which get non-zero vacuum expectation values. A specific vacuum alignment is required and it is shown to be a natural results of the minimization of the scalar potential and, moreover, to be stable under the corrections from the higher order terms.
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1. Introduction

By now there is convincing evidence that the solar and the atmospheric neutrino anomalies can be explained by the neutrino oscillations. The Δm^2 values and mixing angles are known with good accuracy [1–3]. The latest best values for Δm^2 are $\Delta m^2_{\text{atm}} \sim 2.4 \times 10^{-3}$ eV^2 and $\Delta m^2_{\text{sol}} \sim 7.7 \times 10^{-5}$ eV^2. For the mixing angles, two are large and one is extremely small: the

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atmospheric angle θ_{23} is compatible with a maximal value, but the accuracy admits relatively large deviations, indeed at 2σ errors it is $0.366 \leqslant \sin^2\theta_{23} \leqslant 0.602$ with central value 0.466; the solar angle θ_{12} is large, $0.278 \leqslant \sin^2\theta_{12} \leqslant 0.352$ with central value 0.312, but about 5σ errors far from the maximal value; the reactor angle θ_{13} is strongly bounded and at present it has an upper limit of $\sin^2\theta_{13} \leqslant 0.036$. We underline that there are contrasting indications for a vanishing value of the reactor angle: in [2] there is a suggestion for a positive value which, at 1.6σ, is $\sin^2\theta_{13} \simeq 0.016 \pm 0.010$, while in [3] the authors find a best fit value consistent with zero within less than 1σ. Therefore we need a confirmation by the future experiments like DOUBLE CHOOZ [4], Daya Bay [5] and MINOS [6] in the ν_e appearance channel.

From the theoretical point of view, the developments about neutrino masses and mixing angles cannot satisfy: there is a so large number of existing models, that can be interpreted as a lack of a unique and compelling theoretical picture. However a series of models based on some discrete non-Abelian groups seems to be extremely attractive due to their predictions: indeed it is possible to achieve as the lepton mixing matrix the Tri-bimaximal (TB) pattern [7],

$$U_{\mathrm{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

which represents a very good approximation of the experimental data [2], providing the following values for the mixing angles:

$$\sin^2\theta_{13}^{\mathrm{TB}} = 0, \qquad \sin^2\theta_{23}^{\mathrm{TB}} = 1/2, \qquad \sin^2\theta_{12}^{\mathrm{TB}} = 1/3. \tag{2}$$

It is just the TB pattern which can suggest the type of symmetry that best describes the lepton mixings: it is a very well known result [8] that a maximal value for the atmospheric angle can be recovered only with a non-exact symmetry; explaining the indication for a non-vanishing, but still very small, value for θ_{13}, it is necessary to provide the TB pattern at the leading order (LO), invoking corrections from the higher order terms; the solar angle is predicted to be very close, less than 2°, to the measured value and therefore the corrections has to be relatively small. As a result, a realistic lepton flavour symmetry has to be broken at a certain level, predicting at the LO the TB pattern and providing corrections at the next-to-the-leading-order (NLO) at most of about $\theta_c^2 \approx 2^\circ$, where θ_c stands for the Cabibbo angle, which is a convenient hierarchical parameter for both the sectors.

There is a series of models based on the symmetry group A_4 [8–15], which are extremely attractive from this point of view, fulfilling all the previous requirements. A_4 is the group of the even permutations of four objects and has 12 elements and four irreducible representations, which are three singlets, 1, $1'$ and $1''$, and one triplet 3. These models manage in deriving the TB mixing by assuming that the A_4 symmetry is realized at a very high energy scale Λ and that leptons transform in a non-trivial way under this symmetry. Afterward the group is spontaneously broken by a set of scalar multiplets ϕ, the flavons, whose vacuum expectation values (VEV) receive a specific alignment. It is a non-trivial task to explain how to get the expected vacuum alignment in a natural way and we consider it a fundamental requirement for a competitive model. Moreover the TB mixing is corrected by the higher order terms by quantities of the order of $\langle\phi\rangle/\Lambda < 1$ and as a result the reactor angle is no longer vanishing and becomes proportional to $\langle\phi\rangle/\Lambda$.

The common aspect of many of this projects is the structure of the neutrino mass matrix. The most general mass matrix for the neutrinos which can be diagonalized by the TB mixing is the

following

$$m_\nu \sim \begin{pmatrix} a+2c & b-c & b-c \\ b-c & b+2c & a-c \\ b-c & a-c & b+2c \end{pmatrix} \tag{3}$$

and it is $\mu \leftrightarrow \tau$ invariant, yielding to a maximal atmospheric mixing angle and to a vanishing θ_{13}, and it satisfies the relation $m_{\nu_{11}} + m_{\nu_{13}} = m_{\nu_{22}} + m_{\nu_{23}}$, which gives the Tri-maximal solar angle [17]. For $\theta_{13} = 0$ there is no CP violation from the Dirac phase, and there are only Majorana phases. If we disregard them, we can restrict our considerations to real parameters. Usually this pattern can be obtained constructing the Lagrangian in such a way that the usual Weinberg operator, which we can write as $\ell\ell$ implying $\ell h_u \ell h_u$, is forbidden at the leading order, but appears only at higher orders with additional flavons. Most of the models based on the A_4 flavour symmetry, are characterized by $b = 0$[1]; however, different realizations with other relations between a, b and c have been studied, see for instance [15]. In the pattern with $b = 0$, the factors a in Eq. (3) come from the term $\ell\ell F_1$ and the factors c from $\ell\ell F_3$, where F_1 and F_3 are flavons transforming respectively as a singlet 1 and as a triplet 3 of A_4. Presenting the same flavour structure for the neutrino mass matrix, it is extremely difficult to distinguish one model from all the others by the use of only observables connected to the neutrino oscillations. Some improvements in this direction has been recently performed in [18], where the authors develop an analysis on some lepton flavour violating processes, which can be tested in the future experiments.

Moreover the great difficulty of this kind of models is to describe correctly the quark sector. First of all the quark mixing matrix is completely different from its lepton counterpart: the first shows little angles and, in the contrary, the second presents two large angles. As a result, while the lepton mixing matrix can be fairly achieved through a discrete flavour symmetry, the quark mixings seem to be better described by some continuous symmetry, like $U(2)$ [19]. Indeed, according to the left and right-handed quark representation assignments, a discrete non-Abelian flavour symmetry tends to predict no mixing at all in the quark sector, $V_{\mathrm{CKM}} = \mathbb{1}$, or too large mixing angles. On the other hand, the results obtained by the $U(2)$-based models for the quarks suggest that the use of the doublet representation in the quark sector should help in describing quark mixing. However, this possibility is prevented in the A_4-based models, since there are not doublet representations. The solutions which have been proposed consist in the possibility of add several Z_n symmetries [12], in order to suppress the unwanted terms, or in adopting a larger group, which manages in reproducing the structure of A_4 in the lepton sector and possesses some doublet representations useful to describe quarks, like for example the discrete group T' [20,21]. In our opinion, a good candidate to be the flavour symmetry group describing leptons and quarks has to be as small as possible and has not to need of numerous additional elements in order to reproduce correct fermion masses and mixing angles. Following this prejudice, we looked for a

[1] The pattern with $b = 0$ can be obtained with A_4 as flavour symmetry only with a particular flavon spectrum: the singlets $1'$ and $1''$ of A_4 have not to couple to the term $\ell\ell$. In fact with the left-handed leptons, ℓ, transforming as triplet of A_4, we can switch on the entries corresponding to b by coupling $\ell\ell$ to the flavons $F_{1'}$ and $F_{1''}$, singlets $1'$ and $1''$ respectively, as can be checked by looking in [10] and already underlined in the Appendix of [16]. Moreover, in this case, the two couplings give two distinct contributions and as a result the mass matrix is not diagonalizable by the TB pattern any more. To be diagonalized by this mixing scheme, it is necessary to impose the condition $y_1\langle F_{1'}\rangle = y_2\langle F_{1''}\rangle$, where y_i are the coupling constants of the two operators. However, keeping the model as natural as possible, i.e. without fine-tuning, it is necessary to prevent the couplings of the two singlets $F_{1'}$ and $F_{1''}$ with $\ell\ell$. The commonly used solution consists in not introducing such flavons.

proposal which manages in describing both the sectors in a realistic way, keeping as simple as possible the symmetry content.

It has been recently claimed [22], through group theoretical arguments, that the minimal flavour symmetry naturally related to the TB mixing is S_4 [2][23,24]. The group S_4 is the group of the permutations of four objects and it has 24 elements divided into five irreducible representations: two singlets 1_1 and 1_2, one doublet 2 and two triplets 3_1 and 3_2 (a more detailed description of S_4 can be found in Appendix A). We approve of the result in [22] as an indication and we present a model based on the discrete non-Abelian symmetry group S_4, which predicts the TB mixing in the lepton sector and a CKM matrix close to the experimental one in the quark sector (the group S_4 has already been studied in literature [25], but with different aims and different results). Moreover we introduce an additional Z_5 symmetry, which plays a similar role of the total lepton number avoiding some dangerous terms, and a continuous $U(1)_{\mathrm{FN}}$ [26], that helps to provide the correct fermion hierarchies. S_4 contains as a subgroup A_4, but it has a doublet representation, which can be used in order to describe quarks. It has the same number of elements of T', but the representations are different: in particular T' can derive only the same neutrino mass matrix of the A_4-based models. On the other hand, the mass matrix which can be constructed in a S_4-based model is exactly that one in Eq. (3) and therefore it is more general with respect to the previous case. From this point of view we can say that T' constraints the neutrino sector in a stricter way than S_4. We underline that the model cannot be embedded into a GUT context, because of the different transformation properties of leptons with respect to quarks.[3]

The presence of the doublet representation, not only represents the new expedient in order to describe the quark sector, but also introduces a new feature in the neutrino mass matrix: indeed the terms which contribute to m_ν are $\ell\ell F_1$, $\ell\ell F_3$ and the new $\ell\ell F_2$, where F_2 represents a flavon transforming as a doublet 2. In Eq. (3), this last contribution is represented by the term b. This result corresponds to the neutrino mass matrix in [24]: however the presence of three parameters in order to describe three masses prevents any predictions on the neutrino hierarchy type. For these reasons we conclude that the S_4-based model in which a singlet F_1, a doublet F_2 and also a triplet F_3 couple to $\ell\ell$ is not phenomenologically interesting. However it is not restrictive to construct a model in which only a singlet and a doublet contribute to the neutrino mass matrix, but in this case $m_1 = m_3$ and it would be spoiled out by the experimental observations. Moreover it is possible to think about a model in which only a singlet and a triplet contribute to the neutrino mass matrix: we have verified that such a model can be built, with a natural vacuum alignment. This model provides exactly the neutrino mass matrix with $b = 0$ and therefore it has the same predictions in the lepton sector as of the A_4-based models. For this reason in this paper we study the case in which only a doublet and a triplet couple to the term $\ell\ell$ and as a result we get an unusual neutrino mass matrix

$$m_\nu \sim \begin{pmatrix} 2c & b-c & b-c \\ b-c & b+2c & -c \\ b-c & -c & b+2c \end{pmatrix} \tag{4}$$

[2] We agree with the conclusions of the group theoretical analysis, but, in our opinion, it must not be considered a constraint for the model realization: from the model building point of view, the most economical realization which naturally provides the TB pattern as the neutrino mixing matrix is based on the A_4 symmetry group.

[3] When we were completing our work, the following paper appeared [23], in which the authors present a model based on the symmetry group $SU(5) \times S_4$. However in this model it is not possible to explain completely the VEV alignment and as a result the mass hierarchies and some mixings have to be fine-tuned.

which can still be diagonalized by the TB mixing. This new pattern provides different predictions for the $0\nu2\beta$-decay and thus this model can be distinguished from all the others which predict the TB mixing, just looking at some observables related to the neutrino oscillations.

In the following we first provide a phenomenological analysis of the new neutrino mass matrix, underlining the connections with the $0\nu2\beta$-decay. Subsequently, in Section 3, we present the model which naturally develops the TB mixing in the lepton sector and an acceptable CKM matrix in the quark sector in addition to realistic mass hierarchies between all the fermions. In Section 4, we show how to get in a natural way the special vacuum alignment, used throughout the paper. In Section 5, we present a study on the corrections introduced by the higher order terms. Finally, we summarize the results in the conclusions. Details on the group S_4, like the conjugacy table, the complete list of the elements in a particular basis of the generators and the respective Clebsch–Gordan coefficients, can be found in Appendix A. The complete NLO analysis of the vacuum stability is presented in Appendix B.

2. Phenomenological analysis

The neutrino mass matrix in Eq. (4) can be diagonalized by the TB mixing and the eigenvalues are given by

$$m_\nu^{\text{diag}} = (3c - b, 2b, 3c + b)\frac{v_u^2}{\Lambda}. \tag{5}$$

We can now write the neutrino oscillation parameters Δm^2_{atm} and Δm^2_{sol} as follows:

$$\Delta m^2_{\text{atm}} = |m_{\nu_3}|^2 - |m_{\nu_1}|^2 = 12|b||c|\cos\zeta\,\frac{v_u^4}{\Lambda^2}, \tag{6}$$

$$\Delta m^2_{\text{sol}} = |m_{\nu_2}|^2 - |m_{\nu_1}|^2 = 3|b|^2 - 9|c|^2 + 6|b||c|\cos\zeta\,\frac{v_u^4}{\Lambda^2}, \tag{7}$$

where the angle ζ is the relative phase between b and c. This phase is related to the Majorana CP phase α_{21}, which is defined as follows

$$U_\nu = U_{\text{TB}} \cdot \text{diag}\big(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\big). \tag{8}$$

We can express $|b|$ and $|c|$ as functions of Δm^2_{atm}, Δm^2_{sol} and ζ and as a result we get constraints on the type of the neutrino spectrum, on the value of the lightest neutrino mass and on the $0\nu2\beta$ parameter $|m_{ee}|$ directly from the experimental data. In Fig. 1 on the left, we plot $|m_{ee}|$ as a function of the lightest neutrino mass eigenstate, $m_{\nu1}$ in the normal hierarchy (NH) case and $m_{\nu3}$ in the inverse hierarchy (IH) one. On the right, we present $|m_{ee}|$ as a function of the Majorana phase α_{21}. We observe from this last plot that considering the Heidelberg–Moscow [27] experiment, which provides the lowest present bound on $|m_{ee}|$ of about 0.35 eV, the exact CP-conserving Majorana phase $\phi_{21} = 0$ is excluded in our model. Moreover, from Fig. 1 on the left, we conclude that the NH region falls in the quasi Degenerate Case (DC) band and therefore we cannot speak properly of NH in this model.

Restricting our discussion to the IH case, we find a lower bound for the $0\nu2\beta$ parameter, $|m_{ee}| > 14.4$ meV, for the lightest neutrino mass, $|m_{\nu_3}| > 0.72$ meV, and for the sum of the neutrino masses, $\sum_i |m_i| > 89.4$ meV, all of them corresponding to $\zeta = 0$. Moreover we have a prediction for $|m_{ee}|$ in function of Δm^2_{atm}, $r \equiv \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ and ζ

$$|m_{ee}|^2 = \frac{1}{36}\Big[-(1+r)\Delta m^2_{\text{atm}} + \sqrt{\big(\Delta m^2_{\text{atm}}\big)^2\cos^2\zeta\big(3(r-1)^2 + (r+1)^2\cos^2\zeta\big)}\sin^2\zeta\Big].$$

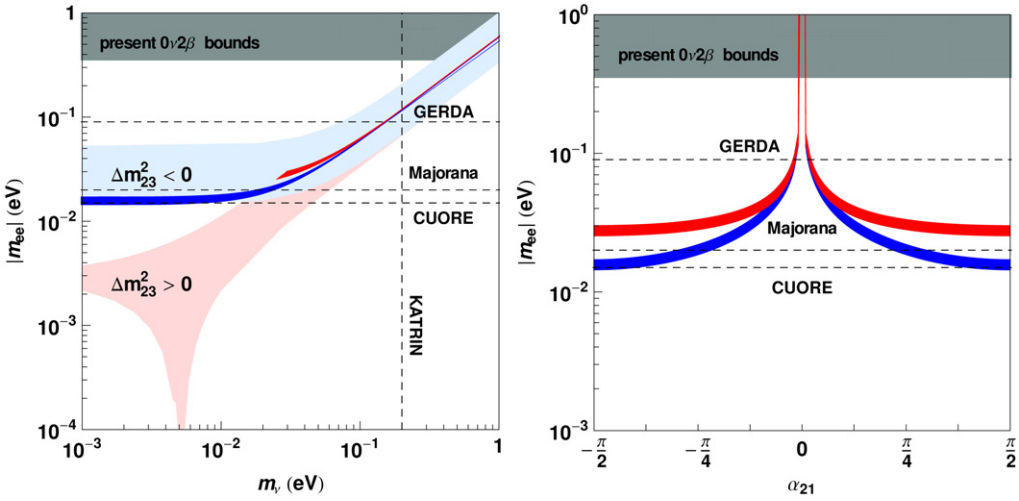

Fig. 1. On the left it is plotted $|m_{ee}|$ as a function of the lightest neutrino mass, m_{ν_1} in red in the case of the NH and m_{ν_3} in blue in the case of the IH. The light colored bands represent the possible regions considering only the exact TB pattern, while the dark colored ones are the predictions of our model. The present bound from the Heidelberg–Moscow experiment is shown in dark gray and the future sensitivity of CUORE (∼ 15 meV), Majorana (∼ 20 meV), and GERDA (∼ 90 meV) experiments are represented by the horizontal dashed lines, while the future sensitivity of 0.2 eV of KATRIN experiment is shown by the vertical dashed line. On the right, $|m_{ee}|$ as a function of the physical Majorana phase α_{21}: in red the NH case and in blue the IH one. The dark gray region and the dashed horizontal lines corresponds to the previous plot. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

We observe that our model can be distinguished from that one in [8,10], based on A_4, and that one in [20], based on T', looking to the lower bound on $|m_{ee}|$: in fact those models predict a lower bound for $|m_{ee}|$, which is about 0.005 eV. Our predictions are quite close to the future experimental sensitivity, which are expected to reach the values of 0.090 eV [28] (GERDA), 0.020 eV [29] (Majorana), 0.050 eV [30] (SuperNEMO), 0.015 eV [31] (CUORE) and 0.024 eV [32] (EXO).

3. The model

The discrete group S_4 is given by the permutations of four objects and it is composed by 24 elements. It can be defined by two generators S and T that satisfy

$$S^4 = T^3 = \left(ST^2\right)^2 = \mathbb{1}. \tag{9}$$

The three relations reported above directly indicate which are the discrete Abelian subgroups of S_4: Z_4, Z_3, Z_2 respectively. Indeed the 24 elements of S_4 belong to five classes reported in Appendix A: the elements of $\mathcal{C}_{2,4}$ define two different sets of Z_2 subgroups of S_4, corresponding to S^2 and ST^2 respectively, those of the class $\mathcal{C}_4$ a set of Z_3 Abelian discrete symmetries associated to T and those belonging to $\mathcal{C}_5$ a set of Z_4 Abelian discrete symmetries corresponding to S. From the three relations that define the group S_4 we see that it contains also a non-Abelian subgroup, S_3. Indeed defining $S' = S^2$ and using $S^2TS^2 = T^2$ we get the relations that define S_3, namely

$$T^3 = S'^2 = (S'T)^2 = 1. \tag{10}$$

Table 1
Transformation properties of the matter fields in the lepton sector and of all the flavons of the model. We distinguish the flavon fields on their role and thus we can consider ψ and η mainly connected to the charged lepton sector and Δ and φ to the neutrino sector. All these fields together to ξ' are present in the quark sector. The FN field, θ, provides the correct mass hierarchy.

	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	θ	ψ	η	Δ	φ	ξ'
S_4	3_1	1_2	1_2	1_1	1_1	1_1	3_1	2	3_1	2	1_2
Z_5	ω	ω^3	1	ω^2	1	1	ω^2	ω^2	ω^3	ω^3	1
$U(1)_{\rm FN}$	0	1	0	0	0	-1	0	0	0	0	0

Furthermore, S_4 presents 5 irreducible representations: two singlets, 1_1, 1_2, one doublet, 2, and two triplets, 3_1 and 3_2. All the technical details are reported in Appendix A.

3.1. The lepton sector

In this part we illustrate the model in the lepton sector, predicting an exact TB mixing at the LO and a realistic charged lepton mass hierarchy, by the use of flavour group G_f in addition to the gauge group of the SM. The complete flavour group is $G_f = S_4 \times Z_5 \times U(1)_{\rm FN}$, where the three factors play different roles: the spontaneous breaking of S_4 down to its subgroup $Z_2 \times Z_2$ in the neutrino sector is directly responsible for the TB mixing[4]; the Z_5 factor plays a similar role of the total lepton number, avoiding some dangerous terms, and, together to the $U(1)_{\rm FN}$, is responsible for the hierarchy among the charged fermion masses. In Table 1, we can see the lepton sector fields of the model and their transformation properties under G_f. We treat the model in a supersymmetric scenario, because the minimization of the scalar potential is simplified, but this is not a constraint from the construction of the model itself.

The superpotential for the leptons can be written as

$$w_\ell = \sum_{i=1}^{4} \frac{\theta}{\Lambda} \frac{y_{e,i}}{\Lambda^3} e^c (\ell X_i)' h_d + \frac{y_\mu}{\Lambda^2} \mu^c (\ell \psi \eta)' h_d + \frac{y_\tau}{\Lambda} \tau^c (\ell \psi) h_d + \text{h.c.}, \tag{11}$$

$$w_\nu = \frac{x_d}{\Lambda^2} (\ell h_u \ell h_u \varphi) + \frac{x_t}{\Lambda^2} (\ell h_u \ell h_u \Delta) + \text{h.c.}, \tag{12}$$

where

$$X = \{\psi\psi\eta, \psi\eta\eta, \Delta\Delta\xi', \Delta\varphi\xi'\} \tag{13}$$

using $(\cdots)$ to refer to the contraction in 1_1 and $(\cdots)'$ to the contraction in 1_2. It is interesting to underline that the first contributions containing e^c would be

$$\frac{\theta}{\Lambda} \frac{y'_{e,1}}{\Lambda^2} e^c (\ell \Delta \Delta)' h_d + \frac{\theta}{\Lambda} \frac{y'_{e,2}}{\Lambda^2} e^c (\ell \Delta \varphi)' h_d, \tag{14}$$

which would dominate with respect to the terms in Eq. (11). However an explicit computation will show that these two terms are vanishing, once we assume that the flavons get this specific

[4] This breaking is extremely unusual, indeed the common preserved subgroup is Z_2. Here $Z_2 \times Z_2$ provides the same flavour structure for the neutrino mass matrix as Z_2 in the A_4 based models and it is associated to one element of the class $\mathcal{C}_2$ and one of the class $\mathcal{C}_4$. The complete list of the elements are present in Appendix A.

VEV:

$$\begin{aligned}
&\langle\psi\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} v_\psi, \qquad \langle\eta\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_\eta, \\
&\langle\Delta\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_\Delta, \qquad \langle\varphi\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} v_\varphi, \\
&\langle\xi'\rangle = v_{\xi'}, \qquad \langle\theta\rangle = v_\theta.
\end{aligned} \tag{15}$$

We will demonstrate that this particular VEV alignment is a natural solution of the scalar potential in the following sections; moreover we will see that all the VEVs are of the same order of magnitude and for this reason we will parameterize the ratio VEV/Λ by the parameter u. The only VEV which originates with a different mechanism with respect to the others is v_θ and we indicate the ratio v_θ/Λ by the parameter t.

With this setting, in the basis of canonical kinetic terms,[5] the mass matrix for the charged leptons is ($m_\ell \sim R^c L$, $m_\nu \sim L^T L$)

$$m_\ell = \begin{pmatrix} y_e^{(1)} u^2 t & y_e^{(2)} u^2 t & y_e^{(3)} u^2 t \\ 0 & y_\mu u & 0 \\ 0 & 0 & y_\tau \end{pmatrix} u v_d, \tag{16}$$

where the $y_e^{(i)}$ are the result of all the different contributions of the $y_{e,i}$. For the neutrinos we get the following mass matrix, which is exactly diagonalized by the TB pattern,

$$m_\nu = \begin{pmatrix} 2c & b-c & b-c \\ b-c & b+2c & -c \\ b-c & -c & b+2c \end{pmatrix} \frac{v_u^2}{\Lambda}, \tag{17}$$

where $b = 2x_d \frac{v_\varphi}{\Lambda}$ and $c = 2x_t \frac{v_\Delta}{\Lambda}$. In order to find the lepton mixing matrix we need to diagonalize the charged lepton mass matrix and, performing a double expansion in the parameters u and t, we get

$$m_\ell^{\text{diag}} \equiv U_{\ell^c}^\dagger m_\ell U_\ell = \left(y_e u^2 t, y_\mu u, y_\tau\right) u v_d, \tag{18}$$

where the unitary U_ℓ results to be the unity matrix. As a consequence we get that

$$U_{\text{PMNS}} \equiv U_\ell^\dagger U_{\text{TB}} = U_{\text{TB}}. \tag{19}$$

When we introduce the NLO terms in the Lagrangian and the corrections in the VEVs of the flavons, we expect corrections to the TB mixing of relative order u, as we will discuss in the next sections. As a consequence, it provides an upper bound on the parameter u, indeed the maximum deviation from the TB pattern, which we can accept, is 0.05. For $u > 0.05$ the model provides a θ_{12} angle which is not in agreement at 2σ error with respect to the experimental data. The hierarchy of the charged leptons comes directly from the symmetry of the model and it is possible to get a constraint on the parameters u and t: indeed, for a very low $\tan\beta$ value, the requirement for the Yukawa of the τ lepton to be in the perturbative regime ($y_\tau < 4\pi$) corresponds to a lower

[5] It has been shown in a series of papers [33] that the corrections, from the transformations needed to move in the basis of canonical kinetic terms, appear at most as NLO deviations.

Table 2
Transformation properties of all the fields in the quark sector.

	D_q	q_3	u^c	d^c	c^c	s^c	t^c	b^c	θ	ψ	η	Δ	φ	ξ'
S_4	2	1_1	1_2	1_2	1_2	1_1	1_1	1_2	1_1	3_1	2	3_1	2	1_2
Z_5	ω^4	ω^3	1	1	ω^2	ω^2	ω^2	ω^2	1	ω^2	ω^2	ω^3	ω^3	1
$U(1)_{\mathrm{FN}}$	0	0	2	1	0	0	0	0	-1	0	0	0	0	0

bound for u of about 0.001. However, using this value for u, we require a particularly large value for y_μ in order to fulfill the measured value for the ratio m_μ/m_τ: from the requirement that also y_μ remains in the perturbative regime, the lower bound on u is raised and we fix it at 0.01. In order to explain the ratio m_e/m_μ, we get a range of values for the parameter t, which is to be similar to that for u. Finally we can write

$$0.01 < u, t < 0.05. \tag{20}$$

3.2. The quark sector

In this part we illustrate the model in the quark sector, getting a good approximation of the experimental quark mixing matrix. In Table 2, we can see the quark sector fields of the model and their transformation properties under $S_4 \times Z_5 \times U(1)_{\mathrm{FN}}$. The superpotential in the quark sector can be written as

$$\begin{aligned}
w_q = {} & y_t t^c q_3 h_u + \frac{y_b}{\Lambda} b^c q_3 \xi' h_d \\
& + \sum_{i=1}^{2} \frac{y_{tc,i}}{\Lambda^2} t^c \big(D_q X_i^{(1)}\big) h_u + \sum_{i=1}^{2} \frac{y_{bs,i}}{\Lambda^2} b^c \big(D_q X_i^{(1)}\big)' h_d \\
& + \sum_{i=1}^{6} \frac{y_{tu,i}}{\Lambda^3} t^c \big(D_q X_i^{(2)}\big) h_u + \sum_{i=1}^{6} \frac{y_{bd,i}}{\Lambda^3} b^c \big(D_q X_i^{(2)}\big)' h_d \\
& + \sum_{i=1}^{2} \frac{y_{c,i}}{\Lambda^2} c^c \big(D_q X_i^{(1)}\big)' h_u + \sum_{i=1}^{2} \frac{y_{s,i}}{\Lambda^2} s^c \big(D_q X_i^{(1)}\big) h_d \\
& + \frac{y_{ct}}{\Lambda} c^c q_3 \xi' h_u + \sum_{i=1}^{3} \frac{y_{sb,i}}{\Lambda^2} s^c \big(q_3 X_i^{(3)}\big) h_d \\
& + \sum_{i=1}^{6} \frac{y_{cu,i}}{\Lambda^3} c^c \big(D_q X_i^{(2)}\big)' h_u + \sum_{i=1}^{6} \frac{y_{sd,i}}{\Lambda^3} s^c \big(D_q X_i^{(2)}\big) h_d \\
& + \sum_{i=1}^{2} \frac{y_{u,i}}{\Lambda^2} \frac{\theta^2}{\Lambda^2} u^c \big(D_q X_i^{(4)}\big) h_u + \sum_{i=1}^{2} \frac{y_{d,i}}{\Lambda^2} \frac{\theta}{\Lambda} d^c \big(D_q X_i^{(4)}\big) h_d \\
& + \sum_{i=1}^{4} \frac{y_{ut,i}}{\Lambda^3} \frac{\theta^2}{\Lambda^2} u^c \big(q_3 X_i^{(5)}\big) h_u + \sum_{i=1}^{4} \frac{y_{db,i}}{\Lambda^3} \frac{\theta}{\Lambda} d^c \big(q_3 X_i^{(5)}\big) h_d,
\end{aligned} \tag{21}$$

where

$$X^{(1)} = \{\eta\eta + \psi\psi\},$$

$$X^{(2)} = \{\eta\eta\xi', \psi\psi\xi', \Delta\Delta\Delta, \Delta\Delta\varphi, \Delta\varphi\varphi, \varphi\varphi\varphi\},$$
$$X^{(3)} = \{\psi\Delta, \eta\varphi, \xi'\xi'\},$$
$$X^{(4)} = \{\varphi\varphi, \Delta\Delta\},$$
$$X^{(5)} = \{\psi\psi\Delta, \psi\psi\varphi, \psi\eta\Delta, \eta\eta\varphi\}.$$

With this setting, the mass matrix for the up quarks is

$$m_u = \begin{pmatrix} y_u u^2 t^2 & y_u u^2 t^2 & y_{ut} u^3 t^2 \\ y_{cu} u^3 & y_c u^2 & y_{ct} u \\ y_{tu} u^3 & y_{tc} u^2 & y_t \end{pmatrix} v_u, \tag{22}$$

and for the down quarks is

$$m_d = \begin{pmatrix} y_d ut & y_d ut & y_{db} u^2 t \\ y_{sd} u^2 & y_s u & y_{sb} u \\ y_{bd} u^2 & y_{bs} u & y_b \end{pmatrix} u v_d, \tag{23}$$

where the Yukawas are the sum of all the different terms, which appear in the superpotential.

These mass matrices can be diagonalized by the following transformations:

$$m_u^{\text{diag}} \equiv U_{u^c}^\dagger m_u U_u = \left(y_u u^2 t^2, y_c u^2, y_t\right) v_u,$$
$$m_d^{\text{diag}} \equiv U_{d^c}^\dagger m_d U_d = (y_d ut, y_s u, y_b) u v_d, \tag{24}$$

where the unitary matrices can be written in terms of order of magnitude of u and t as

$$U_u = \begin{pmatrix} 1 & O(u) & O(u^3) \\ -O(u) & 1 & O(u^2) \\ -O(u^3) & -O(u^2) & 1 \end{pmatrix}, \qquad U_d = \begin{pmatrix} 1 & O(u) & O(u^2) \\ -O(u) & 1 & O(u) \\ -O(u^2) & -O(u) & 1 \end{pmatrix},$$
$$U_{u^c} = \begin{pmatrix} 1 & O(t^2) & -O(ut^2) \\ -O(t^2) & 1 & O(u) \\ -O(ut^2) & -O(u) & 1 \end{pmatrix}, \qquad U_{d^c} = \begin{pmatrix} 1 & O(t) & O(ut) \\ -O(t) & 1 & O(u) \\ -O(ut) & -O(u) & 1 \end{pmatrix}. \tag{25}$$

The resulting quark mixing matrix is

$$V_{\text{CKM}} \equiv U_u^\dagger U_d \simeq \begin{pmatrix} 1 & \left(\frac{y_{sd}}{y_s} - \frac{y_{cu}}{y_c}\right) u & \left(\frac{y_{bd} y_c - y_{bs} y_{cu}}{y_b y_c}\right) u^2 \\ -\left(\frac{y_{cu}}{y_c} - \frac{y_{sd}}{y_s}\right) u & 1 & \frac{y_{bs}}{y_b} u \\ \left(y_{bs} y_{sd} - \frac{y_{bd} y_s}{y_b y_s}\right) u^2 & -\frac{y_{bs}}{y_b} u & 1 \end{pmatrix}. \tag{26}$$

In order to fit the experimental values of the mixing angles we need to invoke a moderate fine-tuning in some parameters. The (23) entry of V_{CKM} has to be of order $\theta_c^2 \simeq 0.05$ and therefore suggests for u a value close to its upper bound. However this is not a strict constraint because this value can be well explained for the entire range of u considering the Yukawas. On the other hand, the entry (12) requires an accidental enhancement of the combination $(\frac{y_{sd}}{y_s} - \frac{y_{cu}}{y_c})$ of order $1/\theta_c \sim 4$ in order to describe the correct Cabibbo angle. It is possible to explain such an enhancement considering particular values of the relative phase, ζ_q, between $\frac{y_{sd}}{y_s}$ and $\frac{y_{cu}}{y_c}$, which is connected to the CP violating phase: if $\zeta_q = \pi$, then the two factors sum up and the required values are easily explained.

Table 3
Transformation properties of the flavons and the driving fields.

	Δ	φ	Δ^0	φ^0	ψ	η	ψ^0	ξ'	ξ'^0
S_4	3_1	2	3_2	2	3_1	2	3_1	1_2	1_2
Z_5	ω^3	ω^3	ω^4	ω^4	ω^2	ω^2	ω	1	1

4. The vacuum alignment

In the following we present the mechanism to get the particular VEV alignment used in the previous sections. In Table 3 we illustrate all the flavon fields of the model and a set of new fields, the driving fields, defined as scalar fields with vanishing VEV, which are used only to select the particular solutions of the scalar potential. In order to distinguish between the matter fields, the flavons and the driving fields we introduce an additional $U(1)_R$, under which the fields have quantum number 1, 0 and 2 respectively. The usual R-parity, useful to avoid FCNC in the supersymmetric extensions of the SM, is a discrete group of this $U(1)_R$.

The driving superpotential is

$$w_d = g_1\left(\Delta^0\Delta\varphi\right) + g_2\left(\varphi^0\Delta\Delta\right) + g_3\left(\varphi^0\varphi\varphi\right) + f_1\left(\psi^0\psi\psi\right) + f_2\left(\psi^0\psi\eta\right) + M_{\xi'}\xi'^0\xi' + h_1\xi'^0(\eta\varphi)'. \tag{27}$$

The equations for the minimum of the scalar potential are obtained deriving w_d by the driving fields:

$$g_1(\varphi_1\Delta_2 - \varphi_2\Delta_3) = 0, \tag{28a}$$
$$g_1(\varphi_1\Delta_1 - \varphi_2\Delta_2) = 0, \tag{28b}$$
$$g_1(\varphi_1\Delta_3 - \varphi_2\Delta_1) = 0, \tag{28c}$$

$$g_2\left(\Delta_3^2 + 2\Delta_1\Delta_2\right) + g_3\varphi_1^2 = 0, \tag{29a}$$
$$g_2\left(\Delta_2^2 + 2\Delta_1\Delta_3\right) + g_3\varphi_2^2 = 0, \tag{29b}$$

$$2f_1\left(\psi_1^2 - \psi_2\psi_3\right) + f_2(\eta_1\psi_2 + \eta_2\psi_3) = 0, \tag{30a}$$
$$2f_1\left(\psi_2^2 - \psi_1\psi_3\right) + f_2(\eta_1\psi_1 + \eta_2\psi_2) = 0, \tag{30b}$$
$$2f_1\left(\psi_3^2 - \psi_1\psi_2\right) + f_2(\eta_1\psi_3 + \eta_2\psi_1) = 0, \tag{30c}$$

$$M_{\xi'}\xi' + h_1(\eta_1\varphi_2 - \eta_2\varphi_1) = 0. \tag{31}$$

The equations can be divided into almost separated groups. The first five equations, (28a)–(29b), are satisfied by the alignment

$$\langle\Delta\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_\Delta, \qquad \langle\varphi\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} v_\varphi, \tag{32}$$

which is a stable solution of the scalar potential, with

$$v_\Delta^2 = -\frac{g_3}{3g_2}v_\varphi^2, \qquad v_\varphi \text{ undetermined.} \tag{33}$$

The three equations (30a)–(30c), almost separated from the others, are satisfied by two different patterns: the first is

$$\langle\psi\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} v_\psi, \qquad \langle\eta\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_\eta, \tag{34}$$

with

$$v_\psi = -\frac{f_2}{2f_1}v_\eta, \qquad v_\eta \text{ undetermined,} \tag{35}$$

and the second is

$$\langle\psi\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_\psi, \qquad \langle\eta\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix} v_\eta, \tag{36}$$

with v_η and v_ψ undetermined. Only the first solution provides the results presented in the previous sections and we need of some soft masses in order to discriminate it as the lowest minimum of the scalar potential. We manage in doing it, considering some Z_5-breaking soft terms involving ψ and η, which in the most general form can be written as

$$m_\psi^2|\psi|^2 + m_\eta^2|\eta|^2 + \tilde{m}_\psi^2\psi\psi + \tilde{m}_\eta^2\eta\eta. \tag{37}$$

Assuming that $m_{\psi,\eta}^2 < 0$ the first two terms stabilize the potential for both the vacuum configurations. On the other hand the last two terms vanish for the first vacuum configuration and get a value different from zero in the second one. With an apposite choice of the soft parameters, these contributions can be positive, distinguishing the two configurations of VEVs and assuring that one in Eq. (34) as the setting with the corresponding lowest minimum.

Acting on the configurations of Eq. (32) or Eq. (34) with elements of the flavour symmetry group S_4, we can generate other minima of the scalar potential. These new minima are physically equivalent to those of the original sets, but it is not restrictive to analyze the model by choosing as local minimum exactly those ones in Eqs. (32) and (34) (it is possible to show that the different scenarios are related by field redefinitions).

The last equation (31) connects all the sectors and fixes the VEV of ξ'

$$\langle\xi'\rangle = v_{\xi'} = \frac{h_1}{M_{\xi'}}v_\eta v_\varphi. \tag{38}$$

For the flavon field θ, related to the Froggatt–Nielsen symmetry, the non-vanishing VEV is determined by the D-term associated with the $U(1)_{\mathrm{FN}}$ symmetry (see [13] for more details). The D-term in the potential is given by:

$$V_D = \frac{1}{2}\left(M_{\mathrm{FI}}^2 - g_{\mathrm{FN}}|\theta|^2 + \cdots\right)^2, \tag{39}$$

where g_{FN} is the gauge coupling constant of $U(1)_{\mathrm{FN}}$ and M_{FI}^2 is the Fayet–Iliopoulos term. The vanishing of V_D requires

$$g_{\mathrm{FN}}|\theta|^2 = M_{\mathrm{FI}}^2. \tag{40}$$

Assuming that $M_{\rm FI}^2/g_{\rm FN}$ is positive, this condition fixes the VEV of θ, given in Eq. (20).

5. NLO corrections

We now study the deviations to the LO results. We first present the analysis for the VEV alignment and then we move to the mass matrices.

5.1. The VEV alignment

Here we only summarize the results for the vacuum alignment, while a detailed study is developed in Appendix B. The part of the superpotential depending on the driving fields Δ^0, φ^0, ψ^0 and ξ'^0 is modified into

$$w_d = w_d^0 + \delta w_d, \tag{41}$$

where w_d^0 corresponds to Eq. (27) and δw_d is the most general quartic, S_4-invariant polynomial linear in the driving fields:

$$\delta w_d = \frac{1}{\Lambda}\left(\sum_{i=1}^{5} x_i I_i^{\Delta^0} + \sum_{i=1}^{6} w_i I_i^{\varphi^0} + \sum_{i=1}^{7} s_i I_i^{\psi^0} + \sum_{i=1}^{2} v_i I_i^{\xi'^0}\right) \tag{42}$$

where x_i, w_i, s_i and v_i are coefficients and $\{I_i^{\Delta^0}, I_i^{\varphi^0}, I_i^{\psi^0}, I_i^{\xi'^0}\}$ represents a basis of independent quartic invariants (the list of all the $I_i^{\Phi^0}$ are present in Appendix B). The new minimum is obtained by searching for the zeros of the F terms, looking for a solution that perturbs Eq. (15) to first order in the $1/\Lambda$ expansion: denoting the general flavon field with Φ, we can write the new VEVs as

$$\langle \Phi_i \rangle = \langle \Phi_i \rangle^{(\rm LO)} + \delta\Phi_i. \tag{43}$$

All the perturbations are non-vanishing, a part $\delta\eta_1$ and $\delta\eta_2$ and one of the perturbations in the neutrino sector, which remains undetermined. On the other hand the NLO terms fixes the relation between v_φ and v_η. We can conclude that the VEV alignment in Eq. (15) is stable under the NLO corrections and the deviations are of relative order u with respect the LO results.

5.2. The mass matrices

In this part we present the corrections to the mass matrices due to the higher order terms in the matter superpotential and the deviations to the VEV alignment.

5.2.1. Lepton sector

The superpotential for the charged leptons can be written as

$$w_\ell = w_\ell^0 + \delta w_\ell, \tag{44}$$

where w_ℓ^0 corresponds to Eq. (11) and δw_ℓ contains all the NLO terms. We note that the LO operators related to e^c completely fill in the first line of m_ℓ and, as a result, the corrections can be reabsorbed in the LO parameters. For this reason, we avoid to specify the NLO operators of δw_ℓ related to e^c, reporting only those ones connected to μ^c and τ^c: denoting Δ and φ with Φ_ν

and ψ and η with Φ_ℓ, we can write

$$\frac{\tau^c}{\Lambda^2}(\ell\Phi_\ell\Phi_\ell\Phi_\nu + \ell\Phi_\ell\xi'\xi'), \qquad \frac{\mu^c}{\Lambda^3}(\ell\Phi_\nu\Phi_\nu\Phi_\nu + \ell\Phi_\ell\Phi_\ell\xi'). \tag{45}$$

These corrections have to be added to those ones originated by w_ℓ^0 considering the deviations at the NLO to the vacuum alignment. Finally the corrected charged lepton mass matrix has the following structure

$$m_\ell = \begin{pmatrix} O(u^2t) & O(u^2t) & O(u^2t) \\ O(u^2) & O(u) & O(u^2) \\ O(u) & O(u) & O(1) \end{pmatrix} u v_d, \tag{46}$$

where only the order of magnitude of the single entries are reported. As a consequence the unitary matrix U_ℓ, which corresponds to the transformation of the charged leptons used to diagonalized m_ℓ, is modified in the following way:

$$U_\ell = \begin{pmatrix} 1 & T_{12}^e u & T_{13}^e u \\ -T_{12}^e u & 1 & T_{23}^e u \\ -T_{13}^e u & -T_{23}^e u & 1 \end{pmatrix}, \tag{47}$$

where the parameters T_{ij}^e are factors of order one.

A similar analysis can be performed for the neutrino superpotential

$$w_\nu = w_\nu^0 + \delta w_\nu \tag{48}$$

where w_ν^0 corresponds to Eq. (12) and δw_ν contains the only NLO operator,

$$\frac{x_d'}{\Lambda^3}(\ell h_u \ell h_u \varphi)'\xi'. \tag{49}$$

In addition to this correction, we have to consider those ones from w_ν^0, with the deviations at the NLO to the VEVs. As a consequence the neutrino mass matrix is corrected by terms of relative order u in every entry. Now the TB pattern has to be modified in order to diagonalize m_ν and we can write

$$U_\nu = U_{\mathrm{TB}} + \delta U_\nu u, \tag{50}$$

where δU_ν can be parameterized by three angles, T_{12}^ν, T_{23}^ν and T_{13}^ν, in a similar way as in Eq. (47).

Finally, summarizing all the corrections from the higher order terms, deviations to the neutrino mixing matrix of relative order u with respect the LO results are generated. The corrected neutrino mixing angles are modified as follows:

$$\tan\theta_{23} = -1 - 2u\left(T_{23}^e + \frac{\sqrt{2}T_{13}^\nu - 2T_{23}^\nu}{\sqrt{3}}\right), \tag{51}$$

$$\tan\theta_{12} = \frac{1}{\sqrt{2}} - \frac{3u}{4}\left(\sqrt{2}\left(T_{12}^e + T_{13}^e\right) - \sqrt{3}\left(T_{12}^\nu + T_{13}^\nu\right)\right), \tag{52}$$

$$\tan\theta_{13} = \frac{u}{2\sqrt{3}}\left(\sqrt{6}\left(T_{12}^e - T_{13}^e\right) + T_{13}^\nu + 2\sqrt{2}T_{23}^\nu - 3T_{12}^\nu\right). \tag{53}$$

We can conclude that the NLO corrections originate deviations to the TB mixing angles of order u.

5.2.2. Quark sector

The analysis for the up and down quark mass matrices is simpler than the previous case, because m_u and m_d do not have any vanishing entry at LO and therefore the corrections from the NLO operators of the superpotential and from the deviations to the VEVs introduce correcting factors of relative order u in each entry of the mass matrices. As a result the quark mixing angles receive deviations of relative order u, which do not spoil the LO results.

6. Conclusions

The aim of a flavour model is getting the correct mixing angles and mass hierarchies of both leptons and quarks, without inducing not observed processes, like FCNC and proton decays. Moreover in the context of non-Abelian flavour discrete symmetries, we face off the further problem of keeping and preserving a different VEV alignment for the flavons and also this requirement has to be naturally fulfilled.[6] All these points are separated one from each other and it seems very hard to get all of them at the same time using a single flavour symmetry group. However if a model manages in doing it, it will be considered as the most promising model in order to describe nature. Trying to understand what is the best candidate, many models have been proposed based on a product of different symmetry groups. However only few of them appear interesting: we consider fundamental aspects the lack, or at least a moderate amount, of fine-tunings, the smallness of the number of elements in the complete flavour symmetry group and in the list of the new scalar fields, the flavons.

Following these points, in this paper we have presented a model for fermion masses and mixing angles based on the flavour symmetry group $S_4 \times Z_5 \times U(1)_{\mathrm{FN}}$. The main aspect is the spontaneous breaking of S_4, which guarantees the TB pattern as the neutrino mixing matrix at LO. This feature is common to other models, like for example those based on the group A_4 [8–15] or those containing A_4, as the group T' [20,21]. The choice of S_4 has been suggested by the recent work by C.S. Lam [22]: S_4 results to be the only group (with all the groups containing S_4) which predicts the TB mixing in a natural way, namely without *ad hoc* assumptions, from the group theory point of view. This result is completely apart from all the possible realizations of a model based on S_4 and predicting the TB pattern: indeed, from the model building point of view, the most economic group which realizes this particular neutrino mixing matrix is A_4. However, there are other reasons which enforce the use of S_4: in the A_4-based models, it seems very difficult and unnatural to generate the correct mass hierarchies and mixings for quarks. S_4 represents a viable solution to this problem, because it contains a doublet representation more than A_4, which can be used in order to describe quarks.

This is not the first attempt in this direction: in [20] the group T' has been studied with good results, getting the TB mixing for leptons and a realistic quark mixings together to correct mass hierarchies. Unfortunately, this model suffers of a fine-tuning in order to generate the up-quark mass and the (12) entry of the CKM matrix: these negative aspects have been already underlined in the papers by Barbieri et al., studying the continuous group $U(2)$ [19], and it is connected to the fact that the first two quark families, both left- and right-handed, transform as doublets. In our model, we followed a different strategy, letting only the left-handed quarks of the first two families transform as a doublet, while the right-handed transform as singlets of S_4. We manage in getting the correct up quark mass, but we ask to some parameters to combine in such a way to

[6] Some attempts in which the VEV alignment problem in not present can be found in [34].

bring an unjustified factor of order $1/\theta_c$ or, from an alternative point of view, we ask to the phase ζ_q to be close to π. From this point of view, we only partially overcome to the problems of the previous models based on $U(2)$ and T'.

In the lepton sector, our model predicts a neutrino mass matrix which can be diagonalized by the TB pattern and a realistic charged lepton mass hierarchy. With respect to the A_4-based models, we predict an inverted hierarchy for the Majorana neutrinos and we get some interesting bounds on $|m_{ee}|$, which dominates the $0\nu2\beta$-decay, on the lightest neutrino mass and on the sum of the neutrino masses:

$$|m_{ee}| > 14.4 \text{ meV}, \qquad |m_{\nu_3}| > 0.72 \text{ meV}, \qquad \sum_i |m_{\nu i}| > 89.4 \text{ meV}. \tag{54}$$

Moreover we have a prediction for $|m_{ee}|$ in function of Δm^2_{atm}, r and the phase ζ, defined in Section 2,

$$|m_{ee}|^2 = \frac{1}{36}\Big[-(1+r)\Delta m^2_{\text{atm}} + \sqrt{\left(\Delta m^2_{\text{atm}}\right)^2 \cos^2\zeta \left(3(r-1)^2 + (r+1)^2\cos^2\zeta\right)}\sin^2\zeta\Big].$$

Our predictions are quite close to the future experimental sensitivity, which are expected to reach the values of 0.090 eV [28] (GERDA), 0.020 eV [29] (Majorana), 0.050 eV [30] (SuperNEMO), 0.015 eV [31] (CUORE) and 0.024 eV [32] (EXO). These aspects can distinguish our model from all the others using only observables linked to the neutrino oscillations. Furthermore, other studies, like on some lepton flavour violating precesses, can be performed in order to complete this analysis and better characterize our proposal.

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Appendix A. The group S_4

The character table of the group S_4 is Table 4. The generators, S and T, obey to the following rules

$$S^4 = T^3 = \left(ST^2\right)^2 = \mathbb{1}, \tag{A.1}$$

Table 4
Character table of S_4. C_i are the conjugacy classes, n the number of elements in each class, h the smallest value for which $\chi^h = \mathbb{1}$. In the last column we have reported an example of the elements for each class.

	n	h	χ_1	$\chi_{1'}$	χ_2	χ_3	$\chi_{3'}$	Example
C_1	1	1	1	1	2	3	3	$\mathbb{1}$
C_2	3	2	1	1	2	-1	-1	S^2
C_3	8	3	1	1	-1	0	0	T
C_4	6	2	1	-1	0	1	-1	ST^2
C_5	6	4	1	-1	0	-1	1	S

and can be written in the different representations as

$$
\begin{aligned}
&\text{representation } 1_1\text{:} \quad S = 1, \qquad T = 1,\\
&\text{representation } 1_2\text{:} \quad S = -1, \qquad T = 1,\\
&\text{representation } 2\text{:} \quad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix},\\
&\text{representation } 3_1\text{:} \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix},\\
&\text{representation } 3_2\text{:} \quad S = \frac{1}{3}\begin{pmatrix} 1 & -2\omega & -2\omega^2 \\ -2\omega & -2\omega^2 & 1 \\ -2\omega^2 & 1 & -2\omega \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}.
\end{aligned}
$$

The 24 elements of the group belong to five conjugacy classes

$$
\begin{aligned}
&\mathcal{C}_1\text{:} \quad \mathbb{1},\\
&\mathcal{C}_2\text{:} \quad S^2, T^2T^2, S^2TS^2T^2,\\
&\mathcal{C}_3\text{:} \quad T, T^2, S^2T, S^2T^2, STST^2, STS, S^TS^2, S^3TS,\\
&\mathcal{C}_4\text{:} \quad ST^2, T^2S, TST, TSTS^2, STS^2, S^2TS,\\
&\mathcal{C}_5\text{:} \quad S, TST^2, ST, TS, S^3, S^3T^2.
\end{aligned}
$$

In the 2-dimensional representation the elements are

$$
\begin{aligned}
&\mathcal{C}_{1,2}\text{:} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\\
&\mathcal{C}_3\text{:} \quad \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix},\\
&\mathcal{C}_{4,5}\text{:} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix},
\end{aligned}
$$

while for the 3-dimensional representation 3_1 the elements are

$$
\mathcal{C}_1\text{:} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

$$
\begin{aligned}
\mathcal{C}_2\text{:}\quad & \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & -1 & 2\omega^2 \\ 2\omega^2 & 2\omega & -1 \end{pmatrix}, \\
\mathcal{C}_3\text{:}\quad & \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \\
& \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2 & -\omega^2 & 2\omega \\ 2 & 2\omega^2 & -\omega \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2 & -\omega & 2\omega^2 \\ 2 & 2\omega & -\omega^2 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2\omega^2 & -\omega^2 & 2\omega^2 \\ 2\omega & 2\omega & -\omega \end{pmatrix}, \\
& \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega^2 & -\omega & 2 \\ 2\omega & 2 & -\omega^2 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & -\omega^2 & 2 \\ 2\omega^2 & 2 & -\omega \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2\omega & -\omega & 2\omega \\ 2\omega^2 & 2\omega^2 & -\omega^2 \end{pmatrix}, \\
\mathcal{C}_4\text{:}\quad & \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & 2 & -\omega \\ 2\omega & -\omega^2 & 2 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}, \\
& \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \omega \\ 0 & \omega^2 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \omega^2 \\ 0 & \omega & 0 \end{pmatrix}, \\
\mathcal{C}_5\text{:}\quad & \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2 & 2\omega^2 & -\omega \\ 2 & -\omega^2 & 2\omega \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2\omega & 2\omega & -\omega \\ 2\omega^2 & -\omega^2 & 2\omega^2 \end{pmatrix}, \\
& \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2 & 2\omega & -\omega^2 \\ 2 & -\omega & 2\omega^2 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega^2 & 2\omega & -1 \\ 2\omega & -1 & 2\omega^2 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2\omega^2 & 2\omega^2 & -\omega^2 \\ 2\omega & -\omega & 2\omega \end{pmatrix},
\end{aligned}
$$

and finally for the 3-dimensional representation 3_2, the matrices representing the elements of the group can be found from those just listed for the representation 3_1: for $\mathcal{C}_{1,2,3}$ are the same, while for $\mathcal{C}_{4,5}$ are the opposite. It is connected with the generator S, which changes sign in the 3_1 and 3_2 representations: the elements in $\mathcal{C}_{1,2,3}$ contain an even number of S, while those in $\mathcal{C}_{4,5}$ contain an odd number of it.

We now report the Clebsch–Gordan coefficients for our basis. In the following we use α_i to indicate the elements of the first representation of the product and β_i to indicate those of the second representation.

We start with all the multiplication rules which include the 1-dimensional representations:

$$
\begin{aligned}
& 1_1 \otimes \eta = \eta \otimes 1_1 = \eta \quad \text{with } \eta \text{ any representation}, \\
& 1_2 \otimes 1_2 = 1_1 \sim \alpha\beta, \\
& 1_2 \otimes 2 = 2 \sim \begin{pmatrix} \alpha\beta_1 \\ -\alpha\beta_2 \end{pmatrix}, \\
& 1_2 \otimes 3_1 = 3_2 \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix},
\end{aligned}
$$

$$1_2 \otimes 3_2 = 3_1 \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix}.$$

The multiplication rules with the 2-dimensional representation are the following:

$$2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2 \quad \text{with} \quad \begin{cases} 1_1 \sim \alpha_1\beta_2 + \alpha_2\beta_1, \\ 1_2 \sim \alpha_1\beta_2 - \alpha_2\beta_1, \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 \\ \alpha_1\beta_1 \end{pmatrix}, \end{cases}$$

$$2 \otimes 3_1 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix}, \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix}, \end{cases}$$

$$2 \otimes 3_2 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix}, \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix}. \end{cases}$$

The multiplication rules with the 3-dimensional representations are the following:

$$3_1 \otimes 3_1 = 3_2 \otimes 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2$$

$$\text{with} \quad \begin{cases} 1_1 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2, \qquad 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \end{pmatrix}, \\ 3_1 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}, \qquad 3_2 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}, \end{cases}$$

$$3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2$$

$$\text{with} \quad \begin{cases} 1_2 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2, \qquad 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ -\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \end{pmatrix}, \\ 3_1 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}, \qquad 3_2 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}. \end{cases}$$

Appendix B. The vacuum alignment at NLO

In this section there is the analysis for the corrections to the vacuum alignment introduced by the higher-dimensional operators. In Table 5 there is a summary of the transformation properties of the flavons and of the driving fields.

Table 5
Transformation properties of the flavons and the driving fields.

	Δ	φ	Δ^0	φ^0	ψ	η	ψ^0	ξ'	ξ'^0
S_4	3_1	2	3_2	2	3_1	2	3_1	1_2	1_2
Z_5	ω^3	ω^3	ω^4	ω^4	ω^2	ω^2	ω	1	1

The part of the superpotential depending on the driving fields Δ^0, φ^0, ψ^0 and ξ'^0 is modified into

$$w_d = w_d^0 + \delta w_d. \tag{B.1}$$

The leading order contribution is

$$\begin{aligned} w_d = {} & g_1\big(\Delta^0\Delta\varphi\big) + g_2\big(\varphi^0\Delta\Delta\big) + g_3\big(\varphi^0\varphi\varphi\big) \\ & + f_1\big(\psi^0\psi\psi\big) + f_2\big(\psi^0\psi\eta\big) + M_{\xi'}\xi'^0\xi' + h_1\xi'^0(\eta\varphi)' \end{aligned} \tag{B.2}$$

and the minimum is

$$\begin{aligned} &\langle\Delta\rangle \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_\Delta, \qquad \langle\varphi\rangle \sim \begin{pmatrix} 1 \\ 1 \end{pmatrix} v_\varphi, \\ &\langle\psi\rangle \sim \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} v_\psi, \qquad \langle\eta\rangle \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_\eta, \\ &\langle\xi'\rangle \sim v_{\xi'}, \end{aligned} \tag{B.3}$$

where

$$v_\Delta^2 = -\frac{g_3}{3g_2}v_\varphi^2, \qquad v_\psi = -\frac{f_2}{2f_1}v_\eta, \qquad v_{xi'} = \frac{h_1}{M_{\xi'}}v_\eta v_\varphi. \tag{B.4}$$

The remaining part, δw_d, is the most general quartic, S_4-invariant polynomial linear in the driving fields:

$$\delta w_d = \frac{1}{\Lambda}\left(\sum_{i=1}^{5} x_i I_i^{\Delta^0} + \sum_{i=1}^{6} w_i I_i^{\varphi^0} + \sum_{i=1}^{7} s_i I_i^{\psi^0} + \sum_{i=1}^{2} v_i I_i^{\xi'^0}\right), \tag{B.5}$$

where x_i, w_i, s_i and v_i are coefficients and $\{I_i^{\Delta^0}, I_i^{\varphi^0}, I_i^{\psi^0}, I_i^{\xi'^0}\}$ represents a basis of independent quartic invariants:

$$\begin{aligned} &I_1^{\Delta^0} = \big(\Delta^0(\Delta\varphi)_{3_1}\big)'\xi', && I_4^{\Delta^0} = \big(\big(\Delta^0\eta\big)_{3_1}(\psi\psi)_{3_1}\big), \\ &I_2^{\Delta^0} = \big(\Delta^0(\Delta\Delta)_{3_1}\big)'\xi', && I_5^{\Delta^0} = \big(\big(\Delta^0\psi\big)_2(\eta\eta)_2\big), \\ &I_3^{\Delta^0} = \big(\big(\Delta^0\psi\big)_2(\psi\psi)_2\big), \end{aligned}$$

$$\begin{aligned} &I_1^{\varphi^0} = \big(\varphi^0(\Delta\Delta)_2\big)'\xi', && I_4^{\varphi^0} = \big(\varphi^0\eta\big)(\psi\psi), \\ &I_2^{\varphi^0} = \big(\varphi^0(\varphi\varphi)_2\big)'\xi', && I_5^{\varphi^0} = \big(\varphi^0\eta\big)(\eta\eta), \\ &I_3^{\varphi^0} = \big(\big(\varphi^0\eta\big)_2(\psi\psi)_2\big), \end{aligned}$$

$$
\begin{aligned}
&I_1^{\psi^0} = \big(\big(\psi^0\psi\big)_2\eta\big)'\xi', && I_4^{\psi^0} = \big(\big(\psi^0\varphi\big)_{3_1}(\Delta\Delta)_{3_1}\big),\\
&I_2^{\psi^0} = \big(\big(\psi^0\Delta\big)_2(\Delta\Delta)_2\big), && I_5^{\psi^0} = \big(\big(\psi^0\Delta\big)_2(\varphi\varphi)_2\big),\\
&I_3^{\psi^0} = \big(\psi^0\Delta\big)(\Delta\Delta), && I_6^{\psi^0} = \big(\psi^0\Delta\big)(\varphi\varphi),\\
&I_1^{\xi'^0} = \xi'^0\xi'\xi'\xi', && I_2^{\xi'^0} = \xi'^0\xi'(\varphi\eta),\\
&I_2^{\xi'^0} = \xi'^0\xi'(\Delta\psi).
\end{aligned}
\tag{B.6}
$$

The new minimum for Δ, φ, ψ, η and ξ' is obtained by searching for the zeros of the F terms, the first derivative of $w_d + \delta w_d$, associated to the driving fields Δ^0, φ^0, ψ^0 and ξ'^0. We look for a solution that perturbs Eq. (B.3) to first order in the $1/\Lambda$ expansion: denoting the general flavon field with Φ, we can write the new VEVs as

$$\langle\Phi_i\rangle = \langle\Phi_i\rangle^{(\mathrm{LO})} + \delta\Phi_i. \tag{B.7}$$

The minimum conditions become equations in the unknown $\delta\Phi_i$, v_φ and v_η. By keeping only the first order in the expansion, we see that the equations can be separated into different groups: the first five concern only the neutrino sector, the second three only the charged lepton one and the last one connects the two sectors. Finally all the perturbations are non-vanishing, a part $\delta\eta_1$ and $\delta\eta_2$ and one of the perturbations in the neutrino sector, which remain undetermined. On the other hand the NLO terms fixes the relation between v_φ and v_η. We can conclude that the VEV alignment in Eq. (B.3) is stable under the NLO corrections and the deviations are of relative order u with respect to the LO results.

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