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Perceiving the Width and Height of a Hand-Held Object by Dynamic Touch

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The haptic perceptual subsystem of dynamic touch is prominent in manipulating and transporting objects, providing a nonvisible awareness of their linear dimensions. The hypothesis that perceptions of object width and height by dynamic touch are different functions of the inertia tensor is addressed. In two experiments heights and widths of nonvisible wielded objects were judged separately. Experiment 1 used solid rectangular parallelepipeds of different sizes; Experiment 2 used objects of identical mass and linear dimensions but nonidentical inertia ellipsoids. Width and height perceptions of comparable reliability and accuracy were found to vary as distinct functions of the objects' inertial eigenvalues. Discussion focused on the notion of tangible shape and on the selectivity of attention within dynamic touch.

In everyday manipulatory and transport activities, objects are grasped with the hand in contact with only a part (and often only a small part) of the object. Objects contacted in this partial manner are subjected to a wide variety of three-space motions as they are raised, lowered, pushed, pulled, carried, inserted, turned, and so on. Of particular importance to the perceptual control of these actions with partially held objects is dynamic touch, a haptic subsystem defined by the fact that extensions, compressions, and shearings of muscles, tendons, and ligaments underlie its perceptual capabilities more so than the deformations of skin and the articulations of joints (Gibson, 1966). Commonly, the perceptual contributions of dynamic touch to manipulatory and transport skills escape notice because attention is directed at the movements, and what is seen tends to predominate over what is felt. It is the case, however, that deformations of muscles, tendons, and ligaments are inevitable accompaniments of manipulation with the consequence that the role of dynamic touch in the control of manipulatory activity may be both more continuous and fundamental than that of vision.

Although wielding an object held firmly in the hand may involve any of the arm's joints, singly or in combination, the wrist joint is always involved, and experiments have shown that it is the rotational inertia defined about this joint that constrains perception by dynamic touch (see Pagano, Fitzpatrick, & Turvey, 1993). In short, a point O in the wrist is the relevant fixed point for mechanical analyses. The equation of 3-D motion about a fixed point is given by

$$I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3) = N_1, \qquad (1)$$

together with two similar equations obtained by cyclic permutation of 1, 2, and 3. In Equation 1, the subscripts refer to the principal axes (see below), N is torque, ω is angular velocity, and $\dot{\omega}$ is angular acceleration. The preceding time-dependent quantities contrast with I_1, I_2, and I_3, the eigenvalues of the inertia tensor I_{ij}, which is a time-independent parameter of the three-space rotational dynamics. This parameter is represented mathematically by a symmetric 3 × 3 matrix (e.g., Borisenko & Taparov, 1979; Goldstein, 1980) in which the diagonal terms (I_{xx}, I_{yy}, I_{zz})—referred to as *moments of inertia*—quantify the object's rotational inertia with respect to the three orthogonal axes of rotation. The off-diagonal terms (I_{xy}, I_{xz}, I_{yz}, I_{yx}, I_{zy})—referred to as *products of inertia*—quantify the object's rotational inertia in directions perpendicular to the axial rotations. When I_{ij} is rendered in diagonal form, the three components are the principal moments or eigenvalues I_k (where k = 1, 2, 3; e.g., Borisenko & Taparov, 1979; Goldstein, 1980). Diagonalization is a process that refers the rotational inertia to the three principal axes or eigenvectors $\mathbf{e}_k$, about which the off-diagonal terms disappear. The same

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$\mathbf{e}_k$ with lengths I_k result from diagonalization regardless of the coordinate system at O and therefore the particular moments and products composing I_{ij}.

As quantities that are invariant over time and coordinate systems, the eigenvectors $\mathbf{e}_k$ could play a significant role in the dynamic-touch perception of various object properties and hand–object relations of relevance to the control of manipulatory activities. Experimental investigations have suggested that perceived "magnitudes," such as object length (e.g., Fitzpatrick, Carello, & Turvey, 1994; Pagano et al., 1993; Pagano & Turvey, 1993; Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989a, 1989b) and object weight (Amazeen & Turvey, 1996) are functions of I_k, that perceived "directions," such as the orientation of an object to the hand (Pagano & Turvey, 1992; Turvey, Burton, Pagano, Solomon, & Runeson, 1992), the location of the hand relative to a wielded object (Pagano, Kinsella-Shaw, Cassidy, & Turvey, 1994), and the direction of a limb or limb segment (Pagano, Carello, & Turvey, 1996; Pagano, Garrett, & Turvey, 1996; Pagano & Turvey, 1995) are functions of the directions of $\mathbf{e}_k$, and that perceived "magnitudes in particular directions" (such as the lengths of the object segments fore and aft of the grasp) are functions of both the lengths and directions of $\mathbf{e}_k$ (Carello, Santana, & Burton, 1996; Pagano et al., 1996; Turvey, Carello, Fitzpatrick, Pagano, & Kadar, 1996). The dependence of perception on the time-invariant quantities of rotational dynamics and its independence from the time-varying quantities are corroborated further by experiments in which perception was unchanged over explicit manipulations of torque (Amazeen & Turvey, 1996; Solomon & Turvey, 1988). In these experiments the angular acceleration of wielding was controlled so that each object was judged (for weight in Amazeen & Turvey, 1996; for length in Solomon & Turvey, 1988) under three distinct and very different average levels of torque. The experiments found the judgment made on each object to be the same at each torque level.

The focus of the present research is on the nonvisible perception of an object's spatial dimensions. Many manipulatory activities, particularly those involving tools and instruments, would seem to depend on the haptic perception of both the length and the width of the manipulated object. A number of previous experiments have been directed at the perception of object length by dynamic touch, as noted above, but none so far has examined the perception of object width. In our research we asked how I_k constrains the perceptions of both the lengthwise and sidewise dimensions of a wielded object. Because the definitions of the eigenvalues are relative to the biases in the mass distribution of an object relative to its fixed rotation point, we adopted the convention of using I_1 and I_2 for the principal moments of inertia about the eigenvectors that are roughly perpendicular to the object's y axis and I_3 for the principal moment of inertia about the eigenvector that is roughly parallel to the object's y axis, as depicted in Figure 1. Inspection of Figure 1 suggests that the major eigenvalue I_1 (or I_2 given that $I_1 = I_2$ for objects of cylindrical symmetry) is the primary constraint on the perception of an object's extent lengthwise to the hand and the minor eigenvalue I_3 is the primary constraint on the perception of an object's extent sidewise to the hand. Previous research has confirmed the expectation concerning I_1. Our major goals in this research were to determine whether dynamic touch has access both to the lengthwise and the sidewise dimensions of an object and, if so, to determine how these two spatial perceptions are differentiated in their dependencies on the inertial eigenvalues.

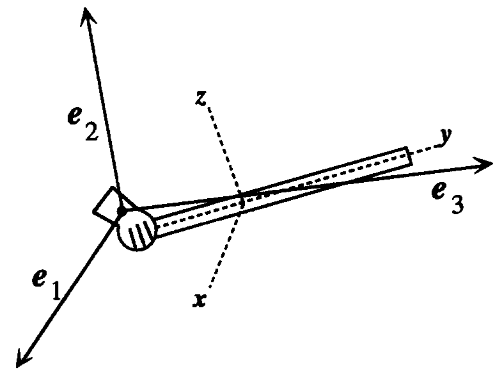

Figure 1. A typical relation between the eigenvectors $\mathbf{e}_k$ and the spatial axes x, y, and z of a wielded object. The origin of $\mathbf{e}_k$ is the point of rotation, which for wielding by hand is at the wrist joint.

Experiment 1

On each trial of Experiment 1, observers made separate judgments about the width and the height of a nonvisible rectangular object wielded by means of a handle attached to its base. In simple terms, the ability to perceive width as well as length would mean that participants should judge rectangular parallelepipeds as being wider than they were high when that was indeed the case, as being higher than they were wide when that was indeed the case, and as being equal in height and width when that was indeed the case. The judgments of width should be constrained primarily by I_3, in contrast to the known primary dependency of length judgments on I_1.

Method

Participants. Sixteen undergraduates at Seton Hall University participated as a means of obtaining extra course credit. All participants were women, although this was not a stipulation of recruitment. One participant reported being ambidextrous; the remainder were right-handed.

Materials. The objects for the experiment had to be representative in size of the kinds of small and medium-sized objects that might commonly be wielded by a single hand. The objects chosen are identified in Table 1. They fall into three subsets. Each subset included three rectangular objects of equal mass, averaging 0.39 kg, 0.89 kg, and 1.67 kg (the most massive block in each subset was greater than the least by no more than 1% in each case). Anchored securely in the center of the base of each block was a handle of 13.4 g, 12.7 cm in length and 0.95 cm in radius. When grasped, with the base of the handle flush with the base of the fist, approximately 2 cm of the handle on the average (i.e., for the average-sized fist) separated the hand from the base of the rectangular block. The

Table 1
Linear Dimensions of Rectangular Objects and Inertial Magnitudes ($\times 10^4$) of Rectangular Objects Plus Handles in Experiment 1 Together With Mean Perceived Extents and Standard Deviations Averaged Over Participants

	Linear dimension			Inertial magnitude		Perceived extent: Height (cm)		Perceived extent: Width (cm)	
Size	Height (cm)	Width (cm)	Mass (g)	I_1 (g cm^2)	I_3 (g cm^2)	*M*	*SD*	*M*	*SD*
Light	13.3	8.9	392	15.45	0.55	11.5	1.9	10.8	1.4
	18.1	7.6	391	19.28	0.47	13.4	2.9	10.5	0.8
	10.2	10.2	390	13.13	0.67	11.2	2.2	11.1	1.4
Medium	9.5	15.9	893	31.03	3.54	13.1	1.3	17.4	2.5
	20.3	10.8	881	50.12	1.85	17.4	2.4	12.8	1.7
	13.3	13.3	882	36.60	2.58	13.8	2.0	15.6	2.9
Heavy	10.2	21.0	1,660	62.80	11.46	17.7	3.3	21.8	3.2
	27.9	12.7	1,676	132.90	4.89	24.9	3.1	15.5	1.8
	16.5	16.5	1,674	83.22	7.54	20.1	4.1	19.7	3.2

Note. Eigenvalues (I_1 and I_3) were computed with the origin in the wrist.

handles inserted into each solid rectangular parallelepiped could be easily placed into and removed from a cuff in a metal stand, which supported the object for a given trial in an upright position for presentation to the participant.

Each subset included one parallelepiped equal in height and in both base dimensions (i.e., a cube), one parallelepiped with its height roughly twice as great as a base side, and one other. For the 0.89-kg and 1.67-kg subsets, the other parallelepiped was smaller in height than in its base side. Thus, for these two subsets that differed in mass and overall size, the objects were so designed as to satisfy height greater than width, height equal to width, and height less than width. For the lightest–smallest subset (0.39 kg), however, the other parallelepiped had to be higher than its base side (see first row of Table 1) to accommodate the handle of constant size that was inserted into each of the rectangular blocks. This difference between the lightest–smallest subset and the other two did not compromise the predictions with respect to I_k, but it did restrict the planned analysis of variance (ANOVA) of Subset $\times$ Object $\times$ Judged Dimension to the 0.89-kg and 1.67-kg subsets.

Across the nine objects, height and width were uncorrelated ($r^2 = .09$, $p > .05$). In addition to the nine experimental objects, there was also a demonstration block, 11.75 cm in height with a 12.7-cm $\times$ 12.7-cm base. For each rectangular block plus handle, the eigenvalues I_k were computed about a fixed point in the wrist. This point was a perpendicular distance of 7 cm from the handle in the grasp of the hand, that is, from the object's longitudinal axis.

The experiment was conducted in a large room that contained equipment unrelated to the current study. The report apparatus consisted of a 3-D metal frame with a shelf 84 cm from the ground. Two indicators were attached to the frame by loops of string. The vertical indicator could be moved along the right-hand side of the front face of the frame (to a maximum height of 92 cm above the shelf), and the horizontal indicator could be moved along the entire 84-cm length of the shelf. Both indicators were attached to weighted strings that hung along the side of the frame, invisible to the participant, allowing the experimenter to read the positions of the indicators. The observer was separated from the presentation stand by an opaque curtain. The observer's right arm and hand fit through a slit in the curtain. The object could be grasped by the handle and removed from the stand without its edges or surfaces being touched. The reporting device was on the left side of the curtain, in full view of the observer.

Procedure. The task for the participant on each trial was to wield the unseen object and to report its height and width separately, using the two indicators. The height to be perceived was defined for the participant in terms of "how far does the object extend above the hand?" Similarly, the width to be perceived was defined in terms of "how far does the object extend on either side of the hand?" Participants were shown the demonstration object, so they were aware of the rectangular form of the object attached to the handle and understood the nature of the two different extents. The order of reports was counterbalanced across the trials, and on each trial the participant was allowed to wield for as long as needed, and in any manner as needed (e.g., turning, shaking, twisting), to arrive at a confident judgment. The nine rectangular objects were presented three times each in a completely randomized order. On each trial the stimulus object was grasped by its handle such that the bottom of the handle was flush with the bottom of the hand. Nine practice trials (one for each object) preceded the experimental trials. Participants were not aware of the nature of the practice trials, nor were they permitted to see any of the objects other than the demonstration object.

Results and Discussion

Mean perceived width and height in relation to actual width and height. Inspection of the individual data revealed that judgments of width and height increased with object size for all but one participant, whose judgments of height decreased pronouncedly with object size. Whereas the mean perceived heights for the other participants were 11.60, 14.27, and 20.18 cm for the 0.39-kg, 0.89-kg, and 1.67-kg blocks, respectively, the corresponding perceptual values for the "odd" participant were 29.22, 18.37, and 16.70 cm, respectively. It was not possible to determine after the fact whether this participant misinterpreted the instructions or whether the participant's data were incorrectly classified. Given this uncertainty, and given the severity of the contrast, it seemed most prudent to exclude the anomalous participant's data from the analyses.

Table 1 presents mean perceived height (H_P) and mean perceived width (W_P) as a function of actual object height

(H_A) and actual object width (W_A) for the 15 remaining participants. Inspection of Table 1 suggests that, in the mean, participants judged objects as being wider than they were high when that was indeed the case, as being higher than they were wide when that was indeed the case, and as being equal in height and width when that was indeed the case. It is also apparent from Table 1 that although heights and widths were both over- and underestimated, participants nevertheless made their judgments within the range of the objects' actual dimensions in the absence of foreknowledge of that range and with the opportunity to report heights and widths up to approximately 1 m. The implication is that the scaling of perceived extent to actual extent was neither absolute (meaning a perfect match) nor relative (meaning that the perceived magnitudes were properly ordered but arbitrary; Gogel, 1977). Bingham's (1993) term for such scaling was *definite*, meaning that the perceived magnitudes were both properly ordered and within a marginal tolerance of the actual magnitudes.

An ANOVA involving subset (0.89 kg and 1.67 kg), object (height greater, height and width equal, width greater), and judged dimension ("how high" vs. "how wide") was conducted (the 0.39-kg subset was precluded for the reason described in the *Materials* section). There was no main effect of judged dimension ($W_P = 17.1$, $H_P = 17.8$), $F(2, 14) < 1$, but there was a main effect of subset, with the heavier subset (composed of higher and wider objects) associated with larger judgments than the lighter subset (19.9 cm vs. 15.0 cm), $F(1, 14) = 88.59$, $MSE = 12.24$, $p < .0001$. Subset and dimension interacted, $F(1, 14) = 10.95$, $MSE = 5.96$, $p < .01$, with $W_P > H_P$ for the smaller subset and $W_P < H_P$ for the larger subset. Of more importance was an interaction of object and judged dimension, as shown in Figure 2, confirming that the relations between H_P and W_P conformed to the relations between H_A and W_A, $F(2, 28) = 25.83$, $MSE = 18.80$, $p < .0001$. Simple effects tests corroborated that $H_P > W_P$ when $H_A > W_A$, $F(1, 14) = 25.58$, $p < .0001$, that $H_P = W_P$ when $H_A = W_A$, $F(1, 14) < 1$, and that $H_P < W_P$ when $H_A < W_A$, $F(1, 14) = 8.76$, $p < .01$. (In corroboration of the Object × Judgment interaction, the two perceptual measures were found to be uncorrelated [$p > .05$] for 12 of the 15 participants; for the 3 participants exhibiting the correlation, the r values ranged from .48 to .81.) A significant three-way interaction indicated that the preceding relations between objects and judgments were more pronounced with the larger subset of objects, $F(2, 28) = 4.51$, $MSE = 4.09$, $p < .05$.

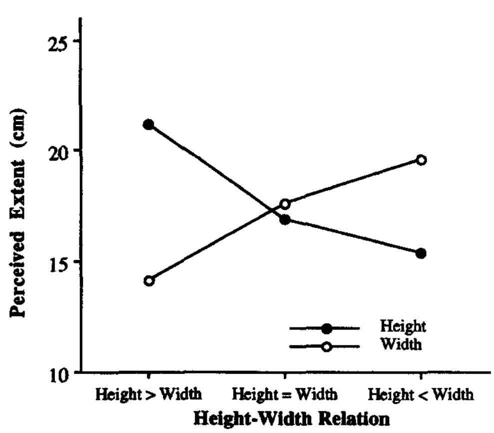

Figure 2. The interaction of judged dimension and the height–width relation in Experiment 1.

For the mean perceptions of all nine objects, linear regressions revealed the following: $H_P = 0.57\ H_A + 7.10$, $r^2(8) = .57$, $p < .01$; $W_P = 0.94\ W_A + 2.76$, $r^2(8) = .96$, $p < .0001$. These regressions reflected the regressions of H_A and W_A on I_1 and I_3, respectively: For H_A on I_1, $r^2(8) = .51$, and for W_A on I_3, $r^2(8) = .87$. The relation between H_A and I_1 was contaminated more by the length of the handles attached to the objects than the relation between W_A and I_3 was contaminated by the diameter of the handles. At the level of the individual participants, the linear regression of mean H_P on H_A for the nine objects was significant ($p < .05$) for 9 of the 15 participants, and the linear regression of mean W_P on W_A for the nine objects was significant ($p < .05$) for all 15 participants.

Variability, reliability, and accuracy of perceived width and height. Standard deviation was calculated for each participant for each judged dimension as the sample standard deviation (i.e., division was by $N - 1$). The standard deviations of the perceptual measures per object averaged over the standard deviations of the individual participants are summarized in Table 1. The ANOVA found no difference due to judged dimension ($W_P = 2.55$, $H_P = 2.72$), $F(2, 14) < 1$, but it did find a difference due to subset, with judgments on the larger subset more variable (3.13 vs. 2.14), $F(1, 14) = 12.29$, $MSE = 3.61$, $p < .01$. As with the ANOVA on means, the ANOVA on standard deviations revealed interactions of judged dimension with subset, $F(1, 14) = 9.83$, $MSE = 1.61$, $p < .01$, and with objects, $F(2, 28) = 6.65$, $MSE = 1.37$, $p < .01$. In sum, the means and standard deviations were similarly affected by the experimental manipulations.

Additional analyses allowed a comparison of observers' reliability and accuracy. Reliability for a given length was expressed as the average absolute deviation of the reported extents relative to the mean for that length (multiplied by 100, these values are understood as a percentage of the mean perceived extent). The reliabilities for the nine object lengths were then averaged to produce an overall reliability estimate for each participant (Norman, Todd, Perotti, & Tittle, 1996). Overall reliability measures were similarly calculated for the nine widths. The mean reliability measures for each participant are shown in Table 2. The reliabilities ranged from 6% to 15% for judgments of width and from 8% to 17% for judgments of height. The corresponding accuracy measures were provided by the root-mean-square (RMS) errors. These revealed how much a participant's judgments of width and height varied from the actual magnitudes. The percentage RMS error was calculated according to the following equation multiplied by 100:

$$\text{RMS error} = \frac{\dfrac{\Sigma\Sigma\sqrt{(\text{perceived} - \text{actual})^2}}{\text{actual}}}{\text{objects} \times \text{repetitions}}, \qquad (2)$$

Table 2
Reliability and Error Measures for Height and Width Judgments in Experiment 1

Participant	Reliability[a]		RMS error[b]	
	Height	Width	Height	Width
1	12.1	10.2	17.4	49.9
2	16.3	13.1	31.8	38.6
3	8.0	7.3	19.5	33.9
4	13.3	14.1	26.1	25.2
5	12.2	10.1	17.6	43.1
6	9.8	7.9	18.9	24.6
7	8.1	5.7	21.1	24.6
8	17.3	11.8	31.3	42.2
9	12.2	15.1	24.2	35.2
10	9.7	11.1	30.3	30.9
11	10.2	11.1	17.5	16.5
12	14.1	5.9	43.4	48.3
13	8.6	7.7	21.5	26.0
14	12.7	12.4	43.0	32.9
15	13.2	9.2	19.0	29.3

Note. RMS = root-mean-square.
[a]Average deviation expressed as a percentage of the mean perceived extent. [b]Expressed as a percentage of the actual extent.

where summation is over the number of objects and the number of trials or repetitions. Following the logic of Norman et al. (1996), if a participant judged an object's dimensions correctly, apart from random fluctuations, then the participant's RMS and reliability for that object should be equal. If, instead, the RMS is greater than the reliability, then it means that the judgments were systematically distorted. Table 2 summarizes the overall RMS errors for the width and height judgments of each participant. Comparison of reliability and RMS errors in Table 2 suggests that participants were less accurate (average 29.5%) than they were reliable (average 11.0%), a suggestion confirmed by ANOVA, $F(1, 14) = 108.30$, $MSE = 5{,}102.35$, $p < .0001$. Norman et al.'s investigation of the visual perception of horizontal distances (analogous, perhaps, to width) and distances in depth (analogous, perhaps, to height) yielded average accuracy and reliability measures of 17.0% and 6.7%, and 28.1% and 7.4%, respectively. Unlike the participants in the present experiment, the participants in Norman et al.'s visual experiment were always aware, when making a perceptual judgment of distance, of the limits on actual distance. The 180-cm × 90-cm surface layout and the response apparatus (an adjustable, oriented line on a computer monitor) were visible simultaneously. It would seem, therefore, that the participants in our experiment perceived physical intervals by dynamic touch almost as well as Norman et al.'s participants perceived physical intervals by vision.

The ANOVA we performed also revealed a main effect of dimension (width = 17.8%, height = 22.6%; that is, the mean of the reliability and accuracy measures was less for width than for height), $F(1, 14) = 10.52$, MSE = 145.70, $p < .01$, and a Measure × Dimension interaction, $F(1, 14) = 5.38$, $p < .04$. There was, therefore, a systematic distortion of the perceptions of width and height that was somewhat greater for height. A systematic but tolerable deviation of perceived from actual is expected in dynamic touch. As detailed earlier, dynamic touch is constrained by an object's inertia tensor—by the second moment of its mass distribution—and not by the object's linear dimensions as such. The systematic distortion of the mapping between actual and perceived extent captured in the contrasting measures of reliability and accuracy is due to the fact that the real mapping is between rotational inertia and perceived extent. The immediate question, therefore, is how did perceived width and perceived height scale to I_k?

Power laws fôr perceived width and perceived height. In their initial investigations of perception by wielding, Solomon and Turvey (1988) found that the perceived length of objects with cylindrical symmetry (two of the three eigenvalues are identical) increased as a curvilinear, concave-downward function of the major eigenvalue I_1 and as a linear function when plotted in double logarithmic coordinates. The results of Solomon and Turvey's (1988) study and Solomon et al.'s (1989a, 1989b) studies suggested a simple power function with an exponent less than 1 governing length perception by dynamic touch—specifically, the perceived lengths of cylindrical objects tend to increase as the cube root of I_1: perceived length $\propto I_1^{1/3}$. With respect to the present experiment, the simple regression of the logarithm of mean H_P on the logarithm of I_1 was highly significant, $r^2(8) = .95$, $p < .0001$, with a slope of .33 and fiducial limits of .26–.40 (the 95% confidence intervals defining the boundaries within which a parameter is considered to be localized). Likewise, the simple regression of the logarithm of mean W_P on the logarithm of I_3 was also highly significant, $r^2(8) = .95$, $p < .0001$, with a slope of .23 and fiducial limits of .18–.27.[1] We have shown above that height perception was more distorted and more weakly correlated with the actual values than was width perception. It should now be apparent that the latter contrasts were, as expected, due to the weaker correlation between I_1 and actual height than between I_3 and actual width. Expressed as a function of the inertial eigenvalue, H_P was no less well constrained than W_P.

The slope of the preceding H_P logarithmic regression was ⅓. Solomon et al. (1989a) argued that the significance of a ⅓-scaling for objects of cylindrical symmetry and uniform density is that perceived length will be proportional to actual length. This conclusion followed from simple dimensional considerations. Because of cylindrical symmetry, volume can be expressed as the product of length and cross-sectional area. Area is proportional to radius squared; thus, Volume $\propto$ Length × (Radius)^2. It follows, therefore, that for a solid cylinder of uniform density, Mass $\propto$ Volume, meaning that Mass $\propto$ Length × (Radius)^2. If the members of a set of objects conforming to cylindrical symmetry are of the same

[1] These regression coefficients are independent of the intercept value, as can be determined by centering the dependent measures and the regressors, that is, dividing each by its mean (see Ryan, 1990). Multiple regression of centered data is often advisable when estimating the coefficients is the major concern (Montgomery & Peck, 1982, 1990).

radius and vary only in length, then object mass will increase in direct proportion to object length. Additionally, given that I_1 is the measure of resistance to rotation about an axis perpendicular to an object's longitudinal axis, it will increase as the mass and squared length of the objects increase, that is, $I_1 \propto$ Mass $\times$ (Length)^2. Consequently, $I_1 \propto$ (Length)^3 given that, for the aforementioned set of objects, Mass $\propto$ Length. Hence, for rods differing in length but not in width, if perceived length grows as $I_1^{1/3}$, then it will increase in proportion to actual length. In experiments using such rods, this has been the observed scaling (e.g., Solomon & Turvey, 1988; Solomon et al., 1989a, 1989b). By the same reasoning about dimensions as above, if the members of a set of objects conforming to cylindrical symmetry are of the same length and vary only in radius, then object mass will increase in direct proportion to object radius. Additionally, given that I_3 is the measure of resistance to rotation about an object's longitudinal axis, it will increase as the mass and squared radius of the objects increase, that is, $I_3 \propto$ Mass $\times$ (Radius)^2. Consequently, $I_3 \propto$ (Radius)^4 given that, for these objects, Mass $\propto$ (Radius)^2. Hence, for rods differing in width but not in length, if perceived width grows as $I_3^{1/4}$, then it will increase in proportion to actual width.

Table 3 summarizes the simple regressions in logarithmic coordinates conducted for each of the 15 participants using their mean judgments. A paired t test comparing the 15 exponents for H_P with the 15 exponents for W_P was significant, $t(14) = 3.18$, $p < .01$, indicating a reliable difference between the mean exponents of .33 and .22, respectively. Comparisons of the exponents with ⅓ found no difference for H_P, $t(14) < 1$, and a highly reliable difference for W_P, $t(14) = -4.96$, $p < .001$. In contrast, comparisons of the exponents with ¼ found no difference for W_P, $t(14) = -1.31$, $p > .05$, and a highly reliable difference for H_P, $t(14) = 3.16$, $p < .01$. In sum, the power functions of the participants taken individually and as a group suggest that $H_P \propto I_1^{1/3}$, $W_P \propto I_3^{1/4}$. Even though the objects on which length and width judgments were made did not conform as a set to the idealizations identified above (e.g., only variation in length), the scalings did conform to those that follow from the idealizations.

The importance of power functions in psychophysics has been argued at length by Stevens (1961, 1962, 1970). They imply that equal stimulus ratios produce equal perceptual ratios, and the sizes of their exponents implicate tendencies of perceptual systems to either "compress" (exponent less than 1) or "expand" (exponent greater than 1) the range of experienced stimuli. Brightness and loudness are perceptual impressions associated with enormous energy ranges. When the stimuli in both cases are quantified in terms of energy, they have a common exponent of approximately ⅓, implicating compression of the stimulus range according to a cube root law (Stevens, 1961). More recent discussions (outside of psychophysics) highlight the links between power laws and major features of complex systems, namely, self-organization, cooperative phenomena, and fractals (e.g., Schroeder, 1991). From this latter perspective, the observation that the perceptual abilities of dynamic touch are expressible as power functions suggests a mode of functioning that is indifferent to scale—a cooperation among neural and muscular components that is the same for all object sizes. The perceptual subsystem of dynamic touch exhibits "self similarity" (Schroeder, 1991). From Stevens's perspective, the observation that the exponents for length perception and width perception are less than 1 suggests that dynamic touch is suited to operating with an extremely large range of rotational inertias (e.g., from those of needles and toothpicks to those of baseball bats and mallets).

Table 3
Exponents of Power Functions Relating Perceived Height to I_1 and Perceived Width to I_3 (Together With Their Fiducial Limits) and Goodness-of-Fit Values for Each Participant in Experiment 1

	Height		Width	
Participant	Exponent	r^2	Exponent	r^2
1	.26 (.14–.37)	.80	.22 (.15–.29)	.90
2	.55 (.41–.70)	.92	.12 (.04–.19)	.65
3	.25 (.13–.37)	.77	.07 (−.01–.15)	.36[a]
4	.43 (.29–.56)	.89	.28 (.14–.42)	.76
5	.30 (.18–.41)	.85	.17 (.11–.23)	.86
6	.35 (.23–.47)	.88	.20 (.12–.28)	.82
7	.27 (.19–.36)	.89	.22 (.15–.29)	.88
8	.47 (.27–.68)	.81	.26 (.19–.33)	.93
9	.21 (.10–.32)	.76	.29 (.19–.39)	.87
10	.39 (.24–.54)	.84	.29 (.22–.36)	.93
11	.25 (.01–.50)	.46	.15 (.04–.26)	.59
12	.32 (.20–.44)	.85	.14 (.03–.24)	.58
13	.20 (.12–.28)	.84	.32 (.26–.39)	.95
14	.35 (.02–.67)	.47	.39 (.22–.52)	.81
15	.36 (.22–.50)	.83	.20 (.12–.28)	.84

[a]Not significant.

Perceived ratio and the notion of tangible shape. The notion of shape refers to the property of a surface or object that depends on neither orientation nor position in space and that is indifferent to uniform scalings. Mathematically speaking, shape is a geometrical structure invariant over isometries (i.e., orientation and position) and homotheities (i.e., scalings; Koenderink, 1990). Burton, Turvey, and Solomon (1990) suggested a definition of tangible shape—tangible, that is, in the manner of dynamic touch—in terms of the largest and smallest elements of I_k. The ratio I_1/I_3 would be independent of coordinate system and unaffected by scale. In Burton et al.'s experiments, participants were asked to identify, from a set of visible candidates, the object corresponding in shape to a differently sized object that was wielded from a fixed point but unseen. There were three major observations. First, participants were able to perform this task at a level significantly exceeding chance. Second, the confusions to which participants were most liable were predicted by the proximity of the I_1/I_3 ratio of one object to that of another. Third, the pattern of confusions was indifferent to object size.

With respect to the objects of the present experiment, shape is expressible by the simple ratio of height to width. Evidence for a rudimentary form of nonvisible shape perception would be provided by a linear dependency of

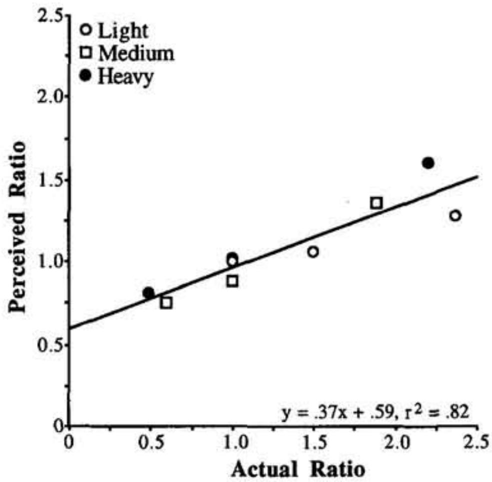

Figure 3. Mean perceived ratio (height to width) as a function of actual ratio and object mass (light = 0.39 kg, medium = 0.89 kg, heavy = 1.67 kg) in Experiment 1.

H_P/W_P on H_A/W_A. Figure 3 shows that the perceived ratio (H_P/W_P) averaged over participants and using the data from all three subsets of the nine objects was linearly dependent on the actual ratio. It is important to note that this dependency was not affected by size, a finding which corroborates the results of Burton et al. (1990) and is in concert with the invariance-over-homotheities criterion for shape. A multiple regression of perceived ratio against actual ratio and size found no contribution of size ($p = .66$). Individual regressions revealed the significant linear dependency of perceived ratio on actual ratio in 11 of the 15 participants (at minimally $p < .05$, for 8 *dfs*). Given the small ranges of heights (9.5–27.9 cm) and widths (7.6–20.9 cm), participants needed to be only a few centimeters off in their reports of either of the two linear dimensions for the perceived ratio to deviate noticeably from the actual ratio. The scaling relations revealed in our data suggest that Burton et al.'s inertial ratio for tangible shape might be more appropriately expressed as $I_1^{1/3}/I_3^{1/4}$.

In sum, it appears that the contrasting height and width dimensions of wielded, nonvisible objects are perceptible by dynamic touch within a reasonable tolerance. Experiment 1 has shown that both of the perceived extents approximated the actual extents and changed across objects in the same way as the actual extents, results that are suggestive of a rudimentary form of nonvisible shape perception (Burton et al., 1990). Additionally, the experiment has shown that the perceptions of object height and object width depend in different ways on the inertial eigenvalues: As expected from simple dimensional considerations of the scaling required for a proportionality between perceived and actual, perceived height increased as the cube root of the major eigenvalue; perceived width increased as the fourth root of the minor eigenvalue.

Experiment 2

In Experiment 2, we took the tack of controlling I_k directly through "tensor objects" (Amazeen & Turvey, 1996). An example of a tensor object is shown in Figure 4. With such tubular constructions the pattern of I_k can be manipulated systematically without introducing variations in the object's lengthwise and sidewise dimensions or in its total mass. In short, it is possible to limit object variations strictly to variations in I_k. Given these conditions, one may ask whether objects identical in I_3 would be perceived as wider if they differed in I_1 and whether objects identical in I_1 would be perceived as higher if they differed in I_3. At issue is the implication of Equation 1 that for any arbitrary wielding of an object in 3-D space, the time-varying motions and the time-varying torques are coupled through all three eigenvalues and that the resultant deformation of the muscles and tendons must be constrained in a time-independent way by all three eigenvalues. It may be hypothesized, therefore, that the perceived dimensions of a hand-held and wielded occluded object should be a function of I_k and not solely a function of either I_1 or I_3 (Fitzpatrick et al., 1994).

The preceding hypothesis is reinforced by geometric considerations. If all possible axes p are passed through the rotation point O, and lengths OA, equal in magnitude to $(I_p)^{-1/2}$, laid off on each axis, the locus of the points A is an ellipsoid called the momental ellipsoid or ellipsoid of inertia of the object at O (see Figure 5). The principal axes of the ellipsoid coincide with the principal axes of inertia $\mathbf{e}_k$ of the object. In short, the inertia tensor is characterized uniquely by a quadric surface with semiaxes of lengths $(I_1)^{-1/2}$, $(I_2)^{-1/2}$, and $(I_3)^{-1/2}$ (e.g., Arnold, 1989; Borisenko & Taparov, 1979; Goldstein, 1980). In the general physical case, the inertia ellipsoid completely determines the rotational characteristics of an object. Two objects with identical inertia ellipsoids will exhibit identical motions for identical initial conditions. From a psychological viewpoint, the inertia ellipsoid may resemble (albeit crudely) the shape of a rigid object (Arnold, 1989) and may thereby provide an appropriate characterization of the property of object shape for dynamic touch (Fitzpatrick et al., 1994; Solomon, 1988). If an object is stretched out along some axis $\mathbf{e}_i$ (consider a rectangular parallelepiped that is longer than it is wide), then

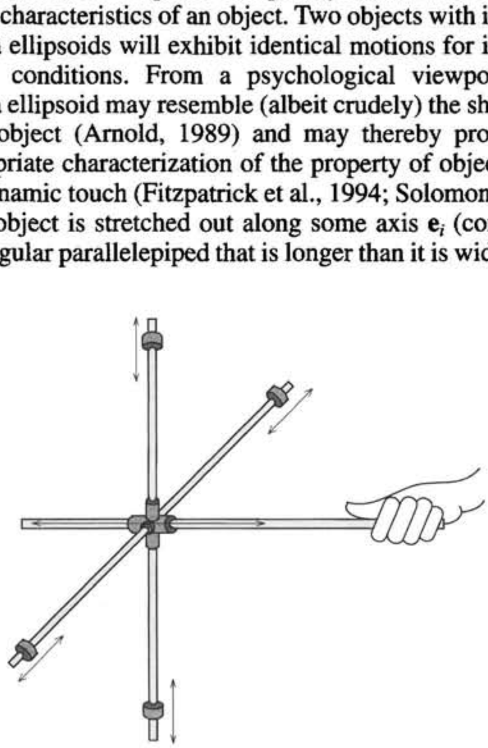

Figure 4. A tensor object. The crossbar can be displaced as can the attached metal rings, thereby permitting explicit control over the mass distribution.

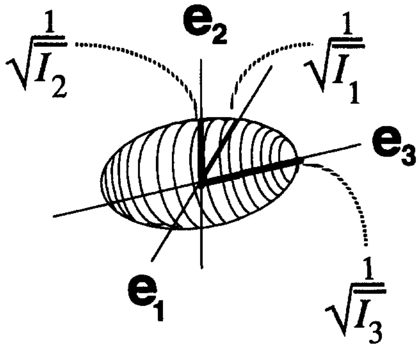

Figure 5. The ellipsoid of inertia. The origin of $\mathbf{e}_k$ is the point of rotation, which for wielding by hand is at the wrist joint.

the moment of inertia I_i with respect to this axis is small, and in consequence the inertia ellipsoid is also stretched out along this axis.

In Experiment 2, 10 tensor objects of the same size and weight were constructed such that five values of the minor eigenvalue I_3 were nested within two considerably larger values of the major eigenvalue I_1 (and, therefore, I_2 given $I_2 \approx I_1$ for our experimental objects that are axially or cylindrically symmetric). If the perception of an object's width is constrained solely by I_3, then perceived width should be the same for both values of I_1. Similarly, if perception of an object's height is constrained solely by I_1, then perceived height should be the same for all five values of I_3. If, however, perception of width and height are constrained by I_k, then both eigenvalues should constrain the perceptions of both dimensions but not, presumably, in the same way. With respect to Experiment 1, although H_P and W_P were primarily constrained by I_1 and I_3, respectively, there were hints of contributions of the other eigenvalue, in agreement with previous research on length perception by Fitzpatrick et al. (1994). Both multiple and stepwise regressions, in which the logarithms of the mean perceptions of the nine objects were regressed on log I_1 and log I_3 simultaneously, found small but reliable effects of log I_3 on log H_P (coefficient = $-.08$, partial $F = 7.7$, $p < .03$) and log I_1 on log W_P (coefficient = $-.12$, partial $F = 13.64$, $p < .01$). These additional contributions of I_k were present to a modest degree at the level of the individual participants. For the individual multiple regressions involving H_P, the sign pattern of positive I_1 and negative I_3 was found for 10 participants, but the negative I_3 coefficient was reliable in only 3 cases. Similarly, for the individual multiple regressions involving W_P, the sign pattern of negative I_1 and positive I_3 was found for 12 participants, but the positive I_1 coefficient was reliable in only 5 cases.

Method

Participants. Ten right-handed graduate students (6 men and 4 women) at the University of Connecticut volunteered to participate.

Materials. There were 10 objects of identical linear dimensions (height and width) and identical mass. Each object consisted of a movable unit of two perpendicular crossbars (each 60-cm long) with a third rod (45 cm) attached as a handle. The crossbar and handle rod were constructed from hollow aluminum cylinders (see Figure 4). For the tensor objects of Amazeen and Turvey (1996), the crossbars were fixed to the end of the stem rather than adjustable as in our objects. The mass of the rods without attachments was 208 g; with the four metal rings attached, the mass was 498 g. The four metal rings, each weighing 50 g, were attached to the crossbars—one at either side of the joint on each of the two crossbars (see Figure 4). I_1, I_2, and I_3 were manipulated systematically by placing the metal rings at computed positions along their respective rod segments and by positioning the crossbars at a computed position along the handle. Displacing the rings on the crossbars away from the joint increased their distances from the hand with a concomitant increase in I_1, I_2, as well as I_3; appropriate positioning of the crossbars on the handle eliminated these variations in I_1 and I_2. The eigenvalues for the 10 stimuli of identical linear dimensions and mass are identified in Table 4.[2]

Apparatus. From the participant's perspective the report device comprised two unmarked strips, one vertical and one horizontal, forming an inverted T, with the crossbar oriented perpendicular to the participant's frontoparallel plane. Each unmarked strip had an independent pulley system along which a pointer could be positioned by the participant anywhere from 0 to 1 m. The two unmarked strips and pulleys were within reach of the left hand. The vertical unmarked strip of the $\perp$ arrangement was aligned with the central stem of the wielded object; the horizontal unmarked strip was at the same height as the bottom of the object's central stem. Height was indicated with the left hand's adjusting the position of the pointer along the vertical branch to match the felt height of the object wielded in the right hand. Width was indicated with the left hand's adjusting the position of two pointers that moved simultaneously along the horizontal branch, one toward the participant and one away from the participant, to match the felt forward and backward extents of the object relative to the right hand. A meter rule on the reverse side of each unmarked strip was used by the experimenter to record the position of the pointers; these rules could not be seen by the participant.

Procedure. Participants sat in a chair with an attached armrest to support the right arm during wielding. The right arm and wielded objects were occluded from view with an opaque curtain. Participants grasped the object firmly by the base of the central stem and were instructed to wield it for as long as needed to perceive both its height and width. The central stem was kept relatively upright to avoid hitting the curtain and to prevent fatigue. The two extents were reported (in whatever order the participant chose) by positioning the appropriate marker relative to the join of the two unmarked strips. Participants were allowed to wield the object and adjust both indicators until satisfied with their judgments. Once the extents had been recorded, the participant repositioned the markers to their extreme maximum or minimum on alternate trials. Each object was presented three times, in a different random order for each participant.

No information about the structure of the objects was provided; participants did not see the tensor objects, and they were given no details about what types of objects (what kinds of shapes, what range of sizes) they would be wielding and judging. There were no practice trials. The participants' task was demonstrated by the experimenter, who simply wielded a clipboard by the bottom edge and arranged the pointers to match its height and width.

[2] Exact equivalence of the five small I_1 values and the five large I_1 values was not practically possible. More precisely, the small and large values were 213,671 ± 9,109 g cm^2 and 643,283 ± 4,264 g cm^2, respectively.

Results and Discussion

Mean perceived extents for each object are shown in columns 2 and 4 of Table 4. The opening question has to do with whether participants could make width and height judgments reliably for 10 objects that varied in neither dimension and that were of identical mass. That is, how consistent were the three judgments made per dimension per object? Reliability of a participant's judgments of a given dimension was measured, as in Experiment 1, by the average deviation of the three judgments expressed as a percentage of the mean (Table 5). Overall, it is apparent that participants' judgments were as reliable on the average in Experiment 2 (average 10.5%) as they were in Experiment 1 (average 11.0%). For two of the present participants (Participants 4 and 7), however, it was obvious that the width-judging task was particularly challenging, and they demonstrated very unreliable width judgments of 21% and 31%, respectively. (Note that RMS error calculations were not appropriate for Experiment 2 because the actual linear dimensions of the tensor objects did not vary.)

The central question was whether both eigenvalues affected both perceptions. A Dimension × I_1 × I_3 ANOVA was conducted. There was a main effect of I_1, $F(1, 9) = 81.20$, $MSE = 80.51$, $p < .0001$, and I_3, $F(4, 36) = 2.68$, $MSE = 12.67$, $p < .05$, but no main effect of dimension ($H_P = 34.2$ cm vs. $W_P = 35.8$ cm; $F < 1$). Most important, both two-way interactions involving dimension were significant: Dimension × I_1, $F(1, 9) = 17.67$, $MSE = 121.20$, $p < .0025$; Dimension × I_3, $F(4, 36) = 12.48$, $MSE = 18.30$, $p < .0001$. With regard to the Dimension × I_1 interaction, there was a 17.9-cm increase in H_P from the smaller to the larger I_1 compared with a 5.1-cm increase in W_P. Simple effects tests showed that both increases were significant: $F(1, 9) = 52.34$, $p < .0001$, and $F(1, 9) = 12.64$, $p < .01$, respectively. With regard to the Dimension × I_3 interaction, as I_3 increased, there was a 3.2-cm decrease in H_P and an 8.4-cm increase in W_P. Simple effects tests showed that both the increase and the decrease were significant: $F(4, 36) = 4.10$, $p < .01$, and $F(4, 36) = 11.57$, $p < .0001$, respectively. The two interactions are shown in Figure 6. (The remaining two-way interaction was not significant, $F < 1$, nor was the three-way interaction, $F < 1$.) As a check, the ANOVA was repeated with the data of Participants 4 and 7 omitted (see above). The same pattern of significant effects was found.

To further evaluate the dependency on I_k, multiple regressions of mean H_P and mean W_P on I_1 and I_3 (all quantities in logarithms) were conducted. For mean H_P, $r^2(9) = .99$, with $p < .0001$ for the .47 coefficient on I_1 (partial $F = 596.2$) and $p < .01$ for the $-.06$ coefficient on I_3 (partial $F = 14.9$); for mean W_P, $r^2(9) = .96$, with $p < .0001$ for the .13 coefficient on I_1 (partial $F = 73.7$) and $p < .0001$ for the .11 coefficient on I_3 (partial $F = 100.0$).[3] Both of the preceding patterns of dependencies were verified by stepwise regressions. Additionally, simple regressions revealed that 69% of

Table 4
Mean Perceived Height (cm) and Mean Perceived Width (cm) Together With Their Respective Standard Deviations Averaged Across Participants as a Function of I_1 and I_3 for Tensor Objects of Experiment 2

	I_1 (g cm^2)			
	21×10^4		64×10^4	
I_3 (g cm^2)	*M*	*SD*	*M*	*SD*
	Perceived height			
2.5×10^4	28.1	4.27	46.0	5.13
3.9×10^4	27.0	2.16	46.1	4.23
6.7×10^4	27.9	4.33	46.5	5.15
11.8×10^4	26.0	4.95	42.5	5.14
19.2×10^4	25.0	2.63	42.7	5.19
	Perceived width			
2.5×10^4	28.7	4.19	32.5	5.26
3.9×10^4	29.6	3.86	34.6	6.20
6.7×10^4	30.8	5.44	36.1	5.59
11.8×10^4	34.9	6.93	40.4	5.79
19.2×10^4	35.3	7.05	40.1	7.24

Note. Eigenvalues (I_1 and I_3) were computed with the origin in the wrist.

Table 5
Reliability for Height and Width Judgments in Experiment 2

	Reliability[a]	
Participant	Height	Width
1	6.3	7.8
2	6.1	5.4
3	7.5	12.1
4	13.2	21.3
5	5.8	5.1
6	7.0	15.5
7	16.0	31.2
8	7.2	6.8
9	8.8	12.4
10	7.2	6.6

[a]Average deviation expressed as a percentage of the mean perceived extent.

[3] In the present experiment the *variance inflation factor* (VIF) was 1.0 for both the H_P and W_P multiple regressions in logarithmic coordinates, well below the value of 10 at which the stability of the estimates of the exponents becomes questionable (Montgomery & Peck, 1990). The VIF is a reliable and informative diagnostic of multicollinearity in which the latter term is understood in the general case as imperfect correlations between regressors. The VIF is given by C_{jj}, defined as $C_{jj} = (1 - R_j^2)^{-1}$, where R_j^2 is the coefficient of determination resulting from regressing x_j on the remaining $p - 1$ regressors (Montgomery & Peck, 1990). If x_j is nearly orthogonal to the remaining regressors, then R_j^2 is small and C_{jj} is close to unity; if x_j is nearly nonorthogonal, then R_j^2 is near unity and C_{jj} is large. A C_{jj} in excess of 10 means that the coefficients are poorly estimated (Montgomery & Peck, 1990). Specifically, C_{jj} is the amount by which the variance of the regression coefficient associated with x_j is inflated due to near collinearity among the regressors; a direct measure of the increase in the coefficient's confidence interval due to near multicollinearity is provided by $C_{jj}^{1/2}$.

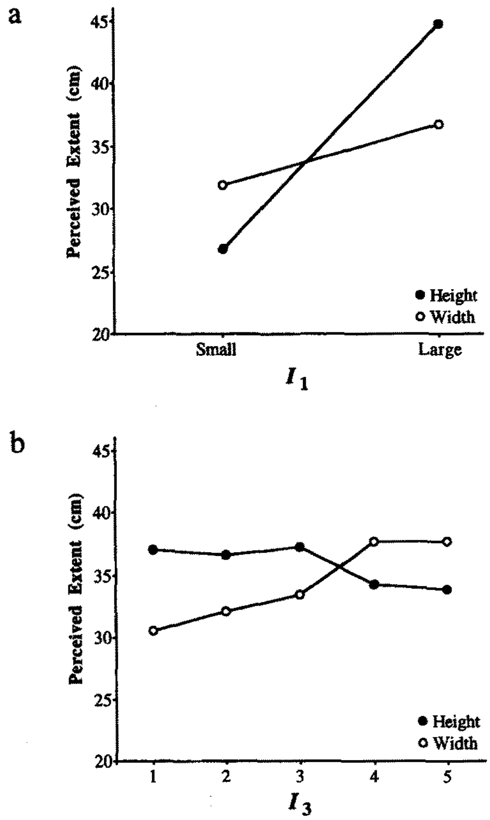

Figure 6. The interaction between (a) judged dimensions and I_1 and (b) judged dimensions and I_3 in Experiment 2.

the residual variance from the regression of log H_P on log I_1 was accommodated by log I_3 and that 92% of the residual variance from the regression of log W_P on log I_3 was accommodated by log I_1. These effects of I_k were present to a limited degree at the level of the individual participants. For the 10 individual participants' multiple regressions involving H_P, 9 were reliable at $r^2(9)$ values ranging from .80 to .99, with the sign pattern of positive I_1 and negative I_3 present for 9 of the 10 participants but with the negative I_3 coefficient reliable in only two cases. Similarly, for the individual multiple regressions involving W_P, 8 were reliable at $r^2(9)$ values ranging from .60 to .95, with the sign pattern of positive I_1 and positive I_3 but with the positive I_1 coefficient reliable in only three cases.

The results of the ANOVA and regression analyses indicated that the perception of height and width in this experiment were constrained by I_k rather than by either I_1 or I_3 alone, in agreement with Fitzpatrick et al.'s (1994) hypothesis. Using objects that varied in material composition, geometry, and the relation of inertial eigenvalues to mass, Fitzpatrick et al. identified that the perception of the longitudinal dimensions of objects with near cylindrical symmetry ($I_1 \approx I_2$) increased with I_1 and decreased with I_3. Our results reinforce the impression that for objects with near cylindrical symmetry, I_3 as well as I_1 can constrain the perception of linear extents and that I_3 can do so without variation in object density or variation in object diameter at the point of grasp that may be coordinate with I_3 (cf. Chan, 1995). What is paramount is variation in I_3, however that variation is brought about (Carello, Fitzpatrick, Flascher, & Turvey, in press). Despite the favorable support our results provide for Fitzpatrick et al.'s hypothesis, I_k's contribution to W_P conforms neither to intuitions from the inertia ellipsoid perspective nor to the results of Experiment 1. An inverse dependency of W_P on I_1 would have been expected rather than the observed direct dependency. A fully coherent outcome of Experiment 2 would have been H_P increasing with I_1 and decreasing with I_3 (which was the case) versus W_P increasing with I_3 and decreasing with I_1 (which was not the case). The failure to obtain this conceptually coherent outcome may have been due to idiosyncrasies of our experimental design. Alternatively, the failure may be evidence that perceiving width is not simply the opposite of perceiving length. For example, the form of the dependence of width perception on I_k may not be uniform across magnitudes of the eigenvalues. Future research will have to resolve this matter.

A remaining question of some significance concerns the relation between the results of Experiment 2, summarized in Table 4, and the results of Experiment 1, summarized in Table 1. The nine wooden rectangular parallelepipeds of Experiment 1 were constructed to differ in their linear dimensions and in their masses and were held by means of handles attached to their bases. The differences in their heights, widths, and mass resulted in differences in their corresponding I_1 and I_3 magnitudes. In contrast, in Experiment 2 we used 10 tensor objects that simulated rectangular parallelepipeds. These objects were of fixed height, width, and mass and were constructed to differ simply in I_1 and I_3. If participants have distinct impressions of a wielded object's spatial dimensions, constrained in a definite manner by I_k, then the superficial differences between the object sets of Experiments 1 and 2 should not matter. It is apparent, however, from a comparison of Tables 1 and 4 that participants in the two experiments did not exhibit perceptions of comparable magnitudes despite considerable overlap in the values of I_k; perceived extents were uniformly larger in Experiment 2. The discrepancy might be due to mechanical aspects of wielding tensor objects versus wielding rectangular parallelepipeds that are not captured by the inertia tensor. Alternatively, it might be due to a procedural contrast between the two experiments. Whereas the participants in Experiment 1 saw the kind of solid rectangular object that they would be wielding prior to the experiment, the participants in Experiment 2 were never shown the experimental stimuli and were completely naive about the construction of the to-be-wielded objects.

In Figure 7, the perceived height data of Experiment 2 are combined with those from Fitzpatrick et al.'s (1994) Experiment 4, in which 15 solid geometric objects were used: three variants each of a cylinder, hemisphere, cube, pyramid, and cone. These objects ranged between 486 g and 1,138 g in mass and between 21.2 and 53.8 cm in height (from the base of the attached handle to the tip of the geometric solid). For our purposes, the special importance of Fitzpatrick et al.'s experiment is that the participants never saw the types of

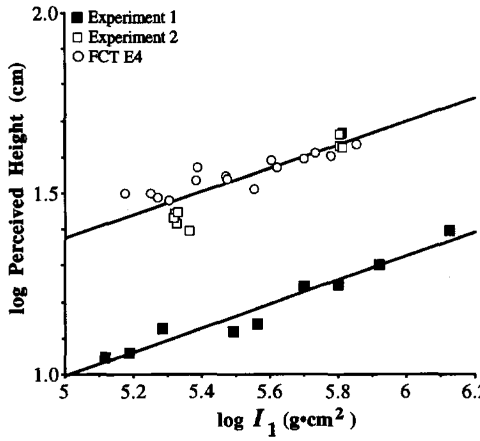

Figure 7. Perceived height as a function of I_1 in logarithmic coordinates for Experiment 2 and FCT E4 (Experiment 4 of Fitzpatrick et al., 1994) and for Experiment 1. The intercept and slope of the upper function are −.24 and .32, respectively, and the intercept and slope of the lower function are −.67 and .33, respectively. These lead to the power functions, perceived length = .58 $I_1^{.32}$ and perceived length = .21 $I_1^{.33}$, respectively.

objects they were required to wield (they had no foreknowledge, visual or otherwise, that these objects were common geometric solids) and were given no information about the size range. Inspection of Figure 7 suggests that the perceived heights in these two experiments (Experiment 2 and Fitzpatrick et al.'s Experiment 4), in which the objects were never visually experienced, might be accommodated by the same power function of I_1. Inspection of Figure 7 also suggests that this latter power function differs from that governing the data of Experiment 1 only in the coefficient or level constant (.58 vs. .21); the exponent scaling perception to I_1 was essentially ⅓ with and without visual experience. Given that participants in Experiment 1 tended to perceive in the actual height range of 9.5–27.9 cm, it would appear that dynamic touch may be significantly tuned by minimal visual experience with the kinds of objects that are to be wielded. Examination of this possibility is an important topic for future research.

General Discussion

A common, everyday experience is that when one grasps a corner of a rectangular solid object such as a book and wields it freely (three dimensionally), there are tangible impressions of how much of the book extends forward of the hand and how much extends to either side of the hand. The two experiments reported here have shown the dependencies of the perception of the extent forward of the hand primarily on I_1 and the perception of the extent sidewise to the hand primarily on I_3. Specifically, Experiment 1 identified the scaling relations: $H_P \propto I_1^{1/3}$, $W_P \propto I_3^{1/4}$. Experiments 1 and 2 also revealed influences (but of lesser degree) of I_3 on length perception and I_1 on width perception. The latter observations are consonant with the implications of Equation 1. When an object is wielded about any arbitrary axis, all distinct eigenvalues contribute to the rotational forces deforming the body's tissues. For the cylindrically symmetrical objects used in our research, two eigenvalues are distinct. Both, therefore, can be expected to affect spatial perceptions by wielding. Our confirmation of this expectation emphasizes the potential importance of the inertia ellipsoid—as originally suggested by Solomon (1988)—to the developing theory of dynamic touch (see summaries by Turvey, 1996; Turvey & Carello, 1995).

In sum, the results of Experiments 1 and 2 suggest that a hand-held object's height and girth are selectively perceptible with reasonable reliability and accuracy by dynamic touch. The capability of perceiving both the lateral and longitudinal dimensions of an object indicates a rudimentary capability of nonvisible shape perception (Burton et al., 1990). This crude shape perception may be sufficient to tune muscular synergies or coordinative structures in the course of manipulatory and instrumental actions conducted with minimal visual control. In what follows we compare tangible shape with visual shape, comment briefly on the perception of linear dimensions by dynamic touch relative to their perception by sight, and highlight the selective-attention capabilities of dynamic touch.

Tangible Shape and Issues of Shape Perception

Three of the most original and thoughtful analyses of the problem of shape perception are those of Marr (1982), Koenderink (1990), and Gibson (1950). These analyses bear on previous (Burton et al., 1990) experimental investigations of shape perception by dynamic touch and on the present investigation of height and width perception. In turn, these particular haptic investigations provide a measure of support for core postulates of Marr, Koenderink, and Gibson that were formulated primarily with vision in mind.

For Marr (1982), the key feature of visual shape perception is its independence from viewpoint. A shape's articulation and its components must be captured in a reference frame that is tied to the shape itself. Consequently, identifying a canonical reference frame within an object is the first step in understanding shape perception, and in the construction of a computational account it prefaces the formulation of the steps that generate the shape description. Marr suggested that for the elected frame of reference to be canonical, it must be based on axes determined by salient geometrical characteristics of the shape, for example, its symmetry. Given the canonical reference frame, the articulation of an object's shape can then be achieved through the aid of two types of primitives, surface based (two dimensional [2-D]) and volumetric (3-D). The latter primitives refer to the spatial distribution of the shape and were considered by Marr to be more directly related to the requirements of shape perception than the former, surface-based primitives (see also Biederman, 1987).

Koenderink (1990) sought to ground the understanding of visual shape perception in the principal curvatures defined at a surface point given the essential requirement of a quantification of shape that is independent of perspective (coordinate system) and scale. Consonant with Marr's (1982) proposal, Koenderink (see also Lehky & Sejnowski, 1990) tied the coordinate system for a shape to the shape itself, specifically, to its principal directions or eigenvectors and their magnitudes, the eigenvalues k_{max} and k_{min} (for applications to haptic touch, see Kappers, Koenderink, & Lichtenegger, 1994; Kappers, Koenderink, & te Pas, 1994).

In dynamic touch, the frame of reference for an object is provided by the principal directions or eigenvectors $\mathbf{e}_k$ of I_{ij} defined about the object's rotation point O. These are the object's symmetry axes about O, and dynamically they constitute the only nonarbitrary reference frame. The suggestions of Marr and Koenderink for shapes examined by eye are dictates for shapes examined by dynamic touch. Within the eigenvector reference frame, the maximum and minimum resistances to rotational acceleration are the analogues of the maximal and minimal surface curvatures in Koenderink's formulation. Shape perception by dynamic touch and shape perception by vision may be founded on similar principles.

The latter point is underscored by Gibson's (1950) insistence that the perception of any particular shape is tied to the detection of a "formless" invariant over transformations. For Gibson, all perceiving begins with a structured energy array—different magnitudes of one or more relevant energy measures in different directions (Gibson, 1979). The structuring is the lawful consequence of environmental properties and the movements of the observer. Dynamic touching makes these intuitions particularly clear. In wielding an object, there is a time-varying structured array of torques given by the time-independent inertia ellipsoid and the time-dependent angular velocities and accelerations (see Equation 1). Perception of the properties that are unchanged during the wielding—such as the dimensions of the hand-held object and the hand's position relative to the object—are tied to those aspects of the structured mechanical energy array that are invariant. Perception of the properties that change during the wielding—such as the hand-held object's displacement relative to the body—are tied to those aspects of the array that are variant. With respect to object shape, the invariant of the structured mechanical energy array, namely, the relation between the major and minor eigenvalues, is revealed over its transformations. Further, this invariant is without form in the sense that it is an intensity (the magnitude of the ratio of primary resistances to rotational acceleration) specific to the shape and not a spatial feature resembling the shape. For Gibson, the perception of shape by vision can be no different from the perception of shape by dynamic touch in respect to its reliance on invariant quantities that are specific to shapes rather than on retinal images that copy a shape's perspective-dependent 2-D forms.

Although they may be grounded in identical principles, shape perception by dynamic touch cannot be expected to match shape perception by vision. The structured array of mechanical energy embodied by Equation 1 is considerably less differentiated than the optic array. The latter—comprising different intensities, different spectral distributions, and different specular highlights, in different directions—is more richly patterned at more nested length scales than the structured array available to the haptic perceptual system in wielding an object. The shape distinctions achievable by dynamic touch cannot exceed the specifying power of I_{ij}.

Lengths Perceived Visually and by Dynamic Touch

The visually perceived length of a physical interval varies with its location and orientation relative to the observer, contrary to the axioms of Euclidian geometry (Norman et al., 1996; Todd, Tittle, & Norman, 1995; Toye, 1986; Wagner, 1985). For example, the perception of a frontoparallel interval has been found to increase with distance, whereas the perception of a longitudinal (in depth) interval has been found to decrease with distance (e.g., Baird & Biersdorf, 1967; Norman et al., 1996). The basis for these visual effects are not known (Norman et al., 1996). The present experiments reveal that the perception of the frontoparallel (width) and longitudinal (height) intervals of a hand-held object by dynamic touch are functions of I_k. For the objects of Experiment 1, the patterning of perceived frontoparallel and longitudinal intervals differed, with the greater perceptual distortion of the latter understandable through the weaker relation between object height and I_1 relative to the relation between object width and I_3. The results of Experiment 1 suggest that these intervals may be perceived with a reliability and an accuracy that are comparable to their perceptions visually. The results of Norman et al. provided the basis for comparison. For frontoparallel intervals, the mean reliabilities were 10.2% (touch) versus 6.7% (vision), and the mean accuracies were 25.5% (touch) versus 17.0% (vision). For longitudinal intervals, the mean reliabilities were 11.9% (touch) versus 7.4% (vision), and the mean accuracies were 33.4% (touch) versus 28.1% (vision). To underscore the potential closeness between the achievements of dynamic touch and vision, we computed the reliability and accuracy for the data of Solomon and Turvey's (1988) Experiment 1 in which participants judged "simple" lengths, namely, the longitudinal dimensions of seven homogeneous aluminum rods. For these longitudinal intervals, the reliability was 8.6% and the accuracy was 16.3%. Clearly, there is a need to examine more carefully and more thoroughly the length perception capabilities of dynamic touch relative to those of vision. As suggested elsewhere, dynamic touch and the grounding of its spatial abilities in the invariants of rotational dynamics is a potential source of insight into the (elusive) basis of vision's spatial abilities (Barac-Cikoja & Turvey, 1993, 1995).

Selectivity of Perception by Dynamic Touch

The present research provides a new demonstration of the selectivity of dynamic touch. In Experiments 1 and 2, participants alternated between wielding an object to perceive how far it extended longitudinally and wielding an object to perceive how far it extended laterally. Clearly, the participants were able to perform these two tasks distinctively: H_P and W_P tended to be uncorrelated, and they were constrained in different ways by I_1 and I_3. These outcomes of

Experiments 1 and 2 complement previous research on selective perception by dynamic touch. When grasping a rod at a position intermediate between its ends, a person can attend selectively to the rod length forward of the hand or to the length of the entire rod, with the two perceptions constrained in different ways by I_{ij} (Carello et al., 1996; see also Turvey et al., 1996). Similarly, when grasping a rod at a position intermediate between its ends, a person can attend selectively to the position of the hand or to the rod length forward of the hand, with the hand position and length perceptions dependent in different ways on I_{ij} (Pagano et al., 1996). When striking a distal surface with a hand-held probe, a person can attend selectively to the nonvisible distance of the surface or to the nonvisible length of the probe without confusion, with the two perceptions constrained by different mechanical quantities (Carello, Fitzpatrick, & Turvey, 1992; Chan & Turvey, 1991). Finally, it is evident that participants can be selective with respect to perceiving hand–object relations rather than properties of the objects themselves. As summarized earlier, the perception of hand–object relations in the wielding of objects is constrained selectively by the directions of $\mathbf{e}_k$. It would seem, therefore, that selective perception and issues of selective attention are as characteristic of dynamic touch, and as central to its understanding, as they are to the perceptual systems of more traditional concerns, namely, vision and audition (Burton & Turvey, 1990; Turvey, 1996).

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