# Bond Graph Modeling of Centrifugal Compression Systems

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## ABSTRACT

*A novel approach to model unsteady fluid dynamics in compressors network by using bond graph is presented. The model is intended in particular for compressor control system development. First, we develop a bond graph model of a single compression system. Bond graph modeling offers a different perspective to previous work by modeling the compression system based on energy flow instead of fluid dynamics. Analysing the bond graph model explains the energy flow during compressor surge. Two principal solutions for compressor surge problem are identified: upstream energy injection and downstream energy dissipation. Both principal solutions are verified in bond graph modelings of single compression system equipped with surge avoidance system (SAS) and single compression system equipped with active control system. Moreover, the bond graph model of single compressor equipped with SAS is able to show the effect of recycling flow to the compressor upstream states which improves the current available model. The bond graph model of single compression system is then used as the base model and combined to build compressors network models. Two compressor networks are modeled: serial compressors and parallel compressors. Simulation results show the surge conditions in both compressor networks.*

**Keywords:** bond graph, compressor modeling, compressor surge, surge avoidance system, active surge control system, serial compressors, parallel compressors.

## 1 Introduction

A centrifugal compressor is basically used to increase gas pressures. The compressor operating area is described by a plot of the compressor pressure against the flow for different compressor speeds known as a compressor map. Figure 1 shows an example of a compressor map. The stable compressor operating area is limited by a surge line for lower flow. Operating the compressor at a flow lower than the surge line would cause the compressor to go into an unstable condition known as surge. It is an axisymmetric oscillation of the compressor flow and the compressor produced pressure followed by severe vibrations. The vibrations may reduce the reliability of the compression system and large amplitude vibrations may lead to compressor damage in particularly to the compressor blades and bearings, and also damages on pipe connections.

Surge phenomena is of interest as higher compressor efficiency operating points are located near the surge line and some processes are requiring low mass flow at high pressure. However, operating in such points may endanger the compressor because a disturbance could bring the compressor into the surge area. There is a trade-off between operating the compressor at a higher efficiency and the stability. Two methods have been developed to overcome the surge problem:

a) Surge avoidance system (SAS)
SAS works by introducing a surge control line located at certain margin to the right of the surge line as illustrated in Figure 1. The margin is known as surge margin. The SAS has a recycle line and a recycle valve. The recycle valve is controlled by a controller which compares the compressor inlet mass flow to a reference mass flow. The reference mass flow is the mass flow at the surge control line for a related compressor speed. The recycle valve will open when the inlet flow is less than the reference mass flow. It results in recycling flow from the downstream line to the upstream line and increasing the compressor inlet flow to be the same as the reference mass flow. This mechanism makes the surge control line to be the lower limit for the compressor flow and the compressor operation will not reach the surge line.

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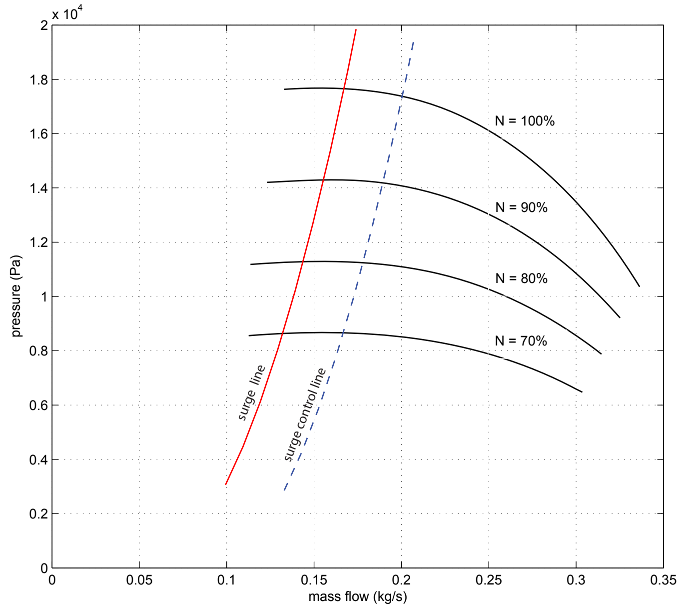


Fig. 1. A typical compressor map for different rotational speeds, N.

Such surge avoidance schemes successfully ensure the safe operation and are widely used for industrial compressors. However, introducing the surge control line reduces the usable size of the compressor map and thereby restricting the compressor operational envelope.

b) Active surge control system (ASCS)
ASCS works by stabilizing surge. It was initialized by [1] who introduced surge stabilization by using an active element. This method could enlarge the operating envelope towards the lower mass flow in the stabilized area. Since then a number of theoretical and experimental results on active surge control systems have been published and several different actuators have been introduced as summarized in [2], examples include: closed couple valve, bleed valve, throttle valve, gas injection, variable plenum, and variable guide vanes. Active surge control by using variable speed drives [3], piston-actuation [4], and active magnetic bearing [5] have also been reported. Although ASCS enlarges the operating envelope which can improve the compressor performance, it has not yet been applied in industrial system. The safety of implementing ASCS is a serious issue, since a failure in ASCS may bring the compressor into the surge [6].

Natural gas pipeline system is an example of industrial compressor applications. Several compressors are commonly operated simultaneously in such systems. The compressors can be arranged in serial, parallel, or serial-parallel. Serial compressors is used to gain higher pressure and parallel compressors to gain higher flow. It has been reported that the compressors operation consume a significant portion of the transmitted gas in the pipeline system. Optimizing the compressors operation is a solution that minimize the operating cost. Mathematical models of compressors in pipeline networks are required for design and optimization of the network. Several works on compressors network modeling have been presented [7–14]. The models focused on stable operating area and assumed the compressors would not enter surge as the compressors were equipped with SAS. However, it is still possible that the compressors in the network could enter surge, for an example due to the SAS failure. It is of interest to further investigate surge in such a compressor network. A model of compressors in a network which can simulate surge is required for this purpose, and to the best of our knowledge such a model can not be found in the existing literatures.

A model of a single compression system which is able to predict the transient response of a compression system subsequent to a perturbation from steady operating conditions has been introduced by Greitzer in [15]. The model is then known as the Greitzer compression model. The Greitzer model was developed based on fluid dynamics laws. Another modeling of the compression system by using bond graph has also been presented in [16]. Both modelings resulted in the same system dynamics which can be use to predict the transient response including surge condition. Bond graph is a modeling method based on energy flow which is universal and systematic [17, 18]. The universality provides the same description in modeling a system with various energy domains. Bond graph is systematic such that several simple models can be combined to build a complex model and the dynamic equations of the system is obtained in a straight forward manner by investigating energy flow in the storages elements. The bond graph model of a single compression system can be extended to model more

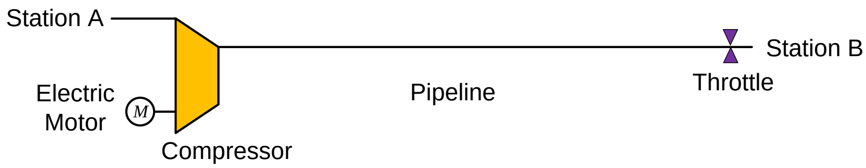


Fig. 2. A pipeline system with a single compressor.

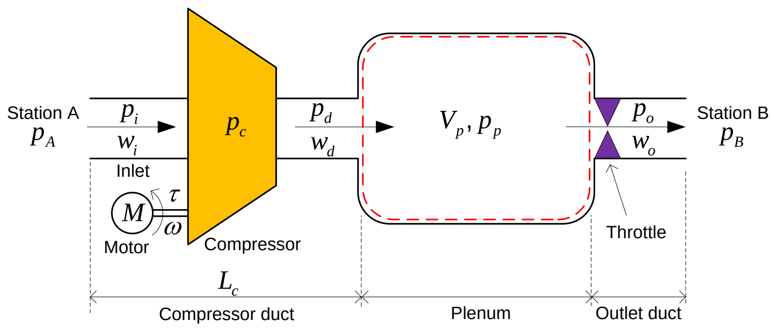


Fig. 3. Model of single compression system.

complex system as well as the dynamic equations. Therefore, we are applying bond graph to model compressor networks. The modeling will be done by combining the bond graph models of single compressor systems. The discussion is limited to incompressible flow and constant compressor speed. This paper is an extension work of [16] by presenting more detail description of bond graph modelings of compression system and compressor networks.

## 2 Single Compression System

A simple diagram of a gas pipeline with a single compressor is shown in Figure 2. The electric motor rotates the compressor blades such that the gas at station A is pulled into the compressor through an inlet. The gas is then accelerated towards the compressor impeller, de-accelerated at the compressor diffuser, and collected in the compressor volute before being discharged. Such processes result in a gas pressure rise. The gas is then discharged and directed to the station B through the pipeline. A throttle is installed before the pipeline outlet to adjust the flow arriving at station B. A model of such compressor system which able to predict the transient response subsequent to a perturbation from steady operating condition was introduced by Greitzer in 1976 [15]. The model is then known as the Greitzer compressor model. The Greitzer model was developed based in fluid dynamics laws (momentum equation and mass conservation) and used the following assumptions: quasi-steady compressor behavior, uniform pressure throughout the plenum, isentropic fluid behaviour in the plenum, incompressible flow in the duct, negligible throttle inertance, quasi-steady flow through the throttle, and constant ambient conditions.

Following the Greitzer model, the pipeline compression system in Figure 2 is modeled as in Figure 3. The model is identical to the Greitzer model except the addition of a motor drive which generates torque $\tau$ to rotate the compressor with rotational speed $\omega$, which is also covered in [19, 20]. Furthermore by neglecting friction flow along the pipeline, the inlet pressure ($p_i$) is equivalent to the pressure at station A ($p_A$), the inlet mass flow ($w_i$) is equivalent to the discharged mass flow ($w_d$), and the outlet pressure ($p_o$) is equivalent to the pressure at station B ($p_B$). The space inside downstream pipeline is modeled as a plenum with volume $V_p$ and pressure $p_p$. It is assumed that the mass flow through the plenum is steady and the pressure is uniform, that there is no pressure drop. A throttle is applied to adjust the outlet flow ($w_o$) and thereby generating throttle pressure drop ($p_{R_T}$) between the plenum and the outlet. The dynamic equations of the Greitzer compression system is given as follows [15]:

$$\dot{w}_i = \frac{A_c}{L_c}(p_c - p_p) \tag{1}$$

$$\dot{p}_p = \frac{a_0^2}{V_p}(w_i - w_o), \tag{2}$$

where $A_c$ is the compressor duct cross-sectional area, $L_c$ is the effective length of the equivalent compressor duct, $p_c$ is the compressor pressure rise, and $a_0$ is the speed of sound. The compressor pressure rise is typically presented in a compressor map as a function of mass flow and compressor speed and provided by the compressor manufacturer. A compressor map can also be obtained by performing a compressor test to collect compressor operating data (e.g.: pressure, mass flow and speed) and approximated by a mathematical function as follows:

a. Compressor pressure rise approximation for a constant compressor speed [21]:

$$p_c(w_i) = p_{c_0} + H\left[1 + \frac{3}{2}\left(\frac{w_i}{W} - 1\right) - \frac{1}{2}\left(\frac{w_i}{W} - 1\right)^3\right], \tag{3}$$

where $p_{c_0}$ is the shut-off value of the axisymmetric characteristic, $W$ is the semi-width of the cubic axisymetric compressor characteristic, and $H$ is the semi-height of the cubic axisymetric compressor characteristic; consult [21] for more detailed definitions.

b. Compressor pressure rise approximation for varying compressor speed [3]:

$$p_c(\omega, w_i) = \left(1 + \frac{\mu d_2^2\omega^2 - 0.5d_1^2(\omega - \alpha w_i)^2 - 4k_f w_i^2}{4c_p T_{01}}\right)^{\frac{\kappa}{\kappa-1}}. \tag{4}$$

where $\omega$ is the compressor speed, $d_1$ is the mean inducer diameter, $d_2$ is the impeller diameter, $k_f$ is the fluid friction constant, $T_{01}$ is the inlet stagnation temperature, $c_p$ is the specific heat at constant pressure, $c_v$ is the specific heat at constant volume, and $\kappa = \frac{c_p}{c_v}$ is the ratio of specific heat. Consult [3] for more detailed definitions of $\mu$ and $\alpha$.

The outlet mass flow is given by:

$$w_o = u_T k_T \sqrt{p_p - p_o} \tag{5}$$

where $u_T$ is the throttle opening with the range value from 0 to 100% and $k_T$ is the throttle constant.

## 3 Bond Graph Modeling

Bond graph is a modeling method based on energy transfer among the components in a system. The energy are transferred through the components port. The energy transfer is shown as a line with a half arrow which is called bond. The half arrow is showing the energy transfer direction. A bond has two variables: effort ($e$) and flow ($f$) to describe energy. Table 1 shows the definitions of effort and flow of various energy domains. Multiplication of effort and flow results in power $P$,

$$P = ef. \tag{6}$$

It is common to placed an effort variable above or on the left of a bond and a flow variable below or on the right of a bond. The direction of effort is shown by a perpendicular bar at the end of a bond which known as the causality. The flow direction is always opposite to the effort direction.

Bond graph modeling defines components in a system as: effort source ($S_e$), flow source ($S_f$), resistance ($R$), inertia ($I$), capacitor ($C$), transformer ($TF$), and gyrator ($GY$). Source effort is a component which supplies effort. Source flow is a component which supplies flow. Resistance is a component which dissipates energy. Inertia is a storage element which integrates the effort. Capacitor is a storage element which integrates the flow. Transformer is a component which converts effort at one port into effort at another port with the transformer ratio $n$ and separately converts the flow in the opposite direction with the same transformer ratio. Gyrator is a component which relates the effort of one port to the flow of another port with the gyrator ratio $r$, and vice versa. If the conversion ratio depends on another variable, the transformer is called as modulated transformer ($MTF$) and the gyrator is called as modulated gyrator ($MGY$).

A bond graph component is connected by a bond to another component or junction. A junction is a connecting point and the energy is neither stored nor dissipated. There are two types of junctions:

a. $0-$ junction

The $0-$ junction connects bonds with equal effort and the total flow is zero.

$$f_1 + f_2 + ... + f_n = 0 \tag{7}$$

$$e_1 = e_2 = ... = e_n \tag{8}$$

b. 1− junction
The 1− junction connects bonds with equal flow, and the total effort is zero.

$$f_1 = f_2 = \dots = f_n \tag{9}$$
$$e_1 + e_2 + \dots + e_n = 0 \tag{10}$$

Dynamic equations of a bond graph model are obtained by effort-flow relation of the storage energy elements ($C$ and $I$). More detail description about bond graph can be found in the bond graph literature, for examples in [17, 18, 22].

Table 1. Effort and flow of various system domains

| Energy domain | Effort ($e$) | Flow ($f$) |
|---|---|---|
| translational mechanics | force $F$ [N] | velocity $v$ [m/s] |
| rotational mechanics | torque $\tau$ [Nm] | angular velocity $\omega$ [rad/s] |
| electrical | voltage $E$ [volt] | current $i$ [ampere] |
| hydraulics | total pressure $p$ [N/m$^2$] | volume flow $q$ [m$^3$/s] |
| pneumatics | total pressure $p$ [N/m$^2$] | volume flow $q$ [m$^3$/s] |
| thermodynamics | temperature $T$ [K] | entropy flow $\dot{S}$ [J/(Ks)] |

## 4 Bond Graph Model of a Single Compression System

The single compression system shown in Figure 2 includes several physical domains. The transmitted gas is in the pneumatic domain, the electric motor transfers energy from the electrical domain into the mechanical domain, the compressor transfers energy from the mechanical domain into the pneumatic domain, and the valve dissipates the pneumatics energy into heat. There are energy transfer from the pneumatic energy into heat due to friction flow along the pipeline and the mechanical energy into heat due to friction in the rotating parts of the compressor. Accounting all of the domains in the system makes in the system modeling becoming extremely complicated. Limiting the modeling scope of the compression ssytem by using several assumptions is necessary as in the Greitzer model. By considering only the pneumatics domain and the mechanical domain of the motor drive, using the same assumptions as in the Greitzer model, and assuming that friction on the piping is insignificant, the components in the single compression system shown in Figure 3 are modeled as follows:

i. Stations A and B are modeled as effort source elements $S_{e_1}$ and $S_{e_2}$ as both are providing pressure $p_A$ and $p_B$, respectively.
ii. The electric motor provides angular velocity to the compressor and is modeled by a flow source element $S_f$.
iii. The compressor converts the angular velocity into fluid pressure is a kind of gyrator. Since the produced pressure is also depending on the mass flow, the compressor is defined as a modulated gyrator $MGY$. The gyrator ratio is given the compressor pressure rise which can be approximated by (3) for a constant compressor speed and (4) for varying compressor speed. This study is only considering compressors at a constant compressor speed.
iv. The fluid inertia along the compressor duct is modeled by an inertia element ($I$).
v. The pipeline volume or plenum provides a capacitance effect and is modeled by a capacitor element ($C$).
vi. The throttle produces flow resistance and is modeled by a resistor element ($R$). Flow resistance due to friction in the pipeline is assumed to be extremely small compared to the throttle resistance and is neglected. The the effort and flow relation in the throttle is given in (5).

Following the bond graph procedures [17, 18], the bond graph model of a single compression system is given in Figure 4. By assuming pressures at station A and B are equivalent to the ambient pressure and the pressures are measured relative to the ambient pressure, the compressor pressure discharge is equal to the compressor pressure rise ($p_d = p_c$). Therefore, the bond graph model can be simplified as shown in Fig. 5. The bonds in the model are identified by index numbers started by number two. Table 2 shows the effort and flow of each bond in the model.

The compressor dynamics are obtained by the effort-flow relation at storage energy elements ($C$ and $I$ ) connected by bonds number 4 and 6 respectively, and given as follows:

$$I\frac{df_4}{dt} = e_3 - e_5 \tag{11}$$

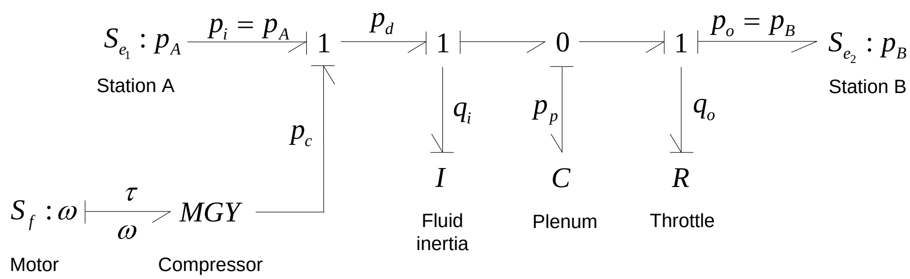


Fig. 4. Bond graph model of the single compression system.

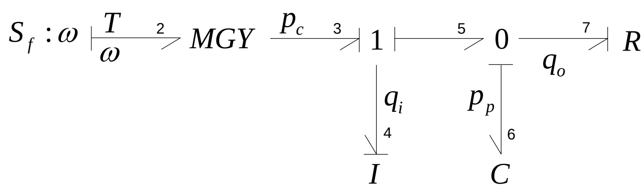

Fig. 5. Simplified bond graph model of the single compression system.

Table 2. Effort and flow of the compressor model

| Bond | Effort | Flow | Note |
|---|---|---|---|
| 2 | $e_2 = \tau$ | $f_2 = \omega$ | - |
| 3 | $e_3 = p_c$ | $f_3 = q_i$ | - |
| 4 | $e_4 = e_3 - e_5$ | $f_4 = q_i$ | Inertia : $e_4 = I\frac{df_4}{dt}$ |
| 5 | $e_5 = p_p$ | $f_5 = q_i$ | - |
| 6 | $e_6 = p_p$ | $f_6 = f_5 - f_7$ | Capacitor : $f_6 = C\frac{de_6}{dt}$ |
| 7 | $e_7 = p_p$ | $f_7 = q_o$ | Resistor : $f_7 = \frac{1}{R}e_7$ |

$$\dot{q}_i = \frac{1}{I}(p_c - p_p) \tag{12}$$

$$C\frac{de_6}{dt} = f_5 - f_7 \tag{13}$$

$$\dot{p}_p = \frac{1}{C}(q_i - q_o). \tag{14}$$

The relation of mass flow and volume flow is given by

$$w = \rho q, \tag{15}$$

where $\rho$ is the fluid density. Since the flow is assumed as incompressible flow, the fluid density is constant and the equations (12) and (14) can be expressed in mass flow as:

$$\dot{w}_i = \frac{\rho}{I}(p_c - p_p) \tag{16}$$

$$\dot{p}_p = \frac{1}{\rho C}(w_i - w_o). \tag{17}$$

By defining $I = \frac{\rho L_c}{A_c}$ and $C = \frac{V_p}{\rho a_0^2}$, the equations (16) and (17) are equivalent to (1) and (2), respectively.

## 5 Energy Analysis of Compressor Surge

Compressor surge is defined as a condition at which the pressure developed by the compressor is less than the pressure in the system (downstream) [23]. It occurs due to inability of the impeller to produce the amount of required energy for the process system [24]. It results in an axisymmetric oscillation of mass flow and pressure as shown by a limit cycle. The chronology of surge is explained as follows refer to Figure 6 [25]:

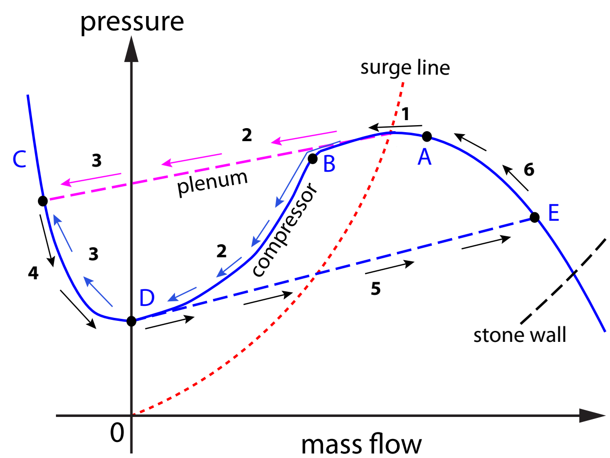


Fig. 6. Compressor surge cycle.

1) A compressor initially operates at steady-state at operating point A located in the stable area, though near the surge line. This operation results in the plenum pressure being the same as the discharge pressure. Due to a disturbance, for an example by reducing the throttle opening for certain value such that the compressor mass flow is reduced and the operating point is moving to B located to the left of surge line.
2) At point B, the compressor discharge pressure is a bit less than the plenum pressure, which makes the mass flow decelerate as formulated in (16), and the compressor produces less pressure discharge according to the compressor characteristic curve. The mass flow continues to decelerate to zero.
3) The mass flow deceleration continues to negative flow (reverse flow). The compressor acts like an orifice in the reverse flow. The stored energy in the plenum accelerates the reverse flow until it reaches the maximum reversal flow which is at point C.
4) After reaching point C, the plenum pressure is decreasing and the reverse flow is decelerated to zero at point D.
5) At point D, the compressor begins to the normal operation by accelerating the flow until point E.
6) The mass flow is then reduced to build plenum pressure. However, the compressor operation from point E is going to point A and then to point B such that the cycle repeats.

Surge occurs due to the compressor pressure discharge ($p_c$) being less than the plenum pressure ($p_p$) which results in decelerating the inlet mass flow such that the operating point crosses the surge line. The bond graph model shown in Figure 5 expresses $p_c$ as $e_3$ at bond 3 and $p_p$ as $e_5$ at bond 5, respectively. Both the bonds are connected together with bond 4 by a 1-junction and the equations are given as follows:

$$e_4 = I\dot{f}_4 = e_3 - e_5 \tag{18}$$
$$f_3 = f_4 = f_5, \tag{19}$$

where $f_4$ is the inlet volume flow. Surge occurs when $e_3 < e_5$ or $e_4$ is negative. It can be eliminated by maintaining $e_4$ non-negative through two following methods:

a. Increasing $e_3$

Effort $e_3$ is the effort output of the gyrator (compressor) where the input-output relation is given by :

$$e_3 = rf_2, \tag{20}$$

where $r$ is the gyrator ratio. Effort $e_3$ (compressor pressure rise) can be increased by increasing $f_2$ (motor speed). It is illustrated in Figure 1 and shown in [3], that the compressor produces a higher pressure at higher motor speeds for the same mass flow.

b. Decreasing $e_5$

Effort $e_5$ comes from the $0-$junction. Based on the junction properties, the connected bonds have the same effort ($e_5 = e_6 = e_7$) and only one bond has an effort inward to the junction, i.e., the bond of the $C-$element ($e_6$). The relation between effort and flow in the $C-$element is given by:

$$e_6 = \frac{1}{C}\int f_6 dt + e_6(0) \tag{21}$$

where the flows equation in the $0-$junction is given by:

$$f_6 = f_5 - f_7. \tag{22}$$

The effort $e_6$ can be reduced by making $f_6$ negative, which can be done by increasing $f_7$. Reducing $f_5$ may also result in $f_6$ becoming negative. However, this is not physically possible because $f_5$ is the compressor mass flow ($w_1$) and reducing the mass flow will bring the compressor into surge. Flow $f_7$ is the flow of the $R-$element (throttle), where the relation between flow and effort is given by:

$$f_7 = \frac{1}{R}e_7. \tag{23}$$

Flow $f_7$ can be increased by reducing the $R$ value which means increasing the throttle opening.

The direct methods to eliminate surge are by increasing the compressor speed and/or the throttle opening. However, those options are not always applicable, for example if surge occurs when shutting down the compressor, where the motor speed and the mass flow should decrease. We introduce terms of upstream energy injection for increasing $e_3$ and downstream energy dissipation for reducing $e_5$ as the two principal for solving the compresor surge. Both principal solutions will be evaluated through a discussion about compressor surge control in the next section.

## 6 Surge Control Solutions

Two methods to overcome compressor surge have been presented, i.e.: surge avoidance system (SAS) and active surge control system (ASCS). We are going to use bond graph to model the systems implementing both methods. The models will show explicitly how to maintain $e_4 > 0$ in practical system or in experimental setup.

### 6.1 Surge Avoidance System

A surge avoidance system (SAS) is the commonly used as surge solution in industrial compressors. SAS introduces a surge control line (SCL) located at certain margin to the right of the surge line (SL). The margin is known as surge margin (SM) and defined as: The distance between SCL and SL is called the surge margin and is defined as [26]:

$$SM = \frac{w_{SCL} - w_{SL}}{w_{SCL}}, \tag{24}$$

where $w_{SL}$ is mass flow at surge line and $w_{SCL}$ is mass flow at surge control line for corresponding compressor speed. SAS has a recycle line and a recycle valve. The recycle valve is controlled by a controller which compares the compressor mass flow to a reference mass flow ($w_{SCL}$). When the compressor operating point is crossing the SCL ($w_i < w_{SCL}$), the SAS controller gives a command to the recycle valve to open such that the plenum fluid is flowing back to the compressor inlet through the recycle line. The recycling flow reduces the plenum pressure and the compressor mass flow is accelerated such that the operating point goes to the SCL ($w_i = w_{SCL}$).

The recycling flow ($w_r$) is giving an additional fluid flow to the upstream such that the inlet states will be affected. The effect is investigated by considering a capacitance effect in the space around the connection of the feeding line, the recycle line, and the inlet line. The space is then called a buffer, as shown in Figure 7.

Bond graph is applied to model the compression system equipped with SAS shown in Figure 7. The same assumptions as in modeling the single compression system are applied. By following the bond graph procedures in particularly on causality,

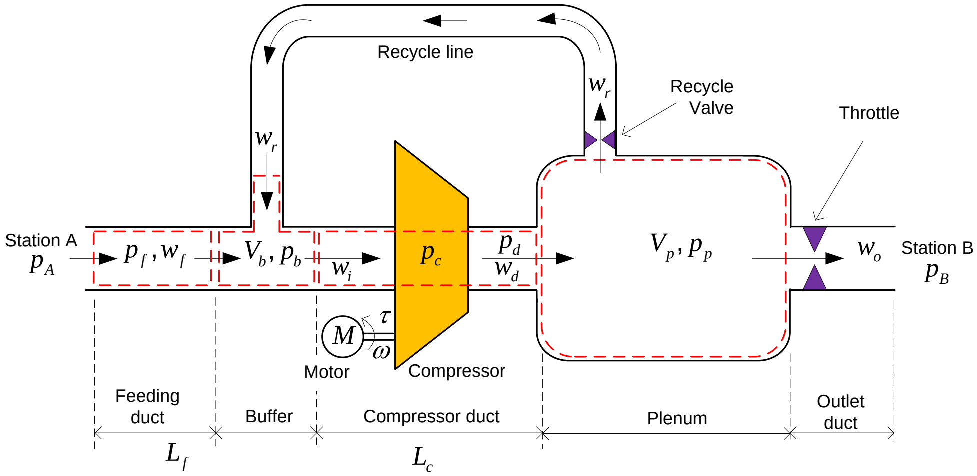


Fig. 7. A compression system equipped with surge avoidance system (SAS).

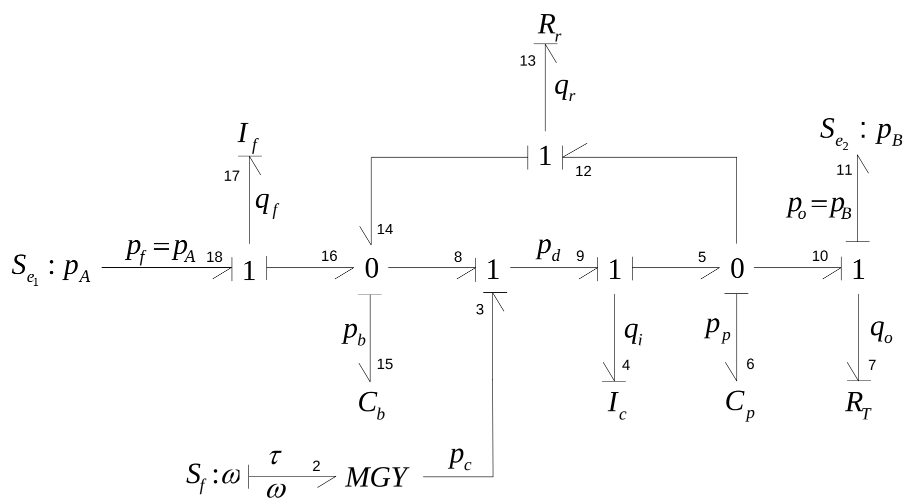

Fig. 8. Bond graph model of the compression system equipped with SAS.

we have to take into account the inertia effect in the feeding line ($I_f$) due to the recycling flow; otherwise, we cannot get a proper causality at the 0-junction connecting the buffer ($C_b$). The resulting bond graph model is shown in Figure 8. The fluid inertia along the compressor duct is modeled by $I_c$, the capacitance effect in the plenum is modeled by $C_p$, the trottle is modeled by $R_T$ and the recycle valve is modeled by $R_r$.

The bonds model shows that the recycle line is decreasing the upstream (plenum) energy and increasing the downstream (inlet) energy. The dynamic equations of the model are obtained by evaluating the effort-flow relation at the storage elements ($I_f$, $C_b$, $I_c$, and $C_p$) and given as follows:

$$\dot{w}_f = -\frac{1}{I_f} p_b \tag{25}$$

$$\dot{p}_b = \frac{1}{C_b}\left[w_f + w_r - w_i\right] \tag{26}$$

$$\dot{w}_i = \frac{1}{I_c}\left[p_b + p_c - p_p\right] \tag{27}$$

$$\dot{p}_p = \frac{1}{C_p}\left[w_i - w_o - w_r\right] \tag{28}$$

Table 3. SIMULATION PARAMETERS

| Parameter | Value | Unit | Parameter | Value | Unit |
|---|---|---|---|---|---|
| $U$ | 68 | m/s | $a_0$ | 340 | m/s |
| $V_b$ | 0.009 | $m^3$ | $V_p$ | 0.9 | $m^3$ |
| $L_f$ | 0.41 | m | $L_c$ | 0.41 | m |
| $A_c = A_f$ | 0.0038 | $m^2$ | $\rho$ | 1.2041 | $kg/m^3$ |
| $p_{c0}$ | 16646 | Pa | $W$ | 0.0775 | kg/s |
| $H$ | 500 | Pa | $k_T$=$k_r$ | 0.75 | $Pa^{-0.5}kg/s$ |
| $k_{Ts}$ | 0.6 | $Pa^{-0.5}kg/s$ | $k_{T_p}$ | 1.5 | $Pa^{-0.5}kg/s$ |
| $w_{SL}$ | 0.155 | kg/s | $w_{SCL}$ | 0.2 | kg/s |

where $I_f = \frac{\rho L_f}{A_f}$, $C_b = \frac{V_b}{\rho a_0^2}$, $I_c = \frac{\rho L_c}{A_c}$, and $C_p = \frac{V_p}{\rho a_0^2}$. The notations $V_b$ is the volume of the buffer, $L_f$ is the length of the feeding duct, and $A_f$ is the cross-sectional area of the feeding duct. Equations (25) and (26) show the effects of recycling flow to the compressor inlet states which improves the model presented in [25] by showing the effect of the recycling flow to the feeding flow.

A recycle valve is similar to a throttle; however, it usually has a faster actuation than the throttle. The recycled mass flow is defined by:

$$w_r = k_r u_r \sqrt{(p_p - p_b)}, \tag{29}$$

where $k_r$ is the recycle-valve constant and $u_r$ is the valve opening control signal with the range value of 0 to 100 %. The recycle valve is controlled by a controller which compares the inlet mass flow to the reference mass flow ($w_{SCL}$) to compute $u_r$. We demonstrate a PI (proportional and integral) controller for the SAS as follows:

$$u_r = K_p w_e + K_i \int w_e dt \tag{30}$$

where $K_p$ is the proportional gain, $K_i$ is the integrator gain, and $w_e$ is the error mass flow. The error mass flow is defined by:

$$w_e = w_{SCL} - w_i. \tag{31}$$

An example simulation of a compression system equipped with SAS using parameters in Table 3 and the control gain $K_p = 5$ and $K_i = 10$ is given as follows. A compressor is initially operating at steady-state at point E where the throttle is fully open ($u_T = 100\%$) as shown in Figure 9. The throttle opening is then reduced to $u_T = 20\%$ at t = 40 s such that the compressor should operate at point F located to the left of the SCL, which is in the surge area. The operating point is moving from E to the left and crossing the SCL such that the SAS controller detects $w_i < w_{SCL}$ and gives a command the recycle valve to open. Figure 9 shows the mass flow and pressure trajectories of the simulation and Figure 10 shows the system time responses. The plenum fluid is recycled to the inlet ($w_r > 0$) such that compressor mass flow stay at the surge control line ($w_{SCL} = 0.2$ kg/s). The pressure fluctuation in the buffer ($p_b$) shows the effect of the recycling flow to the inlet states. The magnitude of the buffer pressure fluctuation is depend on the pipe size and the recycled mass flow. The steady state value of the feeding flow ($w_f$) to the mass flow at point F.

### 6.2 Active Surge Control System

An active surge control system (ASCS) works to stabilize compressor surge such that the compressor can operate in the stabilized surge area. This enlarges the compressor operating envelope to the lower mass flow. The ASCS method has been implemented experimentally by several different actuators and some of them are illustrated together in Figure 11. As described in Section 5 about two basic solutions for compressor surge, the different actuators for ASCS can be classified as:

i. Upstream energy injection

The examples of upstream energy injection are fluid injection [27], close-coupled valve [28], drive torque control [29],

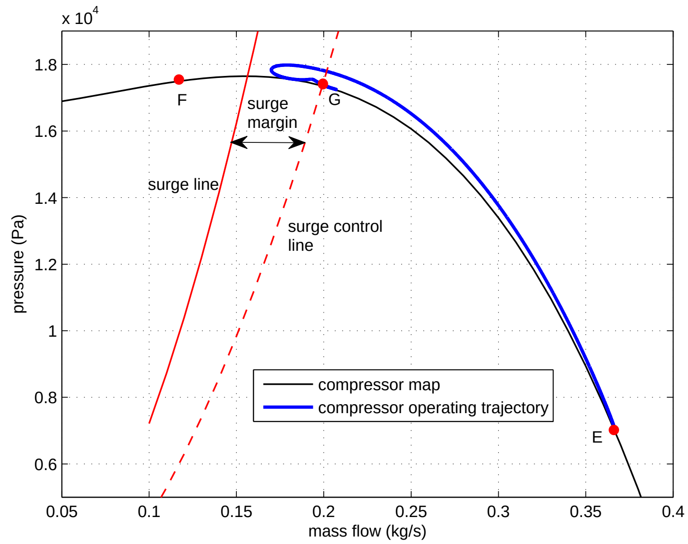


Fig. 9. Trajectory of compression system equipped with SAS when the compressor should be entering surge.

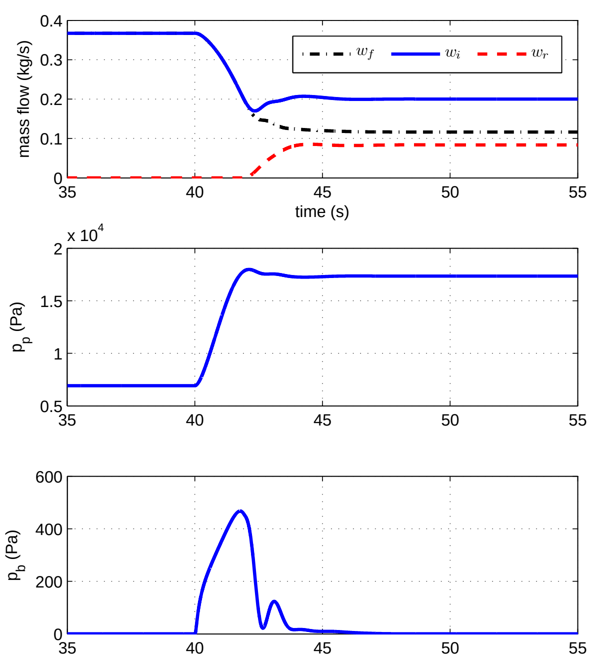


Fig. 10. The time responses of a compression system when the SAS is preventing the compressor of entering surge.

and active magnetic bearing [5]. Defining a new bond graph component $S_u$ as an active component for injecting energy, the bond graph model of a compressor equipped with upstream energy injection ASCS is shown in Figure 12. The model shows that $S_u$ gives additional effort to the 1-junction and the effort equation the junction is given by:

$$e_4 = e_3 + e_{19} - e_5, \tag{32}$$

where $e_{19}$ is an additional effort from the active element to maintain $e_4$ being non-negative. The examples of implementing $e_{19}$ can be found in additional upstream pressure by fluid injection in [27], additional compressor produced pressure

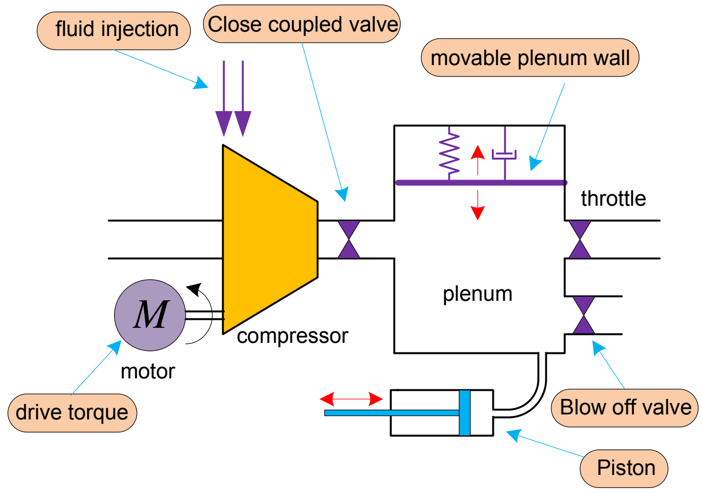


Fig. 11. Several types of active elements applied in active surge control system.

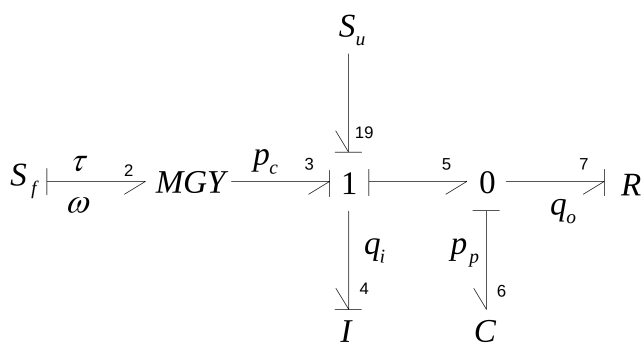


Fig. 12. Bond graph model of active surge control in a class of upstream energy injection.

by controlling the compressor speed through the drive torgue control [29], and additional compressor produced pressure by controlling the axial clearance between the impeller and the static shroud using active magnetic bearing [5].

ii. Downstream energy dissipation

The example of downstream energy dissipation are movable plenum wall [30, 31], piston-actuation [4] and blow off valve [32]. Bond graph model of a compressor equipped with downstream energy dissipation ASCS is shown Figure 13, where $R_u$ is an active component for dissipating energy. The flow equation at the 0-junction is given by:

$$f_5 = f_6 + f_7 + f_{20} \tag{33}$$
$$f_6 = f_5 - f_7 - f_{20} \tag{34}$$

The active element $R_u$ is mainly dissipating the downstream energy by creating additional flow out of the system (plenum control volume). The implementations of $R_u$ can be found in by movable plenum wall [30,31], piston-actuation [4], and blow off valve [32].

The active elements $S_u$ and $R_u$ are not always single components, but they can be a subsystem consisted of several components as given by the following example of ASCS using a piston actuation. Detail of the system is described in [4]. The bond graph model of a compressor equipped with ASCS using piston actuation and the detailed model inside the $R_u$ is shown in Figure 14. The active element $R_u$ consists of a source effort $S_e$ to generate force $F_s$, a transformer $TF$ which transform the force $F_s$ into pressure $p_s$, and an inertia $I_s$ which is the piston mass.

The ASCS using a close-couple valve is in the type of the upstream energy injection. The valve is placed between the compressor discharge and the plenum. Closing the valve will reduce flow from the compressor discharge to the plenum and increase the upstream pressure. The bond graph model is shown in Figure 15.

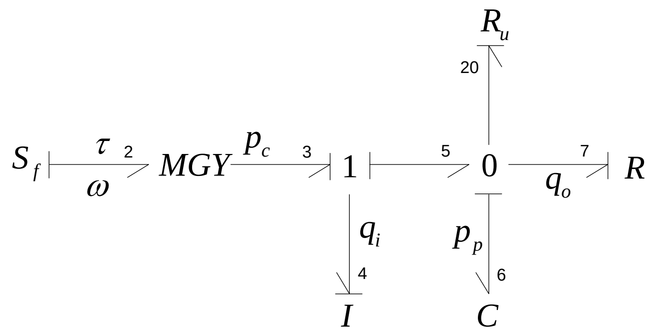


Fig. 13. Bond graph model of active surge control in a class of downstream energy dissipation.

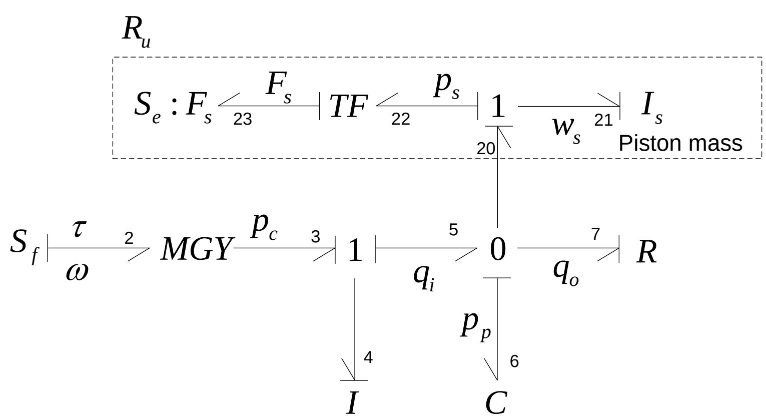


Fig. 14. Bond graph model of active surge control using piston actuation.

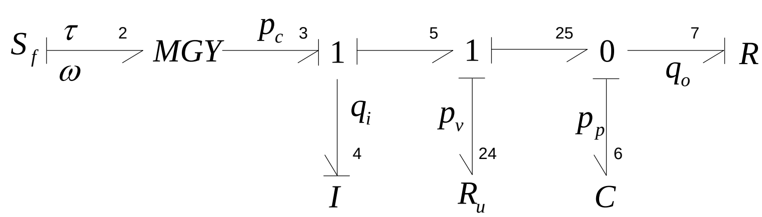


Fig. 15. Bond graph model of active surge control using closed couple valve.

## 7 Modeling of Compressors Networks

A compressors network is commonly applied in the industry to satisfy a process requirement, e.g.: flow and pressure. The compressor network can be in serial, parallel, or serial-parallel configurations. The serial configuration is used to attain a higher pressure rise, while the parallel configuration is to gain higher mass flow, and the serial-parallel configuration is to attain both higher mass flow and higher pressure rise. This section is presenting bond graph modeling of the compressors networks in serial and parallel configurations. The modeling is using the same assumptions as in modeling the single compression system in the previous section.

### 7.1 Bond Graph Modeling of Serial Compressors

Figure 16 shows two compressors in a serial configuration. The first compressor outlet is connected to the second compressor inlet such that the fluid properties are same. A throttle is installed at the outlet of the second compressor to adjust the flow in the both compressors. Pressure drop along the pipeline is denoted as $p_{R_L}$ and assumed to be accumulated at the outlet. Therefore, total pressure drop at the outlet denoted by $p_R$ is an accumulation of pressure drop along the pipeline $p_{R_L}$ and pressure drop due to a throttle $p_{R_T}$ if available. Total pressure drop at the first outlet is solely due to the pipeline pressure drop ($p_{R_1} = p_{R_{L_1}}$) and the total pressure drop at the second outlet is due to the pipeline pressure drop and the throttle pressure drop ($p_{R_2} = p_{R_{L_2}} + p_{R_T}$). Several approximations to calculate pressure drop due to fluid friction along the pipeline can be found in [33].

In this serial connection, we assume that the first outlet states are equal to the second inlet states such that $p_{o_1} = p_{i_2}$ and $w_{o_1} = w_{i_2}$. Bond graph model of the serial compressors is shown in Figure 17.

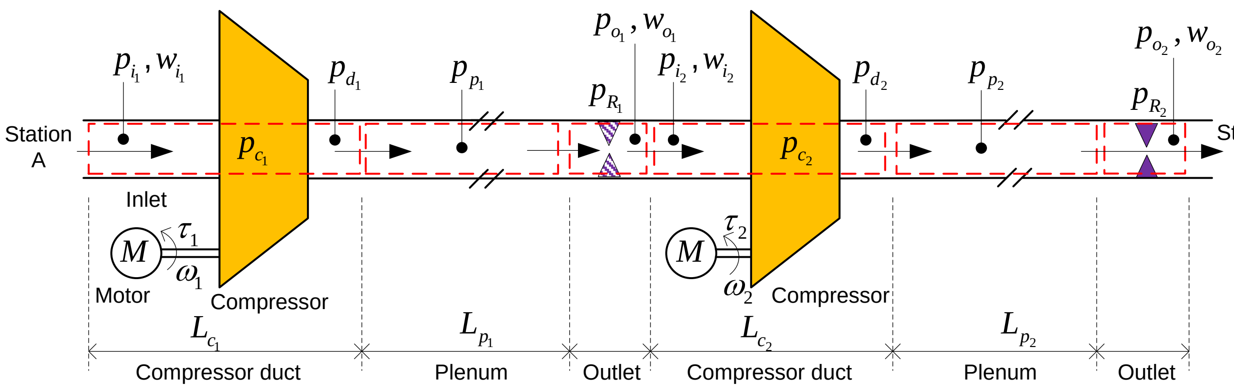


Fig. 16. Two serial compressors.

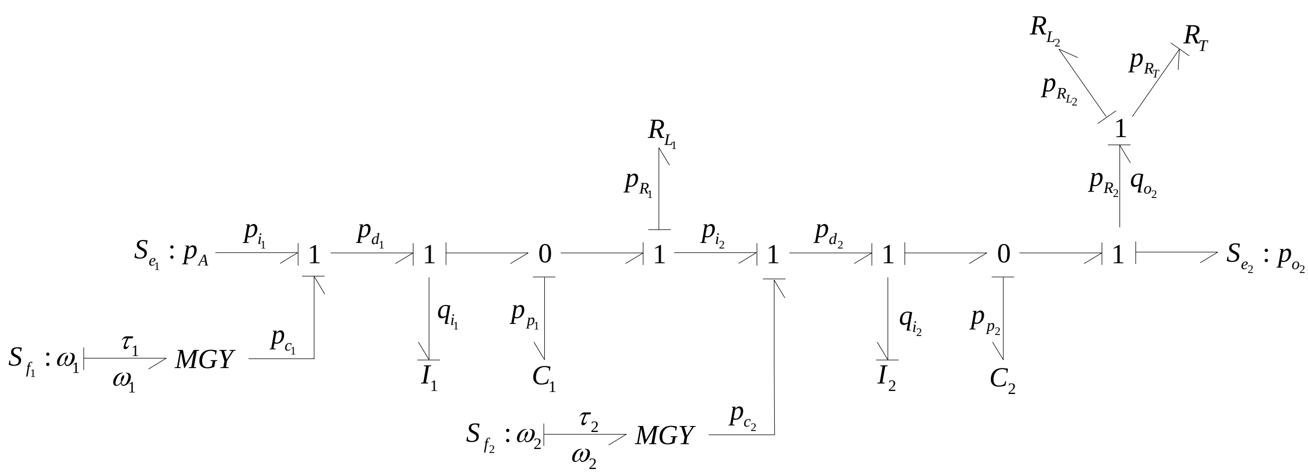


Fig. 17. Bond graph model of two serial compressors.

Dynamics equations of the serial compressor networks are obtained by effort-flow relation of the storage elements ($I_1, C_1, I_2$ and $C_2$) and given as follows:

$$\dot{w}_{i_1} = \frac{A_{c_1}}{L_{c_1}} (p_{i_1} + p_{c_1} - p_{p_1}) \tag{35}$$

$$\dot{p}_{p_1} = \frac{a_0^2}{V_{p_1}} (w_{i_1} - w_{i_2}) \tag{36}$$

$$\dot{w}_{i_2} = \frac{A_{c_2}}{L_{c_2}} (p_{p_1} - p_{R_1} + p_{c_2} - p_{p_2}) \tag{37}$$

$$\dot{p}_{p_2} = \frac{a_0^2}{V_{p_2}} (w_{i_2} - w_{o_2}). \tag{38}$$

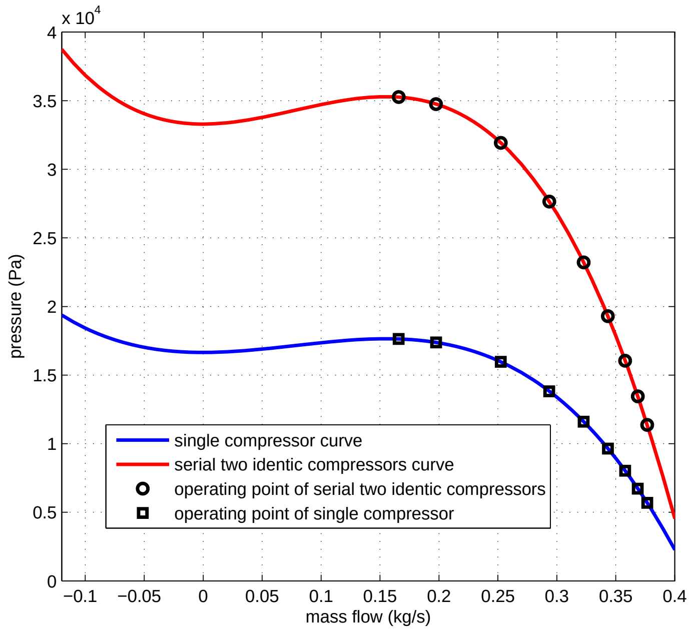


Fig. 18. Compressor map of a single compressor and two identical compressors in serial configuration.

The outlet mass flow of the second compression is defined as:

$$w_{o_2} = k_{T_s} u_T \sqrt{p_{p_2} - p_{R_2} - p_{o_2}}, \tag{39}$$

where $k_{T_s}$ is the throttle constant for the serial compressors configuration. Figure 18 shows the compressor maps of a single compressor and a serial interconnection of two such identical single compressors where the pipeline pressure drop is neglected. Serial compressors are characterized by the same mass flow through each compressor unit and the overall pressure rise is an accumulation of the pressure rise at each units, which is twice in this case. Therefore, serial compressors are applied to produce a higher pressure rise. Serial compressors might be found in a process with a higher pressure requirement or in a long pipeline gas transportation system to compensate the pressure drop. The surge point (the peak of compressor map) of the serial compressor is located at the same mass flow as the single compressor surge point.

Surge simulation of two identical compressor in serial configuration is now presented. The pipeline pressure drop is neglected. The first and second compressors are initially operating steadily at throttle opening $u_T = 100\%$ as the operating points are shown by $Q_1$ and $Q_2$, respectively. The operation points are then changed by reducing the throttle opening to $u = 20\%$ at $t = 40s$. It results the both compressors are entering surge. Figure 19 shows the system trajectories and Figure 20 shows the time responses.

### 7.2 Bond Graph Modeling of Parallel Compressors

Parallel compressors are used to attain a higher total mass flow by connecting several compressors discharge lines into one line. The discharged lines should have the same pressure to avoid back flow. A model of a pipeline system with two compressors in parallel is shown in Figure 21 and the bond graph model is shown in Figure 22. The dynamic equations of the parallel compressors are derived by evaluating the effort-flow relation of the storage elements ($I_1$, $I_2$, and $C$) in the bond graph model and given as follows:

$$\dot{w}_{i_1} = \frac{A_{c_1}}{L_{c_1}} \left( p_{i_1} + p_{c_1} - p_p \right) \tag{40}$$

$$\dot{w}_{i_2} = \frac{A_{c_2}}{L_{c_2}} \left( p_{i_2} + p_{c_2} - p_p \right) \tag{41}$$

$$\dot{p}_p = \frac{a_0^2}{V_p} \left( w_{i_1} + w_{i_2} - w_o \right). \tag{42}$$

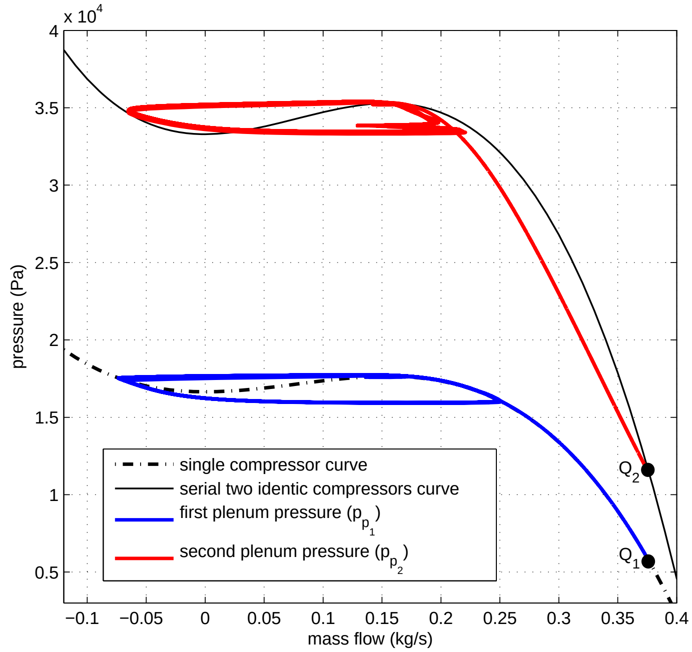


Fig. 19. Serial compressor operating trajectory.

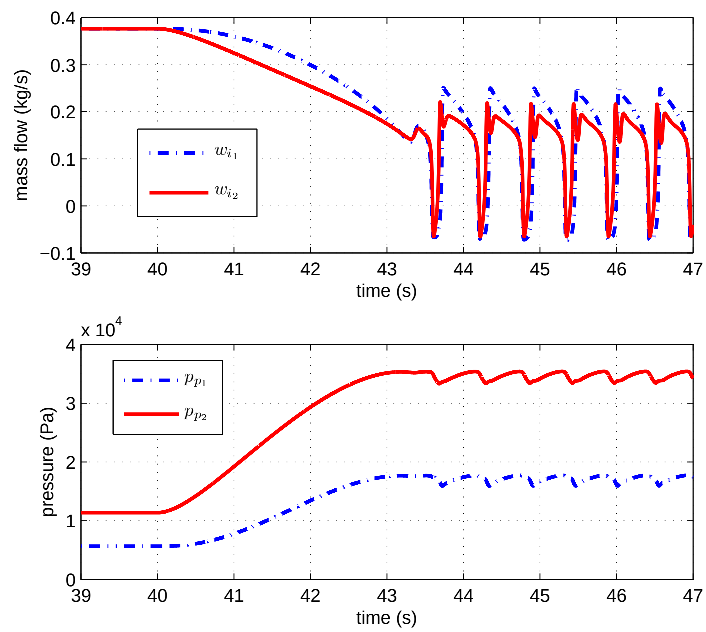


Fig. 20. Mass flow and pressure of serial two compressors in surge condition.

The outlet flow is defined as:

$$w_o = k_{T_p} u_T \sqrt{p_p - p_R - p_o}, \tag{43}$$

where $k_{T_p}$ is the throttle constant for the parallel compressor configuration. Figure 23 shows the compressor maps of a single compressor and the parallel configuration of the two identical single compressors where the pipeline pressure drop is

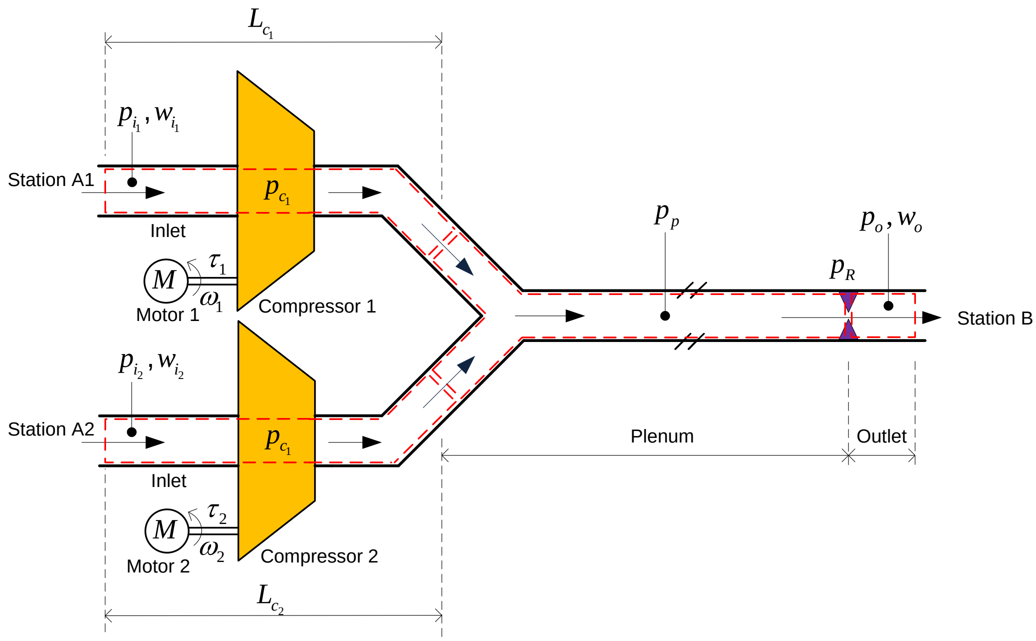


Fig. 21. Two parallel compressors.

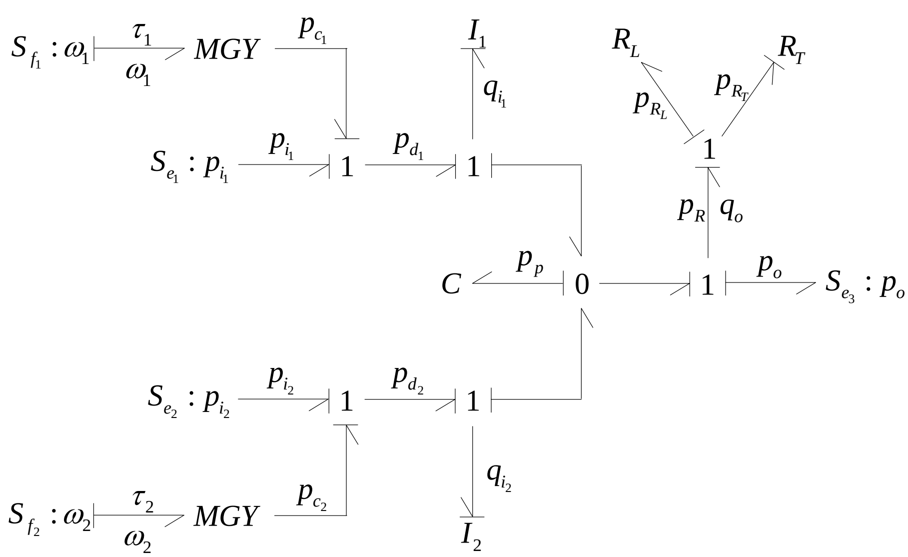


Fig. 22. Bond graph model of two parallel compressors.

neglected.

Surge simulation of two parallel identical compressors by ignoring the pipeline pressure drop is now presented. The both compressors are initially operating steadily at throttle opening $u_T = 100\%$ as the operating points are shown by $P_2$ for the two compressors in parallel and $P_1$ for the single compressor. The operation point is then changed at $t = 40s$ by reducing the throttle opening to $u_T = 20\%$. It results the compressors are entering surge as shown by Figure 24 for the system trajectories and Figure 25 for the time responses.

## 8 Conclusions

Bond graph modeling of compressors networks have been presented. The Greitzer model was used as the reference such that the developed model can represent the transient response of the fluid dynamics. The modeling was started by developing a bond graph model of a single compression system. The bond graph model was used as the basic model to build

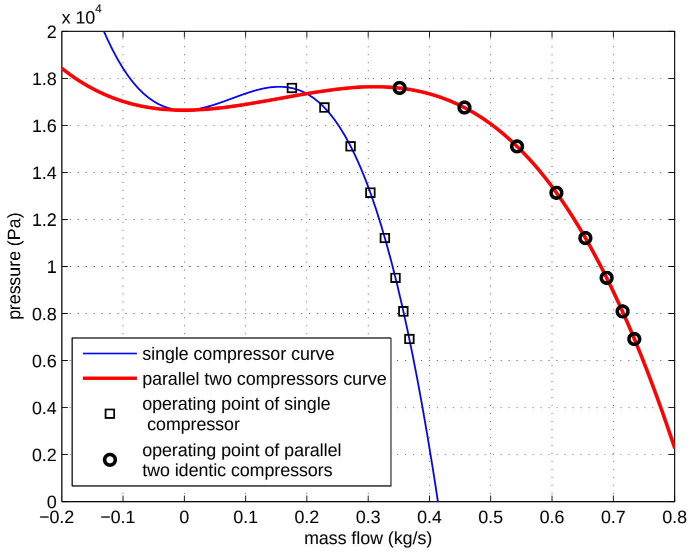


Fig. 23. Compressor map parallel.

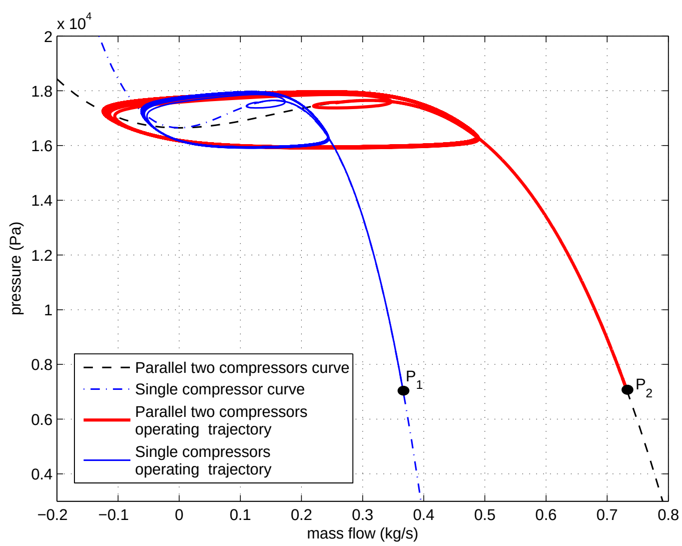


Fig. 24. Operating trajectory of two identical compressors in parallel configuration during surge condition.

more complex compression system models. The bond graph model of single compression system was analysed to explain compressor surge. Solutions to compressor surge control were presented as upstream energy injection and downstream energy dissipation. The bond graph model of compression system with SAS improves the current model by showing the recycling flow to the feeding flow. Several actuators applied in the ASCS were classified into the upstream energy injection and the downstream energy dissipation, and modeled by using bond graph. The compressor networks were modeled by combining the bond graph model of the single compressors. Simulations of the parallel and serial configurations during surge illustrated the complex behavior of these systems.

## 9 Further Works

The developed models are quite simple as the modeling was purposed for control system development. Experiments are necessary to validate the models. The bond graph model is open to be modified such that modeling more complex systems can be done directly based on the presented model, for example: including surge control in the compressor networks.

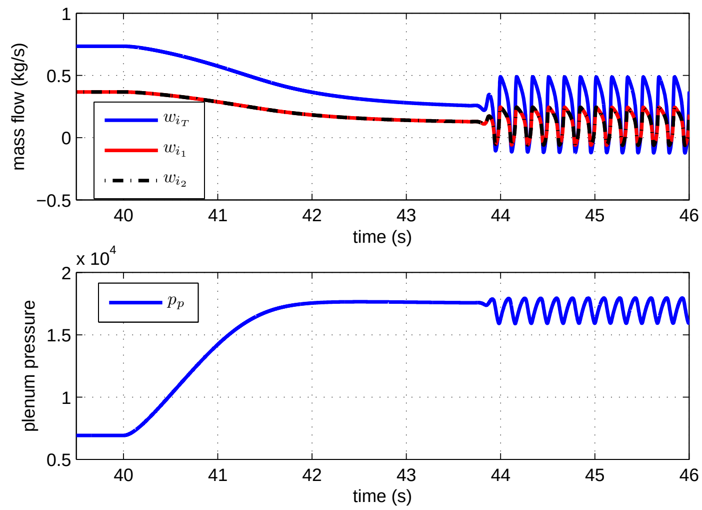


Fig. 25. Mass flow and pressure of two identical compressors in parallel configuration during surge condition.


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