CORE

# Lateral Interception II: Predicting Hand Movements 

Claire F. Michaels<br>University of Connecticut and Vrije Universiteit, Amsterdam

David M. Jacobs and Raoul M. Bongers<br>Vrije Universiteit, Amsterdam


#### Abstract

D. M. Jacobs and C. F. Michaels (2006) concluded that aspects of hand movements in lateral catching were predicted by the ratio of lateral optical velocity to expansion velocity. Their conclusions were based partly on a modified version of the required velocity model of catching (C. E. Peper, R. J. Bootsma, D. R. Mestre, \& F. C. Bakker, 1994). The present article considers this optical ratio in detail and asks whether it, together with a control law, predicts the (often curious) hand trajectories observed in lateral interception. The optical ratio was used to create a succession of target-position inputs for the vector integration to endpoint model of hand movements (D. Bullock \& S. Grossberg, 1988). The model used this succession, initial hand position, and model parameters (fit to 60 trials) to predict hand trajectories on each trial. Predicted trajectories were then compared with observed hand trajectories. Hand movements were predicted accurately, especially in the binocular condition, and were superior to predictions based on lateral ball position, the input variable of the required velocity model. The authors concluded, as did C. E. Peper et al. (1994), that perceivers continuously couple movements to optics.


Keywords: interception, perception-action, model, optics

In our companion article (Jacobs \& Michaels, 2006), we set out to study learning processes in the acquisition of a visually guided interceptive action. For a task, we chose the interception of a ball that passes at some distance to the side of the body but is still within arm's reach. This task seemed an obvious starting point for a study of learning to catch because there was a clearly articulated and empirically supported model explaining how one gets one's hand to the right place at the right time: the required velocity model of Peper, Bootsma, Mestre, and Bakker (1994). Although we were able to use a modified version of the model to discern learning and calibration effects, two important features did not

Claire F. Michaels, Center for the Ecological Study of Perception and Action, Department of Psychology, University of Connecticut; Institute for Fundamental and Clinical Movement Sciences, Vrije Universiteit, Amsterdam, Amsterdam, the Netherlands. David M. Jacobs and Raoul M. Bongers, Institute for Fundamental and Clinical Movement Sciences, Vrije Universiteit, Amsterdam.

Claire F. Michaels is now affiliated only with the University of Connecticut; David M. Jacobs is now at the Faculty of Sport Science, Université de la Méditerranée, Marseille, France; Raoul M. Bongers is now at the Center of Human Movement Sciences, University of Groningen, Groningen, the Netherlands.

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Correspondence concerning this article should be addressed to Claire F. Michaels, Department of Psychology, U-20, University of Connecticut, 406 Babbidge Road, Storrs, CT 06269. E-mail: claire.michaels@uconn.edu
hold up under our attempted replication. First, predictions of movement paths based on the model's input-momentary lateral ball position-turned out to be the poorest of the trajectory variables that were examined. Instead, hand movements appeared to be based on a different optical variable, the ratio of lateral angular velocity of the ball to its angular expansion velocity. Second, even with velocity-to-expansion ratio as input, the amended required velocity model did not always accurately capture the observed hand trajectories (see Figure 7 of Jacobs \& Michaels, 2006). In the present article, we use Jacobs and Michaels's (2006) conclusions about the operative optical variable to further develop a model of lateral interception.

Our article is organized as follows. We begin with a brief review of Peper et al.'s (1994) article and our discrepant findings regarding the information guiding lateral catching movements. We then examine the optical variable that we propose guides catching (the ratio of lateral velocity to expansion) and show how it depends on assumptions and experimental conditions. The core of the article is the modeling section, where we use velocity to expansion ratios as input to vector integration to endpoint (VITE) models (cf. Bullock \& Grossberg, 1988) of hand movement, and show that these ratios accurately predict the trajectories observed by Jacobs and Michaels (2006). Finally, we consider the implications of this study for theories and models concerning lateral interception and for more general issues in information-movement coupling research.

## Peper et al.'s (1994) Conclusions on Lateral Catching

Peper et al. (1994) originated the catching paradigm depicted in Figure 1 (see also Figure 1 of Jacobs \& Michaels, 2006) to test whether perceivers used a particular optical variable, which (under various simplifying assumptions) specified the point at which the ball would cross the catching rail. The variable, $\dot{x} / \dot{r}$, is defined and derived in Figure 2. In their experiments, Peper et al. sought to


Figure 1. Top and side views of the lateral interception task developed by Peper et al. (1994) and used by Jacobs and Michaels (2006). A ball on a string swings down and passes to the right of the catcher. The string is fixed to the attachment rail and at the beginning of the trial the ball is attached to one of the release solenoids. The hand, which is loosely tethered to a horizontal rail, begins at some initial position, shown in gray in the lower left inset. The goal is to intercept the ball as it arrives at some incidence angle to some position on the rail.
determine whether perceivers and actors exploited this ratio in actual catching and in making judgments of whether simulated and real approaching balls were reachable.

Ultimately, Peper et al. (1994) rejected the hypothesis that the ratio was used, because of a combination of participants' errors when making judgments and demonstrated accuracy in catching. It seemed that participants could not make successful judgments of where the ball would pass, but could nevertheless get their hand there at the right time. Examination of the hand trajectories led Peper et al. to conclude that catchers did not exploit the specification of future passing distance offered by $\dot{x} / \dot{r}$ and then move their hand to the predicted interception point, but instead that hand velocity was continuously coupled to the current lateral ball-hand distance and the remaining time to contact, which brought the hand to the right place at the right time.

This was formalized into the required velocity model (Peper et al., 1994). The hand-movement predictions of this model can be appreciated most intuitively for the situation in which the hand is positioned at the future interception point, and the ball approaches that point on a path that is not perpendicular to the eye plane. If the ball comes from the left, the model predicts that the hand will move to the left before returning to the interception point; if the ball comes from the right, the hand will move to the right before returning to the interception point (see also Montagne, Laurent, Durey, \& Bootsma, 1999).

Peper et al.'s (1994) conclusions offered two important and very general lessons for the study of visually guided action. First, the principle of continuous coupling offered a bold alternative to the thesis that interceptive action is predictive in nature. Second, the
optical variables underlying perceptual judgments (e.g., of whether a passing ball can be reached) need not be the same as the optical variables involved in catching itself. The first point is of key importance because continuous coupling does not demand accurate prediction; movement "accuracy is achieved during the unfolding of the act" (Peper et al., 1994, p. 610). Although such continuous coupling had been claimed in cases such as automobile braking (Lee, 1976) and fly ball catching (Chapman, 1968; Michaels \& Oudejans, 1992), in those cases the optical information depends in part on the movement and specifies whether the current movement is appropriate to achieve the goal. The derivative of $\tau$ in the braking case and of vertical optical acceleration in the fly ball case specify whether stopping will be in time or whether the fly ball will be intercepted if current conditions prevail. In the case of moving the hand to catch a ball, the continuous coupling is more general in that the optics are unaffected by the movement (assuming, of course, that the person does not see his or her hand). Finally, Peper et al. concluded that because perfect predictions are not needed, a perceptual system "is not hunting for perfect information, but for useful information to which the action can be geared" (p. 610; see also Michaels, Zeinstra, \& Oudejans, 2001).

In short, we see a great deal at stake surrounding the results reported by Peper et al. (1994) and the interpretations they have drawn from their findings. In our companion article (Jacobs \& Michaels, 2006), we raised doubts about the information perceivers exploit as proposed by the required velocity model and about how that information modulates the catching movement. Given these doubts, Peper et al.'s conclusions about the general nature of


Figure 2. Schematic for showing that $\dot{x} / \dot{r}$ specifies future passing distance. Variables in uppercase letters indicate real distances and sizes; lowercase letters denote optical variables on the projection plane, which is a unit distance from the eye. A disk of width $R$ approaches the eye plane at a constant velocity along a linear path. Two variables are of interest: the image distance of the center of the disk from the straight ahead, $x(t)$, and the image size of the disk, $r(t)$, where $t=0$ is the time at which the ball crosses the eye plane.

$$
\begin{gathered}
x(t)=\frac{X(t)}{Y(t)}, \text { so } \\
\dot{x}(t)=\frac{\dot{X} \times Y(t)-X(t) \times \dot{Y}}{Y(t)^{2}} \text { and } \\
r(t)=\frac{R}{Y(t)}, \text { so } \\
\dot{r}(t)=\frac{-R \times \dot{Y}}{Y(t)^{2}} \text { and } \\
\frac{\dot{x}(t)}{\dot{r}(t)}=\frac{\dot{X} \times Y(t)-X(t) \times \dot{Y}}{R \times \dot{Y}}
\end{gathered}
$$

Substituting $\dot{Y} \times t$ for $Y(t)$ and canceling the $\dot{Y}$ s yields

$$
\frac{\dot{x}(t)}{\dot{r}(t)}=\frac{X(t)-\dot{X} \times t}{R}
$$

The numerator is passing distance, and the denominator is ball size, so the ratio $\dot{x} / \dot{r}$ specifies passing distance in units of ball size (after Peper et al., 1994, Figure 1).
interceptive action might be seen as jeopardized. In the present article, we reevaluate the details of lateral catching to determine whether the more general conclusions still hold. We take as our departure point the conclusion of Jacobs and Michaels (2006) that the variables used by catchers are related to the ratio of angular velocity to angular expansion velocity. Our interest is in describing the relevant optics in more detail and, more so, in describing how the optics can be used to explain hand movements.

## The Ratio of Lateral Velocity to Expansion

In this section, we derive a succession of lateral velocity to expansion ratios. We begin with an analysis of the image properties of a linear, constant-velocity approach of a fixed-size disk and proceed to an optical-angle analysis of the pendular, accelerating approach of a ball whose size can differ on successive trials. It is only for the first of these derivations that the ratio is invariant and specific to future passing distance. In the later derivations, the ratio changes as the ball approaches.

## Linear Approaches

Bootsma and Peper (1992) derived the ratio of lateral image velocity, $\dot{x}$, to rate of optical expansion, $\dot{r}$, as presented in Figure 2 (see also Regan \& Beverly, 1980). The specificity of $\dot{x} / \dot{r}$ to future passing distance depends on several assumptions. Among the more defensible assumptions are an approach on a linear path and an unchanging object size. Such assumptions are realistic for the motion of a ball rolling across a surface toward a catcher. Less realistic assumptions are that the object is flat (e.g., a disk) and parallel to the frontoparallel plane, which means that its image will foreshorten as it approaches (see also Tresilian, 1991). Under these circumstances, the variable $\dot{x} / \dot{r}$ is constant over the time course of the approach, and specifies, in units of disk size, the lateral distance at which the disk will cross the eye plane. The variable is also invariant over different velocities and incidence angles. As briefly noted in Jacobs and Michaels's (2006) article, because passing distance is specified in units of object size, one must either have knowledge of object size (see, e.g., Peper et al., 1994) or be appropriately calibrated to a constant size for interceptions to be successful.

We now change two aspects relevant to the specificity of the ratio of lateral velocity and expansion; a ball, rather than a disk, approaches on a linear path, and optical angles, rather than image sizes, are used. The new formulation and its optical consequences are schematized in Figure 3. Figure 3B shows that $\dot{\theta} / \dot{\phi}$ is invariantly related to the passing distance for the majority of the trajectory. Note that the optical patterns are also invariant over incidence angles for the majority of the trajectory. Both the specificity to distance and the invariance over incidence angle break down late in the trajectory, and more so for larger passing distances. Again, the specificity of $\dot{\theta} / \dot{\phi}$ to passing distance is in units of ball size.

If one were to use $\dot{\theta} / \dot{\phi}$ to guide hand movements to intercept balls under these conditions (specifically when balls arrive on a horizontal linear path at eye level), a variety of predictions would follow. First, to the extent that perceivers are appropriately calibrated, they ought to be able to reliably predict the passing distance of balls and reliably intercept them as they reach the eye plane. Second, a catcher's ability to continuously guide the hand to intercept the ball in the eye plane should be largely unaffected by different interception points, speeds, and incidence angles. Third, a ball size other than that to which the catcher is calibrated should result in an interception error that is proportional to the ratio of calibrated to actual ball sizes. A catcher calibrated to a $5-\mathrm{cm}$ ball should underreach a ball with a diameter of 6 cm , for instance, by a factor of 1.2. Again, a catcher might not make such errors if he or she detected


Figure 3. (A): A revised departure point for computing the values of lateral velocity and optical expansion. A sphere replaces the disk of Figure 2. We define the optical angles $\theta$ as the horizontal direction of the ball from the eye, and $\phi$ as the (one-dimensional) optical angle subtended by the ball. (B): The values of the ratio $\theta / \phi$ for a $5-\mathrm{cm}$ diameter ball starting 8 m away, following a linear path at eye level, and passing at distances of 25,50 , and 75 cm to the right of the eye. Each vertical line is actually two overlapping lines, one representing an approach at an incidence angle of $-5^{\circ}$ and one representing an approach at an incidence angle of $5^{\circ}$. The plot is somewhat unconventional in putting time on the ordinate but is convenient in representing a catcher's-eye view. The traverse begins at time $=$ 0 and takes 1.5 s to reach the eye plane; those values were selected to approximate travel times in Peper et al.'s (1994) article and Jacobs and Michael's (2006) article. By multiplying $\theta / \dot{\phi}$ by ball size, values are scaled to a particular passing distance, as indicated by the second legend on the abscissa.
information about ball size. Without being specific about the optics, we label information that specifies ball size as $\delta$, so a variable that specifies passing distance with changing ball size is $\delta \times \dot{\theta} / \dot{\phi}$.

## Pendular Approach

Let us now examine the optical patterns created by balls following a pendular path, such as those used by Peper et al. (1994) and by Jacobs and Michaels (2006), that is, balls that are released at some point well above the head and that accelerate down on a light line to pass at some distance to the perceiver's side. We calculated the optical patterns created by this situation, approximating the experimental conditions reported by Jacobs and Michaels.

The approach is in three dimensions, so $\theta$ is defined as the azimuth of the spherical coordinates of the ball relative to the origin at the right eye. Figure 4 presents the ratio $\dot{\theta} / \dot{\phi}$ for the pendular approach. There are several noteworthy differences between Figure 4 and the linear approach of 3B. First and least important, note that the ratio is affected more by incidence angle for the pendular approach; a positive incidence angle (lateral distance decreasing) leads to a smaller ratio than a negative incidence angle. This difference is actually an artifact of the mimicked experimental conditions; the plane of interception was 23 cm anterior to the eye plane. Had the two planes been the same, the lines for the two incidence angles would have overlapped as in

Figure 3. Second and more important, the ratio $\dot{\theta} / \dot{\phi}$ for the pendular approach is not invariant; pendular $\theta / \phi$ increases as the ball approaches. This means that the specified passing distance begins well to the left of the actual passing point and goes to its right as the ball approaches. Note also that this change is greater for the larger passing distances.

In light of the changes in $\dot{\theta} / \dot{\phi}$ and, thus, in $\delta \times \dot{\theta} / \dot{\phi}$ as the ball approaches and their curvilinear relation with passing distance as shown in Figure 4, it may appear remarkable, in retrospect, that these variables were successful predictors of judgments and movement variables as reported by Jacobs and Michaels (2006). To anticipate, what we show in our modeling is that hand movements appear to be continuously coupled to these variables. That is, we show that continuous reaching toward the changing locus specified by $\delta \times \dot{\theta} / \dot{\phi}$, especially, leads to accurate predictions of observed lateral hand movements. ${ }^{1}$

## Preliminary Modeling Considerations

Our modeling effort took as its departure point the hypothesis that $\dot{\theta} / \dot{\phi}$ (or $\delta \times \dot{\theta} / \dot{\phi}$ ) provides the informational basis to guide hand movements in lateral catching. We suppose that $\dot{\theta} / \dot{\phi}$ (or $\delta \times \dot{\theta} / \dot{\phi}$ ) specifies (a succession of) target positions. What we now need is a control law that regulates the movement on the basis of those positions. What sort of model should be sought? There are several available models of how a hand gets from Point A to Point B. Among them are computation-intensive models that, for example, determine the needed target joint angles and requisite torques to produce them, while optimizing some variable (e.g., the minimum torque-change model of Uno, Kawato, \& Suzuki, 1989). Our preference is for a model that makes less of a demand on intelligent processes. Thus, we also would bypass models that put heavy demands on memories for possible trajectories (e.g., Rosenbaum, Loukopolous, Meulenbroek, Vaughan, \& Engelbrecht, 1995). Even within models in which trajectories are emergent, rather than planned, there are several possibilities. There are models rooted in the neuromotor mechanisms (e.g., the equilibrium-point model of Feldman, 1986), pure phenomenological models expressed as dynamical equations of motion (e.g., Schöner, 1990; Zaal, Bootsma, \& van Wieringen, 1999), and the vector integration to endpoint (VITE) model of Bullock and Grossberg (1988). We have opted for the VITE model (a) because it is consistent with current understandings of sensory and motor neuroanatomy, (b) because it generates the details of trajectories in an emergent rather than a

[^0]

Figure 4. The ratios of lateral angular velocity to expansion for a pendular approach of a ball to the interception point for two incidence angles. The approach is a pendular (circular) path; however, the anterior-posterior and horizontal aspects closely approximate those in Figure 3B (25-, 50-, and $75-\mathrm{cm}$ passing distances at incidence angles of $-5^{\circ}$ and $5^{\circ}$ ). The origin of the differences lies in the velocity profiles and the vertical component of the motion.
preplanned fashion, and (c) because it predicts a variety of phenomena in movement. The VITE model was originally presented as a model of point-to-point movements, but it can easily make predictions when the movement unfolds along multiple subsequent targets (see Bullock, Bongers, Lankhorst, \& Beek, 1999). In our implementations of the model-and there will be six variationsthe movement target continuously changes in accordance with the value of the input optical variable (see Dessing, Bullock, Peper, \& Beek, 2002, for a different implementation of the VITE model to explain lateral interceptive hand movements).

We opt for the VITE model but are quick to confess that our particular hypothesis-that $\theta / \phi$ continuously guides hand move-ments-if formalized into mathematical expressions on the basis of other models, might not be all that different from the VITE expressions. After all, one has an initial hand position and a time series of an optical variable from which to generate a time series of positions. Given that all these models have a number of free parameters and that there is freedom in precisely how the model is implemented, the accuracy of predications may not depend critically on the underlying model. Thus, we chose the VITE model because of its commitments and credentials, not because we thought it was the only model that would work.

Before looking at the details of our various implementations of the VITE model, it is useful to examine the representative handmovement trajectories presented in Figure 5 and to establish some reference values by which we can evaluate the quality of our predictions of hand trajectory. As for the hand trajectories them-
selves, we note first that movements began some 400 ms after participants' liquid-crystal goggles opened and the ball became visible, which occurred $600-800 \mathrm{~ms}$ after ball release and thus about 500 ms before interception. Movements were smooth and reversals were common, at least in the left-then-right direction. The curious trajectories make them an especially rich phenomenon for modeling.

We computed two benchmarks for evaluating the degree of error in predictions. First, we measured how much the actual hand trajectory deviated from an optimally fit trajectory in which the hand accelerated directly toward the interception point. For this benchmark, we started at the moment the movement began and found the acceleration onset and rate that best fit the observed trajectories for each 60-trial block. As should be obvious from Figure 5, the real trajectories departed substantially from this benchmark's monodirectional start-to-finish movements. We label this the direct baseline. We measured the error as the sum of the absolute momentary errors at each of the $100-\mathrm{Hz}$ movement samples. An error of 1.0, therefore, corresponds to the area of one of the grid squares in Figure 5.

The second baseline measure, the stationary baseline, is the integral of the distance separating the observed trajectory and the hand's initial position. This is a measure of how much movement was observed. Obviously, it is expected that predictions based on the appropriate optics would be less than the direct and stationary baselines.


Figure 5. Six typical hand trajectories from one block of trials from Jacobs and Michaels (2006). Time proceeds from the bottom to the top. The starting hand positions on these trials had horizontal distances of between 40 and 71 cm from the right eye. Movements began 1.00 s to 1.25 s after ball release ( 0.30 to 0.50 s after vision was disoccluded). Interception was at 1.54 s . On the depicted trials, the balls arrived over a range of incidence angles, shown above each hand trajectory. All balls were caught or touched.

## VITE Model With Successive Targets

Bullock and Grossberg's (1988) VITE model yields the trajectories of point-to-point movements. A difference vector, $V$, is established between the present-position command, $P$, of the endeffector and the target $(T)$ of a movement. A separate go signal (GO), which grows at some rate, actuates the movement vector. When the present-position command is updated, a new vector is established, and the movement continues to unfold. To implement the VITE model to predict movement trajectories in catching, we first used optical variables to determine a succession of target positions ( $T \mathrm{~s}$ ) along the catching rail; such successions are plotted in Figure 4. Second, we added a delay parameter to the model; the difference vector was the difference between the current present-position command and the target specified at some fixed interval in the past. We make no claims about the meaning of this interval; it could represent a lower limit of perceptual-motor processing rate, or it could be some temporal calibration. We also added a spatial offset param-eter-a constant distance between the hand marker and the target.

The VITE model has four parameters of its own ( $\alpha, \alpha_{G O}, \beta$, and $G O_{0}$ ), but we found that the fitting of three of the parameters had little effect on the quality of our predictions, so values derived in pilot work were used for all participants and blocks. To simplify our description, we mention those parameters only in passing. The equations used in the modeling can be expressed as follows:

$$
\begin{equation*}
\frac{d}{d t} V_{i}=\alpha\left[-V_{i}+\left(T_{i-d e l a y}-O\right)-P_{i}\right] \tag{1}
\end{equation*}
$$

where $V$ is the difference vector; $\alpha$ was preset at $30 ; T$ is the optically specified target position from some interval earlier (the fitted delay); $O$ is the fitted spatial offset; and $P$ is the presentposition command, which develops as the product of $V$ (if it is positive) and the go signal:

$$
\begin{equation*}
\frac{d}{d t} P_{i}=\left[V_{i}\right]^{+} G O_{i} \tag{2}
\end{equation*}
$$

The go signal, in turn, is a sigmoid function and is related to two components:

$$
\begin{equation*}
\frac{d}{d t} G O 1_{i}=-\alpha_{G O} G O 1_{i}+\left(\beta-G O 1_{i}\right) G O_{0} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d}{d t} G O_{i}=-\alpha_{G O} G O_{i}+\left(\beta-G O_{i}\right) G O 1_{i} \tag{4}
\end{equation*}
$$

(cf. Bullock \& Grossberg, 1988, p. 74), where $\alpha_{G O}$ is the slope of the $G O$ function, which was fitted, and $\beta$ and $G O_{0}$ were preset to 15 and 8.

There are many ways that the VITE model could be implemented to predict movement trajectories in lateral interception. We present six variations. One distinction between the models related to which one of the three optical variables was used to generate targets. The second distinction was whether the movement onset was assumed or predicted. In the former case, we tried to predict where the hand would go once it started to move; in the latter case, we tried to predict the hand position over the entire duration of a trial, including movement onset. Given that standard deviations of

50 ms or more are seen even with choice reaction times initiated by crisp and obvious onsets, we anticipated better success in predicting only the postinitiation movement.

## Assuming Movement Onset

Three sets of predictions were made, one for each of the three input variables. Each of the three variables was used to generate a succession of movement targets (loci on the response rail); the three variables were lateral ball position ( $\chi_{\text {ball }}$ ), which had been implicated by the results of Peper et al. (1994), and two variables based on $\dot{\theta} / \dot{\phi}$, which had been implicated in Jacobs and Michaels's (2006) article. One variable, $\delta \times \dot{\theta} / \dot{\phi}$, included information about ball size, which, it will be recalled, varied from trial to trial. The other variable was $\dot{\theta} / \dot{\phi}$, which was multiplied by average ball size so that it too was a distance on the response rail. Jacobs and Michaels found that the former variable appeared to be exploited in binocular catching, whereas the latter variable appeared to be exploited during monocular catching, though the latter effect was less clear.

For each optical variable, movement trajectories were predicted for each of the 1,920 catching trials of Experiment 2 of the companion article; we omitted Blocks 1 and 6 , on which participants only pretended to catch. The inputs to the model were (a) the initial position of the hand, (b) the moment of movement onset, (c) that trial's time series of the model's optical variable, and (d) the previously determined parameter values for the 60 -trial block in which that trial occurred.

As for the parameters, we determined the best-fitting values of one VITE parameter, $\alpha_{G O}$, and our own two parameters, delay and spatial offset. As noted above, $\alpha, \beta$, and $G O_{0}$, were preset to 30 , 15 , and 8 . To determine the best-fitting combination of the three fit parameters, we compared 308 combinations of parameter values: $\alpha_{G O}$ was varied from 8 to 20 in increments of 4; delay was varied from 300 ms to 540 ms in increments of 40 ms ; and spatial offset was varied from -10 cm to 10 cm in increments of 2 cm . The choice of these values followed careful analyses of the best ranges and densities of parameter sampling. ${ }^{2}$ We computed the prediction error on each trial-as described above for the benchmark mea-sures-for each combination of parameter levels for each block of 60 trials. ${ }^{3}$ We averaged the error over trials and selected the parameter combination that had yielded the smallest average. If any of the best-fitting parameter values on any block were at the extreme of the parameter ranges, the block was rerun with a larger range of values. For the parameter fitting (only), we ignored trials on which the ball was missed.

The trial-by-trial predictions using the VITE model and the above-enumerated inputs proceeded as follows. Modeling began with the moment of movement initiation: The first movement target ( $T$ in Equation 1) was the value of the optical variable being tested, plus the spatial offset, at a fixed interval (delay) before

[^1]initiation. The prediction proceeded forward in time using a fourthorder Runge-Kutta procedure. The target was updated on each $10-\mathrm{ms}$ iteration. The model yielded a hand trajectory analogous to those presented in Figure 5. The measure of error was the sum of the distances between predicted and actual positions over the time slices from movement onset to when the ball arrived at the rail. Later we rationalize our decision to use summed rather than average error, but average error (in cm ) approximates twice the value of our error measure, given that the hand movements lasted about 0.5 s (cf. Figure 5).

The errors averaged over participant, block, and trials are presented by condition and information variable in the upper half of Table 1. The errors averaged over trials were subjected to an analysis of variance (ANOVA) with one between-subjects variable and two within-subjects variables. The between-subjects variable was viewing condition (monocular vs. binocular); the withinsubjects variables were block and model (direct baseline, stationary baseline, VITE with $\chi_{\text {ball }}$ input, VITE with $\delta \times \dot{\theta} / \dot{\phi}$ input, and VITE with $\dot{\theta} / \dot{\phi} \times$ average ball size). In this and other analyses, the $p$ value levels were adjusted according to the Huynh-Feldt correction. Model was a significant source of variance, $F(4,24)=99.93$, $p<.001$; the VITE with $\delta \times \dot{\theta} / \dot{\phi}$ predictions were the best, followed by VITE with $\dot{\theta} / \dot{\phi}$, direct baseline, VITE with $\chi_{\text {ball }}$, and stationary baseline: $1.49,1.75,2.14,2.82$, and 3.93 , respectively. A block effect, $F(3,18)=3.72, p=.03$, revealed a decrease in error over blocks. The Model $\times$ Blocks interaction was the only significant interaction, $F(12,72)=8.46, p<.001$. The biggest decrease in error was in the stationary condition from the first to the second block. This means, simply, that there was less total movement on later blocks. Results from individuals are presented by blocks in Table 2.

Table 1
Mean Prediction Errors (in $\mathrm{cm} \times s$ ) for Post-Onset Analysis and Entire-Trajectory Analysis

| Model | Binocular |  | Monocular |  |
| :---: | :---: | :---: | :---: | :---: |
|  | M | $S D$ | M | $S D$ |
| Postinitiation trajectory |  |  |  |  |
| Direct acceleration | 1.85 | 0.67 | 2.43 | 0.65 |
| Stationary baseline | 3.51 | 0.90 | 4.34 | 0.97 |
| VITE with $\chi_{\text {ball }}$ | 2.54 | 0.61 | 3.11 | 0.65 |
| VITE with $\dot{\theta} / \dot{\phi}$ | 1.76 | 0.40 | 1.76 | 0.42 |
| VITE with $\delta \times \dot{\theta} / \dot{\phi}$ | 1.26 | 0.28 | 1.73 | 0.37 |
| Entire trajectory |  |  |  |  |
| VITE with $\chi_{\text {ball }}$ | 3.58 | 0.74 | 4.27 | 0.67 |
| VITE with $\theta / \phi$ | 2.00 | 0.40 | 2.03 | 0.40 |
| VITE with $\delta \times \theta / \phi$ | 1.52 | 0.31 | 2.03 | 0.40 |

Note. VITE $=$ vector integration to endpoint model; $\chi_{\text {ball }}=$ lateral ball position; $\theta=$ the first temporal derivative of the azimuthal angle between the sagittal plane and the center of the ball; $\dot{\phi}=$ the first temporal derivative of the optical angle of the ball; $\delta=$ information that specifies ball size.

Because $\delta \times \dot{\theta} / \dot{\phi}$ and $\dot{\theta} / \dot{\phi}$ appear to be the critical variables, we performed a new ANOVA with $\dot{\theta} / \dot{\phi}$ and $\delta \times \dot{\theta} / \dot{\phi}$ as the only two levels of the variable model. This analysis yielded a significant effect of model, $F(1,6)=19.86, p<.005$, and a Model $\times$ Viewing Condition interaction, $F(1,6)=15.65, p<.01 ; \delta \times \dot{\theta} / \dot{\phi}$ was better than $\dot{\theta} / \dot{\phi}$ in predicting trajectories in the binocular condition ( 1.26 vs. 1.76, respectively), whereas the two variables did not differ in predicting trajectories in the monocular condition ( 1.73 vs. 1.76 , respectively). There was also a Viewing Condition $\times$ Blocks interaction, $F(1,30)=4.99, p=.01$; more improvement in prediction was seen over blocks in the monocular condition than in the binocular condition.

The first row of Table 3 presents the values of the fitted parameters for the best-fitting optical variable, $\delta \times \dot{\theta} / \dot{\phi}$. The only interesting value is the surprisingly long delay parameter, compared with the typically reported perceptual-motor delay, which is often estimated to be about 110 ms (cf. Bootsma \& van Wieringen, 1991). Nevertheless, the delay parameter is in keeping with the observed delay between the first glimpse of the ball at the opening of the shutter glasses and the moment at which the hand began to move ( 380 ms ). ${ }^{4}$

In summary, the trajectory that a hand will follow after it starts to move on an individual trial can be predicted well on the basis of the hand's initial position, a string of values based on $\dot{\theta} / \dot{\phi}$, three off-the-shelf VITE parameters, and three more parameters fit in 60 -trial blocks. For both the monocular and binocular cases, the accuracy of the predictions was significantly better than (a) fits found with $\chi_{\text {ball }}$, the variable usually considered to be involved in the guidance of lateral catching (Dessing et al., 2002; Montagne, Fraisse, Ripoll, \& Laurent, 2000; Montagne et al., 1999; Peper et al., 1994), (b) fits that ignored $\dot{\theta} / \dot{\phi}$ and accelerated optimally toward the interception point, and (c) fits that supposed no hand movement. As expected from Jacobs and Michaels's (2006) article, the optical variable that led to the best predictions was a variation of that rejected by Peper et al. (1994), the ratio of translation velocity to expansion velocity. The $\delta \times \dot{\theta} / \dot{\phi}$ predictions were superior to the $\dot{\theta} / \dot{\phi}$ predictions, but only in the binocular case. When viewing was monocular, the two had equal predictive value.

## Predicting the Entire Trajectory

In this section, we present our more ambitious attempt to predict the entire movement trajectory, including its onset, on the basis of $\chi_{\text {ball }}, \delta \times \dot{\theta} / \dot{\phi}$, and $\dot{\theta} / \dot{\phi}$. The details of the computations were the same as those reported in the previous section, except that on any trial the determination of the predicted trajectory began with the opening of the goggles. The value of the tested optical variable at that instant formed the first of the succession of VITE targets at an asynchrony equal to the delay parameter. Thus, whereas the previous round of modeling began at some interval (the parameter delay) before the observed movement onset, the current version

[^2]Table 2
Mean Prediction Errors (in $\mathrm{cm} \times \mathrm{s}$ ) for Post-Onset Analysis for Individual Participants and Blocks

| Part | $\underline{\text { Direct acceleration blocks }}$ |  |  |  | Stationary baseline blocks |  |  |  | VITE with $\chi_{\text {ball }}$ blocks |  |  |  | VITE with $\dot{\theta} / \dot{\phi}$ blocks |  |  |  | VITE with $\delta \times \dot{\theta} / \dot{\phi}$ blocks |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Monocular viewing |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 2.08 | 1.99 | 2.12 | 2.07 | 3.52 | 3.22 | 3.60 | 4.14 | 2.03 | 2.78 | 2.76 | 2.92 | 1.76 | 2.02 | 1.80 | 1.77 | 1.61 | 1.77 | 1.55 | 1.55 |
| 2 | 3.47 | 3.46 | 2.56 | 2.27 | 5.82 | 4.81 | 4.10 | 4.37 | 4.15 | 4.16 | 3.75 | 3.27 | 2.38 | 2.48 | 2.44 | 1.98 | 2.22 | 2.45 | 2.25 | 2.17 |
| 3 | 3.66 | 2.71 | 2.32 | 2.19 | 6.99 | 4.24 | 3.78 | 4.58 | 3.38 | 3.26 | 2.55 | 2.63 | 1.77 | 1.55 | 1.38 | 1.22 | 1.91 | 1.65 | 1.51 | 1.45 |
| 4 | 2.69 | 2.49 | 1.54 | 1.38 | 4.91 | 4.31 | 3.33 | 3.71 | 3.54 | 3.76 | 2.55 | 2.32 | 1.66 | 1.58 | 1.26 | 1.09 | 1.59 | 1.51 | 1.29 | 1.17 |
| Binocular viewing |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 1.25 | 1.13 | 1.03 | 1.57 | 3.37 | 2.57 | 2.37 | 3.58 | 2.17 | 2.27 | 1.79 | 2.44 | 1.39 | 1.11 | 1.18 | 1.62 | 0.86 | 0.82 | 0.75 | 1.05 |
| 6 | 2.31 | 2.07 | 2.45 | 2.54 | 4.70 | 3.74 | 3.74 | 3.81 | 2.93 | 3.11 | 2.70 | 2.81 | 1.88 | 1.86 | 1.98 | 2.13 | 1.30 | 1.44 | 1.43 | 1.38 |
| 7 | 1.41 | 1.11 | 1.35 | 1.22 | 2.68 | 2.28 | 2.72 | 2.96 | 1.37 | 1.78 | 2.14 | 2.19 | 1.32 | 1.42 | 1.67 | 1.59 | 1.21 | 1.06 | 1.39 | 1.29 |
| 8 | 3.27 | 2.45 | 2.37 | 2.06 | 5.49 | 3.82 | 3.98 | 4.48 | 3.45 | 3.20 | 3.17 | 3.08 | 2.35 | 2.18 | 2.34 | 2.06 | 1.81 | 1.44 | 1.45 | 1.41 |

Note. VITE $=$ vector integration to endpoint model; $\chi_{\text {ball }}=$ lateral ball position; $\dot{\theta}=$ the first temporal derivative of the azimuthal angle between the sagittal plane and the center of the ball; $\dot{\phi}=$ the first temporal derivative of the optical angle of the ball; $\delta=$ information that specifies ball size.
began when the goggles opened. Entire individual trajectories were predicted on the basis of the initial hand positions, the optical time series, and model parameters fitted anew by blocks; prediction error was again measured as the sum over time of the momentary distances between predicted and observed trajectories on each trial. ${ }^{5}$

Average prediction errors are presented by condition in the lower half of Table 1. Note first that, as expected, predictions of the entire trajectories had larger error than the postinitiation predictions. The whole-trajectory errors were submitted to an ANOVA with one between-subjects variable (viewing condition) and two within-subjects variables (block and the optical variable used in the model: $\chi_{\text {ball }}$ vs. $\delta \times \dot{\theta} / \dot{\phi}$ vs. $\left.\dot{\theta} / \dot{\phi}\right)$. There was one significant effect, that of variable, $F(2,12)=136.64, p<.001$; $\chi_{\text {ball }}$ was a poor predictor of trajectory. Given the reduction of power associated with the high variability in the $\chi_{\text {ball }}$ condition, the analysis was repeated without $\chi_{\text {ball }}$. First, $\delta \times \dot{\theta} / \dot{\phi}$ maintained its superiority over $\dot{\theta} / \dot{\phi}(1.77$ vs. 2.01 , respectively), $F(1,6)=16.28$, $p<.01$. The significant Variable $\times$ Viewing Condition interaction, $F(1,6)=16.78, p<.01$, showed that this superiority held in the binocular condition ( 1.52 vs. 2.00 , respectively) but not the monocular condition ( 2.03 vs .2 .03 , respectively). These findings replicate those of the postinitiation analyses. There was also a marginally significant Block $\times$ Viewing Condition interaction, $F(3,18)=2.70, p=.076$; again there tended to be more improvement in predictions over blocks in the monocular condition. Individuals' results are presented by blocks in Table 4.

To illustrate the quality of the fits in the binocular condition with $\delta \times \dot{\theta} / \dot{\phi}$, we present six sample trials in Figure 6. We chose these particular trials to illustrate differences among participants and to show the types of predictive errors that we observed. For each trial, we present the results of all 4 participants on that trial; this should provide a good overall idea of the variability of trajectories among catchers and of the quality of the predictions.

First, a few reminders about the trials themselves: The hands were to start at the same location on corresponding trials for the 4
participants; the slightly different starting points indicate differences in hand (or marker) placement. The only other difference among the 4 participants on each trial was the variable time of the goggles' opening and closing. Thus, the obvious differences in movement-initiation time among observers may have been due to individual differences, to experimental manipulation, or both. The marker was on the back of the hand, so it need not have been coincident with the ball for a catch or touch to occur. Of the 24 trials presented in Figure 6, the ball was caught on 20 trials and touched on the remaining 4 .

Trials on which large movements were required to reach the interception point, as was the case for the two shown at the top of Figure 6, tended to have the smallest errors. Both movement onset and trajectory were predicted with good accuracy. The predictions on trials in which little movement was required were somewhat less accurate. On some such trials, the participants showed similar movements and predictions were fairly good, as in Trial 199 of Figure 6. Other trials that required a similar net lateral movement sometimes showed deviations in opposite directions, as was the

[^3]Table 3
Parameter Values for Post-Onset Analysis and Entire-Trajectory Analysis for the Model Using $\delta \times \dot{\theta} / \dot{\phi}$

| Parameter | Delay (ms) |  | $\alpha_{G O}$ |  | Offset (cm) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | M | Range | M | Range | M | Range |
| Postinitiation trajectory | 416 | 360 to 447 | 13.8 | 10.7 to 18.7 | -4.2 | -8.2 to -1.6 |
| Entire trajectory | 389 | 327 to 487 | 14.5 | 12.0 to 19.3 | -4.1 | -8.5 to 1.8 |

Note. $\dot{\theta}=$ the first temporal derivative of the azimuthal angle between the sagittal plane and the center of the ball; $\dot{\phi}=$ the first temporal derivative of the optical angle of the ball; $\delta=$ information that specifies ball size; $\alpha_{G O}=$ slope of the $G O$ function.
case in Trial 227. The largest error in the set of 24 trials shown in Figure 6 was due to inaccurate prediction of movement initiation; Participant 7 on Trial 244 began moving almost 200 ms later than predicted, though the predicted movement otherwise showed the correct pattern.

Trials 199 and 207 demonstrate that participants could have very different trajectories on the same trials and that those differences could be well predicted by the model; for 3 of the 4 participants, the predictions on these trials were excellent. Finally, it is worth mentioning that because VITE is actually a model of virtual trajectories, neither it nor our implementation of it takes into account inertial forces, which would affect actual trajectory as the target is redirected. Thus, it was often the case that a fastmoving hand traveled farther to the left than predicted by the model. Examples are seen in Trials 235 and 244.

Although there is visible space between some of the predicted and observed trajectories, we found the quality of the predictions remarkably good given that (a) some of the trajectories are quite complicated, (b) the parameter fitting was done on 60 -trial blocks, and (c) there were a nontrivial number of irregular trajectories and variations in repeated trajectories that would not be predicted by any model. The postinitiation fits would provide more impressive figures, but predicting movement onsets in addition to trajectory shape is a more challenging test of how well the optical variables can account for the observed movements.

Finally, we report the results of a small analysis aimed at assessing the consequence of the size of the blocks over which the parameter values were fit. We selected two binocular participants, the one whose trajectories we predicted with the smallest average error (Participant 5) and the one whose trajectories we predicted with the largest average error (Participant 6). We repeated the simulations exactly as reported above, using the optical variable $\delta \times \dot{\theta} / \dot{\phi}$, except that the 60 -trial blocks were broken into 20 -trial subblocks, and the parameters and average errors were determined for these subblocks. These errors were then averaged over subblocks and compared with the original average errors. If the parameter values remain fairly constant across the three subblocks, the two average errors should be close, but if the parameter values change over subblocks, the average errors over subblocks should be smaller than the original block averages. The results for the two participants are presented in Table 5. The differences in the average errors are small, implying that the quality of the fits would not have been substantially improved had we fit the parameter values over smaller blocks of trials.

From a theoretical perspective, the larger the block over which the parameters are fit, the better. The setting of parameters, which we term calibration in Jacobs and Michaels (2006), must itself be explained. Thus, we do not find it sufficient to show that some model can predict any of several trajectories; a serious test of a model must also show that with the same parameter values, it can

Table 4
Mean Prediction Errors (in $\mathrm{cm} \times s$ ) for Entire-Trajectory Analyses for Individual Participants and Blocks

| Part | VITE with $\chi_{\text {ball }}$ blocks |  |  |  | VITE with $\dot{\theta} / \dot{\phi}$ blocks |  |  |  | VITE with $\delta \times \dot{\theta} / \dot{\phi}$ blocks |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Monocular viewing |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 3.19 | 3.58 | 3.79 | 4.19 | 2.06 | 2.18 | 1.97 | 2.18 | 1.94 | 1.96 | 1.67 | 2.06 |
| 2 | 5.69 | 4.69 | 4.27 | 4.51 | 2.68 | 2.69 | 2.60 | 2.21 | 2.57 | 2.86 | 2.57 | 2.32 |
| 3 | 5.23 | 4.29 | 3.63 | 4.31 | 2.22 | 1.73 | 1.56 | 1.44 | 2.37 | 2.03 | 1.76 | 1.70 |
| 4 | 4.86 | 4.72 | 3.62 | 3.82 | 1.88 | 1.89 | 1.57 | 1.56 | 1.81 | 1.83 | 1.59 | 1.44 |
| Binocular viewing |  |  |  |  |  |  |  |  |  |  |  |  |
| $5$ | 3.37 | 2.97 | 2.60 | 3.76 | 1.67 | 1.31 | 1.38 | 1.83 | 1.18 | 1.01 | 0.87 | 1.32 |
| 6 | 4.26 | 4.12 | 3.96 | 4.19 | 2.20 | 2.20 | 2.41 | 2.48 | 1.61 | 1.71 | 1.74 | 1.92 |
| 7 | 2.48 | 2.53 | 2.75 | 3.12 | 1.74 | 1.56 | 2.00 | 1.90 | 1.64 | 1.30 | 1.64 | 1.52 |
| 8 | 4.68 | 4.05 | 4.16 | 4.29 | 2.37 | 2.42 | 2.54 | 2.12 | 1.94 | 1.76 | 1.75 | 1.41 |

Note. VITE $=$ vector integration to endpoint model; $\chi_{\text {ball }}=$ lateral ball position; $\dot{\theta}=$ the first temporal derivative of the azimuthal angle between the sagittal plane and the center of the ball; $\dot{\phi}=$ the first temporal derivative of the optical angle of the ball; $\delta=$ information that specifies ball size.


Figure 6. Predicted and observed hand trajectories for the 4 participants in the binocular condition on each of six catches. The model predicts the whole trajectory on the basis of the optical variable $\delta \times \dot{\theta} / \dot{\phi}$. The observed trajectories are given as solid lines and the predicted trajectories as dashed lines. The symbol on the pairs of predicted and observed trajectories identifies the participant as shown in the legend. The circle and its location show the ball size and its position at the moment of interception (both scaled to the abscissa). The accompanying arrow indicates the incidence angle. The number in the upper left corner of each panel is the trial number (of the original sequence of 360 ).

Table 5
Effect of Block Size Used in Parameter Fitting on Average Error for 2 Participants

|  | Participant 5 |  |  | Participant 6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Block | 20 blocks | 60 blocks |  | 20 blocks | 60 blocks |
| 1 | 1.14 | 1.18 |  | 1.61 | 1.61 |
| 2 | 0.97 | 1.01 |  | 1.64 | 1.71 |
| 3 | 0.83 | 0.87 |  | 1.67 | 1.74 |
| 4 | 1.24 | 1.32 |  | 1.92 | 1.92 |

predict a lengthy sequence of observed trials. If parameter values are permitted to fluctuate from trial to trial, they become another explanandum for the control law, which must indicate the perceptual information that sets the parameters. Successful prediction with unconstrained parameters is mere hand waving. Indeed, when we fitted parameters 1 trial at a time, one of the parameter combinations almost invariably fit perfectly, save on those rare trials in which a right-left reversal was seen (e.g., Trial 227 of Participant 7, shown in Figure 6).

## General Discussion

Our goal in this study was to determine whether the optical variables that Jacobs and Michaels (2006) identified as being implicated in the guidance of lateral interceptive hand move-ments- $\delta \times \dot{\theta} / \dot{\phi}$ in the binocular case and $\dot{\theta} / \dot{\phi}$ in the monocular case-could be used to accurately predict the temporal evolution of hand movements on individual trials. To start, we needed a model that would permit us to generate a time series of hand positions based on a time series of optical variables. We chose Bullock and Grossberg's (1988) VITE model because of its contact with the phenomena and neuroscience of human movement and because it portrays trajectories as emergent. On the basis of the general model, we presented results for six sets of predictions, all of which used the time series of a variable as a succession of movement targets. The sets of predictions differed both in the information variable used and in whether movement onset was predicted or assumed. Of the four original VITE-model parameters, only one, $\alpha_{G O}$, was fitted for each block; we found manipulation of the other three to have little impact on the quality of the fits. Two parameters were added; one was a spatial offset of the target and the hand marker, and the other was a temporal offset, which captured the delay between the time slice of an optical variable and when it became the target. These parameter values were fit conservatively-to data sets comprising up to 9,300 data points (blocks of 60 trials with 155 recorded positions per trial).

We predicted the observed movement trajectories of individual participants to an average summed error of 0.8 to $1.1 \mathrm{~cm} \times \mathrm{s}$ for the best case (Participant 5 in the binocular, postmovement model using $\delta \times \dot{\theta} / \dot{\phi})$ and to a cumulative error of 2.2 to $2.9 \mathrm{~cm} \times \mathrm{s}$ for the worst case (Participant 2 in the monocular, full-trajectory predictions for both optical variables). The fits seem excellent, especially when compared with the predictions of a simple, optimal movement (the direct baseline). This leads us to conclude that we may have captured the information-movement relation for
lateral interception in this task, at least in the binocular case. We now consider the limitations and implications of our findings, both for the limited scope of interceptive action and for the broader arena of research in information-action relations.

We begin with a few words of caution regarding the generality of our results. First, although the VITE model served us well, we cannot claim that our results support the VITE model, except in the broadest possible terms. The model was overkill insofar as it had many more free parameters than we needed. It is entirely possible that other, simpler models might even be mathematically equivalent to our implementation of the VITE model. However, unlike Dessing et al. (2002), we found no reason to question the success of the VITE model in predicting lateral interceptive movements.

A second caution concerns the optical variables that were used in the model. We identified $\delta \times \dot{\theta} / \dot{\phi}$ (and $\dot{\theta} / \dot{\phi}$ ) as the operative variables in guiding hand trajectories in this task, but obviously any other variable that is perfectly correlated with these variables would do the same. As an example, consider the optics related to the vertical dimension. Taking elevation angle into account in this experiment could not have improved the accuracy of our predictions because the successions of elevation angles were essentially the same on all trials. However, their lack of variation does not exclude the possibility that the operative variable included a vertical aspect; it only means that if a vertical aspect had been involved, our experiment could not have detected its influence. Its possible effect would show up only in an experiment in which there was variation in the vertical component of the trajectory.

In addition, there were no differences in the pattern of speeds of the ball over conditions. This means that the temporal aspects of the movement did not have to be differently tailored to the individual trials; the same general pattern of movement initiation (or urgency) could work on all trials. Had the timing requirements been different from trial to trial, say on the basis of speed of arrival, then an explicit timing aspect might have proven necessary, as has been assumed by Peper et al. (1994) and Dessing et al. (2002).

## Continuous Versus Predictive Control

Following Peper et al. (1994), we were especially interested in comparing the explanatory power of a continuous-control model with that of a predictive model. Peper et al. rejected the predictive model in favor of a continuous control model. Of paramount importance is whether the current results support or undercut their conclusions regarding continuous prospective control over discrete predictive control.

Two facts imply that the control of hand movements was indeed continuous. First, the optical variables were continuously changing, so their momentary values did not specify a unique passing distance. Second, the continuously changing optics was followed by correspondingly changing hand positions; a model in which the hand accelerated directly toward the passing locus yielded errors that were more than $50 \%$ higher than the $\delta \times \dot{\theta} / \dot{\phi}$ model error. However, these observations do not necessarily mean that hand movements were not predictive; it may mean simply that the predictions are continuously updated. Drawing a distinction between (updated) predictive control and continuous control may appear silly at first blush, but at a deeper level, there are issues
worth airing. Bluntly put, there is the possibility that the apparently continuous control shown in this task was an artifact of the pendular ball trajectories.

Constraints grant information (Runeson, 1988); we saw in Figure 3B that for a ball rolling across a tabletop, $\delta \times \dot{\theta} / \dot{\phi}$ is constant and specifies passing distance, at least until the ball gets very close. Swinging balls on strings repeals that specification. The catcher, however, could still use the variable predictively, by detecting a specified future passing location and moving toward it, but would have to continuously update the target, yielding the strange and sometimes reversing hand paths. Other trajectory types (e.g., the parabolic trajectory of a thrown ball) ought to yield other hand paths. We plotted $\delta \times \dot{\theta} / \dot{\phi}$ for parabolic trajectories and observed that under some conditions, the functions were nonmonotonic, which could lead to two movement reversals. Thus, momentary values of $\delta \times \dot{\theta} / \dot{\phi}$ do not specify future passing distance across a broad array of circumstances in the natural ecology of human interception. We therefore must accord to $\delta \times \dot{\theta} / \dot{\phi}$ the same status that Peper et al. (1994) accorded to lateral position or that others have accorded to vertical optical acceleration in the fly ball case: They do not specify future environmental states (e.g., where or when a ball will pass or land) in a way that permits catchers to act predictively without updating; they are variables that can lead to felicitous movement only if continuously coupled to movement production with an appropriate and properly calibrated control law. We therefore endorse what we see as the major conclusion of Peper et al., that lateral interceptive hand movements are not predictive and ballistic, but involve setting up an ongoing relationship that brings the hand to the right place at the right time.

## Resolving the Empirical Conflicts

Two empirical articles in the catching literature have concluded that hand movements in lateral interception can be predicted by $\chi_{\text {ball }}$, or (information about) the lateral position of the ball (Montagne et al., 1999; Peper et al., 1994). These authors reported empirical effects that showed (or were consistent with) the existence of symmetric movement reversals in response to balls with positive and negative incidence angles. These studies are, therefore, at odds with our finding of asymmetric reversals and our finding that movements were guided by $\delta \times \dot{\theta} / \dot{\phi}$. In this subsection, we try to account for the empirical discrepancy among these articles.

Although we used the same apparatus as Peper et al. (1994), there were nevertheless several methodological differences. Their catchers had unrestricted vision, whereas ours did not see the entire approach. We randomized passing distances; Peper et al. blocked passing distances. The most important difference is perhaps that we randomized initial hand position, whereas Peper et al. used a single initial hand position. By holding start and catch positions constant within a block of trials, they may have provided a more sensitive measure of the effects of lateral ball position, results consistent with a required velocity model (see Peper et al.'s, 1994, Figure 9). Our results from the leftmost starting positions approximated those of Peper et al., but the starting positions on the right, which should have shown symmetrical effects if $\chi_{\text {ball }}$ were the operative information variable, did not, as is clear from our Figure 5. We believe that had Peper et al. used starting positions on
the right, they too would have found trajectories inconsistent with the use of lateral ball position.

A second discrepancy is between our results and those of Montagne et al. (1999). Montagne et al. examined hand kinematics in catching when balls approached on linear trajectories at various angles. These authors wanted to test predictions of the lateralposition required velocity model regarding the existence and direction of movement reversals. Recall that this model predicts that when the hand is placed at the future passing point of the ball, a negative approach angle should lead to a left-right movement reversal, whereas a positive approach angle should lead to a right-left movement reversal. Montagne et al. reported such reversals, and they argued that their direction was as predicted by the lateral position of the ball. Balls showing a left-right reversal were more likely to have arrived at negative than positive incidence angles ( $40 \%$ vs. $8 \%$, respectively), whereas balls showing right-left reversals were more likely to have arrived at positive than negative incidence angles ( $42 \%$ vs. $22 \%$, respectively).

We did not find the predicted reversals. In our experiment, the hand was initially positioned at 4 cm or less from the interception point on $21 \%$ of the trials. On these trials, left-right reversals (with reversals defined as when both the maximum leftward and rightward speeds exceeded a speed of $50 \mathrm{~cm} / \mathrm{s}$ ) were more frequent for approaches with negative incidence angles than for those with positive angles ( $41 \%$ vs. $20 \%$ of the trials, respectively). A pairedsamples $t$ test showed that this difference was significant, $t(7)=$ $5.8, p<.01$. However, there were almost no right-left reversals: $0 \%$ versus $2 \%$, respectively, for negative and positive incidence angles, $t(7)=1.3, p>.1$.

The hypothesized exploitation of $\dot{\theta} / \dot{\phi}$ (or $\delta \times \dot{\theta} / \dot{\phi}$ ) predicts an asymmetry in movement reversals, but only for pendular ball approaches. With horizontal approaches, it predicts no reversals (compare Figures 4 and 3B). In short, we do not predict the reversals reported by Montagne et al. (1999). Again, there is a noteworthy methodological difference; Montagne et al. conducted their study in the dark with a luminous ball and glove. It is possible that this manipulation affected which variable catchers attend to. On the other hand, there are aspects of their results that $\dot{\theta} / \dot{\phi}$ (and $\delta \times \dot{\theta} / \dot{\phi}$ ) predict as well or better than does $\chi_{\text {ball }}: \dot{\theta} / \dot{\phi}$ (and $\delta \times$ $\dot{\theta} / \dot{\phi})$ predicts the occurrence of no movement reversals on the 135 trials in which the hand was initially positioned at the central, ball-arrival locus. That prediction was correct on $50 \%$ of the trials. The use of $\chi_{\text {ball }}$, on the other hand, predicts 45 right-left reversals for the $4^{\circ}$ condition, 45 left-right reversals for the $-4^{\circ}$ condition, and 45 nonreversals for the $0^{\circ}$ condition. Those predictions were correct on only $38 \%$ of the trials. Thus, one manner of presenting Montagne et al.'s results appears to favor $\chi_{\text {ball }}$ as the operative variable, and another manner does not. ${ }^{6}$

## An Alternative VITE Model

Ours is not the only study that has used the VITE model to try to predict performance in lateral interceptive catching. Dessing et

[^4]al. (2002) also presented such a model, although their departure points and goals were very different from ours. Dessing et al. started with the required velocity model, as formulated by Peper et al. (1994), and its extension by Bootsma, Fayt, Zaal, and Laurent (1997). Both of these models were derived from the VITE model and take as their inputs the lateral position of the ball and the first-order time to contact. Both adopt a control goal in which hand velocity is pushed to that velocity appropriate to cover the current lateral distance between ball and hand in the allotted time. The goal of Dessing et al.'s modeling was to account for qualitative effects observed in the lateral interception paradigm, in particular, the timing, movement reversals, and incidence-angle effects reported in Montagne et al. (1999, 2000).

Dessing et al. (2002) found that the required velocity model predicted overshoots that were not observed, so the model was modified, first to modulate the go signal of the VITE model according to the first-order time to contact (which they termed the RVITE model [required VITE model]) and later to include a parallel neural circuit embodying a relative velocity vector (the RRVITE model [relative and required VITE model]). In general, Dessing et al.'s models pursue the theory that the catcher controls velocity, whereas in our implementation, velocity is an emergent property, as it was in the original VITE model.

Given the different goals and starting points of the theory, the models are difficult to compare. Our off-the-shelf implementation of VITE attempted to predict not only qualitative effects but also hand trajectories on individual trials. Moreover, we found the VITE model more than up to the task of modeling our data, given that it had more free parameters than we needed. We believe that the VITE-model modifications that Dessing et al. (2002) judged to be necessary were not due to problems with the VITE model but were due to two faulty assumptions: that lateral ball position is the correct input variable and that movement reversals are symmetrical. Both assumptions were called into question by Jacobs and Michaels (2006) and by the present modeling efforts. If our analysis is correct, the elaboration and extension required to create the RRVITE model illustrates how one must (and can) crank up a mechanistic account to make up for misconceptions regarding information and control.

## Monocular Versus Binocular Information

As noted, the modeling presented in the present article supports many findings of our companion article (Jacobs \& Michaels, 2006), such as the surprisingly low predictive value of $\chi_{\text {ball }}$, the superiority of $\delta \times \dot{\theta} / \dot{\phi}$ with respect to $\dot{\theta} / \dot{\phi}$ in the binocular condition, and the finding that the results concerning variable use were less clear in the monocular condition. An apparent discrepancy between dependencies reported here and in our companion article, however, concerns the operative variable in the monocular condition. Jacobs and Michaels (2006) suggested, on the basis of two analyses, that $\dot{\theta} / \dot{\phi}$ was superior to $\delta \times \dot{\theta} / \dot{\phi}$ in predicting the hand movements in the monocular condition. First, $\dot{\theta} / \dot{\phi}$ at the moment the goggles opened had higher correlations with subsequent hand velocity than did $\delta \times \dot{\theta} / \dot{\phi}$. Second, the predictions of hand trajectories using $\dot{\theta} / \dot{\phi}$ as the optical information in a required velocity model seemed to be superior to the predictions using $\delta \times$ $\dot{\theta} / \dot{\phi}$. This apparent superiority of $\dot{\theta} / \dot{\phi}$ over $\delta \times \dot{\theta} / \dot{\phi}$ in the monoc-
ular case was not found in our VITE simulations; the two models, one using $\delta \times \dot{\theta} / \dot{\phi}$ and one using $\theta / \dot{\phi}$, led to the same amount of predictive error.

One key difference between our previous and current analyses concerns the very end of the movement. Whereas the analyses presented here were of entire trajectories, neither of the two analyses reported in our companion article (Jacobs \& Michaels, 2006) included the very end of the movement. The correlation analysis used hand velocity at 0.5 s after the goggles opened and the simulations of movement ended 0.1 s after the goggles closed, leaving approximately 0.1 s (approximately $20 \%$ of the trajectory) unpredicted. We believe that the apparent discrepancy was due to the last segment of the hand trajectory. In particular, monocular participants may have begun exploiting other information that became available as the ball approached, and this shift might have led to more accurate movements than the use of $\dot{\theta} / \dot{\phi}$ alone.

Additional informational constraints must have been operating because monocular catchers caught or touched most of the balls, and $\dot{\theta} / \dot{\phi}$ by itself was not sufficient to guide a hand to the location of interception. Its usage as the sole variable guiding hand movements entails that catchers underreach larger balls and overreach smaller balls. Although there was a tendency in that direction (the hand catch positions of all monocular trials for large to small balls, respectively, averaged $-15.9 \%,-10.3 \%,-7.4 \%,-1.0 \%$, and $6.1 \%$ of the actual passing distance, compared with the binocular range of $-2.6 \%$ to $-1.0 \%$ ), the error in the monocular case was about half that expected on the basis of the ratios of actual to average ball size in combination with the $4.4-\mathrm{cm}$ to $7.4-\mathrm{cm}$ range in ball sizes. It seems that $\dot{\theta} / \dot{\phi}$ is an excellent predictor of the initial phase of the monocular hand trajectories (where better variables might be difficult to detect) but that the variable exploited in the final part of the path more closely approximates $\delta \times \dot{\theta} / \dot{\phi}$. One might have concluded that $\delta \times \dot{\theta} / \dot{\phi}$, by virtue of it superiority in the binocular condition, must be a binocular variable, but it seems that in both the monocular and binocular cases, $\delta$ must be given some interpretation.

Let us consider the binocular case first. By what means might the variable $\delta \times \theta / \phi$ be detected? One possibility would be that mechanisms detecting $\dot{\theta} / \dot{\phi}$ for each eye (cf. Regan \& Kaushal, 1994) feed into a central mechanism. It is easy to show mathematically that two momentary samples of $\dot{\theta} / \dot{\phi}$ separated by a fixed distance, in this case the interocular distance, $I$, are uniquely related to a passing distance, $X$ :

$$
\begin{equation*}
X=\delta \times \dot{\theta} / \dot{\phi} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
X+I=\delta \times \dot{\theta}_{l} / \dot{\phi}_{l} \tag{6}
\end{equation*}
$$

where the subscript $l$ refers to left-eye variables. One simply solves for $\delta$ in Equation 5, substitutes into Equation 6, and $\delta$ drops out. Thus, object size as such need not be registered.

Nevertheless, object size seemed to be perceived; participants not only got their hands to the right place at the right time, but in the binocular case, their hand opening reflected ball size. A Block $\times$ Ball Size $\times$ Vision Condition ANOVA on the maximal hand opening (maximal distance between Optotrak markers on the thumb and forefinger) during a trial yielded two significant effects:
a main effect of ball size, $F(4,24)=6.36, p<.005$, and a Ball Size $\times$ Vision Condition interaction, $F(4,24)=9.85, p<.001$. The interaction showed that the maximal hand openings during binocular viewing were significantly affected by ball size, but those during monocular viewing were not. This implies that information about size was indeed detected in the binocular condition, rather than simply dropping out of the picture.

As for the monocular case, it appears that as the ball came closer, some contribution of $\delta$ information became more available. One possibility is that participants exploited the specification relation between vertical angle and distance. That is to say, because of the constraints on ball trajectories, vertical optical angle, which normally is of no use in specifying the momentary distance of an approaching projectile, in fact, specified distance. Determining whether vertical angle or some other variable contributed the additional constraints that brought monocular performance to a level beyond that expected on the basis of $\dot{\theta} / \dot{\phi}$ alone must await further research. The present modeling results, together with those of our companion article (Jacobs \& Michaels, 2006), indicate that a version of $\delta \times \dot{\theta} / \dot{\phi}$ is detected in both the monocular and binocular cases but that their detection mechanisms differ.

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[^0]:    ${ }^{1}$ Note that $\theta$ is an angle in spherical coordinates, so it is equal to $\tan ^{-1}(x / y)$; it is unaffected by momentary height, $z$. J. C. Dessing (personal communication, November 2004) argued that it should be computed as

    $$
    \theta^{\prime}=\tan ^{-1}\left(\frac{x}{\sqrt{y^{2}+z^{2}}}\right)
    $$

    That version, which we term $\theta^{\prime}$, is in a plane defined by the eyes and the ball, rather than the plane of $\theta$, which is defined by the eyes and the horizon. $\dot{\theta}^{\prime} / \dot{\phi}$ is a very different function from that depicted in Figure 4, and hand movements guided by it would be very different from those we observed. Thus, while $\theta^{\prime}$ might seem more logical given the continuous tracking of the ball by the eyes, its lack of success in explaining hand trajectories may be an important clue about how to measure optical angles.

[^1]:    ${ }^{2}$ Determination of best fits were repeated using Matlab optimization procedures; they usually, but not always, converged on values close to the less sophisticated, linear search described in the text. Because the minimizations appeared to be occasionally caught in local minima, we report only the linear search results.
    ${ }^{3}$ It seems fairly conservative to use such a large number of trials. Later we briefly explore the cost of this procedure for accuracy of prediction.

[^2]:    ${ }^{4}$ Experts on the VITE model might wonder whether the delay parameter might be better used to delay the go signal. Separate analyses showed that this led to very poor prediction. If the go signal is delayed, the current target, and thus the current difference vector, does not draw the hand sufficiently to the left to match the observed paths (e.g., to create the observed movement reversals).

[^3]:    ${ }^{5}$ We can now rationalize our choice of the error integral, rather than average error, as our measure of model fit. In predicting the entire trajectories, predictions started at the moment the goggles opened. The predicted period of no movement overlaps the actual trajectory for as much as half a second. To average over an interval that includes that overlap would underestimate the average error. In fact, averaging made the predictions of the entire trajectory appear better than those of the postinitiation predictions, which should not be possible. If, on the other hand, we averaged over only that range in which the predicted and actual lines diverged (or, say, when either the predicted or observed hand movement began), error would be overestimated because this approach fails to honor that part of the overlap (sometimes considerable) that is a bona fide prediction (e.g., look ahead to Trial 199 in Figure 6). Thus, to make the postinitiation and whole trajectory predictions comparable, and to avoid making an arbitrary assignment of when predictions begin, we opted for the less intuitive, but more principled, error integral.

[^4]:    ${ }^{6}$ In a recent attempt to replicate Montagne et al.'s (1999) experiment, Arzamarski, Harrison, and Michaels (2005) found no reversals.

