

Which Mechanical Invariants Are Associated With the Perception of Length and Heaviness of a Nonvisible Handheld Rod? Testing the Inertia Tensor Hypothesis

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It has been suggested that the inertia tensor governs many instances of haptic perception. However, the evidence is inconclusive because other candidate mechanical parameters (i.e., invariants) were not or were insufficiently controlled for in pertinent experiments. By independently varying all candidate mechanical parameters, the authors were able to test the role of the inertia tensor relative to that of other mechanical parameters. The results showed that length perception during rod wielding is not governed by the inertia tensor alone but also by the static moment. In contrast to previous reports, length perception during rod holding and heaviness perception during rod wielding were found to be unrelated to the inertia tensor and strongly related to the static moment.

Humans routinely manipulate objects without looking at them, as when they drive a golf ball from the tee, play the drums, or hoist a pint. In order to perform such activities successfully, one needs to haptically perceive action-relevant object properties, such as an object's spatial dimensions and weight. In object manipulation, this haptic perception is typically achieved through holding, tossing, or wielding the object. In ecological psychology, this form of perception, which implicates the muscular effort necessary to hold, toss, or wield the object, is termed *dynamic touch* (Gibson, 1966). It has been suggested that, in dynamic touch, the nervous system exploits the physics of rotation to perceive spatial and other properties of objects (Turvey, 1996). In particular, it has been hypothesized that, in its interaction with the physical world, the haptic system extracts mechanical invariants that are specific to, and hence informative about, relevant object properties (Fitzpatrick, Carello, & Turvey, 1994; Pagano, Fitzpatrick, & Turvey, 1993; see Michaels & Carello, 1981, for a discussion on the importance of invariants in perception).

A rigid object's moments of mass distribution constitute potentially relevant mechanical invariants because, together, they specify the dynamics of the object. The zeroth moment of mass distribution is the object's mass (m). The first moment is the mass (m) times the distance (d) between the point of rotation and the object's center of mass. Following Carello, Fitzpatrick, Domaniewicz, Chan, and Turvey (1992), we use the term *static moment* to indicate this invariant object property.¹ The second

moment can be conceived as an object's resistance against angular acceleration. In three dimensions, the second moment is a 3×3 matrix called the inertia tensor (\mathbf{I}). In diagonalized form, \mathbf{I} looks as follows:

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}.$$

The elements I_1 , I_2 , and I_3 are the *eigenvalues* of the inertia tensor and represent the object's resistance against angular acceleration with respect to a coordinate system with three so-called *principal* axes. Around a principal axis, the object can rotate with constant velocity without the need for any torque to be applied in the point of rotation. The first eigenvalue (I_1) is the largest, corresponding to the principal axis around which the resistance against angular acceleration is maximal. Likewise, the third eigenvalue (I_3) is the smallest. The second eigenvalue (I_2) is equal to or larger than I_3 and equal to or smaller than I_1 . With respect to a coordinate system originating at the end point of a homogeneous, cylindrical object like a rod, I_3 represents the rod's resistance against angular acceleration relative to its longitudinal axis. I_1 and I_2 are identical and represent the rod's resistance against angular acceleration relative to the axes perpendicular to its longitudinal axis.

To date, over 50 publications have appeared collectively advancing the hypothesis that the inertia tensor provides the informational basis for the haptic perception of a wide variety of object

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¹ We define static moment as an invariant object property, related to the distribution of mass in the object. Under this definition, static moment is not the gravitational torque, which varies with the object orientation relative to the gravity vector. In a previous study (Kingma et al., 2002), we included the gravitational constant in the static moment. However, it is more consistent with the literature not to include this constant. Consequently, the unit of static moment is $kg \cdot m$ in the present study. Evidently, inclusion of the gravitational constant would not affect the results of the statistical analyses. Note, furthermore, that Stroop, Turvey, Fitzpatrick, and Carello (2000) used static moment to indicate the variable gravitational torque rather than the invariant first moment.

properties. For instance, high correlations have been reported between I_1 (either alone or in combination with I_3) and the perceived length of an object (Carello, Fitzpatrick, Flascher, & Turvey, 1998; Fitzpatrick et al., 1994; Pagano et al., 1993; Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989; Turvey, Burton, Amazeen, Butwill, & Carello, 1998). In addition, perceived object width has been found to correlate with the eigenvalues of the inertia tensor (Turvey et al., 1998). Likewise, perceived heaviness has been found to correlate with the eigenvalues of the inertia tensor (i.e., I_1 and I_3 ; Amazeen & Turvey, 1996; Kloos & Amazeen, 2002), I_1 and I_3 in combination with mass (Amazeen, 1997), and higher order properties of the inertia tensor based on the eigenvalues taken together, again in combination with mass (Shockley, Grocki, Carello, & Turvey, 2001; Turvey, Shockley, & Carello, 1999). Thus, in the evolution of this research, the inertia tensor hypothesis broadened from considerations of I_1 alone, in the pioneering work of Solomon and Turvey (1988), to considerations of the inertia tensor as a whole and compound variables defined over its constituent properties.

In the experiments cited above, object parameters like material density, mass, and mass distribution were varied, and it was concluded that \mathbf{I} was the main mechanical parameter governing perceptual judgments of the object properties of interest. However, as noted by Chan (1994), the inertia tensor is not the only parameter that defines the muscular torque needed to rotate an object. Gravity plays a role as well. For a simplified system of a weightless rod with a point mass m attached at a distance d , Kingma, Beek, and van Dieën (2002) expressed the muscular torque needed to rotate the rod about one of its tips in 2-D as follows:

$$\mathbf{N}_m = I_1 \boldsymbol{\alpha} - \mathbf{M} \mathbf{g} \cos(\theta) = md^2 \boldsymbol{\alpha} - md \mathbf{g} \cos(\theta), \quad (1)$$

where \mathbf{N}_m is the muscular torque, $\boldsymbol{\alpha}$ is the angular acceleration of the object welded, \mathbf{g} is the gravitational acceleration, and θ is the angle of the rod with the horizontal plane. The first term on the right-hand side of Equation 1 can be called the *inertial torque* (and includes the invariant moment of inertia, $I_1 = md^2$), whereas the second term on the right-hand side of the equation is usually called the *static torque* (and includes the invariant static moment, $\mathbf{M} = md$). Defined as md , \mathbf{M} is independent of the object's orientation. Therefore, \mathbf{M} is invariant and should, like I_1 , qualify as a possible source of information for the human haptic system within theoretical perspectives based on information as invariance, such as the theory of direct perception (in the context of which the inertia tensor hypothesis has been developed). When the length of a uniform rod is varied, \mathbf{I} changes more rapidly (i.e., with the length cubed) than \mathbf{M} (i.e., with the length squared). However, there are at least three reasons why \mathbf{M} and/or m may also (besides \mathbf{I}) play a role in haptic perception. The first reason is that the perceptual system is relatively insensitive to changes in \mathbf{I} . The discrimination threshold for I_1 has been found to be on the order of 20%–33%, which is roughly 10 times larger than that for weight discrimination (Kreifeldt & Chuang, 1979). The second reason is that, under certain conditions, extracting information about \mathbf{I} may be harder than extracting information about \mathbf{M} and/or m . For instance, in an experimental condition in which there is no angular acceleration, such as in static holding, the term including \mathbf{I} approaches zero, but participants can still estimate rod length (Chan, 1994; Lederman, Ganeshan, & Ellis, 1996). In a similar vein, it is obvious that heaviness perception is still possible when a rod is held in a fixed

position. Even when a welded rod is held at one of its tips, the muscular torque needed to overcome the static torque may be greater than the muscular torque needed to counteract the inertial torque. Assuming that muscular tension plays an important role in dynamic touch (Fitzpatrick et al., 1994), this means that the signal-to-noise ratio would be larger for the term of Equation 1 containing \mathbf{M} than for the term containing \mathbf{I} . The third reason is that, with respect to length perception, the length of (even) a uniform rod is physically underdefined by \mathbf{I} alone. Physics dictates that at least two parameters from the set $(\mathbf{I}, \mathbf{M}, m)$ are needed to define the length of a uniform rod (Kingma et al., 2002). Although it may be possible to estimate the length of a nonvisible rod on the basis of only a single parameter, such an estimation would necessarily be ambiguous and inaccurate when no information was available about at least one other parameter (i.e., m or \mathbf{M}).

When a change in a property of \mathbf{I} —for instance, I_1 —leads to a change in a perceptual variable, one can only attribute this increase to \mathbf{I} when possible confounding variables are controlled for. In the vast majority of studies performed to date, other parameters, such as \mathbf{M} and/or m , varied together with I_1 in an uncontrolled fashion so that the effect on perceived length could not with certainty be attributed to I_1 . As demonstrated by Kingma et al. (2002), covariation of parameters leads to results that are difficult to interpret. Using simple and multiple regression analyses, it was shown that length and heaviness perception of hand-wielded rods can be explained by different statistical models including one or more parameters from the set $(I_1, I_3, \mathbf{M}, m)$. This turned out to be the case not only for the results of Kingma et al. (2002) but also for the results of many previous studies addressing the inertia tensor hypothesis. Regression models with only \mathbf{M} and m as parameters often turned out to explain results equally well as models including parameters derived from \mathbf{I} and even better than models exclusively based on parameters derived from \mathbf{I} . This shows that a variety of mechanical invariants may constrain haptic perception through dynamic touch. Moreover, it indicates that perceptual judgments may be based on more than one mechanical invariant.

The key to a definite verification or falsification of the inertia tensor hypothesis is to conduct experiments in which all candidate invariants that, from a mechanical point of view, play a role in the wielding or holding of an object, are pitted against each other by being varied one at a time (i.e., while the others are kept constant). Stroop et al. (2000) is the only study to date in which the inertia tensor was varied while variations in both mass and static moment were controlled for. In that study, it was reported that the inertia tensor indeed plays a role in the perception of rod length and heaviness by static holding. However, Stroop et al. did not critically test the inertia tensor hypothesis by independently varying all candidate invariants. The goal of the present study was to perform such a test for length and heaviness perception during wielding and for length perception during holding of a nonvisible rod. In accordance with the first study (Solomon & Turvey, 1988), as well as many subsequent studies relating perception to the inertia tensor (e.g., Burton & Turvey, 1990; Carello, Fitzpatrick, Domaniewicz, et al., 1992; Carello et al., 1998; Carello, Fitzpatrick, & Turvey, 1992; Carello, Flascher, Kunckler-Peck, & Turvey, 1999; Carello, Santana, & Burton, 1996; Chan, 1994, 1995, 1996; Cooper, Carello, & Turvey, 1999; Fitzpatrick et al., 1994; Kingma et al., 2002; Pagano et al., 1993; Stroop et al., 2000; Turvey, Burton, Pagano, Solomon, & Runeson, 1992), a seated posture with for-

ward holding and wielding was selected. A series of three experiments was conducted using specially designed sets of test materials (in the form of rods with weights attached), enabling a complete separation of variation in all parameters of interest (i.e., I_1 , I_3 , \mathbf{M} , and m). Note that in previous work (see above), I_1 and I_3 have been related to the perception of (full) rod length, singly or in combination (recall that for a rod with the origin at one of its tips, $I_1 = I_2$). For heaviness perception, it has recently been suggested that perceptual judgments are constrained by symmetry and volume (e.g., Shockley et al., 2001; Turvey et al., 1999). However, those parameters are based on the eigenvalues of the inertia tensor, and Kingma et al. (2002) have shown for rods that correlations of heaviness perception with symmetry and volume are identical to correlations with I_1 and I_3 . Thus, by manipulating I_1 or I_3 while keeping m and \mathbf{M} constant, one can determine not only whether the eigenvalues as such constrain perception but also whether higher order properties based on these eigenvalues have an effect.

Experiment 1

The first experiment was conducted to determine which of the variables from the set (I_1 , I_3 , \mathbf{M} , and m) are involved in estimating the length of a nonvisible rod that is wielded (Kingma et al., 2002). Participants were asked to estimate rod extent (hereafter called *rod length*) after having wielded rods behind a rigid, opaque screen. It should be noted here that participants were wielding the rods more or less in line with the long axis of the forearm (as in Solomon & Turvey, 1988, and several other studies; see above) rather than perpendicular to the long axis of the forearm (as in several other experiments). Consequently, the rotation center of the wrist was close to the base of the rod.

Method

Participants. Ten participants (9 female and 1 male; 8 right-handed and 2 left-handed; mean age = 23 years, $SD = 3$ years), who neither suffered from any afflictions of the wrist nor from neurological or visual impairments, participated voluntarily in this study after signing a written consent. The selected participants were likely to have had no prior knowledge about the concept of moment of inertia and the results of previous rod-wielding experiments.

Materials. Two sets of five hollow rods were used, one consisting of 1.00-m long rods and one consisting of 0.75-m long rods. All rods had a 0.102-m handle at one end. To maximize the relative variation in the mechanical parameters of interest, the rods were made of lightweight carbon fiber. The outer radius of the carbon fiber rods was 0.0075 m, and the inner radius 0.0060 m.

Two brass weights were attached to each rod. There was one reference rod in each set (Rods 1 and 6 for the long and short rods, respectively). Figure 1 shows how a variation in only one of the parameters (I_1 , I_3 , \mathbf{M} , and m) was obtained while the other parameters were kept constant. I_3 was manipulated by varying the diameter of the brass cylinders that were attached to the rod. The mass and radius of the brass cylinders, as well as the distance at which they were placed relative to the tip of the rod, were chosen in such a way that, in each rod, only one parameter from the set (I_1 , I_3 , \mathbf{M} , or m) varied with respect to the reference rod (Table 1, shown later). Independent variation of the parameters (I_1 , I_3 , \mathbf{M} , or m) can only be achieved relative to a given center of rotation (i.e., the origin of the coordinate system for calculating the inertia tensor and the static moment). The choice of this rotation point is a matter of debate. For reasons outlined

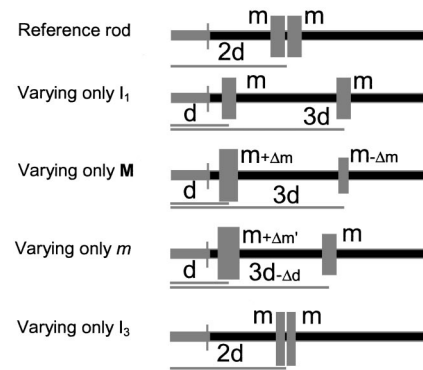


Figure 1. A schematic drawing of the principle that was used to obtain a variation in only one of the invariants (I_1 , \mathbf{M} , m , I_3) within each set of five rods. The principle exploits the fact that, relative to the base of the rod, I_1 of a point mass is related to Mass (m) \times Distance (d) squared, whereas \mathbf{M} is related to $m \times d$. First, I_1 was varied (increased) by shifting both cylinders over an equal distance (one in proximal and one in distal direction). Using the second rod as a starting point, \mathbf{M} was varied (decreased) by reducing the mass of the distal cylinder while increasing the mass of the proximal cylinder by the same amount, until I_1 reached the value of the reference rod. Again using the second rod as a starting point, m was varied by increasing the mass of the proximal cylinder and reducing the distance of the distal cylinder in such a way that both I_1 and \mathbf{M} were equal to the reference rod. Either a constant (in the first four rods) or a changing I_3 was obtained by adapting the length and the radius of the brass cylinders without changing their mass or center location. Finally, cylinder distances and masses were slightly adapted to take into account the I_1 of each cylinder about its own center of mass. I_1 and $I_3 =$ first and third eigenvalues, respectively; $\mathbf{M} =$ static moment.

in the General Discussion, we calculated the inertia tensor relative to the tip (base) of the rod.

The mechanical properties of the rods without cylinders (but with handles) were as follows: $m = 0.134$ kg, $\mathbf{M} = 0.0443$ kg \cdot m, $I_1 = 0.0298$ kg \cdot m², and $I_3 = 0.86 \times 10^{-5}$ kg \cdot m² (for the rods with a length of 1.00 m); $m = 0.111$ kg, $\mathbf{M} = 0.0250$ kg \cdot m, $I_1 = 0.0127$ kg \cdot m², and $I_3 = 0.75 \times 10^{-5}$ kg \cdot m² (for the rods with a length of 0.75 m).

Procedure. As in the original experiment of Solomon and Turvey (1988), the participants sat on a chair with a rigid and opaque screen positioned on their right side. The right arm was positioned on an armrest behind the screen (Figure 2A). A curtain beside the participant's head (not shown in Figure 2) prevented them from seeing their arm. The wrist (tip of the ulna) was positioned on the front edge of the armrest, and the hand was oriented with the thumb upwards. When a rod was put into the participant's hand, the base of the rod was close to the wrist. Before the experimental series started, participants were instructed to grip the rod firmly at all times, to wield about the wrist joint, and to avoid hitting the screen or the floor. One series consisted of wielding 10 rods in random order. The amount of variation that could be realized in each parameter without changing another parameter was relatively small (except for I_3 ; see Table 1). In view of these relatively small variations, each series was repeated six times in random order, with a break of at least 30 min between Series 3 and 4. During Series 1–3 and 4–6, the participants were free to take a break when they desired. After having wielded each rod for as long as they needed to obtain a perception of its length, participants indicated the perceived length of the rod by moving a vertical surface (0.37×0.40 m) along a 1.66-m long horizontal rail on the left side of the screen (Figure 2B). Participants were instructed to move the surface to a position at which, if the surface were to extend through the screen, the tip of the rod would just touch it when the rod was held forward horizontally. A participant could move the surface by

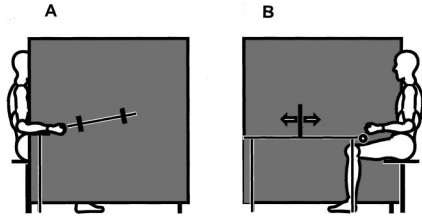


Figure 2. Experimental setup as seen from the right (A) and from the left (B) side of the screen during length perception in Experiments 1 and 2.

rotating a wheel with his or her left hand. One experimenter recorded the length estimates with a resolution of 1 mm. The participants could not see the length scale and were not informed about the position of the surface. After each trial, the experimenter returned the surface to the same starting position.

Results and Discussion

For the two sets of rods combined, a univariate analysis of variance (ANOVA) on I_1 (4 levels), I_3 (4 levels), \mathbf{M} (4 levels), m (4 levels), repetition (6 levels), and participant (10 levels), as well as the interaction between participant and repetition, revealed significant effects of participant, $F(9, 531) > 74.1$, $p < .001$, repetition, $F(5, 531) > 18.4$, $p < .001$, and the Participant \times Repetition interaction, $F(45, 531) > 6.2$, $p < .001$, on perceived length. More important, the ANOVA showed a highly significant effect of \mathbf{M} , $F(2, 531) = 23.7$, $p < .001$, as well as I_1 , $F(2, 531) = 10.5$, $p < .001$, on perceived length. In contrast, m , $F(2, 531) = 1.0$, $p = .375$, and I_3 , $F(2, 531) = 0.03$, $p = .972$, turned out to be unrelated to perceived length. Because of the large systematic variability of the length estimates between participants and over repetitions in length estimates, the effect size (as measured by the partial eta squared, η_p^2), was much larger for the variables participant ($\eta_p^2 = .557$) and repetition ($\eta_p^2 = .148$) and for their interaction ($\eta_p^2 = .343$) than it was for the mechanical invariants that were significant ($\eta_p^2 = .082$ for \mathbf{M} and $.038$ for I_1).²

ANOVAs performed for each set of rods separately confirmed the effects of participant, $F(9, 531) = 45.4$, $p < .001$, repetition, $F(5, 531) = 9.5$, $p < .001$, and the Participant \times Repetition interaction, $F(45, 531) = 4.0$, $p < .001$ (all η_p^2 values slightly higher than for the combined rod sets). Furthermore, significant effects were found for I_1 for the long rods, $F(1, 236) = 9.2$, $p = .003$, $\eta_p^2 = .038$, and for the short rods, $F(1, 236) = 16.9$, $p < .001$, $\eta_p^2 = .067$, and \mathbf{M} for the long rods, $F(1, 236) = 39.9$, $p < .001$, $\eta_p^2 = .145$, and for the short rods, $F(1, 236) = 3.9$, $p = .048$, $\eta_p^2 = .016$, although the effect of \mathbf{M} in the short rod set was small (Figure 3A) and only just significant. In addition, the absence of a significant effect of m for the short rods, $F(1, 236) = 2.8$, $p = .096$, and for the long rods, $F(1, 236) = 0.2$, $p = .678$, and of I_3 (in both sets), $F(1, 236) < 0.06$, $p > .823$, was confirmed. ANOVAs for the combined rod sets were also performed for each participant separately. Probably as a result of the limited number of repetitions, the effect of one (and not more than one) of the variables I_1 , I_3 , \mathbf{M} , and m was significant in only 6 participants. Of those participants, 4 showed a significant effect of \mathbf{M} , all $F_s(2, 45) > 4.5$, $p_s < .017$, $\eta_p^2_s > .160$, and 2 showed a significant effect of I_1 , $F(2, 45) > 3.3$, $p < .044$, $\eta_p^2 > .130$.

Within rod sets, regression analyses were not applicable because each independent variable had only two levels. A regression analysis (in log-log coordinates) over rod sets resulted in $r^2 = .960$, $p < .001$ for I_1 ; $r^2 = .962$, $p < .001$ for \mathbf{M} ; $r^2 = .532$, $p = .017$ for m ; and no significant correlation for I_3 . It should be noted, though, that a regression analysis over rod sets introduces, as in previous studies, confounding covariation among parameters. Results from this analysis should therefore not be considered as evidence for a role of those parameters. Especially for I_1 and \mathbf{M} , there was more variation between than within rod sets (see Table 1). Clearly, this obscures differentiation between parameters when regressions are performed over rod sets. In view of the results of the ANOVA (see also Figure 3A), the significant correlation for m was likely due to covariation with \mathbf{M} and I_1 between rod sets.

Because not only I_1 but also \mathbf{M} induced differences in perceived length in wielding a nonvisible rod, it can be concluded from the results of Experiment 1 that length perception is not uniquely governed by the eigenvalues of the inertia tensor. Contrary to the results of previous experiments (see Kingma et al., 2002), there are no competing models left that could also explain the data. In all likelihood, this is a consequence of the fact that the present experiment is the first in which covariation among the parameters (I_1 , I_3 , \mathbf{M} , m) was completely prevented within rod sets.

Kingma et al. (2002) argued that, when a rod is held in a fixed position instead of being wielded, angular accelerations are probably insufficient to allow for detection of its length through \mathbf{I} . Still, some previous studies have claimed a role of \mathbf{I} in the perception of rod length during rod holding (Carello et al., 1996; Stroop et al., 2000), arguing that minimal rotation would be sufficient to perceive the eigenvalues of \mathbf{I} . The second experiment in the present study was aimed at resolving this issue by asking participants to estimate rod length while holding the rods, using again a design in which the mechanical parameters of interest were manipulated in isolation.

Experiment 2

In the second experiment, we tested length perception by static holding. It is theoretically implausible that the inertia tensor would

² Partial eta squared values (η_p^2) can be interpreted as the proportion of variance in the dependent variable that is attributable to each (significant) effect. An alternative measure of effect size would be the eta squared values (η^2), which have the advantage of adding up to 1 but the (more important) disadvantage of being dependent on the number of other effects and the magnitude of those effects. Over participants and repetitions, the data were highly variable. This is reflected in relatively low η_p^2 values for the mechanical variables that were significant. In the literature, the low explained variance of mechanical variables has received little attention. Data are usually first averaged over participants and repetitions before the explained variance is calculated (in most cases, with the use of regression analysis). This procedure can result in high explained variances for mechanical invariants, as seen in previous studies, as well as in the current study (up to 99% for \mathbf{M} in Experiments 2 and 3). This suggests that the present η_p^2 s are not exceptionally low. As reported in the results of all experiments of the current study, ANOVAs on the data of the individual participants resulted in substantially higher η_p^2 values for the mechanical invariants with a significant effect because, in this case, the variance over participants played no role.

Table 1
Specifications of Mass, Radius, and Center Position Along the Rod of the Two Brass Cylinders Attached to Each Rod in Experiments 1–3

Rod	Length (m)	Cylinder 1			Cylinder 2			Resulting rod properties			
		Mass (kg)	Center dist. (m)	Radius (m)	Mass (kg)	Center dist. (m)	Radius (m)	<i>m</i> (kg)	M (kg · m)	<i>I</i> ₁ (kg · m ²)	<i>I</i> ₃ (kg · m ²)
1. (Ref.)	1.00	0.1500	0.553	0.0218	0.1500	0.567	0.0218	0.434	0.211	0.124	0.886
2. (<i>I</i> ₁ , +44.7%)	1.00	0.1500	0.130	0.0218	0.1500	0.990	0.0218	0.434	0.211	0.179	0.886
3. (M , -22.6%)	1.00	0.2085	0.140	0.0228	0.0915	0.992	0.0195	0.434	0.163	0.124	0.886
4. (<i>m</i> , +39.6%)	1.00	0.3200	0.170	0.0179	0.1521	0.747	0.0144	0.606	0.211	0.124	0.886
5. (<i>I</i> ₃ , +90.3%)	1.00	0.1500	0.557	0.0318	0.1500	0.563	0.0318	0.434	0.211	0.124	1.686
6. (Ref.)	0.75	0.1000	0.427	0.0273	0.1000	0.433	0.0273	0.311	0.110	0.050	0.875
7. (<i>I</i> ₁ , +38.7%)	0.75	0.1000	0.120	0.0273	0.1000	0.740	0.0273	0.311	0.110	0.069	0.875
8. (M , -20.2%)	0.75	0.1365	0.120	0.0287	0.0635	0.743	0.0240	0.311	0.088	0.050	0.875
9. (<i>m</i> , +40.2%)	0.75	0.2400	0.140	0.0211	0.0851	0.616	0.0203	0.436	0.110	0.050	0.875
10. (<i>I</i> ₃ , -57.2%)	0.75	0.1000	0.420	0.0156	0.1000	0.440	0.0156	0.311	0.110	0.050	0.375

Note. The percentage of change induced in one of the mechanical invariants relative to the corresponding reference (Ref.) rod—1 (for Rods 2–5) or 6 (for Rods 7–10)—is given in parentheses following each rod’s number. Bolded values indicate the changed value of the mechanical invariant. Center dist. = distance from the base of the rod to the center of the cylinder.

play a significant role in static holding because detecting an object’s inertial properties requires angular acceleration, which is likely to be negligible in static holding. In accordance, Burton and Turvey (1990), as well as Carello, Fitzpatrick, Domaniewicz, et al. (1992), suggested that the static moment is the primary mechanical invariant in static holding. In contrast, Stroop et al. (2000) recently suggested that nonvisual length perception by static holding may depend on the eigenvalues of the inertia tensor. For the present experiment, it was expected that the static moment would play a major role and that the contribution of the inertia tensor would be nil.

Method

As in Experiment 1, 10 participants (5 female and 5 male; 9 right-handed and 1 left-handed; mean age = 20 years, *SD* = 1 year), who neither

suffered from any afflictions of the wrist nor from neurological or visual impairments, signed an informed consent before taking part in the experiment. None of the participants was familiar with this type of experiment or the rationale behind it. The protocol of the experiment was similar to that of Experiment 1, except that the participants were now instructed to hold the rod stationary while pointing it forward (rather than to wield it). The experimenters saw to it that the participant complied with this instruction. After having positioned their arm on the armrest, participants were instructed to open their hand. One experimenter then carefully placed the rod into the hand of the participant, while holding it horizontally and pointing it forward relative to the participant. The participant was asked to grip the rod firmly and to hold it stationary before the experimenter released it. This procedure minimized movement rather than completely eliminating it. Note that it has been suggested that the remaining movement enables detection of **I**, allowing for its implication in length perception (Stroop et al., 2000). By adopting the same procedure as in previous experiments on

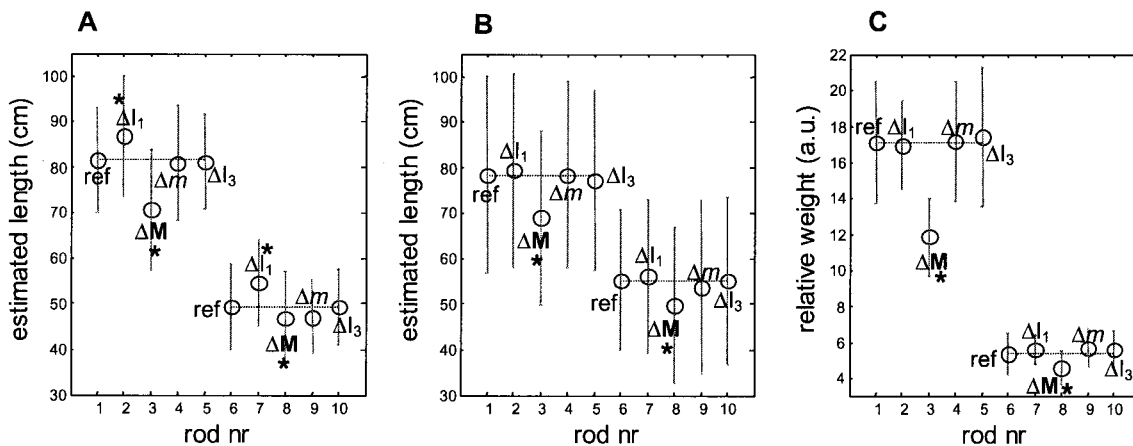


Figure 3. Average length estimates after rod wielding (Experiment 1; A) and after rod holding (Experiment 2; B), and average weight estimates after rod wielding (Experiment 3; C). Significant effects (*p* < .05) are indicated by asterisks. Error bars indicate 1 standard deviation over participants, after averaging over repetitions within participants. Rod numbers (nrs) correspond to the numbers in Table 1. Horizontal lines indicate the length or weight estimated for the reference rod (ref; Rod 1 for the long rods and Rod 6 for the short rods). In all other rods, only one of the variables of interest, first eigenvalue (ΔI_1), static moment (ΔM), mass (Δm), or third eigenvalue (ΔI_3), had been changed relative to the reference rod. a.u. = arbitrary units.

static holding (including the study by Stroop et al., 2000), we were able to test this suggestion and compare our results with those obtained in other studies on static holding. To improve the analysis of the individual data, eight repetitions were performed (instead of six, as in Experiment 1). To prevent participant fatigue, four of the repetitions were performed on 1 day, and the other four were performed on a 2nd day (either 1 or 2 days later). The 40 trials (4 repetitions \times 10 rods) within each session were fully randomized. During the sessions, participants were allowed to take a break when they needed one. The participants were not told how many rods there were, nor were they informed that loads had been attached to them.

Results and Discussion

Besides significant effects of participant, $F(9, 711) = 398.7$, $p < .001$, $\eta_p^2 = .835$, repetition, $F(7, 711) = 10.4$, $p < .001$, $\eta_p^2 = .094$, and the Participant \times Repetition interaction, $F(63, 711) = 3.7$, $p < .001$, $\eta_p^2 = .248$, an ANOVA over all participants and rods (see Experiment 1 for a description of the model used) revealed a highly significant effect of **M** on estimated rod length during rod holding, $F(2, 711) = 35.2$, $p < .001$, $\eta_p^2 = .090$. In contrast, m , I_1 , and I_3 were unrelated to estimated rod length, all $F_s(2, 711) < 0.7$, $ps > .493$.

ANOVAs on the separate sets of rods showed effects of participant, repetition, and the Participant \times Repetition interaction that were comparable to those for the combined rod sets. Furthermore, the effects of **M** were evident for the long rods, $F(1, 316) > 53.1$, $p < .001$, $\eta_p^2 = .144$, and for the short rods, $F(1, 316) > 23.5$, $p < .001$, $\eta_p^2 = .069$. Also, the absence of a significant effect of m , I_1 , or I_3 was confirmed in both sets of rods (for all three parameters and both sets of rods, $F_s(1, 316) < 1.9$, $ps > .172$; see also Figure 3). ANOVAs on the length estimates of the individual participants (for the pooled rod sets) confirmed that **M** was the main parameter associated with length estimation during rod holding. A significant effect of **M** on estimated rod length was found in 8 out of the 10 participants (for all 8 participants, $F_s(2, 63) > 3.7$, $ps < .030$, η_p^2 s = .106–.365). In 2 of those 8 participants, m was also significant, and in 1 other participant, I_3 was also significant. Significant effects of I_1 were not observed in any of the participants. As in Experiment 1, regressions over rod sets (resulting, in log-log coordinates, in $r^2 = .910$, $p < .001$ for I_1 ; $r^2 = .994$, $p < .001$ for **M**; $r^2 = .589$, $p = .010$ for m ; and no significant correlation for I_3) should not be considered as evidence for a role of those parameters. Figure 3B suggests that the correlation for I_1 was entirely a consequence of the covariation with **M** between rod sets. This experiment clearly showed that the eigenvalues of the inertia tensor were unrelated to length perception during rod holding. In fact, **M** appeared to be the only parameter that was consistently associated with length perception under these conditions. These findings contradict those of previous reports (Carello et al., 1996; Stroop et al., 2000). The difference with the study of Stroop et al., in which **M** was controlled by holding it constant, is discussed at greater length in the General Discussion.

Kingma et al. (2002) found a strong correlation between **M** and the perceived heaviness of rods. This stood in contrast with the results of other studies, in which heaviness perception was attributed to the eigenvalues of **I** alone (Amazeen & Turvey, 1996; Kloos & Amazeen, 2002), to the eigenvalues of **I** in combination with m (Amazeen, 1997), or to m in combination with two other parameters derived from **I** (i.e., ellipsoid volume and symmetry; Shockley et al., 2001; Turvey et al., 1999). Although those three-

parameter models predicted heaviness perception as well as **M** in the Kingma et al. (2002) study, it was argued there that, when correlation coefficients are equal (or only marginally different), a one-parameter model should be preferred over a three-parameter model. Furthermore, it was argued that the correlation of the three-parameter model might well have been a result of covariation with other relevant parameters. Patently, a conclusive answer to this problem can only be obtained when the mechanical parameters of interest are varied independently. Therefore, our final experiment was aimed at resolving the issue of the role of mechanical parameters in heaviness perception.

Experiment 3

Method

This experiment was performed using the same two sets of rods as in Experiments 1 and 2. As in Experiment 2, 10 participants (2 female and 8 male; 9 right-handed and 1 left-handed; mean age = 22 years, $SD = 2$ years), who neither suffered from any afflictions of the wrist nor from neurologic or visual impairments, took part in the experiment after signing an informed consent form. All participants performed eight series of heaviness estimation (four series on 1 day and four series 1 or 2 days later), with the 10 rods presented to them in randomized order. Participants were asked to estimate the heaviness of each rod relative to a nonexperimental benchmark rod after having wielded both. The benchmark rod was a 0.82-m long aluminum rod, with the same lightweight handle as the other rods, and a cylindrical weight attached to it at a distance of 0.515 m, resulting in an m of 0.355 kg, an **M** of 0.170 kg \cdot m, an I_1 of 0.0881 kg \cdot m², and an I_3 of $0.349 \cdot 10^{-4}$ kg \cdot m². After the participants had positioned their forearm on the arm support behind the screen, they were first given the benchmark rod before each measurement. They were asked to wield it, and they were told that its heaviness was 10. Immediately thereafter, they were given an experimental rod, and after they had wielded it, they were asked to estimate its heaviness relative to the benchmark rod. For example, a rod that was perceived to be half as heavy as the benchmark rod was to be rated as 5; a rod that was perceived to be twice as heavy as the benchmark rod was to be rated as 20.

Results and Discussion

The results of the ANOVA for heaviness perception during rod wielding were quite similar to the results for length perception during rod holding obtained in Experiment 2. Besides a significant effect of participant, $F(9, 711) = 18.4$, $p < .001$, $\eta_p^2 = .189$, repetition, $F(7, 711) = 3.0$, $p < .004$, $\eta_p^2 = .029$, and the Participant \times Repetition interaction, $F(63, 711) = 1.4$, $p < .012$, $\eta_p^2 = .116$, an ANOVA over all participants and rods (see Experiment 1 for a description of the model used) revealed a highly significant effect of **M** on perceived heaviness of the rods, $F(2, 711) = 59.2$, $p < .001$, $\eta_p^2 = .143$. None of the parameters m , I_1 , and I_3 had a significant effect on estimated heaviness (for all three parameters, $F_s[2, 711] < 0.19$, $ps > .737$). Apart from the effect of repetition in the short rod set, which was not significant, the effects of participant, repetition, and the Participant \times Repetition interaction were similar, though somewhat stronger in separate rod sets. Furthermore, the dominant role of **M** was confirmed: The effect of **M** was highly significant in each rod set for the long rods, $F(1, 316) = 94.4$, $p < .001$, $\eta_p^2 = .230$, and for the short rods, $F(1, 316) = 24.0$, $p < .001$, $\eta_p^2 = .053$, whereas the effects of m , I_1 , and I_3 were nonsignificant for both the long (for all three parameters,

$F_s[1, 316] < 0.31, p_s > .582$) and the short rods (for all three parameters, $F_s[1, 316] < 2.7, p_s > .103$). ANOVAs on the heaviness estimates of the individual participants (for the pooled rod sets) confirmed the unique role of \mathbf{M} in heaviness perception: A significant effect of \mathbf{M} on perceived heaviness was found for all participants (with $F_s[2, 63] > 10.0, p_s < .001, \eta_p^2 > .243$ in 6 of the participants and $3.2 < F_s[2, 63] < 10.0; .011 < p_s < .05, \eta_p^2 > .093$ in the other 4 participants). m showed up as a second significant variable in 1 participant, whereas I_1 and I_3 never had a significant effect on heaviness perception. As in Experiments 1 and 2, regressions over rod sets (resulting, in log-log coordinates, in $r^2 = .901, p < .001$ for $I_1; r^2 = .989, p < .001$ for $\mathbf{M}; r^2 = .633, p = .010$ for m ; and no significant correlation for I_3) should not be considered as evidence for a role of those parameters. Figure 3C suggests that the correlation for I_1 is entirely a consequence of covariation with \mathbf{M} between rod sets. In the studies in which volume and symmetry of the inertial ellipsoid (both higher order invariants calculated from the eigenvalues of \mathbf{I}) were put forward as important invariants in heaviness perception during wielding (i.e., Shockley et al., 2001; Turvey et al., 1999), \mathbf{M} covaried with \mathbf{I} and was not considered as a candidate invariant. In both of our rod sets, symmetry and volume only varied in the rods in which I_1 or I_3 was manipulated with respect to the reference rods (i.e., in Rods 2 and 5 and Rods 7 and 10). In those four rods, the change in volume and symmetry with respect to the reference rods ranged from -27% to 90% . In both rod sets, the changes in volume and symmetry had the same sign in the rod with variation in I_1 and an opposite sign in the rod with variation in I_3 . Because the rods in which I_1 or I_3 was manipulated were not perceived as being of different weight in comparison with the reference rods, we conclude that neither volume nor symmetry was implicated in heaviness perception during rod wielding in the present experiment. The discrepancy between this conclusion and those of Shockley et al. (2001) and Turvey et al. (1999) could well be due to the covariation of volume and symmetry with static moment in those earlier studies.

In sum, the results of this experiment constitute evidence that heaviness perception during rod wielding is uniquely related to \mathbf{M} . Previous studies in which it was suggested that the inertia tensor plays an important role in heaviness perception (Amazeen, 1997; Amazeen & Turvey, 1996; Kloos & Amazeen, 2002; Shockley et al., 2001; Turvey et al., 1999) were probably led astray as a result of confounding variations among relevant mechanical parameters.

General Discussion

This study was conducted to test the role of the inertia tensor versus other invariant mechanical properties of objects in haptic perception. We focused on two perceptual variables, length and heaviness, that have previously been claimed to be (mainly) governed by invariants related to the inertia tensor. By varying only one mechanical parameter at a time, we were able to reveal the role of the different invariants in length and heaviness perception. The results suggest that the role of the inertia tensor may have been overestimated in previous work. Under the conditions tested in this study, I_1 was merely, and not uniquely, related to length perception during rod wielding (Experiment 1). In addition to I_1 , \mathbf{M} was implicated in length perception during rod wielding. Furthermore, \mathbf{M} was found to be uniquely related to length perception during rod

holding (Experiment 2) and to heaviness perception during rod wielding (Experiment 3). These effects were found with a large amount of variance in the data proving to be related to participant and repetition. This large variability was probably related in part to the absence of calibration through feedback in the present experiments (cf. Pagano & Bingham, 1998). In this General Discussion, we first discuss specific aspects and implications of the experiments reported, starting with Experiment 3 and working our way back to Experiment 1, and then discuss their broader theoretical implications.

The results of Experiment 3 show that, in spite of being engaged in wielding activities, participants do not use the resistance against angular acceleration as the mechanical counterpart of heaviness. Rather, participants use the static moment, implying that they base their heaviness judgments on the invariant aspect of the gravitational torque. As stated before, previous claims regarding the role of the inertia tensor in heaviness perception were probably caused by confounding covariation between static moment and the eigenvalues of \mathbf{I} . This holds for the m, I_1 , and I_3 model proposed by Amazeen (1997), as well as for the more recently proposed mass, volume, and symmetry model of Turvey et al. (1999) and Shockley et al. (2001). However, we do not claim that the static moment is uniquely responsible for heaviness perception in all possible conditions. For instance, when an object is held at its center of mass, variations in \mathbf{M} without changes in I_1 and m cannot be detected because they do not lead to a change in muscular torque. Therefore, other parameters (e.g., I_1 or m) may play a role in such situations.

For rod holding, the absence of a role of \mathbf{I} in length perception is consistent with the consideration that, in the absence of a substantial angular acceleration, the signal-to-noise ratio (i.e., the contribution to muscular tension) of the static torque in Equation 1 will be much larger than that of the inertial torque. Small angular accelerations are unlikely to allow for detection of the eigenvalues of \mathbf{I} with sufficient accuracy to estimate rod length during rod holding. However, it should be emphasized that in situations in which the muscular torque (which, in the case of rod holding, equals the static torque) never reaches the level of the static moment $\times \mathbf{g}$, such as when rods are held at a substantial angle relative to the horizontal plane (Carello, Fitzpatrick, Domaniewicz, et al., 1992) or at varying locations along their length (Burton & Turvey, 1990), other parameters like static torque or mass may be implicated. For instance, when rods are offered in a constrained vertical position, such that a change of mass position along a rod does not cause a change in muscular torque, participants have been shown to use rod mass to estimate rod length (Lederman et al., 1996). Furthermore, the present results are inconsistent with the study by Stroop et al. (2000). Although confounding covariation was still present in their Experiments 1 (m and \mathbf{M}) and 3 (m), Stroop et al. found different perceived lengths while controlling for both m and \mathbf{M} in their Experiment 2, which was interpreted as an effect of differences in \mathbf{I} . We did not find such an effect of \mathbf{I} in our Experiment 2 (length perception in static holding). This discrepancy is striking because the number of repetitions (10 participants \times 2 rod sets \times 8 repetitions in the current Experiment 2), as well as the range of variation of I_1 (45% and 39% in the current study), were larger in the current study than in the study by Stroop et al. (9 participants \times 5 rod sets \times 3 repetitions and 13%–20% variation in I_1). This implies that the current Experiment 2 was at

least as sensitive in detecting effects of I_1 as was Experiment 2 of Stroop et al. The association of \mathbf{I} with length perception in the latter experiment suggests that angular accelerations were sufficient to be detectable by the haptic system. Although speculative, two plausible explanations for the occurrence of such angular accelerations can be provided. First, angular accelerations can be induced at the moment the support of the rod is transferred from the experimenter to the participant. The experimenter can inadvertently induce significant angular acceleration by releasing the rod too abruptly. Second, in rods with a smaller I_1 , small fluctuations in the torque applied by the participant will result in greater angular accelerations. Because the rods of Stroop et al. had smaller values for I_1 compared with the rods in the present study, this provides another possible explanation for the inconsistency found.

The particular range of the mechanical properties in the present study follows from the novel experimental method adopted to independently vary the properties of interest. Recall that we used carbon fiber rods with two brass rings, of which the positions and dimensions were varied in order to control for three mechanical invariants while varying only one. This independent variation necessarily limited the possible range of values for the mechanical invariants of interest, as was evident in Experiment 2 of Stroop et al. (2000). In order to obtain an acceptable range for all mechanical invariants, the relative mass of the brass rings, as well as the length of the rods, was maximized, while keeping the rods manageable for the participants. By constructing rods with mechanical properties in the intermediate (the short rod set) as well as the upper range (the long rod set) with respect to previous studies on dynamic touch, we obtained a larger variation in the mechanical properties of interest as compared with that in Experiment 2 of Stroop et al.

In many studies, it has been stated that length perception during object wielding is mainly governed by the eigenvalues of \mathbf{I} (Fitzpatrick et al., 1994; Pagano et al., 1993; Solomon & Turvey, 1988; Solomon et al., 1989; Turvey et al., 1998). As shown in Table 1, we varied I_1 considerably more within rod sets than we varied \mathbf{M} . In spite of this, the statistical results of Experiment 1 suggested that \mathbf{M} was at least as important as I_1 for perceiving rod length during wielding. Note, however, that the effect of \mathbf{M} was less pronounced in the short rods than in the long rods, whereas the converse was true for the effect of I_1 . From a mechanical perspective, this is understandable, because the same muscular torque induces larger angular accelerations in the short rods than it does in the long rods, thus providing a better basis for detecting I_1 . For this reason, it might well be the case that I_1 plays a more prominent role in the perception of rod length when short rather than long rods are wielded. Our finding that \mathbf{M} constrains length perception during rod wielding stands in contrast with previous studies, and it was achieved by eliminating the confounding covariation between the eigenvalues of \mathbf{I} and \mathbf{M} that was present in most of the studies cited. The finding that length perception during rod wielding is not based on \mathbf{I} alone is consistent with the finding of Kreifeldt and Chuang (1979) that Weber fractions for detecting changes in I_1 are relatively high ($1/5$ to $1/3$) compared with those for detecting weight changes.

Mechanical considerations suggest that, even for a uniform rod, at least two of the parameters from the set (I_1 , m , \mathbf{M}) are needed to uniquely specify its length (see Kingma et al., 2002). It was therefore hypothesized by Kingma et al. (2002) that participants

would use (minimally) two rather than one of these variables and that they might switch between them, depending on experimental conditions. The current study seems to confirm the use of two invariants for rod-length perception during rod wielding, at least in terms of group statistics. However, it remains to be investigated whether this also holds for individual participants. The present results certainly do not rule out the possibility that some participants rely on I_1 and others on \mathbf{M} . For rod holding, mechanical considerations would suggest that participants might use a combination of \mathbf{M} and m to reliably estimate length, but this appeared not to be the case (Experiment 2).

In the present study, we selected the base of the rod as the center of rotation relative to which we controlled the individual variations of the mechanical parameters of interest. Clearly, had we chosen the wrist joint as the center of rotation (as has been done in most studies promoting the inertia tensor hypothesis since Pagano et al., 1993), I_1 and I_3 would not have been constant in the rods that we designed to have only variation in either m or \mathbf{M} . However, we disagree with the choice of the wrist joint as the center of rotation, because calculation of eigenvalues of the inertia tensor with respect to the wrist breaks the logical alignment between the eigenvectors and the orientation of the rod and incorporates hand size and a part of the static moment into the inertia tensor, thereby obscuring the interpretation of results. Therefore, we selected the base of the rod as a reference. Considering the negligible effects of the choice of either the wrist or the end point of the rod in a previous study (Kingma et al., 2002), this choice could be safely expected to be inconsequential for the results of the present study as well. Nevertheless, we recalculated I_1 and I_3 relative to the wrist joint and analyzed our data accordingly. As expected, the resulting changes in I_1 and \mathbf{M} were very small (less than 1% in all rods), due to the (3-cm) translation to the wrist, whereas the changes in I_3 were large (over 100%). The ANOVA results could not be reproduced because, due to the translation of the center of rotation, I_1 and I_3 now varied in each rod. However, simple regression analyses showed that, as a consequence of the translation, r^2 values marginally decreased for I_1 , changed less than 0.2% for \mathbf{M} , and decreased and remained nonsignificant for I_3 .

Turning now to a discussion of the general theoretical implications of our results, it is useful to first make a few statements about the perceptual status of the mechanical invariants examined in the present study and in previous studies on dynamic touch. For this purpose, we recall Gibson's (1966) conclusion that "the stimulus information from wielding can only be an invariant of the changing flux of stimulation in the muscles and tendons, an exterospecific invariant in this play of forces" (p. 127). We further recall that, according to the theory of direct perception (see, e.g., Michaels & Carello, 1981; Turvey, 1990), higher order invariants only constitute information to the extent that they specify properties of the environment (or the environment-actor system). Viewed against this background, mechanical invariants like mass, static moment, and the inertia tensor constrain the stimulus flow but do not specify object properties, such as length and heaviness, and thus, strictly speaking, they constitute no information. The relationship between mechanical invariants and perceptual invariants in the stimulus flow constituting information has remained, up to now, unclear, as can be appreciated from the fact that the dimension of the eigenvalues of the inertia tensor or that of the static moment differ from that of the perceptual judgment (e.g., extent or

weight), without any immediate specification relation holding between them. Therefore, if a certain mechanical invariant is found to be correlated to perception, this implies that the mechanical property is constraining the perceptual information that is detected, not that the mechanical invariant itself constitutes that information. When a combination of two mechanical invariants is found to constrain perception (as suggested by the results of Experiment 1), this implies that both variables somehow constrain the perceptual information used. Similarly, if different mechanical variants are found to constrain the performance of the same perceptual task under different mechanical conditions (compare Experiment 1 with Experiment 2), this implies that the information used is constrained differentially by those mechanical conditions. Finally, if different mechanical invariants are implicated in different perceptual tasks (compare Experiment 1 with Experiment 3), this implies that the relevant information differs across tasks and, hence, is constrained differentially by pertinent mechanical invariants. The upshot of this line of argument for the inertia tensor hypothesis and the prospect of a direct perception theory of dynamic touch is twofold: First, the premise (and also the promise) of the inertia tensor hypothesis that an encompassing theory of dynamic touch may be formulated solely in terms of an object's resistances against angular acceleration is untenable; second, to develop a direct perception theory of dynamic touch based on mass distribution, it is necessary to start addressing how the information used by the haptic system in a broad range of perceptual tasks may be constrained by multiple mechanical invariants.

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