Environmental and Resource Economics **28:** 325–345, 2004.

Optimal Investment in Clean Production Capacity

CAROLYN FISCHER[1,*], CEES WITHAGEN[2] and MICHAEL TOMAN[3]
[1]*Resources for the Future, 1616 P. St. NW, DC 20036, Washington, DC, USA;* [2]*Free University of Amsterdam and Tilburg University, The Netherlands;* [3]*Inter-American Development Bank, Washington, DC, USA;* ***Author for correspondence (e-mail: fischer@rff.org)*

Accepted 8 September 2003

Abstract. For the mitigation of long-term pollution threats, one must consider that both the process of environmental degradation and the switchover to new and cleaner technologies are dynamic. We develop a model of a uniform good that can be produced by either a polluting technology or a clean one; the latter is more expensive and requires investment in capacity. We derive the socially optimal pollution stock accumulation and creation of nonpolluting production capacity, weighing the tradeoffs among consumption, investment and adjustment costs, and environmental damages. We consider the effects of changes in the pollution decay rate, the capacity depreciation rate, and the initial state of the environment on both the steady state and the transition period. The optimal transition path looks quite different with a clean or dirty initial environment. With the former, investment is slow and the price of pollution may overshoot the long-run optimum before converging. With the latter, capacity may overshoot.

Key words: capacity investment, clean technology, pollution accumulation

JEL classification: Q2, Q42

1. Introduction

In ongoing debates over how to mitigate long-term pollution threats, there is common agreement that the adoption of more environmentally friendly technologies is crucial. Environmental economists generally have focused on the creation of appropriate economic incentives for pollution control that would induce technology switching as well as reduced consumption of polluting goods and other mitigating responses. Key questions involve the optimal timing and use of investments in clean technologies. There is, however, less understanding of how this process would occur in practice.

In this paper we construct a model that is simple but nonetheless allows us to consider in some detail the interplay of two dynamic processes, the process of environmental degradation or improvement, and the process of developing clean production capacity. The model incorporates both tradeoffs between consumption benefits and environmental damages, and tradeoffs between investment and operating costs for clean production capacity versus the alternative of reducing pollution-creating consumption directly. Consideration of these tradeoffs allows us to explore how the time path of the pollution shadow price evolves, as well as the path of clean capacity creation and utilization. In particular, we find that the optimal trajectory depends importantly on the initial state of the environment: specifically, itmay depend on whether clean capacity is used to mitigate an immediate environmental problem or to forestall a future problem.

A number of other papers have explored such issues. In some respects our analysis resembles those devoted to exploring the creation of backstop resource production capacity in the face of natural resource exhaustion (see Switzer and Salant 1986; Krautkraemer 1985; Oren and Powell 1985). Our analysis goes beyond these frameworks by bringing in pollution decay, thereby introducing a renewable resource aspect to the problem. A number of papers have considered problems of pollution accumulation and investment with uncertainty and irreversibility (Kolstad 1996; Ulph and Ulph 1997; Fisher and Narain 2003). Although our analysis is deterministic, we focus more explicitly and directly on path dynamics versus steady-state properties. Feichtinger et al. (1994) and Toman and Withagen (2000) incorporate the possibility of nonconvexities and threshold effects in the pollution damage function, but they do not focus on the creation of clean production capacity. Wirl and Withagen (2000) consider a problem similar to ours, but they limit attention to clean technologies with variable operating costs that are lower than those of conventional technologies. In their model, costly investments in capacity and nonlinearities in pollution assimilation may lead to limit cycles in the socially optimal path. In contrast, we consider technologies that are more costly to operate as well as to create. With simple, linear pollution assimilation, although the steady states are themselves stable, we find that the path to that steady state can involve overshooting or undershooting of the ultimate pollution or capacity targets before converging.

By understanding the socially efficient path in this deterministic setting, we can lay the foundation for incorporating additional complexities and policy constraints. Since we do not consider market failures other than the pollution externality, our planning problem can be easily decentralized with an optimal series of emissions taxes. In essence, we focus on the optimal path of those taxes and how it depends on the state of the environment at the time the policy takes effect. The research thus addresses part of the ongoing debate about what portfolio of policies best support socially efficient technology transitions for such problems as climate change, accumulative pollutants like methyl bromide and other ozone depletors, or the protection of water bodies from accumulative pollutants.

In the next section, we present a model of production with two technologies, one with emissions and one with a capacity constraint. We evaluate the steady states without and with the clean technology option. We then characterize the optimal paths toward the steady state. In analyzing the optimal trajectory of the shadow value of emissions, we focus on the role of the initial state of the environment. We find that starting from a relatively clean environment, the optimal path involves an initial interval of time when the backstop technology is put in place but not actually used, followed by a transition to the simultaneous use of both clean and dirty technologies. Starting from a sufficiently large stock of pollutants, the clean technology will be used immediately, but dirty production will be maintained while clean capacity is expanded. Then there may be a stage in which all production is carried out using the clean backstop, but eventually both technologies will be used simultaneously.

Furthermore, there is the possibility of overshooting the steady-state value of pollution. Starting from a clean state, the environment may need to grow dirtier than its final state, to ensure enough rents are created to justify the clean technology. On the other hand, starting with a very dirty environment, one may want to build more clean capacity than is ultimately needed, in order to speed the transition to a cleaner environment; this greater capacity may then allow an undershooting of the steady state. These scenarios are discussion in section 3, and the final section offers concluding remarks.

2. Model

We consider an economy where consumers' preferences are affected by three commodities: consumption, leisure and a stock of pollutants. Consumption commodities are produced by labor according to two linear technologies, a "dirty" technology that produces a pollution externality, and a "clean" technology that emits no pollution, but has a greater labor input requirement and necessitates an investment in production capacity. The commodities are uniform, and consumers cannot distinguish which technology produced it.

Dirty production of the consumer commodity is represented by q. Each unit of output requires c units of the labor input.[1] This production method causes emissions that are proportional to output, with an emissions rate equal to unity.[2]

The stock of pollution S increases with emissions q and assimilates at a rate of $\alpha > 0$.[3] The instantaneous net growth in the stock is

$$\dot{S}(t) = q(t) - \alpha S(t). \tag{1}$$

Production with the nonpolluting backstop technology is denoted by k. Let K be installed clean capacity of which $k \leq K$ is used; clean production is not required always to operate at capacity. The labor input requirement of clean production is b. We are considering clean technologies whose capital and operating costs are higher than those of the dirty technology; thus, we restrict the input costs to $b > c$. This restriction means that clean production is only economically viable when the pollution externality is sufficiently large; furthermore, it allows for the possibility that dirty and clean production will co-exist in equilibrium. Wirl and Withagen (2000) consider a case in which the clean technology has no (and thereby lower) operating costs, but requires a fixed installation cost; an example of this type would be solar power. Therefore these cost assumptions are not innocuous.

Installing additional capacity I incurs a labor cost $f(I)$, which is assumed to satisfy

$$f(0) = 0;\ f(I) > 0,\ I > 0;\ f'(0) = 0;\ f'(I) > 0, I > 0;\ f'' > 0. \quad \text{(A1)}$$

An important assumption is that $f'(0) = 0$; although somewhat unrealistic, it is a major analytical simplification. It can be justified if we think of our model as approximating the choice over a broad range of clean technologies, some of which have low installation costs (e.g., in-house process improvements) but limited capacity.[4] Disinvestment, other than through depreciation, can be safely disregarded, since there is no cost (including no opportunity cost) of maintaining excess capacity.[5]

Let δ be the rate of depreciation in the capital stock. Therefore, the instantaneous change in capacity is

$$\dot{K}(t) = I(t) - \delta K(t). \quad (2)$$

We assume that the initial stock of the backstop technology equals zero: $K(0) = 0$.

Instantaneous welfare of the representative agent at time t is given by $W(C(t), \bar{L} - L(t), S(t))$, in which C is total consumption, L is the labor input, and the initial endowment of labour is $\bar{L}$. In the sequel we will be interested in social welfare in a dynamic utilitarian setting. Therefore the objective function can be written as

$$SW = \int_0^\infty e^{-\rho t} W(C(t), \bar{L} - L(t), S(t))dt$$

where ρ is the rate of time preference. Hence the problem is to maximize SW subject to the constraints

$$C(t) = q(t) + k(t)$$

$$L(t) = cq(t) + bk(t) + f(I(t)) \leq \bar{L}$$

as well as the equations of motion for the stock variables.

The structure of the instantaneous welfare function matters for the outcome: the degree of substitutability between the constituent parts of the instantaneous welfare function matters (see Michel and Rotillon 1995). As discussed in Toman and Withagen (2000), assuming additively separable consumption utility and pollution damage is not innocuous but very convenient analytically. In view of eliciting transparent results from this two-state-variable model, we have chosen to employ this simplification, whereby the welfare function is strongly separable and even quasi-linear:

$$W(C(t), \bar{L} - L(t), S(t)) = U(C(t)) + \bar{L} - L(t) - D(S(t)).$$

The function U represents utility from consumption, with $U'(C) > 0$ for all $C > 0$, $U'(0) = \infty$, $U'(\infty) = 0$ and $U'' < 0$. The function D is the damage flow from the stock of pollution, with $D(0) = 0$, $D'(S) > 0$ for $S > 0$, $D'(0) = 0$ and $D'' > 0$. Utility from leisure is linear. We assume that the labor endowment is sufficient to ensure an interior solution (i.e., that the marginal utility when consuming all possible output is less than the marginal utility of leisure, or $U'(\bar{L}/c) < 1$).

The planner chooses $q(t)$, $k(t)$, and $I(t)$ for $t \in [0, \infty)$ to maximize the discounted value of consumption of the good net of production and pollution costs:

$$\int_0^\infty e^{-\rho t}(U(q(t) + k(t)) - cq(t) - bk(t) - f(I(t)) - D(S(t)))dt \tag{3}$$

subject to (1), (2), $S(0) = S_0$, $K(0) = 0$, $q(t) \geq 0$, $k(t) \geq 0$, $I(t) \geq 0$, and also the capacity constraint: $k(t) \leq K(t)$.

The current-value Hamiltonian for this two-state optimal control problem is

$$\mathcal{H}(S, K, q, k, I, \psi, \varphi) = \\ U(q + k) - cq - bk - f(I) - D(S) + \psi[q - \alpha S] + \varphi[I - \delta K]. \tag{4}$$

Incorporating the capacity constraint, we get the following Lagrangian:[6]

$$\mathcal{L}(S, K, q, k, I, \psi, \varphi, \lambda) = \mathcal{H}(S, K, q, k, I, \psi, \varphi) + \lambda[K - k]. \tag{5}$$

To describe the optimum, we have the first-order complementary slackness conditions with respect to the control variables. In each of the following pairs, one of the equations must hold with equality:

$$q \geq 0, \quad U'(q + k) \leq c + \tau; \tag{6}$$

$$k \geq 0, \quad U'(q + k) \leq b + \lambda; \tag{7}$$

$$I \geq 0, \quad f'(I) \leq \varphi; \tag{8}$$

where $\tau \equiv -\psi$ to express what would be a negative shadow value of pollution as a shadow cost. In view of the Inada conditions on the instantaneous utility function, consumption will be positive at all instants of time. We obtain the remaining necessary conditions for an optimum:

$$\dot{\tau} = (\alpha + \rho)\tau - D'(S), \tag{9}$$

$$\dot{\varphi} = (\delta + \rho)\varphi - \lambda, \tag{10}$$

$$\lambda \geq 0, \quad K \geq k, \quad \lambda[K - k] = 0. \tag{11}$$

This yields the following preliminary results. First, from the maximization of the Hamiltonian, we find that if there is production of the dirty commodity, marginal utility ("price") of the good equals social marginal costs of production, inclusive of the cost of the pollution externality ($c+\tau$). Second, use of the backstop technology requires that marginal utility of the commodity at least equal the direct marginal production costs (b), and the difference between them determines the shadow value of the capacity constraint in that period.

Third, to have positive investment in capacity in any period, (8) says that the marginal cost of installation must equal the shadow value of added capacity. That value is determined by the present value of future capacity constraints to clean production, discounted by the rates of time preference and depreciation. We show that the shadow value of capacity is positive – and thereby so is investment – as long as capacity is constrained at some point now or in the future:

LEMMA 1. *If there exists* t_1 *such that* $I(t_1) > 0$, *then* $I(t) > 0$ *for all* $t < t_1$.

Proof. $I(t_1) > 0$ if and only if $\varphi(t_1) > 0$. If $\varphi(t) \leq 0$ for any $t < t_1$, then according to (10) with $\lambda \geq 0$, $\dot{\varphi}(t+s) < 0$ for all $s > 0$, which contradicts $\varphi(t_1) > 0$. ■

In other words, if one will want to invest and have capacity in the future, one will want to smooth investment costs by installing some capacity all along the way, taking advantage of the low part of the marginal cost curve. The problem at hand is of course interesting only if in the steady state the backstop is actually used (because otherwise we would be back in the case of no backstop), implying positive investments to keep capacity constant. This will happen if the use of the backstop technology is not too costly, and in section 2.1.2, we will make this precise. Then it follows that there will occur positive investments over the entire trajectory.

2.1. STEADY STATE ANALYSIS

In the steady state, the stocks of pollution and clean capacity are constant. Let us define q^* and I^* as the solutions to $\dot{S} = 0$ and $\dot{K} = 0$, respectively:

$$q^* = \alpha S^* \tag{12}$$

$$I^* = \delta K^* \tag{13}$$

where S^* and K^* represent the corresponding solutions to the necessary conditions. Thus, in any steady state with dirty production, emissions just equal the environment's capacity to assimilate and capacity installation just replaces depreciation. Clearly, $I^* > 0$ unless $K^* = 0$, which we show below is not the case.

In the steady state, since K and I are constant, it follows from (8) that φ is constant. With a constant S we can solve the differential equation for τ. If τ goes to infinity, then consumption goes to zero, which is suboptimal. Nor should τ go to $-\infty$. Therefore τ goes to a constant. Thus, from $\dot{\tau} = \dot{\varphi} = 0$, we get

$$\tau^* = \frac{D'(S^*)}{\rho + \alpha}; \tag{14}$$

$$\lambda^* = (\rho + \delta) f'(\delta K^*). \tag{15}$$

Equation (14) states that the steady-state shadow cost of emissions equals the present value of marginal damages, discounted by the rate of time preference and the assimilation rate. Existence of a finite shadow cost and finite pollution stock will be assured by our assumption of a constant α; Toman and Withagen (2000) focus on cases in which the assimilation rate is a function of the pollution stock. They find that if the assimilative capacity of the environment can be depleted ($d\alpha/dS < 0$), an optimal solution involving pollution abatement is not assured to exist or to be unique. We focus instead on the role of the second state variable; Equation (15) reveals that the steady-state shadow value of capacity equals the marginal replacement cost of capital, annualized by the rate of time preference and depreciation. In an equilibrium with both technologies, these values will be linked by the utility of consumption.

2.1.1. *Steady state without backstop*

As a reference case, it is useful to characterize the steady state in the absense of the backstop technology. That is, what is the optimal policy if only dirty production is available, or clean production is prohibitively costly? In fact, we will use this case to define the latter threshold. If the backstop is unavailable or unused, then $K^* = 0$, which implies from (15) no shadow value to capital, $\lambda^* = 0$, and from (7) that $U'(q^*) \leq b$ since $k^* = 0$. In other words, clean production must be too expensive to be useful.

From (14), we see that τ^*, which represents the present value of marginal damages in the steady state, is an upward-sloping function of the pollution stock. From the first-order condition with respect to q we have

$$\tau = U'(\alpha S) - c \tag{16}$$

which is a strictly downward-sloping function of the pollution stock. Thus, these two functions intersect only once and there exists a unique steady state that is asymptotically stable. Define S_D^* as the steady-state stock of pollution in this no-backstop case, satisfying

$$U'(\alpha S_D^*) - c = \frac{D'(S_D^*)}{\alpha + \rho}. \tag{17}$$

The comparative statics are straightforward. We have

$$[D''(S) - \alpha U''(\alpha S)(\alpha + \rho)]dS =$$
$$[U'(\alpha S) - c + U''(\alpha S)(\alpha + \rho)S]d\alpha + [U'(\alpha S) - c]d\rho - [\alpha + \rho]dc. \tag{18}$$

Hence, in view of the properties of the functions involved, higher production costs entail a smaller stock of pollutants in the steady state. A higher rate of time preference leads to greater steady-state pollution. The impact of increasing the assimilation rate is ambiguous, however, since it reduces the marginal damage of current emissions and allows greater consumption, but the marginal utility of that consumption is diminishing. Although more transparent in the single-state problem, these basic results will remain in the two-state problem.

2.1.2. *Steady state with backstop*

We began with the assumption that the clean technology is more costly to use than the dirty one, else itwould be used in the absence of pollution. We now further restrict our consideration to clean production that is also less costly than the social marginal cost of only dirty production. Thus, we assume

$$c < b < c + \tau_D^*. \tag{A2}$$

The interpretation is straightforward. The right-hand side gives the marginal costs of an additional unit of production of the dirty commodity, namely the direct production costs plus the marginal pollution costs, in the absence of a backstop. If the marginal cost of producing an additional unit of the consumption commodity with the clean technology were never less, it would not be optimal to develop clean production. The exception would be when the initial stock of pollutants is higher than S_D^*, in which case the use of clean technologies could be cost-effective during the transition to the steady state. This possibility is discussed later.

We state the following lemmata to characterize the steady state with the clean backstop. First, with $\alpha > 0$, some dirty production must exist in the steady state.

LEMMA 2. $q^* > 0$.

Proof. Suppose $q^* = 0$. From (6), then $U'(k^*) \leq c + \tau^*$. From(12), $S^* = 0$. This implies that $\tau^* = 0$, so that $U'(k^*) \leq c$. But from (7), $U'(k^*) \geq b > c$. ■

In other words, if dirty production ceased, assimilation would eventually cause the environment to become completely clean. In that case, society could save costs by switching some production to the dirty technology, as the burden imposed by the extra emissions would be less than the savings in operating costs.

Next, some clean capacity will exist in the steady state, and all of it will be utilized:

LEMMA 3. $K^* > 0$ *and* $k^* = K^*$.

Proof. Suppose $K^* = 0$. Then $k^* = 0$. We have $q^* = \alpha S^* > 0$. Then from (7) and and (15), $U'(\alpha S^*) \leq b$. Moreover, (6) implies that $U'(\alpha S^*) = c + \tau^*$, with $\tau^* = D'(S^*)/(\rho + \alpha)$. Hence $\tau^* = \tau_D^*$, but by (A2) $b < c + \tau_D^*$, a contradiction. Suppose $k^* < K^*$. Then $\lambda^* = 0 = \varphi^* = K^*$, which would violate $K^* > 0$. ■

Thus, an immediate consequence of having capacity is that no excess capacity exists in the steady state. Otherwise, one could save on installation costs by investing less.

Now we can compare the steady states in the respective cases by means of a graph. From the first-order conditions for both production types, we get

$$U'(K^* + \alpha S^*) = c + \frac{D'(S^*)}{\rho + \alpha}; \tag{19}$$

$$U'(K^* + \alpha S^*) = b + (\rho + \delta) f'(\delta K^*). \tag{20}$$

Let us consider these equations in detail, omitting * when there is no danger of confusion.

Consider (19) first. If $S = 0$, then K satisfies $U'(K) = c$ (because $D'(0) = 0$). If $K = 0$, then S follows from $U'(\alpha S) = c + D'(S)/(\rho + \alpha)$. The locus of intermediate points is shown in Figure 2 as the curve "q f.o.c."

Consider (20). If $S = 0$, then K satisfies $U'(K) = b + (\rho + \delta) f'(\delta K) > c$. If $K = 0$, then S follows from $U'(\alpha S) = b$. The locus of corresponding intermediate points is shown in Figure 2 as the curve "k f.o.c."

Deriving the respective slopes of these curves, we can see that "q f.o.c." is steeper than "k f.o.c.":

$$\left.\frac{dK}{dS}\right|_{\frac{\partial H}{\partial q}=0} = \frac{D''/(\rho + \alpha) - \alpha U''}{U''}, \tag{21}$$

$$\left.\frac{dK}{dS}\right|_{\frac{\partial H}{\partial k}=0} = \frac{-\alpha U''}{U'' - (\rho + \delta)\delta f''}. \tag{22}$$

Note that $D''/(\rho + \alpha) - \alpha U'' > -\alpha U''$ and $U'' - (\rho + \delta)\delta f'' < U'' < 0$.

Thus, the two curves (drawn in Figure 1 as lines) cross, and cross only once. A steady state exists, and it is unique and (at least locally) asymptotically stable.

As noted above, with $c + \tau_D^* > b$, some clean production exists in the steady state. In terms of the figure, if that condition did not hold, the "k f.o.c." line would lie wholly inside "q f.o.c." We also assume that $c < b$, so some dirty production must also exist. Otherwise, "q f.o.c." line would lie wholly inside "k f.o.c." and no intersection would occur.

2.2. CHARACTERIZATION OF OPTIMAL PATHS

From the optimality conditions, we can characterize the paths to the steady state. Let us define two particular steady-state values of capacity which will establish some bounds for the optimal path. First, let $\bar{K}$ satisfy $U'(\bar{K}) = b + (\rho + \delta) f'(\delta \bar{K})$, which is the steady-state value of capacity when the economy only has the clean

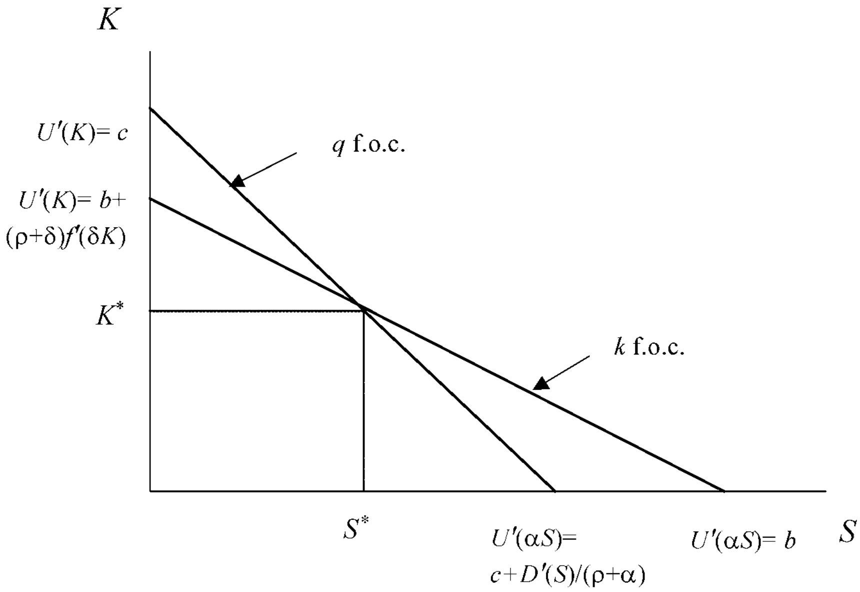

Figure 1. Steady state with backstop.

technology at its disposal. Hence, $\bar{K}$ is an upper bound on installed backstop capacity, also in the case where dirty technology is available. Second, we define $(\hat{K}, \hat{S})$ by $U'(\hat{K} + \alpha\hat{S}) = b$, $D'(\hat{S})/(\rho + \alpha) = b - c$. These represent the steady-state stock values if there were no investment costs ($f = 0$) or no depreciation ($\delta = 0$).

In principle, there are eight possible regimes, as listed in this table.

Regime	K	k	q
I	k	0	0
II	k	0	> 0
III	k	> 0	0
IV	k	> 0	> 0
V	$> k$	0	0
VI	$> k$	0	> 0
VII	$> k$	> 0	0
VIII	$> k$	> 0	> 0

However, only a few regimes are possible for an optimal path. We can quickly eliminate regimes I, II, V, VII, and VIII:

Regimes I and V imply no production at all, which cannot occur along an optimum, since $U'(0) = \infty$. Regime II has only dirty production and no capacity building – it cannot occur along an optimal path that ultimately involves clean production, as it would violate Lemma 1, that some investment smoothing always occurs. Regime VII has excess capacity with no dirty production – it implies that capacity was created that would never be used, which is clearly suboptimal. Formally, if regime VII prevails at time t, then $U'(k(t)) = b$ (since $\lambda(t) = 0$) and $U'(K(t)) < b$. There occurs time s such that $k(s) = K(t)$, because the capacity that is built up must be used at some instant of time. Hence, $b + \lambda(s) = U'(k(s) + q(s)) = U'(K(t) + q(s)) \leq U'(K(t)) < b$, contradicting that $\lambda(s) \geq 0$.

In regime VIII, which has dual production and excess capacity, we have $\lambda = 0$ and $U'(q + k) = b = c + \tau$; hence, $\dot{\tau} = 0$ and $S = \hat{S}$, which implies $k = \hat{K}$. A sufficient condition to exclude this regime is

$$\bar{K} < \hat{K}, \tag{A3}$$

which we thus assume for analytical convenience.[7] Regime VIII would then violate the upper bound on installed capacity.

The analysis thus leads to the conclusion that there are only three regimes to be considered as candidates for an optimal path:

Regime III has only clean production occurring at capacity. It may be possible when starting with a dirty environment, as discussed in section 3.2.

Regime IV has both dirty production and clean production at capacity.

Regime VI has only dirty production but clean capacity available. As shown in section 3.1, this regime can occur when starting from a relatively clean environment, smoothing installation costs in anticipation of needing clean production.

Moreover some transitions between III, IV, and VI can be ruled out. The following lemma will prove useful for the subsequent proposition:

LEMMA 4. *$\tau(t) < b - c$ implies $\dot{\tau}(t) > 0$.*

Proof. From (7), $\tau(t) < b - c$ implies $k(t) = 0$. Hence the system is governed by (6), (9) and (1). If $\tau(t) < \tau_D^*$, as in assumption 2, and $\dot{\tau}(t) < 0$, the system will remain in the region with $\tau(t) < b - c$ forever, which, in the case at hand implies that the backstop will never be used, a contradiction. ■

PROPOSITION 1. *There is no direct transition possible from regime III to regime VI in an optimum.*

Proof. In regime III we have $U'(K) \leq c + \tau$ and $U'(K) = b + \lambda$, implying $\tau \geq b - c$. In regime VI we have $U'(q) = c + \tau$ and $U'(q) \leq b$, implying $\tau \leq b - c$. Suppose there is a transition from regime III to regime VI at time t. Then, since $\dot{\tau} \geq 0$ as long as $\tau \leq b - c$ (lemma 4), we have $\tau = b - c < \tau_D^*$

in view of assumption 2. So, $U'(\alpha S) = \tau + c < \tau_D^* + c$ implying $S > S_D^*$ and $D'(S)/(\alpha + \rho) = b - c < \tau_D^*$ implying $S < S_D^*$, a contradiction. ■

PROPOSITION 2. *There is no direct transition possible from regime VI to regime III in an optimum.*

Proof. Suppose there is a transition from regime VI to regime III at time t. Then, since consumption is continuous over time, $U'(K(t)) = U'(q(t)) = b$. But this implies $K(t) > \bar{K}$, violating the definition of $\bar{K}$ as the upper bound on installed capacity. ■

With those regimes and propositions established, we can describe the equilibrium path. Suppose the initial stock of pollutants is small. Then it is optimal to have an initial interval of time when the backstop technology is put in place but not actually used (regime VI). All consumption comes from the conventional technology. After some time a transition is made to the simultaneous use of both technologies (regime IV).

Suppose the initial stock of pollutants is huge. Then there will be an initial, short, interval of time where there is simultaneous use of both technologies. This is the case because it has been assumed that initially there is no backstop technology installed and $U'(0) = \infty$, so some consumption from the old technology is needed at the outset. In the second stage all production may be carried out in the backstop (regime III). This cannot go on forever, and eventually there will be simultaneous use of both technologies (regime IV).

3. Optimal Trajectories

In this section we describe the optimal trajectories under different circumstances. It turns out that it is important to make a distinction between small and high initial stocks of pollutants.

3.1. SMALL INITIAL STOCK OF POLLUTANTS

The optimal path can be characterized by several phases, described below.[8]

3.1.1. *Dirty production, rising pollution*

If the initial stock of pollutants is small, the shadow price of pollution is also small, and it will not be optimal to use the backstop technology immediately. During this initial period, while $\tau < b - c$, we have regime VI: there is only dirty production and τ rises along with the pollution stock. Any existing capacity will not be utilized since $U'(q) = c + \tau < b$. However, this does not imply that there is no installation of clean production capacity. According to Lemma 1, the existence of *any* positive level of investment at *any* time in the future along the optimal path will trigger some current investment. One always wants to take advantage of negligible costs at low levels of capacity installation.

3.1.2. *Dual production, rising pollution*

The first phase is followed by a period in which there is dual production, characterized by regime IV. After $\tau = b - c$, the existing backstop technology comes online and is used at capacity. At the start of this period, there is a downward jump in dirty production. The stock of pollution will continue to increase in the beginning of this period, but at a slower rate than before. The shadow price of pollution continues to rise, and capacity expansion also continues.

3.1.3. *Dual production, overshooting*

For relatively low rates of capacity depreciation, the shadow value of pollution will actually overshoot its steady-state value. In other words, the environment may have to grow dirtier before it can become cleaner. We develop this intuition by momentarily relaxing the assumption that $\delta > 0$.[9]

LEMMA 5. *If $\delta = 0$, there exists $t_1 > 0$, such that $\tau(t_1) > \tau^*$.*

Proof. If $\delta = 0$, it follows from (19) and (20) that $\tau^* = b - c$. Suppose that $\tau(t) \leq b - c$ for all t. Then $\lambda(t) \leq 0$ for all t. Obviously, $\lambda < 0$ is impossible, and $\lambda = 0$ for all t implies that $\varphi = 0$ (else $\dot{\varphi} = \rho\varphi$ from (10) implies that $\varphi \to \infty$, which is a contradiction). If $\varphi = 0$ for all t then $I = 0$ and $K = 0$ for all time and all production is dirty. However, with only dirty production, the steady state is $\tau_D^* > b - c$ (by A2), which contradicts the supposition. ■

The economic meaning is that the socially optimal price of the dirty good must at some point rise higher than the variable cost of clean technology (b) if installation of clean capacity is ever to be worthwhile. It is that excess price differential that generates the investment rents and the positive shadow value to investment. Lemma 5 shows that in the special case of zero backstop capital depreciation, generating those rents necessitates an overshooting of the steady-state value of the consumption commodity. By continuous dependence on parameters, overshooting also will occur with positive but moderate depreciation rates.

With higher depreciation rates, however, the shadow price may not necessarily overshoot the steady state value, since τ^* itself is well above $b - c$ and high steady-state investment is needed to maintain capacity. This potential case is portrayed in the Figure 3.

It is instructive to go into the economic intuition behind overshooting. The question of overshooting is whether the consumption commodity price would reach the steady-state level before enough time has passed to get sufficient capacity in place. This result is obvious when the initial pollution stock is high enough. If $S_0 = S^*$ and $K_0 = 0$, then it must be that $\tau(0) > \tau^*$, since the dirty output

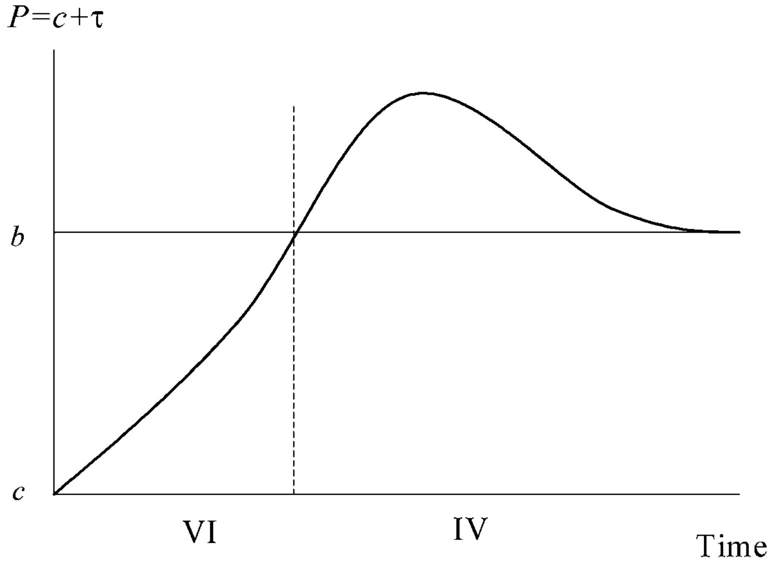

Figure 2. Price path with no depreciation.

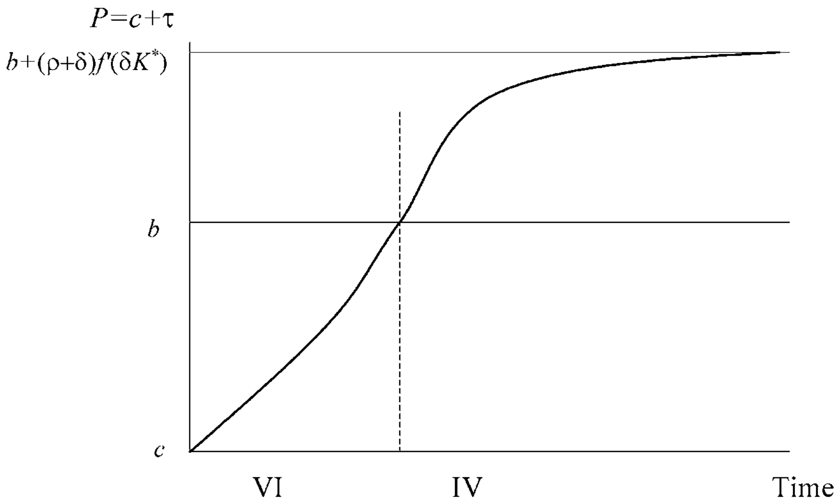

Figure 3. Price path with high depreciation.

level that satisfies consumption at $U'(q) = c + \tau^*$ is greater than q^*, which would drive up the pollution stock. Similarly, for somewhat lower initial pollution stocks, overshooting must also occur.

However, even with a very clean start, overshooting is still possible. Consider the case of a low capital depreciation rate, which also implies a low τ^*. If τ did not rise above τ^*, that would also imply a path of very low shadow values of investment. Still, one could take advantage of low initial installation costs and let capacity build up very slowly over a nearly infinite horizon. However, the fact that the clean option will be there at some point creates incentives to build up pollution more quickly than in the no-backstop case. Let the subscript D denote the path in the case of only dirty production. Since $S_D^* > S^*$, then $S_D(t_d) = S^*$ at some finite t_d. Since $\tau(t) \leq \tau_D(t)$ for all t, $C(t) \geq C_D(t)$ for all t. Let t_c be the point at which clean capacity becomes utilized: $\tau(t_c) = b - c$. Since $C(t) = q(t)$ for all $t < t_c$, then $S(t) \geq S_D(t)$ for all $t < t_c$. Since $S_D(t_c) < S_D(t_d)$, then $t_c < t_d$, and that point is reached in finite time along the optimal path. However, that implies that $K(t_c)$ would still be very small, and negligible clean capacity means greater dirty production continues. If $K(t) < C(t) - CD(t)$ for $t_c < t < t_d$, then it also follows that $S(t_d) > S_D(t_d) = S^*$, which implies overshooting. When τ^* is high, it is more likely that $K(t_c) > C(t_c) - C_D(t_c)$, since more incentive to create capacity is in place, and the price differential between the backstop and no-backstop cases is smaller; then it is possible that the displacement of dirty production at that point is sufficient to stem the growth in pollution and allow for a transition to the steady state without overshooting. However, when τ^* is low, it is more likely that insufficient clean capacity is available compared to demand early on, and pollution is allowed to expand beyond the steady state level, expecting that more dirty production will be displaced later on in the time horizon.

We have here a joint implication, that for the shadow value of pollution to be higher at its peak than in the steady state, it must be that the stock of pollution is greater than the steady-state level at some point between there and the steady state.

PROPOSITION 3. *S overshoots S^* if τ overshoots τ^*, and vice versa.*

Proof. If $\tau(t)$ peaks at $t = t_1$, then $\dot{\tau}(t_1) = 0$. From (9), $\tau(t_1) = D'(S(t_1))/(\alpha + \rho)$. Since $\tau(t_1) > \tau^*$, then $S(t_1) > S^*$: the pollution stock overshoots the steady if its shadow cost does. To see the reverse, suppose that $S(0) < S^*$, $S(t) \leq S^*$ for $0 \leq t < t_2$, and $S(t) > S^*$ for $t_2 \leq t \leq t_3$ for some $t_3 > t_2$. Suppose also that $\tau(t) \leq \tau^*$ for all t. First, we show that this latter assumption implies that clean production is always less than the steady state value. In view of $f'(0) = 0$, $K(t) > 0$ for $t > 0$. If $K(t) > k(t)$, then $\lambda(t) = 0 < \lambda^*$. If $K(t) = K(t)$, then from (6) and (7), $\lambda(t) = c - b + \tau(t) \leq c - b + \tau^*$. Thus, $\lambda(t) \leq \lambda^*$ for all $t \geq 0$. Consider (10); if $\varphi(t) > \varphi^*$ for some $t \geq 0$, then $\varphi(t) \to \infty$ as $t \to \infty$, which is not optimal. Hence, $\varphi(t) \leq \varphi^*$ for all $t \geq 0$, and from (2) and (8), $K(t) \leq K^*$ for all $t \geq 0$. Next, we show that this implies that the pollution stock would grow indefinitely if it overshoots. Since $K(t) \leq K^*$ and $\tau(t) \leq \tau^*$ for all $t \geq 0$, we have $q(t) \geq q^*$ for all $t \geq 0$. This implies that $S(t) \geq S^*$ for all $t \geq t_2$. Moreover, $S(t) > S^*$ for $t_2 \leq t \leq t_3$ for some $t_3 > t_2$ by assumption. It then follows from (9) that $\tau(t)$ becomes negative eventually, which contradicts optimality. ■

In other words, shadow price overshooting goes hand in hand with pollution overshooting. However, we will see next that the timing is not simultaneous.

3.1.4. *Dual production, convergence*

In a final phase the shadow price of pollution starts converging to the steady state. If clean capacity depreciates rapidly enough, the pollution stock may rise monotonically along with the shadow price of pollution. If τ overshoots its steady-state value, it must necessarily hit a peak and then decline as it converges. In this case, the pollution stock must also peak and then decline to its steady-state level.

LEMMA 6. *If $\tau(t)$ peaks at $t = t_1$, then $S(t)$ peaks at $t = t_2 > t_1$.*

Proof. Consider any pairs of points $t_x < t_1 < t_y$ such that $\tau(t_x) = \tau(t_y)$. Since t_x is on the rise to the peak and t_y is on the decline to the steady state, $\dot{\tau}(t_x) > 0$ and $\dot{\tau}(t_y) < 0$. Thus, from (9), $D'(S(t_x)) < D'(S(t_y))$, implying $S(t_x) < S(t_y)$. Taking an infinitely small interval ε, since $S(t_1 - \varepsilon) >> S(t_1 + \varepsilon)$, it must be that S continues to rise after t_1. Since we know it must peak in order to decline to the steady-state stock, we get $t_1 < t_2 < \infty$. ■

In other words, the time before τ peaks is closer to a period of higher pollution stocks, and the time after τ peaks is closer to the steady state of lower pollution stocks. Thus, to have the present discounted value of marginal pollution damages equal at those two points before and after, it must be that the pollution stock is higher at the point after. Correspondingly, the shadow value of pollution begins to decline before the actual pollution stock does, when overshooting occurs.

We also note that convergence to the steady state need not be monotonic. Considering the eigenvalues of the Jacobian, evaluated in the steady state, we find that they can be complex for some parameter values. Consequently, with two eigenvalues having negative real part, this implies a (locally) stable cycle towards the steady state. This phenomenon is interesting because it implies overshooting and undershooting of capacity as well as pollution in the neighborhood of the steady state. The economic intuition is that when pollution is at the steady state value, whereas clean capacity is below its steady state value, it pays to increase pollution and at the same time increase capacity. Once capacity is at the steady state level and pollution is still high, capacity is built up further and pollution can be reduced. But then when pollution is at its steady state value again and capacity is high, capacity as well as pollution can be decreased. The analysis of this section, however, has focused mainly on behavior not too close to the steady state; furthermore, we call attention to how the paths of capacity investment and pollution damages differ depending on whether the initial pollution stock is relatively high or low.

3.2. LARGE INITIAL STOCK OF POLLUTION

Suppose now that one inherits a pollution stock that is relatively large, such that $S(0) > S^*$ along an optimal path. For example, one may begin from a no-policy steady state, S_N, such that $U'(\alpha S_N) = c$. Then one starts to implement both the environmental pricing policy and the capacity investments simultaneously, as the environment had become far too dirty in the absence of intervention. Although the steady state does not vary according to the initial pollution stock, the transition can look quite different.

Starting with a large pollution stock, we have a sequence of several possible stages: dual production and rapid capacity expansion, clean production alone, dual production with capacity overshooting, and convergence.

3.2.1. *Dual production, rapid investment*

With a very dirty initial environment, $\tau(0) > \tau_D^* > b - c$, clean capacity and production are immediately justified. First, there will be an initial period of simultaneous clean and dirty production (regime IV), with much of the weight of environmental recovery carried by curtailment of total (mostly dirty) consumption as its social cost is internalized. Along the path, then, we will see reduction in pollution and shadow price and a rapid buildup in clean capacity. Until (and unless) sufficient capacity exists to displace dirty production, we start and remain in this regime.

3.2.2. *Clean production*

Once enough capacity is built, the economy may pass into a regime III path ($k = K > 0 = q$), relying on clean production to allow environmental recovery. During this time, $c + \tau(t) > U'(K(t))$, and the latter is bounded by $U'(\bar{K})$ at the lowest. Passing through this regime depends on the extent of initial pollution and how slowly the environment recovers compared with how quickly capacity can be built.

3.2.3. *Capacity overshooting and convergence*

Clean capacity may increase monotonically to the steady state, its growth slowed as the environment recovers and some dirty production is reintroduced. But it is also possible that clean capacity will overshoot the steady state. The argument extends from the case in which $c + \tau_D^* = b$, such that dirty production just satisfies demand at the steady state with social costs internalized. Starting from a relatively clean environment, no clean capacity would be built. With a dirty environment, however, $c + \tau(0) > b$, signaling for clean capacity to be built. Later it is depreciated away as the economy converges toward the steady state. Similarly, for other cases with small K^* is but large S_0, it will be important to clean up as soon as possible and we will have some $K(t) > K^*$.

Capacity overshooting is quite possible with either high or low depreciation rates. Low depreciation rates mean that capacity that was installed to speed environmental recovery lingers afterward, allowing for net disinvestment when pollution is sufficiently assimilated. High depreciation rates make capacity installation more like a variable cost of clean production. Consider the extreme case of a very high δ. Starting from S_N, capacity is immediately put in place to displace some or all of the dirty production. That in turn causes the pollution stock to decline,

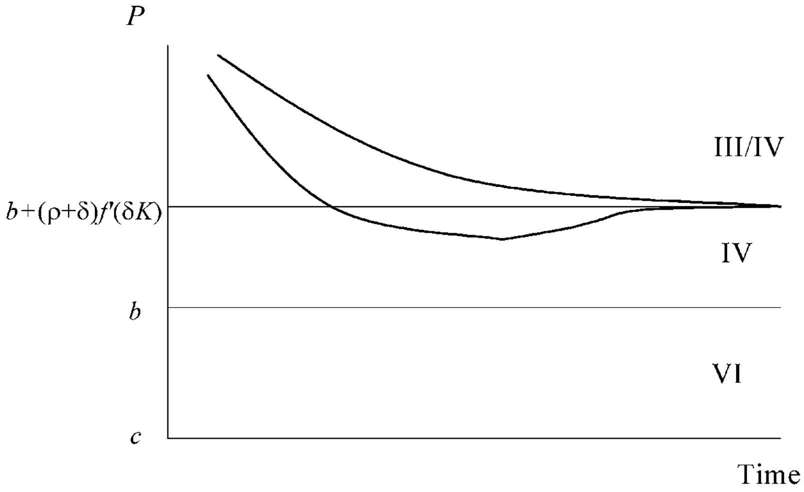

Figure 4. Optimal paths with large S_0.

which induces installation (and thereby capacity) to decline monotonically toward the steady state. Thus, for high rates of depreciation, the path may follow regime IV throughout or pass from III to IV.

It may be possible for pollution to overshoot the steady state as well. We speculate that pollution undershooting could occur in the low depreciation case: clean capacity that was installed when the environment was very dirty lingers a long time, continuing to displace dirty production even once marginal damages are smaller. With $k > k^*$ and declining slowly, we have $q < \alpha S^*$ for longer, possibly long enough to allow τ (and implicitly, but not simultaneously, S) to fall below the steady-state value. With net disinvestment of clean capacity – although some investment always occurs if capacity will be used in the future, it may not replace depreciation – dirty production rises again, converging to the steady state.

Figure 4 portrays possible paths when starting from a dirty pollution stock. As previously noted, a stable cycle to the steady state is also possible.

4. Conclusion

In this paper we have developed as simple a model as possible that still captures the dynamics of pollution accumulation or decay and capital accumulation for clean technology. We look mainly at the case in which both clean and dirty production coexist in the socially optimal steady state. If the clean technology has a dominant cost advantage over the dirty technology and one's pollution damage costs are internalized, then of course the clean technology will displace the dirty technology as soon as capacity can be accumulated (leaving aside other possible market failures related to technology diffusion that are not addressed in this paper). And if utilizing the clean technology makes economic sense only in a highly polluted environment, it naturally follows that this technology will not have a future once the environment recovers.

Although the steady state in our model is invariant with respect to initial conditions (given depreciable capacity and pollution assimilation), the transition paths are very different. The initial state of the environment affects the urgency of action, interacting with the lifespan of capital and creating both immediate and enduring effects on the tradeoffs between environmental protection through curtailed consumption and capacity expansion. As a result, the optimal path may involve over- or undershooting the long-run targets for managing a stock pollutant.

With a clean initial environment, clean capacity is built up gradually, and it is possible that the environmental shadow price and pollution stock will overshoot long-run levels. With low rates of depreciation, the long life of installed capacity leads to more clean production being maintained in equilibrium. Recognizing that marginal damages of pollution will ultimately be lower, society is in less of a hurry to install clean capacity and abate emissions. In fact, the optimal strategy is more likely to be to allow the environment to get dirtier before it gets cleaner, saving investment costs by smoothing capacity expansion, and reducing and displacing dirty production more gradually. On the other hand, if capital depreciates more quickly, less clean capacity will be available in equilibrium. Consequently, marginal damages will be higher, causing the optimal near-term policy to be more aggressive, cutting back on dirty production and building up capacity more quickly, with no need for overshooting.

With a dirty environment, clean capacity will be built up rapidly since it is worth utilizing as soon as the environmental policy begins, while dirty production is immediately curtailed. However, since the immediate needs are greater than the steady-state capacity needs, the clean technology may overshoot its long-run level. If capacity is enduring, it remains long after the initial period of austerity, perhaps allowing the environment to become cleaner for a while, until the extra capacity depreciates and is replaced again with more dirty production.

Dynamic optimization models with two or more state variables always are somewhat technically vexing, and ours is no exception. Although the necessary simplifications leave something to be desired, it is interesting to note that unusual path dynamics can be generated using simple welfare specifications and well-behaved functions. The results can offer some intuition about incorporating more complicated interactions. For example, if dirty production has a capacity constraint as well, those additional incentives would resemble the clean capacity incentives described here, and the path to the steady state would further depend on the initial capacity and lifespan of the polluting technology. Positive marginal investment costs would imply that invesment might not be continuous – the social cost of dirty production would have to pass a positive hurdle price before capacity would begin to be installed. Allowing for production costs to change over time through technological progress would raise other interesting issues – the expectation of lower costs of clean production in the future would tend to exacerbate the overshooting results, unless perhaps progress is highly endogenous, as with learning by doing, which changes the optimal timing of action. Extensions of our results could be obtained by relaxing the simplifying assumptions and using numerical simulations (parameterized to actual industries) to study the resulting investment dynamics.

Acknowledgement

This research benefited from support by the National Science Foundation: 9613035; such support does not imply agreement with the views expressed in the paper.

Notes

1. Effectively, we assume that capacity investment for dirty technology is either unnecessary, costless, or does not depreciate and is sufficiently installed that dirty production is unconstrained when the policy begins. The problem is already quite complex with two state variables. However, intuition about the effects of capacity investment for dirty production can still be drawn from that of the clean capacity investment dynamics.
2. We could assume an emissions rate other than 1, but effectively we are normalizing the pollution stock by the emissions rate.
3. This assumption is made largely for simplicity; we expect our results would apply more generally to assimilation functions $\alpha(S)$ satisfying $\alpha'(S) \geq 0$. The case of $\alpha'(S) < 0$ is more complex, as shown in Toman and Withagen (2000).
4. Equivalently, we could assume that capacity installation requires some investment in terms of the consumption commodity rather than a labor cost. However, this presentation offers a bit more transparency, since we will effectively be normalizing to the marginal utility of leisure.
5. In other words, capacity cannot be converted into consumption, even a cost. Furthermore, we will show in Lemma 1 that investment is always positive along the optimal path, to take advantage of the low marginal costs.
6. For optimal control problems with inequality constraints, see Léonard and Long (1992).
7. Regime VIII could theoretically occur when starting from a dirty environment, when clean capacity needed early on takes longer to depreciate than pollution takes to assimilate. Exploring this possibility further will be the subject of future research. Although such severe undershooting is unlikely, this assumption effectively restricts our consideration to cases in which it is impossible.
8. This case of a small initial stock of pollutants is similar to the optimal overshooting model of Switzer and Salant (1986). Their problem of optimal installation of backstop capacity for an exhaustible resource is analogous to a scenario without decay of the pollution stock or depreciation of the backstop. We discuss a more general case.
9. This also effectively relaxes the assumption A3, but regime VIII is not relevant for this case of starting with a relatively small stock of pollution.

References

Feichtinger, G., A. Novak and F. Wirl (1994), ‘Limit Cycles in Intertemporal Adjustment Models’, *Journal of Economic Dynamics and Control* **18**, 353–380.

Fisher, A. and U. Narain (2002), ‘Global Warming, Endogenous Risk and Irreversibility’, *Environmental and Resource Economics* **25**(4), 395–416.

Kolstad, C.D. (1996), 'Fundamental Irreversibilities in Stock Externalities', *Journal of Public Economics* **60**(2) (May 1996), 221–233.

Krautkraemer, J.A. (1985), 'Optimal Growth, Resource Amenities and the Preservation of Natural Environments', *Review of Economic Studies* **LII**, 153–170.

Léonard, D. and N.V. Long (1992), *Optimal Control Theory and Static Optimization in Economics*. Cambridge, UK: Cambridge University Press.

Michel, P. and G. Rotillon (1995), 'Disutility of Pollution and Endogenous Growth', *Environmental and Resource Economics* **6**(3) (October 1995), 279–300.

Oren, S. and S. Powell (1985), 'Optimal Supply of a Depletable Resource with a Backstop Technology: Heal's Theorem Revisited', *Operations Research* **33**(2) (March–April).

Switzer, S. and S. Salant (1986), 'Optimal Capacity Expansion by Foresighted Extractors and Producers' ('Expansion optimale de capacité par des exploitants prévoyants d'une ressource renouvelable'), in G. Gaudet and P. Laserre, eds., *Ressources naturelles et théorie economique*. Laval, Québec: University of Laval Press.

Toman, M.A. and C. Withagen (2000), 'Accumulative Pollution, "Clean Technology" and Policy Design', *Resource and Energy Economics* **22**, 367–384.

Ulph, A. and D. Ulph (1997), 'Global Warming, Irreversibility and Learning', *Economic Journal* **107**(442) (May 1997), 636–650.

Wirl, F. and C. Withagen (2000), 'Complexities Due to Sluggish Expansion of Backstop Technologies', *Journal of Economics* **72**, 153–174.