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ON THE SUSTAINABLE PROGRAM IN SOLOW'S MODEL

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ABSTRACT. We show that our general result (Withagen and Asheim [1998]) on the converse of Hartwick's rule also applies for the special case of Solow's model with one capital good and one exhaustible resource. Hence, the criticism by Cairns and Yang [2000] of our paper is unfounded.

KEY WORDS: Sustainability, maximin path, Hartwick's rule.

1. Introduction. What characterizes a maximin path in a capital-resource model with one consumption good? The converse of Hartwick's rule answers this question in the following manner: A necessary condition for an efficient constant consumption path is that the revenues from resource depletion are used for the accumulation of man-made capital. In a more general setting it amounts to the result that a necessary condition for an efficient constant utility path is that the value of net investments is equal to zero at all times. The necessity of Hartwick's rule has been addressed earlier by Dixit et al. [1980], Withagen and Asheim [1998] and Mitra [2002], in a rather general setting. Cairns and Yang [2000] concentrate on Solow's model (cf. [1974]) which describes a two-sector economy with one sector exploiting a natural nonrenewable resource and the other one using the raw material from that resource, together with capital, to produce a commodity that can be consumed and invested.

In reference to our paper, Cairns and Yang argue that we "explicitly posit positive utility-discount functions. Discounting utility in this context is contrived and inconsistent with the motivation of sustainability analysis." They thus suggest that Solow's model—which is the basic model in which Hartwick's rule for sustainability was originally derived—falls outside the realm for the main result in Withagen and Asheim [1998]. This view, however, is based on a misunderstanding that stems from confounding discounted utilitarianism as a primary ethical objective with having supporting utility or consumption discount rates in a model where intergenerational equity is the objective.[1]

The main result in Withagen and Asheim [1998] states that, under certain conditions, Hartwick's rule is necessary for sustainability. In the present note we establish in detail how the main result in Withagen and Asheim [1998] (here reproduced as Proposition 1 in the current one-consumption good setting) can be used to obtain the converse of Hartwick's rule in Solow's model. Thereby we show that the criticism of Cairns and Yang is unfounded.

Proposition 1 states that if a constant consumption path maximizes the sum of discounted consumption for some path of supporting consumption discount factors, then the value of net investments is equal to zero at all times. We here supplement Proposition 1 by showing that any maximin path in Solow's model has constant consumption and maximizes the sum of discounted consumption for some path of supporting discount factors. This means that the premise of our general result on the converse of Hartwick's rule is satisfied in the case of Solow's model.

We start in Section 2 by giving a formal presentation of Solow's model, defining the concept of a maximin path, and reproducing Withagen and Asheim's [1998] result as Proposition 1 in the context of Solow's model. We then in Section 3 show that (a) the premise of Proposition 1 is satisfied for any maximin program that is interior and regular, and

(b) that any maximin program in Solow's model indeed is interior and regular provided that the infimum of consumption along the maximin program is positive. We conclude in Section 4 by proving our main results and commenting on Cairns and Yang's analysis.

2. The model. The Solow model describes a two sector economy. One sector exploits a nonrenewable resource, the size of which at time t is denoted by $s(t)$. The initial stock is given and denoted by s_0. The raw material (r) from the resource is used as an input in the other sector, together with capital (k). The production function in this sector is denoted by f. Output is used for consumption (c) and net investments (i). The initial capital stock is k_0. There is no depreciation. We follow Cairns and Yang [2000] in their assumptions concerning the production function.

Assumption 1. *The production function f is concave, nondecreasing and continuous for nonnegative inputs, and it is increasing and twice differentiable for inputs in the interior of the positive orthant. Both inputs are necessary. Finally, denoting partial derivatives by subscripts, $f_k(\infty, r) = 0$ for $r \geq 0$ and $f_r(k, 0) = \infty$ for $k > 0$.*

A quintuple (c, i, r, k, s) is said to be *attainable* if

$$c \geq 0, \quad i \leq f(k,r) - c, \quad r \geq 0, \quad k \geq 0, \quad s \geq 0.$$

A program $\{c(t), i(t), r(t), k(t), s(t)\}_{t=0}^{\infty}$ is said to be *feasible* if, for all t, $(c(t), i(t), r(t), k(t), s(t))$ is attainable and

$$\dot{k}(t) = i(t), \quad \dot{s}(t) = -r(t),$$
$$k(0) = k_0 > 0, \quad s(0) = s_0 > 0.$$

A feasible program is said to be *interior* if, for all t, the quintuple is in the interior of the positive orthant. A feasible program is said to be *efficient* if there is no feasible program with at least as much consumption everywhere and larger consumption on a subset of the time interval with positive measure. A feasible program $\{c(t), i(t), r(t), k(t), s(t)\}_{t=0}^{\infty}$ is said to be *maximin* if $\inf_t c(t) \geq \inf_t \bar{c}(t)$ for all feasible programs $\{\bar{c}(t), \bar{i}(t), \bar{r}(t), \bar{k}(t), \bar{s}(t)\}_{t=0}^{\infty}$.

In Solow's model, it may not be possible to maintain consumption above a positive lower bound forever, even if the initial stocks are positive. Here we simply assume the existence of a maximin program that sustains positive consumption, and refer to Cass and Mitra [1991] for a discussion of sufficient and necessary conditions in terms of the underlying technology.

Assumption 2. *There is a maximin program* $\{c(t), i(t), r(t), k(t), s(t)\}_{t=0}^{\infty}$ *with* $\inf_t c(t) = c^* > 0$.

It follows that any maximin program satisfies $\inf_t c(t) = c^* > 0$.

We end this section by stating our general result on the converse of Hartwick's rule in the setting of Solow's model.

Proposition 1 (Withagen and Asheim [1998, Proposition 2]). *Assume that there are positive consumption discount factors* $\{\pi(t)\}_{t=0}^{\infty}$ *such that maintaining consumption constant and equal to* c^* *forever maximizes* $\int_0^{\infty} \pi(t)c(t)\,dt$ *over all feasible paths, that the maximum principle holds for the corresponding infinite horizon optimal control problem, and that the path of corresponding costate variables* $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$ *is unique. Then, for all* t, $\lambda(t)i(t) = \mu(t)r(t)$.

This reformulation of the main result in Withagen and Asheim [1998] shows that an important step in the following analysis will be to find consumption discount rates for which a maximum program can be implemented as a discounted utilitarian optimum. We will now show how this can be done.

3. Main results. In the current section we use the concept of a 'regular maximin program' to show that our general result in Withagen and Asheim [1998] (restated as Proposition 1 above) on the converse of Hartwick's rule can be applied to demonstrate that along any maximin path in Solow's model the revenues from resource depletion are used for accumulation of man-made capital. Since the concept of a 'regular maximin program' requires the concept of a 'competitive program', we start by introducing the latter.

A feasible program $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is said to be *competitive* at positive consumption discount factors $\{\pi(t)\}_{t=0}^{\infty}$ and nonnegative competitive prices $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$ if, for all t,

$$\begin{aligned}(1)\quad \pi(t)c^*(t) + \lambda(t)i^*(t) - \mu(t)r^*(t) + \dot{\lambda}(t)k^*(t) + \dot{\mu}(t)s^*(t)\\ \geq \pi(t)c + \lambda(t)i - \mu(t)r + \dot{\lambda}(t)k + \dot{\mu}(t)s\end{aligned}$$

for all attainable quintuples (c, i, r, k, s). A program $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ that is competitive at $\{\pi(t)\}_{t=0}^{\infty}$ and $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$ is said to be a *regular maximin path* (cf. Dixit et al. [1980]) if

$$(2)\qquad c^*(t) = c^* \text{ (constant)}$$

$$(3)\qquad \int_0^{\infty} \pi(t)\, dt < \infty$$

$$(4)\qquad \lambda(t)k^*(t) + \mu(t)s^*(t) \longrightarrow 0 \quad \text{as } t \to \infty.$$

It is essential to observe that the path of positive consumption discount factors $\{\pi(t)\}_{t=0}^{\infty}$ solely reflects the rate at which consumption at one point time can be transformed into consumption at some other point in time. In particular, it has no ethical significance since it is derived from the regular maximin program as a price support of the constant consumption path.

We first show as Proposition 2 that the premise of Proposition 1 is satisfied for any maximin program that is interior and regular.

Proposition 2. *If $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is an interior and regular maximin program at consumption discount factors $\{\pi(t)\}_{t=0}^{\infty}$ and competitive prices $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$, then the premise of Proposition 1 is satisfied.*

Secondly, we establish as Proposition 3 that, under Assumptions 1 and 2, any maximin program in Solow's model indeed is interior and regular.

Proposition 3. *Any maximin program in Solow's model is interior and regular at some appropriately chosen consumption discount factors $\{\pi(t)\}_{t=0}^{\infty}$ and competitive prices $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$, provided that Assumptions 1 and 2 are satisfied.*

Together these two main results, which are proven in the following section, demonstrate that Proposition 1 can be applied to show that the converse of Hartwick's rule holds for Solow's model. We have thus established the usefulness of our previous result on the converse of Hartwick's rule, also in the context of Solow's model.

Note that Proposition 3 strengthens similar results that Dasgupta and Mitra [1983] show in discrete time, by not requiring that raw material is, in a certain sense, "important". In a setting that does not explicitly include labor input, raw material could be called important if there is an $\alpha > 0$ such that $f_r(k,r)r/f(k,r) \geq \alpha$ for all $k \geq k_0$ and r sufficiently small (see also Mitra [1978]). Such an assumption, which facilitates showing (3), is not made here.

4. Proofs. Proposition 2 is proven through the following two lemmas. First we observe that, if $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is a regular maximin program, then $\{c^*(t)\}_{t=0}^{\infty}$ maximizes the sum of consumption discounted by $\{\pi(t)\}_{t=0}^{\infty}$.

Lemma 1 (Dixit et al. [1980]). *If a program $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is a regular maximin program at $\{\pi(t)\}_{t=0}^{\infty}$ and $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$, then it maximizes $\int_0^\infty \pi(t)c(t)\,dt$ over all feasible paths.*

Proof. Note that (2) and (3) imply that $\int_0^\infty \pi(t)c^*(t)\,dt < \infty$. It is sufficient to show that

$$\liminf_{T\to\infty} \int_0^T \pi(t)(c(t) - c^*(t))\,dt \leq 0$$

for all feasible programs $\{c(t), i(t), r(t), k(t), s(t)\}_{t=0}^{\infty}$.

$$\begin{aligned}
&\int_0^T \pi(t)(c(t) - c^*(t))\,dt \\
&\quad\leq \int_0^T [\lambda(t)(i^*(t) - i(t)) - \mu(t)(r^*(t) - r(t)) \\
&\qquad + \dot{\lambda}(t)(k^*(t) - k(t)) + \dot{\mu}(t)(s^*(t) - s(t))]\,dt \quad \text{by (1)} \\
&\quad= \int_0^T [d(\lambda(t)(k^*(t) - k(t)) + \mu(t)(s^*(t) - s(t)))/dt]\,dt
\end{aligned}$$

$$
\begin{aligned}
&\quad\text{since } \dot{k}(t) = i(t) \text{ and } \dot{s}(t) = -r(t)\\
&= (\lambda(T)(k^*(T) - k(T)) + \mu(T)(s^*(T) - s(T)))\\
&\quad - (\lambda(0)(k^*(0) - k(0)) + \mu(0)(s^*(0) - s(0)))\\
&\leq \lambda(T)k^*(T) + \mu(T)s^*(T) \quad \text{since } k^*(0) = k(0) = k_0,\\
&\quad s^*(0) = s(0) = s_0, \quad \lambda(T) \geq 0, \quad \mu(T) \geq 0, \quad k(T) \geq 0 \text{ and } s(T) \geq 0.
\end{aligned}
$$

By (4), the result follows. □

Note that, since the consumption discount factors $\{\pi(t)\}_{t=0}^{\infty}$ are positive, Lemma 1 implies that a regular maximin path is efficient.

Secondly, we show that, for any interior and competitive program, the maximum principle holds and the path of costate variables is unique.

Lemma 2. *If an interior program $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is a regular maximin program at $\{\pi(t)\}_{t=0}^{\infty}$ and $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$, then the maximum principle holds for the problem of maximizing $\int_0^{\infty} \pi(t)c(t)\,dt$ and the path of corresponding costate variables is unique and equals $\{\lambda(t), \mu(t)\}_{t=0}^{\infty}$.*

Proof. Since $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is interior and competitive, it follows from (1) that, for all t,

$$
(c^*(t), r^*(t)) \text{ maximizes } \pi(t)c + \lambda(t)(f(k^*(t), r) - c) - \mu(t)r \text{ over all nonnegative } (c, r) \tag{5}
$$

$$
\lambda(t)f_k(k^*(t), r^*(t)) + \dot{\lambda}(t) = 0 \tag{6}
$$

$$
\dot{\mu}(t) = 0. \tag{7}
$$

Since $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ is a regular maximin program and thus, by Lemma 1, maximizes $\int_0^{\infty} \pi(t)c(t)\,dt$ over all feasible paths, it follows that (5)–(7) are necessary conditions for optimality, where

$$
\mathcal{H}(k, s, c, r, \lambda, \mu) = \pi(t)c + \lambda(f(k, r) - c) - \mu r
$$

is the corresponding Hamiltonian function. It follows from (5) that, for all t, $(\lambda(t), \mu(t))$ is uniquely determined from $\pi(t)$ by

$$
\begin{aligned}
\pi(t) - \lambda(t) &= 0\\
\lambda(t)f_r(k^*(t), r^*(t)) - \mu(t) &= 0
\end{aligned}
$$

since the program is interior and f is smooth. □

Proof of Proposition 2. This is a direct consequence of Lemmas 1 and 2. □

By Assumption 2, any maximin program has the property that $\inf_t c(t) = c^*$. Our proof of Proposition 3 is based on three lemmas that derive results from the problem of minimizing resource use subject to, for all t, $c(t) \geq c^*$:

$$\min \int_0^\infty r(t)\, dt \text{ subject to } \dot{k}(t) = f(k(t), r(t)) - c(t) \text{ and } c(t) \geq c^*.$$

It follows from Assumption 2 that this problem has a solution, which we will denote $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^\infty$, and which satisfies

$$\int_0^\infty r^*(t)\, dt \leq s_0.$$

Since, clearly, $c^*(t) = c^*$ for all t, the Hamiltonian function corresponding to the minimum resource use problem can be written

$$\mathcal{H}(k, r, \lambda; c^*) = -r + \lambda(f(k, r) - c^*),$$

from which we can derive the following necessary conditions. For all t,

$$r^*(t) \text{ maximizes } -r + \lambda(t) f(k^*(t), r) \text{ over all nonnegative } r, \tag{8}$$

$$-\dot{\lambda}(t) = \lambda(t) f_k(k^*(t), r^*(t)). \tag{9}$$

Let V denote the value function corresponding to minimum resource use. It follows from Assumption 2 that minimum resource use subject to $c(t) \geq c$ and initial stock k is given as $V(k, c)$ for all $k \geq k_0$ and $c \leq c^*$.

Lemma 3. *If a program* $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^\infty$ *solves the minimum resource use problem subject to* $c(t) \geq c^*$*, then it has constant consumption, and is interior and competitive.*

Proof. Clearly $c^*(t) = c^* > 0$ for all t. Furthermore, since $\lambda(\tau) \leq 0$ would imply $\lambda(t) \leq 0$, $r^*(t) = 0$, and $f(k^*(t), r^*(t)) = 0$ for all $t \geq \tau$, contradicting that $c^*(t) = c^*$ and $k^*(t) \geq 0$ for all t, it follows from (8) and (9) that, for all t, $\lambda(t) > 0$ and $r^*(t) > 0$. We have that, for all t,

$$V(k_0, c^*) = \int_0^t r^*(\tau)\, d\tau + V(k^*(t), c^*)$$

and $\partial V(k^*(t), c^*)/\partial k = -\lambda(t)$. This means that, for all t, $\lambda(t)\dot{k}^*(t) = -dV(k^*(t), c^*)/dt = r^*(t) > 0$, implying that $i^*(t) = \dot{k}^*(t) > 0$ and $k^*(t) \geq k_0 > 0$. Finally, for all t, $\dot{s}^*(t) = -r^*(t) < 0$ and $s^*(t) \geq 0$, implying that $s^*(t) > 0$. Hence, any program that solves the minimum resource use problem subject to $c(t) \geq c^*$ has constant consumption and is interior.

It remains to be shown that any program solving the minimum resource use problem is competitive. To show this, set $\pi(t) = \lambda(t)$ and $\mu(t) = 1$ for all t. It is straightforward to check that the concavity of f implies that (1) is then satisfied for all t. □

Lemma 4. *If a program* $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ *solves the minimum resource use problem subject to* $c(t) \geq c^*$*, then it exhausts the resource and the path of the costate variable* $\{\lambda(t)\}_{t=0}^{\infty}$ *satisfies* $\int_0^\infty \lambda(t)\, dt < \infty$.

Proof. Suppose that $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^{\infty}$ solves the minimum resource use problem subject to $c(t) \geq c^*$, but does not exhaust the resource, i.e., we have that $\int_0^\infty r^*(t)\, dt < s_0$. We will show that it is then possible to construct a feasible program with $\inf_t c(t) > c^*$, contradicting the definition of c^*. Inspired by an argument by Cairns and Yang (see comment at the end of this section), we first show how a uniform reduction in consumption can be achieved by reducing the finite resource input. By the smoothness of the production function this in turn means that a finite increase in resource input can bring about a uniform increase in consumption.

A reduction of the constant rate of consumption by ε can be achieved by keeping the time path of the stock of capital unaltered and by reducing the resource input at time t by $\eta(t;\varepsilon)$, where $0 < \varepsilon < c^*$, and where, for all t,

$$\varepsilon = f(k^*(t), r^*(t)) - f(k^*(t), r^*(t) - \eta(t;\varepsilon)).$$

Note that, for all t, $0 < \eta(t;\varepsilon) < r^*(t)$ since, by Lemma 3,

$$f(k^*(t), r^*(t)) = c^* + i^*(t) > c^* > \varepsilon$$

and $f(k^*(t), 0) = 0$. Furthermore, $\int_0^\infty \eta(t;\varepsilon)\,dt < \int_0^\infty r^*(t)\,dt \le s_0 < \infty$.

Differentiability of f implies

$$\begin{aligned}\varepsilon &= f(k^*(t), r^*(t)) - f(k^*(t), r^*(t) - \eta(t;\varepsilon)) \\ &= f_r(k^*(t), r^*(t))\eta(t;\varepsilon) + O_1(\eta(t;\varepsilon))\end{aligned}$$

where $O_1(\eta(t;\varepsilon))/\eta(t;\varepsilon) \to 0$ as $\eta(t;\varepsilon) \to 0$. Now, instead, consider increasing the resource input at time t by $\eta(t;\varepsilon)$. Differentiability of f implies

$$\begin{aligned}&f(k^*(t), r^*(t) + \eta(t;\varepsilon)) - f(k^*(t), r^*(t)) \\ &\qquad = f_r(k^*(t), r^*(t))\eta(t;\varepsilon) - O_2(\eta(t;\varepsilon)),\end{aligned}$$

where $O_2(\eta(t;\varepsilon))/\eta(t;\varepsilon) \to 0$ as $\eta(t;\varepsilon) \to 0$. Therefore,

$$f(k^*(t), r^*(t) + \eta(t;\varepsilon)) - f(k^*(t), r^*(t)) = \varepsilon - O(\eta(t;\varepsilon)),$$

where $O(\eta(t;\varepsilon)) = O_1(\eta(t;\varepsilon)) + O_2(\eta(t;\varepsilon))$ satisfies $O(\eta(t;\varepsilon))/\eta(t;\varepsilon) \to 0$ as $\eta(t;\varepsilon) \to 0$.

For given ε, $\eta(t;\varepsilon) \to 0$ as $t \to \infty$. Hence, $O(\eta(t;\varepsilon)) \to 0$ as $t \to \infty$, implying that there exists T such that

$$f(k^*(t), r^*(t) + \eta(t;\varepsilon)) - f(k^*(t), r^*(t)) > \varepsilon/2$$

for almost all $t > T$. But then it is possible to increase the constant rate of consumption by $\varepsilon/2$ almost everywhere by adding a finite amount of the resource. In other words, a marginal increase of c^* requires a finite increase of the resource stock. This argument establishes that $\partial V(k_0, c^*)/\partial c < \infty$ and means that it would have been possible to construct a feasible program with $\inf_t c(t) > c^*$ if $\int_0^\infty r^*(t)\,dt < s_0$.

Since the existence of such a program contradicts the definition of c^*, we have that $\int_0^\infty r^*(t)\,dt = s_0$ and $s^*(t) \to 0$ as $t \to \infty$. It also follows from Seierstad and Sydsaeter [1987, p. 217] that

$$\frac{\partial V(k_0, c^*)}{\partial c} = \int_0^\infty \frac{\partial \mathcal{H}(k^*(t), r^*(t), \lambda(t); c^*)}{\partial c}\,dt = \int_0^\infty \lambda(t)\,dt.$$

Hence, $\int_0^\infty \lambda(t)\,dt$ is finite. □

Lemma 5. *If a program $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^\infty$ solves the minimum resource use problem subject to $c(t) \geq c^*$, then $\lambda(t)k^*(t) \to 0$ as $t \to \infty$, where $\{\lambda(t)\}_{t=0}^\infty$ is the path of the costate variable.*

Proof. In view of the concavity of the production function f, the value function V is convex in k, implying that, for all t,

$$V(k(t), c^*) - V(k^*(t), c^*) \geq \frac{\partial V(k^*(t), c^*)}{\partial k} \cdot (k(t) - k^*(t)) \tag{10}$$

for all $k(t) \geq k_0$. Moreover, $\partial V(k^*(t), c^*)/\partial k = -\lambda(t) < 0$ and

$$\lim_{t\to\infty} V(k^*(t), c^*) = \lim_{t\to\infty} \int_t^\infty r^*(\tau)\,d\tau = 0.$$

Since $V(k, c^*) > 0$ for all $k \geq k_0$, it follows that $V(k, c^*) \to 0$ as $k \to \infty$ and $k^*(t) \to \infty$ as $t \to 0$. Take $k(t) = (1/2)k^*(t)$. Then (10) implies

$$V((1/2)k^*(t), c^*) - V(k^*(t), c^*) \geq (1/2)\lambda(t)k^*(t).$$

The left-hand side goes to zero as $t \to \infty$. The right-hand side is nonnegative and therefore goes to zero as well. □

Proof of Proposition 3. If a program $\{c^*(t), i^*(t), r^*(t), k^*(t), s^*(t)\}_{t=0}^\infty$ solves the minimum resource use problem subject to $c(t) \geq c^*$, then, by Lemma 4, it exhausts the resource. Therefore, since there thus does not exist any maximin program not solving the minimum resource use problem, it follows that a program is maximin if and only if it solves the minimum resource use problem subject to $c(t) \geq c^*$.

By Lemma 3, any program solving the minimum resource use problem is interior. Furthermore, it is a regular maximin program since it is competitive with, for all t, $\pi(t) = \lambda(t)$ and $\mu(t) = 1$ by Lemma 3, and satisfies (2) by Lemma 3, (3) by Lemma 4, and (4) by Lemmas 5 and 4. □

One of the steps taken in this paper is to show that, along a program solving a minimum resource use problem, the minimum resource use coincides with the resource stock initially available (s_0) in the original problem. Cairns and Yang also provide an argument to show this. Their argument largely runs parallel to ours in Lemma 4, where we derive

$$f(k^*(t), r^*(t) + \eta(t;\varepsilon)) - f(k^*(t), r^*(t)) = \varepsilon - O(\eta(t;\varepsilon)).$$

Cairns and Yang are less careful in mentioning the time variable and the dependence of η on ε. Then they apply a limit argument on η to show that for η small enough the left-hand side of the expression is larger than $\varepsilon/2$. This is correct for fixed t because, as ε goes to zero, also η goes to zero. But the inequality might not hold for all t.

A second problem with the analysis by Cairns and Yang is their proof that Hotelling's rule, $\dot{f}_r/f_r = f_k$, holds along a program with maximal constant consumption. The proof relies on a set of first order approximations. This method is an excellent tool, in particular in the case at hand, to illustrate what Hotelling's rule is actually saying—namely that there are no subintervals of time where the constant rate of consumption can be maintained, and at the same time the program ends up with larger capital and resource stocks than in the original program. However, such an argument cannot serve as a formal proof.

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ENDNOTES

1. Cairns and Yang also refer to our paper elsewhere. They argue that we "do not show that following Hartwick's rule leads to a unique outcome, much less a maximal level of consumption". Since we were dealing with the necessity of Hartwick's rule, we did not investigate uniqueness, while it was our premise that the program is maximin.

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