

Stability of a diode laser with phase-conjugate feedback

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An exact analysis is presented of the steady-state stability of a semiconductor laser subjected to feedback from a phase-conjugate mirror. Reduced stability occurs at low feedback whenever the effective external delay time is an integer multiple of the relaxation oscillation period. The role of a finite response time of the mirror is to enhance drastically the steady-state stability. © 1998 Optical Society of America

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Optical feedback is known to affect severely the properties of a semiconductor laser. For stabilization purposes feedback from a phase-conjugate mirror is preferred over conventional optical feedback (COF), since in the latter case the laser suffers from extreme sensitivity to mirror-distance variations within an optical wavelength.¹ This sensitivity is due to the fact that with an ordinary mirror the phase of the returning light depends strongly on the mirror position, whereas in phase conjugation there is no such dependence.² However, in the case of phase-conjugate feedback (PCF) one is always confronted with a certain sluggishness of the reflector owing to the finite response time, which should be taken into account when one is analyzing the stability of the laser operation with a phase-conjugate mirror. Most of the previous stability analyses disregarded this sluggishness.^{3,4}

In this Letter we present a linear stability analysis of the steady state of single-frequency operation of a single-mode diode laser with PCF, including the finite response-time effect. Owing to the time-delay term in the rate equations an exponential appears in the characteristic equation $D(s)$, the roots of which determine the stability of the laser. This exponential, which also shows up in the case of COF, complicates the analysis, and several kinds of approximation have been made in stability analysis.^{1,4,5} We report on a stability analysis without any such approximation, the results of which are valid for arbitrary laser parameters. This method was also used by Cohen *et al.*⁶ in analyzing a diode laser with COF.

When multiple external round trips can be ignored, the rate equations for a single-mode semiconductor laser with sluggish PCF are given by⁷

$$\begin{aligned} \dot{E} = & \frac{1}{2} \left(\frac{\xi \Delta N - \epsilon \Gamma_0 P}{1 + \epsilon P} + i \alpha \xi \Delta N \right) E \\ & + \frac{\gamma_p}{t_m} \exp \left[2i \delta_0 \left(t - \frac{\tau}{2} \right) \right] \int_{-\infty}^t E^*(\theta - \tau) \\ & \times \exp \left[- \left(\frac{1}{t_m} + i \delta_0 \right) (t - \theta) \right] d\theta, \end{aligned} \quad (1a)$$

$$\dot{N} = J - \frac{N}{T_1} - \frac{\Gamma_0 + \xi \Delta N}{1 + \epsilon P} P, \quad (1b)$$

where E is the slowly varying amplitude of the optical field with respect to the optical carrier $\exp(i\omega_0 t)$, with ω_0 as the emission frequency of the solitary laser (i.e., the same laser without feedback) at threshold. E is normalized such that $|E|^2 = P$ equals the number of photons inside the cavity. $N = N_{\text{th}} + \Delta N$ is the number of electron–hole pairs (inversion) in the active layer, and N_{th} is the inversion at threshold of the solitary laser. ξ is the differential gain, ϵ is the nonlinear gain parameter, Γ_0 is the photon decay rate, α is the linewidth-enhancement factor, γ_p is the feedback rate, t_m is the response time of the mirror, δ_0 is the detuning of the mirror pump beam with respect to ω_0 , τ is the external-cavity round-trip time, J is the number of carriers injected into the active layer per unit of time by means of an electrical current, and T_1 is the carrier lifetime. Owing to the finite response time t_m the feedback term in Eq. (1a) depends on the optical field at and before time $t - \tau$. In the limit $t_m \rightarrow 0$, the feedback term reduces to the one given by, e.g., Van Tartwijk *et al.*³

The single-frequency steady state is calculated from Eqs. (1) and is characterized by the time-independent frequency, amplitude, and phase of the optical field and by the value of the inversion. It can easily be seen that the laser frequency ω_s must lock to the pump frequency. The calculation of the steady state in analytic form is simplified when we disregard nonlinear gain. In that case we find two solutions for any given set of parameters as long as $|\delta_0| \leq \gamma_p(1 + \alpha^2)^{1/2}$ is satisfied and no solution otherwise. This situation is similar to what is found in the laser with external monochromatic injection.⁸ Below we assume that the steady state is known, with or without nonlinear gain.

The next step involves considering small deviations from the steady state and analyzing their evolution in time. We replace the feedback term in Eq. (1a) with $\gamma_p E_{\text{FB}}$ and add an extra equation for \dot{E}_{FB} . After replacing the complex fields with their power and

phase and linearizing the equations around the steady state, we are left with a system of five coupled linear delay-differential equations.

This system is solved with Laplace-transform techniques. For the small deviations to relax to the steady state, all roots of the characteristic equation $f(s) = 0$ must have a negative real part.

As a consequence of the delay, the function $f(s)$ resembles a polynomial in s but contains exponentials $\exp(-s\tau)$, which makes analytical progress difficult. The exponential is therefore sometimes approximated by others as $1 - s\tau$. However, in many cases this is a bad approximation: Taking a realistic value of the RO of 3 GHz, $\text{Im}(s\tau)$ already equals 1 for a cavity length of less than a centimeter.

Instead of making approximations we apply the so-called principle of the argument.⁹ This theorem states that

$$\frac{1}{2\pi i} \oint_C \frac{f'(s)}{f(s)} ds = N - P, \quad (2)$$

where N and P are the number of zeros and poles of $f(s)$ within the contour C , respectively. By setting $f(s) = u$ we arrive at $\oint_\Gamma u^{-1} du = 2\pi i(N - P)$. In our case $P = 0$ and we choose C to enclose the right half of the complex plane (we choose a semicircle of radius R , close it along the imaginary axis, and let $R \rightarrow \infty$). The laser is stable if $N = 0$, that is, if the contour Γ does not enclose the origin. This criterion leads us to investigate the intersections of a function of a real variable on a segment of the negative real axis, which is most easily done on a computer.

The above-outlined technique is now used for calculating stability diagrams, where we use the parameter values listed in Table 1. These values, which are identical to those in Ref. 7, imply that the laser is pumped 5% above its solitary threshold, and $\omega_R/2\pi = 764$ MHz. Variation of α and ϵ confirms a well-known behavior, i.e., a larger α gives rise to a smaller region of stability, whereas inclusion of nonlinear gain enhances the stability. In the figures presented below nonlinear gain is ignored.

For zero detuning ($\delta_0 = 0$), Fig. 1 shows the stability diagram when γ_p and τ are varied for several values of the mirror-response time. Focusing first on $t_m = 0$ only, one can see that for very small feedback rates the laser is stable. When the feedback is increased, instability sets in when one crosses the bottom dashed curve. Note the periodic modulation of the stability-edge curve with a lower stability limit when the RO matches an external round-trip resonance, i.e., when $\omega_R\tau/2\pi$ is an integer. This result was not seen by Agrawal and Gray,⁴ owing to the above-mentioned lowest-order expansion of the exponential. On the other hand, this result is similar to what was found for a laser with COF, except that in COF high stability tongues were found at half-integer values.^{5,10} As yet we do not have an explanation for the apparent difference.

Comparing PCF from an instantaneously responding mirror with that from a slowly responding mirror, one can see that the stability is enhanced slightly with increasing t_m (for $t_m = 100$ ps the stable re-

gions are shown by the shaded areas at the bottom of Fig. 1). This enhancement is caused by the mirror-induced spectral filtering of the reflected field, which suppresses frequencies larger than $1/t_m$. More striking is the shifted location of the stability peaks, which, in view of the time delay in the mirror, resembles a situation of larger external round-trip length. The effective round-trip length enhancement is not sharply defined, which reduces the quality of the resonance for large t_m . The relative importance of this reduced quality increases when τ gets smaller, which may explain why the peak at $\omega_R\tau/2\pi = 0.5$ is lower for $t_m = 400$ ps than for shorter response times.

We also investigated the behavior at moderate feedback. The result is displayed at the top of Fig. 1. The stability of the laser is greatly enhanced by the sluggish mirror: Over the whole range of τ indicated in the figure the PCF laser is stable with higher feedback for mirror-response times of 100 and 400 ps, whereas for an instantaneous mirror stability is found only in a small region for short cavities. DeTienne *et al.*⁷ found in numerical simulations that the standard deviation of the output power was much smaller for reflectivities

Table 1. Values of the Parameters Used in the Calculations

Quantity	Value	Unit
Γ_0	7.2595×10^{11}	s^{-1}
J	4.0635×10^{17}	s^{-1}
N_{th}	7.74×10^8	—
T_1	2.0×10^{-9}	s
α	3.0	—
ξ	1.19×10^3	s^{-1}
ϵ	3.57×10^{-8}	—

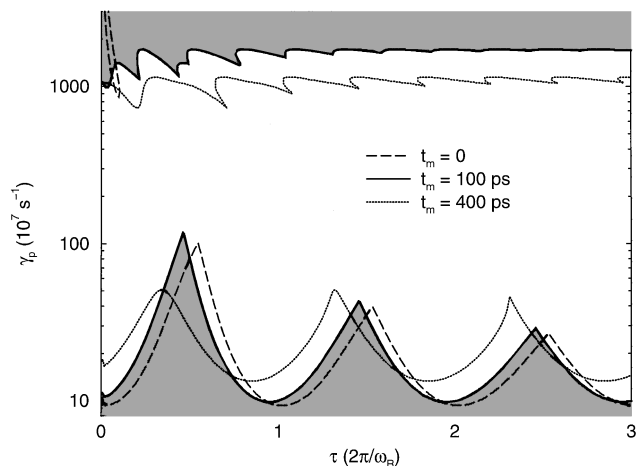


Fig. 1. Feedback rate γ_p at which the laser changes stability as a function of the cavity round-trip time τ , normalized to the RO period (which is constant and equal to 1.3 ns). The shaded regions indicate stable behavior with $t_m = 100$ ps. For low feedback the laser is stable for all mirror-response times. The critical value at which instability sets in depends on τ in an oscillatory way. For moderate amounts of feedback (the top set of curves) a sluggish mirror response gives rise to stable behavior over the whole range of τ investigated, whereas an instantaneous mirror response yields stable behavior in a small region for short cavities only.

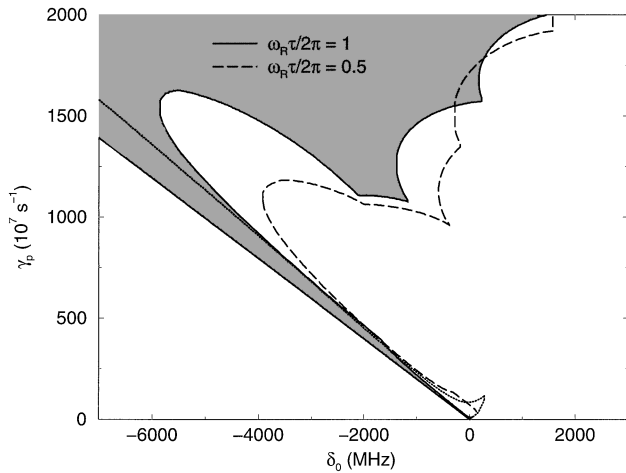


Fig. 2. Stability of a PCF laser as a function of γ_p and the pump detuning δ_0 . $t_m = 100$ ps, except at the dotted curve. The shaded region is the region of stable behavior for a cavity with $\omega_R\tau/2\pi = 1$. The dotted curve represents an instantaneous mirror response at $\omega_R\tau/2\pi = 0.5$; in this case the region of stability shrinks to a narrow stripe. When $|\delta_0| > \gamma_p(1 + \alpha^2)^{1/2}$ steady-state solutions (fixed points) do not exist at all. The line representing $\delta_0 = \gamma_p(1 + \alpha^2)^{1/2}$ is displayed only where it separates stable from nonexistent fixed points.

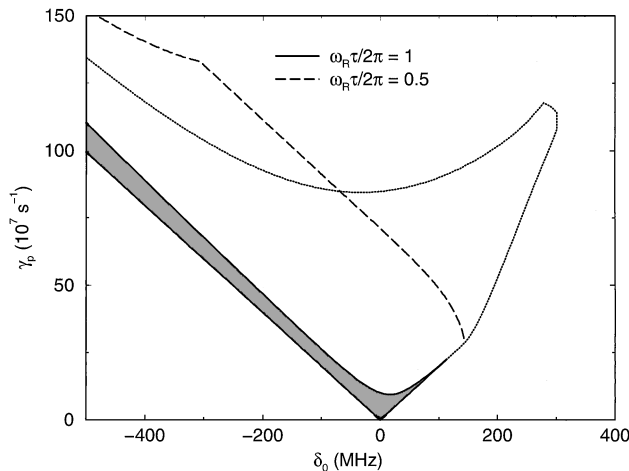


Fig. 3. Magnification of the region near the origin of Fig. 2. Notice the destabilizing influence of the RO when it matches an external round-trip resonance (compare the solid and the dashed curves). The dotted curve is an extension of Fig. 3 of Ref. 3, which was obtained for an instantaneously responding mirror.

of the order of 4%. Here we have proved that the reduction of the standard deviation corresponds to stable steady-state operation of the laser. The structure in the higher-stability boundaries of Fig. 1 is caused by a new frequency Ω instead of the RO frequency. In good approximation Ω is equal to $\gamma_p\alpha/(1 + \alpha^2)^{1/2}$. Each time Ω matches an external round-trip resonance the steady-state stability is decreased. In this sense the top and bottom oscillations in the figure have the same origin.

As a last result we show the influence of the detuning δ_0 in Figs. 2 and 3. For weak feedback one can see a narrow band of stable operation, but this band widens for higher feedback, and finally we find a large region of stable laser output that is consistent with Fig. 1. The general shape of the curves for finite t_m resembles the stability diagram of a diode laser with external optical injection.¹¹ Also, the low-feedback part of the stability diagram with $\omega_R\tau/2\pi = 1$ is very similar to the corresponding part of the injection-laser stability diagram.

All stability diagrams were checked by direct numerical integration of rate equations (1) at several points in the stability diagrams, and no discrepancy between the two methods was found.

In conclusion, a finite mirror-response time tends to stabilize a laser with PCF: The low-feedback stability-edge curve shifts upward with t_m . With a further increase of the feedback rate the PCF laser becomes stable again if the mirror has a finite response time, in sharp contrast with the unstable behavior of a laser with an instantaneously responding mirror. The stability areas for low and moderate feedback are connected in the feedback versus detuning parameter plane.

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