Saturation-induced frequency shift in the noise spectrum of a birefringent vertical-cavity surface emitting laser

Han van der Lem and Daan Lenstra

Department of Physics and Astronomy, Vrije Universiteit, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

Received June 30, 1997

We show by means of analytical and numerical calculations that saturation, in combination with the linewidth-enhancement factor and the finite rate of spin–flip processes, causes a redshift of the spontaneous-emission peak with respect to the off-lasing-mode frequency in the optical spectrum of a quantum-well vertical-cavity surface emitting laser.

Vertical-cavity surface emitting lasers (VCSEL's) are different from conventional edge emitters in their polarization behavior. Whereas edge emitters have anisotropic cavities, which cause the polarization to be fixed (usually TE), VCSEL's have cavities with cylindrical symmetry. VCSEL's are known to emit linearly polarized light preferentially. On the basis of the geometry of the cavity one would expect there to be no preference for the orientation of the linear polarization. However, it turns out that VCSEL's often show a preference for an orientation along two orthogonal axes. Also, polarization switching from one axis to the other was observed when the injection current was altered.

The occurrence of preferred orientations of the polarization indicates the presence of one or more residual cavity anisotropies. Birefringence in the cavity, caused by the elasto-optic[1] and the electro-optic[2,3] effects, was found to be the dominant residual anisotropy responsible for the observed polarization behavior. Martin-Regalado and co-workers[4,5] used a nonlinear rate-equation model developed by San Miguel *et al.*[6] that includes anisotropic gain, spin–flip processes, and a linewidth-enhancement factor to show that the presence of birefringence can explain both the preferred orientations of the polarization and the switching between them.

As a consequence of the presence of birefringence, the emission frequencies of the preferred linearly polarized lasing modes differ. Since the polarization behavior of the VCSEL is significantly influenced by the birefringence (also referred to as linear-phase anisotropy), one would like to be able to measure its magnitude. To do so, Jansen van Doorn *et al.*[7] observed the emitted-field power spectrum. Besides a strong narrow peak that corresponded to the lasing field, they also observed a much weaker and much broader peak, which they attributed to the nonlasing orthogonally polarized mode. However, below the lasing threshold this mode still collects and amplifies spontaneous emission. In this Letter we analyze the power spectrum theoretically in full detail and solve the question of how to extract the magnitude of the birefringence from a measured power spectrum.

It is tempting to assume that the peak frequency in the power spectrum of the nonlasing polarization corresponds to the frequency of the off-lasing mode. However, we show here that this is not the case. Using the nonlinear rate-equation model, we show that the spontaneous-emission-induced peak is redshifted significantly with respect to the corresponding off-lasing-mode emission frequency. In a reasonable approximation we derive a simple analytical expression for this shift that shows that the shift is proportional to (among other quantities) the linewidth-enhancement factor and the lasing output power and inversely proportional to the spin–flip rate. We also show, in the same approximation, that the width of the spontaneous-emission-induced peak is also proportional to the total output power and inversely proportional to the spin–flip rate. Furthermore, we show that a second peak appears centered around a frequency that is the main peak frequency that is mirrored around the (on-) lasing frequency. The analytically obtained results are in good agreement with the results of numerical simulations of the power spectra.

We also comment on recent experiments by Jansen van Doorn *et al.*,[7] reporting a discrepancy between their completely linear model of the VCSEL and their experimentally obtained data. However, their interpretation of the measured power spectra relied on the assumption that the spontaneous-emission peak frequency and the off-lasing-mode frequency are identical. We argue that the discrepancy can be explained by a shift of the spontaneous-emission-induced peak as demonstrated here. Finally, we suggest a new, seemingly easy way of determining the spin–flip relaxation rate.

Polarization-state selection as well as polarization switching in quantum-well VCSEL's was modeled in Refs. 4 and 5 by inclusion of birefringence in a rate-equation model. This model included two important features of semiconductor lasers, nonlinear interaction between the charge carriers and the fields and large-amplitude phase coupling, which is commonly accounted for by linewidth-enhancement factor α (or Henry's α).

We model the influence of spontaneous emission by adding Langevin noise terms to the rate equations:

$$\dot{E}_{\pm} = \kappa(1 + i\alpha)(N \pm n - 1)E_{\pm} - i\gamma_p E_{\mp} + [\beta(N \pm n)]^{1/2}\chi_{\pm}\,, \tag{1}$$

$$\dot{N} = -\gamma(N - \mu) - \gamma N(|E_+|^2 + |E_-|^2) - \gamma n(|E_+|^2 - |E_-|^2), \quad (2)$$

$$\dot{n} = -\gamma_s n - \gamma n(|E_+|^2 + |E_-|^2) + \gamma N(|E_+|^2 - |E_-|^2), \quad (3)$$

where $E_\pm$ are the complex slowly varying dimensionless amplitudes of the left $(-)$ and right $(+)$ circularly polarized vector electric-field components, with corresponding cavity decay rate κ; $N = N_+ + N_-$ is the total normalized population difference between the conduction and the valence bands, with γ the associated decay rate; and $n = N_+ - N_-$ is the inversion difference between levels of opposite sign of magnetic quantum number M_z. The decay rate γ_s associated with n is introduced to model spin–flip relaxation processes. The birefringence of the cavity is accounted for by linear-phase anisotropy parameter γ_p, and μ is the injection current normalized to threshold.

The noise-free ($\beta = 0$) rate equations have two solutions that correspond to linearly polarized emission that are orthogonally polarized[4,5]:

$$E_+ = E_- = \frac{Q}{\sqrt{2}} \exp(-i\gamma_p t), \quad (4)$$

which corresponds to an x-polarized lasing field, and

$$E_+ = -E_- = i\frac{Q}{\sqrt{2}} \exp(i\gamma_p t), \quad (5)$$

which corresponds to a y-polarized lasing field. Both solutions have

$$N = 1, \qquad n = 0, \qquad Q^2 = \mu - 1. \quad (6)$$

Suppose now that the VCSEL emits linearly polarized light oriented along the y axis with an angular frequency $\omega_y = \gamma_p$ [i.e., Eq. (5)]. Here we investigate the response of the VCSEL to perturbations, caused by noise, of this solution. The results obtained are also applicable for the other polarization mode only if γ_p is replaced by $-\gamma_p$.

We denote perturbed variables $E_\pm$, N, and n as follows:

$$E_\pm \to \pm i\left(\frac{Q}{\sqrt{2}} + \delta q_\pm\right)\exp[i(\gamma_p t + \delta\phi_\pm)], \quad (7)$$

$$N \to 1 + \delta N, \quad n \to \delta n. \quad (8)$$

When we substitute the perturbed variables into rate equations (1)–(3), neglecting higher-order terms, we obtain a set of six linear differential equations. This set consists of two independent sets of three differential equations each, one of which describes the behavior of the perturbations of the y-polarized (lasing) field and total population difference N. The order set describes the (nonlasing) x-polarized field and inversion difference n.

Focusing on the nonlasing field, we calculated the eigenvalues[8] that correspond to the latter set. These are the solutions to the polynomial equation[4,5]:

$$\lambda^3 + (\gamma_s + \gamma Q^2)\lambda^2 + (2\kappa Q^2\gamma + 4\gamma_p{}^2)\lambda + 4\kappa\alpha Q^2\gamma\gamma_p + 4(\gamma_s + \gamma Q^2)\gamma_p{}^2 = 0. \quad (9)$$

In Refs. 5 and 6 it is pointed out that there are two regimes of parameters with different behavior of the eigenvalues. In one regime the three eigenvalues are real, and in the other there are one real and two complex-conjugate eigenvalues. To arrive at simple expressions, we focus on the parameter range that satisfies the condition

$$\kappa(\mu - 1)\gamma/\gamma_s \ll \gamma_p \ll \gamma_s, \quad (10)$$

in which case there are one real and two complex-conjugate eigenvalues. The condition corresponds to a VCSEL with a relatively high spin–flip rate that operates not to far above threshold. In this case the eigenvalues can be approximated by $\lambda_1 \approx -\gamma_s$, which corresponds mainly to the decay of the perturbations of n, and

$$\lambda_{2,3} = -\kappa Q^2\gamma/\gamma_s \pm i(2\gamma_p + \kappa\alpha Q^2\gamma/\gamma_s). \quad (11)$$

The imaginary part of the eigenvalues determines the frequency at which resonance peaks in the optical spectrum occur, and the real part determines the width of these peaks. Hence we should expect two peaks in the power spectrum, both having a width

$$\Delta\omega_{\mathrm{FWHM}} = 2\kappa(\mu - 1)\gamma/\gamma_s, \quad (12)$$

where we have used the relation between the injection current and the total output power, $Q^2 = \mu - 1$. Bearing in mind that we linearized around the steady state that oscillates with an angular frequency $+\gamma_p$, we expect the two peaks in the power spectrum to be centered around the angular frequencies

$$\omega_{sp} = -\gamma_p - \kappa\alpha(\mu - 1)\gamma/\gamma_s, \quad (13)$$

$$\omega_{conj} = 3\gamma_p + \kappa\alpha(\mu - 1)\gamma/\gamma_s. \quad (14)$$

As the peak centered around ω_{sp}, which we call the spontaneous-emission peak, is closest to the (off-) lasing-mode frequency, it will be larger than the peak at ω_{conj}.

Hence we find a shift of the spontaneous-emission peak with respect to the off-lasing-mode angular frequency $-\gamma_p$. Since the formula for the shift does not contain γ_p, the shift will be independent of the sign of γ_p, which means that the shift is always in the same direction, the low-frequency direction. Hence, because of the saturation-induced shift, the frequency difference, $\Delta\omega$ between the on-lasing-mode and ω_{sp} will increase when the laser is in the high-frequency mode and decrease when the laser is in the low-frequency mode.

These results are interesting from an experimental point of view for two reasons. First, these results resolve a discrepancy reported by Jansen van Doorn *et al.*,[7] who studied the polarization behavior of a VCSEL under the influence of various values of phase anisotropy.

The experimentally obtained data in Ref. 7 were analyzed with a completely linear model. In most of the experiments the experimental data were in excellent agreement with this linear model. However, in some experiments a discrepancy between the linear model and the experimental data was observed.

In analyzing the spectra Martin-Regalado *et al.* relied on the assumption that the frequency around which the observed peak in the spectrum of the

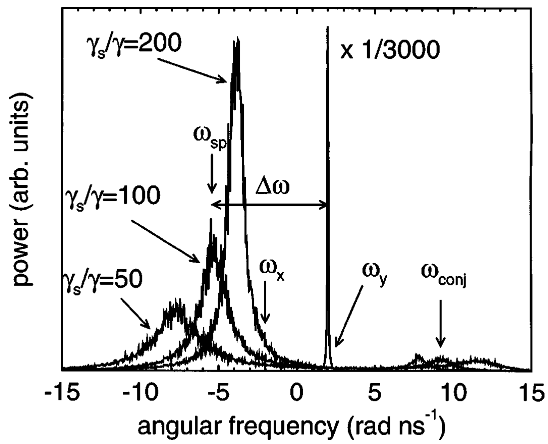

Fig. 1. Simulated spectra of a VCSEL for different values of γ_s. The narrow peak, which corresponds to the y-polarized lasing field, is attenuated by a factor of 3000. Parameter values: $\gamma_p = 2\ \mathrm{ns}^{-1}$, $\gamma = 1\ \mathrm{ns}^{-1}$, $\kappa = 300\ \mathrm{ns}^{-1}$, $\mu = 1.4$, $\alpha = 4$, $\beta = 5 \times 10^{-4}$.

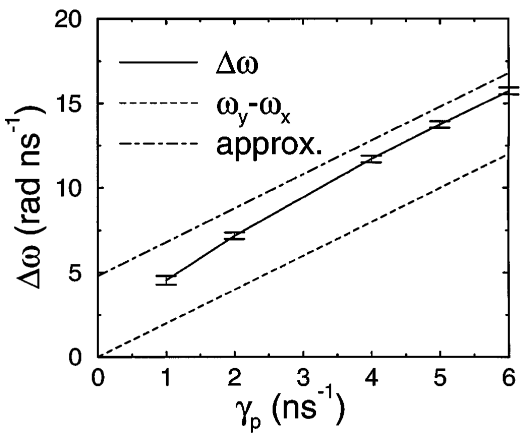

Fig. 2. Comparison of the approximated expression [Eq. (13)] with the numerically calculated value of $\Delta\omega$. $\mu = 1.4$, $\alpha = 4$, $\gamma_s = 100\ \mathrm{ns}^{-1}$, $\kappa = 300\ \mathrm{ns}^{-1}$, $\gamma = 1\ \mathrm{ns}^{-1}$.

nonlasing polarization is centered coincides with the frequency of the off-lasing mode. Our results show that this assumption is not justified. The nonlinear rate-equation model used here provides a natural explanation for the reported discrepancy.

Second, the determination of the shift seems to be an easy way to determine the spin–flip relaxation rate; the unapproximated solutions to Eq. (9) should be used when condition (10) does not apply.

For the numerical integration of Eqs. (1)–(3) we used the Euler scheme, as it allows one to introduce the noise in a proper yet uncomplicated way. To obtain a relatively smooth spectrum, we averaged 64 spectra.

Figure 1 shows numerically simulated power spectra for different values of the spin–flip rate. The two peaks appear symmetrically around the on-lasing-mode frequency $\omega_y = \gamma_p$ as expected. The locations of the peaks correspond to those calculated without approximations with the small-signal analysis.

Figure 2 shows a comparison of the values for $\Delta\omega \equiv \omega_y - \omega_{sp}$ as calculated by numerical simulations and by the approximated result [Eq. (13)]. Also plotted is the frequency splitting between the two lasing modes, $\omega_y - \omega_x = 2\gamma_p$. Figure 2 shows that for the chosen parameter values the approximate expression for ω_{sp} tends to overestimate the value of the saturation-induced frequency shift by an amount that decreases for increasing values of γ_p. The overestimation is due to the fact that we are too high above threshold for condition (10) still to be valid.

In conclusion, we have shown that owing to nonlinear effects the spontaneous-emission peak is redshifted by an amount that, in a reasonable approximation, depends linearly on the linewidth-enhancement factor and the injection current and is inversely proportional to the spin–flip rate. In the same approximation we have shown that the width of this peak shows the same behavior, except that it is independent of α. The saturation-induced shift of the spontaneous-emission peak can explain the above-mentioned discrepancy in Ref. 7. The measurement of the shift might prove to be an easy way to determine the spin–flip rate in quantum-well VCSEL's.

The authors thank J. Martin-Regalado, C. R. Mirasso, and M. San Miguel for many fruitful discussions. This study was supported by project HCM-CHRX-CT94-0594 of the European Union and by FMO/NWO.

References

1. A. K. Jansen van Doorn, M. P. van Exter, and J. P. Woerdman, Appl. Phys. Lett. **69,** 1041 (1996).
2. M. P. van Exter, A. K. Jansen van Doorn, and J. P. Woerdman, Phys. Rev. A **56,** 845 (1997).
3. R. F. M. Hendriks, M. P. van Exter, and J. P. Woerdman, Appl. Phys. Lett. **69,** 869 (1996).
4. J. Martin-Regalado, M. San Miguel, N. B. Abraham, and F. Prati, Opt. Lett. **21,** 351 (1996).
5. J. Martin-Regalado, F. Prati, M. San Miguel, and N. B. Abraham, IEEE J. Quantum Electron. **33,** 765 (1997).
6. M. San Miguel, Q. Feng, and J. V. Moloney, Phys. Rev. A **52,** 1728 (1995).
7. A. K. Jansen van Doorn, M. P. van Exter, A. M. van der Lee, and J. P. Woerdman, Phys. Rev. A **55,** 1473 (1997).
8. We recently heard of a related theoretical study by M. P. van Exter, R. F. M. Hendriks, and J. P. Woerdman, [Phys. Rev. A **56,** 845 (1997)] on the calculation of the eigenvalues that correspond to the linearly polarized solutions by use of the same nonlinear model that we use here.