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Generation of complete coherence in Young's interference experiment with random mutually uncorrelated electromagnetic beams

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The recently developed theory that unifies the treatments of polarization and coherence of random electromagnetic beams is applied to study field correlations in Young's interference experiment. It is found that at certain pairs of points the transmitted field is spatially fully coherent, irrespective of the state of coherence and polarization of the field that is incident on the two pinholes. © 2005 Optical Society of America

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In a recently published series of papers[1–3] a new theory was developed that unifies the treatments of coherence and polarization of random electromagnetic beams. (The term random electromagnetic beam means, of course, that the electric and magnetic field components vary randomly in time. Such beams are, in general, both partially coherent and partially polarized). A key element of this theory is the spectral interference law for the superposition of such beams. If one measures the field intensity, the visibility of the interference fringes formed by light beams superposed in Young's experiment is proportional to the spectral degree of coherence of the electric field at frequency ω at the two pinholes, $Q_1(\boldsymbol{\rho}_1)$ and $Q_2(\boldsymbol{\rho}_2)$. It is given by the expression

$$\eta(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \frac{\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)}{[\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1,\omega)\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}^{(0)}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2,\omega)]^{1/2}}, \qquad (1)$$

where Tr denotes the trace. The electric cross-spectral density matrix that can be used to characterize the state of coherence and polarization of the field at the two pinholes is defined as

$$\overleftrightarrow{\mathbf{W}}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \begin{bmatrix} W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \\ W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \end{bmatrix}, \qquad (2)$$

where

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle, \quad (i,j=x,y). \qquad (3)$$

Here $E_i(\boldsymbol{\rho},\omega)$ is a Cartesian component of the electric field in two mutually orthogonal x and y directions, perpendicular to the direction of propagation of the beam, at a point specified by a position vector $\boldsymbol{\rho}$ at frequency ω, of a typical realization of the statistical ensemble representing the field (Ref. 4, Sec. 4.3). The asterisk denotes the complex conjugate, and the angular brackets denote the ensemble average. The absence of off-diagonal elements of the electric cross-spectral density matrix in definition (1) of the spectral degree of coherence reflects a generalization of the classical Fresnel–Arago interference laws,[5] according to which mutually orthogonal components of the field do not give rise to interference. We emphasize that definition (1) is based on an analysis of an actual interference experiment, unlike definitions suggested in other publications.[6]

In this Letter we apply the unified theory of coherence and polarization to study previously unknown properties of the spectral degree of coherence in Young's interference experiment with random electromagnetic beams (see also Ref. 7).

Consider a random electromagnetic beam propagating close to the z axis, incident on a plane opaque screen containing two pinholes at points $Q_1(\boldsymbol{\rho}_1)$ and $Q_2(\boldsymbol{\rho}_2)$, with

$$\boldsymbol{\rho}_1 = (a,0,0), \qquad (4)$$

$$\boldsymbol{\rho}_2 = (-a,0,0) \qquad (5)$$

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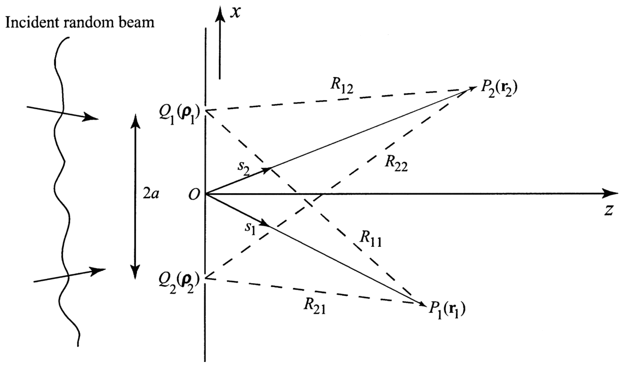

Fig. 1. Notation relating to Young's interference experiment.

(see Fig. 1).

We first derive expressions for the field at two observation points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ in the paraxial approximation. For brevity, we omit from now on the ω dependence of the various quantities. The components of the electric field at P_1 and P_2 are given by

$$E_x(\mathbf{r}_1) = K\left[E_x(\boldsymbol{\rho}_1)\frac{\exp(ikR_{11})}{R_{11}} + E_x(\boldsymbol{\rho}_2)\frac{\exp(ikR_{21})}{R_{21}}\right], \tag{6}$$

$$E_y(\mathbf{r}_1) = K\left[E_y(\boldsymbol{\rho}_1)\frac{\exp(ikR_{11})}{R_{11}} + E_y(\boldsymbol{\rho}_2)\frac{\exp(ikR_{21})}{R_{21}}\right], \tag{7}$$

$$E_x(\mathbf{r}_2) = K\left[E_x(\boldsymbol{\rho}_1)\frac{\exp(ikR_{12})}{R_{12}} + E_x(\boldsymbol{\rho}_2)\frac{\exp(ikR_{22})}{R_{22}}\right], \tag{8}$$

$$E_y(\mathbf{r}_2) = K\left[E_y(\boldsymbol{\rho}_1)\frac{\exp(ikR_{12})}{R_{12}} + E_y(\boldsymbol{\rho}_2)\frac{\exp(ikR_{22})}{R_{22}}\right], \tag{9}$$

respectively, with R_{ij} denoting the distance Q_iP_j (i, j = 1, 2). Factor K is approximately equal to $-i\mathrm{d}A/\lambda$, where $\mathrm{d}A$ is the area of each pinhole and the wavelength $\lambda = 2\pi c/\omega$, with c being the speed of light in vacuum. This expression for K follows from the paraxial approximation to the propagator. For observation points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ in the far zone, with position vectors

$$\mathbf{r}_1 = r_1\mathbf{s}_1\,, \tag{10}$$

$$\mathbf{r}_2 = r_2\mathbf{s}_2\,, \tag{11}$$

with ($s_1^2 = s_2^2 = 1$), we can use the approximation

$$R_{ij} \approx r_j - \boldsymbol{\rho}_i \cdot \mathbf{s}_j\,. \tag{12}$$

Usually one can also make the approximation

$$r_1 \approx r_2 = R\,. \tag{13}$$

Let us consider observation points that satisfy the additional requirement that

$$s_{1x} = s_{2x}\,. \tag{14}$$

These are pairs of points P_1 and P_2 in directions represented by the directional vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ whose end points both lie in the plane x = constant. From the formula

$$s_{ix} = \sin\theta_i \cos\phi_i \qquad (i = 1, 2)\,, \tag{15}$$

it follows that there is an infinite number of pairs of points that satisfy Eq. (14). Among them are points P_1 and P_2, which both lie in the bisecting plane, i.e., the plane that is perpendicular to the screen and bisects the line joining the two pinholes. By making use of Eqs. (3), (6)–(9), and (12) and (13), we obtain the following equations for the elements of the electric cross-spectral density matrix at P_1 ($\mathbf{r}_1$) and P_2 ($\mathbf{r}_2$):

$$W_{xx}(\mathbf{r}_1,\mathbf{r}_2) = |K|^2\frac{\exp[ik(r_2 - r_1)]}{R^2}\{W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1) + W_{xx}^{(0)}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2) + 2\,\mathrm{Re}[W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\exp(i2kas_{1x})]\}\,, \tag{16}$$

$$W_{yy}(\mathbf{r}_1,\mathbf{r}_2) = |K|^2\frac{\exp[ik(r_2 - r_1)]}{R^2}\{W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1) + W_{yy}^{(0)}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2) + 2\,\mathrm{Re}[W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\exp(i2kas_{1x})]\}\,. \tag{17}$$

In a similar way we find that

$$W_{xx}(\mathbf{r}_1,\mathbf{r}_1) = W_{xx}(\mathbf{r}_2,\mathbf{r}_2)\,, \tag{18}$$

$$= \frac{|K|^2}{R^2}\{W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1) + W_{xx}^{(0)}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2) + 2\,\mathrm{Re}[W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\exp(i2kas_{1x})]\}\,, \tag{19}$$

and

$$W_{yy}(\mathbf{r}_1,\mathbf{r}_1) = W_{yy}(\mathbf{r}_2,\mathbf{r}_2)\,, \tag{20}$$

$$= \frac{|K|^2}{R^2}\{W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1) + W_{yy}^{(0)}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2) + 2\,\mathrm{Re}[W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\exp(i2kas_{1x})]\}\,. \tag{21}$$

On substituting from Eqs. (16)–(21) into Eq. (1), it follows that the spectral degree of coherence is given by the expression

$$\eta(P_1,P_2,\omega) = \exp[ik(r_2 - r_1)] \quad (\text{if } s_{1x} = s_{2x})\,. \tag{22}$$

This result shows that, at pairs of points P_1 and P_2 for which $s_{1x} = s_{2x}$, the spectral degree of coherence is unimodular at each frequency, irrespective of the state of coherence and the state of polarization of the field incident on the two pinholes. That is, the field is spectrally completely coherent at such a pair of points.

Next consider a pair of points P_1 and P_2 that both lie in the bisecting plane and are a mirror image of each other in the plane $y = 0$ (i.e., $s_{1y} = -s_{2y}$). It follows from Eq. (22) that one then has

$$\eta(P_1, P_2, \omega) = 1 \quad (s_{1x} = s_{2x}, \quad s_{1y} = -s_{2y}). \tag{23}$$

This implies that the field at P_1 and P_2 is fully coherent and cophasal. According to the spectral interference law for the superposition of random electromagnetic beams,[1] this result has a clear physical implication: If light from two such points is superposed in a second Young's experiment, then the visibility of the resulting fringe pattern will be unity regardless of the state of coherence and the state of polarization of the field at the two pinholes. We emphasize that this would be so even if, for example, each pinhole were illuminated by a different laser.

It is useful to compare our results with the predictions of the van Cittert–Zernike theorem (see Ref. 4, Sec. 4.4.4). This theorem elucidates how a completely incoherent source can generate a field in the far zone that, in certain regions, is highly coherent. However, in contrast with our analysis, the theorem predicts only the existence of pairs of points whose degree of coherence is less than unity. Moreover, the theorem concerns scalar fields and does not take into account the polarization properties of the field as our analysis does. Finally, we note the recently introduced concept of the spectral degree of coherence of an electromagnetic field—as expressed in Eq. (1)—has not been applied previously to the study of interfering electromagnetic beams.

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