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# Spatial dimensions of environmental policies for transboundary externalities: a spatial price equilibrium approach

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**Abstract.** In this paper we present a framework for analyzing spatial aspects of environmental policies in the regulation of transboundary externalities. A spatial price equilibrium model for two regions is constructed, where interactions between these regions can occur via trade, via mutual environmental spillovers as a result of the externality that arises from production, and via uncoordinated taxes when the regions do not behave cooperatively. The additional complications arising from emissions caused by the endogenous transport flows are also considered explicitly. We consider the performance of production taxes, consumption taxes, and the combination of these two, both with and without optimal policy coordination.

## 1 Introduction

Transboundary environmental externalities, including global externalities such as greenhouse gas emissions, have recently received much attention in international negotiations on environmental protection as well as in the environmental economics literature. Apart from various barriers that often frustrate environmental policymaking in general, additional impediments to the effective and efficient regulation of transboundary externalities are the partly public character of the ‘bad’ in question, and the resulting incentives for individual countries to free ride on other countries’ achievements (Carraro, 1997). This is even more serious because of the potentially far-reaching, global consequences of excessive levels of these emissions in case of insufficiently stringent policies.

Apart from free-riding problems for transboundary externalities (see, for instance, Markusen, 1975a), there are additional factors that complicate the design of environmental policies when the international dimension is considered explicitly. One important matter concerns the potential negative effects of local environmental policies on the international competitiveness, and hence the profitability, of local producers. The relations between competitiveness, trade, and environment, and therefore also between trade policies and environmental policies, have consequently received growing attention in the literature [see, for instance, Jaffe et al (1995) for an assessment of empirical evidence, van Beers and van den Bergh (1996) and Ulph (1997) for general surveys, Krutilla (1999) for a survey on partial equilibrium models, and Steininger (1999) for a discussion of general equilibrium models]. To complicate matters further, trade usually requires transportation, which in itself can also give rise to emissions affecting the global environment. At the same time, trade often indirectly affects foreign production and hence emission levels. Consequently, a country should not ignore the derived environmental effects of local policies, such as the induced impacts on international transportation flows as well as on foreign production in addition to the direct impact on local production and emission levels. Various types of interaction that will simultaneously affect different countries’ behavior in terms of environmental policies can thus be distinguished when considering the international dimension.

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In this paper we aim to offer a framework in which these interactions can be studied simultaneously. The central question concerns the first-best and second-best tax rules for open economies concerned with a transboundary environmental externality, and the relative efficiency of various policy alternatives, taking account of spatioeconomic interdependencies between economies. The analysis explicitly pays attention to: (1) the partly public character of transboundary environmental externalities; (2) the relation between trade policies, transport policies, and environmental policies through their joint impacts on various elements making up a region's social surplus (producer surplus, consumer surplus, environmental quality, and tax incomes); (3) the possibility of countries or regions behaving in a noncooperative manner; (4) the role of emissions caused by transport in evaluating environmental policies from a spatial perspective; and (5) the use of consumption taxes, production taxes, or the combination of these two in a multiregion context. Given these features, the model setting is, for instance, applicable to the case of regional or national environmental policies for the regulation of transboundary emissions from energy use in an international context.

The analysis is based on the spatial price equilibrium (SPE) methodology, and can therefore be characterized as a partial equilibrium model [see van Beers and van den Bergh (1996) for a discussion on the use of partial versus general equilibrium models in this context]. Among the growing number of contributions in this field, ours is probably most closely related to Krutilla (1991); we also present a partial equilibrium model allowing for the simultaneous consideration of terms-of-trade and environmental effects of tax policies for a competitive sector, and we ignore the effects of local environmental policies on firm relocation (see, for instance, Markusen et al, 1993) or on the spatial diffusion of environmental technologies (Ulph, 1996b). The analysis is nevertheless quite different from the one in Krutilla (1991) because of the explicit consideration of the simultaneous behavior of two regions, so that the benefits of policy coordination can be considered explicitly [Krutilla (1991) focuses on one single open economy]; by including transport and transport emissions in the analysis; by considering a transboundary externality, so that induced effects on emissions by transport and by the 'foreign' suppliers also affect the efficiency of environmental policies; and by explicitly considering production taxes, consumption taxes, and their combination for the regulation of production induced externalities.

Without claiming to offer an exhaustive overview, we would like briefly to mention some more related studies. Some studies which explicitly focus on the market power of producers and the strategic behavior of producers and/or governments—typically using game-theoretic approaches—are Barret (1994), Conrad (1993), Kennedy (1994), Markusen (1975b), Rauscher (1994), and Ulph (1996a; 1996b). Some recent studies based on general equilibrium approaches are Copeland (1994), Frederiksson (1997), Markusen et al (1993), and Rauscher (1994). To the best of our knowledge, the present setting (in which a partial equilibrium analysis is given of a situation where emissions from local and foreign production as well as from trade-induced transport affect local welfare) has no precedent in the literature, and further extends earlier partial equilibrium studies by considering the simultaneous effects on overall spatial efficiency of second-best taxes used by different interacting regions.

The use of a partial equilibrium approach without a trade balance expression in a model *with* trade of course presupposes that factor income effects are not of interest and that trade imbalances on a sectoral level may persist over the relevant planning period. Krutilla (1999) observes in this respect that:

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“In the trade-and-environment context, partial equilibrium models are particularly useful for studying the consequences of terms-of-trade effects, and for indicating how such factors as a country’s commodity trade balance, and the type of externality problem, affect the normative properties of environmental policy actions” (page 404). These are exactly the type of questions addressed in this paper.

The plan of the paper is as follows. In section 2, we start with a presentation of the general model and in section 3 we present general analytical solutions for the first-best and second-best (‘spatially noncooperative’) policies. In section 4, we discuss the overall system effects of such noncooperative policies and finally, in section 5 we offer some concluding remarks.

## **2 The spatial model of the firms, the markets, and the externalities**

It is clear that, when studying a topic as broad as that sketched in the previous section, a large variety of modelling approaches can be followed [see van Beers and van den Bergh (1996) for an overview]. A clear trade-off, for instance, exists between the level of generality of the model, and the extent to which it is capable of producing tractable analytical results. Our aim is to keep the model as general as possible, but to economize on complexity wherever this does not seem overly restrictive, so that interpretable analytical results can still be obtained.

For that reason, we confine ourselves in the first place to a setting with only two independent regions—for example, countries. Furthermore, because of the relative complexity of the issue at hand, any other market or government failures (apart from the public environmental externality) and failures to coordinate policies optimally, are assumed to be nonexistent. This ensures that additional second-best distortions resulting from, for instance, the market power of firms or consumers, inefficient labor markets, dynamics and uncertainty, or imperfect information will not affect the analyses to follow. Although we recognize that such other market failures may undoubtedly play an important role in reality, we leave their treatment in the current context to future research, and wish to concentrate solely on the economic principles underlying the efficiency and effectiveness of various types of environmental regulation of a global externality in an otherwise ‘first-best’ spatial world.

Finally, as many ‘other things’ as possible are held constant, for the same reason. For instance, the technologies used—including abatement technologies—are assumed to be fixed; although we intend to consider endogenous technologies in our model in the near future [a nonspatial version of the type of model we plan to use can be found in Verhoef and Nijkamp (1997)]. Emission coefficients are therefore fixed, and a tax on production can in principle be as efficient as a tax on polluting inputs, for example energy, and as a tax on emissions, for instance CO<sub>2</sub>. Therefore, policy-induced technological progress is ignored in our model.

Firms are assumed to be profit maximizing price-takers on both the output and the input markets. Firms are assumed to be identical within a region, but may be different between regions. We have emphasized the importance of considering heterogeneity of firms in evaluating environmental policies elsewhere (1997; 1999); in this model, such heterogeneity would be caused by spatial factors. The assumption of identical price-taking firms within a region allows a simple derivation of regional supply functions and locally optimal environmental policies [Verhoef and Nijkamp (1999) discuss complications that arise with heterogeneity of firms within a single closed economy].

One homogeneous good is considered, which is produced and consumed in both regions, which are labeled A and B. It is assumed that the production of the good in either region, as well as transport between the regions, gives rise to environmental emissions that affect welfare in both regions. The externality is therefore public in nature.

The case of a purely public externality—where the extent to which welfare in a region is affected depends on the unweighted sum of all emissions—is a limiting case of the present specification. When the externality is not purely public, heavier weights for local emissions reflect localized externalities. The average and marginal emissions of production may vary between the regions, and the regional valuation of the total emissions may also vary.

For the modeling of the spatial market, we will use the SPE approach, which was first presented in a seminal paper by Samuelson (1952) and, later on, extensively studied and further developed by Takayama and others (see Labys and Yang, 1997; Takayama and Judge, 1971; Takayama and Labys, 1986). The SPE framework has already been proved to be a useful tool in investigating environmental policies in spatial systems in earlier papers which focused primarily on transport (Verhoef and van den Bergh, 1996; Verhoef et al, 1997). Krutilla (1989) used an SPE setting when analyzing a different type of market failure in a comparable context, namely market power by the transportation sector.

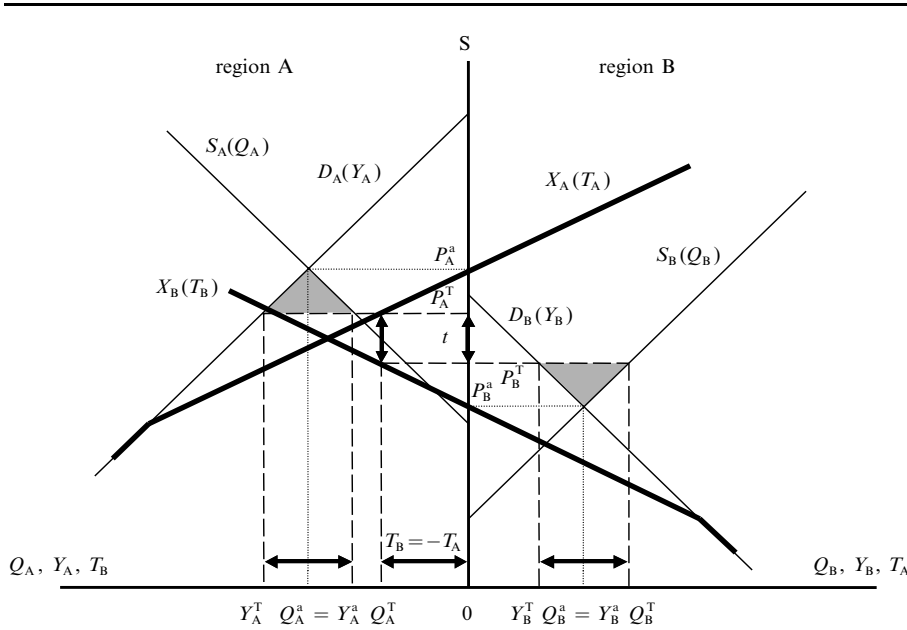
The two regions themselves are modeled as spaceless nodes, in the sense that trade within a region requires no transport (or at least, this transport can safely be ignored).<sup>(1)</sup> Trade between the regions, however, does require transport. Transport services are assumed to be offered by ‘footloose outside transport suppliers’, who reside in neither region A nor region B. Average and marginal transport costs per unit of good traded are constant and given by  $t$  (there is no congestion); and, apart from its environmental externality, transport is efficiently costed at price  $t$ . Hence, the transport supplier(s) make zero profits, so that their footloose status does not lead to a ‘welfare surplus leak’ in the model.

Figure 1 [taken from Verhoef et al (1997)] gives a diagrammatic representation of SPE. The left-hand panel shows the local inverse demand and industry supply curves  $D_A(Y_A)$  and  $S_A(Q_A)$  for the good in region A (note that figure 1 is a back-to-back diagram, so that  $Y_A$  and  $Q_A$  increase when moving leftwards from the origin). In autarky (denoted by superscripts a), equilibrium arises with a production and consumption level  $Q_A^a = Y_A^a$ , and a local price  $P_A^a$ . The right-hand panel shows the same for region B, where the autarky equilibrium is given by  $Y_B^a$ ,  $Q_B^a$ , and  $P_B^a$ .

Now suppose that the per-unit-of-good transport cost between the two regions equals  $t$ , which is smaller than the autarky price difference. It is then profitable and efficient to transport some goods from the lower to the higher price region. In figure 1, we assume that  $P_A^a - P_B^a > t > 0$ . To determine the after-trade equilibrium for both regions (denoted by superscripts T), an excess demand/supply curve,  $X(T)$  is constructed by horizontal subtraction of the industry supply curve from the demand curve. For each after-trade local price  $P_A^T > P_A^a$ ,  $P_A^T = X_A$  thus implies the net export  $T_A$  that region A would supply to B. For  $P_A^T < P_A^a$ , negative values of  $T_A$  imply positive net imports. A comparable curve is also shown for region B [labeled  $X_B(T_B)$ ].

The after-trade equilibrium in a closed system is then given by  $T_A = -T_B$  and  $|P_A^T - P_B^T| = t$ . From figure 1, it can be seen that  $P_A^T - P_B^T = t$  and that  $Q_B^T - Y_B^T = T_B = -T_A = Y_A^T - Q_A^T$ ; therefore region B is the net exporter. By observing that  $D_R$  and  $S_R$  give the marginal benefits of consumption and the marginal costs of production in region  $R$ , respectively, the two shaded areas can be identified as the net benefits of trade and transport. Both regions gain, as they should with voluntary trade. Moreover, assuming price-taking behavior and ignoring the environmental externality,

<sup>(1)</sup>That is to say, private and environmental costs of intraregional transport are not considered explicitly, but can be assumed to be included in those of local production, in which case the costs of transport for trade would actually reflect those owing to the *additional* kilometers driven for interregional trade compared with intraregional trade.



**Figure 1.** A graphical representation of the spatial price equilibrium model.

the after-trade equilibrium is Pareto efficient.<sup>(2)</sup> However, in region A the consumers benefit and the producers lose, whereas the opposite occurs in region B.

Evidently, as soon as environmental externalities enter the picture, the unregulated spatial market is unlikely to produce a Pareto-optimal outcome. For the evaluation of various policy alternatives in that case, the formal model consistent with figure 1 has to be solved. For this purpose we define the following additional variables:  $Z_{O,D}$  is the net delivery from region O (origin) to D (destination);  $T = |T_A| = |T_B|$  is the total level of transport;  $E_R(Q_A, Q_B, T)$  denotes the valuation of the total emissions by region R; and  $\tau_{PR}$  and  $\tau_{CR}$  give local production and consumption taxes, respectively, in region R.

Note that neither import/export tariffs nor taxes on transport—which are actually indistinguishable in the present model with constant technologies—will be considered. There are a number of reasons for this. First, these were already considered in Verhoef and van de Bergh (1996) and Verhoef et al (1997), where the attention was focused explicitly on transport, and on second-best transportation taxes. Second, with local production and consumption taxes present, the model would actually have an excessive number of regulatory taxes if local transportation taxes were also allowed. No unique solutions for the three tax expressions could then be derived. Therefore, the implicit transportation taxes or tariffs in the present case can be found as the difference between local production and local consumption taxes. Third, we wish to place consumption taxes in the spotlight. Such an instrument may have the advantage that the international competitiveness of local firms is not affected as strongly as in the case of taxes on production, because the relative terms of competition between local and domestic suppliers remain equal both on the local market and on the foreign market. In contrast, the local producer would suffer a comparative disadvantage on both markets from an environmental tax on production. Because the effects on

<sup>(2)</sup> This can be verified by solving the total social welfare problem (for both regions jointly), as given in equation (3) below, for the case where the valuation of emissions is zero. All regulatory taxes then become equal to zero; see equations (4a)–(4d).

international competitiveness are often put forward as an important objection against producer-oriented taxes, the consumption tax may in some cases offer an attractive alternative, and therefore certainly deserves attention.

Continuing with the definition of variables,  $\delta_{AB}$  is a dummy variable that takes on the value of 1 if  $Z_{AB} > 0$ , and 0 otherwise. Likewise,  $\delta_{BA} = 1$  ( $\delta_{AA} = 1, \delta_{BB} = 1$ ) if  $Z_{BA} > 0$  ( $Z_{AA} > 0, Z_{BB} > 0$ ), and 0 otherwise. Observe that in the SPE model,  $\delta_{AB} + \delta_{BA} \leq 1$ , cross-hauling does not occur in the standard SPE model (see also Labys and Yang, 1997). This may seem a weakness, in particular since cross-hauling often seems to occur in reality—at least if the definition of a ‘homogeneous good’ is taken to include a sufficiently large number of product variants and brands. However, under the standard ‘benchmark’ assumptions of a completely homogeneous good and perfect information, cross-hauling cannot be consistent with utility and profit-maximizing behavior. Nevertheless, the consideration of heterogeneous products, possibly close substitutes, offers an interesting question for follow-up research. In particular, it would be interesting to see how such cross-elasticities would affect the second-best tax rules to be presented below.

Throughout the paper, it is assumed that all relevant functions are differentiable, that marginal benefits are nonincreasing in consumption, that marginal costs as well as marginal emissions are nondecreasing in production, and that the marginal valuation of total emissions is nondecreasing in the total level of emissions.

Throughout the paper, we define social welfare in region  $R$  as the social surplus; here, the sum of the Marshallian consumers’ surplus (total benefits minus expenditures) *plus* the producers’ surplus (total revenues minus total costs) *plus* the tax revenues *minus* the value of the environmental effect:

$$\begin{aligned}
 W_R = & \int_0^{Y_R} D_R(y_R)dy_R - D_R(Y_R)Y_R + \tau_{CR}Y_R - \int_0^{Q_R} S_R(q_R)dq_R + S_R(Q_R)Q_R \\
 & + \tau_{PR}Q_R - E_R(Q_A, Q_B, T). \tag{1}
 \end{aligned}$$

The following equalities and inequalities will hold in any decentralized market equilibrium (with or without taxes—each  $\tau$  could, of course, be equal to zero):

$$Y_A = Z_{AA} + Z_{BA}; \tag{2a}$$

$$Y_B = Z_{AB} + Z_{BB}; \tag{2b}$$

$$Q_A = Z_{AA} + Z_{AB}; \tag{2c}$$

$$Q_B = Z_{BA} + Z_{BB}; \tag{2d}$$

$$Z_{AA} \geq 0; S_A + \tau_{PA} + \tau_{CA} - D_A \geq 0; Z_{AA}(S_A + \tau_{PA} + \tau_{CA} - D_A) = 0; \tag{2e}$$

$$Z_{AB} \geq 0; S_A + \tau_{PA} + t + \tau_{CB} - D_B \geq 0; Z_{AB}(S_A + \tau_{PA} + t + \tau_{CB} - D_B) = 0; \tag{2f}$$

$$Z_{BA} \geq 0; S_B + \tau_{PB} + t + \tau_{CA} - D_A \geq 0; Z_{BA}(S_B + \tau_{PB} + t + \tau_{CA} - D_A) = 0; \tag{2g}$$

$$Z_{BB} \geq 0; S_B + \tau_{PB} + \tau_{CB} - D_B \geq 0; Z_{BB}(S_B + \tau_{PB} + \tau_{CB} - D_B) = 0. \tag{2h}$$

Equations (2a)–(2h) give a straightforward description of a decentralized equilibrium with price-takers, where all markets clear and where positive deliveries imply that the marginal benefits must be equal to the marginal cost.

This completes the discussion of the general model. In the next section, the policy rules are derived under various circumstances.

**3 First-best and the second-best taxes**

In this section, we provide the analytical solutions to the model outlined above for first-best and second-best production and consumption taxes. In section 3.1 we present the intuitive result that to obtain the first-best spatioeconomic configuration, Pigouvian taxes should apply throughout. In the subsequent sections we provide the solutions for various second-best situations, which have in common that the two regions do not coordinate their policies, but instead set taxes so as to optimize regional welfare only. In section 3.2 we consider uncoordinated second-best taxes for a region using both a production and a consumption tax. In sections 3.3 and 3.4 we subsequently consider the use of a production tax and a consumption tax in isolation.

**3.1 First-best taxes**

First we consider the solution to the problem of maximizing total welfare in the two economies considered. This gives us the Pareto-efficient configuration, and the first-best taxes that would apply if either a supraregional regulator could control the entire system, or if the two regions could coordinate their policies optimally. The solution can be found by maximizing total welfare for both regions as given in equation (1), subject to constraints posed by individual optimizing behavior as given by equations (2e)–(2h), under the assumption that the regulator can use all possible taxes, and therefore is not constrained to use imperfect second-best taxes. Using the dummy variables  $\delta$  introduced above, we can write the associated Lagrangian as:

$$\begin{aligned}
 A = & \int_0^{Y_A} D_A(y_A)dy_A - D_A(Y_A)Y_A + \tau_{CA}Y_A - \int_0^{Q_A} S_A(q_A)dq_A + S_A(Q_A)Q_A \\
 & + \tau_{PA}Q_A - E_A(Q_A, Q_B, T) + \int_0^{Y_B} D_B(y_B)dy_B - D_B(Y_B)Y_B + \tau_{CB}Y_B \\
 & - \int_0^{Q_B} S_B(q_B)dq_B + S_B(Q_B)Q_B + \tau_{PB}Q_B - E_B(Q_A, Q_B, T) \\
 & + \delta_{AA}\lambda_{AA}[S_A(Q_A) + \tau_{PA} + \tau_{CA} - D_A(Y_A)] \\
 & + \delta_{AB}\lambda_{AB}[S_A(Q_A) + \tau_{PA} + t + \tau_{CB} - D_B(Y_B)] \\
 & + \delta_{BA}\lambda_{BA}[S_B(Q_B) + \tau_{PB} + t + \tau_{CA} - D_A(Y_A)] \\
 & + \delta_{BB}\lambda_{BB}[S_B(Q_B) + \tau_{PB} + \tau_{CB} - D_B(Y_B)]. \tag{3}
 \end{aligned}$$

The specification in equation (3) reveals that the structure of the optimization problem depends on the exact trade regime that applies (that is, the questions of whether the various deliveries  $Z_{ij}$  are either zero or positive), and hence that different first-order conditions will apply depending on the prevailing trade regime. The problem could be written in a Kuhn–Tucker form to capture these complications, but for the present purpose—where we wish to present optimal tax rules for each trade regime—this is not necessary. Moreover, we want to avoid complications due to nonnegativity restrictions on Kuhn–Tucker multipliers (see also Verhoef and van den Bergh, 1996).

The Lagrangian equation (3) can be solved by taking partial derivatives with respect to the variables  $Z$ , by using equations (2a)–(2d),  $\tau$ , and  $\lambda$ . The first-order conditions are not presented for reasons of space; they are analogous to those for the Lagrangian in equation (5) below, the main two differences being that, in this case, four taxes instead of two can be set optimally, and that the global instead of the regional

marginal valuation of externalities should be considered. The following first-best taxes can then be derived:

$$\tau_{CA} + \tau_{PA} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_B}{\partial Q_A}; \quad (4a)$$

$$\tau_{PA} + \tau_{CB} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_B}{\partial Q_A} + \frac{\partial E_A}{\partial T} + \frac{\partial E_B}{\partial T}; \quad (4b)$$

$$\tau_{CA} + \tau_{PB} = \frac{\partial E_A}{\partial Q_B} + \frac{\partial E_B}{\partial Q_B} + \frac{\partial E_A}{\partial T} + \frac{\partial E_B}{\partial T}; \quad (4c)$$

$$\tau_{CB} + \tau_{PB} = \frac{\partial E_A}{\partial Q_B} + \frac{\partial E_B}{\partial Q_B}. \quad (4d)$$

Equations (4a)–(4d) show that each delivery, intraregional or interregional, should be taxed at a rate equal to the marginal external cost associated with that particular type of delivery, which is the standard Pigouvian tax. Note that each tax is defined as the sum of a tax on the producers and a tax on the consumers involved. Because, at most, three equations from the set (4a)–(4d) apply simultaneously, whereas four taxes are available, there is always at least one redundant tax. Therefore, the four taxes can always be set such that the relevant conditions among equations (4a)–(4d) are satisfied.

### 3.2 Second-best taxes (I): optimizing regional welfare using both production and consumption taxes

The policy rules derived above will result in a Pareto-efficient spatioeconomic configuration where the joint welfare of the two regions is maximized. Hence, if the regions fully cooperated by optimally coordinating their policies and sharing tax revenues according to rules such that both regions gain from marginal total (for both regions) welfare improvements, then tax rules according to equations (4a)–(4d) would apply. Note also that ‘partial’ welfare losses due to optimal taxation, in terms of reduced consumers’ or producers’ surpluses in either region, compared with any other feasible configuration, could be more than compensated for. This follows from the tautological observation that first-best taxes lead to a maximization of total welfare.

Nevertheless, regulatory taxes as given in equations (4a)–(4d) do not normally constitute a Nash equilibrium. In particular, there will often be incentives for a regional government to deviate from the tax rules implied by these equations. In this subsection, we will investigate this matter by deriving the tax rules to be adopted by a regional government that is solely concerned with local regional welfare when setting consumption and production taxes. Such a regulator fully ignores welfare in the other region, and thus may even ‘exploit’ foreign consumers or producers whenever this would improve the welfare in the home region.

In the absence of policy coordination, we assume that both regions’ governments follow a Nash strategy. The ‘game’ is defined such that both governments set the taxes they control so as to maximize local surplus, treating the tax levels in the other region parametrically (as given), and taking into account that all producers and consumers exhibit price-taking behavior that leads to the spatial price equilibrium as described in equations (2a)–(2h).



Let us consider the relevant maximization problem for the regulator in region A. The associated Lagrangian can be written as:

$$\begin{aligned}
A = & \int_0^{Y_A} D_A(Y_A) dY_A - D_A(Y_A) Y_A + \tau_{CA} Y_A - \int_0^{Q_A} S_A(q_A) dq_A + S_A(Q_A) Q_A \\
& + \tau_{PA} Q_A - E_A(Q_A, Q_B, T) + \delta_{AA} \lambda_{AA} [S_A(Q_A) + \tau_{PA} + \tau_{CA} - D_A(Y_A)] \\
& + \delta_{AB} \lambda_{AB} [S_A(Q_A) + \tau_{PA} + t + \tau_{CB} - D_B(Y_B)] \\
& + \delta_{BA} \lambda_{BA} [S_B(Q_B) + \tau_{PB} + t + \tau_{CA} - D_A(Y_A)] \\
& + \delta_{BB} \lambda_{BB} [S_B(Q_B) + \tau_{PB} + \tau_{CB} - D_B(Y_B)]. \tag{5}
\end{aligned}$$

The following first-order conditions can now be derived (primes denote derivatives for functions in one single argument, in particular the demand and supply functions):

$$\begin{aligned}
\frac{\partial A}{\partial Z_{AA}} = & -D'_A Y_A + \tau_{CA} + S'_A Q_A + \tau_{PA} - \frac{\partial E_A}{\partial Q_A} \\
& + \delta_{AA} \lambda_{AA} (S'_A - D'_A) + \delta_{AB} \lambda_{AB} S'_A - \delta_{BA} \lambda_{BA} D'_A = 0; \tag{6}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial A}{\partial Z_{AB}} = & S'_A Q_A + \tau_{PA} - \frac{\partial E_A}{\partial Q_A} - \frac{\partial E_A}{\partial T} \\
& + \delta_{AA} \lambda_{AA} S'_A + \delta_{AB} \lambda_{AB} (S'_A - D'_B) - \delta_{BB} \lambda_{BB} D'_B = 0; \tag{7a}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial A}{\partial Z_{BA}} = & -D'_A Y_A + \tau_{CA} - \frac{\partial E_A}{\partial Q_B} - \frac{\partial E_A}{\partial T} \\
& - \delta_{AA} \lambda_{AA} D'_A + \delta_{BA} \lambda_{BA} (S'_B - D'_A) + \delta_{BB} \lambda_{BB} S'_B = 0; \tag{7b}
\end{aligned}$$

$$\frac{\partial A}{\partial Z_{BB}} = -\frac{\partial E_A}{\partial Q_B} - \delta_{AB} \lambda_{AB} D'_B + \delta_{BA} \lambda_{BA} S'_B + \delta_{BB} \lambda_{BB} (S'_B - D'_B) = 0; \tag{8}$$

$$\frac{\partial A}{\partial \tau_{CA}} = Y_A + \delta_{AA} \lambda_{AA} + \delta_{BA} \lambda_{BA} = 0; \tag{9}$$

$$\frac{\partial A}{\partial \tau_{PA}} = Q_A + \delta_{AA} \lambda_{AA} + \delta_{AB} \lambda_{AB} = 0; \tag{10}$$

$$\frac{\partial A}{\partial \lambda_{AA}} = \delta_{AA} (S_A + \tau_{PA} + \tau_{CA} - D_A) = 0; \tag{11}$$

$$\frac{\partial A}{\partial \lambda_{AB}} = \delta_{AB} (S_A + \tau_{PA} + t + \tau_{CB} - D_B) = 0; \tag{12a}$$

$$\frac{\partial A}{\partial \lambda_{BA}} = \delta_{BA} (S_B + \tau_{PB} + t + \tau_{CA} - D_A) = 0; \tag{12b}$$

$$\frac{\partial A}{\partial \lambda_{BB}} = \delta_{BB} (S_B + \tau_{PB} + \tau_{CB} - D_B) = 0. \tag{13}$$

Because of the absence of cross-hauling, at most either one of the equations (7a) and (7b) will apply, and equations (6) and (8) only apply if  $Z_{AA} > 0$  and  $Z_{BB} > 0$ .

However, in order to restrict the number of cases to be considered, it will be assumed from now on that  $Z_{AA} > 0$  and  $Z_{BB} > 0$ , and hence that  $\delta_{AA} = 1$  and  $\delta_{BB} = 1$ . Second-best tax rules for the more restrictive cases where either of the two regions lacks operative producers or consumers can be derived in a similar manner as the tax rules to be derived below, and can be expected to be straightforward variations on these rules, with terms relating to the absent consumers or producers either removed or amended in an appropriate way.

Under the assumption that  $\delta_{AA} = 1$  and  $\delta_{BB} = 1$ , three trade regimes remain to be considered: A is a net exporter (regime I:  $\delta_{AB} = 1$  and  $\delta_{BA} = 0$ ); A is a net importer (II:  $\delta_{AB} = 0$  and  $\delta_{BA} = 1$ ); and the no-trade regime (III:  $\delta_{AB} = \delta_{BA} = 0$ ). The second-best taxes for these three regimes are presented below. Note that the inefficiency owing to the public nature of the externality is already reflected in equations (6)–(8) by the absence of terms related to  $E_B$ ; the regulator in region A is, without policy coordination, not concerned with the environmental effects in region B.

### 3.2.1 Region A is a net exporter

In case I, equations (7b) and (12b) become irrelevant. The remaining equations can then be solved for the following second-best optimal taxes:

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_A}{\partial T} - \frac{\partial E_A}{\partial Q_B} \left/ \left( 1 + \frac{S'_B}{-D'_B} \right) \right. + (Q_A - Y_A) \left/ \left( -\frac{1}{D'_B} + \frac{1}{S'_B} \right) \right.; \quad (14)$$

$$\tau_{CA} + \tau_{PA} = \frac{\partial E_A}{\partial Q_A}. \quad (15)$$

First, observe that both taxes will generally be nonzero. Next, note that equation (15) could easily be solved for  $\tau_{CA}$  alone by substitution of equation (14); the present representation merely facilitates the interpretation. In particular, equation (15) shows that the sum of  $\tau_{CA}$  and  $\tau_{PA}$  should be such that local deliveries are taxed according to the standard Pigouvian rule based on the local valuation of marginal emissions from local production. Hence, local production is regulated according to the ‘quasi-first-best’ tax rule, which only ignores the transboundary effects of local emissions.

Equation (14) shows the remaining net tax on exports. Apart from the same quasi-first-best tax rule on marginal emissions from local production, three additional terms appear. These reflect various second-best considerations that are relevant in the present setting. The second term in equation (14), for instance, captures the local impact of emissions from transport, and has the same quasi-first-best structure as the right-hand side of equation (15). As one might suspect, these external costs are treated in exactly the same manner as those from local production.

Next come two terms that both have a typical second-best structure. The numerator represents a local welfare component affected by the production tax—the marginal local valuation of foreign emissions for the third term, and a terms-of-trade effect for the fourth. These terms are multiplied by a weighting factor that may vary between 0 and 1 for the third term and between 0 and infinity for the fourth term. The weighting factor represents the relative importance of these welfare components for the optimal use of the tax, compared with the components that are directly affected by the tax (local and transport emissions). Such second-best taxes have been discussed extensively in the literature; Krutilla (1999), for instance, discusses production taxes and trade tariffs. We therefore refrain from in-depth discussions of all tax expressions to be derived in this paper. However, to demonstrate the sort of interpretation given to such tax components, we will discuss the two final terms in equation (14) in some depth.

The third term in equation (14) is related to the local valuation, in region A, of emissions from production in region B, and captures the marginal impact that local

taxation of exports will have on emissions by foreign producers. A higher tax leads to a comparative disadvantage of the local suppliers on the foreign market, and therefore generally leads to an increase in foreign production for the foreign market, and hence to increased emissions. The weighting factor,  $w = -[1 + (S'_B / -D'_B)]^{-1}$ , is nonpositive and generally negative ( $D'_B$  denotes the slope of the demand curve in B and is non-positive,  $S'_B$  gives the slope of the supply curve and is nonnegative), and may run from 0 to 1. This indicates that in the two limiting cases the local valuation of emissions by foreign producers can be completely ignored ( $w = 0$ ), or (when  $w = 1$ ) should be considered equally important, but opposite in sign, as the tax on emissions by local producers and transport. It is instructive to consider these limiting cases somewhat further; for intermediate cases where  $0 < w < 1$ , the various processes to be indicated below are traded off according to the expression for  $w$ .

First,  $w = 0$  can occur when the foreign demand is perfectly elastic in the second-best optimum:  $D'_B = 0$ . Local taxes in A cannot then affect the intraregional delivery in B, owing to the insensitivity of the market price in region B. Second,  $w = 0$  can occur when the foreign supply is perfectly inelastic:  $S'_B = \infty$ . Also then, the intraregional delivery in B will not be affected by policies in A, because the foreign production level is given. In either case, the policies in A will leave the local production in B and its emissions unaffected, so that the potential interaction can be ignored completely in the second-best tax rule.

In contrast,  $w = 1$  occurs in the opposite cases where either foreign demand is perfectly inelastic ( $D'_B = -\infty$ ), or its supply is perfectly elastic ( $S'_B = 0$ ) in the second-best optimum. In both cases, on the market in B, every unit of good produced by A that is 'priced off the market' as a result of the regulation in A itself will be replaced by a unit of good produced in B. With  $D'_B = -\infty$ , this results directly from the inelasticity of the demand. With  $S'_B = 0$ , the market price in B cannot be influenced by region A, and therefore neither can the total consumption in B. Recalling that intraregional deliveries in region A itself are fine-tuned using equation (15), the result that, in cases where  $w = 1$ , the tax on exports should be corrected by fully subtracting the marginal external costs caused by production in region B then follows directly. Owing to the complete substitution, the 'net' marginal external cost of production in A for the foreign market is  $\partial E_A / \partial Q_A + \partial E_A / \partial T - \partial E_A / \partial Q_B$ . Indeed, if production in B for the local market is more polluting than production in A and transporting goods to B, the sum of the first three terms in equation (14) becomes negative, showing that subsidization of exports is, in such cases, beneficial from an environmental point of view.

The fourth and last term in equation (14) represents the so-called 'terms-of-trade' effect. Krutilla (1999) provides an in-depth discussion of this element in second-best tax rules. In the present case, where A is a net exporter, this term represents the revenues that can be extracted from the foreign market by taxing local production for the purpose of driving up prices in region B. The numerator shows that the advantage of doing so increases linearly with the exports to region B. The denominator shows that if either the demand or the supply in the foreign market is perfectly elastic ( $D'_B = 0$  or  $S'_B = 0$ ), this term vanishes in the second-best tax rule. In that case, taxation for the purpose of affecting the terms of trade becomes useless; prices are given in region B, and taxes would only distort the essentially efficient market process in A (note that for the terms-of-trade effect per se, inefficiencies as a result of environmental externalities do not play a role). The more inelastic the excess demand in region B, the larger the 'market power' enjoyed by the regulator in A, and hence the more strongly he would be inclined to exploit this power.

### 3.2.2 Region A is a net importer

In case II, equations (7a) and (12a) become irrelevant among the first-order conditions [equations (6)–(13)]. By solving the remaining set of equations, the following second-best optimal taxes are yielded:

$$\tau_{CA} = \frac{\partial E_A}{\partial T} + \frac{\partial E_A}{\partial Q_B} \bigg/ \left( 1 + \frac{S'_B}{-D'_B} \right) + (Y_A - Q_A) \bigg/ \left( \frac{1}{-D'_B} + \frac{1}{S'_B} \right); \quad (16)$$

$$\tau_{CA} + \tau_{PA} = \frac{\partial E_A}{\partial Q_A}. \quad (17)$$

Because equation (17) is identical to equation (15), intraregional deliveries in region A are again taxed according to the ‘quasi-first-best’ tax rule. The difference between this and case I is that now the local consumption tax [equation (16)] is adapted to deal with second-best aspects as a result of the openness of the economy. This is what one could expect for a net importer.

The first term in equation (16) shows how the local valuation of emissions from transport should, according to the quasi-first-best principle, enter the tax rule for imports. The second term shows how the local valuation of emissions from foreign production should be added to this. Apart from differing in sign—because now a local consumption tax instead of a production tax is used to influence emissions abroad—this term is identical to the third term in equation (14). Finally, the third term again represents the terms-of-trade effect. In this case, a higher tax on imports will have a depressing effect on the market price in region B, and will improve region A’s terms of trade.

### 3.2.3 No trade

Finally, in the no-trade equilibrium, equations (7a), (7b), (12a), and (12b) become irrelevant. The first-order conditions can then be solved to yield:

$$\tau_{CA} + \tau_{PA} = \frac{\partial E_A}{\partial Q_A}. \quad (18)$$

Local deliveries should be taxed according to the quasi-first-best Pigouvian principle. Because no direct interactions exist between the two regions, other than environmental spillovers, there is obviously no ground for adopting local tax policies for the purpose of affecting emissions from transport or from production in the other region, or for the purpose of improving the terms of trade.

### 3.3 Second-best taxes (II): optimizing regional welfare using production taxes only

The tax scheme presented in section 3.2 involves the joint use of two different taxes. Under the assumption of noncooperation, this yields the optimal outcome for a region, since taxes can be differentiated for the two commodity flows (intraregional deliveries and interregional trade) that the region can directly affect. In practice, however, such a scheme may be considered as too complicated, and regulators may rely on simpler solutions, involving the use of one single tax only. We will therefore now present the second-best tax rules for production taxes (this subsection) and consumption taxes (section 3.4).

In order to find the second-best production taxes without interregional policy coordination, the Lagrangian in equation (5) and the first-order conditions in equations (6)–(13) should be adapted by setting the term  $\tau_{CA}$  equal to zero and removing equation (9). The implied system of equations can then be solved for the three possible trade regimes. Again, assuming that for both regions intraregional deliveries are positive in all second-best optima, the tax rules given below can then be found:

case I—region A is a net exporter,

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_A}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_A}{-D'_B} + \frac{-D'_A}{S'_B} \right)^{-1} \right] \right. - \frac{\partial E_A}{\partial Q_B} \left/ \left( 1 + \frac{S'_B}{-D'_B} + \frac{S'_B}{-D'_A} \right) \right. + (Q_A - Y_A) \left/ \left( \frac{1}{-D'_A} + \frac{1}{-D'_B} + \frac{1}{S'_B} \right) \right.; \quad (19)$$

case II—region A is a net importer,

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} - \frac{\partial E_A}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_A}{-D'_B} + \frac{-D'_A}{S'_B} \right)^{-1} \right] \right. - \frac{\partial E_A}{\partial Q_B} \left/ \left( 1 + \frac{S'_B}{-D'_B} + \frac{S'_B}{-D'_A} \right) \right. - (Y_A - Q_A) \left/ \left( \frac{1}{-D'_A} + \frac{1}{-D'_B} + \frac{1}{S'_B} \right) \right.; \quad (20)$$

case III—no trade,

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A}. \quad (21)$$

First of all, comparing equation (21) with equation (18) reveals that in a second-best optimum without trade, the unavailability of the second tax—the consumption tax in this case—creates no additional constraints, and hence no additional welfare losses compared with the case discussed in section 3.2. This is consistent with the redundancy of one of the two taxes in equation (18). The interpretation of equation (21) is therefore identical to that of equation (18).

Equations (19) and (20) show that in second-best equilibria with trade, however, things will change when the consumption tax can no longer be used. Evidently, the local welfare level achieved will normally be below, and never above, the level obtained from the combinations of equations (14) and (15) for a net exporter, and equations (16) and (17) for a net importer. This follows from the fact that the second-best optimal consumption taxes [equations (15) and (16)] can always be set at zero.

The taxes in equations (19) and (20) are, after nullifying the transport and foreign pollution components, consistent with tax rules (13) and (27.1) in Krutilla (1991) and (1999), respectively. Note that equation (19) is in fact a ‘weighted average’ of the tax rules in equations (14) and (15), that also apply when a separate consumption tax is available. One has to be careful with the term ‘weighted average’, because it should, of course, not be ignored that the terms involved in the optimal tax rules may differ for the respective second-best equilibria, owing to the possible variability of the terms involved. Still, when the local demand is perfectly inelastic ( $D'_A = -\infty$ ), equation (19) becomes identical to equation (14), which is the net tax on exports when both taxes are available. In contrast, when the local demand is perfectly elastic ( $D'_A = 0$ ), the last three terms vanish and equation (19) becomes identical to equation (15), which is the net tax on intraregional deliveries when both taxes are available. In that way, the two subgoals of regulating intraregional deliveries and exports are traded off when only one tax is available, and the relative weights given to these subgoals depend on the elasticity of the local demand.

The same reasoning actually holds for the tax rule in equation (20), which is applied, for case II, where region A is a net importer. This can be verified by substituting equation (16) into equation (17). It then also follows immediately that when the local demand is perfectly inelastic ( $D'_A = -\infty$ ), equation (20) becomes identical to  $\tau_{PA}$  [implied by equations (16) and (17)], and when the local demand is perfectly elastic ( $D'_A = 0$ ), equation (20) becomes identical to the net tax on intranodal deliveries [equation (17)].

### 3.4 Second-best taxes (III): optimizing regional welfare using consumption taxes only

Finally, in order to find the second-best consumption tax without interregional policy coordination, the Lagrangian equation (5) and the first-order conditions given in equations (6)–(13) should be adapted by setting  $\tau_{PA}$  equal to zero and by removing equation (10). The implied system of equations can again be solved for the three possible trade regimes. Maintaining the assumption that for both regions, intraregional deliveries are positive in all second-best optima, the following tax rules can then be found:

Case I—region A is a net exporter

$$\tau_{CA} = \frac{\partial E_A}{\partial Q_A} \left/ \left( 1 + \frac{S'_A}{-D'_B} + \frac{S'_A}{S'_B} \right) \right. - \frac{\partial E_A}{\partial T} \left/ \left[ 1 + \left( \frac{S'_A}{-D'_B} + \frac{S'_A}{S'_B} \right)^{-1} \right] \right. \\ + \frac{\partial E_A}{\partial Q_B} \left/ \left( 1 + \frac{S'_B}{-D'_B} + \frac{S'_B}{S'_A} \right) \right. - (Q_A - Y_A) \left/ \left( \frac{1}{-D'_B} + \frac{1}{S'_A} + \frac{1}{S'_B} \right) \right.; \quad (22)$$

Case II—region A is a net importer

$$\tau_{CA} = \frac{\partial E_A}{\partial Q_A} \left/ \left( 1 + \frac{S'_A}{-D'_B} + \frac{S'_A}{S'_B} \right) \right. + \frac{\partial E_A}{\partial T} \left/ \left[ 1 + \left( \frac{S'_A}{-D'_B} + \frac{S'_A}{S'_B} \right)^{-1} \right] \right. \\ + \frac{\partial E_A}{\partial Q_B} \left/ \left( 1 + \frac{S'_B}{-D'_B} + \frac{S'_B}{S'_A} \right) \right. + (Y_A - Q_A) \left/ \left( \frac{1}{-D'_B} + \frac{1}{S'_A} + \frac{1}{S'_B} \right) \right.; \quad (23)$$

Case III—no trade

$$\tau_{CA} = \frac{\partial E_A}{\partial Q_A}. \quad (24)$$

The interpretation of equations (22)–(24) can be given along the same lines as that of equations (19)–(21). Now the important additional factor, compared with the case where both taxes are available, is the sensitivity of local supply in the second-best optimum, reflected by the slope of the supply curve  $S'_A$ . With perfectly inelastic local supply ( $S'_A = \infty$ ), equation (22) becomes identical to the expression for  $\tau_{CA}$  according to equations (14) and (15), and equation (23) becomes identical to equation (16); with perfectly elastic supply ( $S'_A = 0$ ), both equations (22) and (23) become identical to equations (15) and (17). Finally, it is again found that in a no-trade second-best equilibrium, local deliveries should be taxed according to the quasi-first-best rule of equation (24).

### 3.5 One tax only: production tax or consumption tax?

The final question we want to address in this section concerns the question of which of the two above taxes—production or consumption—would be preferable in a case where the regulator is, for whatever reason, restricted to making this choice. Evidently, if regulation were costless, it is always preferable to use both; as argued above, one of the two can always be set at a zero level, and higher levels of regional welfare generally result when equations (14) and (15), or (16) and (17), are applied simultaneously. However, when, for instance, administrative costs are considered (which are not modeled explicitly here) the situation may become different and, especially when raising an additional tax involves some fixed costs, it may be preferable to use only one tax. In such cases, this question becomes relevant.

A general answer to this question does not exist. That is, it is *not* the case that either production taxes or consumption taxes by definition always allow the regulator to obtain a higher level of regional welfare than would be achievable with the other tax.

In general, however, the tendency will be that net exporters prefer to use production taxes, and net importers prefer consumption taxes. The intuition is that they can then directly affect a larger share of the total production in the entire system. Moreover, the

various subgoals reflected in the second-best tax rules then require, at least to a larger degree, equally signed tax terms. For a net importer, this is evident because equation (23) consists of four positive terms, whereas equation (20) has one positive term and three negative ones. For a net exporter, the difference is somewhat more subtle: equation (19) has three positive terms and one negative one, and equation (22) two positive terms and two negative ones. In general, different signs in a second-best tax rule indicate counteracting forces in the determination of the second-best optimal tax rule—the various subgoals require opposite incentives. The more this can be avoided, the more efficiently the tax can be used. In the extreme case, where the various subgoals would exactly cancel and sum up to a second-best tax equal to zero, the instrument is entirely ineffective and inefficient. The best strategy, then, is not to use the tax at all, since any value different from zero—be it positive or negative—implies a lower welfare level than that obtained when the tax is not used and is set equal to zero.

Along this same line of reasoning, it is also possible to demonstrate that it is not generally true that a net exporter would prefer production taxes, and a net importer consumption taxes. To do so, it is sufficient to construct a counterexample in terms of a set of parameter values, for which the second-best production tax for the net exporter would be equal to zero whereas the second-best consumption tax would not, and conversely, the second-best consumption tax for the net importer would be equal to zero whereas the second-best production tax would not.

For the net exporter, this would, for instance, be the case when equation (19) is equal to zero whereas equation (22) is not—for instance, if the net exporter's local demand is perfectly elastic ( $D'_A = 0$ ), and the local producers do not emit any local externalities ( $\partial E_A / \partial Q_A = 0$ ). The production tax then is indeed an entirely inefficient instrument. Because of the perfectly elastic local demand, the after-production-tax producers' price in the exporting region cannot be affected by the production tax. This also implies that the trade and transport flow cannot be affected. Hence, the only delivery that can be affected is the local intraregional delivery. However, because local production causes no local externalities at all, it is actually welfare reducing to affect this intraregional delivery. In particular, any deviation from the market outcome with a zero production tax would imply a welfare deterioration, because a production tax would drive a (welfare-reducing) wedge between regional marginal benefits and regional marginal costs of local deliveries. Since equation (22) is certainly not necessarily equal to zero when  $D'_A = 0$  and  $\partial E_A / \partial Q_A = 0$ , it follows that a local consumption tax may yield regional welfare improvements, and hence would be preferable to a local production tax under the described circumstances.

Likewise, one can construct a comparable situation where, for a net importer, the tax rule in equation (23) would produce a second-best optimal consumption tax equal to zero, implying no possible welfare gain whatsoever, whereas the second-best production tax rule in equation (20) would be unequal to zero, indicating that welfare improvements can be achieved. This would involve the situation where the net importer's local supply is perfectly elastic ( $S'_A = 0$ ), and the local producers do not emit any local externalities ( $\partial E_A / \partial Q_A = 0$ ). A consumption tax would then be unable to affect the (after-tax) producers' price, implying that the trade and transport volume is given. The same sort of reasoning as given above therefore again applies that only the intraregional delivery—not the trade and transport volume—can be affected. However, this delivery is optimized already under unregulated free-market conditions.

We therefore conclude that if, for whatever reason, a regulator is restricted to using only one tax instrument, the tendency will be that net exporters prefer to use production taxes, and net importers to use consumption taxes. However, counterexamples where the opposite case holds can easily be constructed, and hence this inference is not generally true.

#### 4 The overall system effects of second-best policies

If the two regions cooperate, they would, of course, be best off applying the best tax scheme given in equations (4a)–(4d). Provided that the regions also negotiate on the distribution of the tax revenues, compensation schemes, by definition, must exist that also maximize the individual regional welfare levels. Therefore, both from an overall system's perspective and from the regional perspective, such a tax scheme could be welfare maximizing. On the other hand, the schemes discussed in sections 3.2–3.4 describe situations where regions, under various restrictions, aim to maximize only regional welfare, taking taxes in the other region as given. The question then is how these policies compare, from the overall system's perspective, to the two benchmarks one could distinguish, namely the nonintervention outcome and the first-best situation. In other words, the question is what implicit price is paid for the failure of the two governments to cooperate in formulating joint welfare-maximizing policies.

Based on the general specifications used in this paper, no conclusive answers can be given to this question, since comparative static analyses can be performed only after an explicit model has been specified. This would have the obvious disadvantage that the level of generality is brought down considerably. However, based on the tax rules presented in section 3, some first inferences can still be made. For that purpose, it can, in the first place, be noted that, even if we restrict ourselves to equilibria with trade, no fewer than seventeen regimes can be considered: the optimal one presented in section 3.1; and the  $4 \times 4$  regimes where both regions can employ no tax at all, both second-best taxes as in section 3.2, a production tax as in section 3.3, or a consumption tax as in section 3.4. Another five additional cases become relevant when we also consider equilibria without trade: optimal taxes; and four noncoordinated tax regimes, where both regions can choose whether or not to apply the second-best noncoordinated tax.

Among the resulting twenty-two possible schemes, we limit ourselves in the first place only to equilibria with trade. Table A1 (in the appendix) shows the possible taxes that both regions (the net exporter, E, and the net importer, I) can apply. The tax rules shown in table A1 follow directly from those derived in section 3. The table shows the marginal effect of a tax on the three types of delivery (intraregional deliveries within both regions, and the unidirectional interregional trade flow), and thus enables one to assess the marginal tax rates in each of the seventeen regimes with trade that would apply for each of these deliveries (rows representing 'zero-taxes' are left out of the table). Note that for the determination of the marginal tax on the interregional delivery, one should sum two expressions (from the two columns) because both regions will normally tax (or subsidize) this delivery.

Next, we limit ourselves further by considering only four tax regimes among these seventeen. These are, in the first place, the two benchmarks, whether either the optimal taxes or no taxes at all apply. We concentrate, however, on two second-best regimes, namely the case where both regions apply both taxes, and the case where the net exporter uses a production tax and the net importer a consumption tax. By using table A1, one can then derive the marginal tax rates for each of the three deliveries— $Z_{EE}$ ,  $Z_{II}$ , and  $Z_{EI}$ —in each of the three regimes with taxes. These are presented in table A2. Therefore, the tax rates shown in table A2 indicate the differences between these three tax regimes, and the situation with no taxes at all.

For the interpretation of the second-best tax rules, in particular, it is tempting to compare these rules directly with those that apply in the first-best situation. It is important to emphasize, however, that this exercise has only a limited comparative static relevance, because many of the terms involved in the various tax rules may vary between equilibria (for instance, the optimum compared to one of the second-best equilibria). Therefore, differences between the expressions in table A1 and table A2



(that are shown in table A3) would straightforwardly give the difference in actual equilibrium tax rates only if all terms involved were constants. Although this could still be true for the various marginal external costs and the slopes of the relevant supply and demand curves, it is hard to see how this could be the case for the trade balance,  $T$ . Consequently, the expressions presented in tables A1 and A2 should be compared only with sufficient care, and differences should be interpreted only as listings of relevant terms lacking from (or 'falsely' included in) the second-best marginal tax rules, compared with optimal rules.

Now, if we first consider the regime with both taxes (regime BB), and focus on taxes on both intraregional deliveries, it appears that the relevant expressions suggest tax rates between those applying in the no-tax situation, and best taxes. Although the cross-effects in the optimal taxes, accounting for spatial externality spillovers, are lacking, the remaining terms still indicate positive taxes. For the tax on intraregional deliveries, however, things are more complex. The resulting rule may imply a negative tax (when all externalities except  $\partial E_E / \partial Q_i$  are zero and  $S'_E = S'_I = 0$ ), may take on a nonnegative value below the optimal level, or may suggest a tax exceeding the first-best tax (for instance when environmental externalities are absent or negligible). As long as the tax rule suggests a positive tax below the optimal level, one would therefore expect the non-cooperative equilibrium, where both regions apply both taxes, to result in a below-optimal welfare level, but still an improvement compared with the situation where no taxes at all apply. However, if the intraregional delivery is relatively important, and if the relevant parameters dictate an overall tax on this delivery that is either negative or exceeds the optimal rule, it could in theory be possible that the total welfare in the system even falls below the level with no taxes. Hence, a general conclusion is not possible.

If we next look at the marginal tax rules applying in the situation where the net exporter uses production taxes and the net importer consumption taxes (regime PC), it can be noted that the variability increases compared with the situation where both regions apply both taxes. The tax on the intraregional delivery in the exporting region may become negative, may be positive but below the tax in regime BB, may be between the tax in regime BB and the optimal tax, or may exceed the optimal tax. The tax on the intraregional delivery in the importing region, although strictly nonnegative, may be below the tax in region BB or may exceed it, and could also exceed the optimal tax. Finally, if we compare the tax on the interregional delivery in regimes PC and BB, it can be seen that the former contains one extra positive term, but that the denominators in the subsequent terms are usually larger. Again, therefore, no general conclusion on the relative size of this tax, compared with other regimes, can be given.

Clearly, if we do not consider a full equilibrium model, and compare the marginal tax rules in the various regimes, no general conclusion on the relative performance of the two second-best regimes can be given. The question of how the BB and PC regimes compare with each other is an open one. However, it can be noted that for most parameter combinations, one would expect a welfare level exceeding the level resulting in the no-tax regime for both the BB and the PC regimes. Nevertheless, given the relative complexity of the various tax rules, and given the neglect of the endogeneity of the relevant terms present in these rules, it seems worthwhile to consider these questions further by using a numerical simulation model.

## 5 Conclusion

The international and spatial dimension of environmental pollution and environmental policymaking has recently received ample attention in international negotiations on environmental protection, as well as in the environmental economics literature. In this paper, we presented an analytical framework for analysing spatial aspects of

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environmental policies in the regulation of transboundary externalities. A spatial price equilibrium model for two regions was discussed, where interactions between the regions can occur via trade and transport, via mutual environmental spillovers owing to the externality that arises from production, and via uncoordinated taxes when the regions do not behave cooperatively. Also, the additional complications arising from emissions caused by the endogenous transport flows are considered explicitly in the model presented. The first-best and second-best tax rules were derived and interpreted. The first-best, here, refers to the situation where both regions coordinate their policies optimally so as to maximize their joint welfare, whereas second-best taxes apply in those situations where policy coordination is lacking and regions aim to maximize only their own welfare.

The analyses showed that, whereas first-best taxes still take on the standard Pigouvian form, second-best tax expressions can become quite complex, even in the relatively simple setting chosen. The general forms clearly reflect the various subgoals that a region has to trade off in the maximization of the local welfare. Because local consumers and producers are price-takers and therefore maximize the consumers' and producers' surpluses through their market behavior, the remaining subgoals for a regulator to consider are: the local valuation of emissions by local producers, by transport, and by foreign producers; and the region's terms of trade. These goals are then traded-off in different manners in the various second-best tax rules discussed, depending of course on how the tax intervenes in the economic process. Generally, the tax rules are weighted averages of terms reflecting the four subgoals mentioned, where the weights are made up of terms related to the elasticities of the various relevant demand and supply functions.

The analyses reveal that general conclusions are elusive. For instance, if a region can apply only one tax, then a net exporter will generally prefer a production tax and a net importer a consumption tax, but counterexamples can be given where the opposite holds. Furthermore, uncoordinated second-best policies need not always lead to a welfare improvement compared to the nonintervention situation. Moreover, when comparing two relatively likely second-best tax configurations—namely one where both regions use both taxes, and one where the net exporter uses a production tax and the net importer a consumption tax—it appeared that from an overall system's perspective, no general welfare ranking can be given. It was shown, however, that from the perspective of a single region, the use of both taxes is strictly preferable when there are no administrative costs of regulation.

Notwithstanding the ambiguities, these findings are, of course, valuable as they demonstrate the complexity of the issues at hand. Moreover, the various tax rules presented are generally valid, and enhance our insight into the broad problems studied. These tax rules, as well as the general modeling framework, will prove to be useful in further research into this important area. We intend to address two important extensions of the present analyses in the near future, including the construction of a numerical simulation model in which the full equilibrium effects—as opposed to marginal tax rules—of first-best and various second-best regulatory schemes can be investigated, and the consideration of endogenous environmental technologies in the present framework.

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**References**

- Barret S, 1994, "Strategic environmental policy and international trade" *Journal of Public Economics* **54** 325–338
- Carraro C (Ed.), 1997 *International Environmental Negotiations: Strategic Policy Issues* (Edward Elgar, Cheltenham, Glos)
- Conrad K, 1993, "Taxes and subsidies for pollution intensive industries as trade policy" *Journal of Environmental Economics and Management* **25** 121–135
- Copeland B, 1994, "International trade and the environment: policy reform in a polluted small open economy" *Journal of Environmental Economics and Management* **26** 44–65
- Frederiksson P, 1997, "The political economy of pollution taxes in a small open economy" *Journal of Environmental Economics and Management* **33** 44–65
- Jaffe A B, Peterson S R, Portney P R, 1995, "Environmental regulation and the competitiveness of US manufacturing: what does the evidence tell us?" *Journal of Economic Literature* **33** 132–163
- Kennedy P, 1994, "Equilibrium pollution in open economies with imperfect competition" *Journal of Environmental Economics and Management* **27** 49–63
- Krutilla K, 1989, "Tariff burdens and optimal tariffs under alternative transport cost and market structures" *Economics Letters* **31** 381–386
- Krutilla K, 1991, "Environmental regulation in an open economy" *Journal of Environmental Economics and Management* **20** 127–142
- Krutilla K, 1999, "Partial equilibrium models of trade and the environment", in *Handbook of Environmental and Resource Economics* Ed. J C J M van den Bergh (Edward Elgar, Cheltenham, Glos) pp 404–415
- Labys W C, Yang C W, 1997, "Spatial price equilibrium as a foundation to unified spatial commodity modeling" *Papers in Regional Science* **76** 199–228
- Markusen J, 1975a, "Cooperative control of international pollution and common property resources" *Quarterly Journal of Economics* **89** 618–632
- Markusen J, 1975b, "International externalities and optimal tax structures" *Journal of International Economics* **5** 15–29
- Markusen J R, Morey E R, Olewiler N D, 1993, "Environmental policy when market structure and plant locations are endogenous" *Journal of Environmental Economics and Management* **24** 69–86
- Rauscher M, 1994, "On ecological dumping" *Oxford Economic Papers* **46** 822–840
- Samuelson P A, 1952, "Spatial price equilibrium and linear programming" *American Economic Review* **42** 283–303
- Steininger F, 1999, "General models of environmental policy and foreign trade", in *Handbook of Environmental and Resource Economics* Ed. J C J M van den Bergh (Edward Elgar, Cheltenham, Glos) pp 416–432
- Takayama T, Judge G G, 1971 *Spatial and Temporal Price and Allocation Models* Volume 73 in the series *Contributions to Economic Analysis* (North-Holland, Amsterdam)
- Takayama T, Labys W C, 1986, "Spatial equilibrium analysis", in *Handbook of Regional and Urban Economics* Ed. P Nijkamp (Elsevier, Amsterdam) pp 171–199
- Ulph A, 1996a, "Environmental policy instruments and imperfectly competitive international trade" *Environmental and Resource Economics* **7** 333–355
- Ulph A, 1996b, "Environmental policy and international trade when governments and producers act strategically" *Journal of Environmental Economics and Management* **30** 265–281
- Ulph A, 1997, "International trade and the environment: a survey of recent economic analysis", in *The International Yearbook of Environmental and Resource Economics 1997/1998* Eds H Folmer, T Tietenberg (Edward Elgar, Cheltenham, Glos) pp 205–242
- van Beers C, van den Bergh J C J M, 1996, "An overview of methodological approaches in the analysis of trade and environment" *Journal of World Trade* **30** 143–167
- Verhoef E T, Nijkamp P, 1997, "The adoption of energy efficiency enhancing technologies; market performance and policy strategies in case of heterogeneous firm", DP TI 97-077/3, Tinbergen Institute, Amsterdam–Rotterdam, available at [www.tinbinst.nl/papers/TI97077.pdf](http://www.tinbinst.nl/papers/TI97077.pdf)
- Verhoef E T, Nijkamp P, 1999, "The regulation of input related externalities with heterogeneous firms" *Energy Economics* **21** 111–134
- Verhoef E T, van den Bergh J C J M, 1996, "A spatial price equilibrium model for environmental policy analysis of mobile and immobile sources of pollution", in *Recent Advances in Spatial Equilibrium Modelling: Methodology and Applications* Eds J C J M van den Bergh, P Nijkamp, P Rietveld (Springer, Heidelberg) pp 201–220
- Verhoef E T, van den Bergh J C J M, Button K J, 1997, "Transport, spatial economy, and the global environment" *Environment and Planning A* **29** 1195–1213

**Table A1.** Second-best tax rules for net exporters (region E) and net importers (region I).

Delivery Net exporter

**Optimum**

$$Z_{EE} \quad \frac{\partial E_E}{\partial Q_E}$$

$$Z_{II} \quad \frac{\partial E_E}{\partial Q_I}$$

$$Z_{EI} \quad \frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T}$$

**Noncoordination**

*Both taxes*

$$Z_{EE} \quad \frac{\partial E_E}{\partial Q_E}$$

$$Z_{II} \quad 0$$

$$Z_{EI} \quad \frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} - \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} \right) \right. + T \left/ \left( \frac{1}{-D'_I} + \frac{1}{S'_I} \right) \right.$$

*Production taxes*

$$Z_{EE} \quad \frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_E}{-D'_I} + \frac{-D'_E}{S'_I} \right)^{-1} \right] \right. - \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} + \frac{S'_I}{-D'_E} \right) \right. \\ + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{-D'_I} + \frac{1}{S'_I} \right) \right.$$

$$Z_{II} \quad 0$$

$$Z_{EI} \quad \frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_E}{-D'_I} + \frac{-D'_E}{S'_I} \right)^{-1} \right] \right. - \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} + \frac{S'_I}{-D'_E} \right) \right. \\ + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{-D'_I} + \frac{1}{S'_I} \right) \right.$$

*Consumption taxes*

$$Z_{EE} \quad \frac{\partial E_E}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_I} + \frac{S'_E}{S'_I} \right) \right. - \frac{\partial E_E}{\partial T} \left/ \left[ 1 + \left( \frac{S'_E}{-D'_I} + \frac{S'_E}{S'_I} \right)^{-1} \right] \right. + \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} + \frac{S'_I}{S'_E} \right) \right. \\ - T \left/ \left( \frac{1}{-D'_I} + \frac{1}{S'_E} + \frac{1}{S'_I} \right) \right.$$

$$Z_{II} \quad 0$$

$$Z_{EI} \quad 0$$

**Table A1** (continued).

Net importer

$$\frac{\partial E_1}{\partial Q_E}$$

$$\frac{\partial E_1}{\partial Q_1}$$

$$\frac{\partial E_1}{\partial Q_E} + \frac{\partial E_1}{\partial T}$$

0

$$\frac{\partial E_1}{\partial Q_1}$$

$$\frac{\partial E_1}{\partial T} + \frac{\partial E_1}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} \right) \right. + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{S'_E} \right) \right.$$

0

$$\frac{\partial E_1}{\partial Q_1} - \frac{\partial E_1}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_1}{-D'_E} + \frac{-D'_1}{S'_E} \right)^{-1} \right] \right. - \frac{\partial E_1}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{-D'_1} \right) \right. - T \left/ \left( \frac{1}{-D'_1} + \frac{1}{-D'_E} + \frac{1}{S'_E} \right) \right.$$

0

0

$$\frac{\partial E_1}{\partial Q_1} \left/ \left( 1 + \frac{S'_1}{-D'_E} + \frac{S'_1}{S'_E} \right) \right. + \frac{\partial E_1}{\partial T} \left/ \left[ 1 + \left( \frac{S'_1}{-D'_E} + \frac{S'_1}{S'_E} \right)^{-1} \right] \right. + \frac{\partial E_1}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{S'_1} \right) \right. \\ + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{S'_1} + \frac{1}{S'_E} \right) \right.$$

$$\frac{\partial E_1}{\partial Q_1} \left/ \left( 1 + \frac{S'_1}{-D'_E} + \frac{S'_1}{S'_E} \right) \right. + \frac{\partial E_1}{\partial T} \left/ \left[ 1 + \left( \frac{S'_1}{-D'_E} + \frac{S'_1}{S'_E} \right)^{-1} \right] \right. + \frac{\partial E_1}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{S'_1} \right) \right. \\ + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{S'_1} + \frac{1}{S'_E} \right) \right.$$

**Table A2.** Marginal tax rates in three regimes.

Delivery Tax rates

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<b>Optimum</b>	
$Z_{EE}$	$\frac{\partial E_E}{\partial Q_E} + \frac{\partial E_I}{\partial Q_E}$
$Z_{II}$	$\frac{\partial E_E}{\partial Q_I} + \frac{\partial E_I}{\partial Q_I}$
$Z_{EI}$	$\frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} + \frac{\partial E_I}{\partial Q_E} + \frac{\partial E_I}{\partial T}$
<b>Both regions use both taxes (BB)</b>	
$Z_{EE}$	$\frac{\partial E_E}{\partial Q_E}$
$Z_{II}$	$\frac{\partial E_I}{\partial Q_I}$
$Z_{EI}$	$\frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} - \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} \right) \right. + T \left/ \left( \frac{1}{-D'_I} + \frac{1}{S'_I} \right) \right. + \frac{\partial E_I}{\partial T} + \frac{\partial E_I}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} \right) \right. + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{S'_E} \right) \right.$
<b>Net exporter uses production taxes and net importer uses consumption taxes (PC)</b>	
$Z_{EE}$	$\frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_E}{-D'_I} + \frac{-D'_E}{S'_I} \right)^{-1} \right] \right. - \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} + \frac{S'_I}{-D'_E} \right) \right. + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{-D'_I} + \frac{1}{S'_I} \right) \right.$
$Z_{II}$	$\frac{\partial E_I}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E} \right) \right. + \frac{\partial E_I}{\partial T} \left/ \left[ 1 + \left( \frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E} \right)^{-1} \right] \right. + \frac{\partial E_I}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{S'_I} \right) \right. + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{S'_I} + \frac{1}{S'_E} \right) \right.$
$Z_{EI}$	$\frac{\partial E_E}{\partial Q_E} + \frac{\partial E_E}{\partial T} \left/ \left[ 1 + \left( \frac{-D'_E}{-D'_I} + \frac{-D'_E}{S'_I} \right)^{-1} \right] \right. - \frac{\partial E_E}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_I} + \frac{S'_I}{-D'_E} \right) \right. + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{-D'_I} + \frac{1}{S'_I} \right) \right. + \frac{\partial E_I}{\partial Q_I} \left/ \left( 1 + \frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E} \right) \right. + \frac{\partial E_I}{\partial T} \left/ \left[ 1 + \left( \frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E} \right)^{-1} \right] \right. + \frac{\partial E_I}{\partial Q_E} \left/ \left( 1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{S'_I} \right) \right. + T \left/ \left( \frac{1}{-D'_E} + \frac{1}{S'_I} + \frac{1}{S'_E} \right) \right.$

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**Table A3.** Differences in tax rules.

Delivery Optimum minus second-best tax rules

<b>Both regions use both taxes</b>	
$Z_{EE}$	$\frac{\partial E_I}{\partial Q_E}$
$Z_{II}$	$\frac{\partial E_E}{\partial Q_I}$
$Z_{EI}$	$\frac{\partial E_E}{\partial Q_I} / \left(1 + \frac{S'_I}{-D'_I}\right) + \frac{\partial E_I}{\partial Q_E} - \frac{\partial E_I}{\partial Q_E} / \left(1 + \frac{S'_E}{-D'_E}\right) - T / \left(\frac{1}{-D'_E} + \frac{1}{S'_E}\right) - T / \left(\frac{1}{-D'_I} + \frac{1}{S'_I}\right)$
<b>Net exporter uses production taxes and net importer uses consumption taxes</b>	
$Z_{EE}$	$\frac{\partial E_E}{\partial Q_I} / \left(1 + \frac{S'_I}{-D'_I} + \frac{S'_I}{-D'_E}\right) - \frac{\partial E_E}{\partial T} / \left[1 + \left(\frac{-D'_E}{-D'_I} + \frac{-D'_E}{S'_I}\right)^{-1}\right] + \frac{\partial E_I}{\partial Q_E} - T / \left(\frac{1}{-D'_E} + \frac{1}{-D'_I} + \frac{1}{S'_I}\right)$
$Z_{II}$	$\frac{\partial E_E}{\partial Q_I} + \frac{\partial E_I}{\partial Q_I} - \frac{\partial E_I}{\partial Q_I} / \left(1 + \frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E}\right) - \frac{\partial E_I}{\partial Q_E} / \left(1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{S'_I}\right) - \frac{\partial E_I}{\partial T} / \left[1 + \left(\frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E}\right)^{-1}\right] - T / \left(\frac{1}{-D'_E} + \frac{1}{S'_I} + \frac{1}{S'_E}\right)$
$Z_{EI}$	$\frac{\partial E_E}{\partial Q_I} / \left(1 + \frac{S'_I}{-D'_E} + \frac{S'_I}{-D'_I}\right) + \frac{\partial E_E}{\partial T} - \frac{\partial E_E}{\partial T} / \left[1 + \left(\frac{-D'_E}{-D'_I} + \frac{-D'_E}{S'_I}\right)^{-1}\right] - \frac{\partial E_I}{\partial Q_I} / \left(1 + \frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E}\right) + \frac{\partial E_I}{\partial Q_E} - \frac{\partial E_I}{\partial Q_E} / \left(1 + \frac{S'_E}{-D'_E} + \frac{S'_E}{S'_I}\right) + \frac{\partial E_I}{\partial T} - \frac{\partial E_I}{\partial T} / \left[1 + \left(\frac{S'_I}{-D'_E} + \frac{S'_I}{S'_E}\right)^{-1}\right] - T / \left(\frac{1}{-D'_E} + \frac{1}{S'_E} + \frac{1}{S'_I}\right) - T / \left(\frac{1}{-D'_E} + \frac{1}{D'_I} + \frac{1}{S'_I}\right)$